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Chapter 4 problems> 4.1(a),(d) 4.4(a) 4.6(a)

4.1> a) Z = The company should be unlocked
A : Mr. Jones is in the office
B : Mr. Evans is in the office
C : Company is open for business
D : Security guard is present
$Z = (A+B) \cdot C \cdot D$

d) Z = The elevator door should open
A : Elevator has stopped
B : It is level with the floor
C : Timer has not expired
D : Button is pressed
$Z = A \cdot B \cdot C + A \cdot B \cdot D$

4.4> a) There are $4(2^2)$ switching functions for two variables (x, y)
That is because there are two possible states (0 and 1)
for each variable

4.6> a) There are two Xs in F.

case 1> both are Os
$F = A'B'C' + ABC' + ABC = A'B'C' + AB$

case 2> first x : 1 second x : 0
$F = A'B'C' + A'B'C + ABC' + ABC = A'B' + AB$

case 3> first x : 0 second y : 1
$F = A'B'C' + AB'C + ABC' + ABC = A'B'C' + AB'C + AB$

case 4> both are 1s
$F = A'B'C' + A'B'C + AB'C + ABC' + ABC = A'B' + AB + AB'C$

Simplest Expression : $AB' + AB$

4.8)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

a) $F(A,B,C,D) = m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_8 + m_9 + m_{12}$

$= \sum m(0,1,2,3,4,5,6,8,9,12)$

b) $F(A,B,C,D) = M_7 \cdot M_{10} \cdot M_{11} \cdot M_{13} \cdot M_{14} \cdot M_{15}$

$= \Pi(7,10,11,13,14,15)$

4.9) $F(a,b,c) = ab c' + b'$

a	b	c	F	F'
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

a) $F(a,b,c) = \sum m(0,1,4,5,6)$

b) $F(a,b,c) = \Pi M(2,3,7)$

c) $F'(a,b,c) = \sum m(2,3,7)$

d) $F'(a,b,c) = \Pi M(0,1,4,5,6)$

4.10) $F(a,b,c,d) = (a+b+d)(a'+c)(a'+b'+c')(a+b+c'+d')$

$= (a+b+cc'+d)(a'+bb'+c+dd')(a'+b'+c'+dd')(a+b+c'+d')$

$= (a+b+c+d)(a+b+c'+d)(a'+b+c+d)(a'+b+c+d')(a'+b'+c+d)(a'+b'+c+d')$
$(a'+b'+c'+d)(a'+b'+c'+d')(a+b+c'+d')$

$= \Pi M(0,2,3,8,9,12,13,14,15)$

a) $F(a,b,c,d) = \Sigma m(1,4,5,6,7,10,11)$

b) $F(a,b,c,d) = \Pi M(0,2,3,8,9,12,13,14,15)$

c) $F'(a,b,c,d) = \Sigma m(0,2,3,8,9,12,13,14,15)$

d) $F'(a,b,c,d) = \Pi M(1,4,5,6,7,10,11)$

4.11) a) A = minuend B = Subtrahend C = Borrow

A	B	C	D	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$D = A'B'C + A'BC' + AB'C' + ABC$

