Tensor Network

Introduction

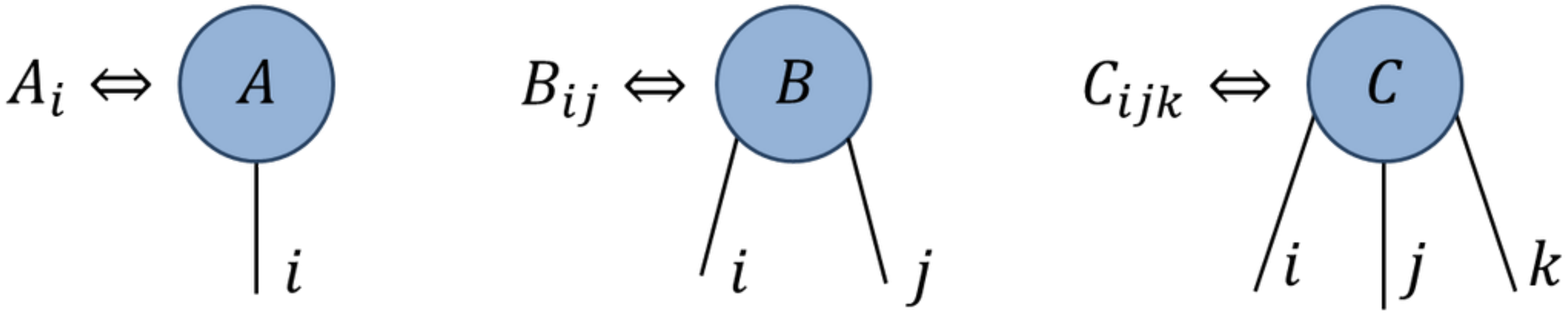
- What are tensor networks used for?
- To represent sets of correlated data
- Applications:
- To study quantum many-body systems, tensor networks are used to encode the coefficients of a state wave function
- To study classical many-body systems,tensor networks are used to encode statistical ensembles of microstates (partition function)
- Big data analytics, tensor networks can represent multidimensional data

- Why tensor networks?
- Offer a greatly compressed representation of a large structured data set
- Potentially allow for a better characterization of structure within a data set, particularly in terms of the correlations,diagrammatic notation used to represent networks can provide a visually clear and intuitive understanding of this structure
- Offer a distributed representation of a data set, many manipulations can be performed in parallel
- Allow a unified framework for manipulating large data sets
- Well suited for operating with noisy or missing data, decompositions are robust

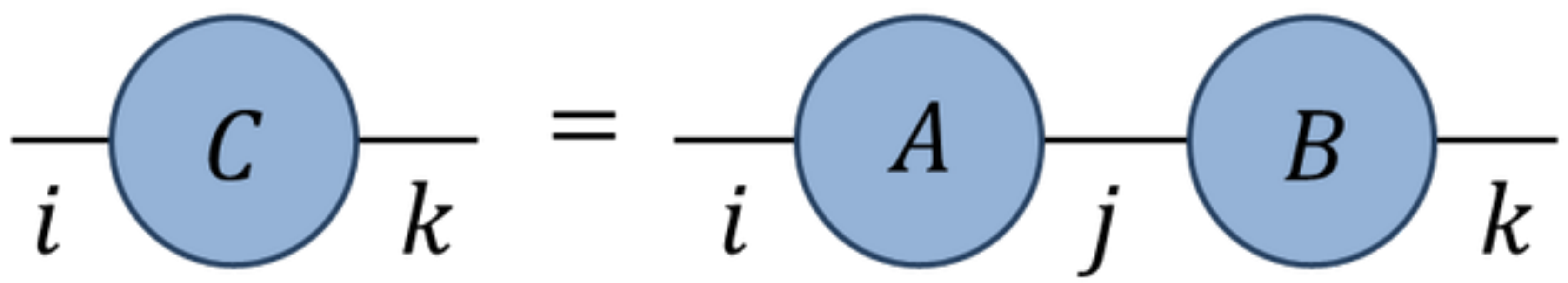
The fundamentals of Tensor Networks

- A tensor can be understood as a multi-dimensional array of numbers.
- Diagrammatic notation for tensors:a solid shape with a number of legs corresponding its order

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix} \qquad C = \begin{bmatrix} C_{111} & \cdots & C_{1n1} \\ \vdots & \ddots & \vdots \\ C_{m11} & \cdots & C_{mn1} \end{bmatrix}$$

