Computing Mathematics 2

Chapter 7:
Normal Distribution

Objectives:

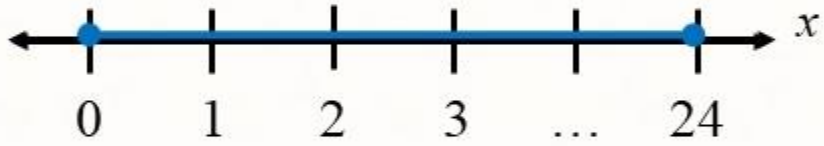
At the end of this lesson, the student should be able to:

1. describe the characteristics of a normal distribution including its shape and the relationship among its mean, median and mode
2. define normal random variable and standard normal random variable
3. compute normal probabilities using standard normal tables
4. use the normal probability distribution to approximate the binomial probabilities (including correction for continuity)

7.1 Introduction

Continuous Random Variable

- Uncountable number of possible outcomes
- E.g. x = time taken to complete a job in a day.



Continuous Random Variable → can be modelled using the normal distribution.

- Examples :
- (a) Students' examination scores
- (b) Height and weight of people

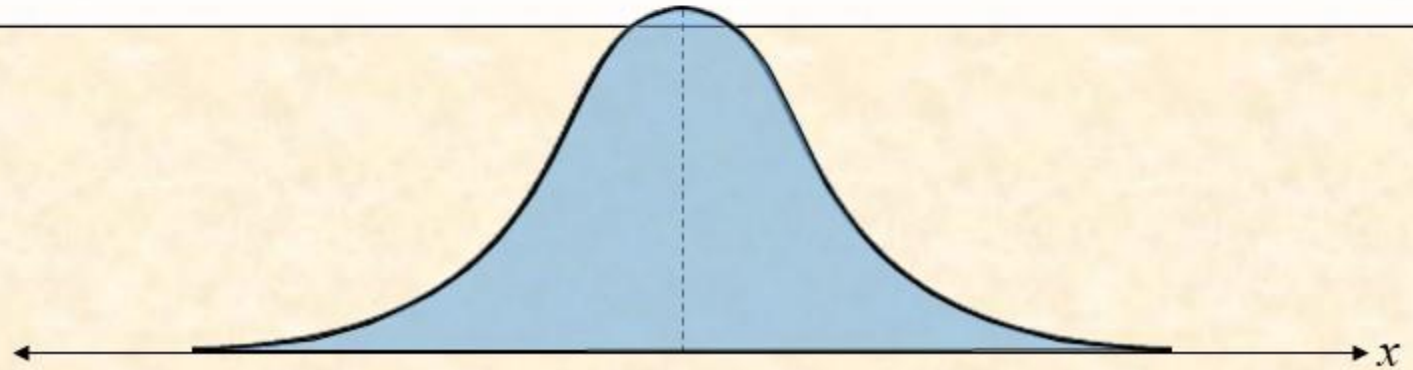
What is a Normal Distribution?

$\mu = 5,\ \sigma = 4$

Normal Distribution

$X \sim N(5, 16)$

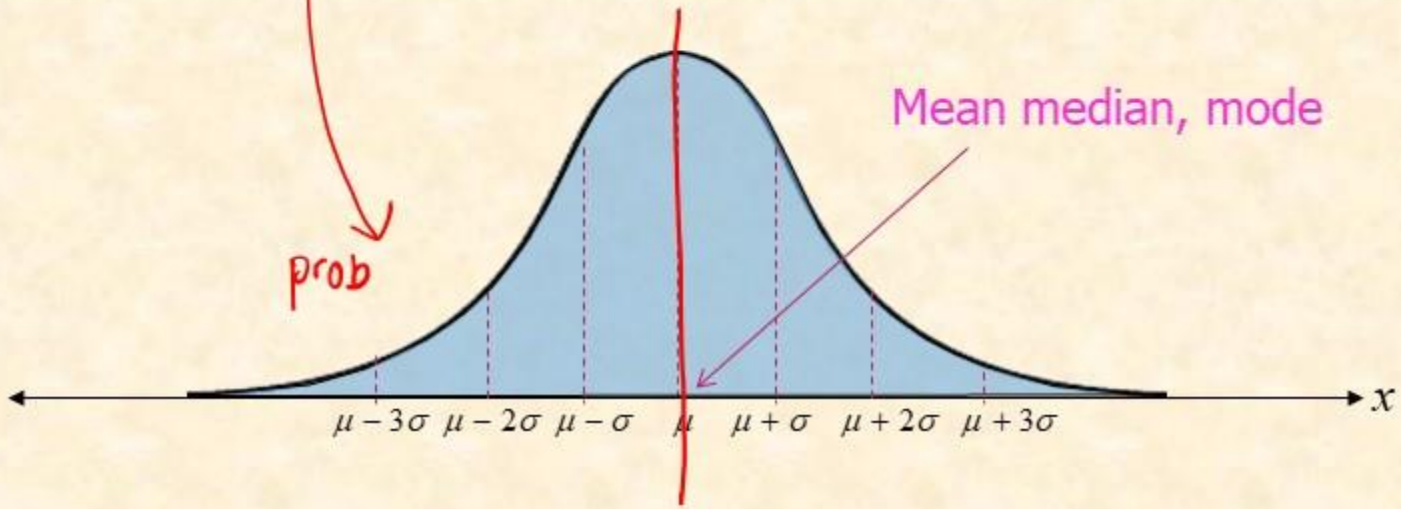
- can be described using two features : *mean* μ and *variance* σ^2 . Notation: $X \sim N\left(\mu, \sigma^2\right)$
- Most important continuous probability distribution in statistics.
- Graph of a normal distribution is called the **normal curve**.



