Real Exchange Rate and Net Trade Dynamics: Financial and Trade Shocks*

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Abstract

This paper studies the drivers of the US real exchange rate (RER), with a particular focus on its comovement with net trade (NT) flows. We consider the entire spectrum of frequencies, as the low-frequency variation accounts for 61 and 64 percent of the unconditional variance of the RER and NT, respectively. We develop a generalization of the standard international business cycle model that successfully rationalizes the joint dynamics of the RER and NT while accounting for the major puzzles of the RER. We find that, while financial shocks are necessary to capture high frequency variation in the RER, trade shocks are essential for the lower frequency fluctuations.

JEL Classifications: E30, E44, F30, F41, F44
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1 Introduction

The real exchange rate (RER)[1] and net trade (NT)[2] flows are central variables for the transmission of business cycles across countries. While the RER reflects the prices that clear the international goods and asset markets, NT flows are the quantities traded in those markets. A comprehensive theory of international business cycles should thus capture both the RER and NT dynamics, particularly in the context of general equilibrium. However, the literature has predominantly focused on understanding RER dynamics, with limited attention given to NT.

The emphasis on the RER stems from the challenge that standard models face in explaining its behavior and the lack of connection with macro fundamentals at the high frequency – a feature known as exchange rate disconnect (Obstfeld and Rogoff, 2000). A recent strand of the literature has proposed a theory relying on shocks in financial markets that goes a long way in explaining the disconnect. (Devereux and Engel, 2002; Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021a). However, this approach has two key limitations due to its abstraction from lower-frequency variations.

First, it overlooks that the RER and NT present different patterns of comovement at high and low frequencies.[3] Figure 1 shows the path of the RER (blue) and NT (red) for the US. The trend of NT after applying the HP filter (solid red) closely follows that of the RER (solid blue), with a lag of around 6 quarters. That is, while the RER and NT exhibit a weak comovement at higher frequencies, they are highly correlated at lower frequencies.[4] Second, it fails to attribute the source of variation to the appropriate frequency. It is clear that the trend (solid red) of the RER drives a large share of its fluctuations. More precisely, our spectrum analysis shows that 61 percent of RER variance arises from lower-frequency movements.[5] Although

[1]The RER is defined as $\mathcal{Q}_t = \mathcal{E}_t P_t^*/P_t$ where $\mathcal{E}_t$ the nominal exchange rate (the price of home currency per unit of foreign currency), P_t^* is the foreign price level, and P_t the home price level. An increase in $\mathcal{Q}_t$ indicates a depreciation of the home RER.

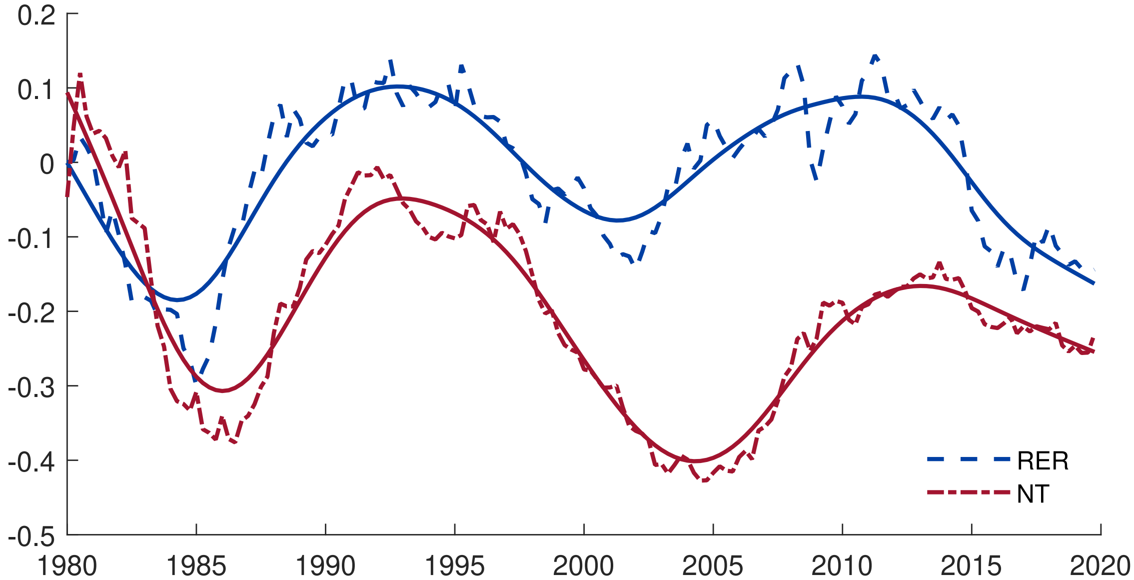
[2]We use the export-import ratio as a measure of NT, as opposed to trade balance as a share of GDP, because the former gives NT controlling for the scale of trade. The trade balance as a share of GDP can be written as $\frac{(X-M)}{Y} = \frac{(X-M)}{(X+M)} \times \frac{(X+M)}{Y}$, where $\frac{(X-M)}{(X+M)}$ is approximated by $0.5 \log X/M$ using the first-order Taylor approximation. We use $\log X/M$ to measure NT, addressing the concern that the changes in trade balance as a share of GDP are primarily due to the changes in the scale of trade (Alessandria and Choi, 2021; Alessandria, Bai and Woo, 2024). This approach aligns with our abstraction from average trade cost and emphasis on relative costs.

[3]Throughout the paper, whenever we refer to low or lower frequencies we specifically mean lower *than* business cycles, i.e. cycles that last longer than 32 quarters.

[4]The delayed movement is consistent with the so-called J-curve from the trade literature and has been robustly documented for the US and many other countries (Baldwin and Krugman, 1989; Rose and Yellen, 1989; Backus, Kehoe and Kydland, 1994; Fitzgerald, Yedid-Levi and Haller, 2019; Hooper, Johnson and Marquez, 2000; Alessandria et al., 2024). For example, Backus et al. (1994) document that in 11 developing countries, the comovement between NT and the terms of trade increases over time. Hooper, Johnson and Marquez (2000) document the delayed comovement in G7 countries. Alessandria, Bai and Woo (2024) also show using a panel of 36 countries during the period of 1970-2019 that the comovement is small in the short run but grows larger in the long run.

[5]This finding aligns with Rabanal and Rubio-Ramirez (2015), who find that 77 percent of the variance in the US RER is from the low-frequency. In Table F.1, we show the same pattern is found in a very large number of economies. We discuss the spectrum analysis in Section 5.3 and provide further details in Appendix B.

Figure 1: Real Exchange Rate and Net Trade Flows



Notes: RER is the log of the quarterly real exchange rates of the United States. Normalized with 1980q1=0. Effective exchange rate indices, Real, Narrow (BIS). NT is the log of Exports to Imports ratio for the United States. Exports and Imports are from Quarterly National Accounts (OECD). Solid lines plot the trend component of each variable from the Hodrick–Prescott filter with a smoothing parameter of 1600.

these two patterns are well-established and robustly observed in the US and many other countries, they have been largely overlooked in the current state-of-the-art international business cycle models. We argue that integrating lower-frequency variation is important for identifying the distinct drivers of the RER and NT.

In this paper, we provide a unified framework for studying the dynamics of the RER and NT flows across all frequencies. We generalize the standard two-country international business cycle model of Backus et al. (1994) by incorporating financial shocks following Itskhoki and Mukhin (2021a), shocks to the cost of trading goods across countries, and dynamic trade.[6] Our model captures the differential comovement between the RER and NT flows at different frequencies, the frequency decomposition of the RER and NT variance observed in the data, and the RER disconnect, along with the major business cycle moments. The omission of any key feature – financial shocks, trade shocks, or dynamic trade – results in the inability to simultaneously account for these empirical patterns. Using this framework we find that, consistent with the recent literature, financial shocks drive most of the variance of the RER at high frequencies. However, trade shocks are the dominant drivers at lower frequencies, precisely where most of the variance of the RER arises.[7]

[6]The framework in Backus et al. (1994) is similar to that in Stockman and Tesar (1995), and has been extended to various asset market structures by Heathcote and Perri (2002, 2014).

[7]Importantly, this result is not an artifact of a particular calibration resulting in identifying a higher variance or persistent of

We model trade shocks as stochastic iceberg trade costs, providing a tractable representation of changes in trade barriers. These barriers have been changing dramatically in the last decades, leading to a global trade integration. However, these changes have occurred at different timings and paces across countries. The time series of gross trade flows not only display an overall upward trend, but also present considerable fluctuations and differences in the growth rates among countries.[8] We focus on the differential component between outward and inward trade costs for the US and the rest of the world (ROW) and abstract from the average cost, as the latter would have no effect on relative measures such as the RER and NT.[9,10]

Trade shocks capture many different sources of fluctuations in barriers to trading goods and services across countries. For instance, numerous episodes of trade liberalization, including those of China, have been accompanied by substantial reduction in both tariff and non-tariff barriers like quotas and sanctions (Obstfeld and Rogoff, 2000; Delpeuch, Fize and Martin, 2021). There have also been asymmetric trade reforms, like GATT rules, and temporary policies, such as Reagan's export restraints on Japanese automobiles. Expectations and uncertainty about future policy can also act as a barrier to trade by affecting firms' investment and exporting decisions (Caldara, Iacoviello, Molligo, Prestipino and Raffo, 2020). Furthermore, technological advancements in shipping and transportation have considerably reduced the cost of international trade (Burstein, Neves and Rebelo, 2003; Corsetti and Dedola, 2005; Corsetti, 2016). More recently, geopolitical conflicts have led to the blockage of trade routes and fluctuations in oil prices. In a recent paper, Itskhoki and Mukhin (2022) find that changes in trade barriers are an important driver of the ruble exchange rate following Russia's invasion of Ukraine in 2022. The COVID-19 pandemic and environmental issues such as droughts affecting the Panama Canal have also contributed to changes in trade costs. Given the abundance of these incidents, the relative size of trade costs across countries fluctuates significantly at a high frequency, emerging as an essential source of NT and RER variation.

Incorporating trade shocks along with financial shocks enables the model to accurately capture the high-frequency dynamics of the RER and NT. Absent trade shocks, where the RER and NT are dominated by financial shocks, the model fails to generate the low correlation between the two variables and result in excessively volatile NT flows.[11] When financial shocks generate a higher return on bonds for the US

trade shocks relative to financial shocks, but an implication of the general equilibrium effects of the two different shocks. We explain this in detail in Section 6.2.

[8] See Figure F.1.

[9] In our quantitative exercise, the ROW aggregate includes Canada, Finland, Germany, Ireland, Italy, Japan, Republic of Korea, Spain, Sweden and United Kingdom. This set of countries represents 60 percent of total US trade on average. The estimated moments from the data are robust to having an unbalanced panel that includes China since 1990. For more details, see Appendix A.

[10] We also show that our quantitative results are robust to incorporating the average trade costs in Section 7.

[11] Itskhoki and Mukhin (2021a) show that adding Dollar currency pricing and nominal rigidities has almost no effect on the dynamics of NT and the RER induced by the financial shock.

relative to the ROW, savings in the US increase, and the excess savings is exported to the ROW (US NT increases). At the same time, due to the fall in aggregate demand in the US, the final good price falls (the US RER depreciates). Hence, financial shocks induce a positive correlation between the RER and NT on impact. On the other hand, a trade shock that raises the relative cost of exporting for the US leads to a decline in its NT. With less intermediate goods exported and more imported, the supply of final goods in the US increases, causing its price to fall (the US RER depreciates). Consequently, trade shocks induce a negative correlation between the RER and NT on impact, offsetting the positive effect of financial shocks. Furthermore, trade shocks generate a higher volatility of the RER relative to that of NT, while the opposite happens under financial shocks. Hence, having both shocks allows the model to capture the high frequency properties of the RER and NT flows.

We incorporate dynamic trade following Alessandria and Choi (2007, 2021) by assuming that intermediate producers are heterogeneous in their idiosyncratic productivity and decide whether to participate in the export market or not, subject to a fixed cost of exporting.[12] We assume that the fixed cost is lower for incumbents than for new exporters, which makes the exporting decision forward-looking. Consequently, the distribution of exporters evolve slowly in response to shocks and aggregate trade flows respond gradually over time. This allows the model to capture the differential short- and long-run comovement between the RER and NT. It also contributes to accounting for the frequency decomposition of the variance of the RER. In our benchmark model, the share of the variance of the RER attributed to the low-frequency variation is 70 percent, closely matching the 61 percent observed in the data. Without dynamic trade, this share increases to 75 percent. This arises from dynamic trade making quantities in the short run more inelastic than under static trade. As a consequence, prices in the short run, relative to the long run, have a stronger response, redistributing the share of variation in the RER from lower to higher frequencies.[13]

Finally, we use our quantitative model to study the relative importance of financial and trade shocks in inducing variation in the RER across different frequencies. We compute the contribution of shocks to the forecast error variance of the RER at different horizons. We find that financial shocks explain 63 percent of the one-quarter ahead error forecast variance, with trade shocks explaining 35 percent. However, when focusing at the eighty-quarters ahead, trade shocks explain 65 percent, while financial shocks account for 26 percent. The more persistent equilibrium effect induced by trade shocks over financial shocks arises from the propagation of the former through the resource constraint, rather than differences in the proper-

[12] Alessandria and Choi (2007, 2021) extends the sunk cost model of exporting of Dixit (1989), Baldwin and Krugman (1989) and Das, Roberts and Tybout (2007) to a general equilibrium framework. Other papers that extended the framework in Backus et al. (1994) to have dynamic trade are Drozd and Nosal (2012), Erceg, Guerrieri and Gust (2006) and Engel (2011).

[13] This is consistent with the "Excess Persistence Puzzle" documented in Rabanal and Rubio-Ramirez (2015), which refers to static trade models having an excess share of low frequency variation in the RER.

ties of the processes. Since most of the variation in the RER arises at lower frequencies, we conclude that trade shocks are crucial for capturing the overall dynamics of the RER.

The remainder of the paper is structured as follows. Section 2 reviews the literature. Section 3 presents our benchmark model, while Section 4 discusses the calibration and identification strategy. Section 5 demonstrates the success of the benchmark model in capturing targeted and untargeted moments related to the RER and NT dynamics at all frequencies. Section 6 studies the role of different shocks in explaining the variation of the RER. Section 7 discusses the robustness of our result to alternative specifications. Finally, Section 8 presents the concluding remarks.

2 Literature Review

Our paper bridges the gap between the studies in international finance and international trade, by developing a theory that is consistent with both the RER and NT dynamics. On one hand, there is a growing literature emphasizing the role of financial shocks for understanding the dynamics of exchange rates, with a focus on the macro and financial disconnect (Devereux and Engel, 2002; Gabaix and Maggiori, 2015; Farhi and Gabaix, 2016; Itskhoki and Mukhin, 2021a).[14] On the other hand, a series of papers have explored the role of trade barriers in explaining the variation in trade and financial flows across countries (Obstfeld and Rogoff, 2000; Eaton, Kortum and Neiman, 2016; Reyes-Heroles, 2016; Alessandria and Choi, 2021; Sposi, 2021; Alessandria, Bai and Woo, 2024).[15] In our study, we generalize the framework in Backus et al. (1994) by integrating financial shocks, trade shocks, and dynamic trade.[16] This unified approach not only enhances our understanding of the outcomes presented in both strands of the existing literature but also deepens our comprehension of the economic dynamics at play. As emphasized in the financial literature, we find that financial shocks are important for high-frequency fluctuations of the RER and the financial disconnect. On the other hand, dynamic trade and trade shocks are crucial for accounting for low-frequency movements of the RER and its comovement with NT.

A distinctive feature of our work is that we incorporate the low frequency variation in the RER and NT. Only a limited number of papers studying the dynamics of the RER in general equilibrium have focused on this frequency. Rabanal and Rubio-Ramirez (2015) show that a reduced form dynamic trade model with

[14] While this literature discusses the dynamics of both the real and nominal exchange rates, we limit our interest to real variables.

[15] Ayres, Hevia and Nicolini (2020) explore the role of commodity prices in driving the variation of the RER and the Backus-Smith-Kollmann correlation in developed economies. Our framework does not include a commodity sector, but variation originated in this sector is most likely to be captured as changes in the trade costs in our model, as they reflect changes in the cost of trading intermediate goods across countries.

[16] Our work is also related to that in Heathcote and Perri (2014), which provides a comprehensive analysis of the Backus et al. (1994) framework under different parametrizations and various asset structures, Heathcote and Perri (2002), Stockman and Tesar (1995) and Baxter and Crucini (1995).

non-stationary cointegrated productivity shocks is able to capture the spectrum of the RER.[17] Gornemann, Guerrón-Quintana and Saffie (2020) propose an alternative mechanism relying on endogenous spillovers that amplify stationary fluctuations.[18] We share with these papers the focus on the low frequency variation of the RER and the importance of dynamic trade.[19] We differ from them in the way we model dynamic trade, which we do with a microfoundation based on firms' dynamic exporting decisions. Moreover, we propose an alternative mechanism to account for the low-frequency variation observed in the RER. Our explanation relies on shocks to the cost of trade that induce persistent changes in NT flows and international relative prices, as opposed to productivity shocks.

Finally, our paper is related to the literature on the measurement of trade wedges. Levchenko, Lewis and Tesar (2010), Fitzgerald (2012) and Alessandria, Kaboski and Midrigan (2013a) measure trade wedges based on the Armington model to study the role of trade costs and asset market frictions for international risk sharing. Head and Mayer (2014) explore different methods of estimating the gravity equation. We contribute to this literature by considering a specification of trade costs that allows for a within-ROW component, and highlight its implications for the comovement of the RER and macro aggregates.

3 Model

We build on the two-country international business cycle model of Backus et al. (1994) and Itskhoki and Mukhin (2021a). The two countries are the ROW and the US, each producing a perfectly competitive non-traded final good. The non-traded final good is made of a mix of tradable intermediates, using a CES technology with home bias.[20] The final good can be consumed or invested by the household, and capital accumulation is subject to capital adjustment costs.

There is a unit mass of intermediate good producers in each country, producing differentiated varieties. They are subject to aggregate productivity shocks and are heterogeneous in their idiosyncratic productivity. They make decisions on entering, staying or exiting the export market, subject to the fixed costs that depends on the experience in the export market as in Dixit (1989), Baldwin and Krugman (1989), Das et al.

[17] Drozd, Kolbin and Nosal (2021) show that dynamic trade is a key feature to improve the model's ability to account for the trade comovement puzzle, i.e. the significant relationship in the data between countries' business cycles synchronization and trade flows.

[18] Corsetti, Dedola and Viani (2012) also study the RER dynamics at the frequency domain through spectral analysis, but focus on the low frequency disconnect between the RER and relative consumption (Backus-Smith-Kollmann Puzzle). Cao, Evans and Luo (2020) study the medium to long run dynamics of the US-UK RER and highlight the role of persistent productivity shocks, incomplete financial markets and a high Armington elasticity in accounting for its dynamics.

[19] We also share with Gornemann et al. (2020) the importance of using trade data to discipline the model parameters.

[20] Itskhoki and Mukhin (2021a) emphasizes the importance of incomplete pass-through of the financial shock mechanism, which they model using a Kimball aggregator. Even though we use a CES aggregator, we model incomplete pass-through by adding frictions in the pricing to market behavior of firms.

(2007), Alessandria and Choi (2007), and Alessandria and Choi (2021). Intermediate firms set destination specific prices, and use labor and capital as inputs of production. Optimal prices are set as a markup over the marginal cost. We introduce time-varying markups, capturing pricing to market frictions in a reduced form, which leads to persistent deviations from the law of one price. Intermediate firms also face stochastic iceberg trade costs, depicted as only a fraction of goods shipped arriving at the destination.

On the asset side, there is an internationally traded bond, denominated in dollars. The ROW household is subject to a bond adjustment cost, which induces stationarity of the model and captures portfolio re-balancing costs in a reduced form. The ROW household is also subject to a financial shock, capturing the shock to uncovered interest parity of Itskhoki and Mukhin (2021a). We describe below the model from the point of view of ROW agents.

Households

The representative household in the ROW maximizes the discounted expected utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\eta} (1 - L_t)^{1-\eta}\right]^{1-\sigma}}{1 - \sigma}$$

where C_t is consumption, L_t is labor, η is the weight on consumption, β is the discount factor, and $1/\sigma$ is the intertemporal elasticity of substitution. The flow budget constraint is given by

$$P_t (C_t + I_t) + B_{t+1} + \frac{\mathcal{E}_t B^*_{t+1}}{e^{\psi_t}} + \mathcal{E}_t \frac{\chi}{2} \left(B^*_{t+1} - \bar{B}\right)^2 \leq W_t L_t + R^k_t K_t + B_t(1 + i_{t-1}) + \mathcal{E}_t B^*_t (1 + i^*_{t-1}) + \Pi_t$$

where P_t is the price index, I_t is investment, B_{t+1} is the quantity of ROW bonds (zero net supply), K_t is capital, i_{t-1} is the nominal interest rate on ROW bonds purchased at $t - 1$, and Π_t is aggregate profits of intermediate firms. On the international asset side, B^*_{t+1} is the quantity of the internationally traded bonds held by the ROW household, i^*_{t-1} is the nominal interest rate on international bonds purchased at $t - 1$, and $\mathcal{E}_t$ is the nominal exchange rate, measured as the price of the ROW currency per unit of US currency. The term ψ_t is the financial shock, χ is the adjustment cost of internationally traded bonds, and $\bar{B}$ is the steady state level of net foreign assets.[21]

The stock of capital in each country follows the law of motion,

$$K_{t+1} = (1 - \delta) K_t + \left[I_t - \frac{\kappa}{2} \frac{(\Delta K_{t+1})^2}{K_t}\right],$$

[21]The financial shock ψ_t only affects the ROW household, hence generating a differential return on internationally traded bonds for ROW and US households. Our result is invariant to whether the shock ψ_t affects the adjustment cost of debt or not. Our results are also invariant to whether the the nominal exchange rate is part of the adjustment cost term or not.

where the parameter κ governs the adjustment cost of capital.

The solution of the ROW household can be characterized by the labor supply condition and the Euler equations for ROW and international bonds and capital. The stochastic discount factor of the ROW household between t and $t+1$ is given by

$$\Omega_{t,t+1} \equiv \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^{\eta}(1-L_{t+1})^{1-\eta}}{C_t^{\eta}(1-L_t)^{1-\eta}} \right)^{1-\sigma} \frac{C_t}{C_{t+1}} \right].$$

From the log-linearized Euler equations of the ROW household for ROW and international bonds, we can derive an equation for the deviations from the uncovered interest parity (UIP) condition,

$$i_t - i_t^* - \mathbb{E}_t \left[\Delta e_{t+1} \right] = \psi_t - \chi \cdot \left(B_{t+1}^* - \bar{B} \right) \tag{1}$$

where $\mathbb{E}_t \left[\Delta e_{t+1} \right] \equiv \mathbb{E}_t \left[\ln \mathcal{E}_{t+1} - \ln \mathcal{E}_t \right]$ is the expected change of the nominal exchange rate. The financial shock ψ_t propagates to the economy by inducing deviations to the UIP condition. While we model the financial shock as an exogenous shock, the derived UIP condition is equivalent to those in models with segmented financial markets, noisy traders or limits to arbitrage (Itskhoki and Mukhin, 2021a; De Long, Shleifer, Summers and Waldmann, 1990; Jeanne and Rose, 2002; Gabaix and Maggiori, 2015).[22] Finally, the second term on the right hand side captures the deviations from UIP that arise endogenously through the effects on the net foreign assets.[23]

Aggregation Technology

A competitive retail sector combines intermediate goods from the ROW and the US with a constant elasticity of substitution (CES) to produce the final good, D_t, which can be consumed or invested. The CES aggregator is given by

$$D_t = \left[Y_{Rt}^{\frac{\gamma-1}{\gamma}} + \omega^{\frac{1}{\gamma}} Y_{Ut}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$$

where Y_{Rt} is the quantity of domestic goods consumed in the ROW, Y_{Ut} is the quantity of imported goods from the US consumed in the ROW, ω captures the home bias, and γ is the Armington elasticity between domestic and imported composite goods.

[22]Financial shocks can also be microfounded by risk-premia (Verdelhan, 2010; Colacito and Croce, 2013; Farhi and Gabaix, 2016) or heterogeneous beliefs and expectational errors (Evans and Lyons, 2002; Gourinchas and Tornell, 2004; Bacchetta and Van Wincoop, 2006).

[23]While we discipline with data the size of the adjustment cost χ in Section 4, we do not find that the endogenous component of the deviations from UIP is quantitatively important.

The total expenditure in the retail sector is given by

$$P_t D_t = P_{Rt} Y_{Rt} + P_{Ut} Y_{Ut}$$

where P_{Rt} is the price of domestic goods in the ROW, and P_{Ut} is the price of imported goods in the ROW.