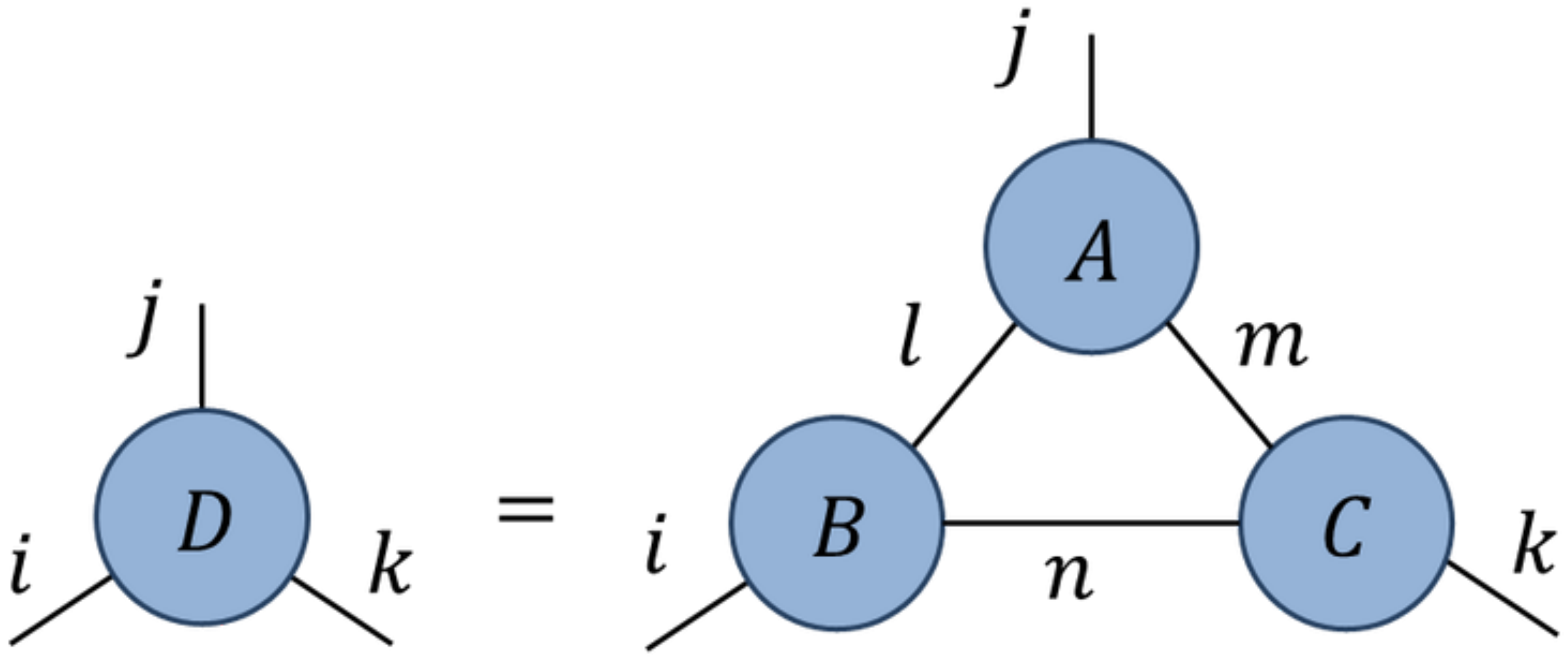


- Form networks comprised of multiple tensors,index shared by two tensors denotes a contraction over this index