Properties of a Normal Distribution

Normal Distribution

- 1) Mean, median, and mode are equal.
- 2) Bell-shaped and symmetric about the mean.
- 3) Total area under the curve = 1
- 4) About 95% of the distribution lies within 2 standard deviations of the mean. This is known as the "2σ rule".

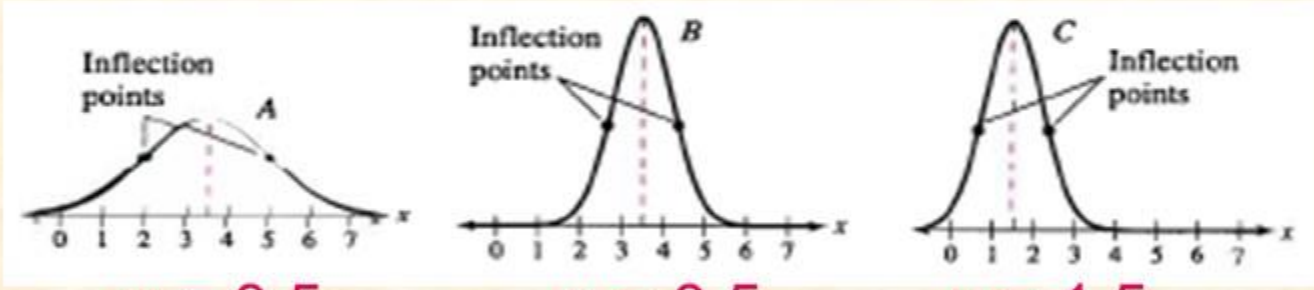


Examples of normal distributions

- There are *infinitely* many normal distributions, *each* with its own *mean* and *standard deviation*.

  Mean μ : location of the line of symmetry.

  Standard deviation σ : spread of the data.

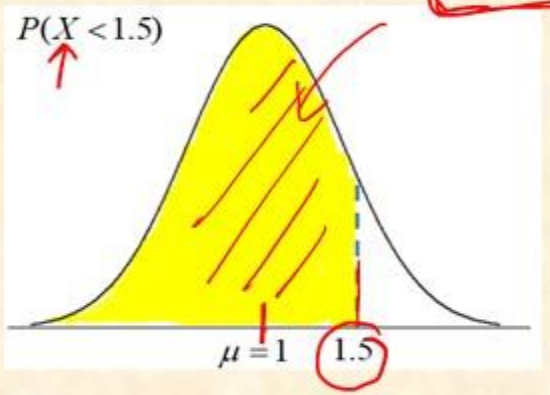


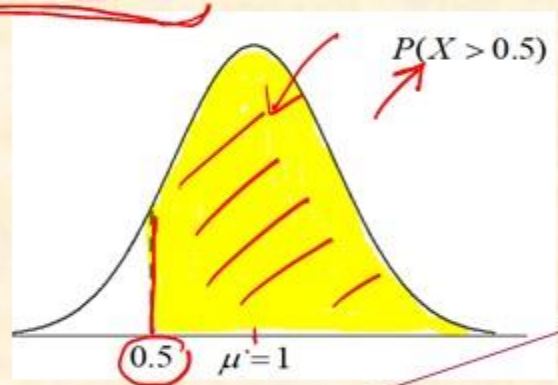
μ = 3.5	μ = 3.5	μ = 1.5
σ = 1.5	σ = 0.7	σ = 0.7

7.2 Probability for normal distribution

For normal distribution, the probability is interpreted as area under the curve.

