Real Exchange Rate and Net Trade Dynamics: Financial and Trade Shocks*

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This Draft: December 30, 2022
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Abstract

This paper studies the drivers of the US real exchange rate (RER), with a particular focus on its comovement with net trade flows. We consider the entire spectrum of frequencies, as the low-frequency movements account for 83% of the RER's unconditional variance. We introduce a model with heterogeneous firms facing sunk costs of exporting, financial shocks, and trade shocks. The model can fully capture the comovement of the RER and net trade flows at all frequencies, without compromising other major moments at the business cycle frequency. While financial shocks are necessary to capture the RER movements at higher frequencies, trade shocks are essential for lower frequency variation.

JEL Classifications: E30, E44, F30, F41, F44

Keywords: Exchange Rates, Trade Balance, Trade Dynamics, International Business Cycles

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*We are greatly indebted to George Alessandria for his guidance and support throughout this project. We are also very grateful to Yan Bai, Mark Bils, Gaston Chaumont, Rafael Guntin, Oleg Itskhoki, Narayana Kocherlakota, Roman Merga, Joseph Steinberg and Walter Steingress for numerous comments and suggestions. Moreover, we would like to thank Thuy Lan Nguyen, Katheryn Russ, Mike Sposi and participants of Midwest Macroeconomics Meeting Fall 2022, Economics Graduate Students' Conference in Washington University in St. Louis and the seminars in University of Rochester. Any errors in the paper are entirely those of the authors.

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1 Introduction

The real exchange rate (RER) is a central variable in many, if not most, models of international economics.[1] Nonetheless, it has been challenging to account for the dynamics of the RER. This arises from the fact that the RER follows a near-random walk process and features large volatility relative to other macro fundamentals. Thus, it is hard to find a connection between the RER and other macroeconomic variables, such as output, consumption, and interest rates.[2] Such *disconnect* is rather surprising since international real business cycle models based on Backus, Kehoe and Kydland (1994) usually predict a robust connection between the RER and these variables.

However, there is a strand of the literature which has been successful in accounting for the disconnect of the RER. To reproduce the disconnect, the literature relies on shocks in incomplete financial markets (Devereux and Engel, 2002; Itskhoki and Mukhin, 2021b). These shocks are based on various microfoundations, including risk-premia induced by financial intermediaries that bear the risk of trading bonds in different currencies, errors in expectations, and liquidity needs. Effectively, they create a wedge between returns of domestic and international bonds.

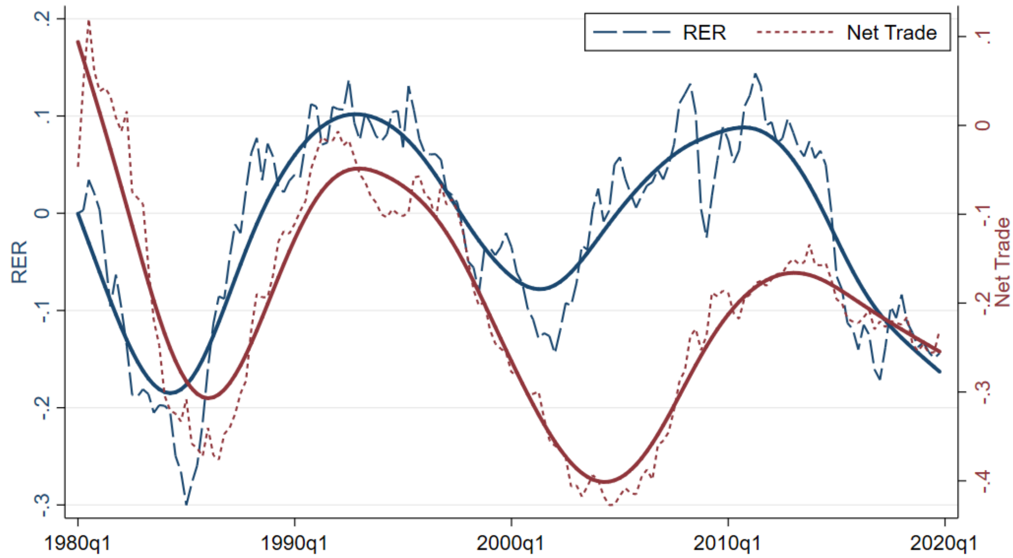
Despite the success of the models with financial shocks in generating the disconnect, the literature has an important caveat: it only considers variation at the business cycle frequency (cycles between 8 and 32 quarters). This approach provides only a partial understanding of the RER, since most of its variation is at frequencies lower than business cycles (cycles of more than 32 quarters). Figure 1 shows the path of the US RER in the dashed blue line, along with its trend component after applying the HP filter in the solid blue line. It is clear that the trend component drives a large part of the fluctuations in the RER. Confirming this observation, a spectrum analysis shows that 83 percent of the unconditional variance is assigned to low-frequency movements.[3] That is, most of the variation of the RER is at frequencies lower than business cycles. Thus, focusing on the business cycle movements provides only a partial understanding of the dynamics of the RER.

[1] The RER is defined as $q_t = e_t P_t^*/P_t$ where e_t the nominal exchange rate (the price of local currency per unit of foreign currency), P_t^* is the foreign price level, and P_t the local price level. An increase in q_t indicates a depreciation of the RER.

[2] More precisely, we are referring to three puzzles in the RER. For a detailed discussion of the puzzles, see Section 6.2.

[3] The spectrum analysis measures how much of the unconditional variance is attributed to the variation in cycles of different lengths. The business cycle frequency is defined as cycles that last between 8 to 32 quarters. Our findings are consistent with Rabanal and Rubio-Ramirez (2015), who find that low-frequency movements account for 77 percent of the US RER variance in their sample. They also show that the result that low-frequency movements account for the bulk of the volatility of the RER is robust across a sample of developed countries.

Figure 1: Real Exchange Rate and Export-Import Ratio



Notes: RER is the log of the quarterly real exchange rates of the United States. Normalized with 1980q1=0. Effective exchange rate indices, Real, Narrow (BIS). Net trade is the log of Exports to Imports ratio for the United States. Exports and Imports are from Quarterly National Accounts (OECD). Solid lines plot the trend component of each variable from the Hodrick–Prescott filter with a smoothing parameter of 1600.

Moreover, focusing only on business cycle frequencies shifts the attention away from the robust comovement between the RER and net trade flows at lower frequencies. In Figure 1, we plot the path of US net trade in the dashed red line, as well as its trend in the solid red line. We use the export-import ratio as a measure of net trade, as opposed to trade balance as a share of GDP, because the export-import ratio gives net trade controlling for the scale of trade.[4] This addresses the concern that the changes in trade balance as a share of GDP are primarily due to the changes in the scale of total trade, rather than asymmetries across countries (Alessandria and Choi, 2021; Alessandria, Bai and Woo, 2022).

Similarly to the RER, the path of net trade exhibits two big cycles. The trend of net trade closely follows that of the RER, with a lag of around 6 quarters. The delayed comovement between these two variables is captured by a long-run elasticity of net trade to relative prices that is larger (1.2) than the short-run elasticity (0.2).[5] This delayed effect, so called J-curve, has been the subject of

[4] The trade balance as a share of GDP can be written as $\frac{(X-M)}{Y} = \frac{(X-M)}{(X+M)} \times \frac{(X+M)}{Y}$, where $\frac{(X-M)}{(X+M)}$ is approximated by $0.5 \log X/M$ using the first-order Taylor approximation. Since we focus on trade balance controlling for the scale of total trade $\frac{(X+M)}{Y}$, we use $\log X/M$ to measure net trade flows.

[5] We estimate these elasticities in Section 5.

much research (Baldwin and Krugman, 1989; Rose and Yellen, 1989; Backus et al., 1994; Fitzgerald et al., 2019).

We point out that models in which the RER is mainly driven by financial shocks miss the comovement between the RER and net trade flows at all frequencies. At the high frequency, it generates a near-perfect contemporaneous correlation between the RER and net trade flows, contradicting the small correlation observed in the data. It also exhibits an excess volatility of net trade flows relative to the RER, a counterfactual result that is highlighted by Miyamoto et al. (2022). At the low frequency, it cannot reproduce the delayed comovement of the RER and net trade, and the long-run elasticity is similar to the short-run. Therefore, it is challenging to claim success of these models as the trade block of the general equilibrium framework is counterfactual.

In this paper, we provide a unified treatment of the dynamics of the RER at all frequencies. In order to correctly account for the comovement between the RER and net trade flows at both high and low frequencies, without compromising the model's ability to account for the RER disconnect, we need to consider two additional features to the financial shock model of Itskhoki and Mukhin (2021a): trade shocks and dynamic trade.[6] Our main result is that trade shocks play an important role in accounting for the low-frequency movements in the RER. Since 83 percent of the unconditional variance of the RER is at low frequencies, we argue that trade shocks are crucial to account for the overall dynamics of the RER. On the other hand, financial shocks are important to account for movements in the RER at the business cycle frequency.

We model trade shocks as iceberg trade cost shocks, a tractable way of modeling trade barriers, including tariffs, quotas and embargoes, political sanctions, and logistics costs (Obstfeld and Rogoff, 2000). We focus on the difference in trade costs across countries, instead of the level of country-specific costs, since only the difference affects the RER and net trade flows. In fact, Cuñat and Zymek (2022) show that the asymmetries in trade frictions account for most of the variation in trade imbalances.

In our model, the differential trade costs capture asymmetric changes in trade barriers that arise from many different sources. For example, sequential changes in trade policy, such as protectionist

[6]We use a specification of financial shocks equivalent to that in Itskhoki and Mukhin (2021a). That is, financial shocks affect the RER by inducing a wedge in the UIP condition, which affects aggregate savings and interest rates.

attacks during a trade war, can induce fluctuations in asymmetric barriers (Delpeuch et al., 2021). Moreover, a series of papers have shown that fiscal policy can be a source of fluctuations in trade barriers (Monacelli and Perotti, 2010; Bluedorn and Leigh, 2011; Bussière et al., 2010). Finally, trade costs also capture changes in the shipping technologies and the market structure of global trade (Burstein et al., 2003; Corsetti and Dedola, 2005; Corsetti, 2016).

While the specification of differential trade costs has been previously used (Waugh, 2011; Gornemann et al., 2020; Alessandria and Choi, 2021), we consider a more general specification of differential costs. In the literature, it is common to assume non-zero trade costs between countries, but the costs within a country are assumed to be zero. We relax this constraint by allowing trade costs within the rest of the world (ROW) to be non-zero. The within-ROW cost capture trade costs between countries that compose the ROW.[7] For example, during the period we consider (1980-2019) the Asia-Pacific Economic Cooperation in the 1990s and the creation of the European Union generated significant changes in trade barriers among the countries in the ROW. Using bilateral data on trade flows and prices, we find external evidence supporting our specification of trade shocks.[8]

Adding trade shocks allows the model to generate a weak high-frequency correlation with net trade flows. With financial shocks, there is an almost perfect correlation between the RER and net trade flows on impact. When financial shocks generate an excess return on bonds for the ROW relative to the US, savings in the ROW increase, and the excess saving in the ROW is exported to the US (net trade increases). At the same time, due to the fall in aggregate demand, the final good price in the ROW falls (the RER increases). Hence, financial shocks induce a positive correlation on impact. On the other hand, trade shocks can offset the large correlation by acting as a wedge that moves the two variables in opposite directions. Trade shocks that raise the relative cost of exporting make net trade to decrease. With less intermediate goods exported but more imported, aggregate final good increases, causing its price to fall (the RER goes up). Thus, trade shocks generate a negative relation between the RER and net trade flows on impact. The high-frequency comovement between the RER and net trade flows is useful to identify the volatility of trade shocks relative to

[7] In our model, we consider two countries: ROW and US. Our ROW aggregate includes Canada, Finland, Germany, Ireland, Italy, Japan, Republic of Korea, Spain, Sweden and United Kingdom. This set of countries represents 60% of total US trade on average.

[8] We explain this in detail in Section 7.

that of financial shocks. Once we include trade shocks, there is no tension in the model between capturing the net trade flows moments at the high frequency while simultaneously accounting for the RER disconnect.

The second ingredient we incorporate is dynamic trade. We model dynamic trade as in Alessandria and Choi (2007, 2021), which extends the sunk cost model of exporting of Dixit (1989), Baldwin and Krugman (1989) and Das et al. (2007) to a general equilibrium framework. In our model, producers are heterogeneous in their idiosyncratic productivity and decide whether to participate in the export market or not. When producers export they pay a fixed cost. Following the literature, we assume that the fixed cost is lower for incumbent exporters than for new exporters. This makes the exporting decision to be forward-looking, since the past export status becomes a state variable. This implies that there will be exporters that are less productive than some non-exporters, a well documented feature known as exporter hysteresis. Moreover, since the distribution of exporters evolves gradually over time in response to shocks, so does aggregate trade. This is of particular importance since it alters the propagation mechanism of shocks, a feature that we exploit to identify the underlying processes of the shocks.

Dynamic trade allows the model to capture the low-frequency dynamics of the RER. Only when we incorporate dynamic trade, the model is able to capture a trade elasticity to prices that is larger in the long-run than in the short-run. Moreover, dynamic trade is crucial to account for the frequency decomposition of the RER variance. In the static trade model, too high a share of the RER fluctuations is assigned to the low-frequency movements (94 percent in the model, as opposed to 83 percent in the data). Dynamic trade makes quantities in the short run more inelastic than under static trade. Therefore, under dynamic trade prices in the short run have a stronger response, so a smaller share of the variance of the RER is attributed to low frequency fluctuations (i.e. relative prices vary less in the long run relative to the short run under dynamic trade). This result is consistent with the "Excess Persistence Puzzle" documented in Rabanal and Rubio-Ramirez (2015), who argue that static trade models generate an excess share of low frequency variation in the RER. In our baseline model, variations at the low frequency explain 87 percent of the variance in the RER, close to the 83 percent we observe in the data.

In brief, by incorporating trade shocks and dynamic trade, our model can account for the comovement between the RER and net trade flows at all frequencies. First, it captures the weak comovement at the high frequency. Second, it generates a comovement between the RER and net trade flows stronger in the long run than in the short run. Finally, it generates a decomposition of the spectrum of the RER that close to the data. It is important to notice that the model successfully reproduces these features while simultaneously accounting for the disconnect of the RER with macro variables, such as output, consumption and interest rates.

Using our baseline model, we evaluate the role of financial and trade shocks in explaining the dynamics of the RER. We show that financial shocks are necessary to account for the disconnect between the RER and interest rates. Moreover, financial shocks are important to capture movements in the RER at the business cycle and higher frequencies (cycles less than 32 quarters). However, trade shocks are essential to capture lower-frequency movements.