$\Updownarrow$

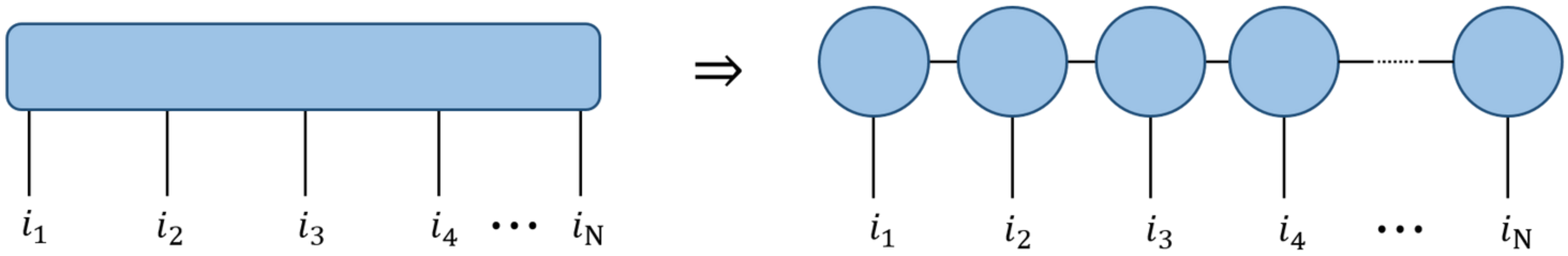
$$C_{ik} = \sum_{j} A_{ij} B_{jk}$$



$\Updownarrow$

$$D_{ijk} = \sum_{lmn} A_{ljm} B_{iln} C_{nmk}$$

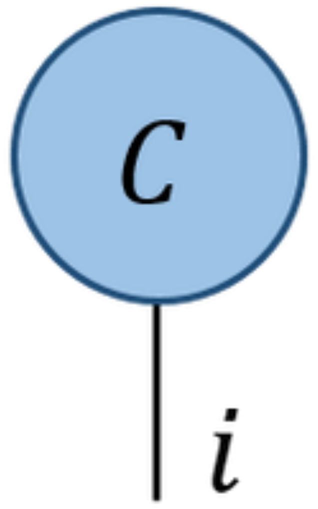
- The goal is to approximate a single high-order tensor as a tensor network composed of many low-order tensors.the total dimension of a tensor grows exponentially with its order.



Tensor networks for quantum many-body systems

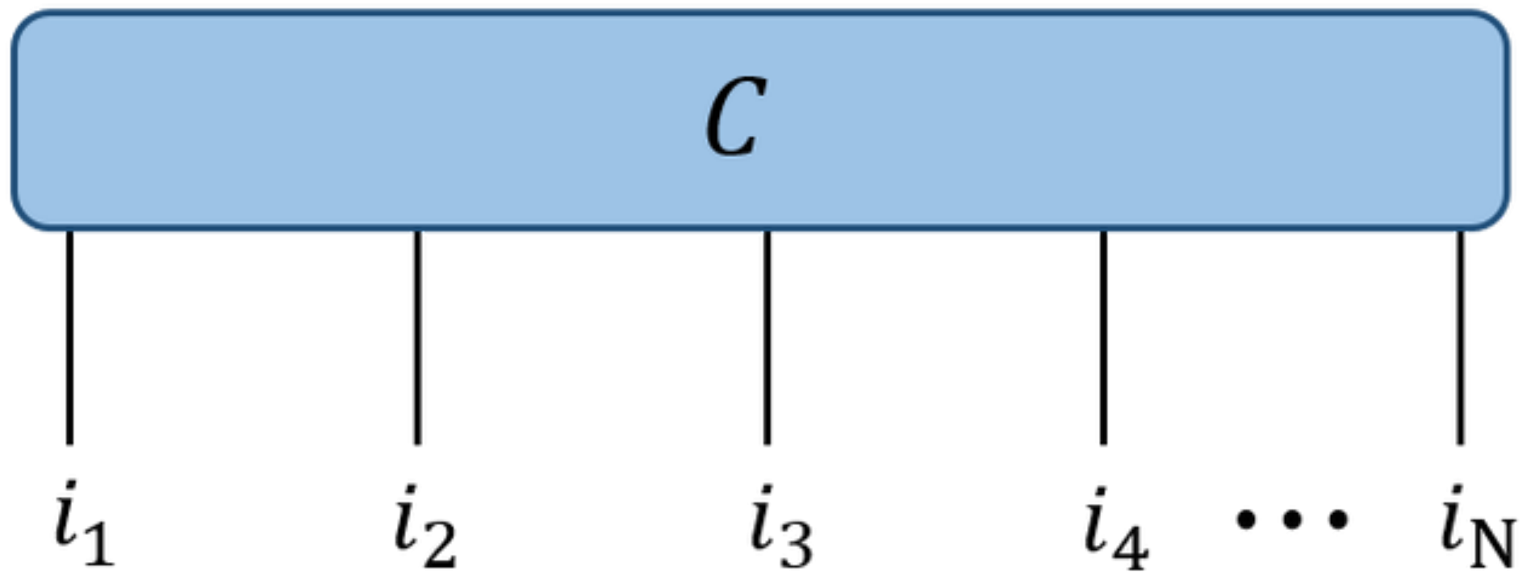
- Tensor networks most commonly are used for representing quantum wave functions
- A (pure) state $|\phi\rangle$ for the system is represented as a vector in the Hilbert space whose dimension is d

$$|\phi\rangle = C_1|1\rangle + C_2|2\rangle + \cdots + C_d|d\rangle = \sum_{i=1}^{d} C_i|i\rangle$$

