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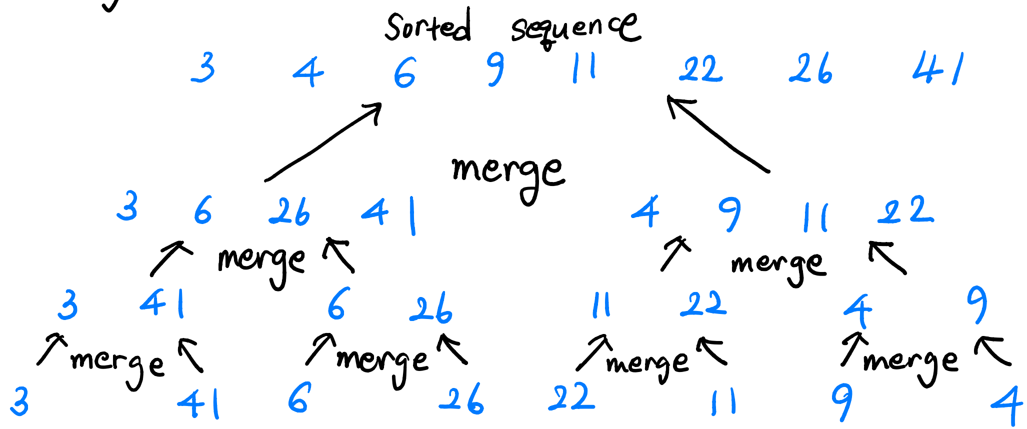
· Problem Solving manually

1)



2) a) SELECTION SORT (A, n)