$= A'(B'C + BC') + A(B'C' + BC)$

$= A'(B \oplus C) + A(B \oplus C)'$

$= A \oplus B \oplus C$

$B_{out} = A'B'C + A'BC' + A'BC + ABC$

$= A'B'C + A'BC' + BC$

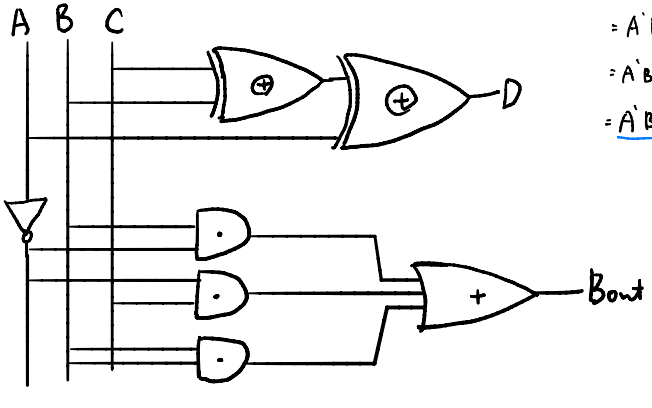
$= A'B'C + A'BC' + BC(1+A')$

$= A'B'C + A'BC' + BC + A'BC$

$= A'B'C + A'B + BC$

$= A'B'C + A'B(1+C) + BC$

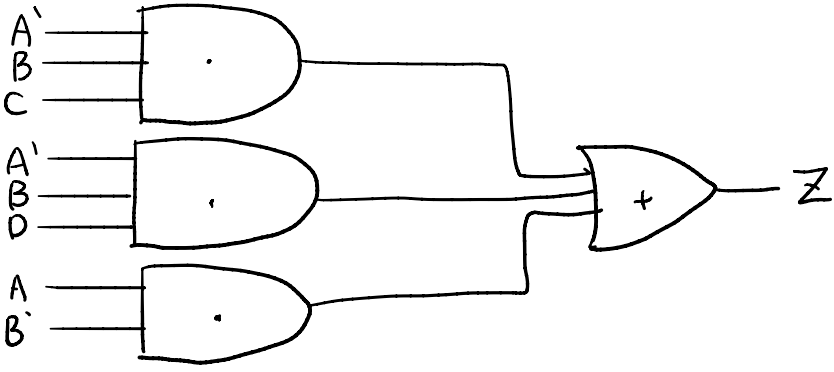
$= A'B'C + A'B + A'BC + BC = A'C + A'B + BC$



4.14)

A	B	C	D	Z
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1

$Z = A'BC'D + A'BCD' + A'BCD + AB'C'D'$
$+ AB'C'D + AB'CD' + AB'CD$
$= A'B(C'D + CD' + CD) + AB'(C'D' + C'D + CD' + CD)$
$= A'B(C'D + C) + AB'(C' + C)$
$= A'B(C'D + C) + AB' = A'B(C'D + C(1+D)) + AB'$
$= A'B(C'D + C + CD) + AB' = A'B(C + D) + AB' = A'BC + A'BD + AB'$



4.17)

A	B	C	D	W	X	Y	Z
0	0	0	0	1	0	0	1
0	0	0	1	1	0	0	0
0	0	1	0	0	1	1	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	0	0
0	1	1	0	0	0	1	1
0	1	1	1	0	0	1	0
1	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

a) i) $W = \sum m(0,1) + \sum d(10,11,12,13,14,15)$
ii) $X = \sum m(2,3,4,5) + \sum d(10,11,12,13,14,15)$
iii) $Y = \sum m(2,3,6,7) + \sum d(10,11,12,13,14,15)$
iv) $Z = \sum m(0,2,4,6,8) + \sum d(10,11,12,13,14,15)$

b) i) $W = \prod M(2,3,4,5,6,7,8,9) \cdot \prod D(10,11,12,13,14,15)$
ii) $X = \prod M(0,1,6,7,8,9) \cdot \prod D(10,11,12,13,14,15)$
iii) $Y = \prod M(0,1,4,5,8,9) \cdot \prod D(10,11,12,13,14,15)$
iv) $Z = \prod M(1,3,5,7,9) \cdot \prod D(10,11,12,13,14,15)$

4.20)

A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$Z = A'BC + AB'C + ABC' + ABC$

$= A'BC + AB'C + AB(C + C') = A'BC + AB'C + AB$

$= A'BC + AB'C + ABC(1 + C) = A'BC + AB'C + ABC + AB = BC(A' + A) + AB'C + AB$

$= BC + AB + AB'C = BC(1 + A) + AB + AB'C = AB + BC + ABC + AB'C$

$= AB + BC + AC(B + B') = AB + BC + AC$

4.21) a) $Z = \Sigma m(1, 2, 3, \cdots, 30) = \Pi M(0, 31) = (A' + B' + C' + D' + E')(A + B + C + D + E)$

b)

A	B	C	D	E	Y
0	0	1	0	1	1

$Y = A'B'CD'E$

4.23) $F_1 = \Pi M(0, 4, 5, 6)$ $F_2 = \Pi M(0, 4, 7)$

A	B	C	F_1	F_2	$F_1 \cdot F_2$
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	0	1	0
1	1	0	0	1	0
1	1	1	1	0	0