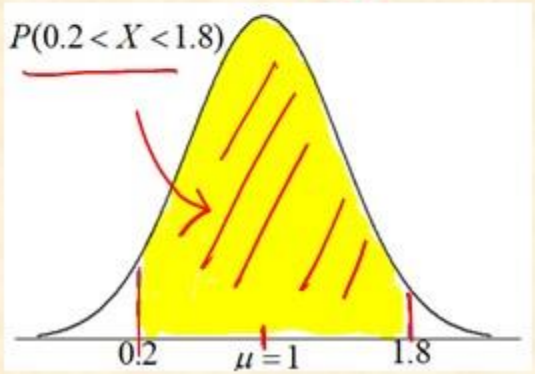
For example, $X \sim N(1, 2)$:

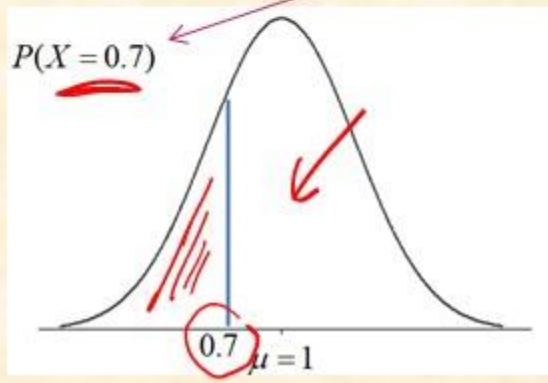




Note :

