整数量子霍尔效应

2DEG，简并度，平台

2020.5.31 喻舜尧

整数量子霍尔效应

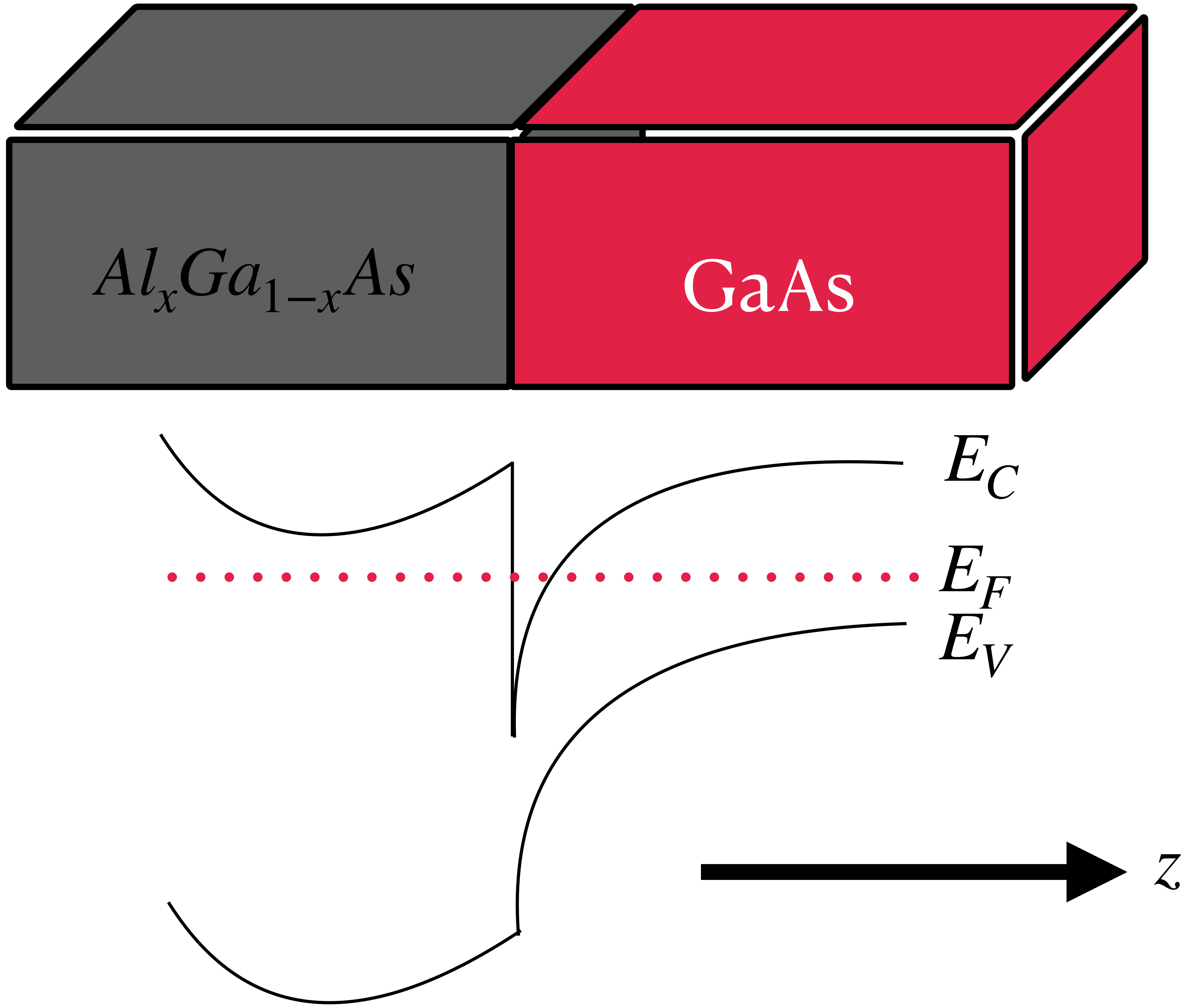
异质结与二维电子气的形成

$GaAs$:a=0.5653nm

Band gap:1.441eV

$Al_xGa_{1-x}As$:a=0.5660nm

Band gap:2.12eV



整数量子霍尔效应

朗道能级-朗道规范-不可压缩

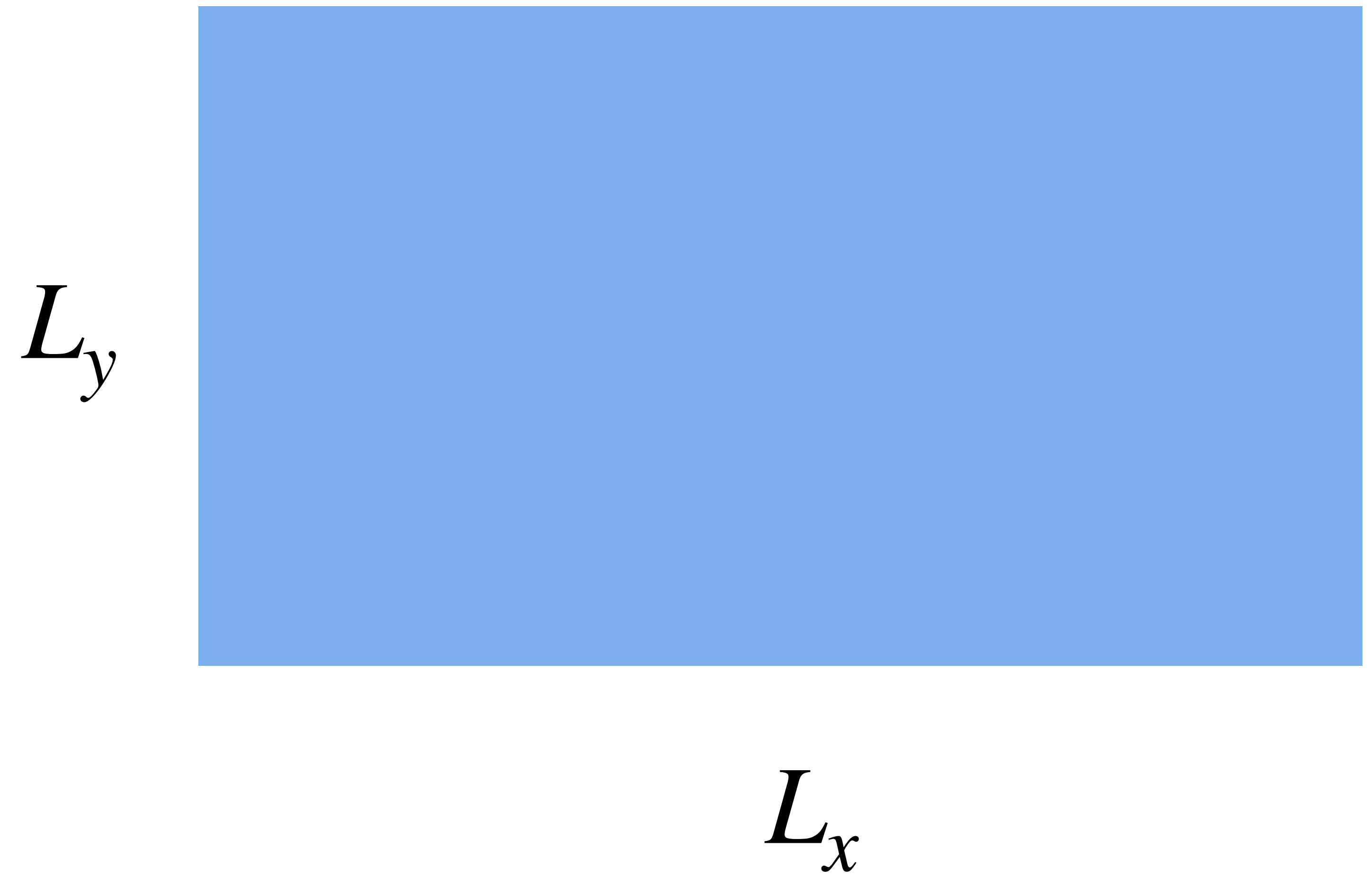
$$\hat{H} = \frac{1}{2m}(\hat{p} + e\hat{A})^2 \quad A_x = -By \quad A_y = 0$$

$$\hat{H} = \frac{\hat{p}_y{}^2}{2m} + \frac{1}{2m}(\hat{p}_x + eBy)^2 \quad [\hat{H}, \hat{p}_x] = 0$$

$$\psi_{k,n} = exp(ikx)\psi_n(y + kl_B^2) \quad l_B^2 = \frac{\hbar}{eB}$$

$$\varepsilon_n = (n + \frac{1}{2})\hbar\omega_c$$

$$0 \le x \le L_x \quad 0 \le y \le L_y \quad -\frac{L_y}{l_B^2} \le k \le 0 \qquad N = \frac{L_x}{2\pi}\int_{-\frac{L_y}{l_B^2}}^{0} dk = \frac{L_xL_y}{2\pi l_B^2} = \frac{BA}{h/e} = \frac{\Phi}{\Phi_0}$$