To show these, we mute each shock and compute the resulting spectrum decomposition. Without financial shocks, the share of the variance in the RER assigned to business cycle and higher frequencies decreases from 13 percent to 7 percent. This means that financial shocks contribute relatively more to variation at higher frequencies than at lower frequencies. On the other hand, absent trade shocks, the share for low frequency decreases from 87 to 78 percent. Hence, trade shocks matter more for generating low-frequency movements. Since most of the unconditional variance of the RER is at these lower frequencies, we argue that trade shocks are crucial for understanding the overall dynamics of the RER.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents our benchmark two-country international macro model with heterogeneous producers, sunk cost of exporting, and financial and trade shocks. Section 4 presents the calibration and our identification strategy. Section 5 shows that the benchmark model is able to capture the differential comovement between the RER and net trade flows as well as the frequency decomposition of the variance of the RER. Section 6 studies the role of different shocks in explaining the data. Section 7 provides empirical evidence of trade costs consistent with the model's specification. Section 8 discusses the robustness of our result for alternative specifications. Finally, Section 9 concludes.

2 Literature Review

International real business cycle models, building on the seminal work of Backus, Kehoe and Kydland (1994), provide a general equilibrium framework for understanding the international transmission of shocks and the dynamics of the RER and net trade. These studies focus on explaining business cycle fluctuations, as done in the recent work by Heathcote and Perri (2014). While we also build on the these framework, we incorporate additional features to the model to study the low-frequency dynamics.

Only a limited number of papers have focused on low-frequency movements of the RER. Rabanal and Rubio-Ramirez (2015) propose a model with a reduced-form specification of dynamic trade, following Erceg et al. (2006), and non-stationary productivity shocks.[9] They show that dynamic trade is necessary to capture the spectrum of the RER when the model is driven by productivity shocks. Gornemann et al. (2020) model dynamic trade in a similar way but with an endogenous growth that amplifies stationary productivity fluctuations. They show that the model is able to capture the low-frequency dynamics of the RER, and that productivity shocks are an important source of the RER variation. We share with these papers the dynamic trade feature, but we model it with a microfoundation based on firms' exporting costs. This allows us to to discipline the frictions using micro data on exporter characteristics. Furthermore, we introduce a generalized specification for iceberg trade costs that considers a within-ROW component. We find that the model is able to reproduce the movements of the RER at different frequencies and that trade shocks are an important source of fluctuations at the low frequency.

Moreover, our paper offers a bridge between the studies of exchange rates in international finance and international trade. On the one hand, there is a growing literature emphasizing the role of shocks originated in the financial market for understanding the dynamics of exchange rates (Devereux and Engel, 2002; Gabaix and Maggiori, 2015; Farhi and Gabaix, 2016; Itskhoki and Mukhin, 2021a). This literature focuses on financial shocks that generate movements in the exchange rates by inducing deviations to uncovered interest parity (UIP) condition. In particular, Devereux and

[9] Drozd, Kolbin and Nosal (2021) adopt the same reduced-form specification and show that dynamic trade is a key feature to improve the model's ability to account for the trade comovement puzzle, i.e. the significant relationship in the data between countries' business cycles synchronization and trade flows.

Engel (2002) argue that violations of the UIP condition are informative about shocks originated in the financial market and that these shocks can explain the excess volatility of the RER. Moreover, Itskhoki and Mukhin (2021a) show that financial shocks go a long way in accounting for both the macro and the financial disconnect. While this literature discusses the dynamics of both the real and nominal exchange rates, we limit our interest to real variables only. We consider the real version of the UIP deviations, i.e. the relationship between the RER and real interest rate differentials. We show that in our data, the real and nominal versions of the UIP deviations are very similar.[10]

On the other hand, a series of papers have explored the role of trade barriers in aggregate fluctuations and capital flows (Obstfeld and Rogoff, 2000; Eaton, Kortum and Neiman, 2016; Reyes-Heroles, 2016; Alessandria and Choi, 2021; Sposi, 2021; Alessandria, Bai and Woo, 2022). In particular, Reyes-Heroles (2016) and Sposi (2021) study the interplay between trade integration and borrowing-lending in a perfect foresight economy. Alessandria, Bai and Woo (2022) focus on the coincident widening of the current account dispersion and an increase in international trade across a broader set of countries. They show that this is consistent with falling international trade barriers and easier international borrowing and lending.

In our model, the structure of the financial market and financial shocks are comparable to that of Itskhoki and Mukhin (2021b). At the same time, we also incorporate frictions and shocks in trade based on the trade literature. Our finding reconciles those of the two stands of the literature. As emphasized in the financial literature, we find that financial shocks are important for high-frequency fluctuations of the RER. Trade shocks, however, are crucial for accounting for low-frequency movements of the RER and its relationship with trade flows.

Finally, our paper is related to the literature on the measurement of trade wedges. Levchenko et al. (2010) and Fitzgerald (2012) use measured trade wedge based on the Armington model to study the role of trade costs and asset market frictions for international risk sharing. Head and Mayer (2014) explore different methods of estimating the gravity equation. We contribute to this literature by considering a generalized specification of trade costs. We show, both theoretically and empirically, how allowing for trade cost within the aggregate of the ROW generates different implications for

[10]This arises from the fact that in our sample the inflation was very low in the countries included in our analysis, so that the NER and RER are highly correlated. This is also the case for the samples considered in the cited literature.

the comovement of the RER and macro aggregates.

3 Model

We build on the standard two-country international business cycle model of Backus et al. (1994). We model the home country as the aggregate of the Rest of the World (ROW) and the foreign country as the US. Each country produces a perfectly competitive non-traded final good. The non-traded final good is made of a mix of tradable intermediates, using a CES technology with home bias. The final good can be consumed or invested by the household, while capital accumulation is subject to a capital adjustment cost.

There is a unit mass of intermediate good producers in each country producing a differentiated proudcts. They are subject to aggregate productivity shocks and are heterogeneous in their idiosyncratic productivity. They make a decision of entering and exiting the export market subject to the fixed costs that depends on the experience in the export market as in Dixit (1989), Baldwin and Krugman (1989) and Das et al. (2007). Firms producing intermediate goods set destination specific prices, and use labor and capital as inputs to production. Optimal prices are set as a markup over the marginal cost. We introduce time-varying markups that are reduced-form pricing to market friction, which leads to persistent deviations from the law of one price.

Intermediate firms also face a stochastic iceberg trade costs. These shocks are a tractable way of modeling trade barriers, depicted as only a fraction of goods shipped arriving at the destination. Furthermore, we assume that ROW intermediate producers also face a stochastic trade cost of shipping within the ROW. As mentioned in the introduction, this is a reduced form of capturing the evolution of trade integration among the countries that compose the ROW aggregate during during the period in consideration (1980 to 2019).

On the asset side, there is an internationally traded bond, denominated in foreign prices. The ROW household is subject to a bond adjustment cost, which induces stationarity of the model and can be interpreted as a reduced form of capturing costs of portfolio re-balancing. The ROW household is also subject to a financial shock, modeled as wedge in the return of the international bond. This captures the financial shock considered in Itskhoki and Mukhin (2021a). To derive deviations from the uncovered interest parity (UIP), which is the main channel through which the financial shocks affect the RER, we introduce a bond that is traded only in the ROW. This bond is priced but not traded in equilibrium. We describe below the model from the point of view of ROW agents.

Households

A representative household in the ROW maximizes the discounted expected utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\eta} (1 - L_t)^{1-\eta}\right]^{1-\sigma}}{1 - \sigma}$$

where $0 < \eta < 1$, β is the discount factor and the elasticity of substitution is $\frac{1}{\sigma}$. The flow budget constraint is given by

$$P_t C_t + P_t I_t + B_{t+1} + \frac{q_t B^*_{t+1}}{e^{\psi_t}} + q_t \frac{\chi}{2} \left(B^*_{t+1} - \bar{B}\right)^2 \leq W_t L_t + R^k_t K_t + B_t(1 + i_{t-1}) + q_t B^*_t(1 + i^*_{t-1}) + \Pi_t$$

where L_t is labor, C_t is consumption, P_t is the consumer price index, I_t is investment, K_t is capital, B_{t+1} is the quantity of ROW bonds, i^*_{t-1} is real interest rate on internationally traded bonds purchased at $t - 1$, and Π_t is aggregate profits of intermediate firms. For the international asset block, B^*_{t+1} is the quantity of the internationally traded bond hold by the ROW household, i^*_{t-1} is the real interest rate on domestic bonds purchased at $t - 1$, and q_t is the real exchange rate (RER), which is defined as a relative price of a basket of US to ROW goods. The term ψ_t is the financial shock, which generate a wedge in the UIP condition.[11]

Under a log-linearization, we can combine the Euler equations of ROW households for domestic and international bonds to derive an equation for the deviations of UIP,

$$i_t - i^*_t - \mathbb{E}_t \left[\Delta q_{t+1}\right] = \psi_t - \chi \cdot \left(B^*_{t+1} - \bar{B}\right)$$

where $\mathbb{E}_t \left[\Delta e_{t+1}\right] \equiv \mathbb{E}_t \left[\ln(q_{t+1}) - \ln(q_t)\right]$ is the expected depreciation of the RER, χ governs the adjustment cost on internationally traded bonds incurred by ROW household and $\bar{B}$ is the steady state level of net foreign assets. This is the same equilibrium condition as derived by Itskhoki and

[11] Our results are invariant to whether the shock ψ_t affects the adjustment cost of debt or not.

Mukhin (2021a) under incomplete segmented financial markets and noisy traders.[12]

Capital stock in each country follows the law of motion,

$$K_{t+1} = (1-\delta)K_t + \left[I_t - \frac{\kappa}{2}\frac{(\Delta K_{t+1})^2}{K_t}\right],$$

where the parameter κ governs the adjustment cost of capital.

Aggregation Technology

A competitive retail sector combines composite goods from ROW and the US with a constant elasticity of substitution (CES) to produce the final good. The CES aggregator is given by

$$D_t = \left[Y_{Rt}^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} Y_{Ut}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

where γ captures the home bias, ρ is the Armington elasticity between domestic and imported composite goods, Y_{Rt} is the quantity of domestic goods consumed in the ROW, Y_{Ut} is the quantity of imported goods from the US consumed in the ROW.

The total expenditure in the retail sector is given by

$$P_t D_t = e^{\xi_{Rt}} P_{Rt} Y_{Rt} + e^{\xi_{Ut}} P_{Ut} Y_{Ut}$$

where P_t is the aggregate price, P_{Rt} is the price of domestic goods in the ROW, P_{Ut} is the price of imported goods in the ROW, ξ_{Rt} is iceberg cost for domestic trade within ROW countries, and ξ_{Ut} is iceberg cost for international trade. Note that the expenditure of the retail sector includes the iceberg costs of $e^{\xi_{Rt}}, e^{\xi_{Ut}}$.

The problem of the retail sector would be to maximize the production of final goods by choosing the quantities of composite goods $\{Y_{Rt}, Y_{Ut}\}$ taking the prices and trade costs as given. The final good is used as either consumption of investment of households, so that $D_t = C_t + I_t$.

Note that we are considering a general case of iceberg costs that allow for the iceberg cost ξ_{Rt} for

[12] This can also be microfounded by risk-premia (Verdelhan 2010, Colacito and Croce 2013, Farhi and Gabaix 2016) or heterogeneous beliefs and expectational errors (Evans and Lyons 2002, Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006)

trade within ROW to be nonzero. This takes into account that the ROW is, in reality, an aggregate of multiple countries that trade with each other. In order to capture the average trade cost within the ROW countries, we relax the constraint of a standard specification with zero domestic iceberg costs.[13]

Solving this maximization problem yields the demand functions for ROW and US intermediate goods, given by

$$Y_{Ut} = \gamma \left(\frac{e^{\xi_{Ut}} P_{Ut}}{P_t} \right)^{-\rho} (C_t + I_t) \quad \text{and} \quad Y_{Rt} = \left(\frac{e^{\xi_{Rt}} p_{Rt}}{P_t} \right)^{-\rho} (C_t + I_t).$$

where P_t is the ideal price index in ROW

$$P_t = \left[\left(e^{\xi_{Rt}} P_{Rt} \right)^{1-\rho} + \gamma \left(e^{\xi_{Ut}} P_{Ut} \right)^{1-\rho} \right]^{1/(1-\rho)}.$$

The domestic and imported goods, Y_{Rt} and Y^*_{Rt}, are the composite of varieties produced by heterogeneous producers. The aggregators are

$$Y_{Rt} = \left(\int_0^1 y_{j,Rt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \qquad Y_{Ut} = \left(\int_{j \in \mathcal{E}^*_t} y_{j,Ut}^{\frac{\hat{\theta}_t-1}{\hat{\theta}_t}} di \right)^{\frac{\hat{\theta}_t}{\hat{\theta}_t-1}} \tag{1}$$

where θ and $\hat{\theta}_t$ are the elasticity of substitution across varieties, and $\mathcal{E}^*_t$ is the set of exporting firms in the US. Firms set destination specific prices, subject to the market-specific demands.

We let the elasticity across imported varieties to be a function of the RER with $\hat{\theta}_t = \theta q_t^{\zeta}$ (and $\hat{\theta}^*_t = \theta q_t^{-\zeta}$ for exported varieties). The time-varying elasticity captures the pricing to market behavior of firms in a reduced form, leading to persistent deviations from the law of one price.[14] When there is a depreciation of the RER for the ROW, markups charged by US firms to ROW importers fall. This is consistent with the findings in Alessandria and Kaboski (2011), which shows that firms price to income, that is, firms charge higher prices to higher income destinations. Absent this friction, the

[13]We explain in more detail the role of the within country iceberg cost when we present the shock processes.

[14]The pricing to market friction generates time-varying markups in a similar way as with a Kimball aggregator, as in Itskhoki and Mukhin (2021a). See Edmond et al. (2018) for a study of heterogeneous firm with the Kimball aggregator. On the other hand, Drozd and Nosal (2012) provide an alternative model of pricing to market where firms invest in marketing activities in order to accumulate customers.

terms of trade is more volatile than the RER, contrary to what is observed in the data. Furthermore, the pricing to market generates the incomplete pass-though of exchange rates to prices.

The price indices for the composite goods are given by

$$P_{Rt} = \left(\int_{j=0}^{1} p_{j,Rt}{}^{1-\theta} \right)^{\frac{1}{1-\theta}} \qquad P_{Ut} = \left(\int_{j \in \mathcal{E}_t^*} p_{j,Ut}{}^{1-\hat{\theta}_t} \right)^{\frac{1}{1-\hat{\theta}_t}} .$$

The problem of the US retailers is given in a symmetric form

$$\max_{\{Y_{Ut}^*, Y_{Rt}^*\}} P_t^* \left(C_t^* + I_t^*\right) - \left[e^{\xi_{Ut}^*} P_{Ut}^* Y_{Ut}^* + e^{\xi_{Rt}^*} P_{Rt}^* Y_{Rt}^*\right]$$

subject to the CES aggregator, resulting in the demand functions of

$$Y_{Rt}^* = \gamma \left(\frac{e^{\xi_{Rt}^*} P_{Rt}^*}{P_t^*} \right)^{-\rho} \left(C_t^* + I_t^*\right) \quad \text{and} \quad Y_{Ut}^* = \left(\frac{P_{Ut}^*}{P_t^*} \right)^{-\rho} \left(C_t^* + I_t^*\right).$$

Intermediate Firms

There is a continuum of heterogeneous firms $j \in [0, 1]$ in each country, specializing in production of a differentiated intermediates. Their output is produced under monopolistic competition among these firms. The firms are subject to aggregate and firm-specific shocks. The firm j's production is given by

$$y_{jt} = e^{a_t + \mu_{jt}} L_{jt}^{\alpha} K_{jt}^{1-\alpha},$$

where α is the capital share of income, a_t is the productivity shock, and μ_{jt} is a idiosyncratic firm-specific shock.