$F_1 \cdot F_2 = \Pi M(0, 4, 5, 6, 7)$

Prove by using general forms

$F_1 = \prod_{i=0}^{n} (a_i + M_i)$ $F_2 = \prod_{j=0}^{n} (b_j + M_j)$

$F_1 \cdot F_2 = \prod_{i=0}^{n} (a_i + M_i) \prod_{j=0}^{n} (b_j + M_j) = \prod_{i=j=0}^{n} (a_i + b_i + M_i + M_j)$ $(i = j \Rightarrow M_i + M_j = M_i)$

$= \sum_{i=0}^{n} (a_i + b_i + M_i)$ which means that $F_1 \cdot F_2$ contains all the terms in F_1 and F_2 with common terms written only once.

Accordingly $F_1 \cdot F_2 = \Pi M(0, 4, 5, 6, 7)$

4.25) a)

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$F(A,B,C,D) = \Sigma m(5,6,7,10,11,13,14,15)$
$= \Pi M(0,1,2,3,4,8,9,12)$

b)

A	B	C	D	G
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$G(A,B,C,D) = \Sigma m(0,2,4,6)$
$= \Pi M(1,3,5,7,8,9,10,11,12,13,14,15)$

c)

A	B	C	D	H
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$H(A,B,C,D) = \Sigma m(7,11,13,14,15)$
$= \Pi M(0,1,2,3,4,5,6,8,9,10,12)$

d)

A	B	C	D	J
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$J(A,B,C,D) = \Sigma m(4,8,12,13,14)$

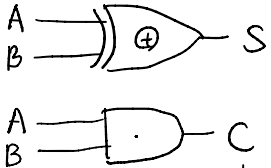
$= \Pi M(0,1,2,3,5,6,7,9,10,11,15)$

4.40)

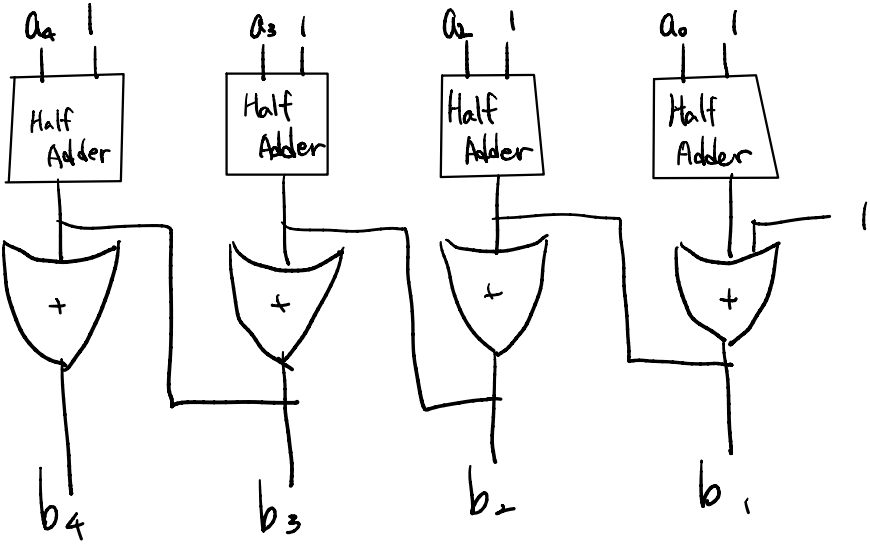
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$S = \Sigma m(1,2) = A'B + AB' = A \oplus B$

$C = \Sigma m(3) = AB$

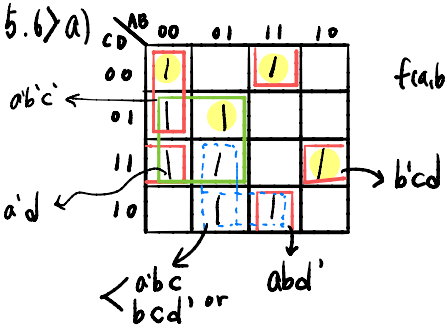


Since $a \oplus 1 = a'$, a circuit which finds 2's complement of a 4-bit binary number is as below.