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1. for i ← 1 to n
       do max ← A[i]
          max_index ← i
          for j ← i+1 to n
              if A[j] > max
                  do max ← A[j]
                     max_index = j
       do temp ← A[i]
          A[i] ← A[max_index]
          A[max_index] ← A[i]
```

b) When finding a right index for $(n-2)$th element, we should compare elements in $(n-1)$th, (n)th indices. So, nth element will be at an appropriate index after sorting only $(n-1)$ elements.

c) No matter how elements were placed in original array, Selection sort always performs the same number of comparisons. So, the best-case and worst-case running times of selection sort are same.

Selecting the largest element requires scanning n elements ($n-1$ comparisons)

Selecting the second largest element requires scanning $(n-1)$ elements ($n-2$ comparisons) and so on.

Therefore, the total number of comparisons is $\sum_{i=0}^{n-1} i = 1+2+\cdots+(n-2)+(n-1) = \frac{(n-1)n}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$

$\frac{1}{2}n^2 - \frac{1}{2}n \le \frac{1}{2}n^2$ for all $n \ge 0$ (since, $\frac{1}{2}n \ge 0$ for all $n \ge 0$). Therefore $\frac{1}{2}n^2 - \frac{1}{2}n = O(n^2)$

Accordingly the best-case and worst-case running times of selection sort are both $\Theta(n^2)$

d)

(a)

13	16	12	21	7	8	25	32

(b)

32	16	12	21	7	8	25	13

(c)

32	25	12	21	7	8	16	13

(d)

32	25	21	12	7	8	16	13

(e)

32	25	21	16	7	8	12	13

(f)

32	25	21	16	13	8	12	7

(g)

32	25	21	16	13	12	8	7

3> a) $2n^2+2\lg n \le 2n^2+2n \le 2n^2+2n^2 \le 4n^2$ for all $n \ge 1$.

$2n^2+2\lg n = O(n^2)$

b) $3n^3+5n+5 \le 3n^3+5n^3+5n^3 \le 13n^3$ for all $n \ge 1$

$3n^3+5n+5 = O(n^3)$

4> Let's say $3n^5-n^3+2n^2-2n+2$, $f(n)$

Since $3n^5-n^3+2n^2-2n+2 \le 3n^5-n^5+2n^5-2n^5+2n^5 = 4n^5$ for all $n \ge 1$, $f(n) \le 4(n^5)$ for all $n \ge 1$

Accordingly $f(n) = O(n^5)$

$f(n) = 3n^5-n^3+2n^2-2n+2 = 3n^5-(n^3-2n^2+2n-2) = 3n^5-(n(n-1)^2+n-2)$. Since $n(n-1)^2+n-2 \ge 0$ for all $n \ge 2$,

$f(n) \ge n^5$ for all $n \ge 2$. Accordingly, $3n^5-n^3+2n^2-2n+2 = \Omega(n^5)$

Since, $f(n) = O(n^5) = \Omega(n^5)$, $3n^5-n^3+2n^2-n+2 = \Theta(n^5)$

5> When $n=1$, $\sum_{i=0}^{1} ar^i = a+ar = a(r+1)$

$a(r^2-1)/r-1 = \frac{a(r+1)(r-1)}{r-1} = a(r+1)$ It shows that the equation is true for $n=1$.

Let's assume that $\sum_{i=0}^{n} ar^i = a(r^{n+1}-1)/(r-1)$ for all $n \ge 0$.

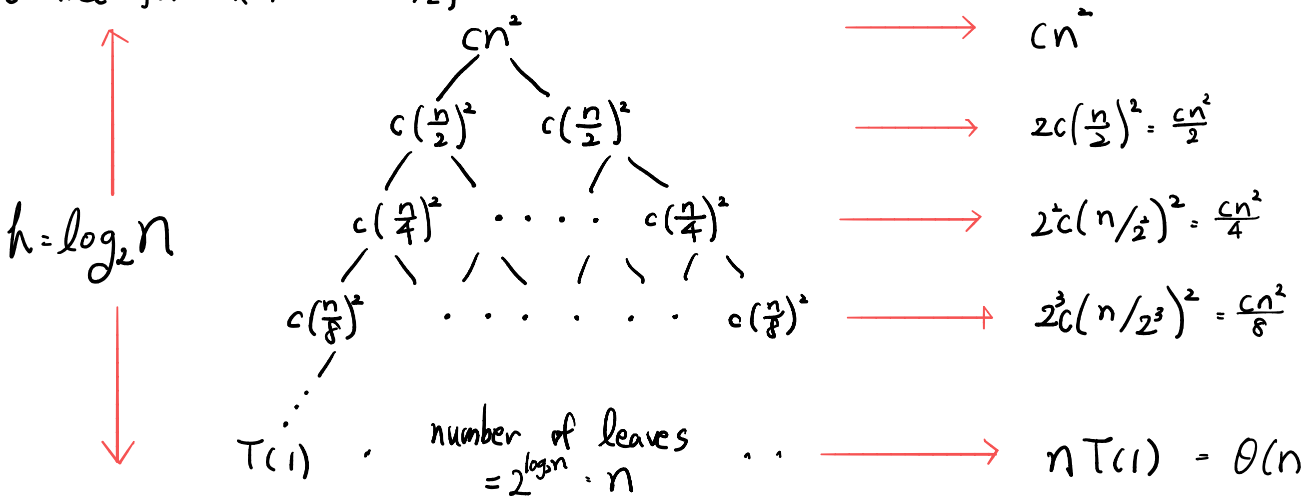
The sum for $n+1$ is obtained by adding ar^{n+1}.

$\sum_{i=0}^{n+1} ar^i = \sum_{i=0}^{n} ar^i + ar^{n+1}$ Since we are assuming that $\sum_{i=0}^{n} ar^i = a(r^{n+1}-1)/(r-1)$,

We obtain $\sum_{i=0}^{n} ar^i + ar^{n+1} = a(r^{n+1}-1)/(r-1) + ar^{n+1} = a\left(\frac{r^{n+1}-1+r^{n+2}-r^{n+1}}{r-1}\right) = a(r^{n+2}-1)/(r-1)$

It shows that $a(r^{n+1}-1)/(r-1) + ar^{n+1} = a(r^{n+2}-1)/(r-1)$.
Therefore, $\sum_{i=0}^{n+1} ar^i = a(r^{n+2}-1)/(r-1)$. And the proof is complete.

6) Recursive Tree for $T(n) = 2T(n/2) + cn^2$



$$T(n) = cn^2 + \frac{cn^2}{2} + \frac{cn^2}{4} + \frac{cn^2}{8} + \cdots + cn^2\left(\frac{1}{2}\right)^{\log_2 n - 1} + \Theta(n)$$

$$= \sum_{i=0}^{\log_2 n - 1} \left(\frac{1}{2}\right)^i cn^2 + \Theta(n) < \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i cn^2 + \Theta(n) = cn^2 \frac{1}{1-\frac{1}{2}} + \Theta(n) = 2cn^2 + \Theta(n) = O(n^2)$$

Verify by substitution method

Let's assume $T(n) = O(n^2)$ as we found above through recursion tree.

Since $T(n) = 2T(\frac{n}{2}) + cn^2$,

$T(n) \le 2\left(s\left(\frac{n}{2}\right)^2\right) + cn^2 = \frac{s}{2}n^2 + cn^2 = (\frac{s}{2}+c)n^2$ (for $s \ge 1$, $n \ge 2$)

Therefore $T(n) = O(n^2)$

7) a) $T(n) = 9T(n/3) + n$

$n = O(n^{\log_3 9 - 1}) = O(n)$

n grows polynomially slower than $n^{\log_3 9}$ by n factor.

Accordingly $T(n) = \Theta(n^{\log_3 9}) = \Theta(n^2)$

b) $T(n) = 9T(\frac{n}{3}) + n^2$

$n^2 = \Theta(n^{\log_3 9}) = \Theta(n^2)$

n^2 and $n^{\log_3 9}$ grow at similar rates

Accordingly, $T(n) = \Theta(n^{\log_3 9} \lg n) = \Theta(n^2 \lg n)$

c) $T(n) = 9T(\frac{n}{3}) + n^3$

$n^3 = \Omega(n^{\log_3 9 + 1}) = \Omega(n^3)$

n^3 grows polynomially faster than $n^{\log_3 9}$ by n factor,

and $9(\frac{n}{3})^3 = \frac{n^3}{3} \le cn^3$ for some constant $c < 1$ which means

that n^3 satisfies the regularity condition.

$T(n) = \Theta(n^3)$