The firm-specific shock is iid, $\mu \overset{iid}{\sim} N(0, \sigma_\mu^2)$. All firms sell their products in their own country, while some of them choose to export. The resource constraint for each good is given by $y_{jt} = y_{j,Rt} + m_{jt} y_{j,Rt}^*$ where $y_{j,Rt}$ is ROW variety used domestically, $y_{j,Rt}^*$ is ROW variety exported to the US, and $m_{jt} \in \{0, 1\}$ is the current export status of firm j, with 1 being export and 0 not export.

In order to export the firms pay a fixed export cost. The cost for starting to export differs from the cost to stay in the export market. To start exporting, a firm pays a relatively high cost of $W_t f_0$, while an existing exporter pays the continuation cost of $W_t f_1$, where $f_1 < f_0$. That is, there is a sunk cost associated with export participation. This is to capture the exporter hysteresis and thus slow response of aggregate export.

The dynamic problem of a firm is

$$V(\mu, m_{-1}, k, S) = \max \; p_R y_R + mqp_R^* y_R^* - Wl - R^k k - mWf_{m_{-1}} + \mathbb{E}\Omega V'(\mu', m, k', S')$$

where q is the real exchange rate, $S = \{a, K\}$ is the aggregate state, and Ω is firm's stochastic discount factor.

Subject to the ROW retailer's demand for ROW intermediates ($y_{j,Rt}$), the US Retailer's demand for ROW intermediates ($y_{j,Rt}^*$), and the resource constraint, the optimality conditions of the firm are given by the optimal demand for inputs and optimal pricing,

$$W_t = (1-\alpha)\frac{y_{jt}}{L_{jt}} \quad \text{and} \quad R_t^k = \alpha\frac{y_{jt}}{K_{jt}}$$
$$p_{j,Rt} = \frac{\theta}{\theta - 1}MC_{jt} \quad \text{and} \quad q_t p_{j,Rt}^* = \frac{\theta q_t^{-\zeta}}{\theta q_t^{-\zeta} - 1}MC_{jt}$$

where the $MC_{jt} = \frac{1}{e^{a_t+\mu_{jt}}}\frac{(R_t^k)^{\alpha}(W_t)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$ is the marginal cost. Note that firms set different prices across destinations since they face different demand elasticities at home and foreign.

Also note that the fixed cost $f_{m_{-1}}$ depends on the exporting status of the previous period m_{-1}. Given the previous exporting status, the value of a firm is monotonically increasing and continuous in its productivity μ. We can solve for the threshold productivity for exporting decisions for non-exporters and exporters. The thresholds η_{0t} and η_{1t} satisfy

$$W_t f_m - \pi_t = \Omega_t \mathbb{E}_t[V_{t+1}(\mu_{t+1}, 1) - V_t(\mu_{t+1}, 0)]$$

for $m \in \{0, 1\}$. That is, at the threshold, a firm is indifferent between exporting and not exporting, and firms will move in and out of exporting depending on its previous exporting status and its own idiosyncratic shocks. The mass of exporting firm N_t evolves as

$$N_t = N_{t-1}P[\mu > \mu_{1t}] + (1 - N_{t-1})P[\mu > \mu_{0t}].$$

The aggregate of firm level inputs are given by

$$L_t = \int_{j=0}^{1} L_{jt} + f_0 \cdot (1 - N_{t-1}) \cdot P[\mu > \mu_{0t}] + f_1 \cdot N_{t-1} \cdot P[\mu > \mu_{1t}]$$
$$K_t = \int_{j=0}^{1} K_{jt}.$$

Note that the aggregate labor includes the fixed cost of exporting of all firms because the costs are in terms of labor.

Shock Processes

Productivity shocks features a common and differential component,[15]

$$\begin{bmatrix} a_t \\ a_t^* \end{bmatrix} = \begin{bmatrix} a_{ct} + a_{dt}/2 \\ a_{ct} - a_{dt}/2 \end{bmatrix}$$

where the common component a_{ct} and the differential component a_{dt} each follow and AR(1) process,

$$a_{ct} = \rho_a^c a_{ct-1} + \varepsilon_{at}^c \qquad \varepsilon_{at}^c \sim N(0, \sigma_a^c)$$
$$a_{dt} = \rho_a^d a_{dt-1} + \varepsilon_{at}^d \qquad \varepsilon_{at}^d \sim N(0, \sigma_a^d)$$

We assume trade shocks only have a differential component ξ_t which follows an AR(1) process. In Waugh (2011) and Alessandria and Choi (2021), they consider common and differential component of trade shocks. Here we abstract from the effect of a common component, since the common component primarily affects the level of gross trade, but does not have a first order effect on the RER and net trade.

The trade costs are given by

$$\xi_{Rt}^* = \frac{\xi_t}{2} \qquad \xi_{Ut} = -\frac{\xi_t}{2} \tag{2}$$
$$\xi_{Rt} = \tau \frac{\xi_t}{2} \qquad \xi_{Ut}^* = 0 \tag{3}$$

[15] Alternatively country-specific shocks can be written as a combination of these orthogonal shocks.

where $\tau \in \mathbb{R}$ and

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}, \quad \varepsilon_{\xi t} \sim N(0, \sigma_\xi).$$

If τ equals zero, we recover the standard iceberg cost shocks. We allow for the within ROW trade cost to potentially be non-zero, as captured by the case $\tau \neq 0$. The parameter τ would capture the *elasticity* of the average within ROW trade cost to the differential trade cost. We allow for the within ROW trade cost to vary over time to capture the evolution of trade integration among the countries that compose the ROW aggregate.[16] For example, during the time period we consider (1980-2019), the Asia-Pacific Economic Cooperation in the 1990s and the creation of the European Union generated significant changes in trade barriers among the countries in the ROW. More recently, though not included in our sample, trade sanctions were applied against Russia due to the Russian-Ukraine War since February 2022. This is an example of an incident that the trade cost increases between the US and Russia, as well as within the ROW countries.

Conditional on a positive iceberg cost shock, as τ increases the higher is the within country trade cost for the ROW. As fewer intermediate goods produced in the ROW can be aggregated to produce the final good in the ROW this induce a negative effect on GDP in the ROW. The strength of the negative effect on GDP is increasing in τ, and so is the effect on domestic absorption. Therefore, the cross country correlation of domestic absorption will vary with τ. In the quantitative exercise in Section 4.2 we show that the cross country correlation of domestic absorption identifies τ.[17] In Section 4.2 we present a detailed analysis on role of τ in the response of aggregate variables to trade shocks.

Finally, we assume the financial shock affecting follows an AR(1) process,

$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi t} \quad \text{where} \quad \varepsilon_t^\psi \sim N(0, \sigma_\psi).$$

[16]While the specification of domestic iceberg trade cost is a generalization of standard case, for values of τ close enough to the home bias parameter γ, it generates a qualitatively similar mechanism as the relative demand shocks in Pavlova and Rigobon (2007). They use a CES function of the form $C_t + I_t = \left[(1-\gamma)^{\frac{1}{\rho}} \left(e^{-\gamma \xi_t}\right)^{\frac{1}{\rho}} C_t^{r,r\,\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} \left(e^{(1-\gamma)\xi_t}\right)^{\frac{1}{\rho}} C_t^{u,r\,\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$.

[17]In the quantitative exercise we assume that the within country component is only present in the ROW. We do this to account for the fact that the within trade cost in the US relative to the within ROW marginally changed in our sample. However, our results are robust to also including the within cost in the US.

Market Clearing

Goods market clearing for each firm j requires that its production is split between supply to the ROW and the US and satisfies the local demand in each market:

$$y_{jt} = e^{\xi_{Rt}} y_{j,Rt} + e^{\xi^*_{Rt}} y^*_{j,Rt}.$$

With the aggregation presented in Equation 1, this leads to the aggregate market clearing condition where the total production of the ROW $Y_t = \int_{j=0}^{1} y_{jt}$ is split between demand for composite goods in the ROW and the US:

$$Y_t = e^{\xi_{Rt}} Y_{Rt} + e^{\xi^*_{Rt}} Y^*_{Rt}.$$

Lastly, combining the household budget constraint with aggregate intermediate profits as well as the market clearing conditions above, to obtain the ROW country budget constraint:

$$\frac{q_t B^*_{t+1}}{e^{\psi_t}} + q_t \frac{\chi}{2} \left(B^*_{t+1} - \bar{B}\right)^2 - q_t B^*_t (1 + i^*_{t-1}) = NX_t \quad \text{with} \quad NX_t = q_t e^{\xi^*_{Rt}} P^*_{Rt} Y^*_{Rt} - e^{\xi_{Ut}} P_{Ut} Y_{Ut}$$

The US country budget constraint is satisfied by the Walras Law.

Final Goods Price Normalization

We fix the final good prices in both countries P_t, P^*_t to one. Implicitly we are assuming that the monetary authority in each country perfectly stabilizes inflation. Note that the RER is defined as the relative price of a basket of ROW to US goods, $q_t = e_t P_t / P^*_t$ where e_t is the nominal exchange rate. Therefore, the RER q_t is same as the nominal exchange rate e_t, which is the price of local currency per unit of foreign currency.

Definition of Recursive Competitive Equilibrium

A recursive competitive equilibrium is defined by a sequence for $t = 0, 1, \ldots, \infty$ of aggregate prices $\{W_t, W^*_t, R^k_t, R^{k*}_t, Q_t, P_{Rt}, P^*_{Rt}, P_{Ut}, P^*_{Ut}, i_t, i^*_t\}$, firm-level prices $\{p_{j,Rt}, p^*_{j,Rt}, p_{j,Ut}, p^*_{j,Ut}\}$, aggregate allocations $\{C_t, C^*_t, L_t, L^*_t, I_t, I^*_t, B^*_{t+1}, B_{t+1}, Y_{Rt}, Y^*_{Rt}, Y_{Ut}, Y^*_{Ut}, \}$, firm-level allocations $\{y_{j,Rt}, y^*_{j,Rt}, y_{j,Ut}, y^*_{j,Ut}\}$, firm-level input choices and export decisions, and the mass of exporters $\{N_t, N^*_t\}$ such that

- Given prices $\{W_t, W^*_t, R^k_t, R^{k*}_t, Q_t, i_t, i^*_t\}$, $\{C_t, L_t, I_t, B_{t+1}, B^*_{t+1}\}$ solves the problem of the ROW households, and $\{C_t^*, L_t^*, I_t^*, B_{t+1}^*\}$ correspondingly for the US households.

- Given prices $\{p_{j,Rt}, p_{j,Rt}^*, p_{j,Ut}, p_{j,Ut}^*\}$, $\{y_{j,Rt}, y_{j,Rt}^*, y_{j,Ut}, y_{j,Ut}^*\}$ solves the problem in the final retail sectors in the ROW and the US.
- Firm-level input choices, prices, and export decisions solve their dynamic programming problems.
- The market clearing conditions for goods, labor and bonds are satisfied.
- Rationality/consistency so that the laws of motions are consistent with firms' decisions rules.

4 Calibration

In this section, we describe the data, discuss our calibration procedure and present the targeted moments. We define three set of calibrated parameters. First, we calibrate parameters that are standard in the literature, some of which are externally calibrated and some internally (i.e. to match some moment). Second, we calibrate the parameters that are specific to the export behavior of firms. To do so, we appeal to microdata on US firms. Third, we jointly calibrate the parameters related to the shocks processes and adjustment costs to match a set of equal number of moments (i.e. just identified). Finally, we show that the baseline model is able to reproduce the targeted moments.

4.1 Data

We use quarterly data during the period of 1980-2019 for the US and ROW. The ROW is a weighted average of 10 countries,[18] where the weights are based on the trade weights calculated by the Federal Reserve Board. GDP, consumption, investment, exports and imports come from Quarterly National Accounts of OECD. For the real exchange rate we use Narrow Real Effective Rate from the BIS. For the interest rates, we use money market rates from multiple sources (IMF; OECD; BOJ) and ex-post inflation rate of consumer price index (OECD). In the case of the US, the money market rate corresponds to the effective federal funds rate. To construct the interest rate for the ROW, we take

[18] Canada; Finland; Germany; Ireland; Italy; Japan; Republic of Korea; Spain; Sweden; United Kingdom. 60% of total US trade. The estimated moments from the data are robust to having an unbalanced panel that includes China since 1990.

the trade-weighted average of country-specific rates. More detailed description of data construction is provided in Appendix A.

4.2 Calibration

To parametrize our model, we proceed in three stages: *(i)* standard parameters, *(ii)* parameters related to the trade block of intermediate producers, and *(iii)* parameters related to the shocks and adjustment costs.

Standard Parameters

The standard parameters are displayed in the first panel of Table 1. The time unit in the model is a quarter, and we choose the discount factor of $\beta = 0.99$ and the depreciation rate of $\delta = 0.02$. The risk aversion is $\sigma = 2$, which is the value frequently used in related business cycle studies. The capital share of $\alpha = 0.36$ is consistent with the labor share in the US. The preference weight on consumption $\eta = 0.36$ is set to match the steady state labor of 1/4. The elasticity of substitution between ROW and US goods θ is set to be 1.5, following the estimates in Feenstra et al. (2018). The home bias, governed by γ, is set to match the average trade share of 14% in the US during our sample period. We assign these values symmetrically to the US and the ROW. Finally, we set the persistence of the common and differential productivity shocks, ρ_{a_c} and ρ_{a_c}, to be equal to 0.98 to match the persistence of output growth.

Producer Trade Parameters

We calibrate four additional parameters related to the export block: fixed trade costs f_1 and f_0, the volatility of idiosyncratic productivity shocks σ_η, and the reduced form pricing to market ζ. The fixed costs and the volatility are set to jointly match exporter dynamics. In particular, it generates an export participation of 25 percent, the quarterly exporter exit rate of 3.5 percent, and exporters are 75 percent larger than non-exporters, consistent with the US trade and exporter characteristics in the early 1990s. The pricing-to-market parameter is to set to match the exchange rate pass-through of 60 percent as in Itskhoki and Mukhin (2021a). The values of these four calibrated parameters are displayed in the second panel of Table 1.

