Real Exchange Rate and Net Trade Dynamics: Financial and Trade Shocks*

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This Draft: June 2023
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Abstract

This paper studies the drivers of the US real exchange rate (RER), with a particular focus on its comovement with net trade flows. We consider the entire spectrum of frequencies, as the low-frequency variation accounts for 61% of the RER's unconditional variance. We build a model with heterogeneous firms facing sunk costs of exporting, financial shocks, and trade shocks. Our model fully captures the comovement of the RER and net trade flows at all frequencies, without compromising other major moments at the business cycle frequency. While financial shocks are necessary to capture the RER movements at higher frequencies, trade shocks are essential for lower frequency variation.

JEL Classifications: E30, E44, F30, F41, F44

Keywords: Exchange Rates, Trade Balance, Trade Dynamics, International Business Cycles

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1 Introduction

The real exchange rate (RER) is a central variable in many, if not most, models of international economics.[1] For many years, the literature studying the dynamics of the RER in general equilibrium struggled to account for its behavior and lack of connection with macro fundamentals, a feature known as exchange rate disconnect (Obstfeld and Rogoff, 2000; Itskhoki and Mukhin, 2021). Nonetheless, a recent strand of literature has proposed a theory relying on shocks in financial markets that go a long way in explaining the disconnect (Devereux and Engel, 2002; Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021).

Despite this success, the literature has an important caveat: it considers variations up to business cycle frequencies (cycles shorter than 32 quarters). This approach presents two important limitations. First, it misses most of the variation in the RER, which arises at frequencies lower than business cycles.[2] Figure 1 shows the path of the US RER (dashed blue) along with their trends after applying the HP filter (solid blue). It is clear that the trend of the RER drives a large share of its fluctuations. In fact, a spectrum analysis shows that 61 percent of its unconditional variance arises from lower-frequency movements.[3] Second, it overlooks the strong comovement with net trade at lower frequencies. The trend of net trade (solid red) closely follows that of the RER (solid blue), with a lag of around 6 quarters.[4] As a consequence, while the RER and net trade exhibit a weak comovement at higher frequencies, they are highly correlated at lower frequencies.[5]

In this paper, we provide a unified framework for studying the dynamics of the RER at all frequencies. We extend the two-country model with financial shocks of Itskhoki and Mukhin (2021) by incorporating trade shocks and dynamic trade. Our model closely matches the frequency

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[1]The RER is defined as $Q_t = \mathcal{E}_t P_t^*/P_t$ where $\mathcal{E}_t$ the nominal exchange rate (the price of home currency per unit of foreign currency), P_t^* is the foreign price level, and P_t the home price level. An increase in Q_t indicates a depreciation of the home RER.

[2]Throughout the paper, whenever we refer to low or lower frequencies we specifically mean lower *than* business cycles, i.e. cycles of more than 32 quarters.

[3]The spectrum analysis measures how much of the unconditional variance is attributed to the variation in cycles of different lengths. We provide details on the estimation methodology in Appendix B. Our findings are consistent with Rabanal and Rubio-Ramirez (2015), who find that 77 percent of the US RER variance is from the low-frequency.

[4]We use the export-import ratio as a measure of net trade, as opposed to trade balance as a share of GDP, because the former gives net trade controlling for the scale of trade. The trade balance as a share of GDP can be written as $\frac{(X-M)}{Y} = \frac{(X-M)}{(X+M)} \times \frac{(X+M)}{Y}$, where $\frac{(X-M)}{(X+M)}$, is approximated by $0.5 \log X/M$ using the first-order Taylor approximation. We use $\log X/M$ to measure net trade flows, addressing the concern that the changes in trade balance as a share of GDP are primarily due to the changes in the scale of trade (Alessandria and Choi, 2021; Alessandria, Bai and Woo, 2022).

[5]The delayed effect captures the so called J-curve, which has been documented in the trade literature (Baldwin and Krugman, 1989; Rose and Yellen, 1989; Backus, Kehoe and Kydland, 1994; Fitzgerald, Yedid-Levi and Haller, 2019).

Figure 1: Real Exchange Rate and Net Trade
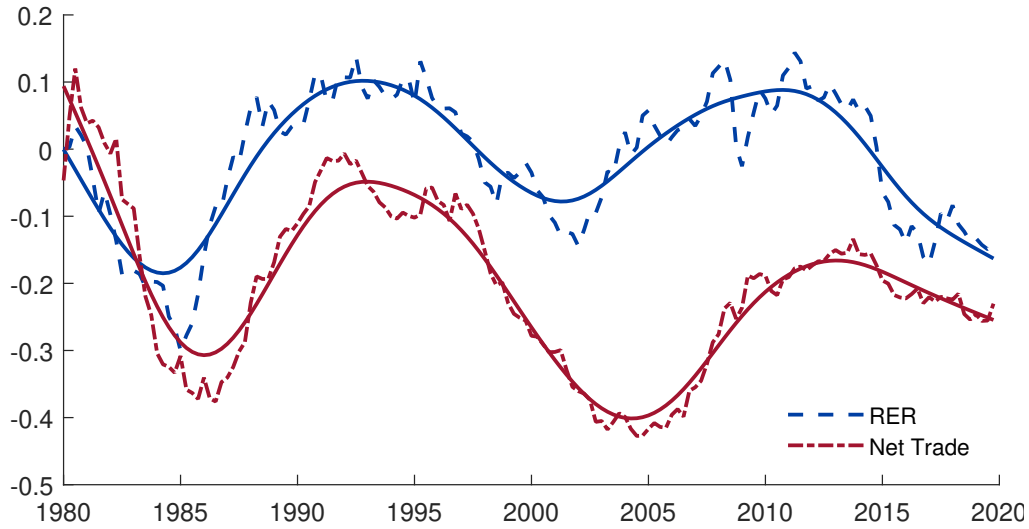
Notes: RER is the log of the quarterly real exchange rates of the United States. Normalized with 1980q1=0. Effective exchange rate indices, Real, Narrow (BIS). Net trade is the log of Exports to Imports ratio for the United States. Exports and Imports are from Quarterly National Accounts (OECD). Solid lines plot the trend component of each variable from the Hodrick–Prescott filter with a smoothing parameter of 1600.

decomposition of the RER variance observed in the data. Moreover, it effectively captures the differential comovement between the RER and net trade at different frequencies. At the same time, our model successfully accounts for the exchange rate disconnect with macro variables. Omitting any of the key features - financial shocks, trade shocks, or dynamic trade - leads to an inability of simultaneously accounting for these empirical patterns.

We model trade shocks as stochastic iceberg trade costs, providing a tractable representation of the trade barriers. This shock encompasses variations such as changes in tariffs and non-tariff barriers (Obstfeld and Rogoff, 2000; Delpeuch, Fize and Martin, 2021), shipping technologies and the global market structure of trade (Burstein, Neves and Rebelo, 2003; Corsetti and Dedola, 2005; Corsetti, 2016), and fiscal policy (Monacelli and Perotti, 2010; Bluedorn and Leigh, 2011; Bussière, Fratzscher and Müller, 2010). Trade shocks affect the relative cost of shipping goods between countries, represented in our model by the US and the rest of the world (ROW).[6] Additionally, we allow for non-zero trade costs within the ROW to better capture the evolution of trade barriers within the ROW, which contributes to the ability of the model to account for the comovement of

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[6]In our quantitative exercise, the ROW aggregate includes Canada, Finland, Germany, Ireland, Italy, Japan, Republic of Korea, Spain, Sweden and United Kingdom. This set of countries represents 60% of total US trade on average. The estimated moments from the data are robust to having an unbalanced panel that includes China since 1990. For more details, see Appendix A.

expenditure between the US and the ROW.[7] This is important since the dynamics of the RER reflect changes in supply and demand of goods and assets across countries.

Incorporating trade shocks along with financial shocks enables the model to capture the weak high-frequency correlation between the RER and net trade. When financial shocks generate an excess return on bonds for the US relative to the ROW, savings in the US increase, and the excess savings is exported to the ROW (US net trade increases). At the same time, due to the fall in aggregate demand in the US, the final good price falls (the US RER depreciates). Hence, financial shocks induce a positive correlation between the RER and net trade on impact. On the other hand, a trade shock that raises the relative cost of exporting for the US leads to a decline in US net trade flows. With less intermediate goods exported and more imported, the supply of final goods in the US increases, causing its price to fall (the US RER depreciates). Consequently, trade shocks induce a negative correlation between the RER and net trade on impact, offsetting the positive effect of financial shocks.

We incorporate dynamic trade following Alessandria and Choi (2007, 2021) and assume that intermediate producers are heterogeneous in their idiosyncratic productivity, and decide whether to participate in the export market or not, subject to a fixed cost of exporting.[8] We assume that the fixed cost is lower for incumbents than for new exporters, which makes the exporting decision forward-looking. Consequently, aggregate trade flows evolve gradually over time in response to shocks, due to the slow adjustment of the distribution of exporters. This allows the model to capture the differential short- and long-run comovement between the RER and net trade. It also contributes to account for the frequency decomposition of the variance of the RER. In our benchmark model, the share of the variance of the RER attributed to the low-frequency variation is 69 percent, close to the 61 percent in the data. Absent dynamic trade, the share increases to 80 percent. This arises from dynamic trade making quantities in the short run more inelastic than under static trade. As a consequence, prices in the short run, relative to the long run, have a stronger response under dynamic trade, redistributing the share of variation in the RER from lower to higher frequencies.[9]

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[7]Using bilateral data on trade flows and prices, we find external evidence supporting our specification of trade shocks. We discuss this in detail in Appendix C.

[8]Alessandria and Choi (2007, 2021) extends the sunk cost model of exporting of Dixit (1989), Baldwin and Krugman (1989) and Das, Roberts and Tybout (2007) to a general equilibrium framework.

[9]This is consistent with the "Excess Persistence Puzzle" documented in Rabanal and Rubio-Ramirez (2015), which refers to static trade models having an excess share of low frequency variation in the RER.

Using our calibrated model, we find that trade shocks play an important role in accounting for the low frequency movements in the RER. To show this, we compute the contribution of each shock to the forecast error variance of the RER. We find that financial shocks explain 62% of the one-quarter ahead error forecast variance, with trade shocks explaining 45%. However, when focusing at the eighty-quarters ahead error forecast variance of the RER, trade shock explain 63%, while financial shocks account for 26%. Therefore, trade shocks matter more than financial shocks for the variation in the RER at lower frequencies. Since 61% of the overall variance of the RER is at lower frequencies, trade shocks are crucial for capturing its overall dynamics.

The remainder of the paper is structured as follows. Section 2 reviews the literature. Section 3 presents our benchmark model, while Section 4 discusses the calibration and identification strategy. Section 5 demonstrates the success of the benchmark model in capturing targeted and untargeted moments related to the RER dynamics at all frequencies. Section 6 studies the role of different shocks in explaining the variation of the RER. Section 7 discusses the robustness of our result to alternative specifications. Finally, Section 8 presents the concluding remarks.

2   Literature Review

Only a limited number of papers studying the dynamics of the RER in general equilibrium have focused on the low-frequency variation. Rabanal and Rubio-Ramirez (2015) show that a reduced form dynamic trade model with non-stationary cointegrated productivity shocks is able to capture the spectrum of the RER.[10] Gornemann, Guerrón-Quintana and Saffie (2020) propose an alternative mechanism relying on endogenous spillovers that amplify stationary fluctuations.[11] We share with these papers the focus on the low frequency variation of the RER and the importance of dynamic trade. We differ from them in the way we model dynamic trade, which we do with a microfoundation based on firms' dynamic exporting decisions. Moreover, our primary contribution lies in proposing an alternative mechanism to account for the low-frequency variation observed in the

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[10]Drozd, Kolbin and Nosal (2021) shows that dynamic trade is a key feature to improve the model's ability to account for the trade comovement puzzle, i.e. the significant relationship in the data between countries' business cycles synchronization and trade flows.

[11]Corsetti, Dedola and Viani (2012) also study the RER dynamics at the frequency domain through spectral analysis, but focus on the low frequency disconnect between the RER and relative consumption (Backus-Smith Puzzle). Cao, Evans and Luo (2020) study the medium to long run dynamics of the US-UK RER and highlight the role of persistent productivity shocks, incomplete financial markets and a high Armington elasticity in accounting for its dynamics.

RER. Our explanation relies on shocks to the cost of trade that induce persistent changes in net trade and international relative prices.

Our paper also offers a bridge between the studies of exchange rates in international finance and international trade. On the one hand, there is a growing literature emphasizing the role of financial shocks for understanding the dynamics of exchange rates, with a focus on the macro and financial disconnect (Devereux and Engel, 2002; Gabaix and Maggiori, 2015; Farhi and Gabaix, 2016; Itskhoki and Mukhin, 2021).[12] On the other hand, a series of papers have explored the role of trade barriers in explaining the variation in trade and financial flows across countries (Obstfeld and Rogoff, 2000; Eaton, Kortum and Neiman, 2016; Reyes-Heroles, 2016; Alessandria and Choi, 2021; Sposi, 2021; Alessandria, Bai and Woo, 2022). In our study, we enhance the depth of analysis by integrating financial shocks, trade shocks, and dynamic trade within a cohesive framework. This unified approach not only enhances our understanding of the outcomes presented in both strands of the existing literature but also deepens our comprehension of the economic dynamics at play. As emphasized in the financial literature, we find that financial shocks are important for high-frequency fluctuations of the RER and the financial disconnect. On the other hand, dynamic trade and trade shocks are crucial for accounting for low-frequency movements of the RER and its comovement with net trade.

Finally, our paper is related to the literature on the measurement of trade wedges. Levchenko, Lewis and Tesar (2010) and Fitzgerald (2012) measure trade wedges based on the Armington model to study the role of trade costs and asset market frictions for international risk sharing. Head and Mayer (2014) explore different methods of estimating the gravity equation. We contribute to this literature by considering a specification of trade costs that allows for a within-ROW component, and highlight its implications for the comovement of the RER and macro aggregates.

3   Model

We build on the two-country international business cycle model of Itskhoki and Mukhin (2021). The two countries are the ROW and the US, each producing a perfectly competitive non-traded final good. The non-traded final good is made of a mix of tradable intermediates, using a CES

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[12]While this literature discusses the dynamics of both the real and nominal exchange rates, we limit our interest to real variables only.

technology with home bias.[13] The final good can be consumed or invested by the household, and capital accumulation is subject to a capital adjustment cost.

There is a unit mass of intermediate good producers in each country, producing differentiated varieties. They are subject to aggregate productivity shocks and are heterogeneous in their idiosyncratic productivity. They make decisions on entering, staying or exiting the export market, subject to the fixed costs that depends on the experience in the export market as in Dixit (1989), Baldwin and Krugman (1989), Das et al. (2007), Alessandria and Choi (2007), and Alessandria and Choi (2021). Intermediate firms set destination specific prices, and use labor and capital as inputs of production. Optimal prices are set as a markup over the marginal cost. We introduce time-varying markups, capturing pricing to market frictions in a reduced form, which leads to persistent deviations from the law of one price. Intermediate firms also face stochastic iceberg trade costs, depicted as only a fraction of goods shipped arriving at the destination.

On the asset side, there is an internationally traded bond, denominated in dollars. The ROW household is subject to a bond adjustment cost, which induces stationarity of the model and captures portfolio re-balancing costs in a reduced form. The ROW household is also subject to a financial shock, capturing the shock to the uncover interest parity of Itskhoki and Mukhin (2021). We describe below the model from the point of view of ROW agents.

Households

The representative household in the ROW maximizes the discounted expected utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\eta} (1 - L_t)^{1-\eta} \right]^{1-\sigma}}{1 - \sigma}$$

where C_t is consumption, L_t is labor, η is the weight on consumption, β is the discount factor, and $1/\sigma$ is the intertemporal elasticity of substitution. The flow budget constraint is given by

$$P_t C_t + P_t I_t + B_{t+1} + \frac{\mathcal{E}_t B_{t+1}^*}{e^{\psi_t}} + \mathcal{E}_t \frac{\chi}{2} \left(B_{t+1}^* - \bar{B} \right)^2 \leq W_t L_t + R_t^k K_t + B_t (1 + i_{t-1}) + \mathcal{E}_t B_t^* (1 + i_{t-1}^*) + \Pi_t$$

where P_t is the consumer price index, I_t is investment, K_t is capital, B_{t+1} is the quantity of ROW

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[13]Itskhoki and Mukhin (2021) emphasizes the importance of incomplete pass-through of the financial shock mechanism, which they model using a Kimball aggregator. Even though we use a CES aggregator, we model incomplete pass-through by adding frictions in the pricing to market behavior of firms.

bonds (zero net supply), i_{t-1} is the nominal interest rate on ROW bonds purchased at $t - 1$, and Π_t is aggregate profits of intermediate firms. On the international asset side, B_{t+1}^* is the quantity of the internationally traded bonds held by the ROW household (zero net supply), i_{t-1}^* is the nominal interest rate on domestic bonds purchased at $t - 1$, and $\mathcal{E}_t$ is the nominal exchange rate (NER), measured as the price of the ROW currency per unit of US currency. The term ψ_t is the financial shock, χ is the adjustment cost of internationally traded bonds, and $\bar{B}$ is the steady state level of net foreign assets.[14]

From the log-linearized Euler equations of the ROW household for domestic and international bonds, we can derive an equation for the deviations from the uncovered interest partiy (UIP) condition,

$$i_t - i_t^* - \mathbb{E}_t \left[\Delta e_{t+1} \right] = \psi_t - \chi \cdot \left(B_{t+1}^* - \bar{B} \right) \tag{1}$$

where $\mathbb{E}_t \left[\Delta e_{t+1} \right] \equiv \mathbb{E}_t \left[\ln \mathcal{E}_{t+1} - \ln \mathcal{E}_t \right]$ is the expected changes of the NER. The financial shock generates a wedge in the UIP condition. While we model the financial shock as an exogenous shock, the derived UIP condition is equivalent to those in models with segmented financial markets, noisy traders or limits to arbitrage (Itskhoki and Mukhin, 2021; De Long, Shleifer, Summers and Waldmann, 1990; Jeanne and Rose, 2002; Gabaix and Maggiori, 2015).[15]

The stock of capital in each country follows the law of motion,

$$K_{t+1} = (1 - \delta)K_t + \left[I_t - \frac{\kappa}{2} \frac{(\Delta K_{t+1})^2}{K_t} \right],$$

where the parameter κ governs the adjustment cost of capital.

The solution of the ROW household can be characterized by the labor supply condition and the Euler equations for ROW and Foreign currency bonds and capital. The stochastic discount factor

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[14]The financial shock ψ_t only affects the ROW household, hence generating a differential return on internationally traded bonds for ROW and US households. Our result is invariant to whether the shock ψ_t affects the adjustment cost of debt or not. It is also invariant to whether the RER is part of the adjustment cost term or not.