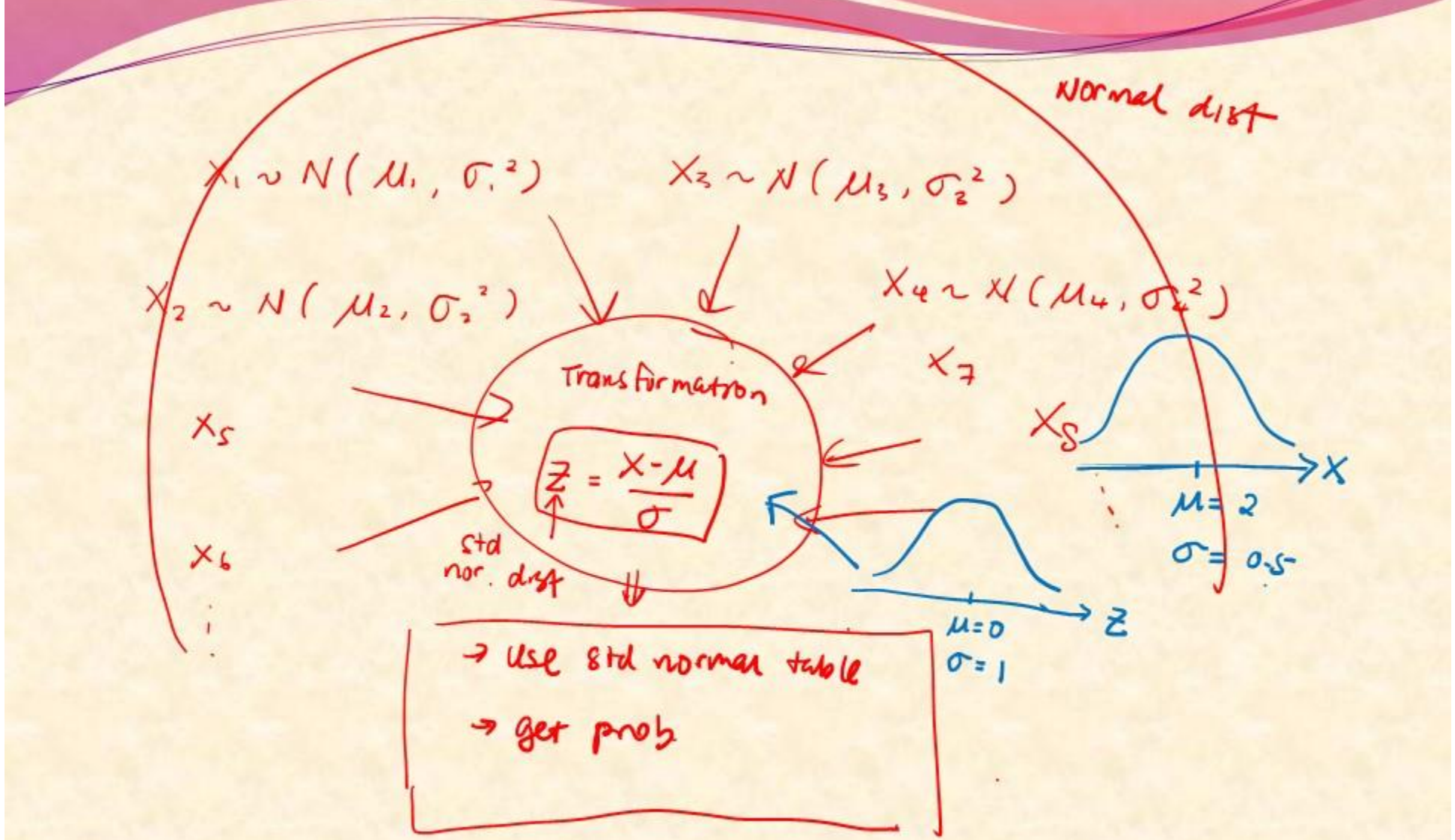
$$P(X = k) = 0$$





Hence,
for normal distribution,

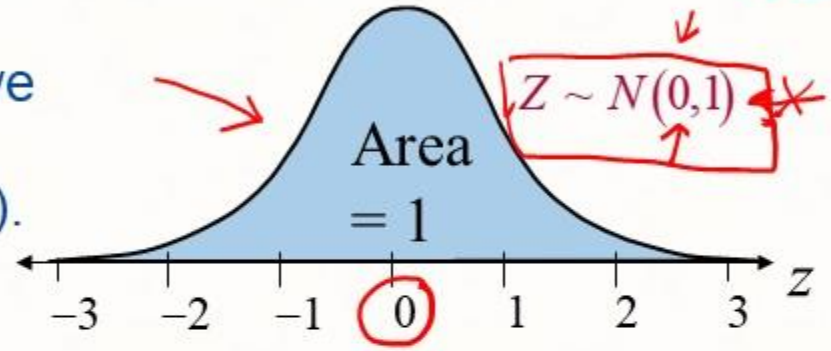
$$P(X \leq k) = P(X < k)$$


Normal dist
$X_1 \sim N(\mu_1, \sigma_1^2)$
$X_3 \sim N(\mu_3, \sigma_3^2)$
$X_2 \sim N(\mu_2, \sigma_2^2)$
$X_4 \sim N(\mu_4, \sigma_4^2)$
X_7
X_5
X_6
Transformation
$Z = \frac{X-\mu}{\sigma}$
Std nor. dist
X_5
$\mu = 2$
$\sigma = 0.5$
$\mu = 0$
$\sigma = 1$
Z
→ use std normal table
→ get prob

7.3 Standard Normal Distribution

Standard Normal Distribution

When a normal random variable has mean = 0 and variance = 1, we call it a *standard normal random variable* (denoted by Z). i.e. $Z \sim N(0,1)$.



- The horizontal scale of the graph of the standard normal distribution corresponds to z-scores.
- A z-score is a measure of position that indicates the number of standard deviations a value lies from the mean.
- To convert from $X \sim N(\mu, \sigma^2)$ to $Z \sim N(0,1)$, we apply the formula:

$$Z = \frac{X - \mu}{\sigma}$$

- This procedure is known as standardization.

Example 7.3-1

Given that X ~ N (2,5), rewrite the following probabilities in the form $P(Z \le k)$.

μ σ^2

normal dist

(a) $P(X \le 3)$

std normal dist

$= P\left(Z \le \dfrac{3-2}{\sqrt{5}}\right)$

$= P(Z < 0.4472)$

$= P(Z < 0.45)$

round off to 2 decimal places

$$Z = \frac{X - \mu}{\sigma}$$

In normal distribution,

$P(Z \le k) = P(Z < k)$

Example 7.3-1

μ σ^2

Given that X ~ N (2,5), rewrite the following probabilities in the form $P(Z \le k)$.

$$Z = \frac{X - \mu}{\sigma}$$

(b) $P(X \ge 1.5)$

$$= P\left(Z \ge \frac{1.5 - 2}{\sqrt{5}}\right)$$

$$= P(Z \ge -0.2236)$$

$$= 1 - P(Z < -0.22)$$

round off to 2 decimal places

Example 7.3-1

μ σ^2

Given that X ~ N (2,5), rewrite the following probabilities in the form $P(Z \le k)$.

$$Z = \frac{X - \mu}{\sigma}$$

(c) $P(1.5 < X < 3)$

$= P\left(\frac{1.5 - 2}{\sqrt{5}} < Z < \frac{3 - 2}{\sqrt{5}}\right)$

$= P(-0.22 < z < 0.45)$

$= P(z < 0.45) - P(z < -0.22)$

Note : $P(a < X < b) = P(X < b) - P(X < a)$

7.4 Standard Normal Table

To calculate probabilities involving normal distribution, we will obtain the probability value via the **standard normal table** (on pages 109 and 110).

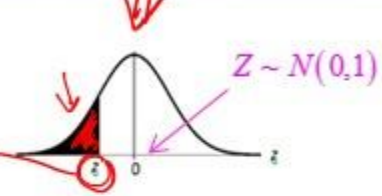
Step 1: Apply standardization from $X \sim N(\mu, \sigma^2)$ to $Z \sim N(0,1)$.

Step 2: Ensure the probability is expressed in the form $P(Z \leq k)$.

Step 3: Obtain the required probabilities' value from the standard normal table.