The problem of the retail sector is to minimize expenditure on intermediate goods subject to the CES aggregator, by choosing quantities $\{Y_{Rt}, Y_{Ut}\}$. The final good is used by households for either consumption or investment so that $D_t = C_t + I_t$. Solving this maximization problem yields the demand functions for ROW and US composite goods, given by

$$Y_{Ut} = \omega \left(\frac{P_{Ut}}{P_t}\right)^{-\gamma} (C_t + I_t) \quad \text{and} \quad Y_{Rt} = \left(\frac{P_{Rt}}{P_t}\right)^{-\gamma} (C_t + I_t)$$

where P_t is given as

$$P_t = \left[P_{Rt}^{1-\gamma} + \omega P_{Ut}^{1-\gamma}\right]^{1/(1-\gamma)}.$$

The domestic and imported goods, Y_{Rt} and Y_{Ut}, are the composite of varieties produced by heterogeneous producers. The aggregators are

$$Y_{Rt} = \left(\int_0^1 y_{j,Rt}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} \qquad Y_{Ut} = \left(\int_{j\in\mathcal{H}_t^*} y_{j,Ut}^{\frac{\hat{\theta}_t-1}{\hat{\theta}_t}} dj\right)^{\frac{\hat{\theta}_t}{\hat{\theta}_t-1}} \tag{2}$$

where θ and $\hat{\theta}_t$ are the elasticity of substitution across varieties, and $\mathcal{H}_t^*$ is the set of exporting firms in the US. Thus the demand function for each variety is given by

$$y_{j,Rt} = \left(\frac{p_{j,Rt}}{P_{Rt}}\right)^{-\theta} Y_{Rt} \qquad y_{j,Ut} = \left(\frac{p_{j,Ut}}{P_{Ut}}\right)^{-\hat{\theta}_t} Y_{Ut}. \tag{3}$$

The price indexes for the composite goods are given by

$$P_{Rt} = \left(\int_{j=0}^1 p_{j,Rt}{}^{1-\theta}\right)^{\frac{1}{1-\theta}} \qquad P_{Ut} = \left(\int_{j\in\mathcal{H}_t^*} p_{j,Ut}{}^{1-\hat{\theta}_t}\right)^{\frac{1}{1-\hat{\theta}_t}}.$$

Note that firms set destination specific prices, subject to the demands that differ across destinations due to the time-varying elasticity for the imported varieties. We let the elasticity of substitution across imported varieties to vary with the RER. Specifically, we set $\hat{\theta}_t = \theta Q_t^{\zeta}$, where Q_t is the RER, and symmetrically the elasticity of substitution across exported varieties is $\hat{\theta}_t^* = \theta Q_t^{-\zeta}$. This captures pricing-to-market behavior of firms in reduced form, where an appreciation of the US RER increases the markups charged by ROW firms to exports to the US. This is consistent with the findings in Alessandria and Kaboski (2011) that shows that firms price to income, charging higher prices to higher income destinations.[24] The pricing-to-market allows the model to capture the incomplete pass-through of exchange rates to prices, which trigger persistent deviations from the law of one price, and to generate a volatility of the terms of trade that is smaller than that of the RER, as in the data.[25,26]

The problem of the US retailers is given in a symmetric form

$$\max_{\{Y^*_{Ut}, Y^*_{Rt}\}} P^*_t (C^*_t + I^*_t) - [P^*_{Ut} Y^*_{Ut} + P^*_{Rt} Y^*_{Rt}]$$

subject to the CES aggregator, resulting in the demand functions of

$$Y^*_{Rt} = \omega \left(\frac{P^*_{Rt}}{P^*_t} \right)^{-\gamma} (C^*_t + I^*_t) \quad \text{and} \quad Y^*_{Ut} = \left(\frac{P^*_{Ut}}{P^*_t} \right)^{-\rho} (C^*_t + I^*_t).$$

Intermediate Firms

There is a continuum of heterogeneous firms indexed by $j \in [0, 1]$ in each country, specializing in the production of a differentiated intermediate good. There is monopolistic competition among these firms. The firms are subject to aggregate and firm-specific shocks. The firm j's production function is given by

$$y_{jt} = e^{a_t + \mu_{jt}} l^{\alpha}_{jt} k^{1-\alpha}_{jt},$$

where α is the capital share of income, a_t is the productivity shock, and μ_{jt} is a idiosyncratic firm-specific shock, $\mu \overset{iid}{\sim} N(0, \sigma^2_\mu)$. All firms sell their products in their own country, while some of them choose to export. The resource constraint of a firm is given by

$$y_{jt} = e^{\xi_{Rt}} y_{j,Rt} + m_{jt} e^{\xi^*_{Rt}} y^*_{j,Rt}$$

where $y_{j,Rt}$ is ROW variety used domestically, $y^*_{j,Rt}$ is ROW variety exported to the US, ξ_{Rt} is the stochastic iceberg cost for domestic trade within the ROW countries, ξ^*_{Rt} is the stochastic iceberg cost for ROW

[24] The pricing-to-market generates time-varying markups in a similar way as with a Kimball aggregator, as in Itskhoki and Mukhin (2021a), and can be microfounded with search frictions. See Edmond, Midrigan and Xu (2018) for a study of heterogeneous firm with the Kimball aggregator. On the other hand, Drozd and Nosal (2012) provide an alternative model of pricing to market where firms invest in marketing activities in order to accumulate customers.

[25] See Raffo (2008) for an analysis on the counterfactual dynamics of the terms of trade in the standard two-country international business cycle model.

[26] Omitting the pricing to market friction do not change the main results of the paper.

exports to the US, and $m_{jt} \in \{0,1\}$ is the current export status of firm j, with $m_{jt} = 1$ being export and $m_{jt} = 0$ not export. Note that we are considering a case of iceberg costs that allows for the iceberg trade cost within the ROW, ξ_{Rt}, to be nonzero. This takes into account that the ROW is an aggregate of multiple countries that trade with each other. In order to capture the average trade cost within the ROW countries, we relax the constraint of a standard specification with zero domestic iceberg costs.[27,28]

In order to export, firms must pay a fixed cost, denominated in units of labor. The fixed cost for starting to export differs from the fixed cost to stay in the export market. To start exporting, a firm pays a cost of $W_t f^0$, while an incumbent exporter pays the continuation cost of $W_t f^1$, with $f^1 < f^0$. That is, there is a sunk cost associated with export participation, capturing exporter hysteresis and the slow response of aggregate exports to shocks.

An intermediate good producer in the ROW is described by its idiosyncratic productivity and past export status, (μ_{jt}, m_{jt-1}). The aggregate state which includes the aggregate productivity, trade and financial shock, and the endogenous assets and distribution of exporters and non-exporters is subsumed in the time subscript of the value function. The dynamic problem of a firm is,[29]

$$V_t(\mu_{jt}, m_{jt-1}) = \max_{\left\{m_{jt}, p_{j,Rt}, p^*_{j,Rt}, l_{jt}, k_{jt}\right\}} p_{j,Rt} y_{j,Rt} + m_{jt} \mathcal{E}_t p^*_{j,Rt} y^*_{j,Rt} - W_t l_{jt} - R^k_t k_{jt} - m_{jt} W_t f^{m_{jt-1}} + \mathbb{E}_t \Omega_{t,t+1} V_{t+1}(\mu_{jt+1}, m_{jt})$$

subject to the ROW retailer's demand for ROW intermediates, $y_{j,Rt}$, the US retailer's demand for ROW intermediates, $y^*_{j,Rt}$, and the resource constraint. The static optimality conditions of the firm are given by the optimal demand for inputs and optimal pricing,

$$\begin{aligned} W_t &= (1-\alpha)\frac{y_{jt}}{l_{jt}} \quad \text{and} \quad R^k_t = \alpha \frac{y_{jt}}{k_{jt}} \\ p_{j,Rt} &= e^{\xi_{Rt}} \frac{\theta}{\theta-1} MC_{jt} \quad \text{and} \quad \mathcal{E}_t p^*_{j,Rt} = e^{\xi^*_{Rt}} \frac{\theta Q_t^{-\zeta}}{\theta Q_t^{-\zeta} - 1} MC_{jt} \end{aligned} \tag{4}$$

where the $MC_{jt} = \frac{1}{e^{a_t+\mu_{jt}}} \frac{\left(R^k_t\right)^{\alpha} (W_t)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$ is the marginal cost. Note that firms set different prices across destinations, since they face different demands at home and foreign. Moreover, note that the pricing to market friction, ζ, generates deviations from the law of one price that are proportional to the RER.[30]

[27]We explain in more detail the role of the within country trade cost when we present the shock processes.

[28]In Appendix E.6, using a three country model we show that this shock operates qualitatively in the same way as a trade shock between two ROW countries.

[29]Intermediate firms discount the future using the household stochastic discount factor.

[30]In particular, the deviations from the law of one price are given by $\frac{\ln\left(Q_t p^*_{j,Rt}/p_{j,Rt}\right)}{\ln Q_t} \propto \frac{\zeta}{\theta-1}$. This implies an exchange rate pass-through of $\left[\frac{(\theta-1)-\zeta}{(\theta-1)} \times 100\right]$ percent at the steady state.

Furthermore, the fixed cost $f^{m_{jt-1}}$ that a firm pays depends on its exporting status in the previous period m_{jt-1}. Thus, we can solve for the threshold productivity to participate in the export market depending on its previous status: μ_t^1 and μ_t^0 for those who were exporting and were not in the previous period, respectively. At the threshold, a firm is indifferent between exporting and not exporting. Hence, a firm will decide to participate in the export market only if its productivity is above the threshold. The thresholds satisfy

$$W_t f^m - \pi^* (\mu_t^m) = \mathbb{E}_t \left[\Omega_{t,t+1} \left(V_{t+1}(\mu_{t+1}, 1) - V_{t+1}(\mu_{t+1}, 0)\right)\right], \qquad m \in \{0, 1\}$$

where $\pi^* (\mu_t^m)$ is the static profit from exporting for a firm with idiosyncratic productivity $\mu_{jt} = \mu_t^m$, given as

$$\pi^*(\mu_{jt}) = \mathcal{E}_t \; p^*_{j,Rt}(\mu_{jt}) \; y^*_{j,Rt}(p^*_{j,Rt}(\mu_{jt}))$$

with $p_{j,Rt}$ and $y_{j,Rt}$ from Equations 3 and 4 as functions of the idiosyncratic productivity μ_{jt}. Since the fixed cost is higher for a new exporter than for an incumbent exporter, $f^0 > f^1$, the productivity threshold is higher for the former than the latter, $\mu_t^0 > \mu_t^1$.

The presence of sunk costs of exporting generates a slow moving distribution of aggregate exporters, N_t. The law of motion of aggregate exporters is given by,

$$N_t = N_{t-1} \cdot P\left[\mu_{jt} > \mu_t^1\right] + (1 - N_{t-1}) \cdot P\left[\mu_{jt} > \mu_t^0\right].$$

The aggregate labor and capital demands from intermediate firms are given by

$$\begin{aligned} L_t &= \int_{j=0}^{1} l_{jt} + f^0 \cdot (1 - N_{t-1}) \cdot P\left[\mu_{jt} > \mu_t^0\right] + f^1 \cdot N_{t-1} \cdot P\left[\mu_{jt} > \mu_t^1\right] \\ K_t &= \int_{j=0}^{1} k_{jt}. \end{aligned}$$

Note that the aggregate labor demand includes the fixed cost of exporting of all firms because the costs are in terms of labor.

Shock Processes

Productivity shocks feature a common and differential component,[31]

$$\begin{bmatrix} a_t \\ a_t^* \end{bmatrix} = \begin{bmatrix} a_{ct} + a_{dt}/2 \\ a_{ct} - a_{dt}/2 \end{bmatrix}$$

[31] Alternatively country-specific shocks can be written as a combination of these orthogonal shocks.

where the common component, a_{ct}, and the differential component, a_{dt}, each follow an AR(1) process,

$$
\begin{aligned}
a_{ct} &= \rho_a^c a_{ct-1} + \varepsilon_{at}^c & \qquad \varepsilon_{at}^c &\sim N(0, \sigma_a^c) \\
a_{dt} &= \rho_a^d a_{dt-1} + \varepsilon_{at}^d & \qquad \varepsilon_{at}^d &\sim N(0, \sigma_a^d).
\end{aligned}
$$

We assume that the relative trade cost between ROW and US, ξ_t, follows an AR(1) process. This arises from decomposing country-specific trade shocks into common and differential components, as in Waugh (2011) and Alessandria and Choi (2021), and then abstracting from the common component. In our benchmark specification, we do not consider a common trade cost because it primarily affect the level of gross trade, without first order effects on relative variables such as the RER and NT.[32,33] Specifically, the trade cost shocks are given by

$$
\begin{aligned}
\xi_{Rt}^* &= \frac{\xi_t}{2} & \qquad \xi_{Ut} &= -\frac{\xi_t}{2} \\
\xi_{Rt} &= \tau \frac{\xi_t}{2} & \qquad \xi_{Ut}^* &= 0
\end{aligned}
\tag{5}
$$

where $\tau \in \mathbb{R}$ and

$$
\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}, \qquad \varepsilon_{\xi t} \sim N(0, \sigma_\xi).
$$

Note that we are allowing for cost of trading ROW goods within the ROW to potentially be non-zero and impose the general assumption $\tau \in \mathbb{R}$.[34] This model nests the case of only differential trade costs between countries under zero within-ROW cost, i.e. $\tau = 0$.[35] The parameter τ captures the *elasticity* of shipping costs within the ROW to export cost to the US. When mapping the model to the data, τ captures the average trade costs across ROW countries.

[32] The importance of asymmetries in trade costs has also been highlighted by Dix-Carneiro, Pessoa, Reyes-Heroles and Traiberman (2023). They show that this source of variation is an important driver of manufacturing production and trade imbalances in the US due to the emergence of China in international goods markets.

[33] Although average trade costs likely exhibit a trend and change rarely, it is reasonable to assume that *relative* trade costs fluctuate more frequently and are mean-reverting with modest persistence. This is because numerous incidents, including trade policy shifts, advancements in transportation technology, and geopolitical issues, affect trade costs of different countries at different times. In Appendix E.4 we include a common trade cost component and show that our main results are robust to this specification.

[34] We assume that the within-country component is only present in the ROW. This is to account for the fact the other countries in the ROW went through significantly larger changes in trade barriers compare to the regions within the US. However, imposing time varying cost for the within-US trade in a symmetric way delivers the same results.

[35] While we also allow for domestic iceberg trade cost, for values of τ close enough to the home bias parameter ω, it generates a qualitatively similar mechanism as the relative demand shocks, or home bias shocks, in Pavlova and Rigobon (2007). They use a CES function of the form $C_t + I_t = \left[(1-\omega)^{\frac{1}{\gamma}} \left(e^{-\omega \xi_t}\right)^{\frac{1}{\gamma}} Y_{Rt}^{\frac{\gamma-1}{\gamma}} + \omega^{\frac{1}{\gamma}} \left(e^{(1-\omega)\xi_t}\right)^{\frac{1}{\gamma}} Y_{Ut}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$. Hence, this type of relative demand shocks can be capturing changes in trade integration within the ROW aggregate, which we also illustrate using a three country model in Appendix E.6.

This specification allows the within-ROW trade cost to vary over time and capture the evolution of trade integration among the countries that compose the ROW aggregate. In fact, during the time period we consider, many countries implemented trade reforms that jointly lowered the exporting cost to the US and non-US ROW countries, lowering both ξ^*_{Rt} and ξ_{Rt}. For example, the Asia-Pacific Economic Cooperation in the 1990s and the creation of the European Union generated significant changes in trade barriers among the countries in the ROW. Also, countries like China, Korea, and India focused on improving their export efficiency and entering the international market. These events resulted in lower costs of exporting to the US, as well as to other countries in the ROW aggregate.

Larger positive values of τ lead to higher within country trade costs for the ROW, conditional on a positive iceberg cost shock. Since this leaves fewer ROW intermediates to be aggregated to produce the final good, the trade shock induce a negative effect on output in the ROW. The strength of the negative effect on output is increasing in τ, and so is the effect on domestic absorption. Therefore, the cross country correlation of domestic absorption will vary with τ. In Section 7 we present a detailed analysis on the role of τ in the response of aggregate variables to trade shocks and show that the cross country correlation of domestic absorption identifies τ.[36]

Finally, we assume that the financial shock follows an AR(1) process,

$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi t}$$

where ρ_ψ is the persistence and $\epsilon_{\psi t} \sim N(0, \sigma_\psi)$.

Market Clearing and Country Budget Constraint

Goods market clearing for each firm j requires that its production is split between supply to the ROW and the US and satisfies the local demand in each market:

$$y_{jt} = e^{\xi_{Rt}} y_{j,Rt} + e^{\xi^*_{Rt}} y^*_{j,Rt}.$$

With the aggregation presented in Equation 2, this leads to the aggregate market clearing condition where the total production of the ROW is split between demand for composite goods in the ROW and the US:

$$Y_t = e^{\xi_{Rt}} Y_{Rt} + e^{\xi^*_{Rt}} Y^*_{Rt}.$$

[36]We also show in Section 7 that the main results about NT and RER at high and lower frequencies are robust to the case of $\tau = 0$.

Lastly, combining the household budget constraint with aggregate intermediate profits as well as the market clearing conditions above, we obtain the ROW country budget constraint:

$$\frac{\mathcal{E}_t B^*_{t+1}}{e^{\psi_t}} - \mathcal{E}_t B^*_t (1 + i^*_{t-1}) = TB_t - \mathcal{E}_t \frac{\chi}{2} \left(B^*_{t+1} - \bar{B} \right)^2 \quad \text{with} \quad TB_t = \mathcal{E}_t P^*_{Rt} Y^*_{Rt} - P_{Ut} Y_{Ut}$$

where TB_t is the nominal trade balance. The log of NT is the log of real export-import ratio, given by

$$nt_t = \gamma \left(tot_t + q_t \right) + \left(d^*_t - d_t \right) + \left((1 - \theta^*) \xi^*_{Rt} - (1 - \theta) \xi_{Ut} \right) + (1 - \gamma) \left(\frac{1}{1 - \theta} n^*_t - \frac{1}{1 - \theta^*} n_t \right) \tag{6}$$

where tot_t is the log of the terms of trade, $q_t = \log Q_t$ the log of the RER, $d_t = \log D_t$ and $d^*_t = \log D^*_t$ are the log of domestic absorption in the US and the ROW respectively, and n_t and n^*_t are the log of the mass of US and ROW exporters respectively. For a detailed derivation of nt_t see Appendix F. Finally, the budget constraint of the US is satisfied by Walras Law.

Final Goods Price Normalization

We fix the final good prices in both countries, P_t and P^*_t, to one. Implicitly we are assuming that the monetary authority in each country perfectly stabilizes inflation as in Itskhoki and Mukhin (2021a). Note that the RER, Q_t, is the relative price of a basket of ROW to US goods,

$$Q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$$

where $\mathcal{E}_t$ is the nominal exchange rate. Thus the RER, Q_t, is same as the nominal exchange rate, $\mathcal{E}_t$, which is the price of ROW currency per unit of US currency.

Definition of Recursive Competitive Equilibrium

A recursive competitive equilibrium is defined by a sequence for $t = 0, 1, \ldots, \infty$ of aggregate prices $\{W_t, W^*_t, R^k_t, R^{k*}_t, \mathcal{E}_t, P_{Rt}, P^*_{Rt}, P_{Ut}, P^*_{Ut}, i_t, i^*_t\}$, firm-level prices $\{p_{j,Rt}, p^*_{j,Rt}, p_{j,Ut}, p^*_{j,Ut}\}$, aggregate allocations $\{C_t, C^*_t, L_t, L^*_t, I_t, I^*_t, B^*_{t+1}, B_{t+1}, Y_{Rt}, Y^*_{Rt}, Y_{Ut}, Y^*_{Ut},\}$, firm-level allocations $\{y_{j,Rt}, y^*_{j,Rt}, y_{j,Ut}, y^*_{j,Ut}\}$, firm-level input choices and export decisions, and the mass of exporters $\{N_t, N^*_t\}$, such that

1. Given prices $\{W_t, W^*_t, R^k_t, R^{k*}_t, \mathcal{E}_t, i_t, i^*_t\}$, $\{C_t, L_t, I_t, B_{t+1}, B^*_{t+1}\}$ solves the problem of the ROW households, and $\{C^*_t, L^*_t, I^*_t, B^*_{t+1}\}$ correspondingly for the US households.

2. Given prices $\{p_{j,Rt}, p^*_{j,Rt}, p_{j,Ut}, p^*_{j,Ut}\}$, $\{y_{j,Rt}, y^*_{j,Rt}, y_{j,Ut}, y^*_{j,Ut}\}$ solves the problem in the final retail sectors in the ROW and the US.

3. Firm-level input choices, prices, and export decisions solve the firm's dynamic programming problems.

4. The market clearing conditions for goods, labor and bonds are satisfied.

5. Rationality holds, so that the laws of motions are consistent with agents' decision rules.

4 Calibration

We use data for the period 1980Q1-2019Q4 for the US and ROW to discipline our model. The details about the data are in Appendix A.

4.1 Benchmark Model

We have three sets of calibrated parameters. First, we exogenously calibrate parameters that are standard in the literature. Second, we calibrate the parameters that are related to the export behavior of firms using firm level data. Third, we jointly calibrate the parameters related to the shocks processes, the pricing to market friction and adjustment costs to match a set of equal number of moments.

Standard Parameters

The standard parameters that are exogeneously calibrated are displayed in panel A of Table 1. The time unit in the model is a quarter, and we choose a discount factor of $\beta = 0.99$, which implies an annual interest rate of 4 percent. The depreciation rate is set to $\delta = 0.02$. The risk aversion is $\sigma = 2$, a value frequently used in related business cycle studies. The capital share of $\alpha = 0.36$ is consistent with the labor share in the US. The preference weight on consumption is $\eta = 0.36$, set to match the steady state labor of 1/4. The elasticity of substitution between ROW and US goods, γ, is set to be 1.5, following the estimates in Feenstra, Luck, Obstfeld and Russ (2018). The elasticity of substitution across varieties θ is set to 4 to match a producer markup of 33 percent. The home bias, governed by ω, is set to match the average trade share of 14 percent in the US during our sample period. We assign these values symmetrically to the US and the ROW. Finally, we set the persistence of the common and differential productivity shocks, ρ_{a_d} and ρ_{a_c}, to be equal to 0.97, following Itskhoki and Mukhin (2021a).

Producer Trade Parameters

One of the benefits of modeling the dynamic trade with the microfoundations of the sunk exporting cost is that we can directly use exporter data to pin down the producer parameters. We calibrate three parameters related to the export block: fixed costs of exporting for new and incumbent exporters, f^0 and

Table 1: Calibrated Parameters

Parameter		Value	Target Moment
A. Standard Parameters			
Discount factor	β	0.99	Annual interest rate of 4%
Risk aversion	σ	2	Intertemporal elasticity of substitution of .5
Weight on consumption	η	0.36	Hours worked (Frisch elasticity)
Capital share	α	0.36	Capital share of income
Elasticity of substitution across varieties	θ	4	Producer markup of 33%
Elasticity of substitution between H and F	γ	1.5	Long-run price elasticity
Depreciation rate	δ	0.02	
Home bias	ω	0.097	Trade-to-GDP ratio of 14%
Common productivity, persistence	ρ_{a_c}	0.97	GDP persistence
Differential productivity, persistence	ρ_{a_d}	0.97	GDP persistence
B. Producer Trade Parameters			
Fixed cost of new exporters	f^0	0.14	Export participation of 20%
Fixed cost of incumbent exporters	f^1	0.04	Exit rate of 2.5%
Volatility of idiosyncratic productivity	σ_μ	0.15	Exporter premium of 50%
C. Shocks, Adjustment Costs and Pricing to Market			
Common productivity, volatility	σ_{a_c}	0.004	$\sigma(\Delta y)$
Differential productivity, volatility	σ_{a_d}	0.005	$cor(\Delta y, \Delta y^*)$
Financial shock, volatility	σ_ψ	0.002	$cor\,(\Delta c - \Delta c^*, \Delta q)$
Financial shock, persistence	ρ_ψ	0.957	$autocor\,(i - i^*)$
Trade shock, volatility	σ_ξ	0.052	$\sigma(nt)/\sigma(q)$
Trade shock, persistence	ρ_ξ	0.971	$cor\,(\Delta nt, \Delta q)$
Trade shock, within-country share	τ	0.171	$cor(\Delta d, \Delta d^*)$
Adjustment cost of portfolios	χ	0.0137	$autocor(nt)$
Adjustment cost of capital	κ	2.425	$\sigma(\Delta inv)/\sigma(\Delta y)$
Pricing to market parameter	ζ	0.966	$cor(\Delta tot, \Delta q)$

Notes: The table presents the values of calibrated parameters of the benchmark model. When we consider alternative models, some of the parameters are set to a different value while the other parameters are all recalibrated. In a model without trade shocks, $\sigma_\xi = \rho_\xi = 0$. In a model without trade dynamics, $f^0 = f^1 = \sigma_\mu = 0$. In Panel C, the lower cases indicate that variables are in logs, for example, $q \equiv \ln(Q)$ is log of the RER.

f^1, and the volatility of idiosyncratic productivity shocks, σ_η. These parameters are displayed in panel B of Table 1. The fixed costs and the volatility are set to jointly match firm level moments on exporter dynamics. In particular, we target an export participation of 20 percent, a quarterly exporter exit rate of 2.5 percent, and a size of exporters 50 percent larger than non-exporters. These are consistent with the US trade and exporter characteristics in the early 1990s (Bernard and Bradford Jensen, 1999; Alessandria and Choi, 2014).

Shocks, Adjustment Costs and Pricing to Market

The remaining parameters to calibrate are those related to trade, financial, and productivity shocks, the pricing to market friction, and the adjustment costs for capital and debt. There are ten parameters to be estimated. We jointly calibrate them to match ten moments. We present the parameters and moments used for the identification in Panel C of Table 1. We display the values of the calibrated parameters, together with the moment that is most relevant for the identification of each parameter.