[15]Financial shocks can also be microfounded by risk-premia (Verdelhan, 2010; Colacito and Croce, 2013; Farhi and Gabaix, 2016) or heterogeneous beliefs and expectational errors (Evans and Lyons, 2002; Gourinchas and Tornell, 2004; Bacchetta and Van Wincoop, 2006).

of the ROW household is given by

$$\Omega_t \equiv \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^{\eta}(1 - L_{t+1})^{1-\eta}}{C_t^{\eta}(1 - L_t)^{1-\eta}} \right)^{1-\sigma} \frac{C_t}{C_{t+1}} \right].$$

Aggregation Technology

A competitive retail sector combines composite goods from ROW and the US with a constant elasticity of substitution (CES) to produce the final good, D_t, which can be consumed or invested. The CES aggregator is given by

$$D_t = \left[Y_{Rt}^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} Y_{Ut}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

where Y_{Rt} is the quantity of domestic goods consumed in the ROW, Y_{Ut} is the quantity of imported goods from the US consumed in the ROW, γ captures the home bias and ρ is the Armington elasticity between domestic and imported composite goods.

The total expenditure in the retail sector is given by

$$P_t D_t = P_{Rt} Y_{Rt} + P_{Ut} Y_{Ut}$$

where P_t is the aggregate price, P_{Rt} is the price of domestic goods in the ROW and P_{Ut} is the price of imported goods in the ROW.

The problem of the retail sector is to maximize the production of final goods by choosing the quantities of composite goods $\{Y_{Rt}, Y_{Ut}\}$. The final good is used by households for either consumption or investment, so that $D_t = C_t + I_t$.

Solving this maximization problem yields the demand functions for ROW and US intermediate goods, given by

$$Y_{Ut} = \gamma \left(\frac{P_{Ut}}{P_t} \right)^{-\rho} (C_t + I_t) \quad \text{and} \quad Y_{Rt} = \left(\frac{P_{Rt}}{P_t} \right)^{-\rho} (C_t + I_t).$$

where P_t is the ideal price index in the ROW

$$P_t = \left[P_{Rt}^{1-\rho} + \gamma P_{Ut}^{1-\rho} \right]^{1/(1-\rho)}$$

Hence, the RER, Q_t, is defined as

$$Q_t = \frac{\mathcal{E}_t P_t^*}{P_t}$$

where $\mathcal{E}_t$ is the nominal exchange rate, measured as the price of the ROW currency per unit of US currency.[16]

The domestic and imported goods, Y_{Rt} and Y_{Rt}^*, are the composite of varieties produced by heterogeneous producers. The aggregators are

$$Y_{Rt} = \left(\int_0^1 y_{j,Rt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \qquad\qquad Y_{Ut} = \left(\int_{j \in \mathcal{H}_t^*} y_{j,Ut}^{\frac{\hat\theta_t-1}{\hat\theta_t}} dj \right)^{\frac{\hat\theta_t}{\hat\theta_t-1}} \tag{2}$$

where θ and $\hat\theta_t$ are the elasticity of substitution across varieties, and $\mathcal{H}_t^*$ is the set of exporting firms in the US. Firms set destination specific prices, subject to the market-specific demands.

We let the elasticity across imported varieties to be a function of the RER with $\hat\theta_t = \theta Q_t^{\zeta}$ (and $\hat\theta_t^* = \theta Q_t^{-\zeta}$ for exported varieties). The time-varying elasticity captures pricing to market frictions in a reduced form, leading to persistent deviations from the law of one price.[17] When there is a depreciation of the RER for the ROW, markups charged by US firms to ROW importers fall. This is consistent with the findings in Alessandria and Kaboski (2011), which show that firms price to income, that is, firms charge higher prices to higher income destinations. This allows the model to capture the incomplete pass-through of exchange rates to prices. Furthermore, absent this friction the terms of trade are more volatile than the RER, contrary to the data.

The price indices for the composite goods are given by

$$P_{Rt} = \left(\int_{j=0}^1 p_{j,Rt}^{1-\theta} \right)^{\frac{1}{1-\theta}} \qquad\qquad P_{Ut} = \left(\int_{j \in \mathcal{H}_t^*} p_{j,Ut}^{1-\hat\theta_t} \right)^{\frac{1}{1-\hat\theta_t}}.$$

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[16]Whenever we use lower cases we will refer to variables in logs. In particular we define the log of nominal and real exchange rate as $e \equiv \ln \mathcal{E}$ and $q \equiv \ln Q$ respectively.

[17]The pricing to market friction generates time-varying markups in a similar way as with a Kimball aggregator, as in Itskhoki and Mukhin (2021). See Edmond, Midrigan and Xu (2018) for a study of heterogeneous firm with the Kimball aggregator. On the other hand, Drozd and Nosal (2012) provide an alternative model of pricing to market where firms invest in marketing activities in order to accumulate customers.

The problem of the US retailers is given in a symmetric form

$$\max_{\{Y^*_{Ut}, Y^*_{Rt}\}} P^*_t (C^*_t + I^*_t) - [P^*_{Ut} Y^*_{Ut} + P^*_{Rt} Y^*_{Rt}]$$

subject to the CES aggregator, resulting in the demand functions of

$$Y^*_{Rt} = \gamma \left(\frac{P^*_{Rt}}{P^*_t} \right)^{-\rho} (C^*_t + I^*_t) \quad \text{and} \quad Y^*_{Ut} = \left(\frac{P^*_{Ut}}{P^*_t} \right)^{-\rho} (C^*_t + I^*_t).$$

Intermediate Firms

There is a continuum of heterogeneous firms indexed by $j \in [0, 1]$ in each country, specializing in production of a differentiated intermediate good. There is monopolistic competition among these firms. The firms are subject to aggregate and firm-specific shocks. The firm j's production function is given by

$$y_{jt} = e^{a_t + \mu_{jt}} l^{\alpha}_{jt} k^{1-\alpha}_{jt},$$

where α is the capital share of income, a_t is the productivity shock, and μ_{jt} is a idiosyncratic firm-specific shock, $\mu \overset{iid}{\sim} N(0, \sigma^2_\mu)$.

All firms sell their products in their own country, while some of them choose to export. The resource constraint of a firm is given by

$$y_{jt} = e^{\xi_{Rt}} y_{j,Rt} + m_{jt} e^{\xi^*_{Rt}} y^*_{j,Rt}$$

where $y_{j,Rt}$ is ROW variety used domestically, $y^*_{j,Rt}$ is ROW variety exported to the US, ξ_{Rt} is the stochastic iceberg cost for domestic trade within the ROW countries, ξ^*_{Rt} is the stochastic iceberg cost for ROW exports to the US, and $m_{jt} \in \{0, 1\}$ is the current export status of firm j, with $m_{jt} = 1$ being export, and $m_{jt} = 0$ not export. Note that we are considering a case of iceberg costs that allows for the iceberg trade cost within the ROW, ξ_{Rt}, to be nonzero. This takes into account that the ROW is an aggregate of multiple countries that trade with each other. In order to capture the average trade cost within the ROW countries, we relax the constraint of a standard specification with zero domestic iceberg costs.[18]

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[18]We explain in more detail the role of the within country trade cost when we present the shock processes.

In order to export, firms must pay a fixed cost, denominated in units of labor. The fixed cost for starting to export differs from the fixed cost to stay in the export market. To start exporting, a firm pays a cost of $W_t f_0$, while an incumbent exporter pays the continuation cost of $W_t f_1$, with $f_1 < f_0$. That is, there is a sunk cost associated with export participation, capturing exporter hysteresis and the slow response of aggregate exports.

Let $S = \{a, K\}$ be the aggregate state of the economy and Ω the firm's stochastic discount factor,[19] then the dynamic problem of a firm is

$$V(\mu, m_{-1}, S) = \max \; p_R y_R + m \mathcal{Q} p_R^* y_R^* - W l - R^k k - m W f_{m_{-1}} + \mathbb{E}\Omega V'(\mu', m, S')$$

subject to the ROW retailer's demand for ROW intermediates, y_{Rt}, the US Retailer's demand for ROW intermediates, y_{Rt}^*, and the resource constraint.

The static optimality conditions of the firm are given by the optimal demand for inputs and optimal pricing,

$$W_t = (1 - \alpha)\frac{y_{jt}}{l_{jt}} \quad \text{and} \quad R_t^k = \alpha\frac{y_{jt}}{k_{jt}}$$

$$p_{j,Rt} = e^{\xi_{Rt}}\frac{\theta}{\theta - 1}MC_{jt} \quad \text{and} \quad \mathcal{Q}_t p_{j,Rt}^* = e^{\xi_{Rt}^*}\frac{\theta \mathcal{Q}_t^{-\zeta}}{\theta \mathcal{Q}_t^{-\zeta} - 1}MC_{jt} \tag{3}$$

where the $MC_{jt} = \frac{1}{e^{a_t + \mu_{jt}}}\frac{(R_t^k)^\alpha (W_t)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$ is the marginal cost. Note that firms set different prices across destinations, since they face different demand elasticities at home and foreign. Moreover, note that the pricing to market friction, ζ, generate deviations from the law of one price that are proportional to the RER.[20]

Also, the fixed cost $f_{m_{-1}}$ depends on the exporting status of the previous period m_{-1}. Given the previous exporting status, the value of a firm is monotonically increasing and continuous in its productivity μ. We can solve for the threshold productivity for exporting decisions for non-

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[19]ROW intermediate firms use the ROW household stochastic discount factor.

[20]In particular, the deviations from the law of one price are given by $\frac{\ln(\mathcal{Q}_t p_{j,Rt}^*/p_{j,Rt})}{\ln \mathcal{Q}_t} \propto \frac{\zeta}{\theta-1}$. This implies an exchange rate pass-through of $\left[\frac{(\theta-1)-\zeta}{(\theta-1)} \times 100\right]$ percent.

exporters and exporters, μ_{0t} and μ_{1t} respectively. The thresholds satisfy

$$W f_{m-1} - \pi(\mu_m) = \mathbb{E}\left\{ \Omega \left[V'(\mu', 1, S') - V'(\mu', 0, S') \right] \right\}, \qquad m \in \{0, 1\}$$

where $\pi(\mu_m)$ is the static profit of a firm with idiosyncratic productivity $\mu_{jt} = \mu_{mt}$ given as

$$\pi(\mu_m) = p_R y_R + mQ p_R^* y_R^* - Wl - R^k k$$

with the prices defined in Equation 3. At this threshold, a firm is indifferent between exporting and not exporting. Hence, a firm will decide to participate in the export market only if its productivity is above the threshold. Since the fixed cost is higher for nonexporters than exporters, $f_0 > f_1$, the productivity threshold is higher for nonexporters than exporters, $\mu_0 > \mu_1$.

The forward looking decisions of the individual exporters induce the mass of exporting firms, N_t, to evolve gradually, following the law of motion:

$$N_t = N_{t-1} P\left[\mu > \mu_{1t}\right] + (1 - N_{t-1}) P\left[\mu > \mu_{0t}\right].$$

The aggregate labor and capital demands from intermediate firms are given by

$$L_t = \int_{j=0}^{1} L_{jt} + f_0 \cdot (1 - N_{t-1}) \cdot P[\mu > \mu_{0t}] + f_1 \cdot N_{t-1} \cdot P[\mu > \mu_{1t}]$$

$$K_t = \int_{j=0}^{1} K_{jt}.$$

Note that the aggregate labor demand includes the fixed cost of exporting of all firms because the costs are in terms of labor.

Shock Processes

Productivity shocks feature a common and differential component,[21]

$$\begin{bmatrix} a_t \\ a_t^* \end{bmatrix} = \begin{bmatrix} a_{ct} + a_{dt}/2 \\ a_{ct} - a_{dt}/2 \end{bmatrix}$$

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[21]Alternatively country-specific shocks can be written as a combination of these orthogonal shocks.

where the common component, a_{ct}, and the differential component, a_{dt}, each follow an AR(1) process,

$$a_{ct} = \rho_a^c a_{ct-1} + \varepsilon_{at}^c \qquad\qquad \varepsilon_{at}^c \sim N(0, \sigma_a^c)$$

$$a_{dt} = \rho_a^d a_{dt-1} + \varepsilon_{at}^d \qquad\qquad \varepsilon_{at}^d \sim N(0, \sigma_a^d).$$

We assume that the relative trade cost between ROW and US, ξ_t, follows an AR(1) process. This arises from decomposing country-specific trade shocks into common and differential components, as in Waugh (2011) and Alessandria and Choi (2021), and then abstracting from the common component. We do not consider a common trade cost because it primarily affect the level of gross trade, without first order effects on relative variables such as the RER and net trade.

Specifically, the trade cost shocks are given by

$$\xi_{Rt}^* = \frac{\xi_t}{2} \qquad\qquad \xi_{Ut} = -\frac{\xi_t}{2}$$

$$\xi_{Rt} = \tau \frac{\xi_t}{2} \qquad\qquad \xi_{Ut}^* = 0 \qquad\qquad (4)$$

where $\tau \in \mathbb{R}$ and

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}, \quad \varepsilon_{\xi t} \sim N(0, \sigma_\xi).$$

Note that we are allowing for cost of trading ROW goods within the ROW to potentially be non-zero and impose the general assumption $\tau \in \mathbb{R}$.[22] This model nests the case of only differential trade costs between countries under zero within-ROW cost, i.e. $\tau = 0$.[23] The parameter τ captures the *elasticity* of shipping costs within the ROW to export cost to the US. When mapping the model to the data, τ captures the average trade costs across ROW countries.

This specification allows the within-ROW trade cost to vary over time and capture the evolution of trade integration among the countries that compose the ROW aggregate. In fact, during the time

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[22]We assume that the within-country component is only present in the ROW. This is to account for the fact the other countries in the ROW went through significantly larger changes in trade barriers compare to the regions within the US. However, imposing time varying cost for the within-US trade in a symmetric way delivers effectively equivalent results.

[23]While we also allow for domestic iceberg trade cost, for values of τ close enough to the home bias parameter γ, it generates a qualitatively similar mechanism as the relative demand shocks in Pavlova and Rigobon (2007). They use a CES function of the form $C_t + I_t = \left[(1-\gamma)^{\frac{1}{\rho}} \left(e^{-\gamma \xi_t} \right)^{\frac{1}{\rho}} Y_{Rt}^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} \left(e^{(1-\gamma)\xi_t} \right)^{\frac{1}{\rho}} Y_{Ut}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$.

period we consider, many countries implemented trade reforms that jointly lowered the exporting cost to the US and non-US ROW countries, lowering both ξ_{Rt}^* and ξ_{Rt}. For example, the Asia-Pacific Economic Cooperation in the 1990s and the creation of the European Union generated significant changes in trade barriers among the countries in the ROW. Also, countries like China, Korea, and India focused on improving their export efficiency and entering the international market. These events resulted in lower costs of exporting to the US, as well as to other countries in the ROW aggregate.

Larger values of τ lead to higher within country trade costs for the ROW, conditional on a positive iceberg cost shock. Since this leaves fewer ROW intermediates to be aggregated to produce the final good, the trade shock induce a negative effect on output in the ROW. The strength of the negative effect on output is increasing in τ, and so is the effect on domestic absorption. Therefore, the cross country correlation of domestic absorption will vary with τ. In the quantitative exercise in Section 4.1 we show that the cross country correlation of domestic absorption identifies τ, and present a detailed analysis on the role of τ in the response of aggregate variables to trade shocks.

Finally, we assume that the financial shock follows an AR(1) process,

$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi t}$$

where ρ_ψ is the persistence and $\varepsilon_t^\psi \sim N(0, \sigma_\psi)$.

Market Clearing

Goods market clearing for each firm j requires that its production is split between supply to the ROW and the US and satisfies the local demand in each market:

$$y_{jt} = e^{\xi_{Rt}} y_{j,Rt} + e^{\xi_{Rt}^*} y_{j,Rt}^*.$$

With the aggregation presented in Equation 2, this leads to the aggregate market clearing condition where the total production of the ROW is split between demand for composite goods in the ROW and the US:

$$Y_t = e^{\xi_{Rt}} Y_{Rt} + e^{\xi_{Rt}^*} Y_{Rt}^*.$$

Lastly, combining the household budget constraint with aggregate intermediate profits as well

as the market clearing conditions above, we obtain the ROW country budget constraint:

$$\frac{\mathcal{E}_t B^*_{t+1}}{e^{\psi_t}} + \mathcal{E}_t \frac{\chi}{2} \left(B^*_{t+1} - \bar{B} \right)^2 - \mathcal{E}_t B^*_t (1 + i^*_{t-1}) = NX_t \quad \text{with} \quad NX_t = \mathcal{E}_t P^*_{Rt} Y^*_{Rt} - P_{Ut} Y_{Ut}$$

The budget constraint of the US is satisfied by Walras Law.

Final Goods Price Normalization

We fix the final good prices in both countries P_t, P^*_t to one. Implicitly we are assuming that the monetary authority in each country perfectly stabilizes inflation. Note that the RER is defined as the relative price of a basket of ROW to US goods, $Q_t = \mathcal{E}_t P^*_t / P_t$ where $\mathcal{E}_t$ is the nominal exchange rate. Therefore, the RER, Q_t, is same as the nominal exchange rate, $\mathcal{E}_t$, which is the price of ROW currency per unit of US currency.

Definition of Recursive Competitive Equilibrium

A recursive competitive equilibrium is defined by a sequence for $t = 0, 1, \dots, \infty$ of aggregate prices $\{ W_t, W^*_t, R^k_t, R^{k*}_t, \mathcal{E}_t, P_{Rt}, P^*_{Rt}, P_{Ut}, P^*_{Ut}, i_t, i^*_t \}$, firm-level prices $\{ p_{j,Rt}, p^*_{j,Rt}, p_{j,Ut}, p^*_{j,Ut} \}$, aggregate allocations $\{ C_t, C^*_t, L_t, L^*_t, I_t, I^*_t, B^*_{t+1}, B_{t+1}, Y_{Rt}, Y^*_{Rt}, Y_{Ut}, Y^*_{Ut}, \}$, firm-level allocations $\{ y_{j,Rt}, y^*_{j,Rt}, y_{j,Ut}, y^*_{j,Ut} \}$, firm-level input choices and export decisions, and the mass of exporters $\{ N_t, N^*_t \}$, such that

1. Given prices $\{ W_t, W^*_t, R^k_t, R^{k*}_t, \mathcal{E}_t, i_t, i^*_t \}$, $\{ C_t, L_t, I_t, B_{t+1}, B^*_{t+1} \}$ solves the problem of the ROW households, and $\{ C^*_t, L^*_t, I^*_t, B^*_{t+1} \}$ correspondingly for the US households.

2. Given prices $\{ p_{j,Rt}, p^*_{j,Rt}, p_{j,Ut}, p^*_{j,Ut} \}$, $\{ y_{j,Rt}, y^*_{j,Rt}, y_{j,Ut}, y^*_{j,Ut} \}$ solves the problem in the final retail sectors in the ROW and the US.