Standard Normal Table (Pg 109 and Pg 110)

Standard Normal Table

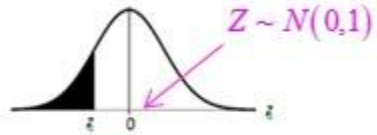


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

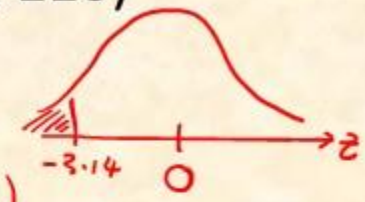
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Standard Normal Table (Pg 109 and Pg 110)

Standard Normal Table

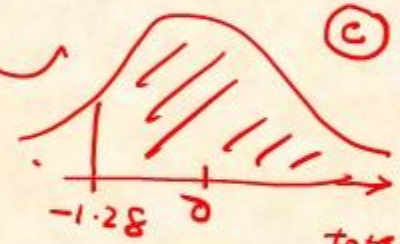


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



$P(Z < -3.14)$

$= 0.0008$



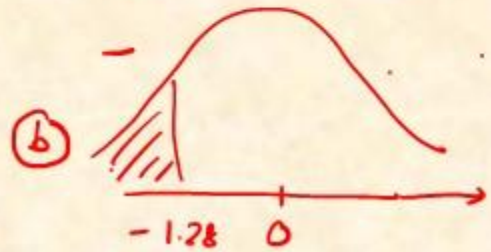
$P(Z > -1.28)$

$= 1 - P(Z < -1.28)$

$= 1 - 0.1003$

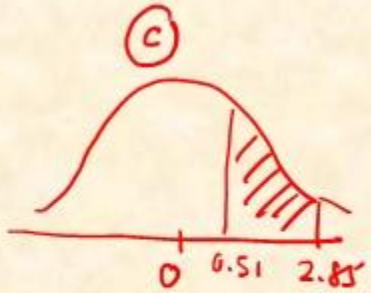
$=$ —

total = 1



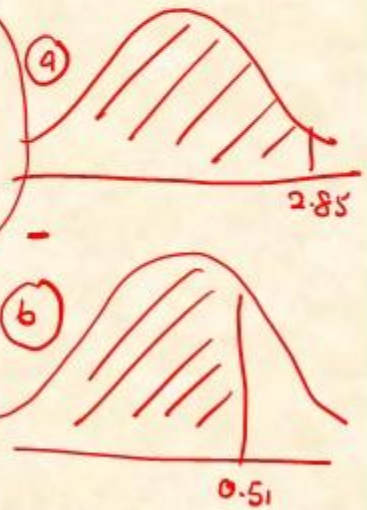
Standard Normal Table (Pg 109 and Pg 110)

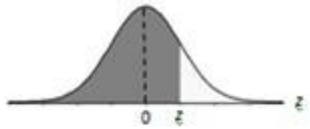
$P(Z < 0.82) = 0.7938$



$P(0.51 < z < 2.85)$

$= P(Z < 2.85) - P(Z < 0.51)$





z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Properties of Standard Normal Distribution

1. The cumulative area is close to 0 for z-scores close to $z = -3.49$.
2. The cumulative area increases as the z-scores increase.
3. The cumulative area for $z = 0$ is 0.5000.
4. The cumulative area is close to 1 for z-scores close to $z = 3.49$.