The volatility of the common productivity shock, identified mainly by the volatility of GDP growth, is found to be 0.004. The estimated volatility of the differential productivity shock, identified by the cross country correlation of the first difference of GDP, is 0.005. Given that both processes have a persistence of 0.97, this implies that the differential component of the productivity shocks slightly dominates the common one.

We follow Itskhoki and Mukhin (2021a) and identify the the volatility of the financial shock using the Backus-Smith-Kollmann correlation. We find a value of 0.002 for the volatility of financial shocks. Hence, the volatility of productivity shocks is estimated to be between 2 and 2.5 times larger than that of financial shocks. This is similar to Itskhoki and Mukhin (2021a), which finds a value between 2.5 and 3.3.[37] The persistence of financial shocks is identified by the autocorrelation of the interest rate differential. We estimate a persistence of 0.957, close to what has been estimated in the literature.

We identify the volatility of trade shocks using the volatility of NT relative to the volatility of the RER, similar to Itskhoki and Mukhin (2017) for the case of foreign demand shocks. The persistence of the trade shock is identified by the contemporaneous correlation between the growth rates of NT and the RER. We find that the volatility and persistence of trade shocks are 0.052 and 0.971, respectively. Hence, trade shocks are found to be more volatile and persistent than productivity and financial shocks. However, the propagation effects of trade shocks depends on the value of the home bias parameters (ω). The ratio $\omega\sigma_\xi/\sigma_\psi$ equals 2.52, similar to the values identified in Itskhoki and Mukhin (2017) for the ratio of the volatility of the foreign demand shock to the financial shock, between 2.4 and 2.7.

[37]Note that the model in Itskhoki and Mukhin (2021a) does not have trade dynamics.

The within-country elasticity of domestic to foreign trade costs, τ, is identified using the cross-country correlation of the growth rates of domestic absorption. Since τ imposes a wedge in the aggregation of intermediate goods, it affects the response of the supply of final goods to trade shocks, ultimately impacting domestic absorption. We present a detailed analysis on the role of τ in Section 7.

The adjustment cost of capital directly affects the volatility of investment relative to that of GDP, while the adjustment cost of debt directly affects the autocorrelation of NT. We find an adjustment cost of capital of 2.42 and and adjustment cost of debt of 0.0137. Finally, we discipline the pricing to market friction using the correlation between the growth rates of the terms of trade and the RER, since this friction induces a wedge between them. We find a value of $\zeta = 0.966$, which implies an exchange rate pass-through of 68 percent, in line with the estimated values in the literature (Gopinath and Itskhoki, 2010).

4.2 Alternative Models

We consider three alternative specifications to our benchmark model to understand the role of each feature of our model: trade shocks, financial shocks, and dynamic trade. We recalibrate models when one of these features is absent. The calibrated values of these models are shown in Table F.2.

For the model without trade shocks, we set to zero the volatility and persistence of trade shocks and the within-ROW trade cost, and recalibrate the remaining parameters. We target the same moments considered before, except for the volatility of NT, its contemporaneous correlation with the RER, and the cross-country correlation of the growth rate of domestic absorption.[38,39]

In the model without financial shocks, we set to zero the volatility and persistence of financial shocks. We drop as targets the contemporaneous correlation between the growth rate of the RER and NT, and their relative volatility.[40]

For the model without dynamic trade, we set to zero the fixed costs of exporting for new and incumbent exporters and the volatility of idiosyncratic shocks. Given these values, the other parameters related to shocks and adjustment costs are estimated in the same way as in the benchmark model. We find a higher volatility and a lower persistence of both financial and trade shocks in the model with no dynamics. Finally,

[38]We exclude the cross-country correlation of domestic absorption from the target since the within-ROW trade cost is absent in this model

[39]We also show in Section 7 that a model without trade shocks but with a more sophisticated financial shock, in particular a mix of two AR(1) processes with different persistence's, is still unable to capture the net trade moments at the high frequency.

[40]Alternatively, in the model without financial shocks we could drop the Backus-Smith-Kollmann correlation and keep the contemporaneous correlation between the growth rate of the RER and NT. However, since trade shocks are able to match the Backus-Smith-Kollmann correlation, due to the role of the within-ROW trade cost, we chose to keep the Backus-Smith-Kollmann correlation and drop the contemporaneous correlation between the growth rate of the RER and NT to show that this model also fails to capture the latter correlation. Hence, conditional on matching the Backus-Smith-Kollmann correlation, in order to match the high frequency comovement between the RER and NT we need both financial and trade shocks.

in the model with no dynamics, the elasticity τ of the within-ROW trade cost is estimated to be around half (0.089) of the value in the benchmark model (0.17). This is because under dynamic trade, the responses of relative domestic absorption and NT, are smaller on impact compared to the static trade model. Since higher τ also generates smaller movements in these variables as discuss above, the model requires a smaller value of τ in this static model.

5 Results

In this section, we present the results of our model. We first show that our benchmark model successfully replicates the targeted moments, including the RER and NT moments at the high frequency. We then show that the model is able to capture the RER and NT dynamics at the whole spectrum of frequencies, in terms of their comovement and the frequency decomposition of the variances. Finally, we show that the model accounts for the RER disconnect puzzles and standard international business cycle moments. Throughout this section, we emphasize the importance of including all three features—financial shocks, trade shocks, and trade dynamics—in the model to effectively capture these patterns.

5.1 The RER and NT at the High Frequency

The results of the benchmark model for the targeted moments are presented in Panel A of Table 2 (column 2). The model closely matches all of the targeted moments, such as the volatility and cross-country correlation of output. It also successfully generates the imperfect correlation between terms of trade and the RER.

More importantly, the benchmark model accurately reproduces the comovement of the RER and NT at high frequencies. First, our model exactly matches the contemporaneous correlation of the RER with NT, $cor(\Delta nt, \Delta q)$. In data, the RER and NT exhibits a relatively weak connection at high frequencies, with a correlation of approximately 0.3. Our model successfully accounts for this weak correlation. Both financial and trade shocks are necessary to capture this pattern. To see this, consider two alternative models: the model without trade shocks (column 3 of Table 2) and the model without financial shocks (column 4 of Table 2). When the model is recalibrated without trade shocks, the correlation between two variables is too high (0.90) compared to data. On the other hand, when financial shocks are absent, the correlation is too low (-0.77). This is because financial shocks generate a positive correlation between the RER and NT upon impact, whereas trade shocks lead to a negative correlation.[41]

[41]When Financial shocks generate an excess return on bonds for the US relative to the ROW, the excess savings is exported to the ROW (US NT increases), US aggregate demand falls, and the US RER depreciates. On the other hand, trade shocks that raise

Second, our model successfully replicates the relative volatility of NT to the RER, $\sigma(nt)/\sigma(q)$. In the data, NT and the RER exhibit roughly equal volatility, with NT being 1.16 times more volatile than the RER. Our model effectively captures this pattern. Again, it requires incorporating both trade and financial shocks: in a model without trade shocks, the volatility of NT relative to the RER becomes too high (1.87), while without financial shocks, this ratio decreases significantly (0.27). That is, financial shocks generate too large volatility in NT relative to the RER. This excess volatility induced by financial shocks has also been noted by Miyamoto, Nguyen and Oh (2022).

Hence, having both shocks is necessary for capturing the high frequency moments related to the RER and NT.[42] For this reason, we focus on the results of models that include both financial and trade shocks in the remaining discussions of the paper.

5.2 Comovement between RER and NT

We show that the model is able to capture the comovement between the RER and NT at the full range of frequencies without directly targeting them. First, we measure on the correlation between the growth rates of the RER and NT at different horizons. Second, we estimate the elasticity of NT to prices in the short and long run using an error correction model. Our main finding is that, conditional on having both financial and trade shocks, dynamic trade is necessary to capture the differential comovement between the RER and NT.

Dynamic Correlation

To capture the differential comovement between the RER and NT, consider the correlation between the growth rates of the RER and NT at different horizons. In Figure 2, we plot the correlation between the h−quarter growth rates of the RER and NT in the data (solid black line). While the contemporaneous correlation at $h = 1$ is 0.30, the correlation gradually increases over the horizon, reaching a value of 0.48 at the 8-quarter growth rate. The growth rates of RER and NT present a stronger comovement in the longer than in the shorter run.

The benchmark model successfully captures the dynamic correlation between the RER and NT, without being targeted except for the contemporaneous correlation (blue line with circles). The contemporaneous correlation in the model is 0.30, while for the 8-period growth rate it is 0.43. Dynamic trade plays an important role for the ability of the model to capture this pattern. To see this, consider a case under no

the relative cost of exporting for the US leads to a decline in its NT, and the supply of final goods in the US increases, causing its price to fall (the US RER depreciates). See Section 6.2 for a detailed discussion on the propagation mechanism of the two shocks.

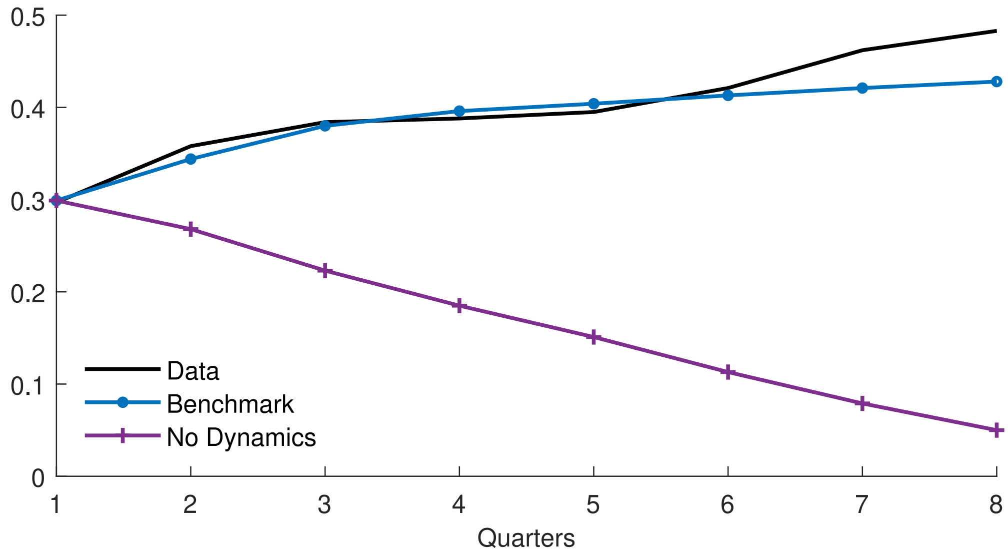
[42] Fukui, Nakamura and Steinsson (2023) also highlight that financial shock alone cannot capture the joint dynamics of RERs and macro aggregates, although they focus on the matching of conditional moments.

Table 2: Model Results

Moments	(1) Data	(2) Benchmark	(3) No Trade Shock	(4) No Financial Shock	(5) No Dynamics
A. Targeted Moments					
$\sigma(\Delta y)$	0.007	0.006	0.007	0.007	0.008
$cor(\Delta y, \Delta y^*)$	0.40	0.40	0.40	0.37	0.40
$cor\,(\Delta c - \Delta c^*, \Delta q)$	-0.10	-0.10	-0.10	-0.10	-0.10
$autocor\,(i - i^*)$	0.87	0.87	0.74	0.93	0.88
$autocor(nt)$	0.98	0.96	0.97	0.98	0.95
$\sigma(\Delta inv^*)/\sigma(\Delta y^*)$	2.59	2.59	2.74	2.60	2.59
$cor(\Delta d, \Delta d^*)$	0.34	0.34	*0.27*†	0.36	0.34
$cor\,(\Delta nt, \Delta q)$	0.30	0.30	*0.90*†	*-0.77*†	0.30
$\sigma(nt)/\sigma(q)$	1.16	1.16	*1.87*†	*0.27*†	1.17
$cor(\Delta tot, \Delta q)$	0.49	0.49	0.49	0.49	0.49
B. Trade Elasticity					
Short run	0.20 (0.05)	0.40	1.16	-0.41	0.36
Long run	1.16 (0.25)	1.00	1.83	-0.28	0.53
Adjustment	0.07 (0.02)	0.04	0.19	0.01	0.04
C1. Frequency Decomposition of RER					
High frequency	0.08	0.07	0.09	0.05	0.06
Business cycle frequency	0.31	0.23	0.27	0.16	0.19
Low frequency	0.61	0.70	0.64	0.79	0.75
C2. Frequency Decomposition of NT					
High frequency	0.06	0.08	0.07	0.06	0.09
Business cycle frequency	0.30	0.30	0.27	0.18	0.24
Low frequency	0.64	0.62	0.66	0.76	0.67
D. Disconnect Puzzles					
$\sigma(q)$	0.10	0.08	0.06	0.16	0.11
$\sigma(\Delta q)/\sigma(\Delta y)$	4.24	3.48	2.95	1.64	3.10
$autocor(q)$	0.97	0.97	0.94	0.99	0.98
β_{fama}	-1.34	0.14	0.08	0.88	-4.34
R^2_{fama}	0.04	0.001	0.001	0.89	0.49
$cor(q, i - i^*)$	-0.30	-0.44	-0.35	-0.06	-0.20
$autocor(i)$	0.93	0.93	0.83	0.96	0.94
$\sigma(i - i^*)/\sigma(\Delta q)$	0.13	0.01	0.02	0.01	0.04

Notes: 'No Trade Shock' presents the result of re-calibrated model only with productivity and financial shocks. 'No Financial Shock' presents the result of re-calibrated model only with productivity and trade shocks. 'No Dynamics' is for the model without fixed exporting costs and producer heterogeneity. Superscript † in Panel A denotes that the moment is not targeted during the calibration procedure.

Figure 2: Dynamic Correlation between RER and NT Flows



Notes: We calculate the dynamic correlations as $cor(\Delta_h q_t, \Delta_h nt_t)$, where q_t and nt_t are log of the RER and the export-import ratio, respectively. and Δ_h denotes h−quarter difference.

trade dynamics (violet line with crosses). Absent trade dynamics, the comovement becomes weaker as the horizon increases, meaning that the growth rates of the RER and NT present a higher correlation in the long than in the short run. Hence, dynamic trade, by inducing a lagged response of NT, allows the model to capture the differential comovement between NT and the RER over time.[43]

We also consider the cases in the absence of shocks. In Figure F.4, we plot the dynamic correlation for the models with no financial and no trade shocks. Absent either financial (dashed red line) or trade shocks (dash-dotted green line), the model fails to capture the differential co-movement, even under dynamic trade. As before, we observe that financial shocks induce an almost perfect correlation across the eight quarter horizon between these variables, while trade shocks induce a strong negative one, although the correlation is increasing in this case as it is in the data. This reinforces our result that both shocks are needed for capturing the comovement. Therefore, conditional on having both financial and trade shocks, dynamic trade is necessary to capture the differential co-movement observed in the data.

Dynamic Trade Elasticity

To examine the predictions of the model regarding the comovement between the RER and NT across all frequencies, we estimate the elasticity of NT to prices in both the short and long run. To do so, we

[43]We also consider using the trade-expenditure ratio as a measure of NT, defined as $TE_t = \log \frac{X_t}{M_t} - \log \frac{D_t^*}{D_t}$. Using the trade-expenditure ratio allows us to isolate the substitution effect that changes in the RER generate on NT from the effect on relative expenditure. As shown in in Figure F.3, the RER and NT present a stronger comovement in the long run than the short run even after controlling for relative expenditure. Our model successfully captures this pattern.

leverage the relationship between prices and NT based on the Armington trade model.

The Armington model, which is also nested within our benchmark model, serves as the basic trade block for almost all multi-good international macro models. From the demand structure of the Armington model, NT can be expressed as a function of the RER, the terms of trade and domestic absorption.[44] We estimate an error correction model of this relationship:

$$\begin{aligned}\Delta nt = \beta + \gamma_{SR}\, \Delta(tot_t + q_t) + \Delta(d_t^* - d_t) \\ - \alpha\left[nt_{t-1} - \gamma_{LR}\,(tot_{t-1} + q_{t-1}) - (d_{t-1}^* - d_{t-1})\right] + \varepsilon_t \end{aligned} \tag{7}$$

where $nt_t = \ln(X/M)$ is log of NT, $tot_t = \ln(p_t^M/p_t^X)$ is the log of the terms of trade, q_t is the log of the RER, and $d_t = \ln(C_t + I_t)$ and $d_t^* = \ln(C_t^* + I_t^*)$ are the log of domestic absorption in the domestic and foreign country. Here, γ_{SR} is the short-run elasticity, γ_{LR} is the long-run elasticity, and α captures the speed of adjustment. The term in square brackets captures the cointegration relationship implied by the Armington model,

$$nt_t = \gamma\,(tot_t + q_t) + (d_t^* - d_t).$$

This type of regression has been widely used in studies of trade dynamics (Hooper et al., 2000; Marquez, 2002; Alessandria and Choi, 2021; Alessandria et al., 2024).

Using the data described in Appendix A, we estimate Equation 7, and present the results in Panel B of Table 2. The short-run elasticity is estimated to be around 0.2, while the long-run elasticity is larger, around 1.2. The estimated values are similar to the estimates from Alessandria and Choi (2021) that covers a similar time period for the US, and are also consistent with Alessandria, Bai and Woo (2024) which uses panel data of a broader set of countries, although their size of the long-run elasticity is slightly larger compared to our estimates.

Using the model simulated data, we conduct the same exercise in our benchmark model (column 2). We estimate a long run elasticity γ_{LR} that is is larger (1.00) than the short run γ_{SR} (0.40), capturing the dynamic adjustment of NT to prices.[45] Trade dynamics are crucial for capturing the difference between short and long run elasticity. In column 6, we present the result for the model without trade dynamics. The short run elasticity is estimated to be 0.36, and the long run elasticity is 0.53. Although there is a small

[44] See Appendix F for the derivation of NT equation in the benchmark model and its comparison with the Armington model.

[45] In Section 7 we present a specification in which we target these elasticities. This alternative specification generates similar results as in our benchmark case.

gap between two elasticities due to the effect of trade shocks, the difference between them is significantly smaller than in the benchmark model. We conclude that dynamic trade, by generating a slow moving distribution of exporters in response to shocks, allows the model to capture the differential comovement between the RER and NT significantly better than in the model without dynamics.[46]

Moreover, similar to our analysis of the correlation of the growth rates of the RER and NT at different horizons, we find that both trade and financial shocks are necessary to capture the differential elasticity. Absent any of these shocks, the model's ability to capture the short and long run elasticity of the data deteriorates. Absent the trade shock, both elasticities are too high, whereas absent the financial shock both are too low. Therefore, conditional on having both financial and trade shocks, dynamic trade is necessary to capture the differential elasticities of NT to prices.

5.3 Spectrum Analysis

We now turn to study the ability of the model to capture the spectrum of the RER and NT flows, which are not targets in our calibration. We consider the spectrum to study the dynamics represented at the frequency domain instead of the time domain. It is useful since it allows to decompose the variance of these variables into different frequencies. That is, the sum of the spectrum for all frequencies equals its unconditional variance. We estimate the spectrum non-parametrically using the modified Bartlett kernel. For the details on this approach, see Appendix B.

Figure 3 shows the spectrum of the RER in the data (black solid line). We find that the spectrum is the highest at the zero frequency, and decreasing as the frequency increases. The standard business cycle frequency, cycles between 8 to 32 quarters, is represented by the shaded grey area. The area under the spectrum for the frequency lower than the business cycle is much larger than that for the frequency of business cycles. In particular, it takes about 61 percent of its variance, as presented in Panel C1 of Table 2 (column 1).

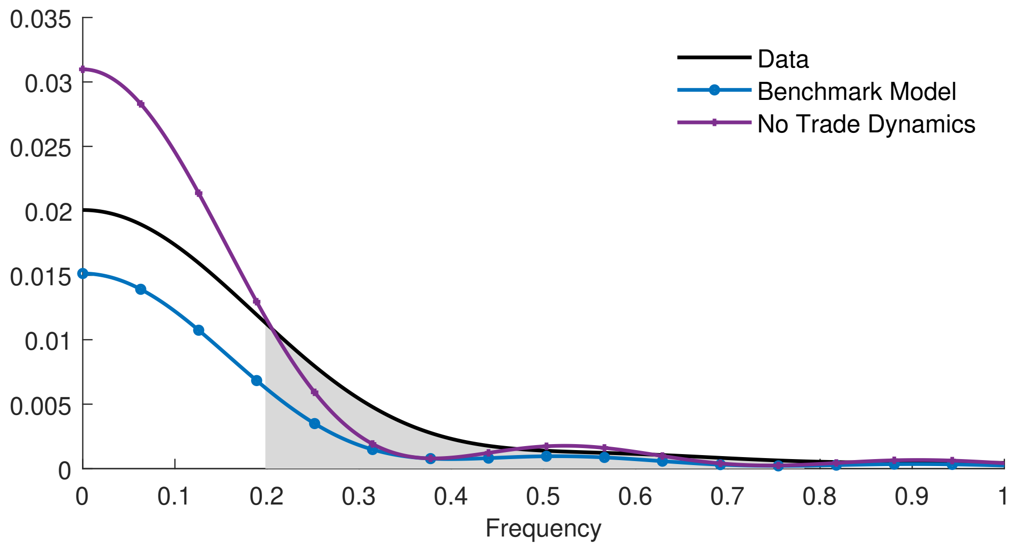
We now turn to estimate the spectrum of the RER using model simulated data.[47] The benchmark model (blue line with circles) captures well the size and shape of the spectrum of the RER, even though it is not a target in our calibration. The model spectrum lies slightly under the data one, reflects that the volatility of the RER is slightly smaller in the model (0.08) than in the data (0.10) (Panel D of Table 2). Moreover, the

[46]We do not consider a model without trade dynamics and trade shocks, since in that case the short and long-run trade elasticities will be the same, and equal to the Armington elasticity of 1.5, as can be inferred from equation 6. For the same reason, the model in Section 7 where we drop the trade shock but allow for a more sophisticated financial shock does not capture the differential trade elasticity.

[47]We simulate the model for 10,000 periods and burn the first half. We show in Sections 7 and E.2 that the result is robust to using multiple samples of shorter periods.

shape of the spectrum is very similar to the data. This can be mapped to the share of the variance of the RER arising at different frequencies, which is displayed in Panel C1 of Table 2. In the model, the largest share (70 percent) of the RER variation is assigned to the low frequency, followed by the business cycle frequency (23 percent), and then the high frequency (7 percent).

Figure 3: Spectrum of the RER



Notes: The graph is enlarged for the frequency of $[0, 1]$, as the spectrum on $[1, \pi]$ is near zero. The full graph is presented in Figure B.1. Gray area shows the range of the frequencies for the business cycle. The blue line with circles shows the result in the benchmark model, while the violet line with crosses shows the result of the model recalibrated with no sunk cost of exporting.

Dynamic trade plays an important role in matching the shape of the RER spectrum. To see this, consider the re-calibrated model without dynamic trade (violet line with crosses). It is evident that the overall size and the shape of the RER spectrum in this model is worse than in our benchmark case. The unconditional volatility in the model is higher than in the data, 0.11 and 0.10 respectively. Moreover, a larger share of the RER variance is attributed to the low frequency (75 percent) than in the benchmark model (70 percent). This result is consistent with the "Excess Persistence Puzzle," documented in Rabanal and Rubio-Ramirez (2015). The intuition for this result is the following. When trade is static, quantities in the short run are more elastic than under dynamic trade. Therefore, prices in the short run have a weaker response absent trade dynamics, so a higher share of the RER variance is assigned to low frequency fluctuations. Once we incorporate dynamic trade, quantities in the short-run are more inelastic and prices need to adjust more to clear the market. This leads to a redistribution from the lower to the higher frequencies.

Finally, we show in Panel C2 of Table 2 the decomposition of the spectrum of NT. The benchmark model generates a spectrum decomposition very close to the data. The low frequency variation in the model is

0.62, very close to the 0.64 in the data. We notice that the model without dynamic trade generates a share of low frequency variation of 0.67, larger than the data.[48]

5.4 Disconnect with Macro Fundamentals

Having shown the success of the benchmark model in accounting for the variation in the RER and NT at the full range of frequencies, we now show that the model also captures the RER disconnect, both with real and financial variables. There are several moments related the RER and macro fundamentals, so called *puzzles*, that have been hard to explain in most models. Our benchmark model is able to account for them. The results are presented in Panel D of Table 2.

First, there is an empirical disconnect between the RER and output, i.e. the real disconnect, that the literature have struggled to reproduce. In particular, in the data the RER follows a near random-walk process and is three to six times more volatile than output (Meese-Rogoff Puzzle). We also find these patterns in our data, since the volatility is around four times larger than output, and the RER is highly persistent. In a standard BKK-type models, the volatility is lower than that of output, and the process is far from a random walk. However, our model successfully reproduces the data patterns. As shown in the second column, the volatility and persistence of the RER are very close to the data.[49] Note that these moments are not targeted during our calibration procedure.

Second, the empirical correlation between the growth rates of relative consumption growth and the RER is negative in the data. However, the risk-sharing condition in BKK-type models usually implies that high relative consumption is associated with a depreciation of the RER, generating a correlation close to one (Backus-Smith-Kollmann Puzzle).[50] It is important to notice that even under under incomplete markets, in the presence of a risk-free bond, the models usually predict an almost perfect correlation between the cross-country consumption growth and changes in the RER. Our model is able to reproduce this puzzle by directly targeting the correlation during the calibration. As shown in Panel A of Table 2, the correlation between cross-country consumption growth and RER growth is -0.10, in both data and the benchmark model.