3. Firm-level input choices, prices, and export decisions solve the firm's dynamic programming problems.

4. The market clearing conditions for goods, labor and bonds are satisfied.

5. Rationality holds, so that the laws of motions are consistent with agents' decision rules.

4 Calibration

We use data for the period 1980Q1-2019Q4 for the US and ROW to discipline our model. The details about the data are in Appendix A.

4.1   Benchmark Model

We have three sets of calibrated parameters. First, we exogenously calibrate parameters that are standard in the literature. Second, we calibrate the parameters that are related to the export behavior of firms using firm level data. Third, we jointly calibrate the parameters related to the shocks processes, the pricing to market friction and adjustment costs to match a set of equal number of moments.

Standard Parameters

The standard parameters that are exogeneously calibrated are displayed in panel A of Table 1. The time unit in the model is a quarter, and we choose a discount factor of β = 0.99, which implies an annual interest rate of 4%. The depreciation rate is set to δ = 0.02. The risk aversion is σ = 2, a value frequently used in related business cycle studies. The capital share of α = 0.36 is consistent with the labor share in the US. The preference weight on consumption is η = 0.36, set to match the steady state labor of 1/4. The elasticity of substitution between ROW and US goods, ρ, is set to be 1.5, following the estimates in Feenstra, Luck, Obstfeld and Russ (2018). The home bias, governed by γ, is set to match the average trade share of 14% in the US during our sample period. We assign these values symmetrically to the US and the ROW. Finally, we set the persistence of the common and differential productivity shocks, ρ_{a_d} and ρ_{a_c}, to be equal to 0.97, following Itskhoki and Mukhin (2021).

Producer Trade Parameters

We calibrate three parameters related to the export block: fixed costs of exporting for new and incumbent exporters, f_0 and f_1, and the volatility of idiosyncratic productivity shocks, σ_η. These parameters are displayed in panel B of Table 1. The fixed costs and the volatility are set to jointly match firm level moments on exporter dynamics. In particular, we target an export participation of 20 percent, a quarterly exporter exit rate of 2.5 percent, and a size of exporters 50 percent larger than non-exporters. These are consistent with the US trade and exporter characteristics in the early 1990s (Bernard and Bradford Jensen, 1999; Alessandria and Choi, 2014).

Shocks, Adjustment Costs and Pricing to Market

The remaining parameters to calibrate are those related to trade, financial, and productivity shocks, the pricing to market friction, and the adjustment costs for capital and debt. There are

Table 1: Calibrated Parameters

Parameter		Value	Target Moment
A. Standard Parameters			
Discount factor	β	0.99	Annual interest rate of 4%
Risk aversion	σ	2	Intertemporal elasticity of substitution of .5
Weight on consumption	η	0.36	Hours worked (Frisch elasticity)
Capital share	α	0.36	Capital share of income
Elasticity of substitution across varieties	θ	4	Producer markup of 33%
Elasticity of substitution between H and F	ρ	1.5	Long-run price elasticity
Depreciation rate	δ	0.02	
Home bias	γ	0.097	Trade-to-GDP ratio of 14%
Common productivity, persistence	ρ_{a_c}	0.97	GDP persistence
Differential productivity, persistence	ρ_{a_d}	0.97	GDP persistence
B. Producer Trade Parameters			
Fixed cost of new exporters	f_0	0.14	Export participation of 20%
Fixed cost of incumbent exporters	f_1	0.04	Exit rate of 2.5%
Volatility of idiosyncratic productivity	σ_μ	0.15	Exporter premium of 50%
C. Shocks, Adjustment Costs and Pricing to Market			
Common productivity, volatility	σ_{a_c}	0.004	$\sigma(\Delta y)$
Differential productivity, volatility	σ_{a_d}	0.005	$\rho(\Delta y, \Delta y^*)$
Financial shock, volatility	σ_ψ	0.002	$\rho(\Delta c - \Delta c^*, \Delta q)$
Financial shock, persistence	ρ_ψ	0.957	$\rho(i - i^*)$
Trade shock, volatility	σ_ξ	0.061	$\sigma(xm)/\sigma(q)$
Trade shock, persistence	ρ_ξ	0.968	$\rho(\Delta xm, \Delta q)$
Trade shock, within-country share	τ	0.174	$\rho(\Delta d, \Delta d^*)$
Adjustment cost of portfolios	χ	0.020	$\rho(xm)$
Adjustment cost of capital	κ	2.529	$\sigma(\Delta inv)/\sigma(\Delta y)$
Pricing to market parameter	ζ	0.971	$\rho(\Delta tot, \Delta q)$

Notes: The table presents the values of calibrated parameters of the benchmark model. When we consider alternative models, some of the parameters are set to a different value while the other parameters are all recalibrated. In a model without trade shocks, $\sigma_\xi = \rho_\xi = 0$. In a model without trade dynamics, $f_0 = f_1 = \sigma_\mu = 0$.

ten parameters to be estimated. We jointly calibrate them to match ten moments. We present the parameters and moments used for the identification in Panel C of Table 1. We display the values of the calibrated parameters, together with the moment that is most relevant for the identification of each parameter.

The volatility of the common productivity shock, identified mainly by the volatility of GDP

growth, is found to be 0.004. The estimated volatility of the differential productivity shock, identified by the cross country correlation of the first difference of GDP, is 0.005. Given that both processes have a persistence of 0.97, this implies that the differential component of the productivity shocks slightly dominates the common one.

As shown in Itskhoki and Mukhin (2021), the Backus-Smith correlation identifies the volatility of the financial shocks, which we find to be 0.002. Hence, the volatility of productivity shocks is estimated to be between 2 and 2.5 times larger than that of financial shocks. This is similar to Itskhoki and Mukhin (2021), which finds a value between 2.5 and 3.3.[24] The persistence of financial shocks is identified by the autocorrelation of the interest rate differential. We estimate a persistence of 0.957, close to what has been estimated in the literature.

We identify the volatility of trade shocks using the volatility of net trade relative to the volatility of the RER, similar to Itskhoki and Mukhin (2017) for the case of foreign demand shocks. The persistence of the trade shock is identified by the contemporaneous correlation between the growth rates of net trade and the RER. We find that the volatility and persistence of trade shocks are 0.061 and 0.968, respectively. Hence, trade shocks are found to be more volatile and persistent than productivity and financial shocks. However, the propagation effects of trade shocks depends on the value of the home bias parameters (γ). The ratio $\gamma \sigma_\xi / \sigma_\psi$ equals 2.82, similar to the values identified in Itskhoki and Mukhin (2017) for the ratio of the volatility of foreign demand to financial shock, between 2.4 and 2.7.

We identify the within-country elasticity τ using the cross country correlation of the growth rate of domestic absorption. To build intuition on the role of τ, Figure 2 displays the impulse response functions of relative domestic absorption, net trade, and the RER to a shock that increases the shipping cost from the ROW to the US (an increase in ξ), for different values of τ, while keeping constant the other calibrated parameters. When τ is zero (the dashed line), an iceberg cost shock that increases the cost of the ROW exports and decreases its import costs, triggers a fall in net trade flows for the ROW. The increase in imports for the ROW raises the supply of final goods in the ROW. This effect is reinforced by an increase in the use of ROW intermediates for the production of final goods, due to the increase in exporting costs. For markets to clear, the final good price in the ROW must fall for domestic absorption to increase, inducing a depreciation of the RER.

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[24]Note that the model in Itskhoki and Mukhin (2021) does not have trade dynamics.

Figure 2: IRFs to Trade Shock for Different Values of τ (%)
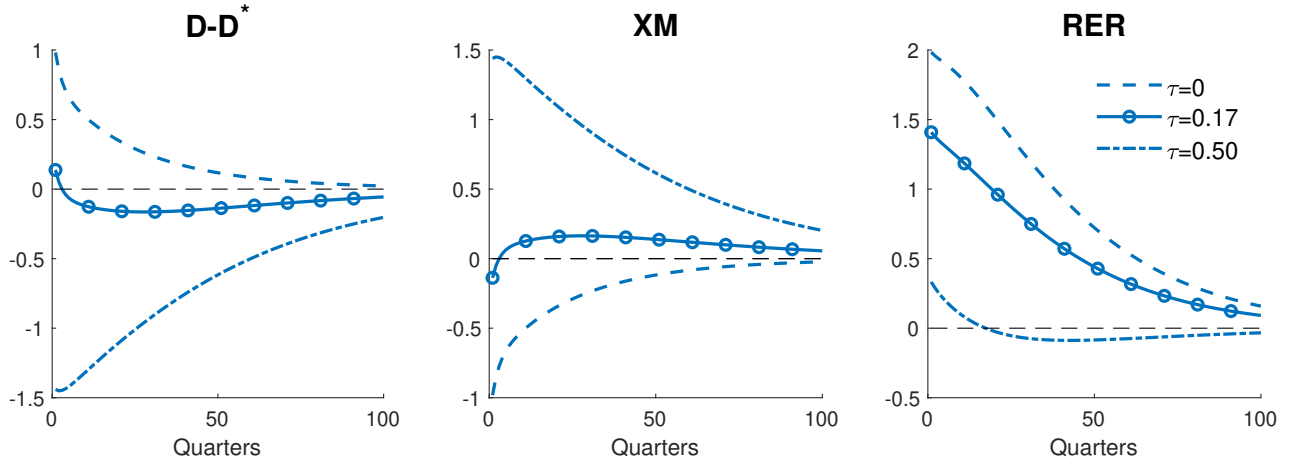
Notes: The parameters other than τ are set as in Table 1.

Now, consider the case of a positive but small value of τ of 0.17 (line with circles). With a positive τ, when the cost of exporting from the ROW to the US increases, there is also a small increase in the marginal trade cost within the ROW, between its intermediate and final good producers. This makes exporting for the ROW relatively more attractive than under zero τ, so that the fall of net exports is smaller. This implies that the fall in the final good price needed to clear the markets is weaker, so that in equilibrium there is a smaller depreciation of the RER and a weaker increase in domestic absorption.[25] If τ is sufficiently high, net trade flows for the ROW can be positive with domestic absorption in the ROW decreasing relative to the US (see dash-dotted line with $\tau = 0.50$).

It is clear that the cross-country correlation of the first difference of domestic absorption is sensitive to the choice of τ. Figure 3 shows this by displaying the cross country-correlation of the first difference of domestic absorption across different values of τ. We use this correlation to identify the size of τ. In our calibration, we find a value of τ of 0.17. Moreover, in Appendix C, using bilateral data on trade flows, we provide external evidence supporting a positive elasticity.

The adjustment cost of capital directly affects the volatility of investment relative to that of GDP, while the adjustment cost of debt directly affects the autocorrelation of net trade. We find an adjustment cost of capital of 2.52 and and adjustment cost of debt of 0.02. Finally, we discipline the pricing to market friction using the correlation between the growth rates of the terms of trade and the RER, since this friction induces a wedge between them. We find a value of $\zeta = 0.971$, which

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[25]A smaller response of relative consumption in the ROW relative to the US also contributes to generating a small value of the Backus-Smith statistic.

implies an exchange rate pass-through of 67%, in line with the estimated values in the literature (Gopinath and Itskhoki, 2010).

Figure 3: Identification of τ
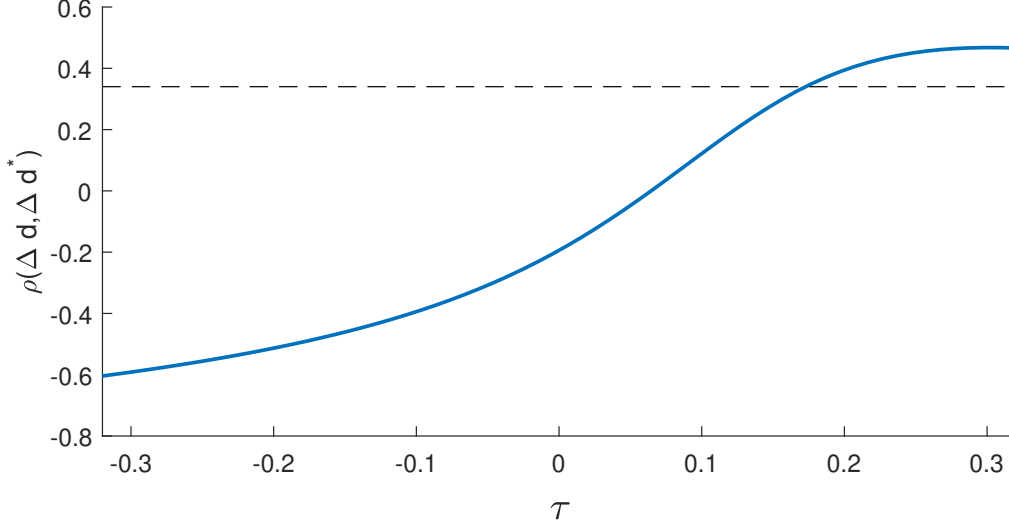
Notes: Correlation of the cross country growth rates of domestic absorption between the US and ROW given different values of τ. The other parameters are set as in Table 1. Based on model simulation of 10,000 periods. Black dashed line is the correlation in the data.

4.2 Alternative Models

We consider three alternative specifications to our benchmark model to understand the role of each feature of our model: trade shocks, financial shocks, and dynamic trade. We recalibrate models when one of these features is absent. The calibrated values of these models are shown in Table F.1.

For the model without trade shocks, we set to zero the volatility and persistence of trade shocks and the within-ROW trade cost, and recalibrate the remaining parameters. We target the same moments considered before, except for the volatility of net trade, its contemporaneous correlation with the RER, and the cross-country correlation of the growth rate of domestic absorption.[26]

For the model without financial shocks, we set to zero the volatility and persistence of financial shocks. We drop as targets the contemporaneous correlation between the growth rate of the RER and net trade, and their relative volatility.[27]

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[26]We exclude the cross-country correlation of domestic absorption from the target since the within-ROW trade cost is absent in this model

[27]Alternatively, for the model without financial shocks we could drop the Backus-Smith correlation and keep the contemporaneous correlation between the growth rate of the RER and net trade. However, since trade shocks are able

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In the model without dynamic trade, we set to zero the fixed costs of exporting for new and incumbent exporters and the volatility of idiosyncratic shocks. Given these values, the other parameters related to shocks and adjustment costs are estimated in the same way as in the benchmark model. As a result, we find a slightly different volatility and persistence for both types of shocks. However, the relative size of the unconditional variance between the two shocks is almost the same as the benchmark model (0.029 in the benchmark and the model with no dynamics). Also, in the model with no dynamics, the elasticity τ of the within-ROW trade cost is estimated to be less than a half (0.07) of the value in the benchmark model (0.17). This is because under dynamic trade, the responses of relative domestic absorption and net trade, are smaller on impact compared to the static trade model. Since higher τ also generates smaller movements in these variables as discuss above, the model requires a smaller value of τ in this static model.

5   Results

In this section, we present the results of our model. We first show that our benchmark model successfully replicates the targeted moments, especially the disconnect between the RER and net trade at the high frequency. We then show that the model is able to capture the RER dynamics at the whole spectrum of frequencies, in terms of the comovement with net trade and the frequency decomposition of the variance. Finally, we show that the model still can account for the RER disconnect puzzles and standard international business cycle moments. Throughout the section we discuss why the model requires all of financial shocks, trade shocks, and trade dynamics.

5.1   The RER and Net Trade at the High Frequency

The results of the benchmark model for the targeted moments are presented in Panel A of Table 2 (column 2). The benchmark model matches the targeted moments, such as the volatility and cross-country correlation of output. More importantly, it captures the net trade moments from the data: the contemporaneous correlation with the RER, $\rho\left(\Delta xm, \Delta q\right)$, and its relative volatility, $\sigma(xm)/\sigma(q)$.

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to match the Backus-Smith correlation, due to the role of the within-ROW trade cost, we chose to keep the Backus-Smith correlation and drop the contemporaneous correlation between the growth rate of the RER and net trade to show that this model also fails to capture the latter correlation. Hence, conditional on matching the Backus-Smith correlation, in order to match the high frequency comovement between the RER and net trade we need both financial and trade shocks.

That is, the benchmark model reproduces the comovement of net trade and the RER at the high frequency.

To see why both financial and trade shocks are necessary to capture the contemporaneous correlation between the RER and net trade, consider two alternative models: the model without trade shocks (column 3 of Table 2), and the model without financial shocks (column 4 of Table 2). When trade shocks are absent, there is an almost perfect correlation between the RER and net trade flows on impact. When financial shocks generate an excess return on bonds for the US relative to the ROW, savings in the US increase, and the excess savings is exported to the ROW (US net trade increases). At the same time, due to the fall in aggregate demand in the US, the final good price falls (the US RER depreciates). Hence, financial shocks induce a positive correlation between the RER and net trade on impact. On the other hand, a trade shock that raises the relative cost of exporting for the US, leads to a decline in US net trade flows. With less intermediate goods exported and more imported, the supply of final goods in the US increases, causing its price to fall (the US RER depreciates). As a result, trade shocks induce a negative correlation on impact, offsetting the positive effect of financial shocks.

Furthermore, the volatility of net trade relative to the RER is lower than in the data absent financial shocks, while higher absent trade shocks.[28] Hence, having both shocks is necessary for capturing the high frequency moments related to the RER and net trade. For this reason, we focus on the results of models that include both financial and trade shocks in the discussions in the remaining of the paper.

5.2 Differential Comovement with Net Trade Flows

We show that the model is able to capture the differential comovement between the RER and net trade at the full range of frequencies. First, we focus on the correlation between the growth rates of the RER and net trade flows at different horizons. Finally, we estimate the elasticity of net trade to prices in the short and long run using an error correction model. Our main finding is that, conditional on having both financial and trade shocks, dynamic trade is necessary to capture the differential comovement between the RER and net trade.