整数量子霍尔效应

朗道能级-对称性规范

$$\hat{H}=\frac{1}{2m}(\hat{p}+e\hat{A})^2 \quad A_x=-\frac{1}{2}By \quad A_y=\frac{1}{2}Bx \quad \hat{H}=\frac{1}{2}[(-i\frac{\partial}{\partial x}-\frac{y}{2})^2+(-i\frac{\partial}{\partial y}+\frac{x}{2})^2] \quad z=x+iy$$

$$\hat{a}=\frac{1}{\sqrt{2}}[(\frac{x}{2}+\frac{\partial}{\partial x})-i(\frac{y}{2}+\frac{\partial}{\partial y})]=\frac{1}{\sqrt{2}}[\frac{1}{2}\bar{z}+2\frac{\partial}{\partial z}] \qquad \hat{a}^\dagger=\frac{1}{\sqrt{2}}[(\frac{x}{2}-\frac{\partial}{\partial x})+i(\frac{y}{2}-\frac{\partial}{\partial y})]=\frac{1}{\sqrt{2}}[\frac{1}{2}z-2\frac{\partial}{\partial \bar{z}}]$$

$$\hat{b}=\frac{1}{\sqrt{2}}[(\frac{x}{2}+\frac{\partial}{\partial x})+i(\frac{y}{2}+\frac{\partial}{\partial y})]=\frac{1}{\sqrt{2}}[\frac{1}{2}z+2\frac{\partial}{\partial \bar{z}}] \qquad \hat{b}^\dagger=\frac{1}{\sqrt{2}}[(\frac{x}{2}-\frac{\partial}{\partial x})-i(\frac{y}{2}-\frac{\partial}{\partial y})]=\frac{1}{\sqrt{2}}[\frac{1}{2}\bar{z}-2\frac{\partial}{\partial z}]$$

$$[\hat{a},\hat{a}^\dagger]=[\hat{b},\hat{b}^\dagger]=1 \qquad H=\hat{a}^\dagger\hat{a}+\frac{1}{2} \qquad |n,m\rangle=\frac{\hat{a}^{\dagger n}\hat{b}^{\dagger m}}{\sqrt{n!m!}}|0,0\rangle$$

$$[\hat{H},\hat{L}_z]=0$$

$$L_z=\hbar(\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b}) \qquad L_z|0,m\rangle=-m\hbar|0,m\rangle$$