Table 1: Calibrated Parameters

Parameter		Value	Target Moment
Standard Parameters			
Discount Factor	β	0.99	Annual interest rate of 4%
Risk Aversion	σ	2	Intertemporal elasticity of substitution of .5
Weight on Consumption	η	0.36	Hours worked (Frisch elasticity)
Capital Share	α	0.36	Capital share of income
Elasticity of Substitution across Varieties	θ	4	Producer markup of 33%
Elasticity of Substitution between H and F	ρ	1.5	Long-run price elasticity
Depreciation Rate	δ	0.02	
Home Bias	γ	0.097	Trade-to-GDP ratio of 14%
Persistence Common Productivity	ρ_{a_c}	0.98	GDP persistence
Persistence Differential Productivity	ρ_{a_d}	0.98	GDP persistence
Producer Trade Parameters			
Fixed cost of new exporters	f_0	0.07	Export participation of 25%
Fixed cost of incumbent exporters	f_1	0.04	Exit rate of 3.5%
Volatility of idiosyncratic productivity	σ_μ	0.08	Exporter premium of 75%
Pricing to market parameter	ζ	1.00	Exchange rate pass-through of 60%
Shocks and Adjustment Costs			
Financial shock, volatility	σ_ψ/σ_{a_c}	0.57	$\rho\left(\Delta c - \Delta c^*, \Delta q\right)$
Financial shock, persistence	ρ_ψ	0.99	$\rho\left(i - i^*\right)$
Trade shock, volatility	σ_ξ/σ_{a_c}	17.01	$\sigma(xm)/\sigma(q)$
Trade shock, persistence	ρ_ξ	0.98	$\rho\left(\Delta xm, \Delta q\right)$
Trade shock, within-country share	τ	0.17	$\rho(\Delta d, \Delta d^*)$
Productivity differentials, volatility	$\sigma_{a_d}/\sigma_{a_c}$	1.24	$\rho(\Delta y, \Delta y^*)$
Adjustment cost of portfolios	χ	0.06	$\rho(xm)$
Adjustment cost of capital	κ	1.59	$\sigma(\Delta inv)/\sigma(\Delta y)$

Notes: The table presents the values of calibrated parameters of the baseline model. When we consider an alternative models, some of the parameters are set to a different value while the other parameters are all recalibrated. In a model without trade shocks, $\sigma_\xi = \rho_\xi = 0$. In a model without trade dynamics, $f_0 = f_1 = \sigma_\mu = 0$.

For the case with no dynamics (i.e. static trade) we set the fixed costs of exporting and the volatility of idiosyncratic productivity shocks to zero, but keep the pricing-to-market and the incomplete exchange rate pass-through.

Shocks and Adjustment Costs: Internally Calibrated

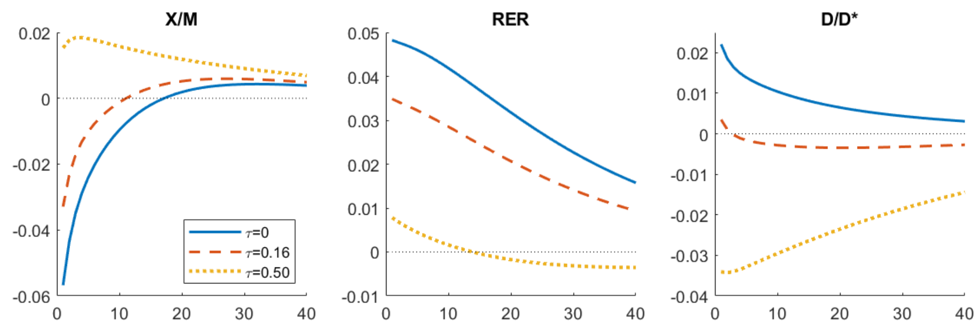
The remaining parameters related to trade, financial, and productivity shocks, and the adjustment cost for capital and portfolios. We normalize the volatility of the common productivity shock to unity, so that the volatility of the other shock processes are relative to that of common productivity. There are eight parameters to be estimated. We jointly calibrate them to match eight moments. The third panel of Table 1 lists the parameters and moments used for the identification.

Each parameters affect all of the moments but not with the same relevance. In the third panel of Table 1 we display the values of the internally calibrated parameters, together with the moment that is most relevant for the identification of each parameter. As shown in Itskhoki and Mukhin (2021a), the Backus-Smith correlation identifies the volatility of the financial shock, and the persistence is identified by the autocorrelation of the interest rate differential.

For the case of the trade shock, we use the volatility of net trade relative to the volatility of the RER to identify the volatility of the trade shock. The persistence of the trade shock is identified by the contemporaneous correlation between the first difference of net trade and the first difference of the RER. Recall that these two net trade moments are the counterfactual results from the financial shock model. Here we aim to target them by adding trade shocks.

Finally, we identify the within-country share τ using the cross country correlation of the first difference of domestic absorption. To build intuition on the role of τ, Figure 2 displays the impulse response functions to a shock that increases the shipping cost from the ROW to the US (an increase in ξ), for different values of τ, while keeping constant the other calibrated parameters. When τ is zero (the blue line), higher shipping cost from the ROW to the US decreases real net trade flows for the ROW. The increase in imports for the ROW, together with the a higher use in the final goods production in the ROW of local intermediates, generates an excess of supply of final goods in the ROW. For markets to clear, the final good price in the ROW must fall, inducing a depreciation of the RER. Higher output in the ROW increases domestic absorption relative to the US, since consumption and investment increase in the ROW. Now, consider the case of a positive but small value of τ of 0.17 (red line). With a positive τ, there is a dead-weight loss in the transactions between the ROW intermediates and final good producers. This makes exporting of the ROW relatively more attractive than under zero τ, so that the fall of net exports is smaller. Moreover, final goods output in the ROW increases by less relative to the zero τ case, implying that the final good price does not need to fall as much (i.e. smaller depreciation). Since there is fewer final goods in the ROW, domestic absorption does not increase as much as in the zero τ case. If τ is sufficiently high, net trade flows for the ROW

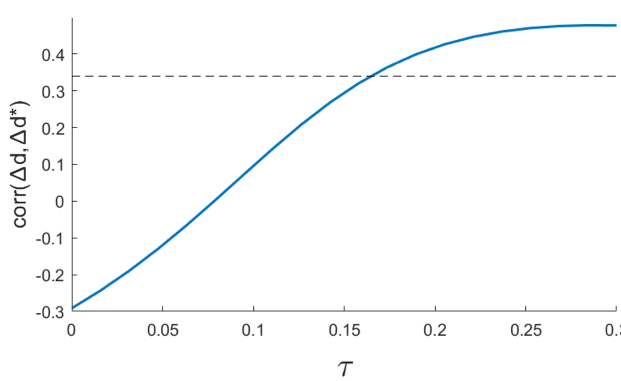
Figure 2: IRFs to ξ shock for different values of τ



can be positive with domestic absorption in the ROW decreasing relative to the US (this is shown for a value of τ of 0.50 under the orange line). It is clear that the cross country correlation of the first difference of domestic absorption is sensitive to the choice of τ, hence providing an identification for τ. Figure 3 shows this by displaying the cross country correlation of the first difference of domestic absorption across different values of τ. In our calibration we find a value of τ of 0.17.

Finally, the volatility of the differential productivity shock is identified by the cross country correlation of the first difference of GDP. Finally, the adjustment cost of capital directly affects the volatility of investment relative to that of GDP, while the adjustment cost of debt is identified by the autocorrelation of net trade.

Figure 3: Identification of τ



Notes: Correlation of domestic absorptions in the US and ROW given different values of τ. The other parameters are set as in Table 1. Based on model simulation of 10,000 periods. Black dashed line is the correlation in the data.

4.3 Alternative Models

We consider alternative specifications to our baseline model to understand the role of two essential features of our model: trade shocks and dynamic trade. That is, we consider a case when either of these features is absent. To consider a model without trade shocks, we set the volatility σ_ξ and persistence ρ_ξ of trade shock to be zero and recalibrate the remaining parameters. For this exercise, we target the same moments we considered before, except for the volatility of net trade and its contemporaneous correlation with the RER. In a case we mute the dynamic trade and consider a model with static trade, we set the fixed costs of exporting f_0, f_1 to be zero and assume that all producers are homogeneous with zero idiosyncratic shocks $\sigma_\mu = 0$. Given these values for the trade related parameters, the other parameters related to shocks and adjustment costs are estimated in the same way as in the baseline case. If we have neither of the features, the model is comparable to the one considered in Itskhoki and Mukhin (2021b).

5 Results

In this section, we show the model successfully captures the dynamics of the RER at all spectrum of frequencies. First, we emphasize the moments related to the net trade are reproduced in our model. Second, we present the low frequency comovement between the RER and net trade that is consistent with data, although they are untargeted. Third, we show the model performance for the spectrum analysis that decomposes the RER into different frequencies. In all of three results, we show that incorporating trade shocks and dynamic trade is crucial for the success of the model.

5.1 Net Trade at the High Frequency

The results of the baseline model for the targeted moments are presented in the first panel of Table 2. The baseline model performs well in matching the targeted moment. In particular, it successfully captures the net trade moments: contemporaneous correlation with the RER $\rho(\Delta xm, \Delta q)$, and its relative volatility $\sigma(xm)/\sigma(q)$. That is, the baseline model reproduces the comovement of net trade and the RER at the high frequency.

For the low contemporaneous correlation, trade shocks play a crucial role. Consider a model

Table 2: Model Results

Moments	Data	Baseline	No Trade Shock	No Dynamics
Targeted Moments				
$\rho\,(\Delta c - \Delta c^{*}, \Delta q)$	-0.10	-0.11	-0.09	-0.06
$\rho\,(i - i^{*})$	0.87	0.88	0.80	0.90
$\rho(\Delta y, \Delta y^{*})$	0.40	0.39	0.41	0.35
$\rho(\Delta d, \Delta d^{*})$	0.34	0.34	0.34	0.39
$\rho(xm)$	0.98	0.93	0.99	0.94
$\sigma(\Delta inv^{*})/\sigma(\Delta y^{*})$	2.59	2.60	2.62	2.60
$\rho\,(\Delta xm, \Delta q)$	0.30	0.29	*0.85*†	0.32
$\sigma(xm)/\sigma(q)$	1.12	1.12	*2.50*†	1.13
Trade Elasticity (Untargeted)				
ρ_{SR}	0.20 (0.05)	0.35	1.31	0.59
ρ_{LR}	1.16 (0.25)	0.80	1.95	0.55
Frequency Decomposition (Untargeted)				
High frequency	0.02	0.03	0.001	0.02
Business cycle frequency	0.15	0.10	0.003	0.04
Low frequency	0.83	0.87	0.995	0.94

Notes: 'No Trade Shock' presents the result of re-calibrated model only with productivity and financial shocks. Superscript † denotes that the moment is not targeted during the calibration procedure. 'No dynamics' is for the model without fixed exporting costs and producer heterogeneity.

that only features productivity and financial shocks, and no trade shocks. We recalibrate the model by targeting all of the moments except $\rho\,(\Delta xm, \Delta q)$ and $\sigma(xm)/\sigma(q)$. As it can be seen in the last column of Table 2, there is an excess volatility of net trade as well as an excess contemporaneous correlation with the RER. This shows that the models driven by financial shocks, although they are successful with the RER puzzles, misses the moments of net trade. Hence, we focus on the results from our baseline model that includes both financial and trade shocks. In the remainder of the paper we do not present the results for the model without trade shocks.[19]

5.2 Net Trade at the Low Frequency

In this section, we show the model successfully captures the low frequency comovements of the RER and net trade, although they are untargeted. This is captured by the long-run elasticity of net trade to prices larger than the short-run elasticity.

An important feature of the comovement between the RER and net trade is there is a strong lead-lag relation. This is captured by the difference between long-run and short-run elasticity of trade with respect to relative prices. The elasticity is small in the short run, because net trade is lagging behind the RER. On the other hand the long-run elasticity, that captures the gradual response, is strong and positive.

To estimate the elasticity from our data, we start from the the decomposition of net trade based on the Armington trade model. The Armington trade model is the basic trade block for almost all multi-good international macro models. In this framework, domestic and foreign goods are imperfect substitutes with a Constant Elasticity of Substitution (CES). Taking a ratio of CES demand functions for exports and imports, we have:

$$xm_t = \rho\ (tot_t + q_t) + (d_t^* - d_t) \tag{4}$$

where $xm_t = \log(X/M)$ is net trade, $tot_t = \log(p_t^M/p_t^X)$ is the terms of trade, q_t is the RER, and $d_t = \log(C_t + I_t)$ is domestic absorption.

Depending on the model, the ratio may include additional terms to the Equation 4. Note that our baseline model also nests the Armington trade block with additional features such as time-varying trade costs and sunk cost of exporting. These features will be reflected by additional terms related to trade shocks and the mass of exporters in Equation 4 (See Appendix C for the derivation).

In order to estimate the short- and long-run elasticity separately, we consider an error correction

[19] We will present the result for the baseline model when we shut down some shocks (without re-calibrating), including the trade shock, but we will not present results for the model that is calibrated under no trade shock.

model of the decomposition in equation 4. In specific,

$$\begin{aligned}\Delta xm = \beta + &\rho_{SR}\, \Delta(tot_t + q_t) + \Delta(d_t^* - d_t) \\ &- \alpha\left[xm_{t-1} - \rho_{LR}\,(tot_{t-1} + q_{t-1}) - (d_{t-1}^* - d_{t-1})\right] + \varepsilon_t \qquad (5)\end{aligned}$$

where ρ_{SR} is the short-run elasticity, ρ_{LR} is the long-run elasticity, and α captures the speed of adjustment. The term in square brackets captures the cointegration relationship. This type of regression has been widely used in studies of trade dynamics (Hooper et al., 2000; Marquez, 2002).

Using the data described in 4.1, we estimate Equation 5. The second panel of Table 2 presents the results. The short-run elasticity is estimated to be around 0.2, and long-run elasticity is larger, around 1.2. The estimated values are similar to the estimates from Alessandria and Choi (2021) that covers similar data period for the US, and are also consistent with Alessandria, Bai and Woo (2022) that uses panel data of a broader set of countries although the size of the long-run elasticity is slightly larger compared to our estimates.

Our model generates similar trade elasticities. Using the model simulated data, we conduct the same exercise.[20] The result is again presented in the second panel of Table 2. Controlling for expenditure, long run elasticity ρ_{LR} is larger than the short run ρ_{SR}, capturing the dynamic adjustment of net trade to prices.

We next show that trade dynamics are necessary to capture the differences between short and long run elasticity. In the last column, we present the result of the model with zero fixed costs of exporting. Without trade dynamics, trade is able to respond by more on impact, so that the short and long run elasticities are very similar.[21] This shows that it is crucial to account for dynamic trade in order to capture the delayed comovement between the RER and net trade.

5.3 Spectrum Analaysis

Next, we show our model successfully captures the spectrum of the RER. Using the spectrum analysis, we decompose the variance of the RER into different frequencies.

[20]For model simulated data we simulate the model for 10,000 periods and burn the first half.

[21]There is a small difference between long-run and short-run elasticities under static trade. The difference is nonzero due to the time-varying trade costs. If we control for trade costs, two elasticities are estimated to be the same.