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[28]The excess volatility of net trade flows relative to the RER induced by financial shocks has also been highlighted by Miyamoto, Nguyen and Oh (2022)

Moments	(1) Data	(2) Benchmark	(3) No Trade Shock	(4) No Financial Shock	(5) No Dynamics
A. Targeted Moments					
$\sigma(\Delta y)$	0.007	0.007	0.007	0.007	0.007
$\rho(\Delta y, \Delta y^*)$	0.40	0.40	0.40	0.37	0.40
$\rho(\Delta c - \Delta c^*, \Delta q)$	-0.10	-0.10	-0.10	-0.10	-0.10
$\rho(i - i^*)$	0.87	0.87	0.74	0.93	0.84
$\rho(xm)$	0.98	0.95	0.97	0.98	0.94
$\sigma(\Delta inv^*)/\sigma(\Delta y^*)$	2.59	2.62	2.75	2.67	2.62
$\rho(\Delta d, \Delta d^*)$	0.34	0.34	$0.27^{\dagger}$	0.36	0.34
$\rho(\Delta xm, \Delta q)$	0.30	0.30	$0.90^{\dagger}$	$-0.76^{\dagger}$	0.30
$\sigma(xm)/\sigma(q)$	1.16	1.16	$1.86^{\dagger}$	$0.26^{\dagger}$	1.16
$\rho(\Delta tot, \Delta q)$	0.49	0.49	0.49	0.49	0.49
B. Frequency Decomposition (Untargeted)					
High frequency	0.08	0.08	0.10	0.05	0.05
Business cycle frequency	0.31	0.23	0.27	0.16	0.15
Low frequency	0.61	0.69	0.63	0.79	0.80
C. Trade Elasticity (Untargeted)					
Short run	0.20 (0.05)	0.39	1.16	-0.41	0.59
Long run	1.16 (0.25)	0.97	1.83	-0.28	0.59
Adjustment	0.07 (0.02)	0.04	0.19	0.01	0.04
D. Disconnect Puzzles (Untargeted)					
$\sigma(q)$	0.10	0.09	0.06	0.15	0.08
$\sigma(\Delta q)/\sigma(\Delta y)$	4.24	3.54	2.96	1.60	2.46
$\rho(q)$	0.97	0.96	0.93	0.99	0.98
β_{fama}	-1.34	0.23	0.12	0.88	-1.77
R^2_{fama}	0.04	0.002	0.001	0.89	0.65
$\rho(q, i - i^*)$	-0.30	-0.44	-0.34	-0.06	-0.20
$\rho(i)$	0.93	0.93	0.82	0.96	0.88
$\sigma(i - i^*)/\sigma(\Delta q)$	0.13	0.02	0.02	0.01	0.12

Notes: 'No Trade Shock' presents the result of re-calibrated model only with productivity and financial shocks. 'No Financial Shock' presents the result of re-calibrated model only with productivity and trade shocks. 'No Dynamics' is for the model without fixed exporting costs and producer heterogeneity. Superscript † denotes that the moment is not targeted during the calibration procedure.

Figure 4: Dynamic Correlation between RER and Net Trade Flows
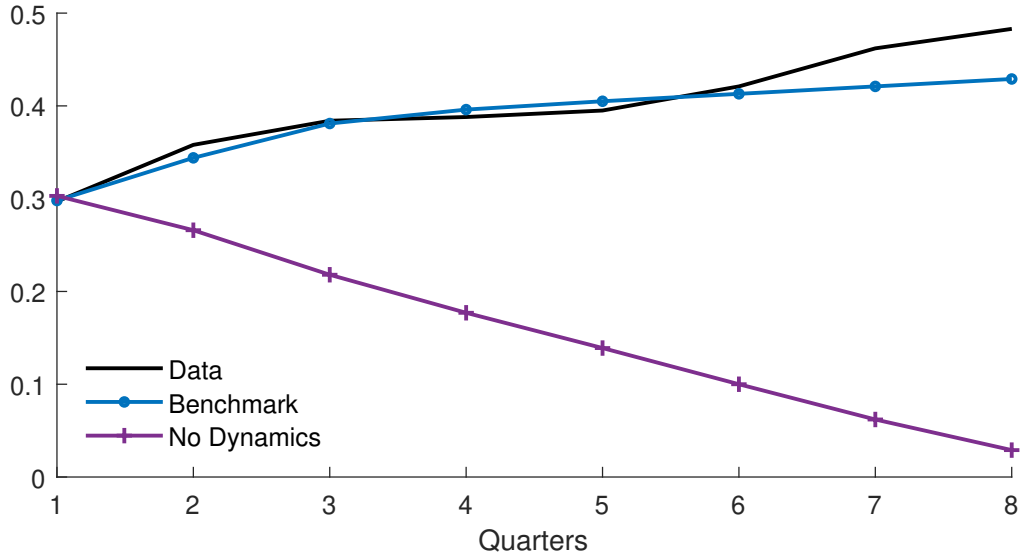
Notes: We calculate the dynamic correlations as $\rho(\Delta_h q_t, \Delta_h xm_t)$, where q_t and xm_t are log of the RER and the export-import ratio, respectively. and Δ_h denotes h–quarter difference.

Dynamic Correlation

To capture the differential comovement between the RER and net trade, consider the correlation between the growth rates of the RER and net trade at different horizons. In Figure 4, we plot the correlation between the h–quarter growth rates of the RER and net trade in the data (solid black line). While the contemporaneous correlation at $h = 1$ is 0.30, the correlation gradually increases over the horizon, reaching a value of 0.48 at the 8-quarter growth rate. The growth rates of RER and net trade present a stronger comovement in the longer than in the shorter run.

The model successfully captures the dynamic correlation between the RER and net trade, without being targeted except for the contemporaneous correlation (blue line with circles). The contemporaneous correlation in the model is 0.30, while for the 8-period growth rate it is 0.43. The benchmark model is able to capture the shape of the differential comovement between the RER and net trade, which is higher in the longer than in the shorter run.

Dynamic trade plays an important role for the ability of the model to capture this pattern. To see this, consider a case under no trade dynamics (violet line with crosses). Absent trade dynamics, the comovement becomes weaker as the horizon increases, meaning that the growth rates of the RER and net trade present a higher correlation in the short than in the long run. Hence, dynamic trade, by inducing a lagged response of net trade, allows the model to capture the differential

comovement between net trade and the RER over time.[29]

We also consider the cases in the absence of shocks. In Figure F.3 we plot the dynamic correlation for the models with no financial and no trade shocks. Absent either financial (dashed red line) or trade shocks (dash-dotted green line), the model fails to capture the differential co-movement, even under dynamic trade. As before, we observe that financial shocks induce an almost perfect correlation between these variables, while trade shocks induce a strong negative one, reinforcing our result of the need of both shocks for capturing the comovement. Therefore, conditional on having both financial and trade shocks, dynamic trade is necessary to capture the differential co-movement observed in the data.

Differential Trade Elasticity

In order to inspect the predictions of the model about the comovement between the RER and net trade at lower frequencies, we estimate the elasticity of net trade to prices. We exploit the relationship between the RER and net trade based on the Armington trade model, which is the basic trade block for almost all multi-good international macro models. Taking the ratio of demand functions for exports and imports implied in the Armington model, we have:

$$xm_t = \rho \left(tot_t + q_t \right) + \left(d_t^* - d_t \right) \tag{5}$$

where $xm_t = \ln(X/M)$ is net trade, $tot_t = \ln(p_t^M / p_t^X)$ is the log of the terms of trade, q_t is the log of the RER, and $d_t = \ln(C_t + I_t)$ is the log of domestic absorption.

Depending on the specifics of a model, the ratio may include additional terms to the Equation 5. Note that our benchmark model also nests the Armington trade block with additional features such as time-varying trade costs and sunk cost of exporting. These features will be reflected by additional terms related to trade shocks and the mass of exporters in Equation 5 (See Appendix E for the derivation).

To estimate the short- and long-run elasticity separately, we consider an error correction model

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[29]We also consider using the trade-expenditure ratio as a measure of net trade, defined as $TE_t = \log \frac{X_t}{M_t} - \log \frac{D_t^*}{D_t}$. Using the trade-expenditure ratio allows us to isolate the substitution effect that changes in the RER generate on net trade from the effect on relative expenditure. As shown in in Figure F.2, the RER and net trade present a stronger comovement in the long run than the short run even after controlling for relative expenditure. Our model successfully captures this pattern.

of the decomposition in Equation 5. Specifically,

$$\Delta xm = \beta + \rho_{SR}\,\Delta(tot_t + q_t) + \Delta(d_t^* - d_t)$$
$$- \alpha\left[xm_{t-1} - \rho_{LR}\,(tot_{t-1} + q_{t-1}) - (d_{t-1}^* - d_{t-1})\right] + \varepsilon_t \tag{6}$$

where ρ_{SR} is the short-run elasticity, ρ_{LR} is the long-run elasticity, and α captures the speed of adjustment. The term in square brackets captures the cointegration relationship. This type of regression has been widely used in studies of trade dynamics (Hooper, Johnson and Marquez, 2000; Marquez, 2002; Alessandria and Choi, 2021; Alessandria et al., 2022).

Using the data described in Appendix A, we estimate Equation 6, and present the results in Panel C of Table 2. The short-run elasticity is estimated to be around 0.2, while the long-run elasticity is larger, around 1.2. The estimated values are similar to the estimates from Alessandria and Choi (2021) that covers a similar time period for the US, and are also consistent with Alessandria, Bai and Woo (2022) which uses panel data of a broader set of countries, although their size of the long-run elasticity is slightly larger compared to our estimates.

Using the model simulated data, we conduct the same exercise in our benchmark model (column 2). We estimate a long run elasticity ρ_{LR} that is is larger (0.97) than the short run ρ_{SR} (0.39), capturing the dynamic adjustment of net trade to prices. Trade dynamics are crucial for capturing the difference between short and long run elasticity. In column 6, we present the result for the model without trade dynamics. The short and long run elasticities in this model are both 0.59, hence the model fails to generate a differential comovement between the RER and net trade flows. This shows that dynamic trade is necessary to capture the differential comovement between the RER and net trade in the short and long run.

Moreover, similar to our analysis of the correlation of the growth rates of the RER and net trade at different horizons, we find that both trade and financial shocks are necessary to capture the overall differential dynamics. Absent any of these shocks, the model does not capture the differential elasticity from the data. Absent the trade shock, both elasticities are too high, whereas absent the financial shock both are to low. Therefore, conditional on having both financial and trade shocks, dynamic trade is necessary to capture the differential elasticities of net trade to prices.

5.3   Spectrum Analysis

We now turn to study the ability of the model to capture the spectrum of the RER, which is not a target in our calibration. We consider the spectrum of the RER to study the dynamics of the RER represented at the frequency domain instead of the time domain. It is useful since it allows to decompose the variance of the RER into different frequencies. That is, the sum of the spectrum for all frequencies equals its variance. We estimate the spectrum non-parametrically using the modified Bartlett kernel. For the details on this approach, see Appendix B.
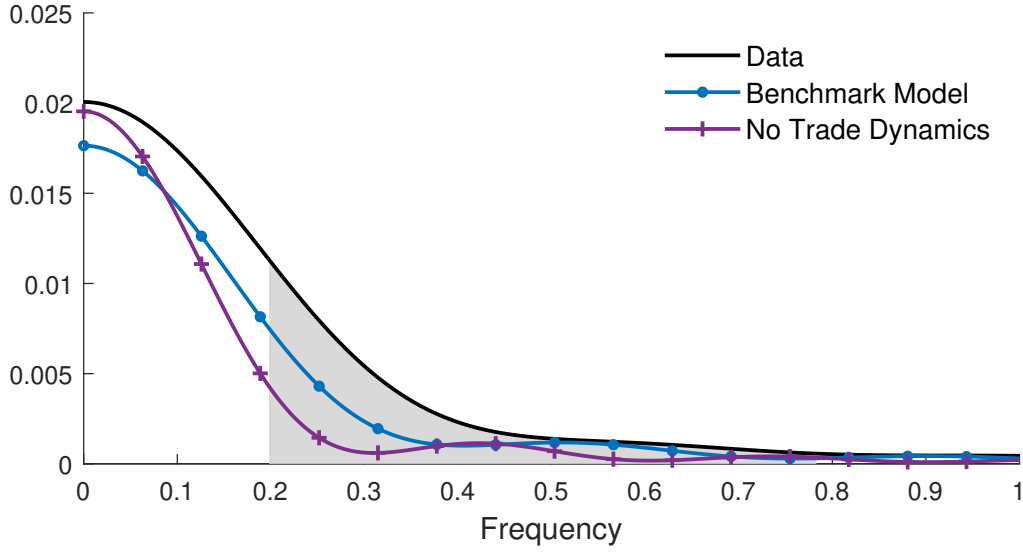
Figure 5 shows the spectrum of the RER in the data (black solid line). We find that the spectrum is the highest at the zero frequency, and decreasing as the frequency increases. The business cycle frequency, cycles that last between 8 to 32 quarters, is represented by the shaded grey area. The area under the spectrum for the frequency lower than the business cycle is much larger than that for the frequency of business cycles. In particular, it takes about 61 percent of its variance, as presented in Panel B of Table 2 (column 1).

We now turn to estimate the spectrum of the RER using model simulated data.[30] The benchmark model (blue line with circles) captures well the size and shape of the spectrum of the RER, even though it is not a target in our calibration. The model spectrum lies slightly under the data one, which reflects that the volatility of the RER is slightly smaller in the model than in the data, 0.09 and 0.10 respectively (Panel D of Table 2). Moreover, the shape of the spectrum is very similar to the data. This can be mapped to the share of the variance of the RER arising at different frequencies, which is displayed in Panel B of Table 2. In the model, the largest share (69 percent) of the RER variation is assigned to the low frequency, followed by the business cycle frequency (23 percent), and then the high frequency (8 percent).

Dynamic trade plays an important role in matching the shape of the RER spectrum. To see this, consider the re-calibrated model without dynamic trade (violet line with crosses). It is evident that the shape of spectrum of this model is worse than in our benchmark case. A larger share of the RER variance is attributed to the low frequency (80 percent). This result is consistent with the "Excess Persistence Puzzle," documented in Rabanal and Rubio-Ramirez (2015). The intuition for this result is the following. When trade is static, quantities in the short run are more elastic than under dynamic trade. Therefore, prices in the short run have a weaker response absent trade

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[30]We simulate the model for 10,000 periods and burn the first half.

Figure 5: Spectrum of the RER

Notes: The graph is enlarged for the frequency of $[0, 1]$, as the spectrum on $[1, \pi]$ is near zero. The full graph is presented in Figure B.1. Gray area shows the range of the frequencies for the business cycle. The blue line with circles shows the result in the benchmark model, while the violet line with crosses shows the result of the model recalibrated with no sunk cost of exporting

dynamics, so a higher share of the RER variance is assigned to low frequency fluctuations. Once we incorporate dynamic trade, quantities in the short-run are more inelastic and prices need to adjust more to clear the market. This leads to a redistribution from the lower to the higher frequency.

5.4 Disconnect with Macro Fundamentals

Having shown the success of the benchmark model in accounting for the variation in the RER and net trade at the full range of frequencies, we now show that the model also captures the RER disconnect, both with real and financial variables. There are several moments related the RER and macro fundamentals, so called *puzzles*, that have been hard to explain in most models. Our benchmark model is able to account for them. The results are presented in Panel D of Table 2.

First, there is an empirical disconnect between the RER and output, i.e. the real disconnect, that the literature have struggled to reproduce. In particular, in the data the RER follows a near random-walk process and is three to six times more volatile than output (Meese-Rogoff Puzzle). We also find these patterns in our data, since the volatility is around four times larger than output, and the RER is highly persistent. In a standard BKK-type models, the volatility is lower than that of output, and the process is far from a random walk. However, our model successfully reproduces

the data patterns. As shown in the second column, the volatility and persistence of the RER are very close to the data. Note that these moments are not targeted during our calibration procedure.

Second, the empirical correlation between the growth rates of relative consumption growth and the RER is negative. However, the risk-sharing condition in these models usually implies that high relative consumption is associated with a depreciation of the RER. This implies that the correlation would be close to one (Backus-Smith Puzzle). It is important to notice that even under under incomplete markets, in the presence of a risk-free bond, the models usually predict an almost perfect correlation between the cross-country consumption growth and changes in the RER. Our model is able to reproduce this puzzle by directly targeting the correlation during the calibration. As shown in Panel A of Table 2, the correlation between cross-country consumption growth and RER growth is negative, near -0.10, in both data and the benchmark model.

The ability of the benchmark model to account for the disconnect between the RER and output and consumption is not affected by the presence of trade shocks or dynamic trade, as can be seen from columns 3 and 4 in Table 2 under Panel D. Hence, introducing these features, which allows us to account for the dynamic comovement between the RER and net trade and the shape of spectrum of the RER, does not generate a tension in matching the real disconnect. Furthermore, the presence of trade shocks marginally improves the quantitative properties of the model, since the autocorrelation of the RER is closer to the data when we include the trade shock. Moreover, the model is able to capture the real disconnect even without financial shocks. This implies that trade and financial shocks alone can account for the disconnect between the RER and consumption and output, although both of them are needed to capture the moments related to net trade flows.

We now turn to the results related to the financial disconnect. It is well known that real interest rate differentials are not well connected to the changes in the RER. The disconnect can be summarized by the regression similar to Fama (1984),

$$\mathbb{E}_t \left[\Delta q_{t+1} \right] = \alpha + \beta_{Fama}(i_t - i_t^*) + u_{t+1}. \tag{7}$$

In standard models, the Fama coefficient would be close to 1, since no arbitrage condition implies that high interest rates predict depreciations of the RER. However, in the data, interest rate differentials tend to predict appreciations of the RER. More importantly, the predictive power of interest

rates is weak, as measured by an R^2 close to zero.[31] Engel, Kazakova, Wang and Xiang (2022) emphasizes that the low R^2 of the Fama regression is a more robust statistic for the financial disconnect than the negative coefficient on the interest rate differentials.

The results are shown in Panel D of Table 2. We find that indeed in our data, both the Fama coefficient is negative, with a large standard error, and an R^2 close to zero. Our benchmark model (column 2) is able to generate a Fama coefficient lower than one and, more importantly, an R^2 near zero, showing that the model is able to account for the financial puzzle. In other words, the financial moments are not compromised by the presence of trade shocks or dynamic trade. Finally, absent the financial shock (column 4), the Fama coefficient and the R^2 increase significantly, showing the importance of financial shocks for capturing the financial disconnect, consistent with the results in Itskhoki and Mukhin (2021).[32][33]

5.5 International Business Cycle Moments

Our benchmark model is also consistent with the standard international business cycle moments. We report the results in Table 3. The model is able to capture a volatility of consumption that is lower than output. It generates a cross country correlation of consumption and investment very close to the data. It also captures the positive correlation between output and exports.[34] Overall, we find that our benchmark model accounts for the dynamics of the RER and net trade at the whole spectrum of frequencies, without compromising the ability to account for the real and financial puzzles, and other international business cycle moments.

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[31]Strictly speaking, the Fama regression is used to show the disconnect in nominal variables, also known as the Forward Premium Puzzle. In this paper we are considering real version of the puzzle. In Table F.3 in Appendix F we present the Fama coefficient we find using both real and nominal data, which is very similar to each other. This arises from the fact that the RER and NER are highly correlated and inflation was low in the countries and period included in our analysis.

[32]Potentially, trade shocks (and productivity shocks) could account for the financial disconnect, since they generate changes in net foreign assets that induce UIP deviations through the adjustment cost of debt (Equation 1). However, we find that this indirect effect of the other shocks is quantitatively small.

[33]For the model without trade dynamics, we find an R^2 of 0.65. This is in part driven by the presence of adjustment costs of capital. Absent capital adjustment costs, the R^2 and β equals 0.02 and -0.85 respectively.