The ability of the benchmark model to account for the disconnect between the RER and output and

[48]Since dynamic trade makes quantities traded across countries more inelastic in the short-run, one would expect NT to experience larger lower frequency variation relative to the static trade case. The results from the spectrum decomposition of NT in Panel C2 of Table 2 seems to suggests the contrary. In Appendix D, we show that this arises from a calibrated lower adjustment cost of net foreign assets in the static trade model. When we set the adjustment cost to be the same as in the dynamic trade model, we find that the low frequency share of variation in NT flows is significantly lower under static trade (0.54) than under dynamic trade (0.62).

[49]Not surprisingly, the model generates a half-life of the RER almost identical to its data counterpart. We show in Figure F.6 that the IRF of the RER from the estimated AR(1) process of the RER in the data and in the model simulated data are very close.

[50]See Backus and Smith (1993) and Kollmann (1995).

consumption is not compromised by the presence of trade shocks or dynamic trade, as can be seen from columns 3 to 5 in Table 2 under Panel D. Furthermore, the presence of trade shocks improves the quantitative properties of the model, particularly those related to the level of the RER. With the addition of trade shocks, not only does the autocorrelation $autocor(q)$ increase and align more closely with the data, but the volatility $\sigma(q)$ also rises, matching that observed in the data. Additionally, the model is able to capture the real disconnect even without financial shocks. Even though the volatility of the first difference of the RER falls absent the financial shocks, the volatility and persistence of the level of the RER increases by even more than in the data. This is not surprising given that trade shocks affect the RER more in the long run. Finally, absent financial shocks, the model still captures the negative Backus-Smith-Kollmann correlation due to the effects of trade shocks under a higher within-ROW trade elasticity.[51] This implies that trade and financial shocks alone can account for most of the disconnect between the RER and consumption and output, although both of them are needed to capture the joint variation of the RER and NT at high frequencies.

We now turn to the results related to the financial disconnect. It is well known that real interest rate differentials are not well connected to the changes in the RER. The disconnect can be summarized by the regression similar to Fama (1984),

$$\mathbb{E}_t\left[\Delta q_{t+1}\right] = \alpha + \beta_{Fama}(i_t - i_t^*) + u_{t+1}. \tag{8}$$

In standard models, the Fama coefficient would be close to 1, since no arbitrage condition implies that high interest rates predict a depreciation of the RER. However, in the data, interest rate differentials tend to predict appreciations of the RER. More importantly, the predictive power of interest rates is weak, as measured by an R^2 close to zero.[52] Engel, Kazakova, Wang and Xiang (2022) emphasizes that the low R^2 of the Fama regression is a more robust statistic for the financial disconnect than the negative coefficient on the interest rate differentials.

The results are shown in Panel D of Table 2. We find in our data that the Fama coefficient is negative, with a large standard error, and the R^2 close to zero. Our benchmark model (column 2) is able to generate a Fama coefficient lower than one and, more importantly, an R^2 near zero. Since the low R^2 is considered a more robust support for the Fama puzzle, we argue that the model does a fairly good job accounting for

[51] We explain this in detail in Section 7 when we discuss the within-ROW trade costs.

[52] Strictly speaking, the Fama regression is used to show the disconnect in nominal variables, also known as the Forward Premium Puzzle. In this paper we consider a real version of the puzzle. In Table F.4 in Appendix H we present the Fama coefficient we find using both real and nominal data, which is very similar to each other. This arises from the high correlation between the RER and the NER, due to the low inflation in our sample.

the financial puzzle. In other words, the financial moments are not compromised by the presence of trade shocks or dynamic trade. Finally, absent the financial shock (column 4), the Fama coefficient and the R^2 increases significantly, showing the importance of financial shocks for capturing the financial disconnect, consistent with the results in Itskhoki and Mukhin (2021a, 2023).[53,54]

5.5 International Business Cycle Moments

Our benchmark model is also consistent with the standard international business cycle moments. We report the results in Table 3. The model is able to capture a volatility of consumption that is lower than output. It generates a cross country correlation of consumption and investment very close to the data. Overall, we find that our benchmark model accounts for the dynamics of the RER and NT at the whole spectrum of frequencies, without compromising the ability to account for the real and financial puzzles, and other international business cycle moments.

Table 3: International Business Cycle Moments

	(1) Data	(2) Benchmark	(3) No Trade Shock	(4) No Financial Shock	(5) No Dynamics
$\sigma(\Delta c)/\sigma(\Delta y)$	0.83	0.62	0.68	0.58	0.67
$cor(\Delta y, \Delta c)$	0.68	0.90	0.95	0.98	0.93
$cor(\Delta y, \Delta z)$	0.77	0.84	0.94	0.97	0.90
$cor(\Delta c, \Delta c^*)$	0.31	0.48	0.32	0.45	0.43
$cor(\Delta inv, \Delta inv^*)$	0.31	0.31	0.25	0.34	0.31
$\sigma(\Delta tot)/\sigma(\Delta q)$	0.46	0.02	0.02	0.12	0.06
$cor(\Delta tot, \Delta q)$	0.49	0.49	0.49	0.49	0.49
$cor(\Delta nt, \Delta tot + \Delta q)$	0.31	0.31	0.90	-0.71	0.25
$cor(nt, tot + q)$	0.30	0.30	0.86	0.85	0.23
$cor(i - i^*, tot + q)$	-0.29	-0.44	-0.34	-0.04	-0.22

[53] Potentially, trade shocks (and productivity shocks) could account for the financial disconnect, since they generate changes in net foreign assets that induce UIP deviations through the adjustment cost of debt (Equation 1). However, we find that this indirect effect of the other shocks is quantitatively small.

[54] For the model without trade dynamics, we find an R^2 of 0.65. This is in part driven by estimating significantly higher adjustment costs of capital in this model. Absent capital adjustment costs, the R^2 and β equals 0.02 and -0.85 respectively.

6 Quantifying the Effect of Financial and Trade Shocks

Using our benchmark model, which provides a unified framework to study the dynamics of the RER at all frequencies, we evaluate the role of trade and financial shocks in shaping the dynamics of the RER. First, we compute the contribution of each shock for the error forecast variance of the RER at different horizons. Second, we present the impulse response functions of variables of interest to trade and financial shocks.

6.1 Conditional Variance Decomposition

We inspect the relevance of each shock for driving the variation in the RER by computing the contribution of each shock to the h−quarter ahead error forecast variance of the RER. The results for the benchmark model are presented in Panel A of Table 4. The main result is that the trade shock explains most of the error forecast variance of the RER in the long-run (i.e. low frequency), while the financial shock is important for the short-run (i.e. high frequency) fluctuations.

In particular, the financial shock explains 63 percent of the one-quarter ahead error forecast variance, with the trade shock explaining around 35 percent. Hence, at the high frequency, financial shocks matter more than trade shocks for explaining the variation of the RER. However, when focusing at the eighty quarters ahead error forecast variance of the RER, the trade shock explains around 65 percent, while the financial shock explains 26 percent. Hence, trade shocks matter more than financial shocks for the variation in the RER at lower frequencies. Since, 61 percent of the overall variance of the RER is at frequencies lower than business cycles, we find that trade shocks are crucial for capturing the overall variation in the RER.

This result is not due to a different persistence of the processes but is driven by different general equilibrium effects. To show that, we consider an alternative case where we set the persistence of the financial shock to be the same as that of the trade shock (0.971). This alternative case is displayed in Panel B of Table 4. Even in this case, we still find that financial shocks dominate in the high frequency and trade shocks in the low frequency. Since we are increasing the persistence of the financial shock, we find stronger effects of this shock across all frequencies, but trade shocks still dominate at the lower frequencies.

We find consistent results using the analysis at the frequency domain. In particular, we conduct a spectral analysis of the model for the counterfactual cases where only the trade or the financial shock is present under the identified parameters of the benchmark model. We find that in the case of only trade shocks, the volatility is only slightly smaller than in the benchmark, shown by the total area below the spectrum. Furthermore, the shape of the spectrum follows very closely that of the benchmark model.

However, having only financial shocks largely misses the size of spectrum. For more details, see Figure F.5 and Table F.3.

Table 4: Conditional Variance Decomposition (%)

	quarters = 1	8	32	80
A. Benchmark				
Financial shock	62.6	46.6	25.8	25.7
Trade shock	34.5	48.4	65.1	64.5
Productivity shock	2.9	5.0	9.1	9.8
B. Alternative: $\rho_\psi = \rho_\xi = 0.971$				
Financial shock	70.1	57.2	35.0	36.2
Trade shock	27.6	38.8	57.0	55.3
Productivity shock	2.3	4.0	8.0	8.5

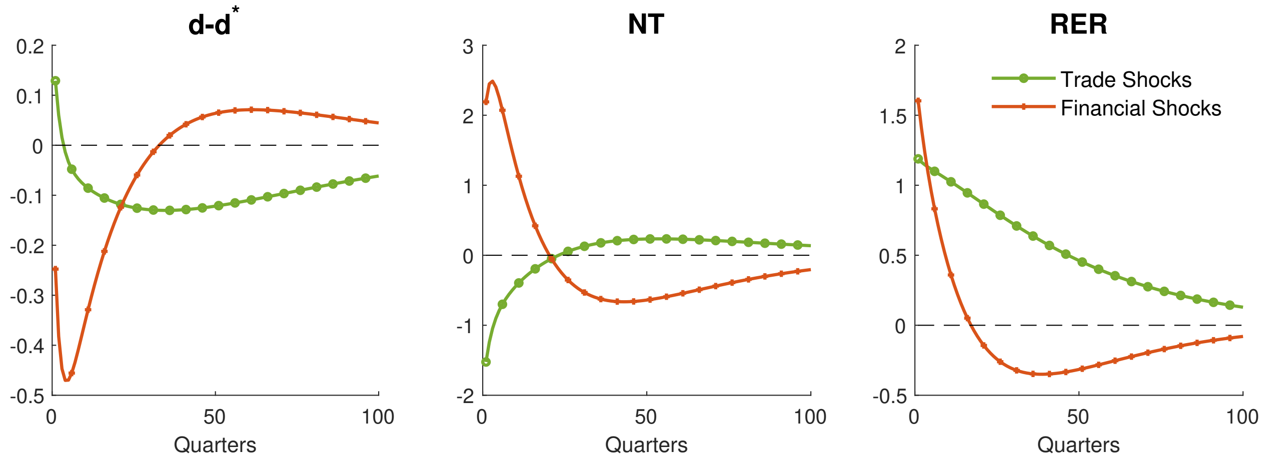
Notes: 'Alternative' presents the result of the model where we set the persistence of the financial process to be the same as the trade process in the Benchmark model, that is $\rho_\psi = \rho_\xi = 0.971$, while keeping all the remaining parameters as in the Benchmark model.

6.2 Inspecting the Financial and Trade Shock Mechanism

We now turn to study the propagation mechanism triggered by financial and trade shocks in more detail. For this purpose, we present the impulse response functions of relative domestic absorption, NT and the RER to the two types of shocks in Figure 4.

First, consider the effect of a financial shock that increases the return on bonds for the ROW (red line with dots). Since households in the ROW face a higher return on bonds, they optimally decide to increase their savings. Hence, domestic absorption in the ROW falls relative to the US. Due to the presence of home bias in expenditure, the fall in demand of ROW households translates into a stronger shortage in demand for intermediate goods in the ROW than in the US. For markets to clear, the price of ROW intermediate goods must fall, so that the US increases its expenditure in ROW intermediates. As a consequence, NT for the ROW increase, while at the same time the RER depreciates. In particular, a one standard deviation financial shock generates a 1.6 percent depreciation of the RER on impact and a 2.19 percent increase in NT.

Figure 4: Selected IRFs to Trade and Financial Shocks (%)



Due to dynamic trade, domestic absorption and trade flows take time to respond, leading to hump shaped responses, peaking two quarters after the shock. On the other hand, prices adjust without any delay. This contributes to a lagged response of NT relative to the RER. Eventually, households in the ROW consume their initial savings, so that NT becomes negative, around 5 years after the shock. Higher domestic absorption in the ROW induce an upward pressure on ROW prices, which translates into an appreciation of its RER relative to the pre-shock value.

Next, we study the effect of a trade cost shock that increases the cost of exporting for the ROW relative to the importing cost (green line with *o*). A higher exporting cost in the ROW generates a fall in NT. Larger inflows of foreign intermediates, together with smaller outflow of domestic intermediates, increases the supply of final goods in the ROW. These effects evolve gradually over time due to the dynamic nature of trade. For markets to clear, the ROW final good price must fall. Consequently, domestic absorption in the ROW increases and the RER depreciates. In particular, a one standard deviation trade shock generates an almost 1.2 percent depreciation of the RER on impact and a 1.52 percent decrease in NT on impact. Note that both magnitudes are smaller (in absolute terms) than under financial shocks, allowing the model to generate an unconditional correlation between the growth rates of NT and the RER that is slightly positive (0.30). This also explains why financial shocks drive most of the variation of the RER at higher frequencies. Over time, domestic absorption in the ROW falls to repay the debt used to finance the negative NT flows in the short run, triggering an increasing path of NT flows.

Finally, as can be seen from the last panel, trade shocks induce more persistent effects on the RER than financial shocks. It is worth noticing that it is the effect of trade shocks that is more persistent and not the process itself. In fact, the calibrated persistence of the financial and trade shocks are very similar

(ρ_ψ = 0.957 for financial shocks; ρ_ξ = 0.971 for trade shocks). Trade shocks induce a more persistent effect on the RER mainly due to its persistent effect on the resource constraint.[55] It is important to notice that the lack of dominance of financial shocks in explaining RER variation in the long-run does not longer hold absent the trade shock: in the model without trade shocks, most of the RER variation is driven by financial shocks across all frequencies, with productivity shocks explaining a small share of its variation.[56] Overall, our results suggest that financial shocks matter more than trade shocks for the dynamics of the RER in the short run, while the opposite is true for the longer run.

7 Sensitivity and Robustness

In this section, we explore the sensitivity and robustness of our quantitative results. First, we provide a detailed analysis of the role of the elasticity of domestic to foreign trade costs, τ, and further consider a model when the elasticity is zero. Next, we consider alternative estimation methods, namely Bayesian estimation and using small sample simulations. We show that we obtain similar estimates of parameters compared to those under the Benchmark model in Section 4.1. We also examine the robustness of our results to different model specifications, including modeling of dynamic trade, a three-country model, common shocks to trade costs, investment adjustment costs, a case where we target the short and long run trade elasticity, and a model without trade shocks but with a more sophisticated financial shock.

Overall, our findings are robust across these models, while the benchmark model tends to better capture the dynamics of key variables compared to the alternative specifications. Moreover, we find that trade shocks drive most of the variation in the RER at low frequencies in all of the alternative models. More detailed descriptions are provided in Appendix E.

Within-ROW Trade Costs

We first analyze in detail the nonzero trade cost within ROW countries and develop intuition on the role of the elasticity τ. Figure 5 displays the IRFs of relative domestic absorption, NT, and the RER to a shock that increases the shipping cost from the ROW to the US (an increase in ξ) for different values of τ, while keeping constant the other calibrated parameters.

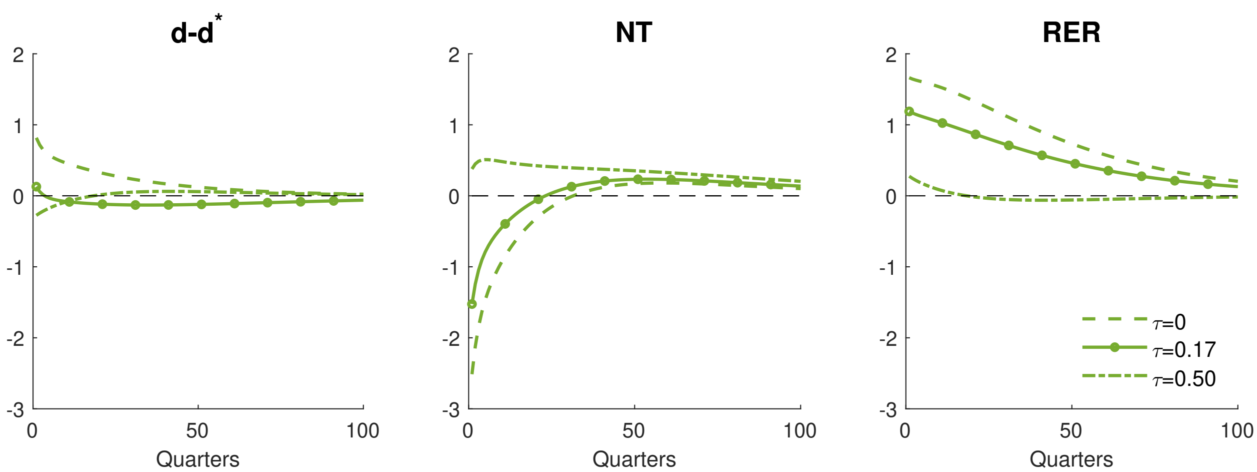
When $\tau = 0$ (the dashed line), a positive trade shock, which increases the cost of the ROW exports and decreases its import costs, triggers a fall in NT for the ROW. The increase in imports for the ROW raises the supply of final goods in the ROW. This effect is reinforced by an increase in the use of ROW intermediates

[55] In Appendix G we present an analytical solution to a simplified model to illustrate the role of the resource constraint channel of trade shocks.

[56] See Table F.5 for the conditional variance decomposition of the RER in the model without trade shocks.

for the production of final goods, due to the increase in exporting costs. For markets to clear, the final good price in the ROW must fall for domestic absorption to increase, inducing a depreciation of the RER. Now, consider the case of a positive but small value of $\tau = 0.17$ (line with circles). With a positive τ, when the cost of exporting from the ROW to the US increases, there is also a small increase in the marginal trade cost within the ROW, between its intermediate and final good producers. This makes exporting for the ROW relatively more attractive than under zero τ, so that the fall of net exports is smaller. This implies that the fall in the final good price needed to clear the markets is weaker, so that in equilibrium there is a smaller depreciation of the RER and a weaker increase in domestic absorption.[57] If τ is sufficiently high, NT for the ROW can be positive with domestic absorption in the ROW decreasing relative to the US (dash-dotted line with $\tau = 0.50$). This explains why the model without financial shocks can generate a negative Backus-Smith-Kollmann correlation. It is due to the calibrated value of the within-ROW trade cost elasticity that is higher than in the benchmark model (0.282, shown in Table F.2.)

Figure 5: IRFs to Trade Shock for Different Values of τ (%)



Notes: The rest of the parameters are set as in Table 1.

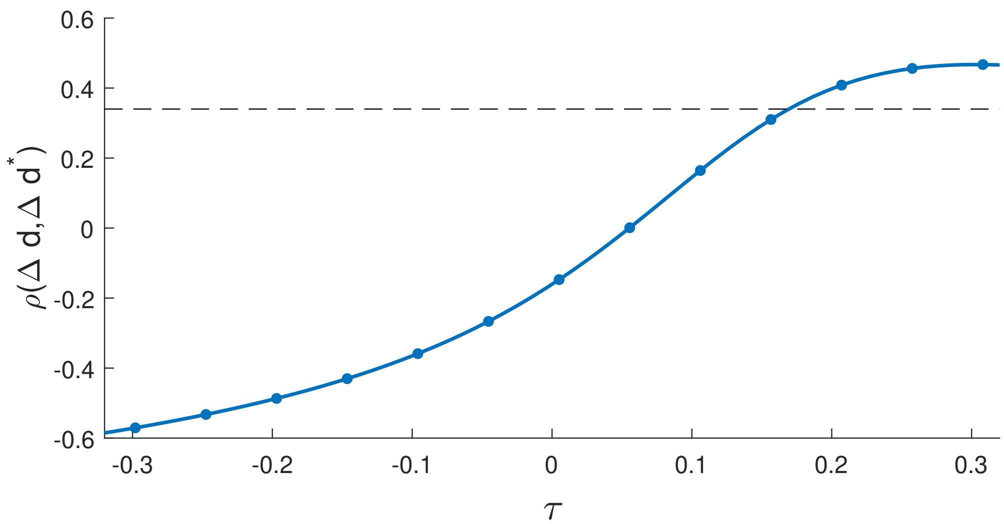
It is clear that the cross-country correlation of the first difference of domestic absorption is sensitive to the choice of τ. Figure 6 shows this by displaying the cross country-correlation of the first difference of domestic absorption across different values of τ. We use this correlation to identify the size of τ, as was briefly mentioned in Section 4. In our calibration, we find a value of τ of 0.17. Moreover, in Appendix C, we provide external evidence supporting a positive elasticity using bilateral data on trade flows.

To see role role of τ in a full model, we look at the results of the nested model with zero domestic trade costs. The details of this exercise are displayed in Appendix E.5. We exogenously set $\tau = 0$ and do

[57] A smaller response of relative consumption in the ROW relative to the US also contributes to generating a small value of the Backus-Smith-Kollmann statistic.

not target the cross country correlation of domestic absorption. The calibrated parameters and resulting moments are reported in Tables E.5 and E.6 under '$\tau = 0$.' This model generates a worse fit for the Backus-Smith-Kollmann correlation and the cross country correlation of domestic absorption. Thus, τ matters for accounting for the Backus-Smith-Kollmann puzzle and the cross country correlation of domestic absorption. In terms of the untargeted moments, we notice that the model is able to generate a differential short and long run elasticity of trade to prices, but both elasticities are higher than in our benchmark model. Finally, the contribution of financial and trade shocks to the variation in the RER are very similar to our benchmark model (see Table E.7).

Figure 6: Identification of τ



Notes: Correlation of the cross country growth rates of domestic absorption between the US and ROW given different values of τ. The other parameters are set as in Table 1. Based on model simulation of 10,000 periods. Black dashed line is the correlation in the data.

Bayesian Estimation

We estimate the model using Bayesian methods along with four data series: US GDP, ROW GDP, NT and the RER. Details about the Bayesian estimation are provided in Appendix E.1. Overall, we find that the estimated parameters are very similar to those obtained from our benchmark model in Section 4.1. We present the estimated parameters in Table E.1. Moreover, we find that the dynamic trade model is preferred to the static trade model. That is, the model with dynamic trade has a better fit, as shown by the log data density higher in the dynamic trade model than the static trade model. This is consistent with our results from Section 5.3 and Section 5.2, where we argue in favor of the dynamic trade model.

To study the role of each shock in shaping the variation of the RER, we consider the counterfactual path of the RER, when it is driven by only one of the estimated shocks. The counterfactual RER under only trade shocks tracks closely the actual path of the RER across the whole time period. With only financial shocks, the RER follows a similar path up to the early 2000s, but not after that. Productivity shocks do not seem to generate a path for the RER closely related to the data. Overall, trade shocks generate a path of the RER that most closely tracks the actual data.[58] Finally, we compute the variance decomposition of the RER using the estimated parameters, and find similar results as in our benchmark model (see Table E.3). Thus, we find that trade shocks are crucial to capture the dynamics of the RER.[59]

Short Sample Simulations

As opposed to our Benchmark case where the simulation is based on one long-period sample, we estimate the model using multiple samples consisting of shorter periods. Specifically, we run simulations of 160 periods, to be consistent with our quarterly data during 1980Q1-2019Q4, and take average over the moments calculated from each simulation. More details are presented in Appendix E.2.

The parameters and their calibrated values are presented in Table E.5 under 'Short.' The estimated parameters are almost the same as the Benchmark case. This is because the moments calculated from multiple short-samples are very similar to those from one long-sample. If anything, the estimates for the autocorrelations are slightly smaller in short samples, due to the well known fact that least square estimates of AR(n) models are biased, although consistent (Marriott and Pope, 1954; Kendall, 1954). However the differences are negligible, and the model result again shows that trade shocks play a crucial role for low fequency dynamics of the real exchange rate.

Specification of Dynamic Trade

To explore the robustness of our specification of dynamic trade, we consider an alternative way of modeling it. In particular, we introduce adjustment costs in the use of imported inputs in the final good aggregator, as in Erceg et al. (2006), Rabanal and Rubio-Ramirez (2015) and Gornemann et al. (2020) (see Appendix E.3 for details.). This is a reduced form way of generating a differential long- and short-run trade elasticity without dynamic decisions of heterogeneous firms. We identify the adjustment cost using the estimated speed of adjustment of NT to prices in the data.[60] The parameters and calibrated values are presented in Table E.5 under 'Input Adj.'

[58] In Table E.2, we show that the correlation between the actual RER and the counterfactual under only trade shocks is 0.83. Under only financial shocks, the correlation is slightly lower, 0.73. The correlation under only productivity shocks is -0.20.

[59] This is consistent with the message in Rios-Rull, Santaeulalia-Llopis, Schorfheide, Fuentes-Albero and Khrysko (2012) that argues that it is not the choice of quantitative methodology that is responsible for empirical findings, but rather the data employed in the identification. Data on NT is key to the identification of parameters relevant to capture the dynamics of the RER at the whole spectrum of frequencies.

[60] The speed of adjustment is captured with the parameter α in the ECM equation 7, which is estimated to be 0.07.

The alternative model generates similar targeted and untargeted moments as in the benchmark model (see Table E.6). It is able to generate a differential short and long run trade elasticity to prices. However, the long-run trade elasticity is lower than in our benchmark model. We find that the low frequency share in the spectrum of the RER is higher than in our benchmark model. Finally, we find a stronger effect of financial shocks on the RER in the short run, relative to our benchmark model, as measured by the variance decomposition of shocks (Table E.7). However, in the long run we find similar effects of trade shocks on the RER as in our benchmark model.