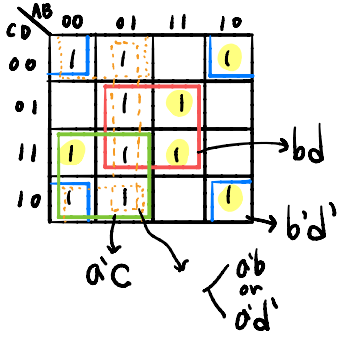
$b_4b_3b_2b_1$ is a 2's complement of $a_1a_2a_3a_4$

5.6) a)



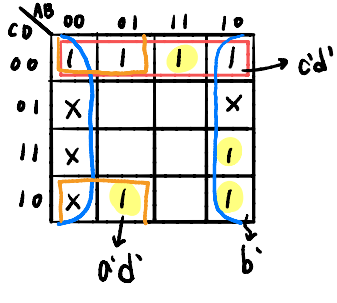
$f(a,b,c,d) = a'd + a'b'c' + abd' + b'cd + \begin{cases} a'bc \\ \text{or} \\ bcd' \end{cases}$

b)



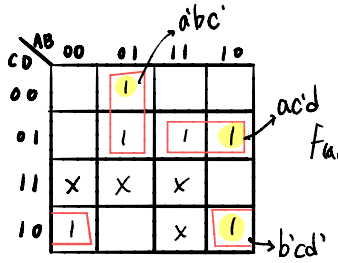
$f(a,b,c,d) = a'c + b'd' + bd + \begin{cases} a'b \\ \text{or} \\ a'd' \end{cases}$

c)

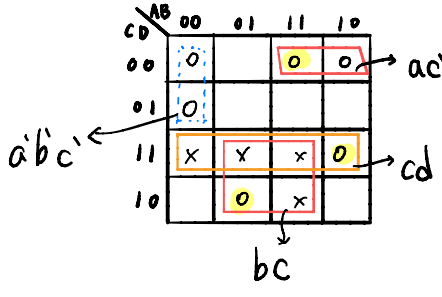


$f(a,b,c,d) = a'd' + c'd' + b'$

5.8) a)

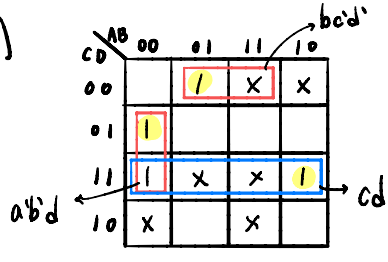


$f(a,b,c,d) = ac'd + b'cd' + a'bc'$

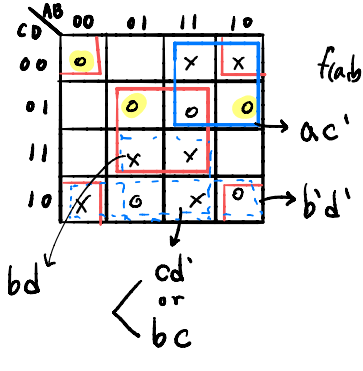


$f(a,b,c,d) = (b'+c')(c'+d')(a'+c+d)(a+b+c)$

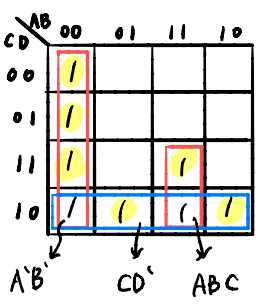
b)



$f(a,b,c,d) = bc'd' + a'b'd + cd$

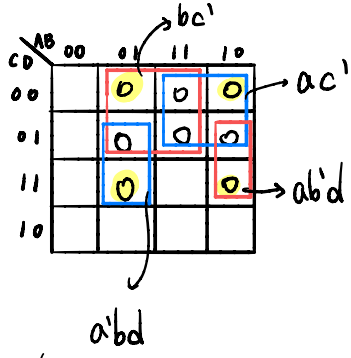


$f(a,b,c,d) = (a'+c)(b+d)(b'+d')$ $\begin{cases} (c'+d) \\ \text{or} \\ (b'+c') \end{cases}$

5.17) a) 