$$|0,0\rangle=ce^{-\frac{\bar{z}z}{4}}$$

$$|0,m\rangle=c\bar{z}^m e^{-\frac{\bar{z}z}{4}}$$

整数量子霍尔效应

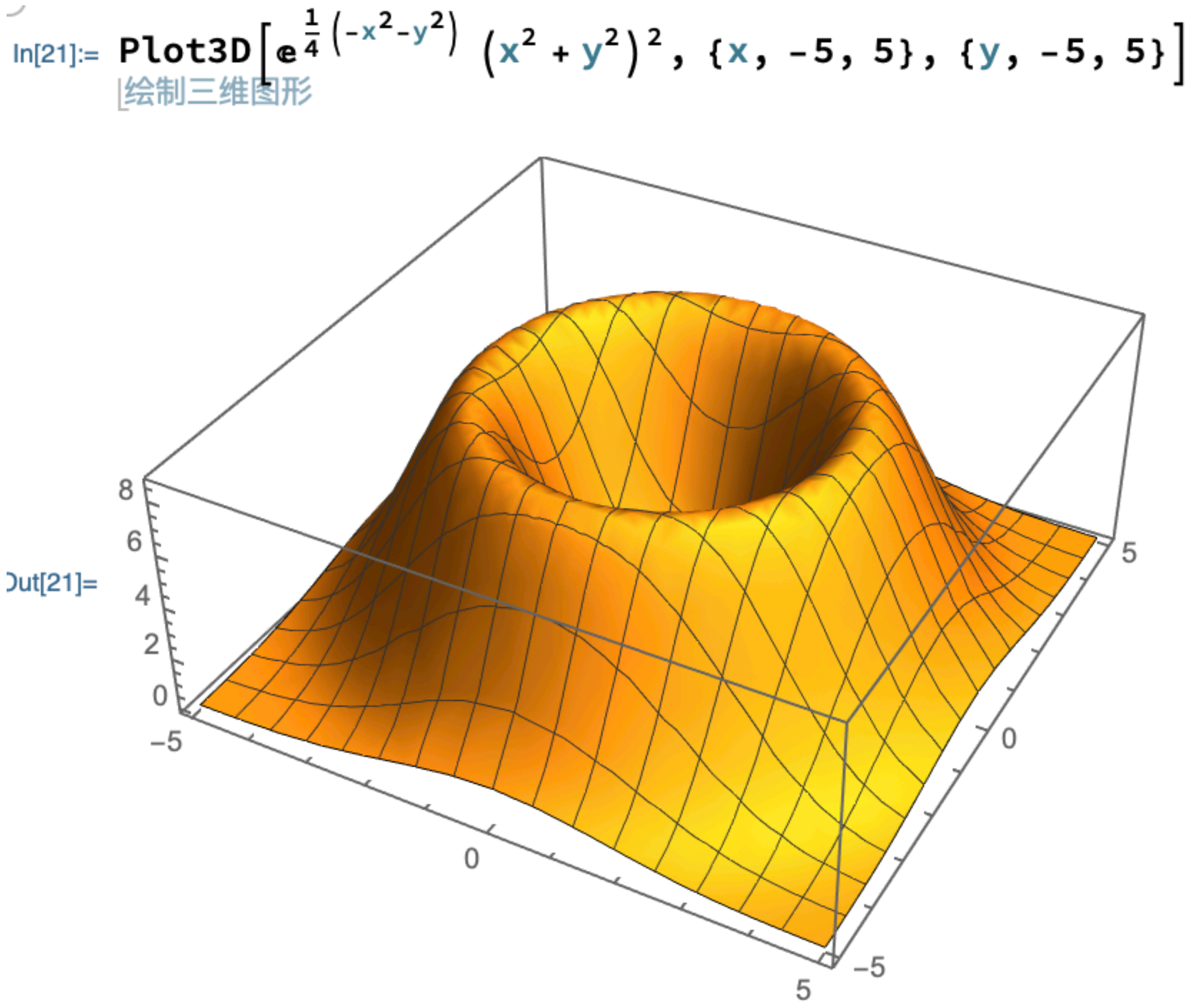
朗道能级-对称性规范-简并度

$$|0,m\rangle = c\bar{z}^m e^{-\frac{\bar{z}z}{4}}$$

$$\langle 0,m|0,m\rangle = |c|^2 r^{2m} e^{-r^2/2}$$