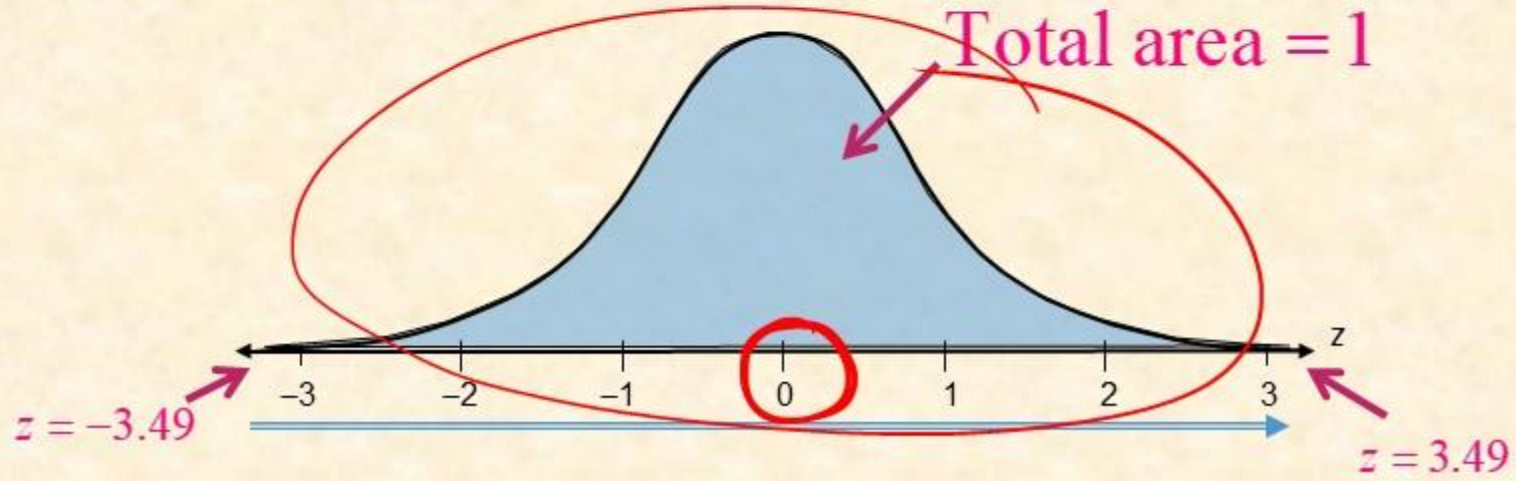


Illustration : how to read the Standard Normal Table

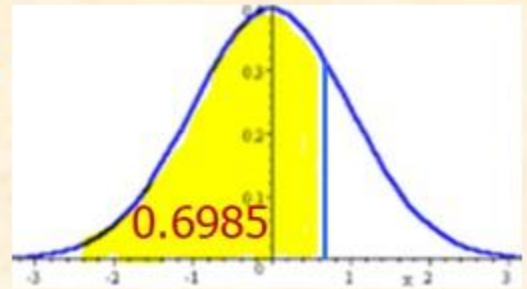
(a) Suppose we want to find $P(Z \leq 0.52)$:

2nd decimal place

z	.00	.01	.02	.03
0.0	.5000	.5040	.5080	.5120
0.1	.5398	.5438	.5478	.5517
0.2	.5793	.5832	.5871	.5910
0.3	.6179	.6217	.6255	.6293
0.4	.6554	.6591	.6628	.6664
0.5	.6915	.6950	.6985	.7019

1st decimal place

Probability value



$$\therefore P(Z \leq 0.52) = 0.6985$$

Left area

Illustration : how to read the Standard Normal Table

(b) Suppose we want to find the value of k such that

$P(Z \le k) = 0.0020$:

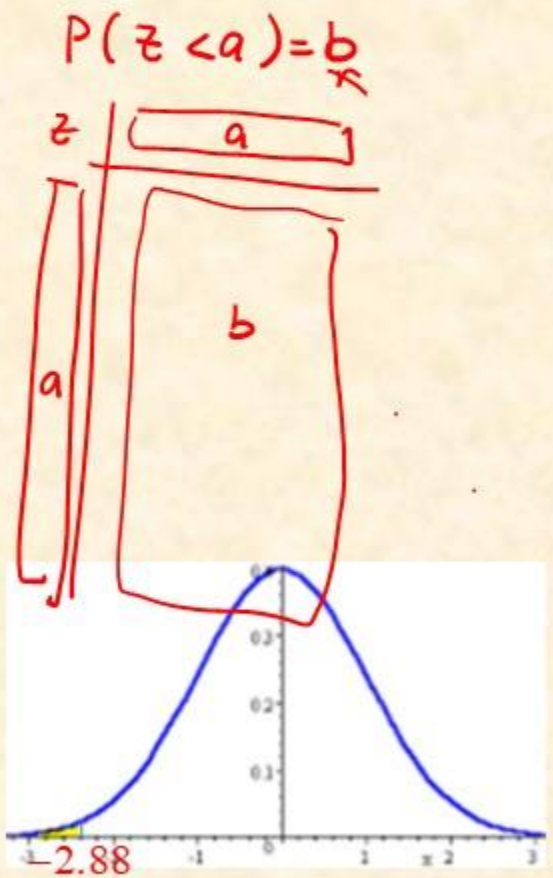
2nd decimal place

z	.09	.08	.07	.06
−3.4	.0002	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004
−3.2	.0005	.0005	.0005	.0006
−3.1	.0007	.0007	.0008	.0008
−3.0	.0010	.0010	.0011	.0011
−2.9	.0014	.0014	.0015	.0015
−2.8	.0019	.0020	.0021	.0021

1st decimal place

Probability value

$\therefore k = -2.88$

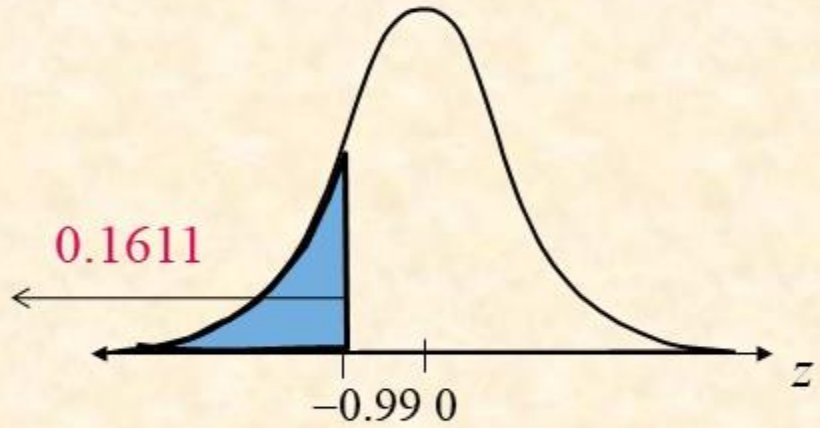


Example 7.4-1

Let $Z \sim N(0,1)$. Use the standard normal table on pages 109 and 110 to evaluate the following probabilities:

