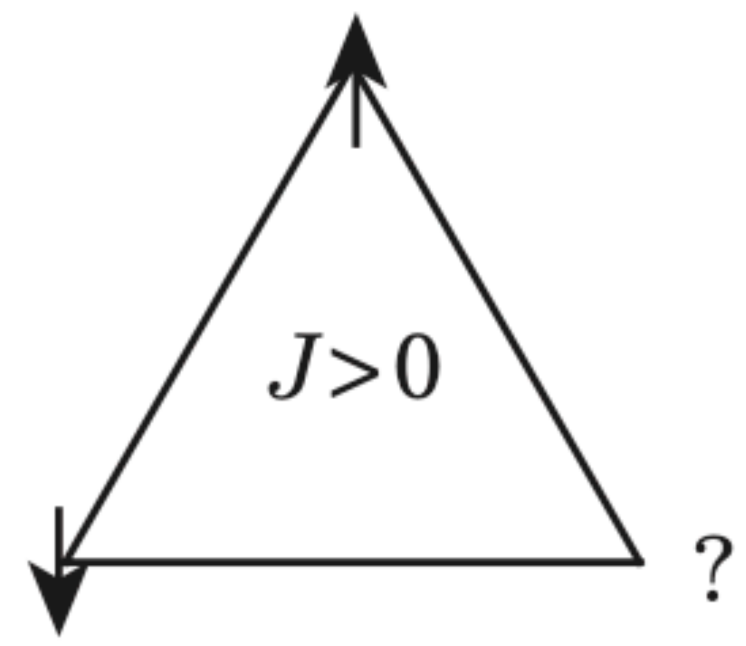
Spin Liquid States

Shunyao Yu 2022

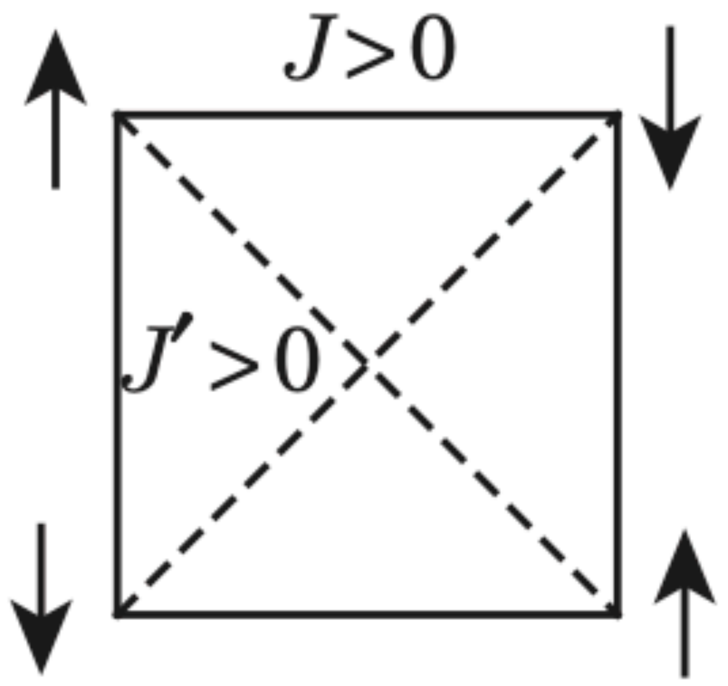
Contents

Frustration, Fluctuations and Disordered Ground States



- Non-bipartite lattice, spins are frustrated

Triangular, Kagome, Pyrochlore ...



- Bipartite lattice lattice, introduce longer interactions

Strong frustration can suppress long-range order.