[34]The correlation between the growth rates of output and imports, $\rho(\Delta y, \Delta m)$, is negative in our model, leading to slightly cyclical net exports, while the opposite it true in the data. This arises from our choice of modeling trade costs as asymmetric (i.e. differential component between the US and ROW) as opposed to including a common component, since the latter has no effect on relative variables such as net trade flows and the RER. Adding a common component would increase the correlation between output and imports, since lower costs of trade will increase imports of intermediate goods, increasing final output. This would come at no cost of matching relative variables. We choose not to add the common component to keep the structure of trade costs as simple as possible for highlighting the effect of trade costs on the RER and net trade.

Table 3: International Business Cycle Moments

	(1) Data	(2) Benchmark	(3) No Trade Shock	(4) No Financial Shock	(5) No Trade Dynamics
$\sigma(\Delta c)/\sigma(\Delta y)$	0.83	0.61	0.68	0.56	0.81
$\rho(\Delta y, \Delta c)$	0.68	0.91	0.95	0.98	0.51
$\rho(\Delta y, \Delta z)$	0.77	0.83	0.94	0.97	0.39
$\rho(\Delta y, \Delta x)$	0.44	0.52	0.35	0.49	0.76
$\rho(\Delta y, \Delta m)$	0.49	-0.11	0.31	0.46	-0.20
$\rho(\Delta c, \Delta c^*)$	0.31	0.48	0.32	0.45	0.49
$\rho(\Delta inv, \Delta inv^*)$	0.31	0.31	0.25	0.33	0.28
$\sigma(\Delta tot)/\sigma(\Delta q)$	0.46	0.02	0.02	0.12	0.13

6   Quantifying the Effect of Financial and Trade Shocks

Using our benchmark model, which provides a unified framework to study the dynamics of the RER at all frequencies, we evaluate the role of trade and financial shocks in shaping the dynamics of the RER. First, we compute the contribution of each shock for the error forecast variance of the RER at different horizons. Second, we present the impulse response functions of variables of interest to trade and financial shocks.

6.1   Conditional Variance Decomposition

We inspect the relevance of each shock for driving the variation in the RER by computing the contribution of each shock to the h–quarter ahead error forecast variance of the RER. The results are presented in Table 4. It is clear that the trade shock explains most of the error forecast variance of the RER in the long-run (i.e. low frequency), while the financial shock is important for the short-run (i.e. high frequency) fluctuations.

In particular, the financial shock explains 62% of the one-quarter ahead error forecast variance, with the trade shock explaining around 45%. Hence, at the high frequency, financial shocks matter more than trade shocks for explaining the variation of the RER. However, when focusing at the

eighty quarters ahead error forecast variance of the RER, the trade shock explains around 63%, while the financial shock explains 26%. Hence, trade shocks matter more than financial shocks for the variation in the RER at lower frequencies. Since, 61% of the overall variance of the RER is at frequencies lower than business cycles, we find that trade shocks are crucial for capturing the overall variation in the RER.

Table 4: Conditional Variance Decomposition (%)

	quarters = 1	8	32	80
Financial shock	62.37	45.42	26.00	26.30
Trade shock	34.71	49.39	64.43	62.84
Productivity shock	2.92	5.19	9.57	10.86

We find consistent results using the analysis at the frequency domain. In particular, we conduct a spectral analysis of the model for the counterfactual cases where only the trade or the financial shock is present under the identified parameters of the benchmark model. We find that in the case of only trade shocks, the volatility is only slightly smaller than in the benchmark, shown by the total area below the spectrum. Furthermore, the shape of the spectrum follows very closely that of the benchmark model. However, having only financial shocks largely misses the size of spectrum. For more details, see Figure F.4 and Table F.2.

6.2 Inspecting the Financial and Trade Shock Mechanism

We now turn to study the propagation mechanism triggered by financial and trade shocks in more detail. For this purpose, we present the impulse response functions of relative domestic absorption, net trade and the RER to the two types of shocks in Figure 6.

First, consider the effect of a financial shock that increases the return on bonds for the ROW (dashed red line). Since households in the ROW face a higher return on bonds, they optimally decide to increase their savings. Hence, domestic absorption in the ROW falls relative to the US. Due to the presence of home bias in expenditure, the fall in demand of ROW households translates into a stronger shortage in demand for intermediate goods in the ROW than in the US. For markets to clear, the price of ROW intermediate goods must fall, so that the US increases its expenditure in

ROW intermediates. As a consequence, net trade flows for the ROW increase, while at the same time the RER depreciates. In fact, a one standard deviation financial shock generates an almost 2% depreciation of the RER on impact and a 2.57% increase in net trade flows.

Figure 6: Selected IRFs to Trade and Financial Shocks (%)
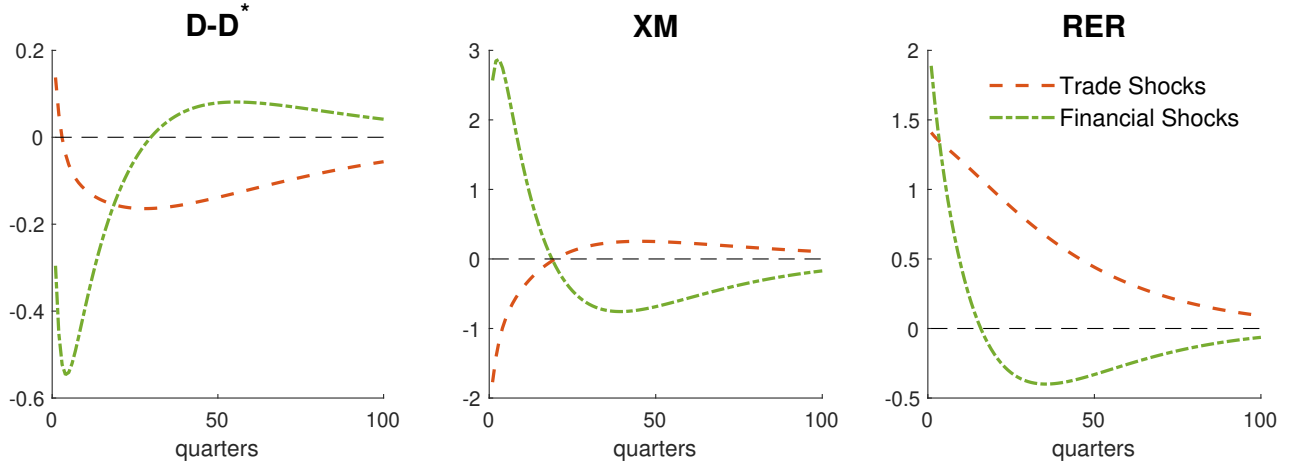
Due to dynamic trade, trade and domestic absorption take time to respond, leading to hump shaped responses, peaking two quarters after the shock. On the other hand, prices adjust without any delay. This contributes to a lagged response of net trade relative to the RER. Eventually, households in the ROW consume their initial savings, so that net trade becomes negative, around 5 years after the shock. Higher relative domestic absorption translates into an appreciation of the RER relative to its pre-shock value.

Next, we study the effect of a trade cost shock that increases the cost of exporting for the ROW relative to the importing cost (dashed green line). A higher exporting cost in the ROW generates a fall in net trade. Larger inflows of foreign intermediates, together with smaller outflow of domestic intermediates, increases the supply of final goods in the ROW. These effects are more gradual than the case with static trade due to the dynamic nature of trade. For markets to clear, the ROW final good price must fall. Consequently, domestic absorption increases and the RER depreciates. In particular, a one standard deviation trade shock generates an almost 1.5% depreciation of the RER on impact and a 1.77% decrease in net trade flows on impact. Note that both magnitudes are smaller (in absolute terms) than under financial shocks. This explains why the model is able to generate an unconditional correlation between the growth rates of net trade and the RER that is slightly positive (0.30).

Over time, domestic absorption in the ROW falls, eventually below its pre-shock level, while net trade become positive. This is due to the effect of the within-ROW trade cost, as discussed in Section 4.1 (see Figure 2). With a trade shock, the within-ROW trade cost increases since the elasticity τ is found to be positive.[35] The increase in the within-ROW trade cost contributes to dampening the effect on both domestic absorption and net trade, making them switch signs (relative domestic absorption eventually falls relative to its pre-shock level, inducing net trade flows to become positive).

Finally, as can be seen from the last panel, trade shocks induce more persistent effects on the RER than financial shocks. With trade shocks, the RER follows a very persistent path. It is worth noticing that it is the *effect* of trade shocks that is more persistent than the financial shock, and not the process itself. In fact, the calibrated persistence of the financial and trade shocks are very similar ($\rho_\psi = 0.957$ for financial shocks; $\rho_\xi = 0.968$ for trade shocks).[36] Overall, our results suggest that financial shocks matter more than trade shocks for the dynamics of the RER in the short run, while the opposite is true for the longer run.

7   Robustness

In this section, we explore the robustness of our quantitative results to alternative specifications. First, we consider estimating the model using Bayesian methods. We show that we obtain similar estimates of parameters than under in the Benchmark model in Section 4.1. Next, we explore the robustness to a different specification of dynamic trade, having zero within-ROW trade costs, and investment adjustment costs. Overall, we find that our benchmark model better captures the dynamics of key variables relative to the alternative specifications. Finally, we find that trade shocks drive most of the variation in the RER at low frequencies, consistent with our benchmark model. All the details are in Appendix D.

Bayesian Estimation

The details about the Bayesian estimation are provided in Appendix D.1. We use four data series to estimate the model: US GDP, ROW GDP, net trade flows and the RER. Overall, we find that the

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[35]See Section 4.1 and Appendix C for the evidence supporting the non-zero, positive elasticity.

[36]If we set the persistence of both shocks to be the same (either both equal 0.957 or 0.968), while keeping all the other parameters constant, we still find stronger effects of financial shocks in the short run (more than 50%), and stronger effects of trade shocks in the long run (more than 50%).

estimated parameters are very similar to those obtained from our benchmark model in Section 4.1. We present the estimated parameters in Table D.1. Moreover, we find that the dynamic trade model is preferred to the static trade model. That is, the model with dynamic trade has a better fit, as shown by the log data density higher in the dynamic trade model than the static trade model. This is consistent with our results from Section 5.3 and Section 5.2, where we argue in favor of the dynamic trade model.

To study the role of each shock in shaping the variation of the RER, we consider the counterfactual path of the RER, when it is driven by only one of the estimated shocks. The counterfactual RER under only trade shocks tracks closely the actual path of the RER across the whole time period. With only financial shocks, the RER follows a similar path up to the early 2000s, but not after that. Productivity shocks do not seem to generate a path for the RER closely related to the data. Overall, trade shocks generate a path of the RER that most closely tracks the actual data.[37] Finally, we compute the variance decomposition of the RER using the estimated parameters, and find similar results as in our benchmark model (see Table D.3). Thus, we find that trade shocks are crucial to capture the dynamics of the RER.[38]

Specification of Dynamic Trade

To explore the robustness of our specification of dynamic trade, we consider an alternative way of modeling it. In particular, we introduce adjustment costs in the use of imported inputs in the final good aggregator, as in Erceg, Guerrieri and Gust (2006), Rabanal and Rubio-Ramirez (2015) and Gornemann et al. (2020) (see Appendix D.2 for details.). This is a reduced form way of generating a differential long- and short-run trade elasticity without dynamic decisions of heterogeneous firms. We identify the adjustment cost using the estimated speed of adjustment of net trade to prices in the data.[39] The parameters and their calibrated values are presented in Table D.4 under 'Input Adj.'

The alternative model generates similar targeted and untargeted moments as in the benchmark model (see Table D.5). It is able to generate a differential short and long run trade elasticity to prices.

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[37]In Table D.2, we show that the correlation between the actual RER and the counterfactual under only trade shocks is 0.83. Under only financial shocks, the correlation is slightly lower, 0.73. The correlation under only productivity shocks is -0.20.

[38]This is consistent with the message in Rios-Rull, Santaeulalia-Llopis, Schorfheide, Fuentes-Albero and Khrysko (2012) that argues that it is not the choice of quantitative methodology that is responsible for empirical findings, but rather the data employed in the identification. Data on net trade flows is key to the identification of parameters relevant to capture the dynamics of the RER at the whole spectrum of frequencies.

[39]The speed of adjustment is captured with the parameter α in the ECM equation 6, which is estimated to be 0.07.

However, the long-run trade elasticity is lower than in our benchmark model. We find that the low frequency share in the spectrum of the RER is higher than in our benchmark model. Finally, we find a stronger effect of financial shocks on the RER in the short run, relative to our benchmark model, as measured by the variance decomposition of shocks (see Table D.6). However, in the long run we find similar effects of trade shocks on the RER as in our benchmark model.

Within-ROW Trade Costs

We evaluate the role of the within-ROW iceberg cost component τ. The details are displayed in Appendix D.3. We calibrate the model assuming that the elasticity of domestic trade costs to international costs is $\tau = 0$, and do not target the cross country correlation of domestic absorption. The calibrated parameters and resulting moments are reported in Tables D.4 and D.5 under 'No Within Cost.'

This model generates a worse fit for the Backus-Smith correlation and the cross country correlation of domestic absorption. Thus, τ matters for accounting for the Backus-Smith puzzle and the cross country correlation of domestic absorption. In terms of the untargeted moments, we notice that the model is able to generate a differential short and long run elasticity of trade to prices. Both the short and run elasticities are higher than in our benchmark model. Finally, both the spectrum decomposition of the RER and the contribution of financial and trade shocks to the variation in the RER are very similar as in our benchmark model (see Table D.6).

Investment Adjustment Cost

We consider investment, as opposed to capital, adjustment costs. The details are presented in Appendix D.4. We consider the specification in Christiano, Eichenbaum and Evans (2005) and calibrate the parameters in the same way as in the benchmark model. The results are presented in Tables D.4 and D.5, under 'Inv Adj.'

Overall, the calibrated parameters and the results of this model are very similar to the benchmark model, including the volatility of investment. We find that this model generates a higher share of the variance of the RER for the low frequency than in the benchmark model. Finally, we find a similar contribution of financial and trade shocks to the variation in the RER at different horizons (see Table D.6). Overall, our main results are robust to this alternative specification of investment adjustment cost.

8 Concluding remarks

In this paper, we have presented a comprehensive analysis of the dynamics of the RER by integrating financial shocks, trade shocks, and dynamic trade within a unified framework. Our findings contribute to a deeper understanding of the behavior of the RER and its connection with net trade flows. We have found that trade shocks are crucial to capture the RER fluctuations at the low frequency. Given that 61 percent of the RER unconditional variance is attributed to the low-frequency movements, trade shocks are essential to account for its overall dynamics. On the other hand, consistent with the existing literature, we found that financial shocks are important for explaining the variation in the RER at higher frequencies, especially for the financial disconnect.

While this study represents a substantial advancement in our comprehension of the factors influencing and shaping the dynamics of the RER across various frequencies, it also highlights avenues for future research that warrant exploration. Specifically, further investigation is necessary to refine the measurement of financial and trade shocks, and shed light on their primary sources of variation. Additionally, while we have treated financial and trade shocks as independent, it is conceivable that they share a common underlying cause, accounting for a portion of their variability. Addressing these unresolved issues will deepen our understanding of the RER dynamics and provide valuable insights into the nature of financial and trade phenomena.

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Appendix

A   Data Description

In this section, we describe the data sources and how we construct the variables for our calibration.

- Period: 1980Q1 - 2019Q4, quarterly

- ROW: Trade-weighted average of 10 Countries

  - Countries: Canada, Finland, Germany, Ireland, Italy, Japan, Republic of Korea, Spain, Sweden, United Kingdom. These countries account for 60 percent of total US trade.

  - Weights: Country-specific average of the sample period (Federal Reserve). While the weights are updated every year, we use the constant weights using country-specific average during our sample period. For countries in Euro Area after 1999, we allocate the weights for the total of Euro Area into these countries using the average distribution within Euro Area during 1980-1999.

  - We check the robustness of the empirical moments across using other weights than average trade. Moreover, we consider adding China into the sample, although data for China is available only after 1990. Table A.1 shows that the moments we consider are similar across these variations.

- US interest rate: Effective federal funds rate (IMF), deflated with consumer price index (OECD)

- ROW interest rate: Money market rates, deflated with consumer price index (OECD)

  - For most countries, money market rates (IMF). In a few cases where the data is not available from the IMF for the whole sample period, we consider different sources as below.

  - China, Germany, UK: Immediate call money/interbank rate (OECD)

  - Canada: Short term interest rate (OECD)

  - Japan: Overnight call rate (Bank of Japan)

- Figure F.1 shows that the interest rate data from different sources we use align very well with the money market rate from the IMF.

- Quarterly National accounts (OECD)

  - US dollars, volume estimates, fixed PPPs, seasonally adjusted

  - Y: Gross domestic product - expenditure approach

  - C: Private final consumption expenditure

  - I: Gross fixed capital formation

  - X: Exports of goods and services

  - M: Imports goods and services

- Real exchange rate: Effective exchange rate, Real, Narrow indices, 2010=100 (BIS)

- Terms of trade: Terms of trade index (BEA, retrieved from FRED)

- US exporter characteristics (Alessandria and Choi 2021)

Table A.1: Empirical Moments with Different Weights

	Mean trade	Output	Trade	With China
$\rho\left(\Delta c - \Delta c^{*}, \Delta q\right)$	-0.10	-0.07	-0.12	-0.09
$\rho\left(i - i^{*}\right)$	0.87	0.86	0.86	0.86
$\rho(\Delta y, \Delta y^{*})$	0.40	0.26	0.30	0.32
$\rho(\Delta d, \Delta d^{*})$	0.34	0.24	0.24	0.11
$\rho(xm)$	0.98	0.98	0.98	0.98
$\sigma(\Delta inv^{*})/\sigma(\Delta y^{*})$	2.59	2.59	2.59	2.59
$\rho\left(\Delta xm, \Delta q\right)$	0.30	0.30	0.30	0.30
$\sigma(xm)/\sigma(q)$	1.16	1.16	1.16	1.16
$\rho(\Delta tot, \Delta q)$	0.49	0.49	0.49	0.49
$\sigma(\Delta y)$	0.007	0.007	0.007	0.007

B   Spectrum Analysis

In this section, we describe our spectrum analysis. For more detailed and rigorous steps, see Hamilton (2020).

To study the RER represented at the spectrum domain, we convert its time-domain representation using the Fourier transform. Given a covariance-stationary process q_t, the spectrum is defined as the Fourier transform of its autocovariance function $C(\tau)$:

$$S(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} e^{-i\omega\tau} \, C(\tau) \tag{8}$$

where

$$C(\tau) = \mathbb{E}(q_t - \mu_q)(q_{t-\tau} - \mu_q).$$

Note that ω is a (angular) frequency measure of radians per period.[40] Given that upper and lower bounds for business cycle frequency are 8 and 32 quarters, the range of frequency that corresponds to the business cycle is

$$\omega \in \left[\frac{2\pi}{32 \text{ quarters}}, \frac{2\pi}{8 \text{ quarters}} \right] = [0.196, 0.785].$$

This is consistent with the range used by Rabanal and Rubio-Ramirez (2015).