The last panel in Table 2 present the result of the spectrum analysis that decomposes the unconditional variance of the RER into different frequencies. In the data (second column), the largest share of the RER variations that is assigned at the frequency lower than the business cycle is largest, being 83 percent, and least is assigned at the higher frequency. In our model (third column), the similar pattern is found. 87 percent of the RER variation is assigned at the low frequency, followed by the business cycle, and least by the high frequency.

Dynamic trade plays an important role in matching of the spectrum of the RER. In a model re-calibrated without dynamic trade (last column), almost all of the share of the RER variance is attributed to the low frequency. When trade is static, quantities in the short-run are more elastic than under dynamic trade. This implies that prices in the short-run have a weaker response under static trade, so that a higher share of the variance of the RER is attributed to lower frequency fluctuations (i.e. relative prices vary more in the long-run relative to the short run). This result for static trade is consistent with the "Excess Persistence Puzzle" documented in Rabanal and Rubio-Ramirez (2015). Once we incorporate dynamic trade, the share of the low-frequency movements is closer to the data, since quantities in the short-run are more inelastic, and thus prices need to adjust more to clear the market. Thus, dynamic trade helps to improve the matching of the spectrum of the RER, by reducing the share of variation in the RER at low frequencies.

6 Role of Trade and Financial shocks

Having shown that the baseline model captures the high and low frequency correlation between the RER and net trade as well as the targeted moments, we now turn to evaluate the role of trade and financial shocks to account for the RER dynamics.

We consider three aspects. First, we study the contribution of each shock for generating the spectrum of the RER. Second, we explain and show the model result for the RER disconnect, and compare the role of two shocks. Finally, we show the success of the model in matching the standard international business cycle moments, and again examine the role of the shocks.

6.1 Contribution of Shocks to the Spectrum of the RER

In this section, we study the contribution of financial and trade shocks to the variation in the RER at different frequencies. To do this, we compute the model's share of the variance of the RER at different frequencies. Specifically, we look at business cycle frequencies (cycles between 8 and 32 quarters), as well as higher and lower frequencies. We present the results in the Panel A of Table 3, for the baseline model as well as the cases where we shut down either the financial or the trade shock.

Column 3 of Table 3 shows that the baseline model generates a decomposition of the unconditional variance of the RER close to the data, as shown in Section 5.3. In Column 4, we shut down trade shocks, and find that the share of the variance attributable to low frequencies fall by 10 percentage points. In other words, trade shocks contribute relatively more to variation in the RER at low frequencies. On the other hand, in Column 5 we shut down financial shocks and find that the low frequency share increases. This implies that financial shocks are more important than trade shocks for inducing movements in the RER at business cycle and higher frequencies.

To see why the trade shock have a more persistent effect on the RER we compare the IRFs of the RER (second panel in Figure 4). The effect of financial shocks die out faster than trade shocks. It is worth noticing that it is the *effect* of trade shocks that is more persistent, and not the process itself. In fact, the calibrated persistence of the financial shock is slightly higher than that of the trade shock. ($\rho_\psi = 0.99$ for financial shocks; $\rho_\xi = 0.98$ for trade shocks). Financial shocks generate less persistent fluctuations in domestic absorption (third panel in Figure 4), and thus in price levels. On the other hand, trade shocks that distort real trade flows generate more persistent effects on domestic absorption and the RER.

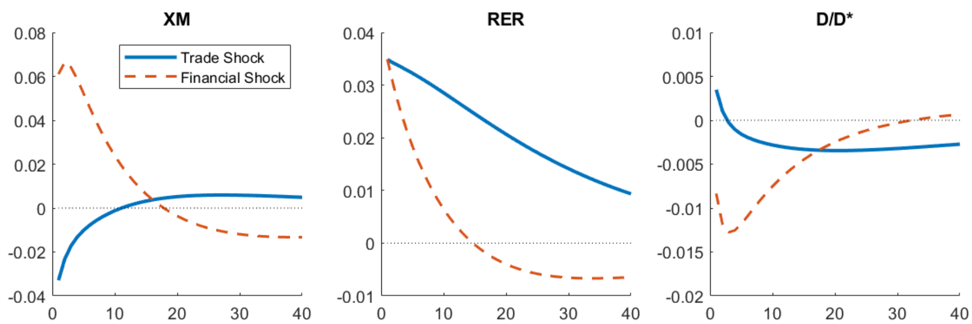
Moreover, the persistent effect of trade shocks is not due to dynamic trade. In Figure D2, we compare the impulse response of the RER with and without trade dynamics. In both cases, trade shocks have much more persistent effects on the RER than financial shocks.

Overall, our results suggest that trade shocks are more important than financial shocks for the low frequency variation in the RER, which accounts for 83 percent of the its unconditional variance. Hence, we argue that trade shocks are crucial to account for the dynamics of the RER.

Table 3: Results with Different Shocks

	Data	Baseline	No Trade Shock	No Financial Shock	No Productivity Shock
A. Frequency Spectrum					
High	0.02	0.03	0.06	0.01	0.03
Business cycle	0.15	0.10	0.16	0.06	0.10
Low	0.83	0.87	0.78	0.93	0.87
B. Real Disconnect					
$\sigma(\Delta q)/\sigma(\Delta y)$	4.24	4.12	3.03	2.89	23.32
$\rho(\Delta q)$	≈0	-0.02	-0.05	0.01	-0.02
$\rho(q)$	0.97	0.96	0.93	0.98	0.96
$\rho\left(\Delta c - \Delta c^{*}, \Delta q\right)$	-0.10	-0.11	-0.16	0.24	-0.65
C. Financial Disconnect					
β^{expd}_{Fama}	-1.34 (0.52)	0.35	-0.22	1.20	0.38
R^2	0.02	0.004	0.001	0.14	0.003
$\sigma(i - i^{*})/\sigma(\Delta q)$	0.13	0.04	0.05	0.04	0.31
$\rho(i - i^{*})$	0.90	0.88	0.89	0.92	0.83
$\rho(i)$	0.97	0.93	0.93	0.97	0.89
D. Business Cycle Moments					
$\sigma(\Delta c^{*})/\sigma(\Delta y^{*})$	0.83	0.65	0.68	0.62	1.74
$\rho(\Delta y^{*}, \Delta c^{*})$	0.65	0.83	0.90	0.93	0.62
$\rho(\Delta y^{*}, \Delta z^{*})$	0.68	0.86	0.98	0.88	0.60
$\rho(\Delta c, \Delta c^{*})$	0.31	0.36	0.37	0.53	0.64
$\rho(\Delta inv, \Delta inv^{*})$	0.31	0.39	0.46	0.42	0.07
$\rho(\Delta tot, \Delta q)$	0.49	0.98	1.00	1.00	0.97
$\sigma(\Delta tot)/\sigma(\Delta q)$	0.46	0.20	0.26	0.18	0.11

Figure 4: IRF of RER to trade and financial shocks



6.2 Disconnect between the RER and Macro Fundamentals

In this section, we discuss the real and financial disconnect. We show that our model is successful in reproducing the disconnect. We also show how financial and trade shocks contribute for generating this result.

First, there is an empirical disconnect between the RER and output, that the literature have struggled to reproduce. In particular, in the data the RER follows a near random-walk process and is three to six times more volatile than output (Meese-Rogoff Puzzle). We also find the pattern in our data, as shown in the Panel B of Table 3. In the second column, we show that the volatility is more than four times larger than output, and the persistence is high in levels and small in growth rates.

In a standard BKK-type model, the volatility is lower than that of output, and the process is far from a random walk. However, our model successfully reproduces the data patterns. As shown in the third column, all of the volatility and persistence of the RER is very close to the data. Note that these moments are not targeted during our calibration procedure.

Second, the empirical correlation between relative consumption growth across countries and the RER growth is negative. However, the perfect risk-sharing condition in these models imply that a country experiencing higher consumption growth relative to another is accompanied one by one by a depreciation of its currency. This implies that the correlation would be close to one (Backus-Smith Puzzle). It is important to notice that even under under incomplete markets the model predicts an almost perfect correlation between the cross-country consumption growth and changes in the RER, for plausible values of the elasticity of substitution between home and foreign varieties.

Our model is able to reproduce this puzzle by directly targeting the correlation during the calibration. In the last row in the Panel C of Table 3, the correlation between cross-country consumption growth and RER growth is negative near -0.10 in both data and the baseline model.

To study the contribution of each shock to the real puzzles we shut down either the trade shock (Column 4) or the financial shock (Column 5). We find that both financial and trade shocks contribute to accounting for the real puzzles. The volatility of the RER is significantly greater than that of macro aggregates when we shut down each shock. It is worth noticing that financial shocks contribute slightly more than trade shocks, since the values under no trade shock are greater than under no financial shock. However, the autocorrelation of the level of the RER is closely matched under no financial shock rather than no trade shock, meaning that trade shocks induce more persistent variation in the RER. This is not surprising since we find that trade shocks contribute more to low-frequency movements in the RER than financial shocks. Finally, we notice that absent trade shocks the Backus-Smith correlation is more negative, meaning that financial shocks are important to account for this correlation. When we shut down the financial shocks, the correlation is positive but small, which implies that trade shocks contribute to the disconnect between the RER and consumption.

We now turn to the results related to the financial disconnect. In our data, real interest rate differentials are not well connected to the expected changes in the RER. The disconnect can be summarized by the regression similar to Fama (1984),[22]

$$\mathbb{E}_t[\Delta q_{t+1}] = \alpha + \beta^{expd}_{Fama}(i_t - i^*_t) + u_t. \tag{6}$$

We present the Fama coefficient and R^2 we find using our data in the Panel C of Table 3. As shown in the second column, the coefficient is negative, and the standard errors are large, consistent with the findings in Engel et al. (2022). More importantly, the predictive power of interest rates is weak, as measured by R^2 close to zero.

However, in BKK-type models, high interest rate predicts the RER depreciation, implying that

[22]Strictly speaking, the Fama regression is used to the show the disconnect in nominal variables, also known as the Forward Premium Puzzle. In this paper we are considering real version of the puzzle. In Table D1 in Appendix D we present the Fama coefficient we find using both real and nominal data, which is very similar to the real case. This arises from the fact that in our sample the RER and NER are almost perfectly correlated and inflation is very low in the countries included in our analysis.

the Uncovered Interest Parity (UIP) condition holds. In this case, β^{expd}_{Fama} should be equal to 1 and the R^2 should be close to 1. The puzzle is that the regression shows very low R^2, and the estimates are often negative with a large standard errors. Our model is able to generate a $\hat{\beta}_{\mathrm{F}}$ of 0.35, less than one (Column 3). More importantly, the R^2 is near zero showing the success of the model in accounting for the financial puzzle.

If we shut down the trade shock, we find that $\hat{\beta}_{\mathrm{F}}$ is negative, meaning that trade shocks are not contributing to a lower coefficient.[23] Moreover, the R^2 of the Fama regression is higher than in the data, which implies that interest rates have an excess predictive power on changes in the exchange rate. On the other hand, absent financial shocks $\hat{\beta}_{\mathrm{F}}$ is around 1 and the R^2 is significantly greater. Thus, we find that financial shocks are key to account for the financial market puzzles, since they generate low $\hat{\beta}_{\mathrm{F}}$ together with low predictive power of interest rates on changes in the exchange rate. Finally, the model matches well the autocorrelation of interest rates, although it generates a lower volatility than in the data. We find that both financial and trade shocks contribute similarly to these moments.

6.3 International Business Cycle Moments

Finally, we show that the dynamic trade model is consistent with the standard international business cycle moments. We report these results in the Panel D of Table 3. Overall, our results suggest that our baseline model can accounts for the real and financial puzzles and the comovement of the RER and net trade at high and low frequency, without compromising the business cycle moments.

The only moment that is significantly off is the correlation between the terms of trade and the RER. However, this have been challenging to match, as also shown in Itskhoki and Mukhin (2021a). To break the link between the terms of trade and the RER we would need to add extra frictions for the pricing of firms, such as nominal rigidities and/or dollar currency pricing.

Finally, as shown in Columns 4 and 5, both financial and trade shocks contribute similarly to the matching of the international business cycle moments. Thus, there is no tension in including financial and trade shocks to the matching of the international business cycle moments.

[23] In principle trade shocks can generate negative a β_F, since they affect the net foreign asset position which, through the adjustment cost of debt generates a wedge in the UIP condition. However, we find that this channel is not quantitatively relevant.

7 Empirical Evidence of Trade Costs

In this section, we provide an external validation for our specification of trade costs. First, we use data on bilateral trade to measure these costs for different pairs of countries. Next, we estimate the elasticity of within-country trade costs and show it is consistent with the specification in our baseline model.

We measure trade costs from data as a wedge in the CES demand, common in any Armington trade model. The demand for country i goods in country j is given by:

$$X_t^{ij} = \left(\frac{e^{\xi_t^{ij}} p_t^{ij}}{P_{jt}} \right)^{-\rho} D_{jt}$$

where X_t^{ij} is bilateral trade flows from country i to j, p_t^{ij} is the price level of exports from country i to j, P_{jt} is the price level of domestic absorption in country j, D_{jt} is the domestic absorption of country j, and ρ is the elasticity of substitution. Our model assumes the same type of CES structure for the demand for differentiated goods. Moreover, it is the basic trade block for almost all studies in trade literature.

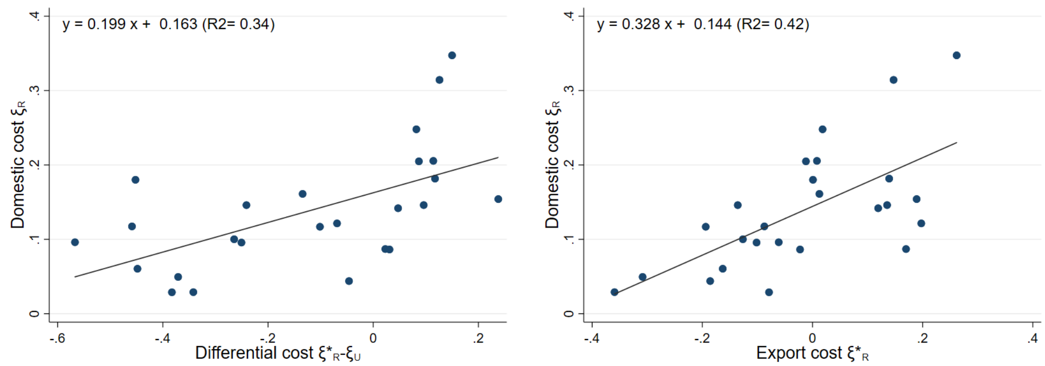
Note that all of the terms in the demand function except for ξ_t^{ij} are observables. Thus, we can recover trade costs ξ_t^{ij} as a gap between actual and predicted trade flows given prices and aggregate demand. In particular, we estimate the above demand function using the following regression

$$\log X_t^{ij} = \beta \log(P_t^{ij}/P_{jt}) + \log D_{jt} + \varepsilon_t^{ij}. \tag{7}$$

and consider the residuals ε_t^{ij} as trade costs. By estimating the demand function, we do not restrict ourselves to a particular value of elasticity. In fact, there is a broad range of values used for the elasticity in the literature, and the estimated elasticity varies greatly depending on the sample and the length of period considered. Also, the estimation by construction minimizes the size of trade costs and lets us take a conservative stance on the role of trade costs.