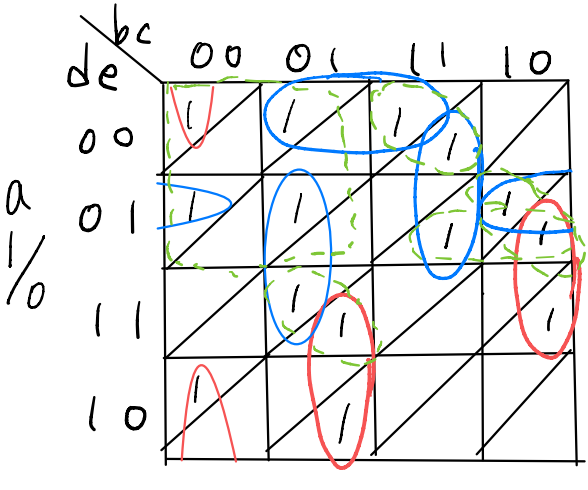
$F(A,B,C,D) = A'B' + CD' + ABC + A'BCD' + ABCD'$

b) $F(A,B,C,D) = A'B' + CD' + ABC$

c) 

$F(A,B,C,D) = (A'+C)(B'+C)(A'+B+D')(A+B'+D')$

5.33) $f(a,b,c,d,e) = \sum m(6,7,9,11,12,13,16,17,18,20,21,23,25,28)$



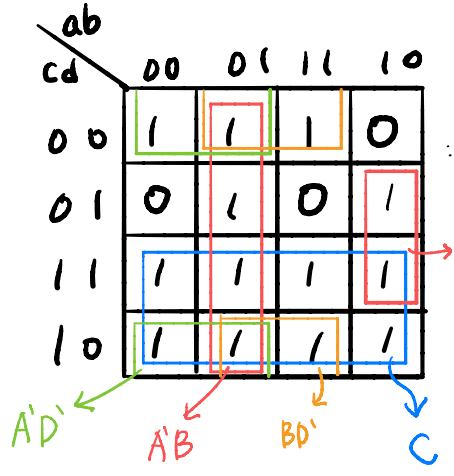
a) essential prime implicants:
: $ab'c'e'$, $a'bc'e$, $a'b'cd$

b) Minimum sum of products
: $ab'c'e' + a'bc'e + a'b'cd + ac'd'e + ab'ce + a'bcd' + acd'e'$

c) All of the prime implicants
: $ab'c'e'$, $a'bc'e$, $a'b'cd$, $ac'd'e$, $ab'ce$, $a'bcd'$, $acd'e'$, $ab'd'$, $bcd'e'$, $a'bd'e$, $bc'd'e$, $b'cde$

5.34) a, b) d: odd-parity bit, f: set if input digit is a prime number

a	b	c	d	f
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	0
1	1	1	0	1



All prime implicants : A'D', A'B, BD', C, AB'D

6.2) a) $f(a,b,c,d) = \Sigma m(1, 5, 7, 9, 11, 12, 14, 15)$

Column I			Column II		
1	0001	✓	1,5	0-01	a'c'd
5	0101	✓	1,9	-001	b'c'd
9	1001	✓	5,7	01-1	a'bd
12	1100	✓	9,11	10-1	ab'd
7	0111	✓	12,14	11-0	abd'
11	1011	✓	7,15	-111	bcd
14	1110	✓	11,15	1-11	acd
15	1111	✓	14,15	111-	abc

All the prime implicants: a'c'd, b'c'd, a'bd, ab'd, abd', bcd, acd, abc

b) $f(a,b,c,d) = \Sigma m(0,1,3,5,6,7,8,10,14,15)$

Column I			Column II			Column III		
0	0000	✓	0,1	000-	a'b'c'			
1	0001	✓	0,8	-000	b'c'd'			
8	1000	✓	1,3	00-1	✓	1,3,5,7	0--1	a'd
3	0011	✓	1,5	0-01	✓	1,5,3,7	0--1	a'd
5	0101	✓	8,10	10-0	ab'd'			
6	0110	✓	3,7	0-11	✓	6,14,7,15	-11-	bc
10	1010	✓	5,7	01-1	✓	6,7,14,15	-11-	bc
7	0111	✓	6,7	011-	✓			
14	1110	✓	6,14	-110	✓			
15	1111	✓	10,14	1-10	acd'			
			7,15	-111	✓			
			14,15	111-	✓			