Using the inverse of Equation (8), we can write the autocovariance function as

$$C(\tau) = \int_{-\pi}^{\pi} e^{i\omega\tau} \, S(\omega) \, d\omega$$

Then with $\tau = 0$, the variance $C(0) = \int_{-\pi}^{\pi} S(\omega)d\omega$ is the sum of spectrum. In this sense, the spectrum decomposes the variance into different frequencies.

Also, we can show that spectrum is symmetric around zero, periodic with a period of 2π, and can be written as

$$S(\omega) = \frac{1}{2\pi} C(0) + \frac{2}{2\pi} \sum_{\tau=1}^{\infty} \cos(\omega\tau) \, C(\tau). \tag{9}$$

In order to estimate the population spectrum given the data sample of T observations, we could

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[40]For an ordinary frequency $\xi = \omega/2\pi$ (Hz), the spectrum is defined as

$$S(\xi) = \int_{-\infty}^{\infty} C(\tau)e^{-2\pi i \xi \tau} d\tau.$$

use the sample autocovariance

$$\hat{C}(j) = \frac{1}{T} \sum_{t=j+1}^{T} (q_t - \bar{q})(q_{t-j} - \bar{q}),$$

where $\bar{q}$ is a sample mean. This yields an estimate of Equation (9), known as the sample periodogram:

$$\hat{S}^{sp}(\omega) = \frac{1}{2\pi} \hat{C}(0) + \frac{2}{2\pi} \sum_{j=1}^{T-1} cos(\omega j) \; \hat{C}(j). \tag{10}$$

However, such estimate is subject to a few limitations. Thus we use a nonparametric estimation instead. That is, we estimate the spectrum by

$$\hat{S}(\omega_j) = \sum_{m=-h}^{h} k(\omega_{j+m}, \omega_j) \; \hat{S}^{sp}(\omega_{j+m}) \tag{11}$$

where $k(\omega_{j+m}, \omega_j)$ is a kernel with a bandwidth h. The idea is to take a weighted average of the sample periodograms $\hat{S}^{sp}(\tilde{\omega})$ for the values $\tilde{\omega}$ around ω, where the distance between ω and $\tilde{\omega}$ determines the kernel, i.e. the weight.

After substituting Equation (10) into Equation (11) and some calculations, it can be shown that Equation (11) is equivalent to

$$\hat{S}(\omega) = \frac{1}{2\pi} \hat{C}(0) + \frac{2}{2\pi} \sum_{j=1}^{T-1} k_j^* \; cos(\omega j) \; \hat{C}(j).$$

where $\{k_j^*\}_{j=1}^{T-1}$ is a weighting sequence corresponding to a kernel function $k(\omega_{j+m}, \omega_j)$. The weight for the modified Bartlett kernel is given as

$$k_j^* = \begin{cases} 1 - \frac{j}{J+1} & \text{for } j = 1, 2, \cdots, J \\ 0 & \text{for } j > J \end{cases}$$

where J is the length of a window for the weight that is related to the kernel bandwidth. This yields the spectrum estimate of

$$\hat{S}(\omega) = \frac{1}{2\pi} \hat{C}(0) + \frac{2}{2\pi} \sum_{j=1}^{J} \left[1 - \frac{j}{J+1} \right] cos(\omega j) \; \hat{C}(j).$$

45

On the other hand, there is no fixed rule for the choice of the bandwidth h (or window J). Hamilton (2020) suggests trying different values and "relying on subjective judgement for the most plausible estimate." For the benchmark exercise we use the window of $J = 16$, and check that other values yield a similar result that is within the range of findings of the literature. Figure B.1 shows the estimated spectrum of the RER for the full range in $[0, \pi]$.

Figure B.1: Spectrum of the RER
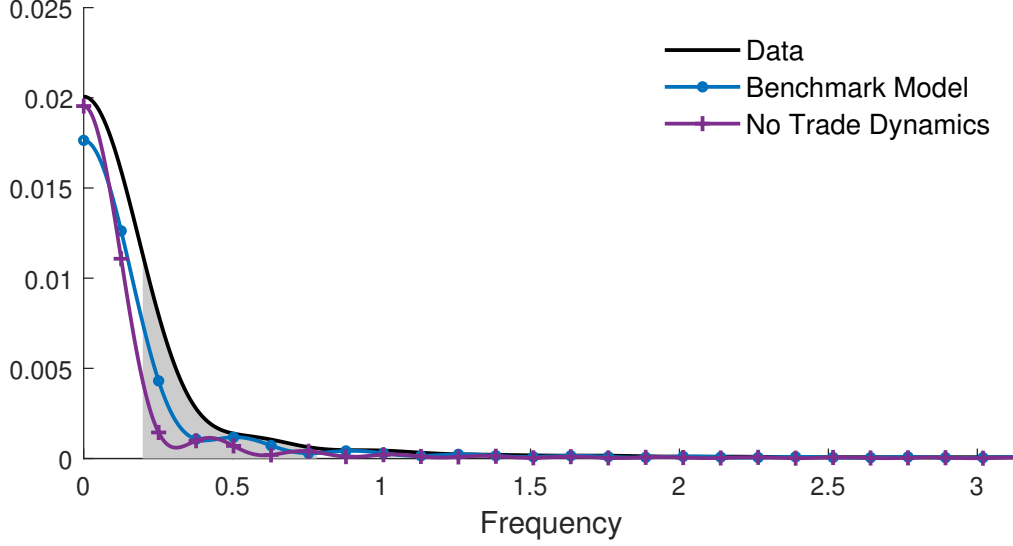
C Empirical Evidence of Trade Costs

In this section, we provide an external validation for our specification of trade costs. First, we use data on bilateral trade to measure these costs for different pairs of countries. Next, we estimate the elasticity of within-country trade costs and show it is consistent with the specification in our benchmark model.

We measure trade costs from data as a wedge in the CES demand, common in any Armington trade model. The demand for country i goods in country j is given by:

$$X_t^{ij} = \left(\frac{e^{\xi_t^{ij}} p_t^{ij}}{P_{jt}} \right)^{-\rho} D_{jt}$$

where X_t^{ij} is bilateral trade flows from country i to j, p_t^{ij} is the price level of exports from country

i to j, P_{jt} is the price level of domestic absorption in country j, D_{jt} is the domestic absorption of country j, and ρ is the elasticity of substitution. Our model assumes the same type of CES structure for the demand for differentiated goods. Moreover, it is the basic trade block for almost all studies in trade literature.

Note that all of the terms in the demand function except for ξ_t^{ij} are observables. Thus, we can recover trade costs ξ_t^{ij} as a gap between actual and predicted trade flows given prices and aggregate demand. In particular, we estimate the above demand function using the following regression

$$\log X_t^{ij} = \beta \log(P_t^{ij}/P_{jt}) + \log D_{jt} + \varepsilon_t^{ij}. \tag{12}$$

and consider the residuals ε_t^{ij} as trade costs. By estimating the demand function, we do not restrict ourselves to a particular value of elasticity. In fact, there is a broad range of values used for the elasticity in the literature, and the estimated elasticity varies greatly depending on the sample and the length of period considered. Also, the estimation by construction minimizes the size of trade costs and lets us take a conservative stance on the role of trade costs.

We estimate the demand function using data for the US and ten other countries for the ROW, as is done in our benchmark quantification. For data on bilateral trade flows, we use annual data from UN Comtrade, converted into real terms using the price levels of the US dollars from Penn World Table 10.0. Domestic absorption and price levels of different countries in our sample also come from Penn World Table 10.0. Our sample period covers the period of 1994-2019, mostly due to data availability of trade flows.[41]

For the trade cost between the US and the ROW, ξ_{Rt}^* and ξ_{Ut}, we aggregate the data on the ten countries and use it as the variables for the ROW. Then we run the regression (12) for the US-ROW pair. On the other hand, for the trade cost within the ROW, ξ_{Rt}, we use bilateral data on each pair of countries in the ROW, and take average of the recovered residuals across countries to construct time series.

Given the path of trade costs, we check the relationship of ξ_{Rt} with ξ_{Rt}^* or $\xi_{Rt}^* - \xi_{Ut}$. We use these estimates to compare with the model analogue. As shown in Equation 4, in our model we allow trade costs within the ROW aggregate, ξ_{Rt}, to be nonzero. We further assume it to be $\xi_{Rt} = \tau^{\frac{\xi_t}{2}}$,

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[41]We also check the robustness with quarterly data during the period of 2008Q1-2019Q4. We find that the path of trade costs is similar to using annual data.

Figure C.1: Empirical Relationship of Trade Costs
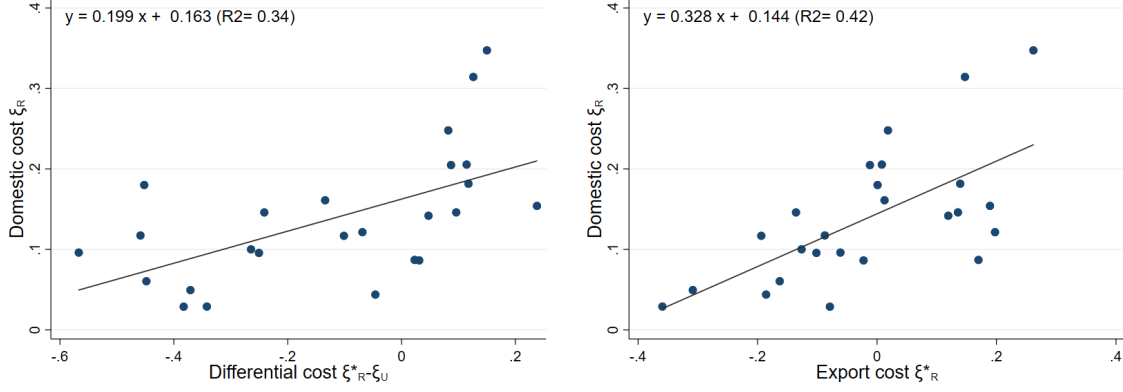
Notes: Each point represents trade costs of each year. The plots corresponds to the first and second columns of Table C.1.

where τ measures the elasticity of the within component respect to the ROW-US trade cost. In the calibration of the benchmark model, displayed in Section 4.1, we find that τ is a small positive number (0.16). Thus ξ_{Rt} is positively correlated with trade costs from ROW to the US, $\xi_{Rt}^* = \frac{\xi_t}{2}$, and also with the difference between exporting and importing costs, $\xi_{Rt}^* - \xi_{Ut} = \xi_t$.

Figure C.1 shows that we do find a consistent pattern in the data. It plots the relationship of ξ_R (left panel) with $\xi_R^* - \xi_U$ and ξ_R^* (right panel). The estimated elasticity is between 0.199 and 0.32.

Finally, table C.1 displays the result with additional controls. Although the size of estimated τ differs slightly, we have the robust result that the estimated τ is positive as in our benchmark model presented in Section 4.1. Moreover, the coefficient of ξ_R^* is always larger than $\xi_R^* - \xi_U$, as specified in our benchmark model.

D Robustness

In this section, we consider alternative specifications to check the robustness of the results of the benchmark model. First, we explore an alternative estimation strategy to identify the parameters and shocks driving the RER: Bayesian methods. We show that we obtain similar estimates of parameters than under our Benchmark model in Section 4.1.

Next, we show that explore alternative specifications to our benchmark model, in particular a reduced form specification of dynamic trade, which has been used in the literature, the role of the within-ROW trade cost, and an alternative model with investment adjustment costs. Overall, we

Table C.1: Empirical Estimates of τ

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Dependent variable: ξ^R				
$(\xi_R^* - \xi_U)$	0.199**		0.546*		0.493***		0.443	
	(0.0581)		(0.223)		(0.100)		(0.304)	
ξ_R^*		0.328***		0.843***		0.583***		0.972**
		(0.0798)		(0.166)		(0.0627)		(0.293)
Country FE			Y	Y			Y	Y
Spending Constraints					Y	Y	Y	Y
Observations	25	25	25	25	25	25	25	25
R-squared	0.338	0.423	0.207	0.530	0.513	0.790	0.0847	0.324

Notes: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. 'Country FE' denotes the fixed effect for origin and destination countries when estimating the demand function for the pair of ROW countries. 'Spending Constraints' are a restriction on the coefficient of domestic absorption to be 1, as predicted in the model with CES demand.

find that our benchmark model better captures the dynamic of key variables in our model relative to the alternative specifications. Moreover, we find that the result that financial shocks matter more for the short run and trade shocks for the long run is robust across the alternative specifications.

D.1 Bayesian Estimation

We explore an alternative estimation strategy to identify the shocks driving the RER: Bayesian methods. First, we show that we obtain similar estimated of parameters than under our benchmark model in Section 4.1. Second, we show that the model with dynamic trade is preferred to that of static trade. Finally, we show that trade shocks are crucial for generating the dynamics of the RER. That is, the counterfactual RER under trade shocks is closer to the RER in the data than under the financial shock. We also present the estimated path of the different shocks and compute the conditional variance decomposition of the RER.

Estimated Parameters

We estimate the same parameters as the ones we internally calibrate in the benchmark case. In particular, we estimate the productivity shock volatility, σ_c and σ_d, financial shock parameters, ρ_ϕ and σ_ϕ, trade shock parameters, $\rho_{\xi d}$, $\sigma_{\xi d}$ and τ, as well as the adjustment costs parameters χ and κ.

Table D.1: Estimated Parameters

	Prior Distribution	Dynamic Trade		Static Trade	
		Post Mean	90% Interval	Post Mean	90% Interval
ρ_ψ	Uniform [0.9,0.999]	0.94	(0.91 , 0.98)	0.96	(0.94 , 0.99)
ρ_{ξ_d}	Uniform [0.9,0.999]	0.99	(0.97 , 0.99)	0.98	(0.96 , 0.99)
σ_c	Inverse gamma (0.01,0.01)	0.004	(0.004 , 0.005)	0.02	(0.02 , 0.02)
σ_d	Inverse gamma (0.01,0.01)	0.006	(0.005 , 0.006)	0.008	(0.007 , 0.009)
σ_ψ	Inverse gamma (0.01,0.01)	0.003	(0.003 , 0.004)	0.002	(0.002 , 0.002)
σ_{ξ_d}	Inverse gamma (0.01,0.01)	0.04	(0.03 , 0.04)	0.09	(0.07 , 0.11)
τ	Uniform [-0.5, 0.5]	0.14	(0.12 , 0.18)	0.10	(0.07 , 0.13)
χ	Uniform [0.00001,0.5]	0.05	(0.01 , 0.10)	0.19	(0.08 , 0.31)
κ	Uniform [0,20]	2.34	(0.01 , 5.06)	1.73	(0.55 , 3.28)
ζ	Uniform [0.85, 1.5]	1.17	(1.10 , 1.23)	1.40	(1.32 , 1.49)
Log data density		1862.88		1592.32	

We impose loose priors, mostly uniform distribution and inverse gamma for volatility parameters. For observables, we use four data series: GDP growth of the US and the ROW, the net trade flows and the RER, with the same sample period as in the benchmark case (1980Q1-2019Q4).

The left panel of Table D.1 reports the prior and posterior distribution. The estimated results are similar to the benchmark case. Both financial and trade shocks show relatively high volatility, that of trade shocks being only slightly larger than financial shocks. The size of the financial shock volatility is the smallest, while the size of trade shock is largest. The within-country trade cost parameter $\tau = 0.14$ is also very close to the benchmark case (0.17).

Dynamic vs Static Trade

To show that dynamic trade model better captures the data on trade and the RER compared to the static mode, we estimate the static model with no fixed cost of exporting. We use the same priors as the before. The result of the static case is presented in the right panel of Table D.1.

We find that the log data density (Laplace Approximation) in the dynamic trade model is 1862.88 while in the static model it is 1592.32, so that the dynamic trade model is preferred over the static

trade model by a Bayes factor of $exp(270.56)$.[42] The value of $exp(270.56)$ provides a strong evidence supporting the hypothesis that the dynamic trade model is statistically better than the model with static trade. This is consistent with our results from Section 5.2 and Section 5.3, where we argue in favor of the dynamic trade model.

Figure D.1: Estimated Shocks
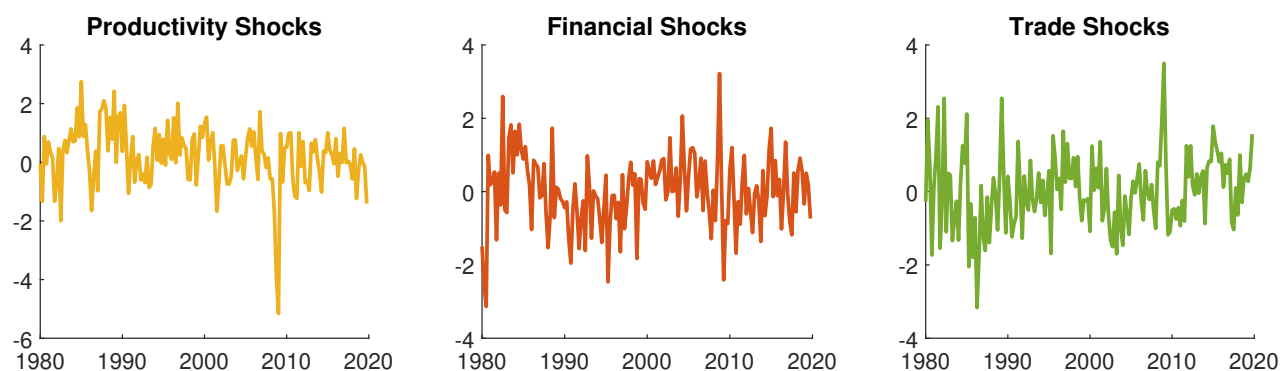
Estimated Shocks

Figure D.1 shows the estimated path of productivity shocks of the ROW, trade shocks, and financial shocks. The trade shocks were most volatile during the 1980s, when the series of different trade policy were implemented in many countries. For example, Uruguay Round launched multilateral trade negotiations. Also, countries like India and Mexico introduced trade reforms and lowered their trade barriers. In recent years trade shocks became more stable, while 2009 marks the period of the highest trade cost.

Counterfactual RER

In Figure D.3, we show the path of the RER in the data, as well as the counterfactual where the RER is driven by only one of the shocks. We present the correlation between the data and counterfactual cases in Table D.2. It is clear that the RER under trade shocks closely tracks the actual RER during the whole sample period. The path generated only with the Trade shocks, shown in green dashed line, very closely follow the data path. The correlation with the data is 0.83. On the other hand, with only financial shocks, the RER follows a similar path up to the early 2000s, but rather departs from data in the later periods. The correlation is 0.73 and lower than the case with only trade shocks. Productivity shocks do not seem to generate a path for the RER that

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[42]The Bayes factor is similar to a likelihood-ratio test.