We estimate the demand function using data for the US and ten other countries for the ROW, as is done in our baseline quantification. For data on bilateral trade flows, we use annual data from UN Comtrade, converted into real terms using the price levels of the US dollars from Penn World Table

Figure 5: Empirical Relationship of Trade Costs



Notes: Each point represents trade costs of each year. The plots corresponds to the first and second columns of Table 4.

10.0. Domestic absorption and price levels of different countries in our sample also come from Penn World Table 10.0. Our sample period covers the period of 1994-2019, mostly due to data availability of trade flows.[24]

For the trade cost between the US and the ROW, ξ^*_{Rt} and ξ_{Ut}, we aggregate the data on the ten countries and use it as the variables for the ROW. Then we run the regression (7) for the US-ROW pair. On the other hand, for the trade cost within the ROW, ξ_{Rt}, we use bilateral data on each pair of countries in the ROW, and take average of the recovered residuals across countries to construct time series.

Given the path of trade costs, we check the relationship of ξ_{Rt} with ξ^*_{Rt} or $\xi^*_{Rt} - \xi_{Ut}$. We use these estimates to compare with the model analogue. Recall that we model the trade cost as a general version of the iceberg costs that are common in trade literature. As shown in equation 2, we allow trade costs within the ROW aggregate, ξ_{Rt}, to be nonzero. We further assume it to be $\xi_{Rt} = \tau\frac{\xi_t}{2}$, where τ measures the elasticity of the within component respect to the ROW-US trade cost. In the calibration of the baseline model, displayed in Section 4.2, we find that τ is a small positive number (0.16). Thus ξ_{Rt} is positively correlated with trade costs from ROW to the US, $\xi^*_{Rt} = \frac{\xi_t}{2}$, and also with the difference between exporting and importing costs, $\xi^*_{Rt} - \xi_{Ut} = \xi_t$.

Figure 5 shows that we do find a consistent pattern in the data. It plots the relationship of ξ_R

[24] We also check the robustness with quarterly data during the period of 2008Q1-2019Q4. We find that the path of trade costs is similar to using annual data.

(left panel) with $\xi_R^* - \xi_U$ and ξ_R^* (right panel). The estimated elasticity is between 0.144 and 0.163, very similar to the calibrated value of 0.16 in Section 4.2. As in our baseline model, ξ_R is positively correlated with with both $\xi_R^* - \xi_U$ and ξ_R^*.

Table 4: Empirical Estimates of τ

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent variable: ξ^R							
$(\xi_R^* - \xi_U)$	0.199**		0.546*		0.493***		0.443	
	(0.0581)		(0.223)		(0.100)		(0.304)	
ξ_R^*		0.328***		0.843***		0.583***		0.972**
		(0.0798)		(0.166)		(0.0627)		(0.293)
Country FE			Y	Y			Y	Y
Spending Constraints					Y	Y	Y	Y
Observations	25	25	25	25	25	25	25	25
R-squared	0.338	0.423	0.207	0.530	0.513	0.790	0.0847	0.324

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: 'Country FE' denotes the fixed effect for origin and destination countries when estimating the demand function for the pair of ROW countries. 'Spending Constraints' are a restriction on the coefficient of domestic absorption to be 1, as predicted in the model with CES demand.

Finally, table 4 displays the result with additional controls. Although the size of estimated τ differs slightly, we have the robust result that the estimated τ is positive as in our baseline model. Moreover, the coefficient of ξ_R^* is always larger than $\xi_R^* - \xi_U$, as specified in our baseline model.

8 Robustness

In this section, we show the robustness of our quantitative results to several alternative specifications. In particular, we consider an alternative specification of dynamic trade, the Armington elasticity, estimation using Bayesian methods, pricing-to-market with the Kimball aggregator, within-ROW trade costs, material inputs, and investment adjustment cost.

Specification of Dynamic Trade

To explore the robustness of our specification of dynamic trade, we consider an alternative specification in a reduced-form. In particular, we introduce adjustment costs in the use of imported inputs in the final good aggregator, as in Erceg et al. (2006), Rabanal and Rubio-Ramirez (2015) and

Gornemann et al. (2020). The details are presented in Appendix B.2.

The CES aggregator of the ROW retail sector is now given by

$$D_t = \left[Y_{Rt}^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} \left(\varphi_t Y_{Ut}\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

where φ_t captures the cost of adjusting the the use of imported inputs in the production of the final good. Its functional form is given by

$$\varphi_t = \left[1 - \frac{\iota}{2} \left(\frac{Y_{Ut}/Y_{Rt}}{Y_{Ut-1}/Y_{Rt-1}} - 1 \right)^2 \right].$$

The parameter ι determines the size of the adjustment cost. For the aggregator of the US retail sector, we assume a symmetric equation.

In this alternative specification of dynamic trade, we estimate the parameter ι, while assuming no fixed exporting cost and idiosyncratic productivity shock. We find that the alternative model result in similar targeted and untargeted moments as in the baseline. Also, the model successfully generates the frequency decomposition similar to the data (82 percent at low frequencies). More importantly, our main result carry on: trade shocks induce relatively more variation at lower frequencies, while financial shocks are more important for variation at the business cycle or higher frequencies. This is shown by the counterfactual exercise where we shut down each of the shocks. When we shut down trade shocks, the share at the low frequency reduces significantly (68 percent). On the other hand, shutting down financial shocks increases the share (97 percent).

Trade Elasticity

We now show that we can improve the fit of the long- and short-run trade elasticity by directly targeting them. To do so, we estimate the Armington elasticity, fixed exporting costs and firm idiosyncratic volatility instead of setting them exogeneously. The details are in Appendix B.3.

The Armington elasticity is a crucial parameter that determines the relationship between relative prices and net trade flows. Yet the estimates for the elasticity tend to vary, and a large range of values are used in trade literature. In our baseline model, we set the Armington elasticity exogenously with $\rho = 1.5$ as in Itskhoki and Mukhin (2021a). However, the long-run trade elasticity that is slightly lower in our baseline model than the data suggests the need for a larger Armington elasticity.

Consistent with our conjecture, the estimated Armington elasticity $\rho = 2.57$ is larger than the baseline case. With the larger Armington elasticity, we are able to generate both the long- and short-run elasticities that are closer to data. Also, the frequency decomposition is similar to data and the baseline model, with the largest share being at the low frequency. However, this calibration would imply lower turnover rate compared to our data. Given the estimated parameters, the entry rate is 0.04%, and the exit rate is 0.1%, whereas in our data the rates are 3.5%, 1.1%, respectively.

Bayesian Estimation

We now show that estimating our baseline model using Bayesian methods delivers similar results as in our calibration strategy. The details are provided in Appendix B.1.

We use four data series to estimate the model: GDP growth of the US and the ROW, the net trade flows and the RER. Overall, we find that the estimated parameters are very similar to those obtained from our baseline model in Section 4.2.

Moreover, we find that the dynamic trade model is preferred to the static trade model. That is, the model with dynamic trade has a better fit, as shown by the log data density higher in the dynamic trade model than the static trade model. This is consistent with our results from Section 5.2 and Section 5.3, where we argue in favor of the dynamic trade model.

Finally, we consider the counterfactual path of the RER where it is driven by only one type of the shocks. It is clear that the RER under trade shocks tracks closely follow the actual path of the RER during all the time period. With only financial shocks, the RER follows a similar path up to the early 2000s, but not after that. Productivity shocks do not seem to generate a path for the RER closely related to the data. Overall, trade shocks generate a path of the RER that most closely tracks the actual data. Thus, trade shocks are crucial to capture the dynamics of the RER.[25]

Within-ROW Trade Costs

We evaluate the role of the within-ROW iceberg cost component τ. The details are displayed in

[25]This is consistent with the message in Rios-Rull et al. (2012) that argues that it is not the choice of quantitative methodology that is responsible for empirical findings, but rather the data employed in the identification. Data on net trade flows is key to the identification of parameters relevant to capture the dynamics of the RER at the whole spectrum of frequencies.

Appendix B.4.

We calibrate the model assuming that the elasticity of domestic trade costs to international costs is $\tau = 0$, and do not target the cross country correlation of domestic absorption. The model falls short in accounting for the Backus-Smith correlation, the value being 0.14 as opposed to -0.10 in the data. Thus, τ matters for accounting for this moment. Furthermore, it misses the cross country correlation of domestic absorption, which has a value of 0.00 in the model as opposed to 0.34 in the data.

On the other hand, our main result for the role of shocks holds in this alternative model with $\tau = 0$. The model generates a share of low frequency variation of 90 percent. When we shut down trade shocks the share at the low frequency reduces significantly (61 percent). On the other hand, shutting down financial shocks increases the share (97 percent). Thus, trade shocks matters more for lower frequency movements, while financial shocks are more important higher frequency variation.

Pricing to Market

We show that our main results are robust to a different specification of the pricing to market friction. In this case, we adopt the general demand structure of the Kimball aggregator (Kimball, 1995) and show that it generates similar result as our baseline case with CES demand structure and the reduced-form pricing to market. The details are presented in Appendix B.5.

In our baseline model, we incorporate the pricing-to-market by making the import demand elasticity a function of the RER. This generates time-varying markups for the imported goods, inducing deviations from the law of one price and an incomplete pass-through of exchange rates to prices. An alternative way of modeling pricing-to-market is using the Kimball aggregator as in Itskhoki and Mukhin (2021a).

We compare the results of two models that differ only in the pricing-to-market specification. For simplicity, we consider a static version of our baseline model, similar to that of Itskhoki and Mukhin (2021a). Specifically, the reduced-form pricing-to-market is the same as our baseline model but with zero fixed export costs. The alternative model is with the Kimball aggregator instead of the reduced-form specification. The two models result in similar values for the targeted and untargeted moments. Also, both models give an excess share for the low frequency movements. That is, they both generate the "excess persistence puzzle" that the share of the RER variance accounted by the low frequency movements are too high. Moreover, capturing pricing-to-market from the Kimball aggregator still generates a similar short and long run trade elasticity.

Intermediate Inputs in Production Function

We show that our results are robust to including intermediate inputs in the production function as in Itskhoki and Mukhin (2021a). The details are presented in Appendix B.6.

Since intermediate inputs of production are bundles of domestic and foreign varieties, we need to re-calibrate the home bias to match the 14% trade share over GDP. This implies that the home bias parameter is greater than before. As shown in Itskhoki and Mukhin (2021a), higher home bias reduces the response of macro aggregates to financial shocks. This affects the ability of financial shocks to generate the disconnect between the RER and macro variables.

We compare the results of two models that differ only in the input of intermediates in the production function. For simplicity, we consider a static version of our baseline model, similar to that of Itskhoki and Mukhin (2021a). The two models result in similar values for the targeted and untargeted moments. Also, both models give an excess share for the low frequency movements. That is, they both generate the "excess persistence puzzle" that the share of the RER variance accounted by the low frequency movements are too high. Furthermore, the model generate a similar short and long run trade elasticity, so this feature is robust to adding intermediate inputs in the production function.

Investment Adjustment Cost

We show the result when we consider the adjustment cost in investment instead of capital. The details are presented in Appendix B.7.

We consider a specification of adjustment cost as in Christiano et al. (2005) and calibrate the parameters in the same way as in the baseline case. The calibrated parameters and the result of the model are very similar to the baseline case, including the volatility of investment. Thus, our result is robust to the specification of adjustment cost related to the capital accumulation.

9 Conclusion

We offer a unified treatment of the dynamics of the RER at all frequencies by introducing a model with heterogeneous firms facing sunk costs of exporting, financial shocks and trade shocks. The model successfully accounts for the comovement of the RER and net trade flows. At the same time, it can generate the disconnect of the RER with other macro fundamentals and is consistent with business cycle moments.

We find, as emphasized in the recent literature, that financial shocks are important for explaining the RER fluctuations at the business cycle frequency, especially for capturing the financial disconnect. Only under financial shocks, interest rate differentials predict appreciations in the RER with low predictive power, as observed in the data. More importantly, we find that trade shocks are crucial to capture the low frequency variation in the RER. Since 83 percent of its unconditional variance is attributed to the low-frequency movements, trade shocks are key to account for the overall dynamics of the RER.

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Appendix

A Data Description

In this section, we describe the source of data and how we construct the variables for our calibration.

- Period: 1980Q1 - 2019Q4, quarterly
- ROW: Trade-weighted average of 10 Countries
  - Countries: Canada, Finland, Germany, Ireland, Italy, Japan, Republic of Korea, Spain, Sweden, United Kingdom. These countries account for 60 percent of total US trade.
  - Weights: Country-specific average of the sample period (Federal Reserve). While the weights are updated every year, we use the constant weights using country-specific average during our sample period. For countries in Euro Area after 1999, we allocate the weights for the total of Euro Area into these countries using the average distribution within Euro Area during 1980-1999.
  - We check the robustness with different sets of countries, such as China, and using GDP weights instead of trade weights. The moments we consider are similar across these variations.
- US interest rate: Effective federal funds rate (IMF), deflated with consumer price index (OECD)
- ROW interest rate: Money market rates, deflated with consumer price index (OECD)
  - For most countries, money market rates are retrieved from IMF
  - Germany and China: Immediate call money/interbank rate from OECD
  - Canada: Short term interest rate from OECD
  - Japan: Overnight call rate from Bank of Japan
  - Figure D.D1 in the appendix shows that the interest rate data from these sources align very well with the money market rate from IMF.
- Quarterly National accounts (OECD)

- US dollars, volume estimates, fixed PPPs, seasonally adjusted
- Y: Gross domestic product - expenditure approach
- C: Private final consumption expenditure
- I: Gross fixed capital formation
- X: Exports of goods and services
- M: Imports goods and services

- Real exchange rate: Effective exchange rate, Real, Narrow indices, 2010=100 (BIS)
- Terms of trade: Terms of trade index (BEA, retrieved from FRED)
- US exporter characteristics (Alessandria and Choi 2021)

B Robustness

We consider alternative specifications to check the robustness of our main result: Bayesian estimation method, dynamic trade specification, Armington elasticity, and Kimball aggregator.

B.1 Bayesian Estimation

In this section, we explore an alternative estimation strategy to identify the shocks driving the RER, in particular the Bayesian methods. First, we show that we obtain similar estimated of parameters than under our Baseline model in Section 4.2. Second, we show that the model with dynamic trade is preferred to that of static trade. Finally, we show that trade shocks are crucial for generating the dynamics of the RER. That is, the counterfactual RER under trade shocks is closer to the RER in the data than under the financial shock. We also present the estimated path of the different shocks and compute the conditional variance decomposition of the RER.