All prime implicants: a'b'c', b'c'd', ab'd', acd', a'd, bc

6.5) $F(A,B,C,D) = \sum m(9,12,13,15) + \sum d(1,4,5,7,8,11,14)$

Column I		Column II		Column III	
1	0001 ✓	1,5	0-01 ✓	1,5,9,13	--01 C'D
4	0100 ✓	4,5	010- ✓	4,5,12,13	-10- Bc'
8	1000 ✓	1,9	-001 ✓	~~1,5,9,13~~	~~--01~~
5	0101 ✓	4,12	-100 ✓	~~4,12,5,13~~	~~-10-~~
9	1001 ✓	8,9	100- ✓	8,9,12,13	1-0- Ac'
12	1100 ✓	8,12	1-00 ✓	~~8,12,9,13~~	~~1-0-~~
7	0111 ✓	5,7	01-1 ✓	5,7,13,15	-1-1 BD
11	1011 ✓	5,13	-101 ✓	~~5,13,7,15~~	~~-1-1~~
13	1101 ✓	9,11	10-1 ✓	9,11,13,15	1--1 AD
14	1110 ✓	9,13	1-01 ✓	~~9,13,11,15~~	~~1--1~~
15	1111 ✓	12,13	110- ✓	12,13,14,15	11-- AB
		12,14	11-0 ✓	~~12,14,13,15~~	~~11--~~
		7,15	-111 ✓		
		11,15	1-11 ✓		
		13,15	11-1 ✓		
		14,15	111- ✓		

All prime implicants: C'D, Bc', Ac', BD, AD, AB

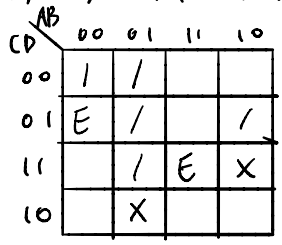
Using Petrick's method

	9	12	13	15	
(1,5,9,13) P_1	X		X		C'D
(4,5,12,13) P_2		X	X		Bc'
(8,9,12,13) P_3	X	X	X		Ac'
(5,7,13,15) P_4			X	X	BD
(9,11,13,15) P_5	X		X	X	AD
(12,13,14,15) P_6		X	X	X	AB

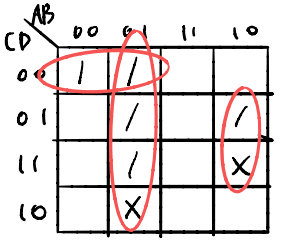
$F_1 = P_1 + P_6$
$= P_2 + P_5$
$= P_3 + P_4$
$= P_3 + P_5$
$= P_3 + P_6$
$= P_5 + P_4$

$F_1 = C'D + AB$, $Bc' + AD$, $Ac' + BD$, $Ac' + AD$, $Ac' + AB$, $AD + AB$

6.6) a) $F(A,B,C,D,E) = \Sigma m(0,4,5,7,9) + \Sigma d(6,11) + E(m_1 + m_{15})$

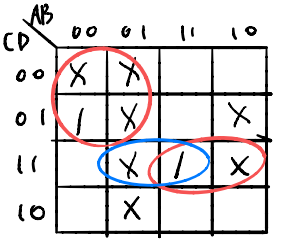


G



E = 0

$MS_0 = A'B + A'C'D' + AB'D$



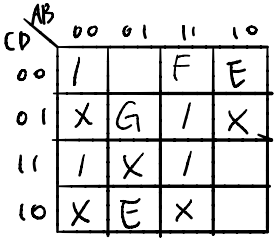
E = 1

$MS_1 = (A'C' + ACD)E$

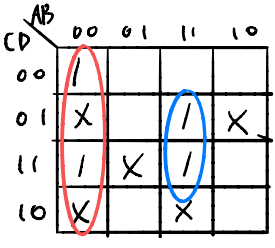
$= (A'C' + BCD)E$

$F = A'B + A'C'D' + AB'D + A'C'E + ACDE = A'B + A'C'D' + AB'D + A'C'E + BCDE$

b) $\Sigma(A,B,C,D,E,F,G) = \Sigma m(0,3,13,15) + \Sigma d(1,2,17,9,14) + E(m_6 + m_8) + Fm_{12} + Gm_5$