Figure D.2: RER Dynamics under Different Shocks
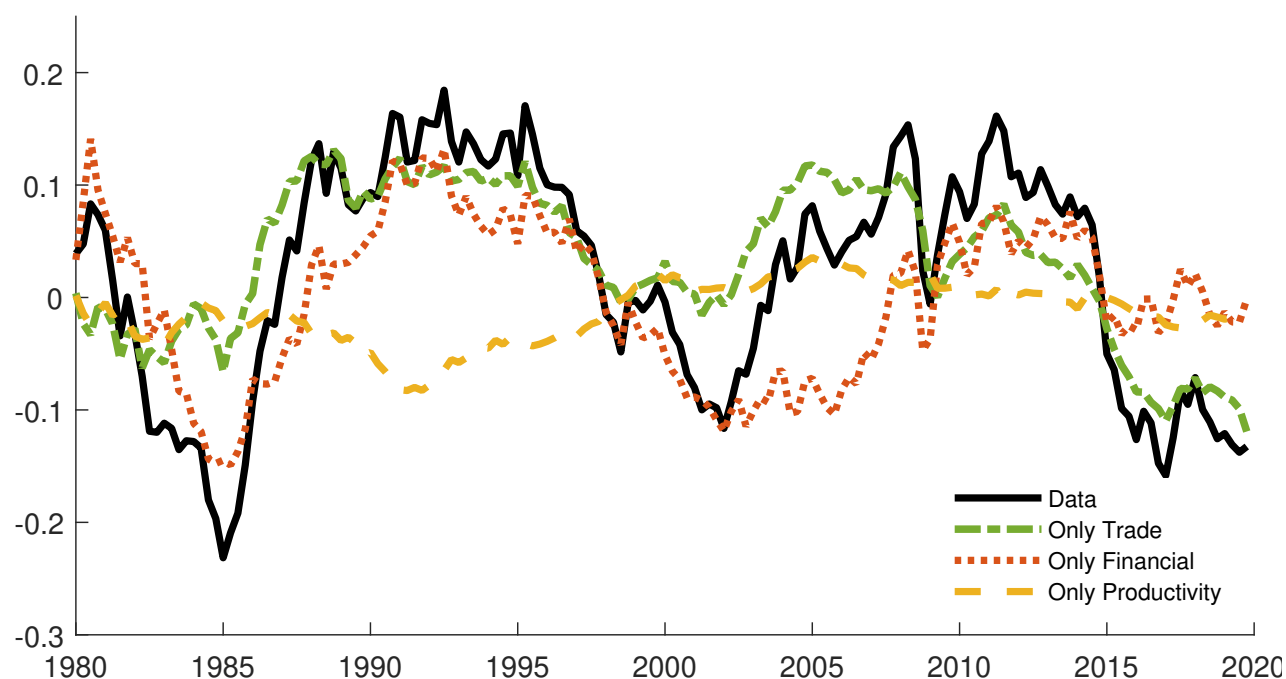
Notes: This figure shows the counterfactual path of RER with only one type of shocks. The productivity shocks include both the differential and common component.

closely related to the data. The correlation in this case is negative. Overall, we conclude that trade shocks generate a dynamics of the RER that more closely track the actual data.

We turn to look at the spectrum of the counterfactual cases with muting each shock. The result is presented in Figure D.3. The shape of the spectrum is disrupted the most when we shut down the trade shocks. Share accounted by the low frequency reduces from 61% in the Benchmark case to 58% without trade shocks. However, without financial shocks, it increases to 63%.

Finally, in Table D.3 we provide the conditional variance decomposition obtained from the Bayesian estimation of the dynamic trade model. In particular, we compute the share of the h–quarter ahead error forecast variance of the RER explained by each shock. It is clear that the trade shock explains most of the forecast error variance of the RER in the long run (i.e. low frequency), while the financial shock is important for the short run (i.e. high frequency) fluctuations.

Table D.2: Correlation between Data and Counterfactuals

	Only productivity	Only trade	Only financial
Corr(data, model)	-0.20	0.83	0.73

Figure D.3: RER Spectrum Under Different Shocks
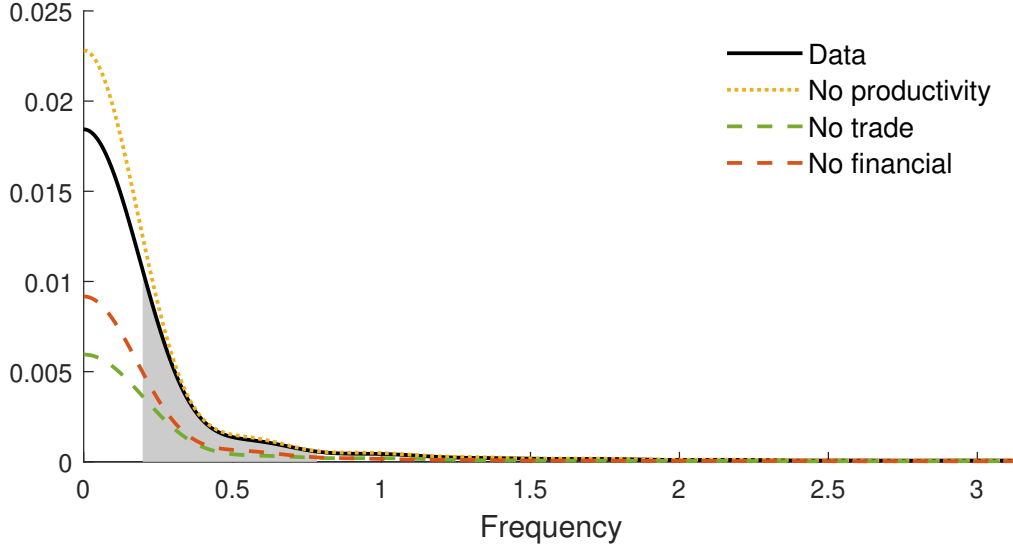
Notes: This figure shows the counterfactual spectrum of RER with only one type of shocks. The productivity shocks include both the differential and common components.

D.2 Dynamic Trade Specification

In this section, we consider the final good aggregator with adjustment costs in the use of imported inputs, as in Erceg et al. (2006), Rabanal and Rubio-Ramirez (2015) and Gornemann et al. (2020). The CES aggregator of the retail sector in each country is now given by

$$D_t = \left[Y_{Rt}^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} \left(\varphi_t Y_{Ut} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \qquad D_t^* = \left[Y_{Ut}^{*\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} \left(\varphi_t^* Y_{Rt}^* \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

where φ_t and φ_t^* is the weight on the use of imported inputs in the production of the final good. Their functional forms are given by

$$\varphi_t = \left[1 - \frac{\iota}{2} \left(\frac{Y_{Ut}/Y_{Rt}}{Y_{Ut-1}/Y_{Rt-1}} - 1 \right)^2 \right] \qquad \varphi_t^* = \left[1 - \frac{\iota}{2} \left(\frac{Y_{Rt}^*/Y_{Ut}^*}{Y_{Rt-1}^*/Y_{Ut-1}^*} - 1 \right)^2 \right].$$

where parameter ι determines the size of the adjustment cost in the use of imported inputs.

We identify the adjustment cost ι using the speed of adjustment of net trade to prices, i.e. the estimated parameter α in the ECM equation 6, which has a value of 0.07 in data. That is, on top of the other targeted moments, we add the speed-of-adjustment parameter to jointly estimate the parameters, including the new parameter ι (11 parameters and 11 moments). Since we compare

Table D.3: Conditional Variance Decomposition (%)

	quarters = 1	8	32	80
	Bayesian Estimation			
Financial shock	70.57	50.75	28.93	23.89
Trade shock	26.47	43.21	60.26	64.93
Productivity shock	2.96	6.03	10.80	11.18
	Benchmark Model			
Financial shock	62.37	45.42	26.00	26.30
Trade shock	34.71	49.39	64.43	62.84
Productivity shock	2.92	5.19	9.57	10.86

this model with our benchmark, we shut down trade dynamics that arises from the fixed costs of exporting , i.e. we set the fixed costs and idiosyncratic productivity to zero.

The parameters and their calibrated values are presented in Table D.4 under 'Input Adj.' The calibrated value of the input adjustment cost parameter ι is 5.45. This implies that when the share of home to foreign inputs $\frac{Y_{Ut}/Y_{Rt}}{Y_{Ut-1}/Y_{Rt-1}}$ deviates 10 percent from the steady state, they weight on foreign inputs is only $\varphi = 1 - \frac{\iota}{2}(0.1)^2 = 0.972$. Also, note that the volatility of shocks are relatively larger compared to the benchmark case. Not having direct amplification mechanism related to firm dynamics, the model requires much larger shocks to generate the volatility in macro variables.

In Table D.5, we label the column for the result of an alternative dynamic specification 'Input Adj.' The model is able to capture the for the speed of adjustment of net trade to prices ($\alpha = 0.07$). However, we note that our benchmark model has a superior performance over this alternative on matching the targeted moments. In particular, the alternative of input adjustment costs cannot replicate the autocorrelation of the interest rate differential nor the cross-country correlation of domestic absorption as well as our benchmark.

When turning into the not targeted moments, we find that this alternative model is able to generate a differential short and long run trade elasticity to prices.[43] The alternative model captures

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[43]Rabanal and Rubio-Ramirez (2015) also shows incorporating trade dynamics using input adjustment cost is crucial for the long-run elasticity that is larger than the short-run elasticity. They measure the trade elasticity based on the impulse response function to a TFP shock over different time horizons.

Parameters	Benchmark	Input Adj	No Within Cost	Inv Adj
Financial shock, volatility σ_ψ	0.002	5e-04	0.002	0.005
Financial shock, persistence ρ_ψ	0.96	0.98	0.91	0.83
Trade shock, volatility σ_ξ	0.06	0.02	0.05	0.09
Trade shock, persistence ρ_ξ	0.97	0.99	0.97	0.986
Trade shock, within-country share τ	0.17	0.09	0.00‡	0.17
Common productivity, volatility σ_{a_c}	0.004	0.005	0.005	0.004
Differential productivity, volatility σ_{a_d}	0.005	0.003	0.006	0.004
Adjustment cost of portfolios χ	0.02	0.01	0.004	$7e^{-04}$
Adjustment cost of capital κ	2.53	11.73	11.78	0.52*
Pricing to market parameter ζ	0.97	1.50	0.90	0.99
Import adjustment cost ι	0‡	5.45	0‡	0‡
Fixed cost of new exporters f_0	0.14	0‡	0.14	0.14
Fixed cost of incumbent exporters f_1	0.04	0‡	0.04	0.04
Volatility of idiosyncratic productivity σ_μ	0.15	0‡	0.15	0.15

Notes: Superscript ‡ denotes that the parameter is exogeneously set. 'Benchmark' shows the same results presented in Secton 5. 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section D.2). 'No Within Cost' is the case with no within-ROW trade cost shocks (Section D.3). 'Inv Adj' is the case with investment adjustment cost (Section D.4). Superscript * specifies that the calibrated adjustment cost is for investment not capital.

a closer short run elasticity of trade to prices, but has a worse fit for the long run elasticity.

Furthermore, we plot in Figure D.4 the spectrum of the RER in the data (solid black line), the benchmark model (dashed blue line) and the alternative input adjustment model (dashed red line). The alternative dynamic trade model does not capture the size of the spectrum as well as the benchmark model. Moreover, in Panel B of Table D.5 we show that the low frequency share in the spectrum of the RER is higher in the alternative dynamic trade model (0.75%). Hence, the benchmark model, where we exploit information from the microdata on firm dynamics, captures the shape of the spectrum of the RER better than the alternative dynamic trade model.

Finally, Table D.6 present the variance decomposition of this alternative model, under 'Input Adj.' We find a stronger role of financial shocks as drivers of the RER in the short run, relative to

our benchmark model. The contribution of financial shocks to the one-quarter ahead error forecast variance of the RER is around 85% in the alternative model, as opposed to 62% in our benchmark. However, in the long-run we find similar results as in our benchmark model for the contribution of financial and trade shocks in explaining the variation of the RER. We find that trade shocks explains around 63% of the 80-quarters ahead error forecast variance in the RER in both models. On the other hand, financial shocks explain around 31% in the alternative model, similar to the 26% found in our benchmark model. Hence, our main result holds: trade shocks are crucial to explain the low frequency variation in the RER, thus being crucial for capturing its overall variation.

Figure D.4: Spectrum Robustness
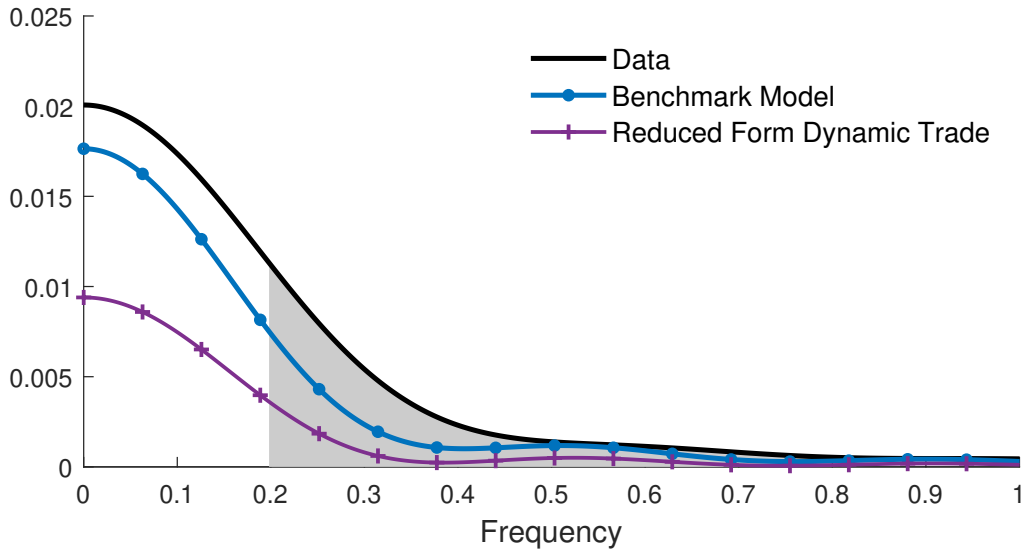
However, our finding about the importance of trade costs in the long run still holds. In Table D.6, the columns labeled 'Input Adj,' we show the variance decomposition of each shock. As in the benchmark case, the financial shocks play a dominant role in the earlier periods, accounting up to 85 percent in the first quarter. However, the share reduces to 31 percent in the 80th quater, and trade shocks account for the largest share of 63 percent.

D.3 Within-ROW Trade Costs

In this section, we evaluate the role of the within-ROW trade cost τ. We set up an alternative model where the elasticity of domestic trade costs to international costs is $\tau = 0$. Then, we calibrate the model by targeting the same moments as in the benchmark model, except the cross country

correlation of domestic absorption.

The calibrated parameters and resulting moments are reported in Tables D.4 and D.5 under 'No Within Cost.' This model generates a worse fit for the Backus-Smith correlation, which is 0.04 in the model as opposed to -0.10 in the data, although it lies within the estimated range in the literature. The model misses the cross country correlation of domestic absorption, being 0.07 in the model and 0.34 in the data. Thus, τ matters for accounting for the Backus-Smith puzzle and the cross country correlation of domestic absorption. Overall, this model has a worse fit into matching the target moments relative to our benchmark model.

In terms of the untargeted moments, we notice that the model delivers a differential short and long run elasticity of trade to prices, but but both significantly higher than in the data. Hence, our benchmark model better captures the differential short and long run elasticity of net trade to prices. The spectrum decomposition of this model is very close to that in the data, with the low frequency share being 0.59 in the model and 0.61 in the data.

In Table D.6 we present the results related to the variance decomposition of the RER, under '$\tau = 0$.' Our main results holds under this specification: financial shocks explain a higher portion of the variation in the RER in the short run, while trade shocks explains most of the variation in the long run. In particular, we find a stronger role of financial shocks in the short run relative to the benchmark model, since the contribution of this shocks to the 1-period ahead error forecast variance of the RER is 71% in this specification, compared to 62% in the benchmark model. however, financial shocks explain 29% of the 80-quarters ahead error forecast variance in the RER, close to the 26% in our benchmark model. Finally, trade shocks explains 59% of the 80-quarters ahead error forecast variance in the RER, close to the 63% in our benchmark model.

D.4 Capital Adjustment Cost

In this section, we consider an adjustment cost in investment as in Christiano et al. (2005). That is, the law of motion for capital is now given by

$$K_{t+1} = (1 - \delta)K_t + \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t$$

where $S(1) = S'(1) = 0$ and $S''(1) > 0$. Here, we consider the functional form of S as

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\tilde{\kappa}}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

To estimate the adjustment cost parameter $\tilde{\kappa}$, we again use the volatility of investment. That is, the targeted moments remain unchanged. The result of the estimated model with the new investment adjustment cost is presented in Tables D.4 and D.5, under 'Inv Adj.'

The estimated parameter for the adjustment cost is smaller than in the benchmark model, since now the adjustment cost is over a flow rather than a stock. This version of the model requires higher standard deviations of financial and trade shocks relative to our benchmark, where the variance of the common and differential productivity shocks are almost the same. We find that the adjustment cost of debt is smaller under investment adjustment costs. Finally, we find the same pricing to market coefficient as in the benchmark, with an implied pass-through of exchange rate to prices of 67%.

The model is able to match the target moments, as well as the untargeted moments. The short and long run elasticity of net trade to prices is also very similar to the benchmark model. Moreover, the investment adjustment cost model generates a higher share of the variance of the RER for the low frequency than in the benchmark model, hence the later captures better this aspect of the variation in the RER.

Finally, in Table D.6 we present the contribution of each shock to the error forecast avriance of the RER. COnsistent with our benchmakr model, we find that financial shocks explains most of the variation in the short run, while trade shocks explain most of it in the long run. In particular, financial shocks explains 68% of the one-quarter ahead error forecast variance, very close to the 62% in our benchmark model. On the other hand, trade shocks explains 86% of the 80-quarters ahead error forecast variance of the RER in this model, a bit higher than the 63% found in the benchmark model. Overall, our main results are robust to this alternative specification of investment adjustment cost.