(a) $P(Z < -0.99)$

left area



z	0.09	0.08	0.07
-1.1	0.1170	0.1190	0.1210
-1.0	0.1379	0.1401	0.1423
-0.9	0.1611	0.1635	0.1660

(a) $P(Z < -0.99)$

$= 0.1611$

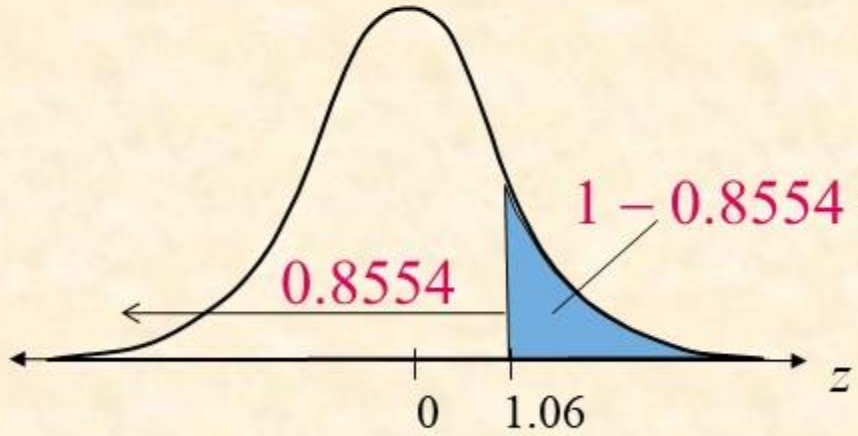
Example 7.4-1

Let $Z \sim N(0,1)$. Use the standard normal table on pages 109 and 110 to evaluate the following probabilities:

(b) $P(Z > 1.06)$

right area

z	0.04	0.05	0.06
0.9	0.8264	0.8289	0.8315
1.0	0.8508	0.8531	0.8554
1.1	0.8729	0.8749	0.8770



$$\begin{aligned}(b)\ P(Z > 1.06) &= 1 - P(Z < 1.06) \\ &= 1 - 0.8554 \\ &= 0.1446\end{aligned}$$

Example 7.4-1

Let $Z \sim N(0,1)$. Use the standard normal table on pages 109 and 110 to evaluate the following probabilities:

(c) $P(-1.5 < Z < -1.25)$.

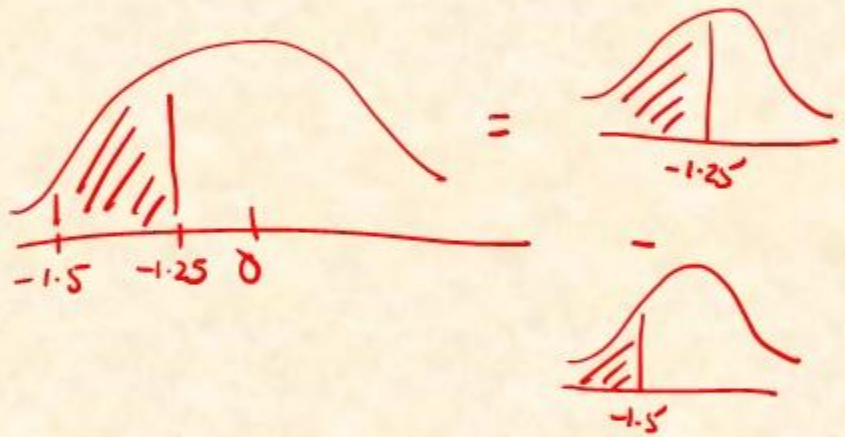
$$P(a < Z < b) = P(Z < b) - P(Z < a)$$

$$P(-1.5 < Z < -1.25)$$

$$= P(Z < -1.25) - P(Z < -1.5)$$

$$= 0.1056 - 0.0668$$

$$= 0.0388$$



Example 7.4-2

Given the normally distributed variable X with mean 20 and standard deviation 4 ,find

$X \sim N(20, 4^2)$

(a) $P(X > 28)$

$= P\left(Z > \frac{28-20}{4}\right)$

$= P(Z > 2.00)$

$= 1 - P(Z < 2.00)$

$= 1 - 0.9772$

$= 0.0228$

$P(X > 28) = P(Z > \frac{28-20}{4})$

$Z = \frac{X-\mu}{\sigma}$

available at the std n. table

Example 7.4-2 (b)