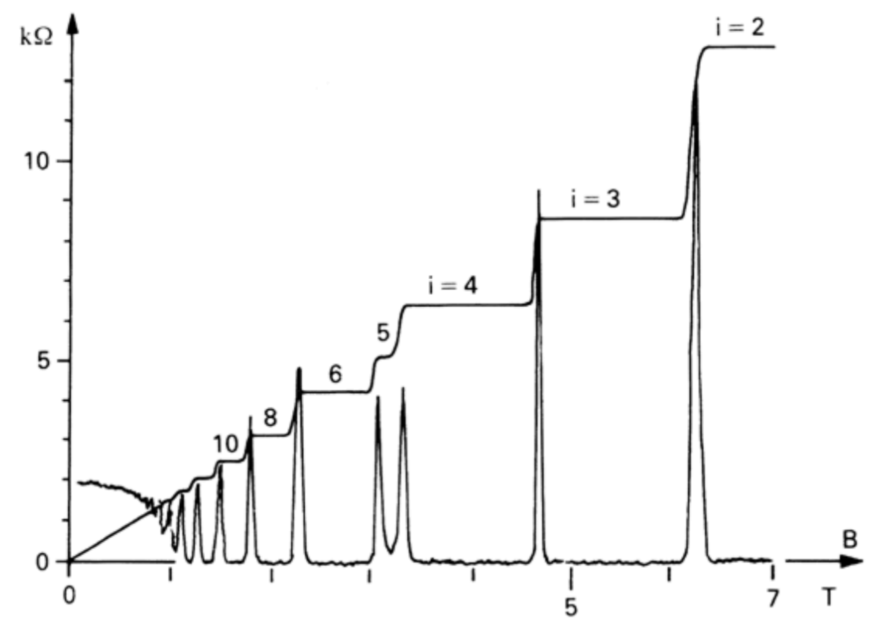
$$r = \sqrt{2m}\, l_B$$

$$m = \frac{R^2}{2l_B^2} = \frac{\Phi}{\Phi_0}$$



整数量子霍尔效应

杂质、局域与量子霍尔平台



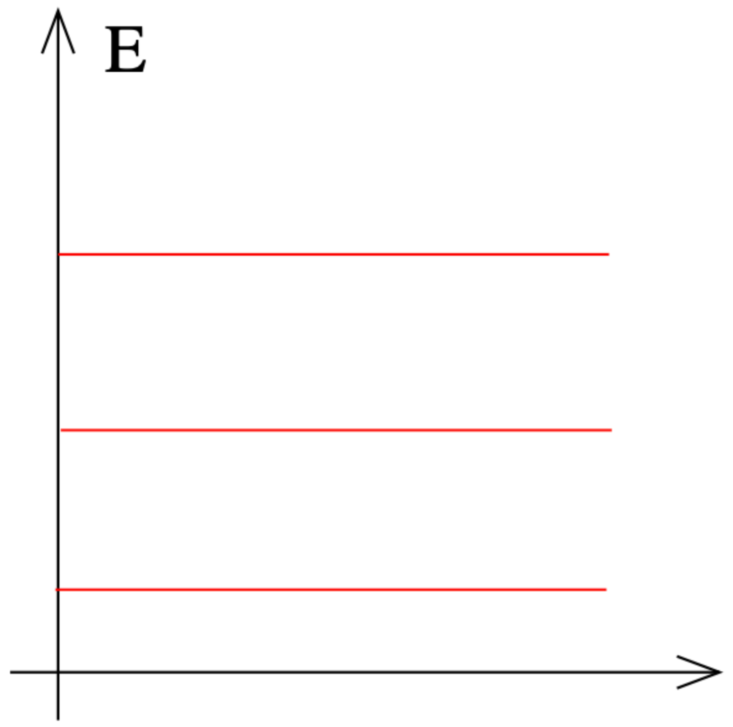


Figure 16: Density of states without disorder...