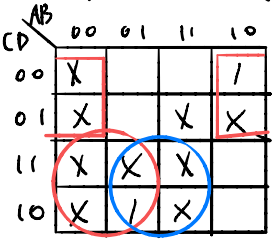


G



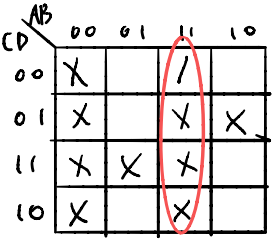
E = F = G = 0

$MS_0 = A'B' + ABD$



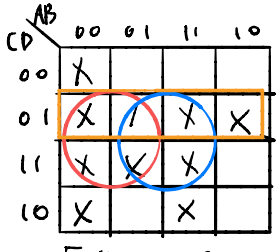
E = 1, F = G = 0

$MS_1 = \left(B'C' + \begin{bmatrix} A'C \\ \text{or} \\ BC \end{bmatrix}\right)E$



E = G = 0 F = 1

$MS_2 = (AB)F$



E = F = 0 G = 1

$MS_2 = \left(\begin{bmatrix} A'D \\ \text{or} \\ BD \\ \text{or} \\ C'D \end{bmatrix}\right)G$

$\Sigma = A'B' + ABD + B'C'E + A'CE + ABF + A'DG$

$= A'B' + ABD + B'C'E + A'CE + ABF + BDG$

$A'B' + ABD + B'C'E + A'CE + ABF + C'DG$

$A'B' + ABD + B'C'E + BCE + ABF + A'DG$

$A'B' + ABD + B'C'E + BCE + ABF + BDG$

$A'B' + ABD + B'C'E + BCE + ABF + C'DG$

6.9) b) $f(a,b,c,d) = \Sigma m(0,1,5,6,8,9,11,13) + \Sigma d(7,10,12)$

Column I

0	0000	✓
1	0001	✓
8	1000	✓
5	0101	✓
6	0110	✓
9	1001	✓
10	1010	✓
12	1100	✓
7	0111	✓
11	1011	✓
13	1101	✓

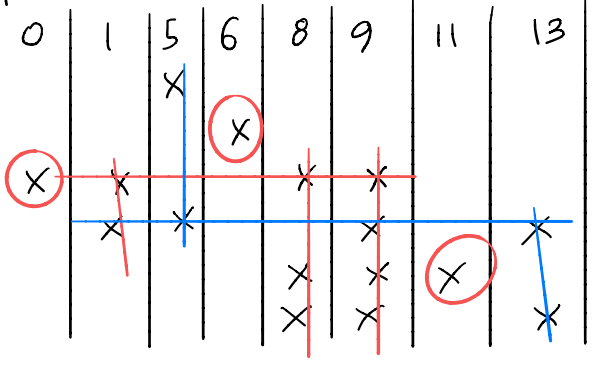
Column II

0,1	000-	✓
0,8	-000	✓
1,5	0-01	✓
1,9	-001	✓
8,9	100-	✓
8,10	10-0	✓
8,12	1-00	✓
5,7	01-1	a'bd
5,13	-101	✓
6,7	011-	a'bc
9,11	10-1	✓
9,13	1-01	✓
10,11	101-	✓
12,13	110-	✓

Column III

0,1,8,9	-00-	b'c'
~~0,1,8,9~~	~~-00-~~	
1,5,9,13	--01	c'd
~~1,9,5,13~~	~~--01~~	
8,10,9,11	10--	ab'
8,12,9,13	1--0	ac'

(5, 7)	a'bd
(6, 7)	a'bc
(0, 1, 8, 9)	b'c'
(1, 5, 9, 13)	c'd
(8, 10, 9, 11)	ab'
(8, 9, 12, 13)	ac'



essential prime implicants : a'bc, b'c', ab'

Minimum sum of products : a'bc + b'c' + ab' + c'd