Three Country Model

We verify the claim that the within-ROW trade cost elasticity captures the cost of trade between countries that compose the ROW. We consider the world consisting of three countries, where one of them is the US and the remaining two countries are aggregated as a ROW. The details are presented in Appendix E.6.

We show that changing the elasticity of trade cost between the two ROW countries to trade costs against the US generates similar results as varying the domestic trade cost elasticity in the two country model. That is, a higher elasticity of trade costs between the ROW countries in response to higher export costs to the US dampens both the effect of trade cost shocks on relative domestic absorption and the RER.

Common Trade Costs

It is well known that the scale of trade as a share of GDP for most countries has been increasing significantly since the fall of the Bretton Woods system in 1973. A large part of this increase can be attributed to the reductions in intratemporal trade frictions across countries, induced by technological progress and policies promoting free trade (Alessandria, Bai and Woo, 2024). The frequent and significant changes in the trade costs of most countries, in fact, are the main reasons for the fluctuations in relative trade costs across countries. While we captured the differences in these costs in our benchmark model, we study the robustness of our specification to include a common component between the ROW and the US.

Specifically, we consider a shock to common trade cost, which affects the US and ROW in tandem, in addition to the shocks to differential trade costs. The sum of common and differential components will be the process of the country-specific trade costs. The details of this robustness check are presented in Appendix E.4. We find that the results are similar to the benchmark model, although the common trade cost shock increases the volatility of macro aggregates relative to the benchmark case, consistent with the findings in Alessandria, Kaboski and Midrigan (2013b) in the absence of inventories.

Investment Adjustment Costs

We consider investment, as opposed to capital, adjustment costs since the two types of costs generate different responses to shocks (e.g. hump-shape investment responses under investment adjustment costs) which potentially matters for the co-movement of variables of interest. The details are presented in Appendix E.7. We consider the specification in Christiano, Eichenbaum and Evans (2005) and calibrate the parameters in the same way as in the benchmark model. The results are presented in Tables E.5 and E.6, under 'Inv Adj'. Overall, the calibrated parameters and the results of this model are very similar to the benchmark model, including the volatility of investment. We find that this model generates a higher share of the variance of the RER for the low frequency than in the benchmark model. Finally, we find a similar contribution of financial and trade shocks to the variation in the RER at different horizons (see Table E.7). Overall, our main results are robust to this alternative specification of investment adjustment cost.

Sunk Exporting Cost and Trade Elasticity

In our benchmark model, the trade elasticity is larger in the long run than in the short run, correctly displaying the J-curve feature. However, because we are restricting the elasticity of substitution to be $\gamma = 1.5$ as in Itskhoki and Mukhin (2021b) and fixed costs of exporting to be consistent with firm level data, there are slight disparities from the values of the short and long run trade elasticity in the data. We show that by varying these three parameters, we can improve the fit of these long- and short-run trade elasticities. To do so, we jointly estimate the elasticity of substitution and fixed exporting costs along with other parameters and target ECM estimates. The details are in Appendix E.8. As shown in Table E.6 under 'TE,' we get the elasticities much closer to data. This is driven by higher sunk costs and a larger elasticity of substitution, as presented in Table E.5. Finally, we find that financial shocks increase their importance in driving variation in the RER across all horizons relative to the benchmark model (see Table E.7), although we still find that trade shocks are the dominant shock in the long run.

A More Sophisticated Financial Process

We show that our result that trade shocks are needed to match the RER and NT moments at the high frequency is robust to considering a more sophisticated financial process. In particular, we allow the financial shock to be the mix of two AR(1) processes, each of them with a different persistence. The details are in Section E.9. This model fails to capture the RER and NT moments at the high frequency because both processes trigger a positive comovement between the RER and NT on impact, as shown in Figure E.6. As a consequence, the model cannot match the weak high frequency correlation. Moreover, conditional on matching the other target moments, the model generates an excess volatility of NT at the high frequency.

Finally, the model fails to generate a short and long-run trade elasticity close to the data. Hence, the main results of the paper hold under this more sophisticated financial process.

8 Concluding remarks

In this paper, we present a comprehensive analysis of the dynamics of the RER and NT flows by integrating *financial shocks*, *trade shocks*, and *dynamic trade* into a standard international business cycle model. Our analysis shows the necessity of incorporating all of these three features to capture the joint dynamics of the RER and NT across the frequency domain, while still accounting for the major RER puzzles and business cycle moments.

In line with existing literature, we find that financial shocks are important for explaining the variation in the RER at higher frequencies, especially for the financial disconnect. However, our novel contribution lies in demonstrating the critical importance of trade shocks in capturing movements of the RER and NT flows. Given that 61 percent of the RER unconditional variance is attributed to the low-frequency movements, trade shocks are essential to account for its overall dynamics.

While this study represents substantial advances in our understanding of the factors shaping the RER and NT dynamics across various frequencies, it also highlights avenues for future research that warrant exploration. Specifically, further investigation is necessary to refine the measurement of financial and trade shocks, and shed light on their sources of variation. Our analysis suggests that extending the work that focuses on the dynamics of trade shocks would be very valuable. Finally, while we have treated financial and trade shocks as independent, it is also conceivable that they share common underlying causes.[61] Addressing these unresolved issues will deepen our understanding of the RER and NT dynamics and provide further insights about their nature.

[61] For example, Costinot, Lorenzoni and Werning (2014) show that intertemporal policy, such as capital controls, have similar implications as intratemporal trade policy in terms of policy outcomes.

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Appendix

A Data Description

In this section, we describe the data sources and how we construct the variables for our calibration.

- Period: 1980Q1 - 2019Q4, quarterly
- ROW: Trade-weighted average of 10 Countries
  - Countries: Canada, Finland, Germany, Ireland, Italy, Japan, Republic of Korea, Spain, Sweden, United Kingdom. These countries account for 60 percent of total US trade.
  - Weights: Country-specific average of the sample period (Federal Reserve). While the weights are updated every year, we use the constant weights using country-specific average during our sample period. For countries in Euro Area after 1999, we allocate the weights for the total of Euro Area into these countries using the average distribution within Euro Area during 1980-1999.
  - We check the robustness of the empirical moments across using other weights than average trade. Moreover, we consider adding China into the sample, although data for China is available only after 1990. Table A.1 shows that the moments we consider are similar across these variations.
- US interest rate: Effective federal funds rate (IMF), deflated with consumer price index (OECD)
- ROW interest rate: Money market rates, deflated with consumer price index (OECD)
  - For most countries, money market rates (IMF). In a few cases where the data is not available from the IMF for the whole sample period, we consider different sources as below.
  - China, Germany, UK: Immediate call money/interbank rate (OECD)
  - Canada: Short term interest rate (OECD)
  - Japan: Overnight call rate (Bank of Japan)
  - Figure F.2 shows that the interest rate data from different sources we use align very well with the money market rate from the IMF.
- Quarterly National accounts (OECD)
  - US dollars, volume estimates, fixed PPPs, seasonally adjusted
  - Y: Gross domestic product - expenditure approach
  - C: Private final consumption expenditure
  - I: Gross fixed capital formation
  - X: Exports of goods and services
  - M: Imports goods and services
- Real exchange rate: Effective exchange rate, Real, Narrow indices, 2010=100 (BIS)
- Terms of trade: Terms of trade index (BEA, retrieved from FRED)
- US exporter characteristics (Alessandria and Choi 2021)

Table A.1: Empirical Moments with Different Weights

	Mean trade	Output	Trade	With China
$cor\left(\Delta c - \Delta c^{*}, \Delta q\right)$	-0.10	-0.07	-0.12	-0.09
$autocor\left(i - i^{*}\right)$	0.87	0.86	0.86	0.86
$cor(\Delta y, \Delta y^{*})$	0.40	0.26	0.30	0.32
$cor(\Delta d, \Delta d^{*})$	0.34	0.24	0.24	0.11
$autocor(nt)$	0.98	0.98	0.98	0.98
$\sigma(\Delta inv^{*})/\sigma(\Delta y^{*})$	2.59	2.59	2.59	2.59
$cor\left(\Delta nt, \Delta q\right)$	0.30	0.30	0.30	0.30
$\sigma(nt)/\sigma(q)$	1.16	1.16	1.16	1.16
$cor(\Delta tot, \Delta q)$	0.49	0.49	0.49	0.49
$\sigma(\Delta y)$	0.007	0.007	0.007	0.007

B Spectrum Analysis

In this section, we describe our spectrum analysis. For more detailed and rigorous steps, see Hamilton (2020).

To study the RER represented at the spectrum domain, we convert its time-domain representation using the Fourier transform. Given a covariance-stationary process q_t, the spectrum is defined as the Fourier transform of its autocovariance function $C(\tau)$:

$$S(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} e^{-i\omega\tau}\ C(\tau) \tag{9}$$

where

$$C(\tau) = \mathbb{E}(q_t - \mu_q)(q_{t-\tau} - \mu_q).$$

Note that ω is a (angular) frequency measure of radians per period.[62] Given that upper and lower bounds for business cycle frequency are 8 and 32 quarters, the range of frequency that corresponds to the business cycle is

$$\omega \in \left[\frac{2\pi}{32 \text{ quarters}}, \frac{2\pi}{8 \text{ quarters}}\right] = [0.196, 0.785].$$

This is consistent with the range used by Rabanal and Rubio-Ramirez (2015).

Using the inverse of Equation (9), we can write the autocovariance function as

$$C(\tau) = \int_{-\pi}^{\pi} e^{i\omega\tau}\ S(\omega)\ d\omega$$

Then with $\tau = 0$, the variance $C(0) = \int_{-\pi}^{\pi} S(\omega) d\omega$ is the sum of spectrum. In this sense, the spectrum decomposes the variance into different frequencies.

Also, we can show that spectrum is symmetric around zero, periodic with a period of 2π, and can be

[62] For an ordinary frequency $\xi = \omega/2\pi$ (Hz), the spectrum is defined as

$$S(\xi) = \int_{-\infty}^{\infty} C(\tau) e^{-2\pi i \xi \tau} d\tau.$$

written as

$$S(\omega) = \frac{1}{2\pi} C(0) + \frac{2}{2\pi} \sum_{\tau=1}^{\infty} cos(\omega\tau)\ C(\tau). \tag{10}$$

In order to estimate the population spectrum given the data sample of T observations, we could use the sample autocovariance

$$\hat{C}(j) = \frac{1}{T} \sum_{t=j+1}^{T} (q_t - \bar{q})(q_{t-j} - \bar{q}),$$

where $\bar{q}$ is a sample mean. This yields an estimate of Equation (10), known as the sample periodogram:

$$\hat{S}^{sp}(\omega) = \frac{1}{2\pi} \hat{C}(0) + \frac{2}{2\pi} \sum_{j=1}^{T-1} cos(\omega j)\ \hat{C}(j). \tag{11}$$

However, such estimate is subject to a few limitations. Thus we use a nonparametric estimation instead. That is, we estimate the spectrum by

$$\hat{S}(\omega_j) = \sum_{m=-h}^{h} k(\omega_{j+m}, \omega_j)\ \hat{S}^{sp}(\omega_{j+m}) \tag{12}$$

where $k(\omega_{j+m}, \omega_j)$ is a kernel with a bandwidth h. The idea is to take a weighted average of the sample periodograms $\hat{S}^{sp}(\tilde{\omega})$ for the values $\tilde{\omega}$ around ω, where the distance between ω and $\tilde{\omega}$ determines the kernel, i.e. the weight.

After substituting Equation (11) into Equation (12) and some calculations, it can be shown that Equation (12) is equivalent to

$$\hat{S}(\omega) = \frac{1}{2\pi} \hat{C}(0) + \frac{2}{2\pi} \sum_{j=1}^{T-1} k_j^*\ cos(\omega j)\ \hat{C}(j).$$

where $\{k_j^*\}_{j=1}^{T-1}$ is a weighting sequence corresponding to a kernel function $k(\omega_{j+m}, \omega_j)$. The weight for the modified Bartlett kernel is given as

$$k_j^* = \begin{cases} 1 - \frac{j}{J+1} & \text{for } j = 1, 2, \cdots, J \\ 0 & \text{for } j > J \end{cases}$$

where J is the length of a window for the weight that is related to the kernel bandwidth. This yields the spectrum estimate of

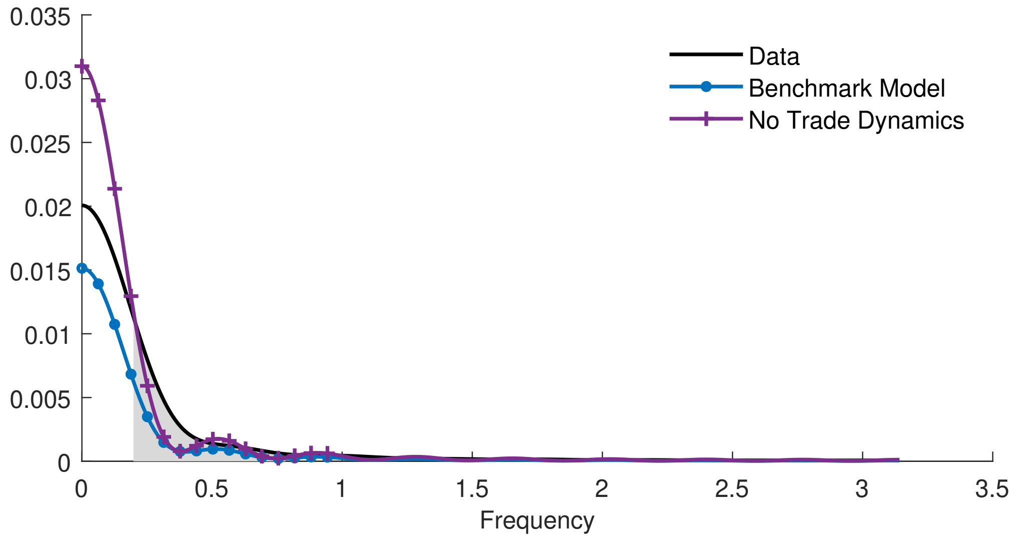
$$\hat{S}(\omega) = \frac{1}{2\pi} \hat{C}(0) + \frac{2}{2\pi} \sum_{j=1}^{J} \left[1 - \frac{j}{J+1}\right]\ cos(\omega j)\ \hat{C}(j).$$

On the other hand, there is no fixed rule for the choice of the bandwidth h (or window J). Hamilton (2020) suggests trying different values and "relying on subjective judgement for the most plausible estimate." For the benchmark exercise we use the window of $J = 16$, and check that other values yield a similar result that is within the range of findings of the literature. Figure B.1 shows the estimated spectrum of the RER for the full range in $[0, \pi]$.

C Empirical Evidence of Trade Costs

In this section, we provide an external validation for our specification of trade costs. First, we use data on bilateral trade to measure these costs for different pairs of countries. Next, we estimate the elasticity of

Figure B.1: Spectrum of the RER



within-country trade costs and show it is consistent with the specification in our benchmark model.

We measure trade costs from data as a wedge in the CES demand, common in any Armington trade model. The demand for country i goods in country j is given by:

$$X_t^{ij} = \left(\frac{e^{\xi_t^{ij}} p_t^{ij}}{P_{jt}} \right)^{-\gamma} D_{jt}$$

where X_t^{ij} is bilateral trade flows from country i to j, p_t^{ij} is the price level of exports from country i to j, P_{jt} is the price level of domestic absorption in country j, D_{jt} is the domestic absorption of country j, and γ is the elasticity of substitution. Our model assumes the same type of CES structure for the demand for differentiated goods. Moreover, it is the basic trade block for almost all studies in trade literature.

Note that all of the terms in the demand function except for ξ_t^{ij} are observables. Thus, we can recover trade costs ξ_t^{ij} as a gap between actual and predicted trade flows given prices and aggregate demand. In particular, we estimate the above demand function using the following regression

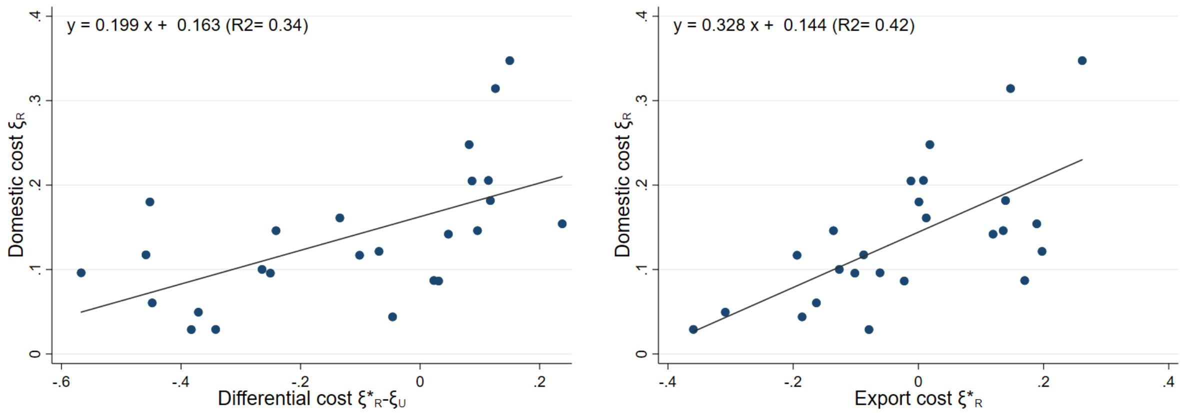
$$\log X_t^{ij} = \beta \log(P_t^{ij}/P_{jt}) + \log D_{jt} + \varepsilon_t^{ij}. \tag{13}$$

and consider the residuals ε_t^{ij} as trade costs. By estimating the demand function, we do not restrict ourselves to a particular value of elasticity. In fact, there is a broad range of values used for the elasticity in the literature, and the estimated elasticity varies greatly depending on the sample and the length of period considered. Also, the estimation by construction minimizes the size of trade costs and lets us take a conservative stance on the role of trade costs.

We estimate the demand function using data for the US and ten other countries for the ROW, as is done in our benchmark quantification. For data on bilateral trade flows, we use annual data from UN Comtrade, converted into real terms using the price levels of the US dollars from Penn World Table 10.0. Domestic absorption and price levels of different countries in our sample also come from Penn World Table 10.0. Our sample period covers the period of 1994-2019, mostly due to data availability of trade flows.[63]

[63] We also check the robustness with quarterly data during the period of 2008Q1-2019Q4. We find that the path of trade costs

Figure C.1: Empirical Relationship of Trade Costs



Notes: Each point represents trade costs of each year. The plots corresponds to the first and second columns of Table C.1.

For the trade cost between the US and the ROW, ξ^*_{Rt} and ξ_{Ut}, we aggregate the data on the ten countries and use it as the variables for the ROW. Then we run the regression (13) for the US-ROW pair. On the other hand, for the trade cost within the ROW, ξ_{Rt}, we use bilateral data on each pair of countries in the ROW, and take average of the recovered residuals across countries to construct time series.

Given the path of trade costs, we check the relationship of ξ_{Rt} with ξ^*_{Rt} or $\xi^*_{Rt} - \xi_{Ut}$. We use these estimates to compare with the model analogue. As shown in Equation 14, in our model we allow trade costs within the ROW aggregate, ξ_{Rt}, to be nonzero. We further assume it to be $\xi_{Rt} = \tau\frac{\xi_t}{2}$, where τ measures the elasticity of the within component respect to the ROW-US trade cost. In the calibration of the benchmark model, displayed in Section 4.1, we find that τ is a small positive number (0.16). Thus ξ_{Rt} is positively correlated with trade costs from ROW to the US, $\xi^*_{Rt} = \frac{\xi_t}{2}$, and also with the difference between exporting and importing costs, $\xi^*_{Rt} - \xi_{Ut} = \xi_t$.

Figure C.1 shows that we do find a consistent pattern in the data. It plots the relationship of ξ_R (left panel) with $\xi^*_R - \xi_U$ and ξ^*_R (right panel). The estimated elasticity is between 0.199 and 0.32.

Finally, table C.1 displays the result with additional controls. Although the size of estimated τ differs slightly, we have the robust result that the estimated τ is positive as in our benchmark model presented in Section 4.1. Moreover, the coefficient of ξ^*_R is always larger than $\xi^*_R - \xi_U$, as specified in our benchmark model.

D Spectrum of NT Flows

Figure D.1 shows the spectrum of the NT flows, normalized by its unconditional variance, in the data and different specifications. As shown in Table 2 Panel C2, the spectrum in the benchmark model (blue line with *o*) is very close to the data (black solid). Moreover, in the model without dynamic trade (violet line with +), there is too much variation arising from lower frequencies, which as we mention in Section 5.3 seems to contradict the intuition. To show that the intuition we provide is correct, we plot the spectrum for the model without dynamic trade but keeping the same adjustment cost of net foreign assets (χ) as in the benchmark model (red line with >).[64] In this case the low frequency variation in NT flows is lower (54 percent) than in the benchmark model (62 percent). Hence, dynamic trade helps to distribute variation in

is similar to using annual data.

[64]We do not re-calibrate.

Table C.1: Empirical Estimates of τ

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent variable: ξ^R							
$(\xi_R^* - \xi_U)$	0.199**		0.546*		0.493***		0.443	
	(0.0581)		(0.223)		(0.100)		(0.304)	
ξ_R^*		0.328***		0.843***		0.583***		0.972**
		(0.0798)		(0.166)		(0.0627)		(0.293)
Country FE			Y	Y			Y	Y
Spending Constraints					Y	Y	Y	Y
Observations	25	25	25	25	25	25	25	25
R-squared	0.338	0.423	0.207	0.530	0.513	0.790	0.0847	0.324

Notes: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. 'Country FE' denotes the fixed effect for origin and destination countries when estimating the demand function for the pair of ROW countries. 'Spending Constraints' are a restriction on the coefficient of domestic absorption to be 1, as predicted in the model with CES demand.

NT flows from higher to lower frequencies, exactly the opposite way as with the RER.[65]

E Robustness

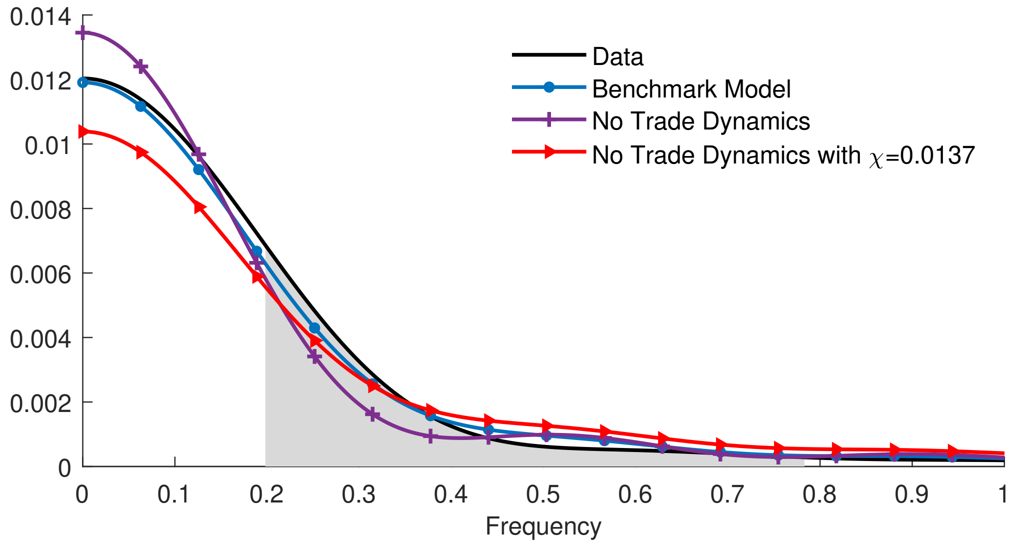
In this section, we consider alternative specifications to check the robustness of the results of the benchmark model. First, we explore an alternative estimation strategy to identify the parameters and shocks driving the RER: Bayesian methods. We show that we obtain similar estimates of parameters than under our Benchmark model in Section 4.1. Next, we show that explore alternative specifications to our benchmark model, in particular an estimation based on short sample simulations, a reduced form specification of dynamic trade, a model with common trade costs, a model with no within-ROW trade costs (i.e. $\tau = 0$), a three-country model, and an alternative model with investment adjustment costs. Overall, we find that our benchmark model better captures the dynamic of key variables in our model relative to the alternative specifications. Moreover, we find that the result that financial shocks matter more for the short run and trade shocks for the long run is robust across the alternative specifications.

E.1 Bayesian Estimation

We explore an alternative estimation strategy to identify the shocks driving the RER: Bayesian methods. First, we show that we obtain similar estimated of parameters than under our benchmark model in Section 4.1. Second, we show that the model with dynamic trade is preferred to that of static trade. Finally, we show that trade shocks are crucial for generating the dynamics of the RER. That is, the counterfactual RER under trade shocks is closer to the RER in the data than under the financial shock. We also present the estimated path of the different shocks and compute the conditional variance decomposition of the RER.

[65]The share of low frequency variation of the RER under this alternative model (no dynamics with same χ as benchmark) is 0.73, hence higher than in the benchmark model (0.70).