Table D.5: Robustness – Model Results

Moments	(1) Data	(2) Benchmark	(3) Input Adj	(4) No Within Cost	(5) Inv Adj
A. Targeted Moments					
$\sigma(\Delta y)$	0.007	0.007	0.007	0.007	0.007
$\rho\,(\Delta c - \Delta c^*, \Delta q)$	-0.10	-0.10	-0.10	0.32	-0.10
$\rho\,(i - i^*)$	0.87	0.87	0.72	0.87	0.87
$\rho(\Delta y, \Delta y^*)$	0.40	0.40	0.40	0.42	0.40
$\rho(\Delta d, \Delta d^*)$	0.34	0.34	0.75	$0.11^\dagger$	0.34
$\rho(xm)$	0.98	0.95	0.96	0.94	0.98
$\sigma(\Delta inv^*)/\sigma(\Delta y^*)$	2.59	2.62	2.64	2.60	2.59
$\rho\,(\Delta xm, \Delta q)$	0.30	0.30	0.31	0.30	0.30
$\sigma(xm)/\sigma(q)$	1.16	1.16	1.13	1.17	1.16
$\rho(\Delta tot, \Delta q)$	0.49	0.49	0.49	0.48	0.49
B. Frequency Decomposition					
High frequency	0.08	$0.08^\dagger$	$0.06^\dagger$	$0.07^\dagger$	$0.06^\dagger$
Business cycle frequency	0.31	$0.23^\dagger$	$0.19^\dagger$	$0.21^\dagger$	$0.18^\dagger$
Low frequency	0.61	$0.69^\dagger$	$0.75^\dagger$	$0.72^\dagger$	$0.76^\dagger$
C. Trade Elasticity					
SR elasticity	0.20 (0.05)	$0.39^\dagger$	$0.32^\dagger$	$0.73^\dagger$	$0.39^\dagger$
LR elasticity	1.16 (0.25)	$0.97^\dagger$	$0.67^\dagger$	$1.19^\dagger$	$1.05^\dagger$
Adjustment	0.07 (0.02)	$0.04^\dagger$	0.07	$0.04^\dagger$	$0.02^\dagger$
D. Disconnect Puzzles					
$\sigma(q)$	0.10	0.09	0.06	0.09	0.20
$\sigma(\Delta q)/\sigma(\Delta y)$	4.24	3.54	1.55	3.01	4.83
$\rho(q)$	0.97	0.96	0.98	0.97	0.98
β_{fama}	-1.34	0.23	-0.16	-1.17	0.83
R^2_{fama}	0.04	0.002	0.02	0.11	0.01
$\rho(q, i - i^*)$	-0.30	-0.44	-0.04	-0.60	-0.43
$\rho(i)$	0.93	0.93	0.82	0.91	0.82
$\sigma(i - i^*)/\sigma(\Delta q)$	0.13	0.02	0.15	0.02	0.01

Notes: Superscript † denotes that the moment is not targeted during the calibration procedure. 'Benchmark' shows the same results presented in Secton 5. 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section D.2). $\tau = 0$' is the case with no within-ROW trade cost shocks (Section D.3). 'Inv Adj' is the case with investment adjustment cost (Section D.4).

Table D.6: Robustness – Variance Decomposition

	Benchmark			Input Adj			No Within Cost			Inv Adj		
Quarters	P	F	T	P	F	T	P	F	T	P	F	T
1	2.92	62.37	34.71	9.35	84.91	5.74	6.21	55.42	38.37	0.71	67.75	31.54
8	5.19	45.42	49.39	8.85	66.61	24.5	8.95	36.35	54.70	1.97	40.43	57.60
32	9.57	26.00	64.43	8.63	40.17	51.20	11.70	18.05	70.25	4.26	17.35	78.49
80	10.86	26.30	62.84	6.54	30.89	62.58	11.78	15.95	72.27	3.74	10.51	85.75

Notes: 'P,' 'F' and 'T' refer to share accounted by productivity shocks, financial shocks, and trade shocks, respectively. 'Benchmark' shows the same results presented in Secton 5. 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section D.2). 'No Within Cost' is the case with no within-ROW trade cost shocks, with $\tau = 0$ (Section D.3).

E   Theoretical Decomposition of Net Trade

In this section, we provide the derivation of net trade in our benchmark model. For simplicity, we omit the time subscript t.

The demand function for aggregate exports of ROW is given by

$$Y_R^* = \gamma \left(\frac{P_R^*}{P^*} \right)^{-\rho} D^*$$

where $P^* = 1$. The demand faced by a producer of each variety j is

$$y_{Rj}^* = \left(\frac{p_{Rj}^*}{P_R^*} \right)^{-\theta} Y_R^* = \gamma \left(\frac{p_{Rj}^*}{P_R^*} \right)^{-\theta} \left(\frac{P_R^*}{P} \right)^{-\rho} D^*$$

where the second equality uses the aggregate demand function. Using that total sales is a sum of sales of all varieties,

$$P_R^* Y_R^* = \int p_{Rj}^* y_{Rj}^* dj = \int \gamma p_{Rj}^{*\,1-\theta} P_R^{*\,\theta-\rho} D^* \, dj$$

$$= \gamma P_R^{*\,1-\rho} D^*.$$

Aggregate exports and imports in nominal terms are given by

$$X^N = Q \int_{j \in \mathcal{H}} p_{Rj}^* y_{Rj}^* dj = Q P_R^* Y_R^* = \gamma Q P_R^{*\,(1-\rho)} D^*$$

$$M^N = \int_{j \in \mathcal{H}^*} p_{Uj} y_{Uj} dj = \gamma P_U^{(1-\rho)} D$$

and the export and import prices are

$$Px = Q \left(\frac{1}{N} \int_{j \in \mathcal{H}} \left(\frac{p_{Rj}^*}{e^{\xi_R}} \right)^{1-\theta^*} dj \right)^{\frac{1}{1-\theta^*}} = Q \, P_R^* \, e^{\xi_R^*(\theta^*-1)} \, N^{\frac{-1}{1-\theta^*}}$$

$$Pm = \left(\frac{1}{N^*} \int_{j \in \mathcal{H}^*} \left(\frac{p_{Uj}}{e^{\xi_U}} \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}} = P_U \, e^{\xi_U(\theta-1)} \, N^{*\frac{-1}{1-\theta}}$$

where N denotes the mass of exporters. In logs,

$$x^N = \log \gamma + (1 - \rho)p_R^* + d^* + q$$

$$m^N = \log \gamma + (1 - \rho)p + d$$

$$px = q + p_R^* + \frac{1}{1 - \theta^*}n - (1 - \theta^*)\xi_R^*$$

$$pm = p_U + \frac{1}{1 - \theta}n^* - (1 - \theta)\xi_U$$

where lower case letters denote variables in logs.

Using that in real terms real exports and real imports are $X = X^N/Px$, $M = M^N/Pm$, respectively, log of exports and imports are given by

$$x = x^N - px = \log \gamma - \rho p_R^* + d^* - \frac{1}{1 - \theta^*}n$$

$$m = m^N - pm = \log \gamma - \rho p_U + d - \frac{1}{1 - \theta}n^*.$$

Therefore, log of Export-Import ratio $xm = \log XM$ is

$$xm = \rho(p_U - p_R^*) + (d^* - d) + \left(\frac{1}{1 - \theta}n^* - \frac{1}{1 - \theta^*}n \right)$$

$$= \rho\left(tot_t + q_t \right) + (d^* - d) + ((1 - \theta^*)\xi_R^* - (1 - \theta)\xi_U) + (1 - \rho)\left(\frac{1}{1 - \theta}n^* - \frac{1}{1 - \theta^*}n \right).$$

where $tot_t = pm - px$ is the terms of trade.

Comparing with Equation 5, the we have additional terms $((1 - \theta^*)\xi_R^* - (1 - \theta)\xi_U)$ and $(1 - \rho)\left(\frac{1}{1-\theta}n^* - \frac{1}{1-\theta^*}n \right)$. These reflect that in our model we have two features, trade shocks and trade dynamics.

F   Additional Figures and Tables

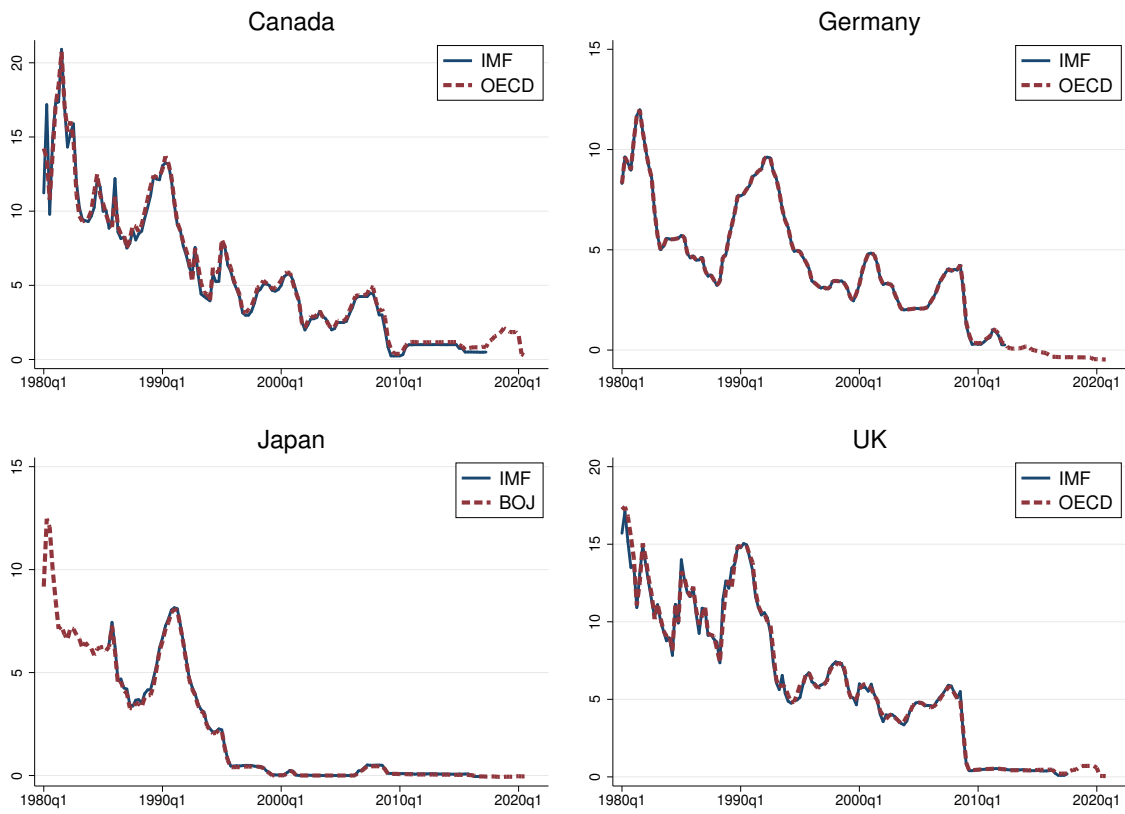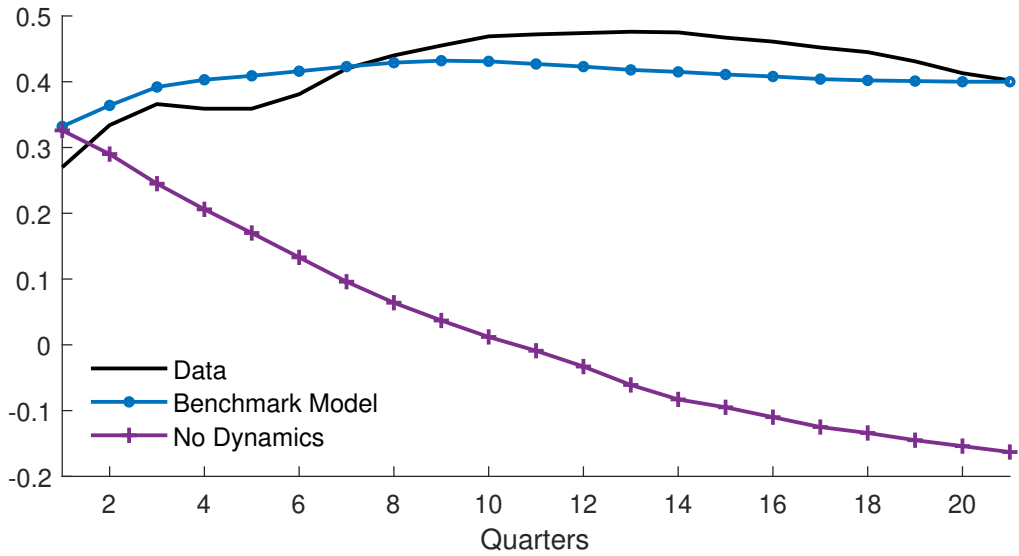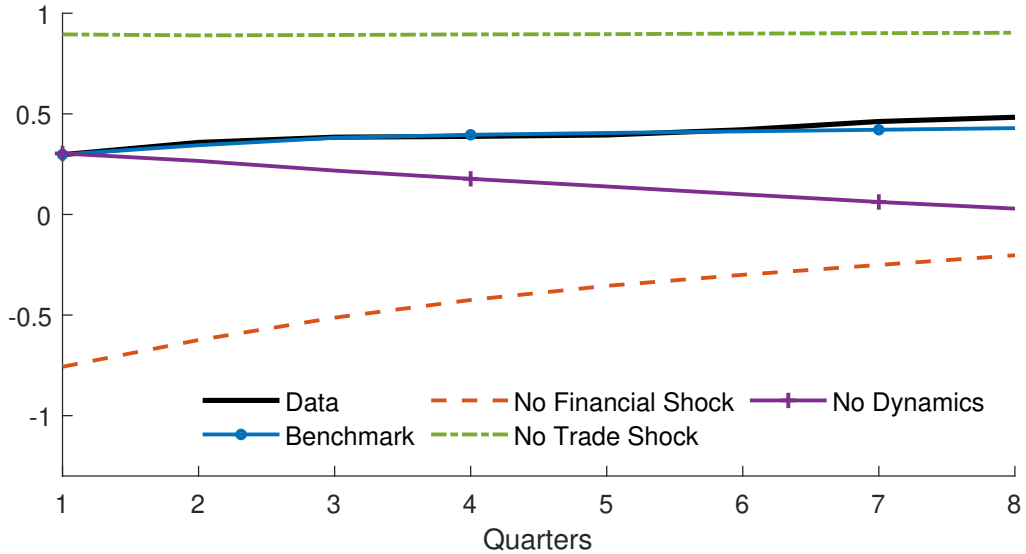Figure F.1: Data Source Comparison

Figure F.2: Dynamic Correlation between RER and Trade-Expenditure Ratio

Notes: The figure presents dynamic correlations as $\rho(\Delta_h q_t, \Delta_h TE_t)$, where q_t and TE_t are log of the RER and the trade-expenditure ratio, respectively. and Δ_h denotes h–period difference.

Figure F.3: Dynamic Correlation between RER and Net Trade

Notes: We calculate the dynamic correlations as $\rho(\Delta_h q_t, \Delta_h xm_t)$, where q_t and xm_t are log of the RER and the export-import ratio, respectively. and Δ_h denotes h–period difference. It present the results for the benchmark model and alternative models: no financial shock, no trade shock, and no trade dynamics.

Figure F.4: Counterfactual Spectrum
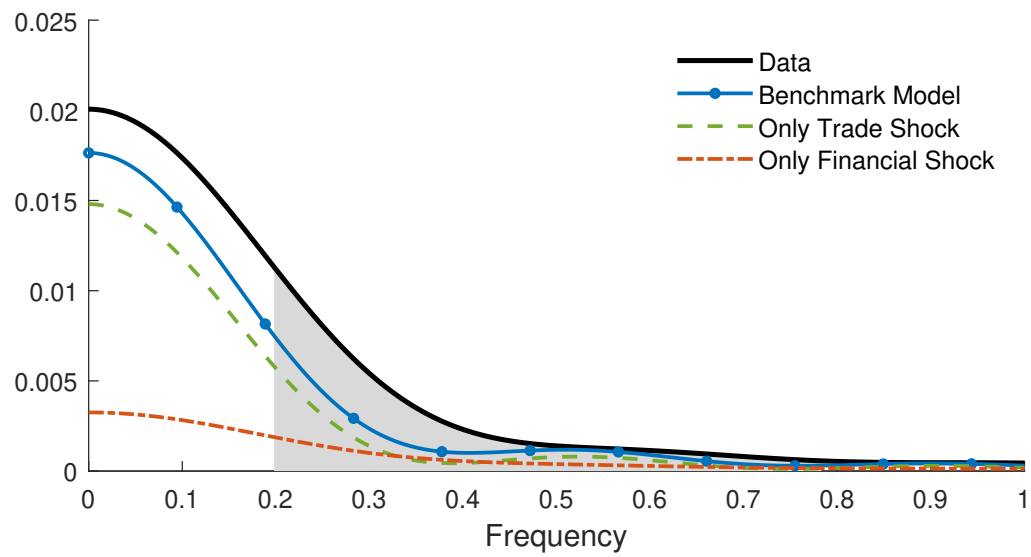
Notes: Spectral analysis of counterfactual models without re-calibrating, as our goal is to use the identified parameters from the benchmark model to perform exercises informative about the role of each shock at different frequencies. The graph is enlarged for the range [0,1] to show better the low and business cycle frequencies.

Table F.1: Calibrated Parameters - Alternative Models

Parameter		(1) Baseline	(2) No Trade Shock	(3) No Financial Shock	(4) No Dynamics
B. Producer Trade Parameters					
Fixed cost of new exporters	f_0	0.14	0.14	0.14	0‡
Fixed cost of incumbent exporters	f_1	0.04	0.04	0.04	0‡
Volatility of idiosyncratic productivity	σ_μ	0.15	0.15	0.15	0‡
C. Shocks, Adjustment Costs and Pricing to Market					
Common productivity, volatility	σ_{a_c}	0.004	0.005	0.004	0.005
Differential productivity, volatility	σ_{a_d}	0.005	0.006	0.005	0.002
Financial shock, volatility	σ_ψ	0.002	0.002	0‡	0.004
Financial shock, persistence	ρ_ψ	0.957	0.960	0‡	0.801
Trade shock, volatility	σ_ξ	0.060	0‡	0.042	0.056
Trade shock, persistence	ρ_ξ	0.968	0‡	0.998	0.970
Trade shock, within-country share	τ	0.170	0‡	0.250	0.074
Adjustment cost of portfolios	χ	0.020	0.006	0.000	0.003
Adjustment cost of capital	κ	2.530	7.682	1.074	12.697
Pricing to market parameter	ζ	0.972	0.969	1.041	1.455

Notes: The table presents the values of calibrated parameters of the benchmark and alternative models. When we consider an alternative models, some of the parameters are set to a different value while the other parameters are all recalibrated. Panel A is same as the baseline case presented in Table 1 for all models.

Table F.2: Share in Counterfactual Spectrum

	(1) Data	(2) Benchmark	(3) Trade Shock Only	(4) Financial Shock Only	(5) Prod Shock Only
Low frequency	0.61	0.69	0.74	0.54	0.75
BC frequency	0.31	0.23	0.20	0.34	0.19
High frequency	0.08	0.08	0.06	0.12	0.06

Table F.3: Fama Estimates in Data

Moments	Nominal	Real
β_{fama}	-1.15	-1.34
	(0.59)	(0.52)
R^2_{fama}	0.02	0.04

Notes: 'Nominal' denotes the results of using nominal data for the Fama regression, $\Delta e_{t+1} = \alpha + \beta_{Fama}(i^n_t - i^{n*}_t) + u_t$, where e is nominal exchange rate, and i^n is the nominal interest rate. 'Real' denotes the result of using real data for the regression (7).