$|\uparrow\downarrow\rangle$

$-\frac{J}{4}\times 4$

Neel correlated with all neighbors

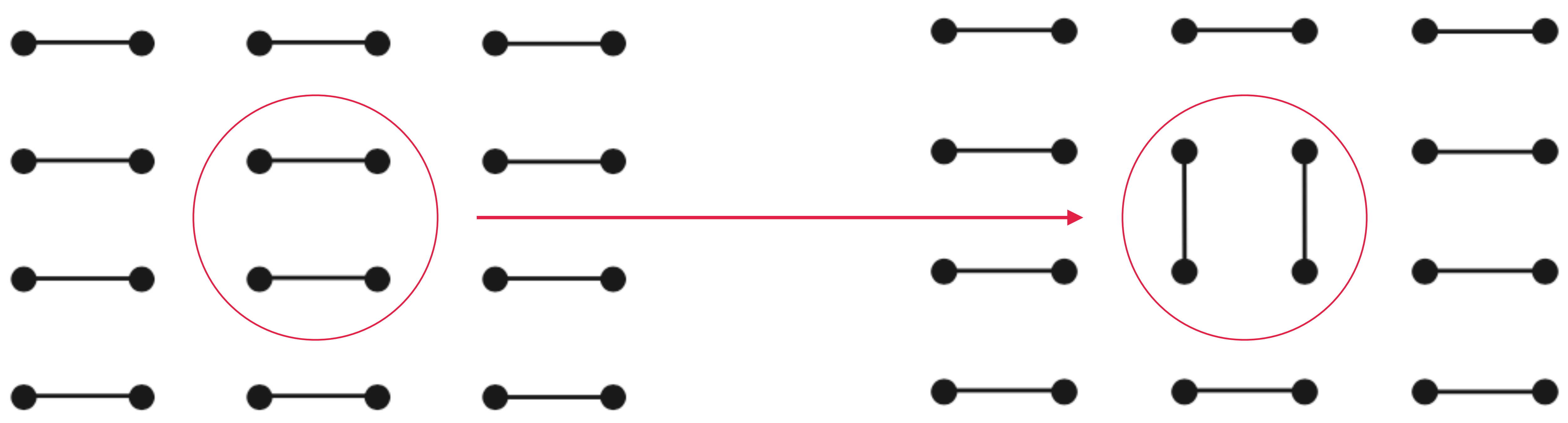
$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$-\frac{3}{4}J\times 2 + 0\times 2$