Figure D.1: Spectrum of NT Flows



Notes: The graph is enlarged for the frequency of $[0, 1]$, as the spectrum on $[1, \pi]$ is near zero. Gray area shows the range of the frequencies for the business cycle. The blue line with circles shows the result in the benchmark model, while the violet line with crosses shows the result of the model recalibrated with no sunk cost of exporting. The red line with > shows the result of the model recalibrated with no sunk cost of exporting, but setting the adjustment cost of net foreign assets (χ) as in the benchmark model (i.e. $\chi = 0.0137$).

Estimated Parameters

We estimate the same parameters as the ones we internally calibrate in the benchmark case. In particular, we estimate the productivity shock volatility, σ_c and σ_d, financial shock parameters, ρ_ϕ and σ_ϕ, trade shock parameters, $\rho_{\xi d}$, $\sigma_{\xi d}$ and τ, as well as the adjustment costs parameters χ and κ. We impose loose priors, mostly uniform distribution and inverse gamma for volatility parameters. For observables, we use four data series: GDP growth of the US and the ROW, the NT flows and the RER, with the same sample period as in the benchmark case (1980Q1-2019Q4).

The left panel of Table E.1 reports the prior and posterior distribution. The estimated results are similar to the benchmark case. Both financial and trade shocks show relatively high volatility, that of trade shocks being only slightly larger than financial shocks. The size of the financial shock volatility is the smallest, while the size of trade shock is largest. The within-country trade cost parameter $\tau = 0.14$ is also very close to the benchmark case (0.17).

Dynamic vs Static Trade

To show that dynamic trade model better captures the data on trade and the RER compared to the static mode, we estimate the static model with no fixed cost of exporting. We use the same priors as the before. The result of the static case is presented in the right panel of Table E.1.

We find that the log data density (Laplace Approximation) in the dynamic trade model is 1862.88 while in the static model it is 1592.32, so that the dynamic trade model is preferred over the static trade model by a Bayes factor of $exp(270.56)$.[66] The value of $exp(270.56)$ provides a strong evidence supporting the hypothesis that the dynamic trade model is statistically better than the model with static trade. This is consistent with our results from Section 5.2 and Section 5.3, where we argue in favor of the dynamic trade model.

[66] The Bayes factor is similar to a likelihood-ratio test.

Table E.1: Estimated Parameters

	Prior Distribution	Dynamic Trade		Static Trade	
		Post Mean	90% Interval	Post Mean	90% Interval
ρ_ψ	Uniform [0.9,0.999]	0.94	(0.91 , 0.98)	0.96	(0.94 , 0.99)
ρ_{ξ_d}	Uniform [0.9,0.999]	0.99	(0.97 , 0.99)	0.98	(0.96 , 0.99)
σ_c	Inverse gamma (0.01,0.01)	0.004	(0.004 , 0.005)	0.02	(0.02 , 0.02)
σ_d	Inverse gamma (0.01,0.01)	0.006	(0.005 , 0.006)	0.008	(0.007 , 0.009)
σ_ψ	Inverse gamma (0.01,0.01)	0.003	(0.003 , 0.004)	0.002	(0.002 , 0.002)
σ_{ξ_d}	Inverse gamma (0.01,0.01)	0.04	(0.03 , 0.04)	0.09	(0.07 , 0.11)
τ	Uniform [-0.5, 0.5]	0.14	(0.12 , 0.18)	0.10	(0.07 , 0.13)
χ	Uniform [0.00001,0.5]	0.05	(0.01 , 0.10)	0.19	(0.08 , 0.31)
κ	Uniform [0,20]	2.34	(0.01 , 5.06)	1.73	(0.55 , 3.28)
ζ	Uniform [0.85, 1.5]	1.17	(1.10 , 1.23)	1.40	(1.32 , 1.49)
Log data density		1862.88		1592.32	

Estimated Shocks

Figure E.1 shows the estimated path of productivity shocks of the ROW, trade shocks, and financial shocks. The trade shocks were most volatile during the 1980s, when the series of different trade policy were implemented in many countries. For example, Uruguay Round launched multilateral trade negotiations. Also, countries like India and Mexico introduced trade reforms and lowered their trade barriers. In recent years trade shocks became more stable, while 2009 marks the period of the highest trade cost.

Counterfactual RER

In Figure E.3, we show the path of the RER in the data, as well as the counterfactual where the RER is driven by only one of the shocks. We present the correlation between the data and counterfactual cases in Table E.2. It is clear that the RER under trade shocks closely tracks the actual RER during the whole sample period. The path generated only with the Trade shocks, shown in green dashed line, very closely follow the data path. The correlation with the data is 0.83. On the other hand, with only financial shocks, the RER follows a similar path up to the early 2000s, but rather departs from data in the later periods. The correlation is 0.72 and lower than the case with only trade shocks. Productivity shocks do not seem

Figure E.1: Estimated Shocks

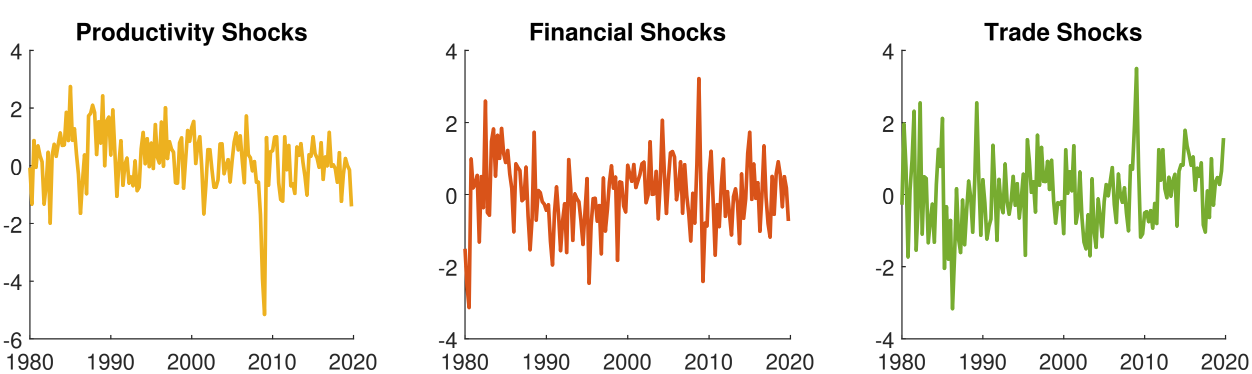
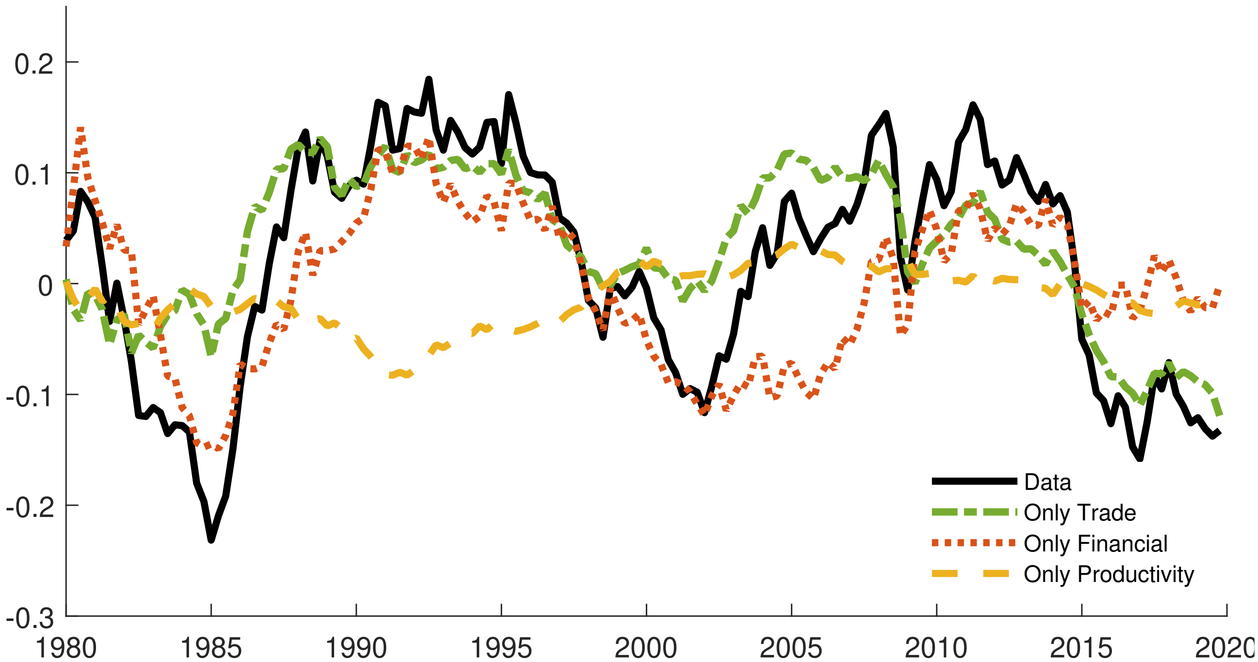


Figure E.2: RER Dynamics under Different Shocks



Notes: This figure shows the counterfactual path of RER with only one type of shocks. The productivity shocks include both the differential and common component.

to generate a path for the RER that closely related to the data. The correlation in this case is negative. Overall, we conclude that trade shocks generate a dynamics of the RER that more closely track the actual data.

We turn to look at the spectrum of the counterfactual cases with muting each shock. The result is presented in Figure E.3. The shape of the spectrum is disrupted the most when we shut down the trade shocks. The share accounted by the low frequency reduces from 61 percent in the Benchmark case to 58 percent without trade shocks. However, without financial shocks, it increases to 63 percent.

Finally, in Table E.3 we provide the conditional variance decomposition obtained from the Bayesian estimation of the dynamic trade model. In particular, we compute the share of the h–quarter ahead error forecast variance of the RER explained by each shock. It is clear that the trade shock explains most of the forecast error variance of the RER in the long run (i.e. low frequency), while the financial shock is important for the short run (i.e. high frequency) fluctuations.

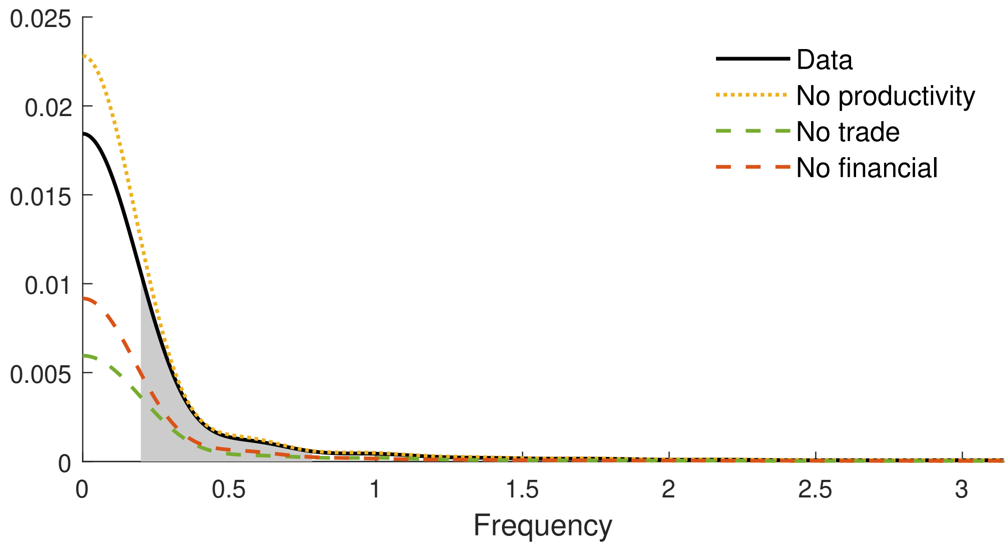
E.2 Simulation with Short Samples

In this section, we take a different approach to estimate the parameters that are jointly pinned down to match the targeted moments. Instead of taking moments from one long-sample simulation, we simulate each sample for 400 periods, to be consistent with our quarterly data during 1980Q1-2019Q4, and take the

Table E.2: Correlation between Data and Counterfactuals

	Only productivity	Only trade	Only financial
cor(data, model)	-0.20	0.83	0.72

Figure E.3: RER Spectrum Under Different Shocks



Notes: This figure shows the counterfactual spectrum of RER with only one type of shocks. The productivity shocks include both the differential and common components.

average of the last 160 observations from multiple runs. The results are presented in Tables E.5 and E.6 under 'Short.'

The estimated parameters, and thus the values of targeted moments, are very similar to those of the Benchmark. However, notice that the moments of persistence tend to be slightly smaller than those from longer samples. To analyze the effect of using different sample periods further, we consider keeping the same parameters as in the Benchmark case and use different periods to compare the calculated moments.

The result of these exercises is presented in Table E.4. First, we consider the effect of the distance from the initial point (or steady state). To see this, we use samples of same length starting at different periods, which are presented in the first three columns of Table E.4. We find that given the same sample lengths, distance from the initial point seems to have no impact on the estimates. Second, we consider using samples of increasing lengths. We find that volatility moments are similar across sample length, but autocorrelations are increasing in sample lengths, especially for lengths<900. This is a case not only for the endogenous variables, like the RER, but also for the shock process such as ψ_h, ξ_d. This is because least square estimates of AR(n) models are downward biased (Marriott and Pope, 1954; Kendall, 1954). The bias is decreasing in the sample length, and the estimate is consistent. However even with very small sizes the difference is negligible leading to a very similar result as using longer samples as in our Benchmark case. Finally, Table E.7 shows the results in terms of the conditional variance decomposition. We find that trade shocks are more important under the short sample identification.

E.3 Dynamic Trade Specification

In this section, we consider the final good aggregator with adjustment costs in the use of imported inputs, as in Erceg et al. (2006), Rabanal and Rubio-Ramirez (2015) and Gornemann et al. (2020). The CES aggregator of the retail sector in each country is now given by

$$D_t = \left[Y_{Rt}^{\frac{\gamma-1}{\gamma}} + \omega^{\frac{1}{\gamma}} \left(\varphi_t Y_{Ut}\right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \qquad D_t^* = \left[Y_{Ut}^{*\frac{\gamma-1}{\gamma}} + \omega^{\frac{1}{\gamma}} \left(\varphi_t^* Y_{Rt}^*\right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$$

Table E.3: Conditional Variance Decomposition (%)

	quarters = 1	8	32	80
	Bayesian Estimation			
Financial shock	68.26	47.22	25.27	19.57
Trade shock	28.04	45.76	63.74	69.72
Productivity shock	3.69	7.02	10.98	10.71
	Benchmark Model			
Financial shock	62.6	46.6	25.8	25.7
Trade shock	34.5	48.4	65.1	64.5
Productivity shock	2.9	5.0	9.1	9.8

where φ_t and φ_t^* is the weight on the use of imported inputs in the production of the final good. Their functional forms are given by

$$\varphi_t = \left[1 - \frac{\iota}{2}\left(\frac{Y_{Ut}/Y_{Rt}}{Y_{Ut-1}/Y_{Rt-1}} - 1\right)^2\right] \qquad \varphi_t^* = \left[1 - \frac{\iota}{2}\left(\frac{Y_{Rt}^*/Y_{Ut}^*}{Y_{Rt-1}^*/Y_{Ut-1}^*} - 1\right)^2\right].$$

where parameter ι determines the size of the adjustment cost in the use of imported inputs.

We identify the adjustment cost ι using the speed of adjustment of NT to prices, i.e. the estimated parameter α in the ECM equation 7, which has a value of 0.07 in data. That is, on top of the other targeted moments, we add the speed-of-adjustment parameter to jointly estimate the parameters, including the new parameter ι (11 parameters and 11 moments). Since we compare this model with our benchmark, we shut down trade dynamics that arises from the fixed costs of exporting , i.e. we set the fixed costs and idiosyncratic productivity to zero.

The parameters and their calibrated values are presented in Table E.5 under 'Input Adj.' The calibrated value of the input adjustment cost parameter ι is 18.9. This implies that when the share of home to foreign inputs, $\frac{Y_{Ut}/Y_{Rt}}{Y_{Ut-1}/Y_{Rt-1}}$, deviates 1 percent from the steady state, then, given $\iota = 18.9$ and $\omega = 0.097$, the home-country output will be 0.001 percent smaller than without the presence of this cost.

In Table E.6, we label the column for the result of this alternative dynamic specification as 'Input Adj.' The model is able to capture the speed of adjustment of NT to prices ($\alpha = 0.07$) that is a target in this case. We find that this alternative model is able to generate a differential short and long run trade elasticity to prices. However, it does not generate a differential elasticity as close to the data as in the benchmark model.

Furthermore, we plot in Figure E.4 the spectrum of the RER in the data (solid black line), the benchmark model (dashed blue line) and the alternative input adjustment model (green line with x). The alternative dynamic trade model does not capture the size of the spectrum as well as the benchmark model. Moreover, in Panel C1 of Table E.6 we show that the low frequency share in the spectrum of the RER is higher in the alternative dynamic trade model (0.76 percent). Hence, the benchmark model, where we exploit information from the microdata on firm dynamics, captures the shape of the spectrum of the RER better than the alternative dynamic trade model.

Finally, Table E.7 present the variance decomposition of this alternative model, under 'Input Adj.' We

Table E.4: Moments with Different Sample Lengths

Length	100	100	100	300	900	2900	6900
Start period	501	1801	7801	7701	7401	6401	3401
End period	600	1900	7900	8000	8300	9300	10300
$\sigma(\Delta y)$	0.682	0.681	0.684	0.685	0.685	0.685	0.685
	(0.0032)	(0.0034)	(0.0033)	(0.002)	(0.0012)	(0.0007)	(0.0004)
$cor(\Delta y, \Delta y^*)$	0.379	0.375	0.385	0.383	0.385	0.386	0.385
	(0.0059)	(0.006)	(0.0063)	(0.0034)	(0.002)	(0.0011)	(0.0007)
$\sigma(\Delta inv^*)/\sigma(\Delta y^*)$	2.523	2.515	2.533	2.526	2.520	2.517	2.517
	(0.0105)	(0.0096)	(0.0097)	(0.0061)	(0.0034)	(0.0018)	(0.0012)
$autocor(nt)$	0.922	0.922	0.919	0.938	0.941	0.942	0.942
	(0.0023)	(0.0028)	(0.0028)	(0.0011)	(0.0006)	(0.0003)	(0.0002)
$autocor\,(i - i^*)$	0.776	0.773	0.767	0.825	0.847	0.853	0.855
	(0.0058)	(0.0059)	(0.0064)	(0.0034)	(0.0019)	(0.0009)	(0.0006)
$cor\,(\Delta c - \Delta c^*, \Delta q)$	-0.080	-0.098	-0.089	-0.088	-0.084	-0.084	-0.087
	(0.0066)	(0.0076)	(0.006604)	(0.00414)	(0.002389)	(0.0013)	(0.000834)
$cor\,(\Delta nt, \Delta q)$	0.298	0.298	0.292	0.289	0.290	0.291	0.292
	(0.0064)	(0.0067)	(0.0066)	(0.004)	(0.0024)	(0.0012)	(0.0008)
$\sigma(nt)/\sigma(q)$	1.491	1.508	1.436	1.260	1.163	1.137	1.130
	(0.0346)	(0.033)	(0.0334)	(0.0182)	(0.01)	(0.0059)	(0.004)
$cor(\Delta d, \Delta d^*)$	0.352	0.353	0.355	0.355	0.356	0.357	0.357
	(0.0065)	(0.0062)	(0.0061)	(0.0034)	(0.0022)	(0.0011)	(0.0007)
$cor(\Delta tot, \Delta q)$	0.500	0.510	0.500	0.500	0.490	0.490	0.500
	(0.01)	(0.01)	(0.01)	(0)	(0)	(0)	(0)
$\sigma(\Delta q)$	5.836	5.887	5.827	5.773	5.774	5.759	5.753
	(0.0452)	(0.0407)	(0.0415)	(0.0221)	(0.0134)	(0.0077)	(0.0051)
$autocor(q)$	0.910	0.906	0.912	0.945	0.955	0.958	0.959
	(0.0032)	(0.0036)	(0.0035)	(0.0014)	(0.0008)	(0.0004)	(0.0003)
$autocor(\psi_h)$	0.912	0.912	0.913	0.946	0.954	0.956	0.956
	(0.0034)	(0.0035)	(0.0033)	(0.001)	(0.0007)	(0.004)	(0.0002)
$autocor(\xi_d)$	0.912	0.912	0.913	0.946	0.954	0.956	0.956
	(0.0034)	(0.0035)	(0.0033)	(0.001)	(0.0007)	(0.004)	(0.0002)

find a stronger role of financial shocks as drivers of the RER in the short run, relative to our benchmark model. The contribution of financial shocks to the one-quarter ahead error forecast variance of the RER is around 89 percent in the alternative model, as opposed to 63 percent in our benchmark. However, in the long run we find similar results as in our benchmark model for the contribution of financial and trade shocks in explaining the variation of the RER. We find that trade shocks explains around 60 percent of the 80-quarters ahead error forecast variance in this alternative model, close to the 63 percent in the benchmark model. On the other hand, financial shocks explain around 37 percent in the alternative model, higher than the 26 percent found in our benchmark model. Hence, our main result holds: trade shocks are crucial to explain the low frequency variation in the RER, thus being crucial for capturing its overall variation.

However, our finding about the importance of trade costs in the long run still holds. In Table E.7, the columns labeled 'Input Adj,' we show the variance decomposition of each shock. As in the benchmark

Table E.5: Robustness – Calibrated Parameters

Parameters	Benchmark	Short	Input Adj	Common	$\tau = 0$	Inv Adj	TE	Sophisticated ψ
Financial shock, volatility σ_ψ	0.002	0.002	7.94E-04	0.007	0.002	0.004	0.002	0‡
Financial shock, persistence ρ_ψ	0.96	0.97	0.99	0.95	0.93	0.87	0.98	0‡
Trade shock, volatility σ_ξ	0.05	0.07	0.05	0.03	0.05	0.07	0.08	0‡
Trade shock, persistence ρ_ξ	0.97	0.99	0.99	0.99	0.97	0.98	0.97	0‡
Trade shock, within-country share τ	0.17	0.16	0.02	0.40	0.00‡	0.17	0.15	0‡
Common productivity, volatility σ_{a_c}	0.004	0.005	0.005	0.005	0.005	0.005	0.005	0.005
Differential productivity, volatility σ_{a_d}	0.005	0.006	0.004	0.005	0.007	0.005	0.004	0.006
Adjustment cost of portfolios χ	0.0137	0.021	0.01	0.01	0.002	$8e^{-04}$	0.01	0.007
Adjustment cost of capital κ	2.42	3.22	9.35	2.32	10.27	1.60*	2.86	4.95
Pricing to market parameter ζ	0.97	0.95	1.4	1.01	0.81	0.98	0.99	0.97
Import adjustment cost ι	0‡	0‡	18.9	0‡	0‡	0‡	0‡	0‡
Fixed cost of new exporters f^0	0.14	0.14	0‡	0.14	0.14	0.14	0.39	0.14
Fixed cost of incumbent exporters f^1	0.04	0.04	0‡	0.04	0.04	0.04	0.08	0.04
Volatility of idiosyncratic productivity σ_μ	0.15	0.15	0‡	0.15	0.15	0.15	0.15	0.15
Common Trade shock, volatility σ_{ξ^c}	0‡	0‡	0‡	0.01	0‡	0‡	0‡	0‡
Common Trade shock, persistence ρ_{ξ^c}	0‡	0‡	0‡	0.99	0‡	0‡	0‡	0‡
Elasticity of Substitution γ	1.5‡	1.5‡	1.5‡	1.5‡	1.5‡	1.5‡	1.7	1.5‡
High Persistence Fin shock, volatility σ_ψ^h	0‡	0‡	0‡	0‡	0‡	0‡	0‡	0.001
High Persistence Fin shock, persistence ρ_ψ^h	0‡	0‡	0‡	0‡	0‡	0‡	0‡	0.94
Low Persistence Fin shock, volatility σ_ψ^l	0‡	0‡	0‡	0‡	0‡	0‡	0‡	0.002
Low Persistence Fin shock, persistence ρ_ψ^l	0‡	0‡	0‡	0‡	0‡	0‡	0‡	0.30
Correlation innovations ϵ_ψ^h and ϵ_ψ^l ρ_ψ^l	0‡	0‡	0‡	0‡	0‡	0‡	0‡	0.03

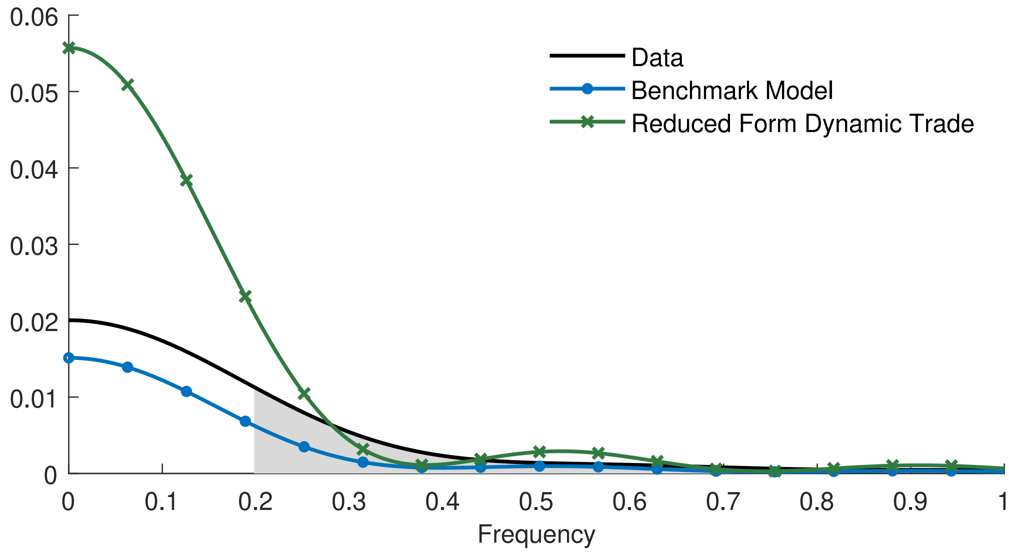
Notes: Superscript ‡ denotes that the parameter is exogeneously set while superscript * specifies that the calibrated adjustment cost is for investment not capital. 'Benchmark' shows the same results presented in Section 5. 'Short' shows the result of the estimation using short period samples (Section E.2). 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section E.3). 'Common' is for the model with common shocks to trade costs (Section E.4). '$\tau = 0$' is the case with no within-ROW trade cost shocks (Section E.5). 'Inv Adj' is the case with investment adjustment cost (Section E.7). 'TE' is when we target short- and long-run elasiticies (Section E.8). 'Sophisticated ψ' is the case of a mix of two AR(1) processes for the financial shock (Section E.9).