Estimated Parameters

We estimate the parameters that we internally calibrate in the baseline case. In particular, we estimate the volatility of the productivity shocks, σ_c, σ_d, financial shock parameters, ρ_ϕ, σ_ϕ, trade shocks parameters, ρ_{ξ^d}, σ_{ξ^d}, as well as the trade cost parameter τ and the adjustment costs parameters χ and κ.

We use four data series as an observation: GDP growth of the US and the ROW, the net trade flows and the RER. The left panel of Table B2 reports the prior distribution and posterior mean. As a robustness check, we also consider using growth rates of consumption instead of the growth rates of GDP. This is to check whether providing observable related to the Backus-Smith correlation affect our results.

To compare the estimated values with our baseline case, we take the posterior mean and calculate the relative volatility. Table B1 presents the results. We find that the estimated parameters are similar to what our baseline calibration in Section 4.2. Moreover, we get similar results from the estimations using output and using consumption.

Table B1: Dynamic Trade: Bayesian Estimation and Baseline Calibration

Parameter		Baseline	Bayesian Estimation	
			Using Δy	Using Δc
Financial shock, volatility	σ_ψ/σ_{a_c}	0.57	0.24	0.12
Financial shock, persistence	ρ_ψ	0.99	0.96	0.97
Trade shock, volatility	σ_ξ/σ_{a_c}	17.01	13.85	11.84
Trade shock, persistence	ρ_ξ	0.98	0.99	0.99
Trade shock, within-country share	τ	0.17	0.08	0.15
Productivity differentials, volatility	$\sigma_{a_d}/\sigma_{a_c}$	1.24	1.31	1.41
Adjustment cost of portfolios	χ	0.06	0.0004	0.0007
Adjustment cost of capital	κ	1.59	1.06	3.21

Dynamic vs Static Trade

To show that dynamic trade model is preferred to the static mode, we estimate the static model with no fixed cost of exporting. We use the same priors as the before. The result of the static case is presented in the right panel of Table B2.

We find that the log data density (Laplace Approximation) in the dynamic trade model is 1833.71, while in the static model it is 1763.76, so that the dynamic trade model is preferred over the static trade model by a Bayes factor of $exp(69.95)$.[26] The value of $exp(69.95)$ provides very strong evidence supporting the hypothesis that the dynamic trade model is a better statistical model than the static trade one. This is consistent with our results from Section 5.2 and Section 5.3, where we argue in favor of the dynamic trade model.

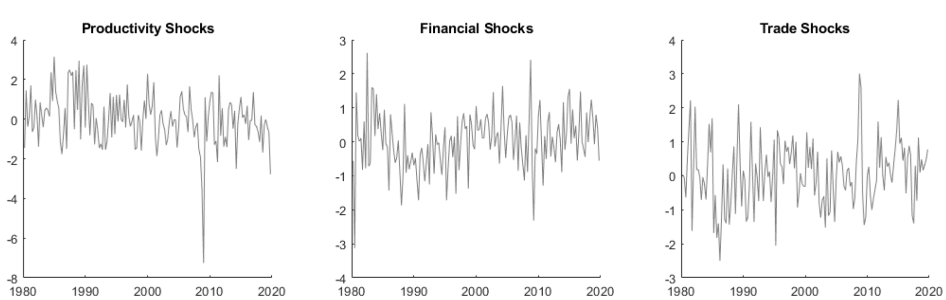
Table B2: Estimated Parameters

		Dynamic Trade		Static Trade	
	Prior Distribution	Posterior Mean	90% Interval	Posterior Mean	90% Interval
ρ_ψ	Uniform [0.9,0.999]	0.96	(0.9597 , 0.9602)	0.97	(0.9662 , 0.9691)
ρ_{ξ_d}	Uniform [0.9,0.999]	0.99	(0.9982 , 0.9986)	0.99	(0.9984 , 0.9990)
σ_c	Uniform (0.005,10)	0.54	(0.5179 , 0.5727)	1.83	(1.7302 , 1.9371)
σ_d	Uniform (0.005,10)	0.71	(0.6542 , 0.7748)	2.98	(2.6635 , 3.3097)
σ_ψ	Uniform (0.01,5)	0.13	(0.1297 , 0.1423)	0.26	(0.2243 , 0.2895)
σ_{ξ_d}	Uniform (0.05,35)	7.48	(7.3708 , 7.5691)	33.82	(33.3357 , 34.2707)
τ	Uniform (-0.5, 0.5)	0.08	(0.0789 , 0.0847)	0.14	(0.1275 , 0.1562)
χ	Uniform [0.00001,1]	0.001	(0.0004 , 0.0023)	0.003	(0.0008 , 0.0055)
κ	Uniform [1,20]	1.44	(1.0692 , 1.8439)	0.21	(0.0507 , 0.3944)
Log data density		1833.71		1763.76	

Estimated Shocks and Counterfactual RER

Figure B1 shows the estimated path of productivity shock of the ROW, trade shocks, and financial shocks. In Figure B2, we show the path of the RER in the data, as well as the counterfactual where the RER is driven by only one of the shocks.[27] It is clear that the RER under trade shocks closely

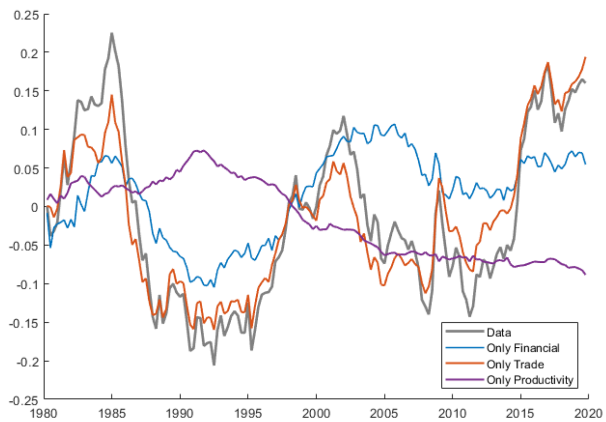
Figure B1: Estimated Shocks



[26]The Bayes factor is similar to a likelihood-ratio test.
[27]The productivity shocks include both the differential and common component.

tracks the actual RER during the whole sample period. With only financial shocks, the RER follows a similar path up to the early 2000s, but not after that. Productivity shocks do not seem to generate a path for the RER that closely related to the data. Overall, we conclude that trade shocks generate a dynamics of the RER that more closely track the actual data.

Figure B2: RER Dynamics Under Different Shocks



Finally, in Table B3 we provide the conditional variance decomposition obtained from the Bayesian estimation of the dynamic trade model. In particular, we compute the share of the h−quarter ahead error forecast variance of the RER explained by each shock. It is clear that the trade shock explains most of the forecast error variance of the RER in the long-run (i.e. low frequency), while the financial shock is important for the short-run (i.e. high frequency) fluctuations.

Table B3: 1. Contribution to h−Quarter ahead FEV of the RER (%)

	h= 1	8	32	80
ξ: Trade Shock	65.33	74.50	86.66	90.81
ψ: Financial Shock	32.31	22.08	8.23	5.08
a: Productivity Shock	2.35	3.43	5.11	4.11

B.2 Dynamic Trade Specification

In this section, we consider the final good aggregator with adjustment costs in the use of imported inputs, as in Erceg et al. (2006), Rabanal and Rubio-Ramirez (2015) and Gornemann et al. (2020). In specific, the CES aggregator of the retail sector in each country is now given by

$$D_t = \left[Y_{Rt}^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} \left(\varphi_t Y_{Ut}\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \qquad D_t^* = \left[{Y_{Ut}^*}^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} \left(\varphi_t^* Y_{Rt}^*\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

where φ_t and φ_t^* capture the cost of adjusting the the use of imported inputs in the production of the final good. Their functional forms are given by

$$\varphi_t = \left[1 - \frac{\iota}{2} \left(\frac{Y_{Ut}/Y_{Rt}}{Y_{Ut-1}/Y_{Rt-1}} - 1 \right)^2 \right] \qquad \varphi_t^* = \left[1 - \frac{\iota}{2} \left(\frac{Y_{Rt}^*/Y_{Ut}^*}{Y_{Rt-1}^*/Y_{Ut-1}^*} - 1 \right)^2 \right]$$

The parameter ι determines the size of the adjustment cost. For the aggregator of the US retail sector, we assume a symmetric equation.

As we have the alternative source of dynamic trade, we assume that the fixed costs of exporting are zero and that intermediate firms are not subject to idiosyncratic shocks. To estimate the new parameter ι, we use the short-run elasticity of trade to prices. That is, we add the short-run elasticity of trade to prices to the set of jointly targeted moments.

The parameters and their calibrated values are presented in Table B4 under 'Input Adj.' Note that the calibrated value of the input adjustment cost parameter ι is 9.67, very close to the value of 10 that is used in Gornemann et al. (2020).

In Table B5 the column 'Input Adj,' we show targeted and untargeted moments of the alternative model. The model is successful in matching all of the targeted moments, as in the baseline case. The untargeted moments are also similar to the data. That is, the long-run elasticity is much larger than the short-run elasticity,[28] and the frequency decomposition is similar to the data and the baseline case.

In fact, the alternative model performs slightly better than our baseline in terms of the moment

[28] Rabanal and Rubio-Ramirez (2015) also shows incorporating trade dynamics using input adjustment cost is crucial for the long-run elasticity that is larger than the short-run elasticity. They measure the trade elasticity based on the impulse response function to a TFP shock over different time horizons.

Table B4: Robustness – Calibrated Parameters

Parameters	Baseline	Input Adj	TE	Static PTM	Kimball	$\tau = 0$	Interm	Inv Adj
Financial shock, volatility σ_ψ/σ_{a_c}	0.57	0.86	0.65	0.30	0.62	1.50	0.66	1.32
Financial shock, persistence ρ_ψ	0.99	0.82	0.98	0.94	0.87	0.77	0.87	0.85
Trade shock, volatility σ_ξ/σ_{a_c}	17.01	3.34	3.58	38.35	14.10	10.84	38.97	25.83
Trade shock, persistence ρ_ξ	0.98	0.99	0.97	0.98	0.97	0.97	0.97	0.96
Trade shock, within-country share τ	0.17	0.17	0.56	0.07	0.12	$0^‡$	0.04	0.18
Productivity differentials, volatility $\sigma_{a_d}/\sigma_{a_c}$	1.24	1.29	1.22	1.26	0.08	1.19	1.33	1.22
Adjustment cost of portfolios χ	0.06	7e-04	0.53	0.02	0.01	0.001	0.50	0.01
Adjustment cost of capital κ	1.59	14.47	3.75	11.97	10.28	14.01	14.55	0.33
Import adjustment cost ι	$0^‡$	9.67	$0^‡$	$0^‡$	$0^‡$	$0^‡$	$0^‡$	$0^‡$
Armington elasticity ρ	$1.5^‡$	$1.5^‡$	2.57	$1.50^‡$	$1.50^‡$	$1.5^‡$	$1.50^‡$	$1.50^‡$
Fixed cost of new exporters f_0	0.07	$0^‡$	0.05	$0^‡$	$0^‡$	0.07	$0^‡$	0.07
Fixed cost of incumbent exporters f_1	0.04	$0^‡$	0.03	$0^‡$	$0^‡$	0.04	$0^‡$	0.04
Volatility of idiosyncratic productivity σ_μ	0.08	$0^‡$	0.02	$0^‡$	$0^‡$	0.08	$0^‡$	0.08
Pricing to market parameter ζ	1.00	1.00	1.00	1.00	$0^‡$	1.00	1.00	1.00
Kimball elasticity ν	-	-	-	-	0.40	-	-	-

Notes: Superscript ‡ denotes that the parameter is exogeneously set. 'Baseline' shows the same results presented in Secton 5. 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section B.2). 'TE' shows the result when we estimate the Armington elasticity (Section B.3). 'Static PTM' is the static model with the reduced-form pricing to market, and 'Kimball' is the alternative static model with Kimball aggregator (Section B.5). '$\tau = 0$' is the case with no within-ROW trade cost shocks (Section B.4). 'Interm' is the static model with intermediate inputs. 'Inv Adj' is the case with investment adjustment cost (Section B.7).

matching. This arises from the fact that the reduced form dynamic trade model is not constrained by, and thus inconsistent with, the dynamics at the exporter level. For example, the alternative model does not have an extensive margin of firm exports, and all firms participate in exporting. We are able to generate the short-run and long-run elasticity closer to data within the framework of our baseline model by considering different values of exporter parameters, which is shown in Appendix B.3.

Finally, we show that the alternative model have same implications for the role of different shocks as our baseline model. We present the spectrum decomposition when we shut down each shock in Table B6. When we shut down trade shocks, the share reduces significantly (68 percent). On the other hand, shutting down financial shocks increases the share (97 percent). Thus, our main result carries on: trade shocks induce relatively more variation at lower frequencies, while financial

Table B5: Robustness – Moments

Moments	Data	Baseline	Input Adj	TE	Static PTM	Kimball	$\tau = 0$	Interm	Inv Adj
$\rho(\Delta c - \Delta c^*, \Delta q)$	-0.10	-0.11	-0.10	-0.03	-0.06	-0.14	0.14	-0.10	-0.11
$\rho(i - i^*)$	0.87	0.88	0.86	0.87	0.90	0.96	0.81	0.90	0.90
$\rho(\Delta y, \Delta y^*)$	0.40	0.39	0.40	0.36	0.35	0.39	0.43	0.40	0.40
$\rho(\Delta d, \Delta d^*)$	0.34	0.34	0.28	0.38	0.39	0.40	$0.00^\dagger$	0.34	0.34
$\rho(xm)$	0.98	0.93	0.96	0.93	0.94	0.98	0.93	0.94	0.91
$\sigma(\Delta inv^*)/\sigma(\Delta y^*)$	2.59	2.60	2.59	2.60	2.60	2.64	2.62	2.59	2.59
$\rho(\Delta xm, \Delta q)$	0.30	0.29	0.30	0.31	0.32	0.27	0.49	0.30	0.29
$\sigma(xm)/\sigma(q)$	1.12	1.12	1.12	1.14	1.13	1.12	1.14	1.13	1.13
ρ_{SR}	0.20	$0.35^\dagger$	0.17	0.19	$0.59^\dagger$	$0.34^\dagger$	$0.89^\dagger$	$0.44^\dagger$	$0.36^\dagger$
ρ_{LR}	1.16	$0.80^\dagger$	$1.14^\dagger$	1.13	$0.55^\dagger$	$0.59^\dagger$	$1.36^\dagger$	$0.56^\dagger$	$0.63^\dagger$
High freq share	0.02	$0.03^\dagger$	$0.06^\dagger$	$0.07^\dagger$	$0.02^\dagger$	$0.02^\dagger$	$0.04^\dagger$	$0.02^\dagger$	$0.03^\dagger$
BC freq share	0.15	$0.10^\dagger$	$0.12^\dagger$	$0.16^\dagger$	$0.04^\dagger$	$0.01^\dagger$	$0.06^\dagger$	$0.04^\dagger$	$0.09^\dagger$
Low freq share	0.83	$0.87^\dagger$	$0.82^\dagger$	$0.77^\dagger$	$0.94^\dagger$	$0.97^\dagger$	$0.90^\dagger$	$0.94^\dagger$	$0.88^\dagger$

Notes: Superscript † denotes that the moment is not targeted during the calibration procedure. 'Baseline' shows the same results presented in Secton 5. 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section B.2). 'TE' shows the result when we estimate the Armington elasticity (Section B.3). 'Static PTM' is the static model with the reduced-form pricing to market, and 'Kimball' is the alternative static model with Kimball aggregator (Section B.5). '$\tau = 0$' is the case with no within-ROW trade cost shocks (Section B.4). 'Interm' is the static model with intermediate inputs. 'Inv Adj' is the case with investment adjustment cost (Section B.7).

shocks are more important for variation at the business cycle or higher frequencies.