Singlet correlated with one neighbor

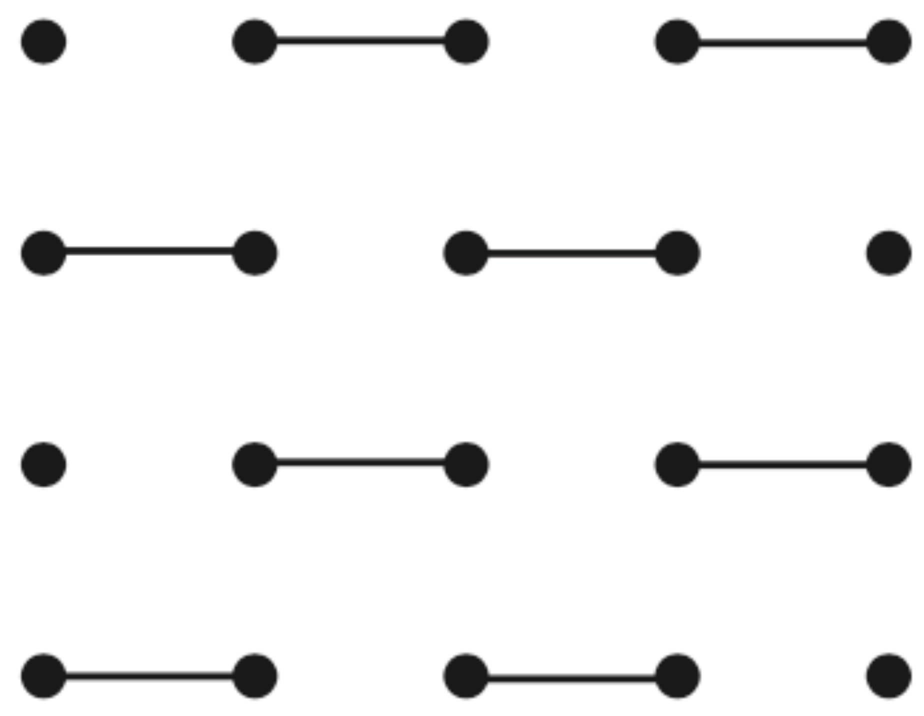
Coordination number compete with energy

Valence-Bond-Solid & Resonating-Valence-Bond

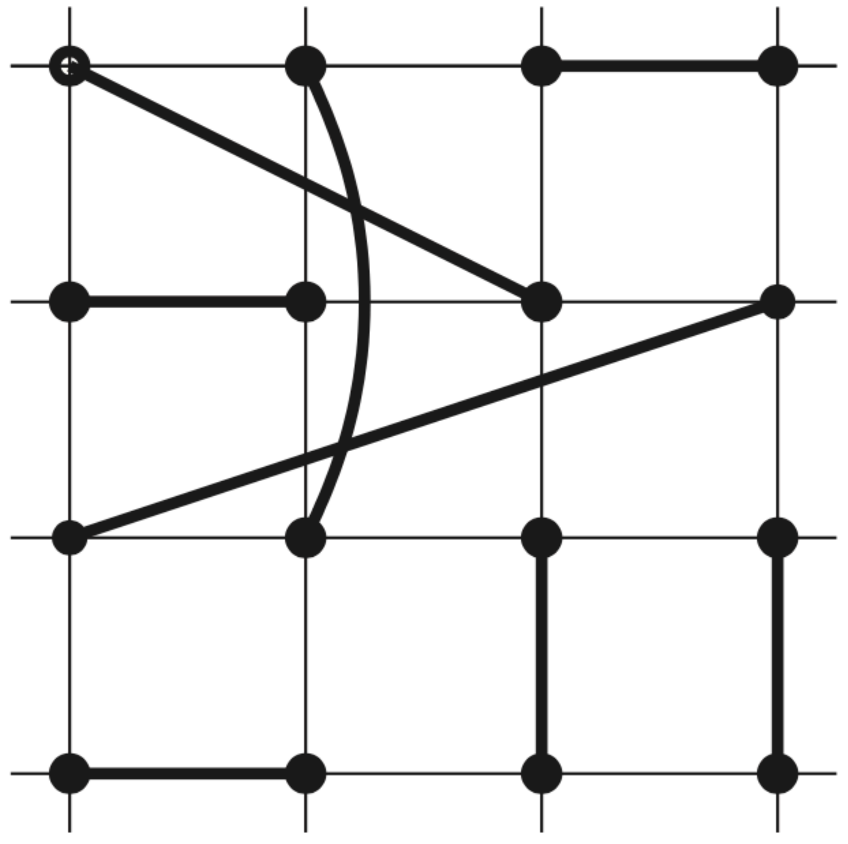


Column VBS

Easily resonant, lower the total energy.



Staggered VBS, higher energy.



$$|(ij)\rangle = \frac{1}{\sqrt{2}}(|\uparrow_i \downarrow_j\rangle - |\downarrow_i \uparrow_j\rangle) \qquad |\psi\rangle = \sum_P \Pi_{pairs} a(i_k, j_k)\,|(i_k j_k)\rangle$$

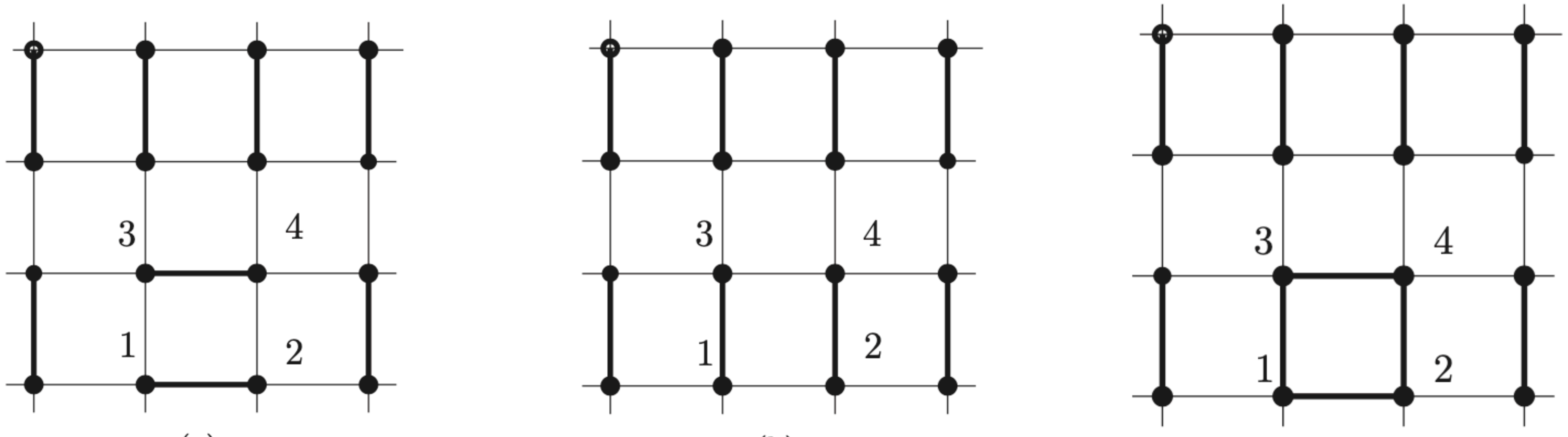
$$a(i_k, j_k) = a(|i_k - j_k|) \sim \frac{1}{|i_k - j_k|^{\sigma}}$$

