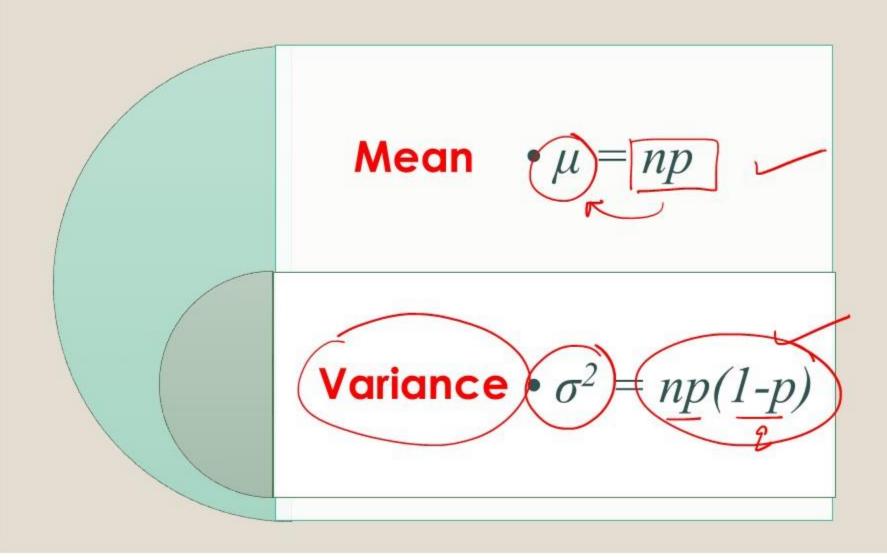


Mean and Variance of the Binomial Distribution



Example 6.2-1

- 5 % of workers at construction sites are known to suffer from hearing impaired problem due to the unhealthy noise level. If we randomly select 28 workers from construction sites, find
- (a) the probability that exactly 4 of them suffer from hearing impaired problem

Let X be the number of workers with hearing impaired problems out of 28 workers. $\times \times B(n, p)$

$$P(X = 4) = {}_{28}C_4 0.05^4 (1 - 0.05)^{28-4}$$

$$= 0.0374$$

Example 6.2-1

(b) the mean and standard deviation of the number of workers suffering from hearing impaired problem.

$$E(X) = np$$

= 28×0.05
= 1.4
 $Var(X), \sigma^2 = np(1-p)$
= 28×0.05×0.95
= 1.33
standard deviation, $\sigma = \sqrt{1.33}$
= 1.1533

Example 6.2-2

The random variable X which follows a Binomial distribution is such that the mean is 2 and variance is $\frac{24}{13}$. Find the values of n and p.

$$X \sim B(n, p)$$

$$E(X) = np = 2$$
 ---- (1)
 $Var(X), \sigma^2 = np(1-p) = \frac{24}{13}$ --- (2)

$$2(1-p) = \frac{24}{13}$$

$$(1-p) = \frac{12}{13}$$

$$p = 1 - \frac{12}{13}$$

$$= \frac{1}{13}$$

Sub back to (1)

$$n\left(\frac{1}{13}\right) = 2$$

$$n = 26$$

Poisson Distribution

Poisson Distribution

A discrete probability distribution that satisfies the following conditions:

- 1. The mean rate of events, μ , occurring in an unit interval / region (eg time/space) is the same for every other unit interval / region.
 - The number of occurrences in one interval is independent of the number of occurrences in other intervals.
 - No two events can occur at the same time.

Poisson Distribution

- Examples of such events:
 - Number of <u>defects per batch</u> in a production process.
 - (2) Number of flaws in a given length of material.
 - (3) Number of telephone calls made to a helpdesk in a given hour.
 - (4) Number of accidents in a factory in one week.

Poisson Probability Formula

The probability of exactly k occurrences in an interval is

$$X \sim Po(\mu)$$

$$P(X = \underline{k}) = e^{-\underline{u}} \underbrace{\underline{u}}_{\underline{k}!}$$

*X

$$k = 0,1,2,...$$

μ = mean rateoccurrences per unit interval k = no. of occurrences per unit interval

Example 6.3-1

The mean number of accidents per month at a certain intersection is (3) What is the probability that in any given month,

- (a) 4 accidents will occur at this intersection?
- (b) more than 1 accidents will occur at this intersection?

Let *X* be the number of accident **per month**

$$(X \sim Po(3))$$
 7 $e^{-\mu} \frac{\mu^{k}}{k!}$

a)
$$P(X = 4) = e^{-3} \frac{3^4}{4!} = 0.1680$$

= 0.8008

Example 6.3-2

brown trout are introduced into a small lake. The lake has a volume of 20000 cubic meters. Find the probability that

(a) 3 brown trout are found on any given cubic meter of the lake.

Let X be the number of trouts per cubic meter of lake

mean number of trouts per cubic meter

$$\mu = \frac{2000}{20000}$$
= 0.1

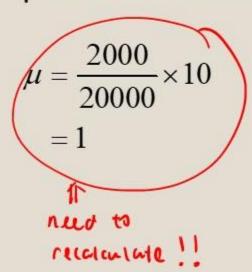
$$X \sim Po(0.1)$$
 $P(X = 3) = e^{-0.1} \frac{0.1^3}{3!}$
 $= 0.0000151$

Example 6.3-2

(b) less than 2 brown trout are found on any 10 cubic meters of the lake.

Let Y be the number of trouts per 10 cubic meter of lake

mean number of trouts per 10 cubic meter



$$Y \sim Po(1)$$

$$P(Y < 2) = P(Y = 0) + P(Y = 1)$$

$$= e^{-1} \frac{1^{0}}{0!} + e^{-1} \frac{1^{1}}{1!}$$

$$= 0.7358$$

Mean and Variance of the Poisson Distribution

Mean • $E(x) \neq \hat{\mu}$ Variance • $Var(x) \neq \mu$

Example 6.4-1

A school "Lost and Found" department receives an average of 3.7 reports per week of lost student ID cards.

(a) Find the probability that at most 2 such reports will be received during a given week by this department.

Let *X* be the number of reports **per week**

$$X \sim Po(3.7)$$

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= e^{-3.7} \frac{3.7^{0}}{0!} + e^{-3.7} \frac{3.7^{1}}{1!} + e^{-3.7} \frac{3.7^{2}}{2!}$$

$$= 0.2854$$

Example 6.4-1

(b) Find the probability that there will be 1 to 3 (inclusive) such reports received during a given week by this department.

$$X \sim Po(3.7)$$

$$P(1 \le X \le 3) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$= e^{-3.7} \frac{3.7^{1}}{1!} + e^{-3.7} \frac{3.7^{2}}{2!} + e^{-3.7} \frac{3.7^{3}}{3!}$$

$$= 0.4694$$

Example 6.4-1

(c) Find the variance and standard deviation of the probability distribution.

Variance,
$$\sigma^2 = \mu = 3.7$$

Standard Deviation,
$$\sigma = \sqrt{\sigma^2} = \sqrt{3.1} = 1.92$$