Given the normally distributed variable X with mean 20 and standard deviation 4 ,find

$$P(a<Z<b)=P(Z<b)-P(Z<a)$$

(b) $P(17.5<X<22.5)$

$$=P\left(\frac{17.5-20}{4}<Z<\frac{22.5-20}{4}\right)$$

$$=P(-0.625<Z<0.625)$$

round off to 2 decimal places

$$=P(-0.63<Z<0.63)$$

$$=P(Z<0.63)-P(Z<-0.63)$$

$$=0.7357-0.2643$$

$$=0.4714$$

z	0.01	0.02	0.03
0.6	0.7291	0.7324	0.7357
0.7	0.7611	0.7642	0.7673
0.8	0.7910	0.7939	0.7967

z	0.01	0.02	0.03
-0.6	0.2709	0.2676	0.2643
-0.5	0.3050	0.3015	0.2981
-0.4	0.3409	0.3372	0.3336

Example 7.4-2

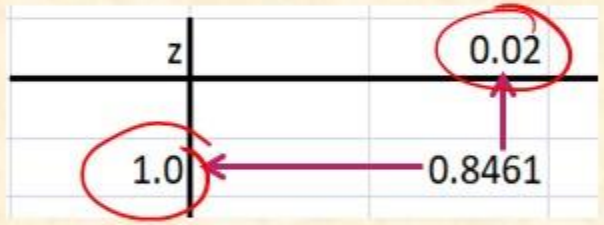
Given the normally distributed variable X with mean 20 and standard deviation 4 ,find

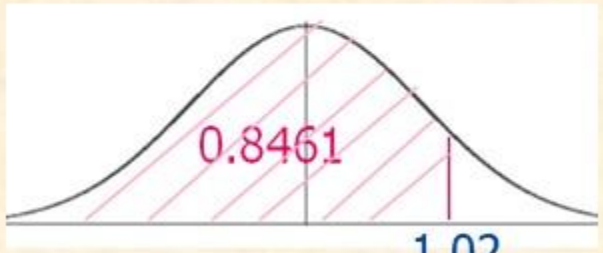
(c) the value of k such that $P(X > k) = 0.1539$ ⇒ $1 - P(X < k) = 0.1539$

left area ↓ $P(X < k) = 1 - 0.1539 = 0.8461$ ← right area

$P(X < k) = 1 - 0.1539$

$$P\left(Z < \frac{k-20}{4}\right) = 0.8461$$





From Standard normal table, $z = 1.02$

$$\frac{k-20}{4} = 1.02$$

$$\therefore k = 1.02(4) + 20 = 24.08$$

Example 7.4-3

The serum cholesterol levels of a certain population of 40-year-olds male adults follow approximately a normal distribution with mean 185 mg/dl and standard deviation 36 mg/dl. If a 40-year-old male adult is chosen at random from this population, what is the probability that he has serum cholesterol level

(a) greater than 195 mg/dl ?

Let X be the cholesterol levels of a 40 year old male

$$X \sim N(185, 36^2)$$

$$Z = \frac{X - \mu}{\sigma}$$

(a) $P(X > 195)$

$$= P\left(Z > \frac{195 - 185}{36}\right)$$

$$= P(Z > 0.28)$$

$$= 1 - P(Z < 0.28)$$

$$= 1 - 0.6103$$

$$= 0.3897$$

Example 7.4-3

σ μ

The serum cholesterol levels of a certain population of 40-year-olds male adults follow approximately a normal distribution with mean 185 mg/dl and standard deviation 36 mg/dl. If a 40-year-old male adult is chosen at random from this population, what is the probability that he has serum cholesterol level

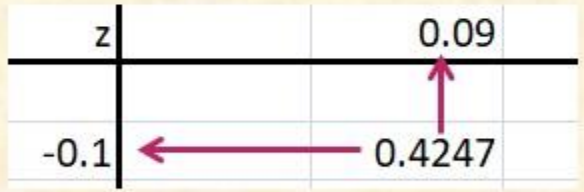
(b) less than 178 mg/dl ?

$$\text{(b) } P(X<178)$$

$$=P\left(Z<\frac{178-185}{36}\right)$$

$$=P(Z<-0.19)$$

$$=0.4247$$



Example 7.4-3

The serum cholesterol levels of a certain population of 40-year-olds male adults follow approximately a normal distribution with mean 185 mg/dl and standard deviation 36 mg/dl. If a 40-year-old male adult is chosen at random from this population, what is the probability that he has serum cholesterol level

(c) Between 178 and 195 mg/dl ?

$$(c)\ P(178 < X < 195)$$

$$= P\left(\frac{178-185}{36} < Z < \frac{195-185}{36}\right)$$

$$= P(-0.19 < Z < 0.28)$$

$$= P(Z < 0.28) - P(Z < -0.19)$$

$$= 0.6103 - 0.4247$$

$$= 0.1856$$