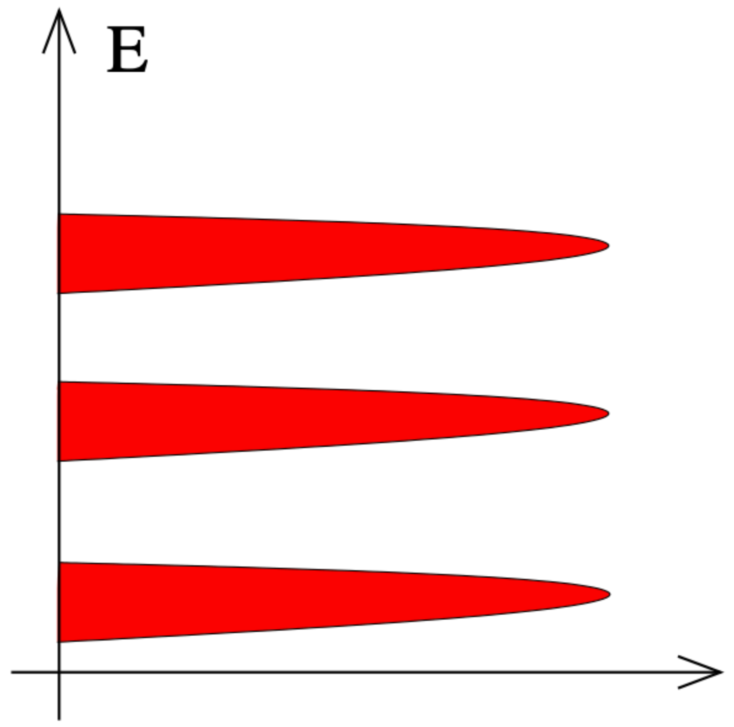
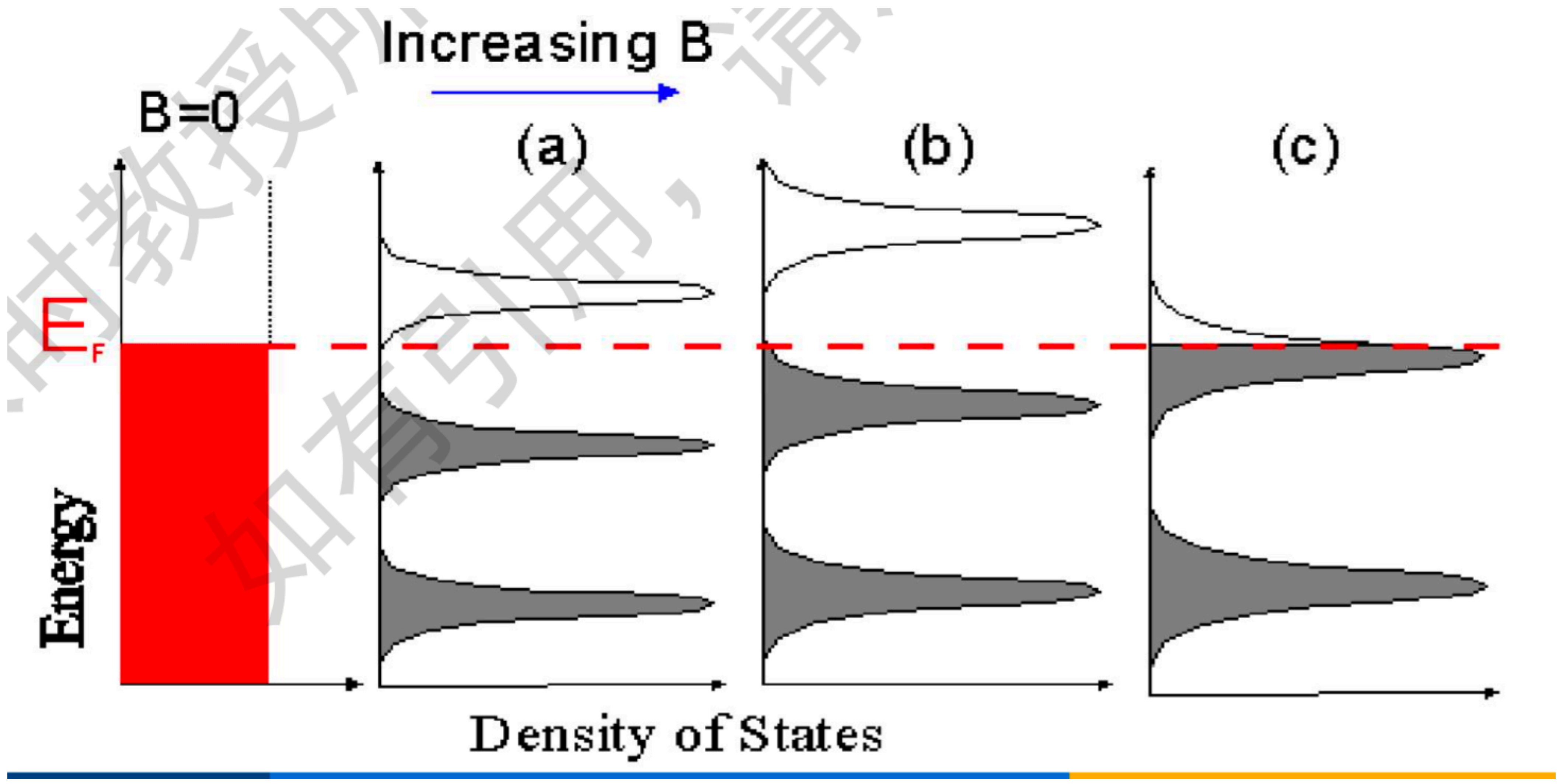


Figure 17: ...and with disorder.



下期预告

整数量子霍尔效应与拓扑

参考资料

David Tong QuantumHallEffect

吴咏时 拓扑序讲座6