COMPUTING MATHEMATICS 2

Chapter 6 : Binomial and PoissonDistribution

Mean and Variance of the Binomial Distribution

Mean

- $\mu = \boxed{np}$

Variance

- $\sigma^2 = np(1\text{-}p)$

Example 6.2-1

0.05 = P

5 % of workers at construction sites are known to suffer from hearing impaired problem due to the unhealthy noise level. If we randomly select 28 workers from construction sites, find

n k

(a) the probability that exactly $\boxed{4}$ of them suffer from hearing impaired problem

Let X be the number of workers with hearing impaired problems out of 28 workers.

$X \sim B(n, p)$

$$X \sim B(\overset{n}{28}, \overset{p}{0.05})$$

$${}_nC_k\, p^k q^{n-k} = {}_nC_k\, p^k (1-p)^{n-k}$$

$$P(X = \overset{k}{\boxed{4}}) = {}_{28}C_4\, 0.05^4 (1-0.05)^{28-4}$$

$$= 0.0374$$

Example 6.2-1

(b) the mean and standard deviation of the number of workers suffering from hearing impaired problem.

$$
\begin{aligned}
E(X) &= np \\
&= 28 \times 0.05 \\
&= 1.4
\end{aligned}
$$

$$
\begin{aligned}
Var(X), \sigma^2 &= np(1-p) \\
&= 28 \times 0.05 \times 0.95 \\
&= 1.33
\end{aligned}
$$

$$
\begin{aligned}
\text{standard deviation, } \sigma &= \sqrt{1.33} \\
&= 1.1533
\end{aligned}
$$

Example 6.2-2

The random variable X which follows a Binomial distribution is such that the mean is 2 and variance is $\frac{24}{13}$. Find the values of n and p.

$np = 2$

$npq = np(1-p) = \frac{24}{13}$

$$X \sim B(n, p)$$

$$E(X) = np = \underline{2} \text{ ----- (1)}$$

$$Var(X), \sigma^2 = \boxed{np}(1-p) = \frac{24}{13} \text{ ---(2)}$$

Sub (1) into (2)

$$2(1-p) = \frac{24}{13}$$

$$(1-p) = \frac{12}{13}$$

$$\boxed{p} = 1 - \frac{12}{13}$$

$$= \frac{1}{13}$$

Sub back to (1)

$$n\left(\frac{1}{13}\right) = 2$$

$$n = 26$$

Poisson Distribution

Poisson Distribution

A discrete probability distribution that satisfies the following conditions:

1. The mean rate of events, μ, occurring in an unit interval / region (eg time/space) is the same for every other unit interval / region.
2. The number of occurrences in one interval is independent of the number of occurrences in other intervals.
3. No two events can occur at the same time.

Poisson Distribution

- Examples of such events:
  (1) Number of defects per batch in a production process.
  (2) Number of flaws in a given length of material.
  (3) Number of telephone calls made to a helpdesk in a given hour.
  (4) Number of accidents in a factory in one week.

Poisson Probability Formula

The probability of exactly k occurrences in an interval is

$X \sim B(n, p)$

8nıfp $e^{\square}$ $\boxed{\ln}$

$$X \sim Po(\mu)$$

$$P(X = k) = e^{-\mu}\frac{\mu^{k}}{k!}$$

**

$k = 0,1,2,\ldots$	μ = mean rate occurrences per unit interval	k = no. of occurrences per unit interval

Example 6.3-1

time

The mean number of accidents per month at a certain intersection is 3. What is the probability that in any given month,

$\mu = 3$

(a) 4 accidents will occur at this intersection?

(b) more than 1 accidents will occur at this intersection?

Let X be the number of accident **per month**

$\boxed{X \sim Po(3)}$

$e^{-\mu}\dfrac{\mu^k}{k!}$

a) $P(X = \overset{k}{4}) = e^{-3}\dfrac{3^4}{4!} = 0.1680$

complement method

b) $P(X > 1) = 1 - P(X = 0) - P(X = 1)$ $1 - P(X \le 1)$

$$= 1 - e^{-3}\frac{3^0}{0!} - e^{-3}\frac{3^1}{1!}$$

$$= 0.8008$$

Example 6.3-2

2000 brown trout are introduced into a small lake. The lake has a volume of 20000 cubic meters. Find the probability that

(a) 3 brown trout are found on any given cubic meter of the lake.

Let X be the number of trouts **per cubic meter** of lake

mean number of trouts per cubic meter

$$\boxed{\mu = \frac{2000}{20000} = 0.1}$$

$$X \sim Po(\underset{=\mu}{0.1})$$

$$P(X = \underset{k}{3}) = e^{-0.1}\frac{0.1^3}{3!}$$

$$= 0.000151$$

Example 6.3-2

(b) less than 2 brown trout are found on any 10 cubic meters of the lake.

Let Y be the number of trouts **per 10 cubic meter** of lake

mean number of trouts per 10 cubic meter

$$\mu = \frac{2000}{20000} \times 10$$
$$= 1$$

need to recalculate !!

$$Y \sim Po(1)$$

$$P(Y < 2) = P(Y = 0) + P(Y = 1)$$
$$= e^{-1}\frac{1^0}{0!} + e^{-1}\frac{1^1}{1!}$$
$$= 0.7358$$

Mean and Variance of the Poisson Distribution

Mean

- $E(x) = \mu$

Variance

- $Var(x) = \mu$

Example 6.4-1

A school "Lost and Found" department receives an average of 3.7 reports per week of lost student ID cards.

$\mu = 3.7$

(a) Find the probability that at most 2 such reports will be received during a given week by this department.

Let X be the number of reports **per week**

$X \sim Po(3.7)$

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= e^{-3.7}\frac{3.7^0}{0!} + e^{-3.7}\frac{3.7^1}{1!} + e^{-3.7}\frac{3.7^2}{2!}$$

$$= 0.2854$$

Example 6.4-1

(b) Find the probability that there will be 1 to 3 (inclusive) such reports received during a given week by this department.

$$X \sim Po(3.7)$$

$$P(1 \le X \le 3) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$= e^{-3.7}\frac{3.7^1}{1!} + e^{-3.7}\frac{3.7^2}{2!} + e^{-3.7}\frac{3.7^3}{3!}$$

$$= 0.4694$$

Example 6.4-1

(c) Find the variance and standard deviation of the probability distribution.

Variance, $\sigma^2 = \mu = 3.7$

Standard Deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{3.7} = 1.92$