Table E.6: Robustness – Model Results

Moments	Data	Benchmark	Short	Input Adj	Common	τ = 0	Inv Adj	TE	Sophisticated ψ
A. Targeted Moments									
$\sigma(\Delta y)$	0.007	0.006	0.007	0.007	0.007	0.007	0.007	0.007	0.007
$cor\,(\Delta c - \Delta c^{*}, \Delta q)$	-0.10	-0.10	-0.09	-0.10	-0.11	0.28	-0.10	-0.08	0.20
$autocor\,(i - i^{*})$	0.87	0.87	0.77	0.85	0.85	0.86	0.87	0.83	0.85
$cor(\Delta y, \Delta y^{*})$	0.40	0.40	0.41	0.40	0.34	0.40	0.40	0.42	0.41
$cor(\Delta d, \Delta d^{*})$	0.34	0.34	0.33	0.34	0.38	0.06†	0.34	0.33	0.31
$autocor(nt)$	0.98	0.96	0.95	0.96	0.98	0.96	0.98	0.99	0.95
$\sigma(\Delta inv^{*})/\sigma(\Delta y^{*})$	2.59	2.59	2.53	2.59	2.61	2.71	2.59	2.60	2.90
$cor\,(\Delta nt, \Delta q)$	0.30	0.30	0.29	0.31	0.29	0.32	0.30	0.34	0.79
$\sigma(nt)/\sigma(q)$	1.16	1.16	1.15	1.17	1.16	1.41	1.16	1.23	1.40
$cor(\Delta tot, \Delta q)$	0.49	0.49	0.48	0.49	0.49	0.45	0.49	0.47	0.47
$autocor(\frac{x+m}{y})$	0.97	0.98†	0.95†	0.98†	0.97	0.98†	0.98†	0.98†	0.98†
$cor(\Delta\frac{x+m}{y}, \Delta y)$	0.32	0.49†	0.20†	-0.09†	0.34	0.53†	-0.03†	0.44†	0.51†
B. Trade Elasticity									
SR elasticity	0.20 (0.05)	0.40†	0.37†	0.34†	0.34†	0.78†	0.40†	0.17	1.13†
LR elasticity	1.16 (0.25)	1.00†	1.19†	0.66†	1.03†	1.44†	1.11†	1.13	1.65†
Adjustment	0.07 (0.02)	0.04†	0.07†	0.07	0.02†	0.03†	0.02†	0.02	0.14†
C1. Frequency Decomposition of RER									
High frequency	0.08	0.07†	0.10†	0.06†	0.06†	0.07†	0.06†	0.07†	0.09†
Business cycle frequency	0.31	0.23†	0.29†	0.18†	0.20†	0.21†	0.18†	0.22†	0.26†
Low frequency	0.61	0.70†	0.61†	0.76†	0.74†	0.72†	0.76†	0.71†	0.65†
C2. Frequency Decomposition of NT Flows									
High frequency	0.06	0.08†	0.10†	0.08†	0.06†	0.08†	0.06†	0.05†	0.08†
Business cycle frequency	0.30	0.30†	0.35†	0.23†	0.24†	0.28†	0.22†	0.22†	0.32†
Low frequency	0.64	0.62†	0.55†	0.69†	0.70†	0.64†	0.72†	0.73†	0.60†
D. Disconnect Puzzles									
$\sigma(q)$	0.10	0.08	0.09	0.15	0.10	0.09	0.16	0.17	0.04
$\sigma(\Delta q)/\sigma(\Delta y)$	4.24	3.48	3.9	3.24	2.90	3.12	4.14	6.13	1.81
$autocor(q)$	0.97	0.97	0.94	0.99	0.97	0.97	0.98	0.97	0.94
β_{fama}	-1.34	0.14	0.71	-0.18	0.13	-1.07	-0.46	1.60	-0.21
R^2_{fama}	0.04	0.001	0.07	0.007	0.001	0.10	0.004	0.07	0.001
$cor(q, i - i^{*})$	-0.30	-0.44	-0.42	-0.09	-0.33	-0.60	-0.42	-0.50	-0.47
$autocor(i)$	0.93	0.93	0.82	0.89	0.90	0.90	0.86	0.90	0.93
$\sigma(i - i^{*})/\sigma(\Delta q)$	0.13	0.01	0.01	0.07	0.01	0.02	0.01	0.01	0.02

Notes: Superscript † denotes that the moment is not targeted during the calibration procedure. 'Benchmark' shows the same results presented in Section 5. 'Short' shows the result of the estimation using short period samples (Section E.2). 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section E.3). 'Common' is for the model with common shocks to trade costs (Section E.4). 'τ = 0' is the case with no within-ROW trade cost shocks (Section E.5). 'Inv Adj' is the case with investment adjustment cost (Section E.7). 'TE' is when we target short- and long-run elasticities (Section E.8). 'Sophisticated ψ' is the case of a mix of two AR(1) processes for the financial shock (Section E.9).

Figure E.4: RER Spectrum Robustness



case, the financial shocks play a dominant role in the earlier periods, accounting up to 85 percent in the first quarter. However, the share reduces to 31 percent in the 80th quater, and trade shocks account for the largest share of 63 percent.

E.4 Common Trade Costs

We extend the trade shock process to include a common trade cost. Specifically, trade cost shocks are given by

$$\begin{aligned} \xi^*_{Rt} &= \frac{\xi_t}{2} + \xi^c_t & \xi_{Ut} &= -\frac{\xi_t}{2} + \xi^c_t \\ \xi_{Rt} &= \tau\frac{\xi_t}{2} & \xi^*_{Ut} &= 0 \end{aligned} \tag{14}$$

where $\tau \in \mathbb{R}$,

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}, \quad \varepsilon_{\xi t} \sim N(0, \sigma_\xi)$$

and

$$\xi_{c,t} = \rho_{\xi_c} \xi_{c,t-1} + \varepsilon_{\xi ct}, \quad \varepsilon_{\xi ct} \sim N(0, \sigma_{\xi_c}).$$

This means we need to discipline two extra parameters: the persistence ρ_{ξ_c} and volatility σ_{ξ_c} of the common trade cost shock. We target the autocorrelation of the share of trade over GDP, $(x + m)/y$, and the correlation between the growth rates of the trade share and GDP to identify these parameters, since the common trade process has a direct effect on the scale of trade.

Table E.5 present the calibrated parameters. The parameters in common with the benchmark model are similar, although we find a higher persistence of the differential trade shock and a higher domestic trade cost elasticity. We find that the persistence of the common trade shock process is quite high, around 0.99, consistent with the fact that the trade share has been increasing since the early 1970s.

The moment matching of the common trade cost model is presented in Table E.6. In general, we find that the model performs similarly as the benchmark model. However, we find a higher volatility of macro aggregates relative to the RER and a slightly higher share of low frequency variation in the RER. Finally,

Table E.7: Robustness – Variance Decomposition

	Benchmark			Input Adj			τ = 0			Inv Adj		
Quarters	P	F	T	P	F	T	P	F	T	P	F	T
1	2.9	62.6	34.5	3.19	89.39	7.42	5.84	60.62	33.54	1.42	66.91	31.67
8	5.0	46.6	48.4	3.41	80.08	16.50	8.48	43.41	48.11	2.78	44.48	52.75
32	9.1	25.8	65.1	3.67	55.22	41.12	11.82	22.60	65.58	5.71	19.93	74.36
80	9.8	25.7	64.5	2.84	37.04	60.13	11.65	20.25	68.09	5.18	12.79	82.03

	Benchmark			Short Sample			Common Trade Cost			TE		
Quarters	P	F	T	P	F	T	P	F	T	P	F	T
1	2.9	62.6	34.5	2.51	51.74	45.76	4.98	57.06	37.96	1.16	74.74	24.11
8	5.0	46.6	48.4	3.85	36.00	60.15	7.32	45.70	46.98	1.74	65.68	32.57
32	9.1	25.8	65.1	5.68	17.27	77.06	11.77	24.97	63.26	3.35	44.45	52.20
80	9.8	25.7	64.5	5.38	15.33	79.29	11.27	23.33	65.40	3.64	42.15	54.22

Notes: 'P,' 'F' and 'T' refer to share accounted by productivity shocks, financial shocks, and trade shocks, respectively. 'Benchmark' shows the same results presented in Section 5. 'Input Adj' shows the result of the model with reduced-form trade dynamics (Section E.3). 'τ = 0' is the case with no within-ROW trade cost shocks, with $\tau = 0$ (Section E.5). 'Short Sample' shows the result of the estimation using short period samples (Section E.2). 'Common' is for the model with common shocks to trade costs (Section E.4). 'TE' is when we target short- and long-run elascicities (Section E.8).

Table E.7 show the results in terms of the conditional variance decomposition of the RER. Consistent with the fact that the estimated persistence of the differential trade shock is higher, we find a stronger effect of the differential trade shock on the RER than in the benchmark model.

E.5 Within-ROW Trade Costs

In this section, we evaluate the role of the within-ROW trade cost τ. We set up an alternative model where the elasticity of domestic trade costs to international costs is $\tau = 0$. Then, we calibrate the model by targeting the same moments as in the benchmark model, except the cross country correlation of domestic absorption.

The calibrated parameters and resulting moments are reported in Tables E.5 and E.6 under '$\tau = 0$.' This model generates a worse fit for the Backus-Smith-Kollmann correlation, which is 0.27 in the model as opposed to -0.10 in the data, although it lies within the estimated range in the literature. The model misses the cross country correlation of domestic absorption, being 0.06 in the model and 0.34 in the data. Thus, τ matters for accounting for the Backus-Smith-Kollmann puzzle and the cross country correlation of domestic absorption. Overall, this model has a worse fit into matching the target moments relative to our benchmark model.

In terms of the untargeted moments, we notice that the model delivers a differential short and long run elasticity of trade to prices, but but both higher than in the data. Hence, our benchmark model better captures the differential short and long run elasticity of NT to prices. The spectrum decomposition of the RER in this model is also worse that in the benchmark model.

In Table E.7 we present the results related to the variance decomposition of the RER, under '$\tau = 0$.' Our main results holds under this specification: financial shocks explain a higher portion of the variation in the RER in the short run, while trade shocks explains most of the variation in the long run. We find a similar role of financial shocks in the short run to the benchmark model, since the contribution of this shocks to the 1-period ahead error forecast variance of the RER is 61 percent in this specification, compared to 63 percent in the benchmark model. Furthermore, financial shocks explain 20 percent of the 80-quarters ahead error forecast variance in the RER, close to the 26 percent in our benchmark model. Finally, trade shocks explains 68 percent of the 80-quarters ahead error forecast variance in the RER, close to the 65 percent in our benchmark model.

E.6 Three Country Model

We extend the static trade model to include and extra country. One of the countries is the US, which has measure 0.5, whereas each of the two extra countries are ROW countries with size 0.5. The aggregate of the ROW is an average of the two ROW countries, and we use the same moments as in the two country model. Trade cost shocks are given by

$$\begin{aligned} \xi_{R1,U} &= \xi_d/2 & \xi_{R2,U} &= \xi_d/2 \\ \xi_{U,R1} &= -\xi_d/2 & \xi_{U,R2} &= -\xi_d/2 \\ \xi_{R1,R2} &= \tau \cdot \xi_d/2 & \xi_{R2,R1} &= \tau \cdot \xi_d/2 \end{aligned}$$

where R1 and R2 denote two countries consisting the ROW, US denotes the US, and ξ_d follows an AR(1) processes as in the benchmark model. To calibrate the model we set the value of τ to the benchmark case (0.174), and discipline the remaining parameters using the same moments as the benchmark case. Table E.8 show the matching of the moments.[67]

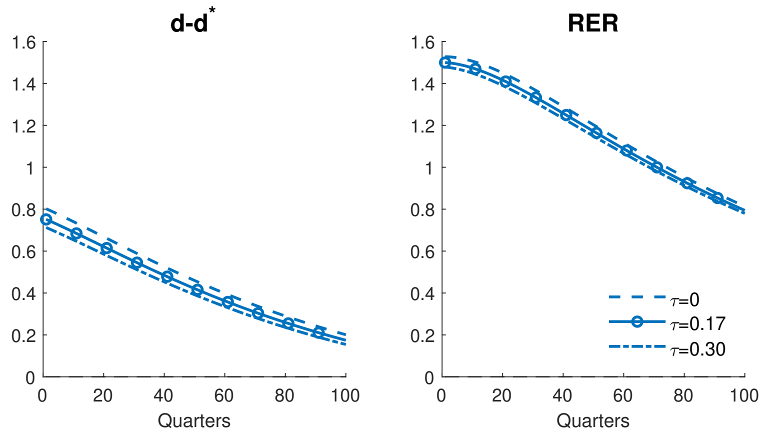
Table E.8: Targeted Moments from Three Country Model

Moments	Data	Benchmark	Three-Country Model
$\sigma(\Delta y)$	0.007	0.006	0.017
$cor\,(\Delta c - \Delta c^*, \Delta q)$	-0.10	-0.10	0.13
$autocor\,(i - i^*)$	0.87	0.87	0.85
$cor(\Delta y, \Delta y^*)$	0.40	0.40	0.83
$cor(\Delta d, \Delta d^*)$	0.34	0.34	0.27
$autocor(nt)$	0.98	0.96	0.98
$cor\,(\Delta nt, \Delta q)$	0.30	0.30	0.27
$\sigma(nt)/\sigma(q)$	1.16	1.16	1.46

Figure E.5, shows the Impulse Response Functions of selected variables to a differential trade cost shock, for different values of τ. As it is the case with the domestic trade cost elasticity in the two country model (Figure 5), a higher elasticity dampens the effect on relative domestic absorption and the RER.

[67]We find that matching the aggregate US and ROW moments in the three country model is harder than in the two country case. However, the model does a reasonable job in matching them. Moreover, the purpose of this exercise is to show that the elasticity of domestic to foreign trade cost in the two country model operates as a trade cost between ROW countries, which we show it does qualitatively.

Figure E.5: IRFs to Trade Shock in Three-Country Model



E.7 Investment Adjustment Costs

In this section, we consider an adjustment cost in investment as in Christiano et al. (2005). That is, the law of motion for capital is now given by

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where $S(1) = S'(1) = 0$ and $S''(1) > 0$. Here, we consider the functional form of S as

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\tilde{\kappa}}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

To estimate the adjustment cost parameter $\tilde{\kappa}$, we again use the volatility of investment. That is, the targeted moments remain unchanged. The result of the estimated model with the new investment adjustment cost is presented in Tables E.5 and E.6, under 'Inv Adj.'

The estimated parameter for the adjustment cost is smaller than in the benchmark model, since now the adjustment cost is over a flow rather than a stock. This version of the model requires higher standard deviations of financial and trade shocks relative to our benchmark, where the variance of the common and differential productivity shocks are almost the same. We find that the adjustment cost of debt is smaller under investment adjustment costs. Finally, we find the a very similar pricing to market coefficient as in the benchmark, with an implied pass-through of exchange rate to prices of 67 percent.

The model is able to match the target moments, as well as the untargeted moments. The short and long run elasticity of NT to prices is also very similar to the benchmark model. Moreover, the investment adjustment cost model generates a higher share of the variance of the RER for the low frequency than in the benchmark model, hence the later captures better this aspect of the variation in the RER.

Finally, in Table E.7 we present the contribution of each shock to the error forecast variance of the RER. Consistent with our benchmark model, we find that financial shocks explains most of the variation in the short run, while trade shocks explain most of it in the long run. In particular, financial shocks explains 67 percent of the one-quarter ahead error forecast variance, very close to the 63 percent in our benchmark model. On the other hand, trade shocks explains 82 percent of the 80-quarters ahead error forecast variance of the RER in this model, more than the 65 percent found in the benchmark model. Overall, our main results are robust to this alternative specification of investment adjustment cost.

E.8 Sunk Exporting Cost and Trade Elasticity

In this section, we improve the performance of the model in generating short- and long-run trade elasticities. To do so, we allow the Armington elasticity and the exporter fixed costs to be estimated jointly along with other internally-calibrated parameters.

The Armington elasticity is a crucial parameter that determines the relationship between relative prices and NT flows. Yet the estimates for the elasticity tend to vary, and a large range of values are used in the trade literature. In our benchmark model, we set the Armington elasticity exogenously with $\gamma = 1.5$ as in Itskhoki and Mukhin (2021a). However, the long-run trade elasticity is slightly lower in our benchmark model than in the data (0.97 in the benchmark model and 1.16 in data, see Table 2), suggesting the need for a larger Armington elasticity. Moreover, since the behavior of individual firms affects aggregate trade flows, re-calibrating the fixed costs of exporting would allow the model to generate a short and long run trade elasticity closer to data.

To estimate these three additional parameters, we add to our targeted moments three ECM estimates, namely, short- and long-run trade elasticities and the speed of adjustment. The result of this exercise is presented in Tables E.5 and E.6, under the column 'TE.' Consistent with our conjecture, the estimated Armington elasticity $\gamma = 1.7$ is slightly larger than the benchmark case. With the estimated fixed costs $f^0 = 0.39, f^1 = 0.08$, we get larger sunk costs, contributing to generating a larger gap between short- and long-run elasticities so that they are closer to data.

The volatility and persistence of trade shocks are estimated to be larger relative to the benchmark model. This arises from firms being subject to larger frictions in their exports. It is also the case that the persistence of financial shocks is higher. Overall, we find similar results as in the benchmark model. However, the persistence of NT and, as a consequence, the low frequency share of variation are higher than in the benchmark model which arises from estimating a higher persistence of trade and financial shocks and higher sunk costs. Finally, as shown in Table E.7, financial shocks explain a higher portion of the variation of the RER at all horizons relative to the benchmark model (due to the higher persistence of its process), although trade shocks are still dominant in the long run.

E.9 A More Sophisticated Financial Process

We show that our result that trade shocks are needed to match the RER and NT moments at the high frequency is robust to considering a more sophisticated financial process. In particular, we allow the financial shock to be the mix of two AR(1) processes, each of them with a different persistence. Assume that there are twp financial processes given by,

$$\psi_t^h = \rho_\psi^h \psi_{t-1}^h + \epsilon_{\psi t}^h \quad \text{and} \quad \psi_t^l = \rho_\psi^l \psi_{t-1}^l + \epsilon_{\psi t}^l$$

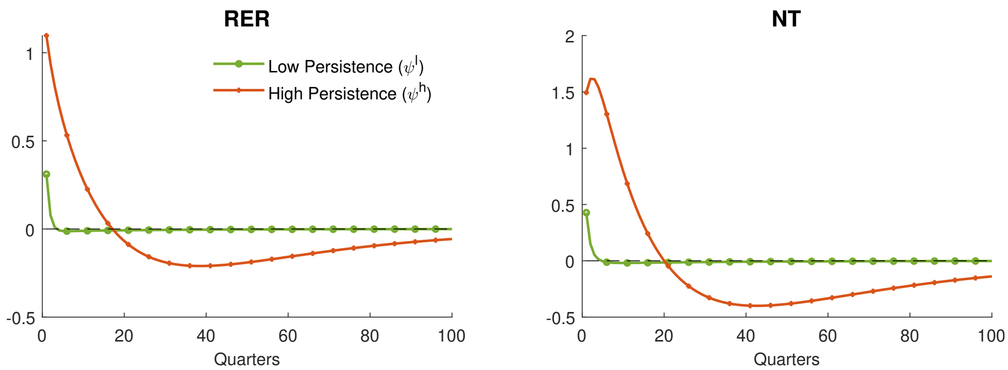
where $\rho_\psi^h \geq \rho_\psi^l$ are the persistence's of the processes, $\epsilon_{\psi t}^h \sim N(0, \sigma_\psi^h)$ and $\epsilon_{\psi t}^l \sim N(0, \sigma_\psi^l)$. We also allow the innovations $\epsilon_{\psi t}^h$ and $\epsilon_{\psi t}^l$ to be correlated. We target the same moments as in the benchmark model (we have the same number of moments than parameters since we include the correlation between the two financial innovations). When we estimate the model we impose the following constraints: $0.5 \leq \rho_\psi^h < 1$ and $0 \leq \rho_\psi^l \leq 0.5$[68]

The estimated parameters are displayed in Table E.5 and the moments in Table E.6, under 'Sophisticated ψ'. This model fails to capture the RER and NT moments at the high frequency because both processes trigger a positive comovement between the RER and NT on impact, as shown in Figure E.6. As a consequence, the model cannot match the weak high frequency correlation. Moreover, conditional on matching the other target moments, the model generates an excess volatility of NT at the high frequency. Finally,

[68]Increasing the upper bound of rho_ψ^l in the estimation did not change the results.

the model fails to generate a short and long-run trade elasticity close to the data. Hence, the main results of the paper hold under this more sophisticated financial process.

Figure E.6: Impulse Response Functions: Two AR(1) Financial Processes



F Theoretical Decomposition of NT

In this section, we provide the derivation of NT in our benchmark model. For simplicity, we omit the time subscript t.

The demand function for aggregate exports of ROW is given by

$$Y_R^* = \omega \left(\frac{P_R^*}{P^*} \right)^{-\gamma} D^*$$

where $P^* = 1$. The demand faced by a producer of each variety j is

$$y_{Rj}^* = \left(\frac{p_{Rj}^*}{P_R^*} \right)^{-\theta} Y_R^* = \omega \left(\frac{p_{Rj}^*}{P_R^*} \right)^{-\theta} \left(\frac{P_R^*}{P} \right)^{-\gamma} D^*$$

where the second equality uses the aggregate demand function. Using that total sales is a sum of sales of all varieties,

$$\begin{aligned} P_R^* \; Y_R^* &= \int p_{Rj}^* \; y_{Rj}^* dj = \int \omega {p_{Rj}^*}^{1-\theta} {P_R^*}^{\theta-\gamma} D^* \; dj \\ &= \omega \; {P_R^*}^{1-\gamma} \; D^*. \end{aligned}$$

Aggregate exports and imports in nominal terms are given by

$$\begin{aligned} X^N &= Q \int_{j \in \mathcal{H}} p_{Rj}^* \; y_{Rj}^* dj = Q P_R^* Y_R^* = \omega \; Q \; {P_R^*}^{(1-\gamma)} D^* \\ M^N &= \int_{j \in \mathcal{H}^*} p_{Uj} \; y_{Uj} dj = \omega \; P_U^{(1-\gamma)} D \end{aligned}$$

and the export and import prices are

$$
\begin{aligned}
Px &= Q\left(\frac{1}{N}\int_{j\in\mathcal{H}}\left(\frac{p^*_{Rj}}{e^{\xi^*_R}}\right)^{1-\theta^*} dj\right)^{\frac{1}{1-\theta^*}} = Q\ P^*_R\ e^{\xi^*_R(\theta^*-1)}\ N^{\frac{-1}{1-\theta^*}} \\
Pm &= \left(\frac{1}{N^*}\int_{j\in\mathcal{H}^*}\left(\frac{p_{Uj}}{e^{\xi_U}}\right)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} = P_U\ e^{\xi_U(\theta-1)}\ N^{*\frac{-1}{1-\theta}}
\end{aligned}
$$

where N denotes the mass of exporters. In logs,

$$
\begin{aligned}
x^N &= \log\omega + (1-\gamma)p^*_R + d^* + q \\
m^N &= \log\omega + (1-\gamma)p + d \\
px &= q + p^*_R + \frac{1}{1-\theta^*}n - (1-\theta^*)\xi^*_R \\
pm &= p_U + \frac{1}{1-\theta}n^* - (1-\theta)\xi_U
\end{aligned}
$$

where lower case letters denote variables in logs.

Using that in real terms real exports and real imports are $X = X^N/Px, M = M^N/Pm$, respectively, log of real exports and imports are given by

$$
\begin{aligned}
x &= x^N - px = \log\omega - \gamma p^*_R + d^* - \frac{1}{1-\theta^*}n \\
m &= m^N - pm = \log\omega - \gamma p_U + d - \frac{1}{1-\theta}n^*.
\end{aligned}
$$

Therefore, NT, measured by log of Export-Import ratio, is

$$
\begin{aligned}
nt &= x - m \\
&= \gamma(p_U - p^*_R) + (d^* - d) + \left(\frac{1}{1-\theta}n^* - \frac{1}{1-\theta^*}n\right) \\
&= \gamma\ (tot + q) + (d^* - d) + ((1-\theta^*)\xi^*_R - (1-\theta)\xi_U) + (1-\gamma)\left(\frac{1}{1-\theta}n^* - \frac{1}{1-\theta^*}n\right). \qquad (15)
\end{aligned}
$$

where $tot_t = pm - px$ is the terms of trade.