B.3 Trade Elasticity

In this section, we calibrate the Armington elasticity and the parameters related to the exporter dynamics in order to generate the long- and short-run elasticity closer to data.

In our baseline model, the long-run trade elasticity is slightly lower than the data, which suggests the need for a larger Armington elasticity. However, altering the size of the Armington elasticity would require different values for the parameters related to the exporter dynamics, because the collective behavior of individual firms also affects the speed of aggregate trade adjustments.

We let the Armington elasticity ρ, exporter fixed costs f_0, f_1, and idiosyncratic volatility σ_μ to be jointly estimated with along with the other internally-calibrated parameters. The result of this

Table B6: Frequency Decomposition – Dynamic Trade Specifications

	Data	Baseline	Reduced-Form Dynamic Trade			
			All Shocks	No Trade Shock	No Fin Shock	No Prod Shock
Low frequency	0.83	0.87	0.82	0.68	0.97	0.80
BC frequency	0.15	0.10	0.12	0.21	0.02	0.13
High frequency	0.02	0.03	0.06	0.11	0.01	0.07

exercise is presented in Tables B4 and B5, under the column 'TE.' Consistent with our conjecture, the estimated Armington elasticity $\rho = 2.57$ is larger than the baseline case (second last row of Table B4). With the larger Armington elasticity, we are able to generate the larger long-run elasticity of trade that is closer to data (second panel of Table B5). Also, the frequency decomposition is similar to data, the largest share being at the low low frequency.

However, this calibration would imply lower turnover rate compared to our data. Given the estimated parameters, the entry rate is 0.04%, and the exit rate is 0.1%, whereas in our data the rates are 3.5%, 1.1%, respectively.

B.4 Within-ROW Trade Costs

In this section, we evaluate the role of the within-ROW trade cost τ. We set up an alternative model where the elasticity of domestic trade costs to international costs is $\tau = 0$. Then, we calibrate the model by targeting the same moments as in the baseline case, except the cross country correlation of domestic absorption.

The calibrated parameters and resulting moments are reported in Tables B4 and B5 under '$\tau = 0$.' The model falls short in accounting for the Backus-Smith correlation: the correlation is positive (0.17) and larger than the negative correlation in the data (-0.10) . Thus, τ matters for accounting for the Backus-Smith puzzle. Furthermore, it misses the cross country correlation of domestic absorption (0.01 in the model, 0.34 in the data).

In Table B7 we present the results related to the frequency decomposition of the RER. The model generates a share of low frequency variation of 90 percent. When we shut down trade shocks the share at the low frequency reduces significantly (60 percent). On the other hand, shutting down financial shocks increases the share (96 percent). Thus, trade shocks matters for lower frequency movements, while financial shocks are more important higher frequencu fluctuations. Therefore, our main result for the role of shocks holds in this alternative model with $\tau = 0$.

Table B7: Frequency Decomposition – Within ROW Trade Cost Specifications

	Data	Baseline	No Within ROW Trade Cost ($\tau = 0$)			
			All Shocks	No Trade Shock	No Fin Shock	No Prod Shock
Low frequency	0.83	0.87	0.90	0.61	0.97	0.90
BC frequency	0.15	0.10	0.06	0.24	0.02	0.06
High frequency	0.02	0.03	0.04	0.15	0.01	0.04

B.5 Pricing to Market

In this section, we consider an alternative preference for the final good, using the Kimball aggregator (Kimball, 1995). We use a basic static model to show that Kimball aggregator generates similar mechanism as in our reduced-form pricing to market specification.

The Kimball aggregator for the final good production is given by

$$\int_0^1 \left[g\left(\frac{Y_{Rt}}{D_t}\right) + \gamma g\left(\frac{Y_{Rt}}{D_t}\right) \right] di = 1$$

where $g(\cdot)$ is the aggregator function with $g' > 0, g'' < 0, and g''(1) = 0, 1$ and $g(1) = g'(1) = 1$. With this aggregator, the demand function of ROW for the ROW and US composite goods are given by

$$Y_{Rt} = h\left(\frac{P_{Rt}}{P_t}\right) D_t \qquad Y_{Ut} = \gamma h\left(\frac{P_{Ut}}{P_t}\right) D_t.$$

where $h(\cdot) = g'^{-1}(\cdot)$ and satisfies $h(1) = 1, h' < 0$.

For the functional form of the demand structure, we consider the demand schedule developed by Klenow and Willis (2016) and Gopinath and Itskhoki (2010). In specific,

$$h(x) = (1 - \epsilon log(x))^{\upsilon/\epsilon}$$

where $\upsilon = -h'(1) > 1$ and $\epsilon > 0$ are elasticity parameters. As $\epsilon \longrightarrow 0$, the demand structure converges to the CES demand. The elasticity of $\upsilon = 0.4$ and $\epsilon = 0.33$ implies the exchange rate passthrough of 60%, as in our baseline case.

We calibrate a static model using same targets as in the baseline case. The calibration and results of the static models using the reduced-form pricing to market and the Kimball aggregators are presented in Tables B5 and B4 under 'Static PTM' and 'Kimball' respectively. As in the reduced-form case, the model with the Kimball aggregator can match the targeted moments. Also, the untargeted moments are similar to the static model with baseline pricing-to-market structure. In particular, it generates the "excess persistence puzzle" that the share of the RER variance accounted by the low frequency movements are too high. Thus, using the Kimball aggregator is comparable to the case with CES aggregator and reduced-form pricing to market specification.

B.6 Intermediate Inputs in Production Function

In this section, we consider intermediate inputs in the production function. We use a basic static model to show that it generates similar results as in the static model case without intermediate inputs of production.

The production function is now

$$Y_t = \left(e^{a_t} K_t^{\alpha} L_t^{1-\alpha}\right)^{1-\phi} X_t^{\phi}$$

where α is the elasticity of the value added with respect to capital and ϕ is the elasticity of output with respect to intermediates, which determines the equilibrium expenditure share on intermediate goods. Following Itskhoki and Mukhin (2021a), we set ϕ to be equal to 0.5. This implies that we need to re-calibrate the home bias parameter γ so that trade over GDP in the steady state equals 0.14.

We calibrate the static model using same targets as in the baseline case, given the values of ϕ and γ. The result of the static model using intermediate inputs are reported in Table B5, under 'Interm.' The model with intermediate inputs can match the targeted moments. Also, the untargeted moments are similar to the static model without intermediate inputs. It generates the "excess persistence puzzle" that the share of the RER variance accounted by the low frequency movements are too high, as well as a similar short and long run trade elasticity. Thus, our results are robust to adding intermediate inputs in the production function.

B.7 Capital Adjustment Cost

In this section, we consider an adjustment cost in investment as in Christiano et al. (2005). That is, the law of motion for capital is now given by

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where $S(1) = S'(1) = 0$ and $S''(1) > 0$. Here, we consider the functional form of S as

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\tilde{\kappa}}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

To estimate the adjustment cost parameter $\tilde{\kappa}$, we again use the volatility of investment. That is, the targeted moments remain unchanged. The result of the estimated model with the new investment adjustment cost is presented in Tables B4 and B5, under 'Inv Adj.' The estimated parameter for the adjustment cost is smaller than in the baseline case, because the investment is more volatile than capital. The targeted moments are very close to both data and baseline model, including the volatility of investment. Also, untargeted moments are also similar. Long-run elasticity is larger than the short-run elasticity, and we get a very similar shape of spectrum as in the baseline case. Thus, our results are robust this alternative specification of investment adjustment cost.

C Theoretical Decomposition of Net Trade

In this section, we provide the derivation of net trade in our baseline model. For simplicity, we omit the time subscript t.

The demand function for aggregate exports of ROW is given by

$$Y_R^* = \gamma \left(\frac{P_R^*}{P^*} \right)^{-\rho} D^*$$

where $P^* = 1$. The demand faced by a producer of each variety i is

$$y_{Ri}^* = \left(\frac{\xi_R^* p_{Ri}^*}{P_R^*} \right)^{-\theta} Y_R^* = \gamma \left(\frac{\xi_R^* p_{Ri}^*}{P_R^*} \right)^{-\theta} \left(\frac{P_R^*}{P} \right)^{-\rho} D^*$$

where the second equality uses the aggregate demand function. Using that total sales is a sum of sales of all varieties,

$$\begin{aligned} P_R^* \; Y_R^* &= \int p_{Ri}^* \; y_{Ri}^* di = \int \gamma \xi_R^{*\,-\theta} p_{Ri}^{*\,1-\theta} P_R^{*\,\theta-\rho} D^* \; di \\ &= \gamma \xi_R^{*\,-\theta} \; P_R^{*\,1-\rho} \; D^*. \end{aligned}$$

Aggregate exports and imports in nominal terms are given by

$$\begin{aligned} X^N &= q \int_{i \in \mathcal{E}} p_{Ri}^* \; (e^{\xi_R^*} \; y_{Ri}^*) di = q \xi_R^* P_R^* Y_R^* = \gamma \; q \; e^{\xi_R^*(1-\theta^*)} P_R^{*\,(1-\rho)} D^* \\ M^N &= \int_{i \in \mathcal{E}^*} p_{Ui} \; (e^{\xi_U} \; y_{Ui}) di = \gamma \; e^{\xi_U (1-\theta)} P^{UR(1-\rho)} D \end{aligned}$$

and the export and import prices are

$$\begin{aligned} Px &= q \left(\frac{1}{N} \int_{i \in \mathcal{E}} p_{Ri}^{*\,1-\theta^*} di \right)^{\frac{1}{1-\theta^*}} = q \; P_R^* \; N^{\frac{-1}{1-\theta^*}} \\ Pm &= \left(\frac{1}{N^*} \int_{i \in \mathcal{E}^*} p_{Ui}^{1-\theta} di \right)^{\frac{1}{1-\theta}} = P_U \; N^{*\,\frac{-1}{1-\theta}} \end{aligned}$$

where N denotes the mass of exporters. In logs,

$$\begin{aligned}
x^N &= \log \gamma + (1-\theta^*)\xi_R^* + (1-\rho)p_R^* + d^* + q \\
m^N &= \log \gamma + (1-\theta)\xi_U + (1-\rho)p + d \\
px &= q + p_R^* + \frac{1}{1-\theta^*} n \\
pm &= p_U + \frac{1}{1-\theta} n^*
\end{aligned}$$

where lower case letters denote variables in logs.

Using that in real terms real exports and real imports are $X = X^N/Px, M = M^N/Pm$, respectively, log of exports and imports are given by

$$\begin{aligned}
x = x^N - px &= \log \gamma + (1-\theta^*)\xi_R^* - \rho p_R^* + d^* - \frac{1}{1-\theta^*} n \\
m = m^N - pm &= \log \gamma + (1-\theta)\xi_U - \rho p_U + d - \frac{1}{1-\theta} n^*.
\end{aligned}$$

Therefore, log of Export-Import ratio $xm = \log XM$ is

$$\begin{aligned}
xm &= \rho(p_U - p_R^*) + (d^* - d) + ((1-\theta^*)\xi_R^* - (1-\theta)\xi_U) + \left(\frac{1}{1-\theta} n^* - \frac{1}{1-\theta^*} n\right) \\
&= \rho\,(tot_t + q_t) + (d^* - d) + ((1-\theta^*)\xi_R^* - (1-\theta)\xi_U) + (1-\rho)\left(\frac{1}{1-\theta} n^* - \frac{1}{1-\theta^*} n\right).
\end{aligned}$$

where $tot_t = pm - px$ is the terms of trade.

Comparing with Equation 4, the we have additional terms $((1-\theta^*)\xi_R^* - (1-\theta)\xi_U)$ and $(1-\rho)\left(\frac{1}{1-\theta} n^* - \frac{1}{1-\theta^*} n\right)$. These reflect that in our model we have two features, trade shocks and trade dynamics.

D Additional Graphs and Tables

Figure D1: Data Source Comparison

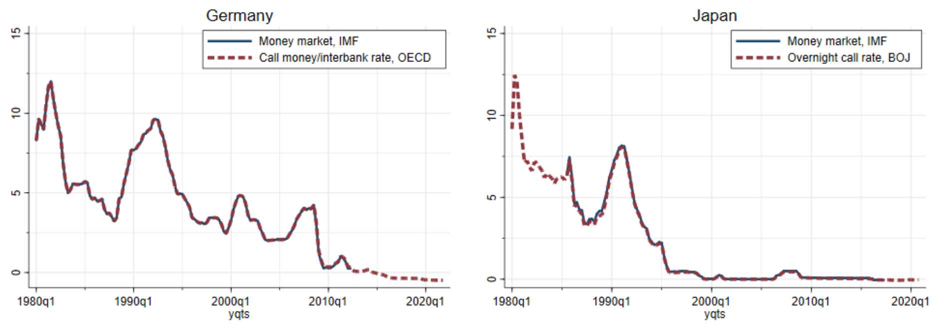


Figure D2: IRF of RER with and without Dynamic Trade

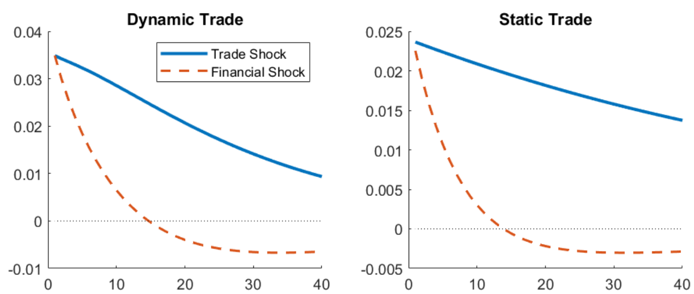


Table D1: Fama Regression in Data

Moments	Nominal	Real
β_{Fama}	-1.17	-0.53
	(0.60)	(0.23)
R^2	0.08	0.03

Notes: 'Nominal' denotes the results of using nominal data for the Fama regression, $\mathbb{E}_t[\Delta e_{t+1}] = \alpha + \beta_{Fama}(i_t^n - i_t^{n*}) + u_t$, where e is nominal exchange rate, and i^n is the nominal interest rate. Here we use realized values for the expected variables. 'Real' denotes the result of using real data for the regression (6).