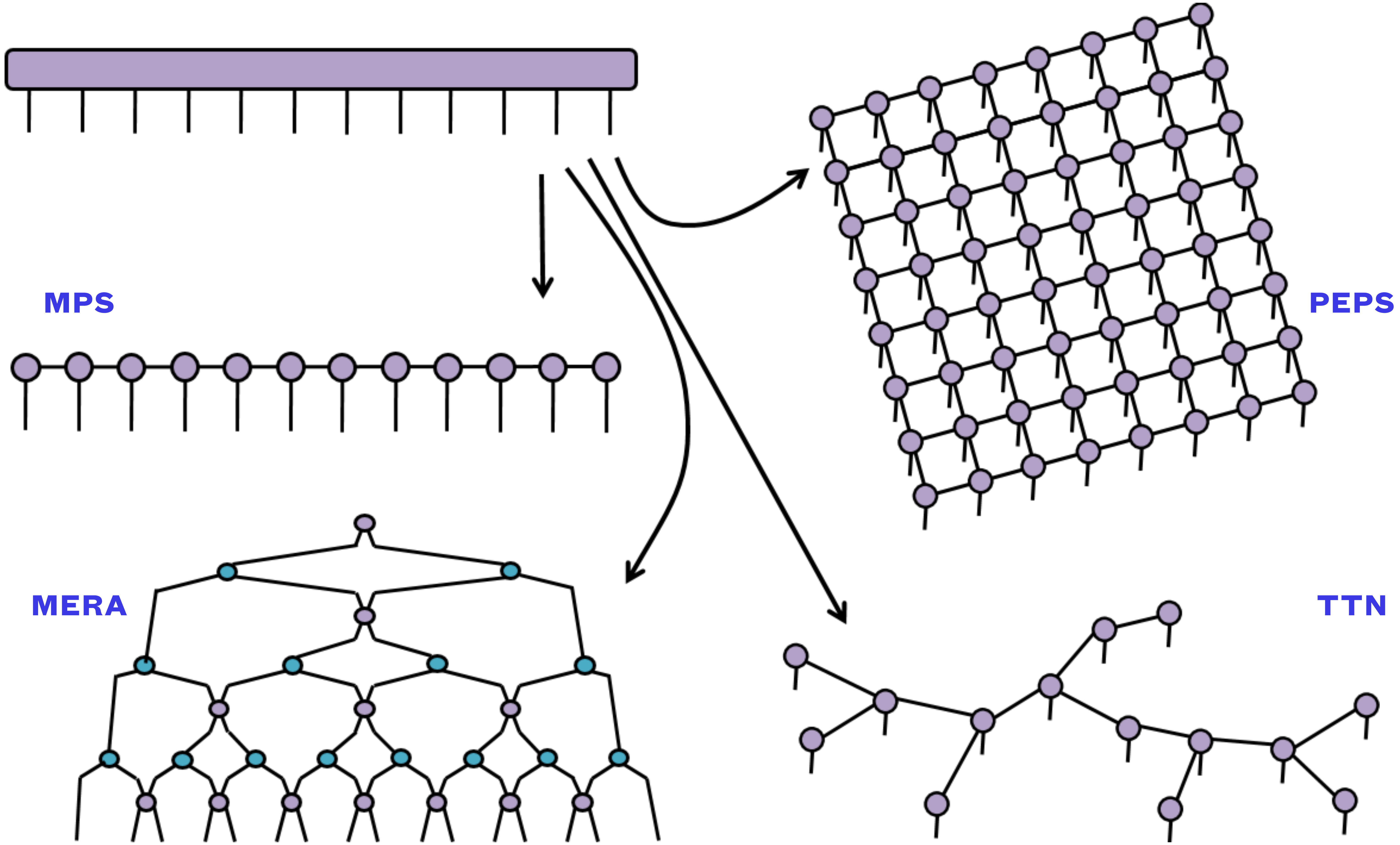


- A quantum many-body system composed of N individual systems of dimension d
- A pure wave function $|\psi\rangle$ of the composite system is given by an order-N tensor

$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N} C_{i_1,i_2,\ldots,i_N}|i_1\rangle|i_2\rangle\cdots|i_N\rangle$$



- In a typical many-body problem, we begin with a Hamiltonian and want to find the ground state.In this setting, tensor networks are most commonly used as ansatz for quantum states
- Common tensor network ansatz include MPS ,TTN,MERA,PEPS



- The choice of best tensor network ansatz for a particular problem depend on the geometry of the problem as well as its physical properties.
- Choose a specific tensor network ansatz,randomly initializing an instance of this network, vary parameters within the tensors by iterations and find a best approximation to the ground state of the system