On the other hand, in the Armington trade model, demand for exports and foreign goods follows a standard CES structure. Taking the ratio of demand functions for exports and imports implied in the Armington model, we have

$$
nt_t = \gamma\ (tot_t + q_t) + (d^*_t - d_t). \qquad (16)
$$

Comparing this equation, Equation 15 for the benchmark model has additional terms $((1-\theta^*)\xi^*_R - (1-\theta)\xi_U)$ and $(1-\gamma)\left(\frac{1}{1-\theta}n^* - \frac{1}{1-\theta^*}n\right)$. These reflect that in our model we have two features, trade shocks and trade dynamics.

G Analytical Solution and Impacts of Shocks on the RER Persistence

In this section, we derive the analytical solution for the RER to study the impact of financial and trade shocks on the RER persistence.

We start with the log-linearized resource constraint with trade shock ξ_t:

$$y_t = (1-\omega)y_{Ht} + \omega(y^*_{Ht} + \xi_t)$$

where the small case denotes log-linearized variables. Using log-linearized NX_t and substituting the solution for prices and quantities, we get

$$nx_t = \omega(y_{Ht} - y_{Ft} - s_t) = \omega(\lambda_q q_t - \lambda_\xi \tilde{\xi}_t)$$

for some coefficients λ_q, λ_ξ and $\tilde{\xi}_t = \xi_t - \xi^*_t$. Notice that we have an additional shock in the resource constraint while the equations for other quantities and prices are same as in Itskhoki and Mukhin (2021a). Also, we are setting productivity shocks $a_{ct} = a_{dt} = 0$ to focus on two other shocks.

Following similar steps as described in Itskhoki and Mukhin (2021a), we end up in a system of two equations, which can be expressed in a matrix form as

$$E_t \begin{pmatrix} 1 & -\hat{\chi}_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} q_{t+1} \\ \hat{b}^*_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} q_t \\ \hat{b}^*_t \end{pmatrix} - \begin{pmatrix} -\hat{\chi}_1 & k(1-\rho) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_t \\ \hat{\xi}_t \end{pmatrix}$$

where k is a coefficient substituted for simplicity, and $\hat{\xi}_t$ is a normalization of $\tilde{\xi}_t$. We use Blanchard-Khan methods to derive the closed-form solution for the RER. That is, we diagonalize the dynamic system of

$$E_t z_{t+1} = Bz_t - C(\psi_t \quad \hat{\xi}_t)'$$

where $z_t = \begin{pmatrix} q_t \\ b^*_t \end{pmatrix}$, $B = \begin{pmatrix} 1+\hat{\chi}_2 & \frac{\hat{\chi}_2}{\beta} \\ 1 & \frac{1}{\beta} \end{pmatrix}$, and C is a coefficient matrix to the vector of shocks. Eigenvalues μ_1, μ_2 of the matrix B are solutions to

$$(1+\hat{\chi}_2 - \mu)\left(\frac{1}{\beta} - \mu\right) - \frac{\hat{\chi}_2}{\beta} = 0.$$

The left eigenvector for an eigenvalue $\mu_2 > 1$ is $v = (1, 1/\beta - \mu_1)$. We pre-multiply the dynamic system by v and get the equilibrium cointegration relationship:

$$\begin{aligned} vz_t &= q_t + \left(\frac{1}{\beta} - \mu_1\right) b_t \\ &= \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \left(\frac{1-\beta\mu_1+\beta(1-\rho)k\mu_1}{1-\beta\rho\mu_1}\right)\hat{\xi}_t. \end{aligned} \tag{17}$$

Combining this with the second dynamic equation for $\hat{b}^*_{t+1}$, we get

$$\begin{aligned} \hat{b}^*_{t+1} - \mu_1\hat{b}^*_t &= q_t + \left(\frac{1}{\beta} - \mu_1\right)\hat{b}^*_t - \hat{\xi}_t \\ &= vz_t - \hat{\xi}_t \\ &= \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{\xi}_t. \end{aligned}$$

Now apply lag operator $(1-\mu_1 L)$ to Equation (17) to finally get

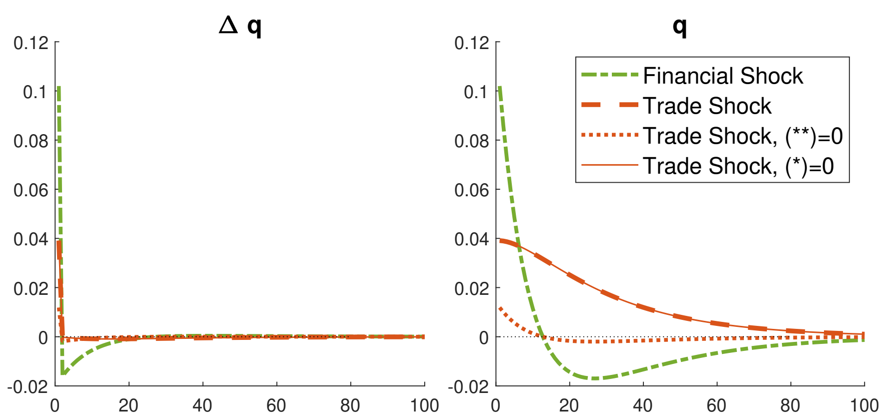
$$(1-\mu_1 L)q_t = \left(1-\frac{1}{\beta}L\right)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \underbrace{\frac{\beta(1-\rho)k\mu_1}{1-\beta\rho\mu_1}}_{(*)}\hat{\xi}_t\right] + \underbrace{\frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)}_{(**)}\hat{\xi}_t.$$

This equation shows that the equilibrium RER follows a stationary ARMA(2,1) process. Note that the term (*) captures the trade shock effect through the UIP deviation, and (**) is for the effects via the resource constraint.

As can be seen from this equation, the main reason that effect of trade shock has a different impact than the financial shock is due to the second term, (**), the mechanism through the resource constraint. Absent of this term, financial and trade shocks differ only in their coefficients but share the same lag operator. Then the impacts of financial and trade shocks become proportional to each other that only differ in their sizes but not persistences. For example, a shock $\varepsilon_{\psi t-1}$ will affect the left-hand-side, $(1-\mu_1 L)q_t = q_t - \mu_1 q_{t-1}$, in a proportional way as a shock $\varepsilon_{\xi t-1}$. Therefore, the autocorrelation of two IRFs are going to be equal. However, due to the existence of the second them, the trade shock has another layer of affecting the left-hand-side. In specific, a shock $\varepsilon_{\xi t-1}$ has a lag operator $(1-\rho\mu_1 L)$ and its effect on the left-hand-side is not proportional to the others anymore.

This can be seen by plotting IRFs using the derived equation. Using the parameter values of the benchmark case, and also checking robustness with other values, we plot the IRFs of two shocks in Figure C1. The result is similar to the one from our quantitative exercise, presented in Figure 4. The calculated autocorrelations of each IRF is 0.96 (trade shock) and 0.86 (financial shock).

Figure C1: IRFs from Analytical Solution

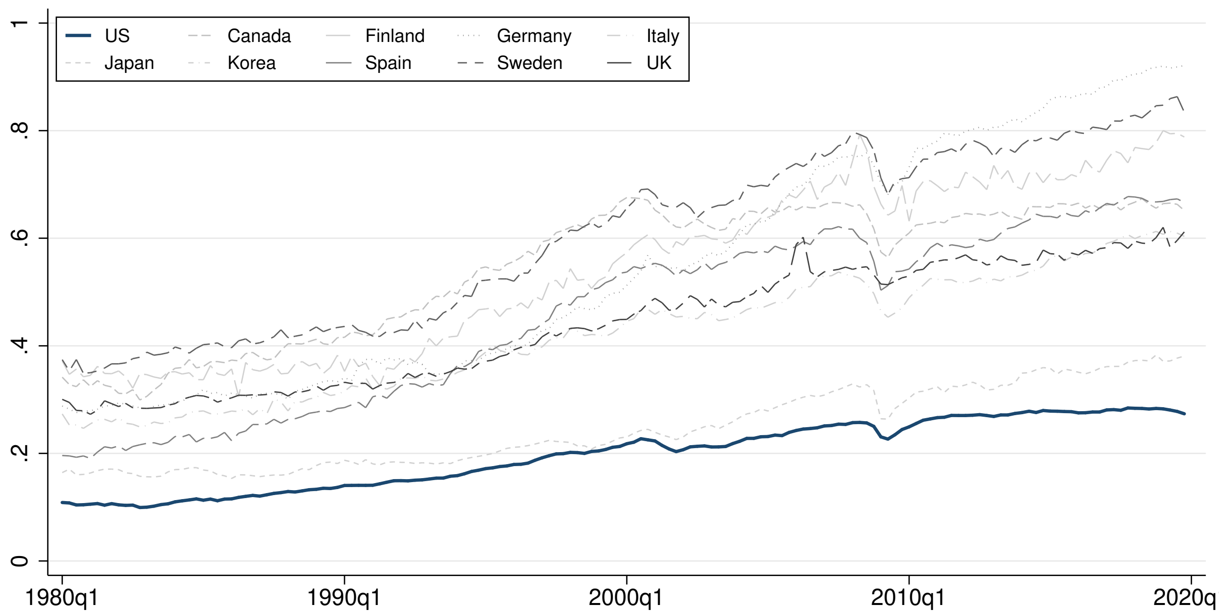


Now consider a case when we shut off the effects through the budget constraint. If we force (**) = 0, the IRF of trade shock becomes much less persistence, and the autocorrelation reduces to 0.86 (red dotted line in Figure C1).

On the other hand, shutting of (*) term has a negligible effect. That is, its IRF coincides with the original case (red solid line in Figure C1). This result is consistent with our quantitative exercise that effect of trade shock through generating the UIP deviation is small.

H Additional Figures and Tables

Figure F.1: Gross Trade to GDP Ratio



Notes: The figure shows the share of gross trade as a share of total output, measured by the ratio of volume estimates of exports plus imports to GDP for each country.

Figure F.2: Data Source Comparison

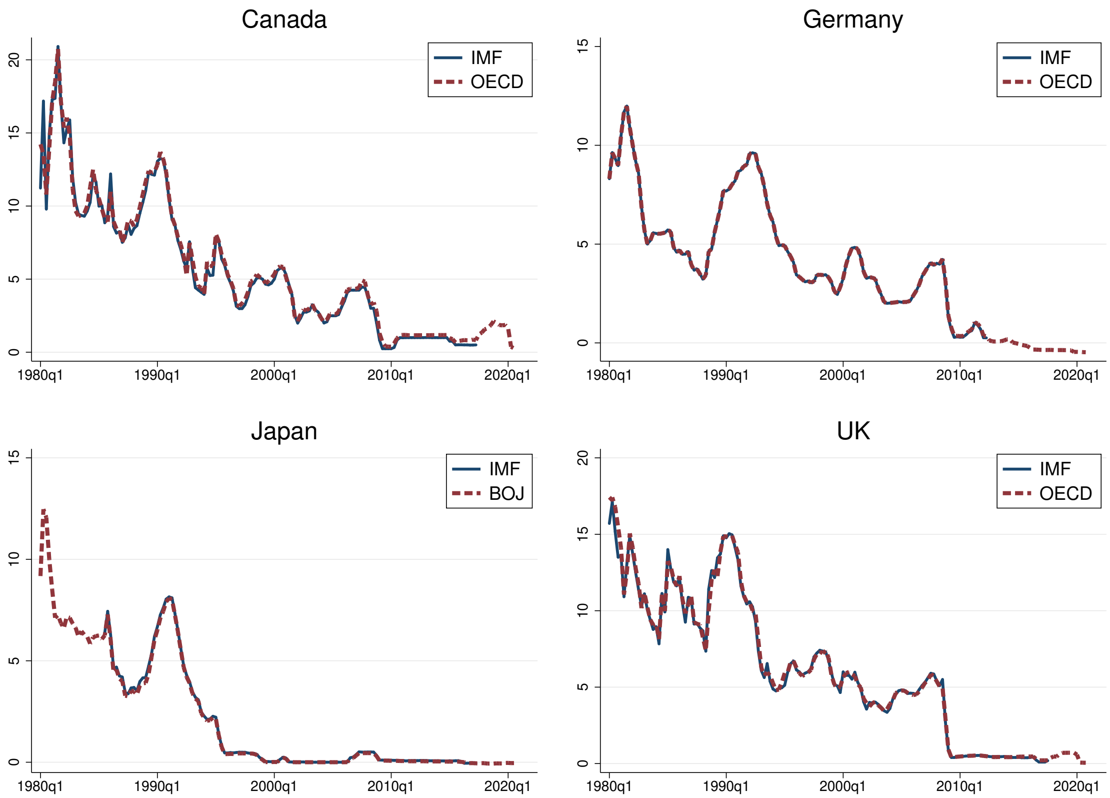
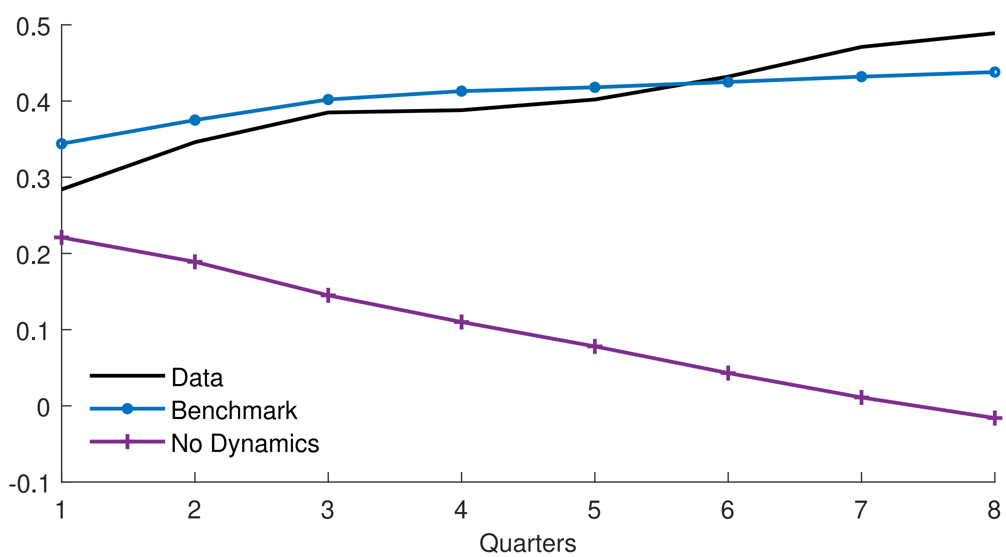
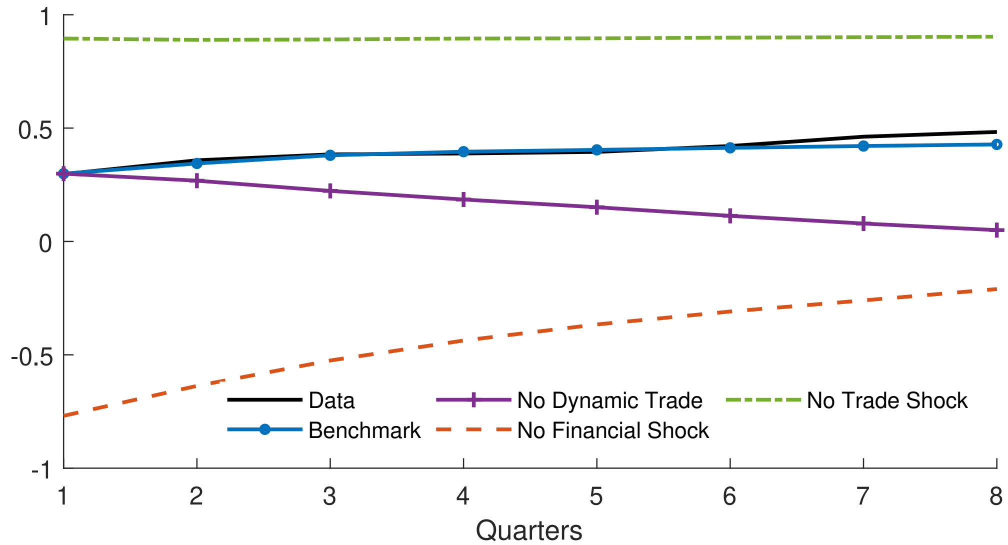


Figure F.3: Dynamic Correlation between RER and Trade-Expenditure Ratio



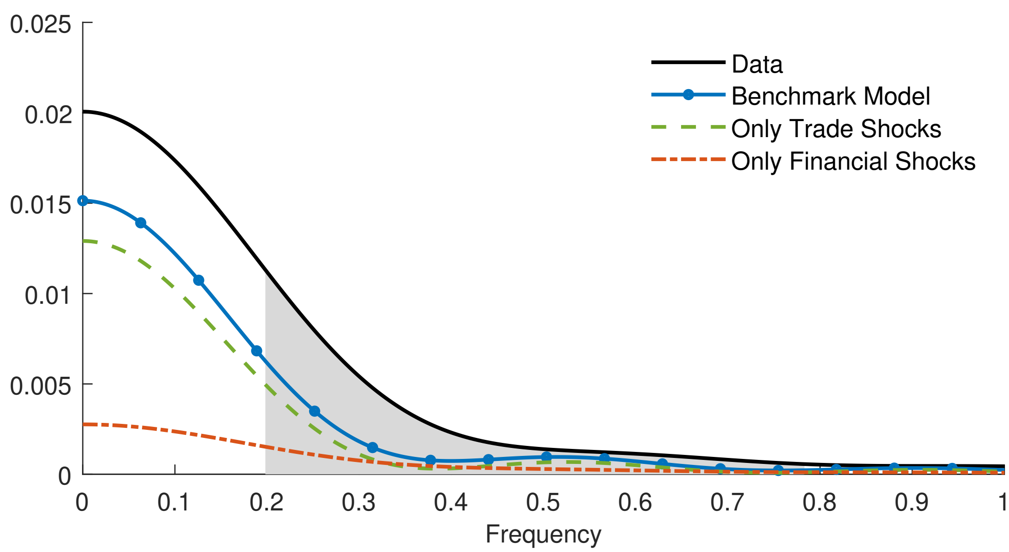
Notes: The figure presents dynamic correlations as $\rho(\Delta_h q_t, \Delta_h TE_t)$, where q_t and TE_t are log of the RER and the trade-expenditure ratio, respectively. and Δ_h denotes $h-$period difference.

Figure F.4: Dynamic Correlation between RER and NT



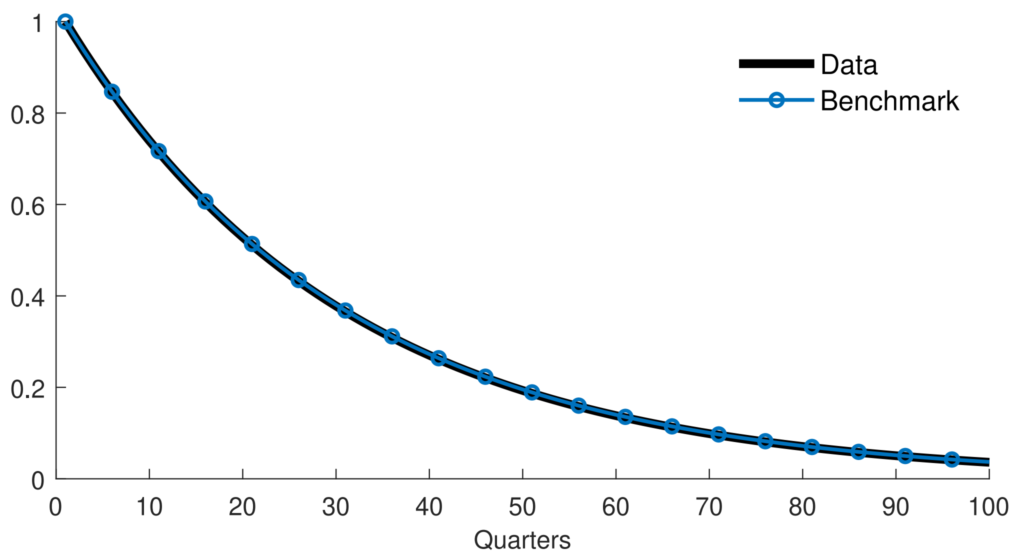
Notes: We calculate the dynamic correlations as $\rho(\Delta_h q_t, \Delta_h nt_t)$, where q_t and nt_t are log of the RER and the export-import ratio, respectively. and Δ_h denotes h−period difference. It present the results for the benchmark model and alternative models: no financial shock, no trade shock, and no trade dynamics.

Figure F.5: Counterfactual Spectrum



Notes: Spectral analysis of counterfactual models without re-calibrating, as our goal is to use the identified parameters from the benchmark model to perform exercises informative about the role of each shock at different frequencies. The graph is enlarged for the range [0,1] to show better the low and business cycle frequencies.

Figure F.6: IRFs from Estimated AR(1) Process of the RER



Notes: the figure presents the impulse response functions of the RER from the estimated AR(1) processes using the data and model simulated data.

Table F.1: Frequency Decomposition of RER – Other Countries

Country	Low	Business cycle	High
Australia	0.60	0.32	0.08
Austria	0.63	0.29	0.07
Belgium	0.54	0.38	0.08
Canada	0.61	0.31	0.08
Chinese Taipei	0.65	0.26	0.09
Denmark	0.64	0.29	0.07
Finland	0.60	0.33	0.07
France	0.47	0.42	0.11
Germany	0.63	0.30	0.07
Greece	0.63	0.28	0.09
Hong Kong SAR	0.61	0.30	0.09
Ireland	0.39	0.43	0.18
Italy	0.64	0.27	0.09
Japan	0.62	0.30	0.08
Korea	0.67	0.25	0.08
Netherlands	0.62	0.31	0.07
New Zealand	0.52	0.36	0.12
Norway	0.58	0.32	0.10
Portugal	0.58	0.34	0.08
Singapore	0.62	0.30	0.07
Spain	0.59	0.31	0.10
Sweden	0.60	0.31	0.09
Switzerland	0.68	0.25	0.07
United Kingdom	0.58	0.33	0.09
United States	0.61	0.32	0.08
Euro area	0.41	0.45	0.14
Average (excl. Euro Area)	0.60	0.32	0.09

Notes: The table presents the share of the RER variance explained by different frequencies for a panel of developed economies and Chinese Taipei. We use the effective exchange rate, real, narrow indices, from BIS.

Table F.2: Calibrated Parameters – Alternative Models

Parameter		Benchmark	No Trade Shock	No Financial Shock	No Dynamics
B. Producer Trade Parameters					
Fixed cost of new exporters	f^0	0.14	0.14	0.14	$0^{\ddagger}$
Fixed cost of incumbent exporters	f^1	0.04	0.04	0.04	$0^{\ddagger}$
Volatility of idiosyncratic productivity	σ_μ	0.15	0.15	0.15	$0^{\ddagger}$
C. Shocks, Adjustment Costs and Pricing to Market					
Common productivity, volatility	σ_{a_c}	0.004	0.005	0.004	0.005
Differential productivity, volatility	σ_{a_d}	0.005	0.006	0.006	0.006
Financial shock, volatility	σ_ψ	0.002	0.002	$0^{\ddagger}$	0.004
Financial shock, persistence	ρ_ψ	0.957	0.964	$0^{\ddagger}$	0.810
Trade shock, volatility	σ_ξ	0.052	$0^{\ddagger}$	0.037	0.095
Trade shock, persistence	ρ_ξ	0.971	$0^{\ddagger}$	0.999	0.965
Trade shock, within-country share	τ	0.171	$0^{\ddagger}$	0.282	0.089
Adjustment cost of portfolios	χ	0.0137	0.008	0.002	0.002
Adjustment cost of capital	κ	2.425	6.976	2.338	8.25
Pricing to market parameter	ζ	0.966	0.969	1.060	1.62

Notes: The table presents the values of calibrated parameters of the benchmark and alternative models. When we consider an alternative models, some of the parameters are set to a different value while the other parameters are all recalibrated. Panel A is same as the baseline case presented in Table 1 for all models.

Table F.3: Share in Counterfactual Spectrum

	Data	Benchmark	Trade Shock Only	Financial Shock Only	Prod Shock Only
Low frequency	0.61	0.70	0.75	0.56	0.75
BC frequency	0.31	0.23	0.19	0.32	0.19
High frequency	0.08	0.07	0.06	0.12	0.06

Table F.4: Fama Estimates in Data

Moments	Nominal	Real
β_{fama}	-1.15	-1.34
	(0.59)	(0.52)
R^2_{fama}	0.02	0.04

Notes: 'Nominal' denotes the results of using nominal data for the Fama regression, $\Delta e_{t+1} = \alpha + \beta_{Fama}(i_t^n - i_t^{n*}) + u_t$, where e is nominal exchange rate, and i^n is the nominal interest rate. 'Real' denotes the result of using real data for the regression (8).

Table F.5: Conditional Variance Decomposition of the RER (%) – Model Without Trade Shocks

	quarters = 1	8	32	80
Financial shock	93.6	88.9	74.5	74.2
Productivity shock	6.4	11.1	25.5	25.8

Notes: This model corresponds to calibration under 'No Trade Shock' in Table F.2.