$\sigma < 5$ there is Neel long range order.

长程RVB的性质讨论

Valence-Bond-Solid & Resonating-Valence-Bond

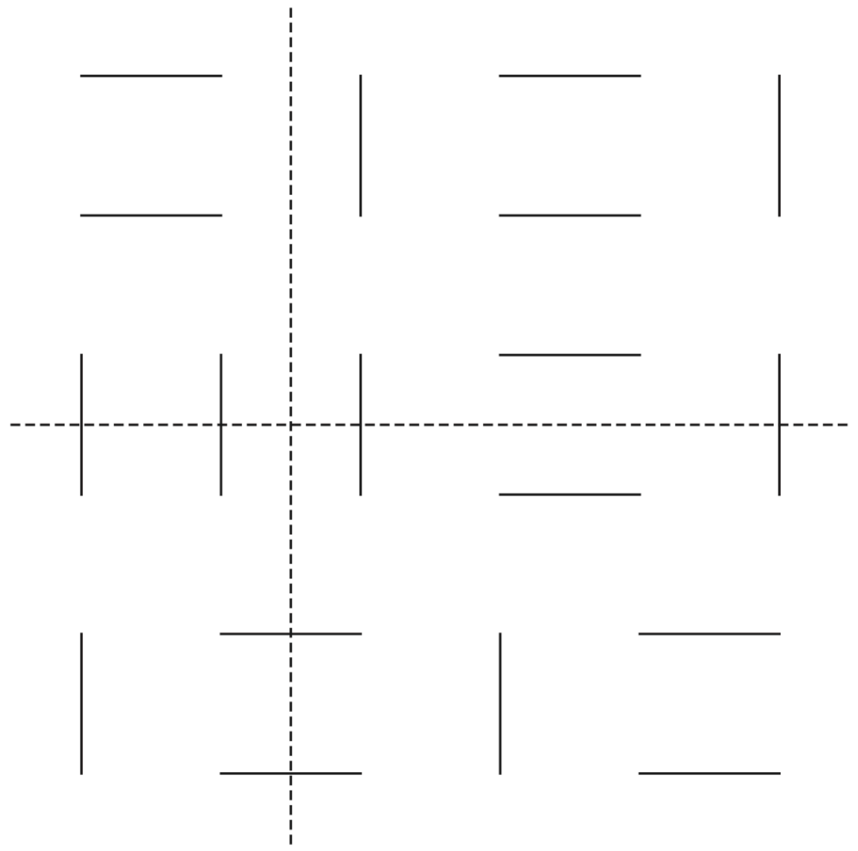
Short range RVB



(a) (b) (c)

$$\langle \psi_a | \psi_b \rangle = 2^{P(a,b)-N/2}$$

$$G(\vec{x}) = 4(-1)^{x_1+x_2} \frac{\langle \Psi | \sigma_z(\vec{0}) \sigma_z(\vec{x}) | \Psi \rangle}{\langle \Psi | \Psi \rangle} \propto e^{-(|\vec{x}|/a_0)\ln 2}$$



Short-range RVB wave functions represent states with total spin equal to zero and exponentially decreasing correlation functions.

RVB spin liquid is the gapped spin liquid with topological order.

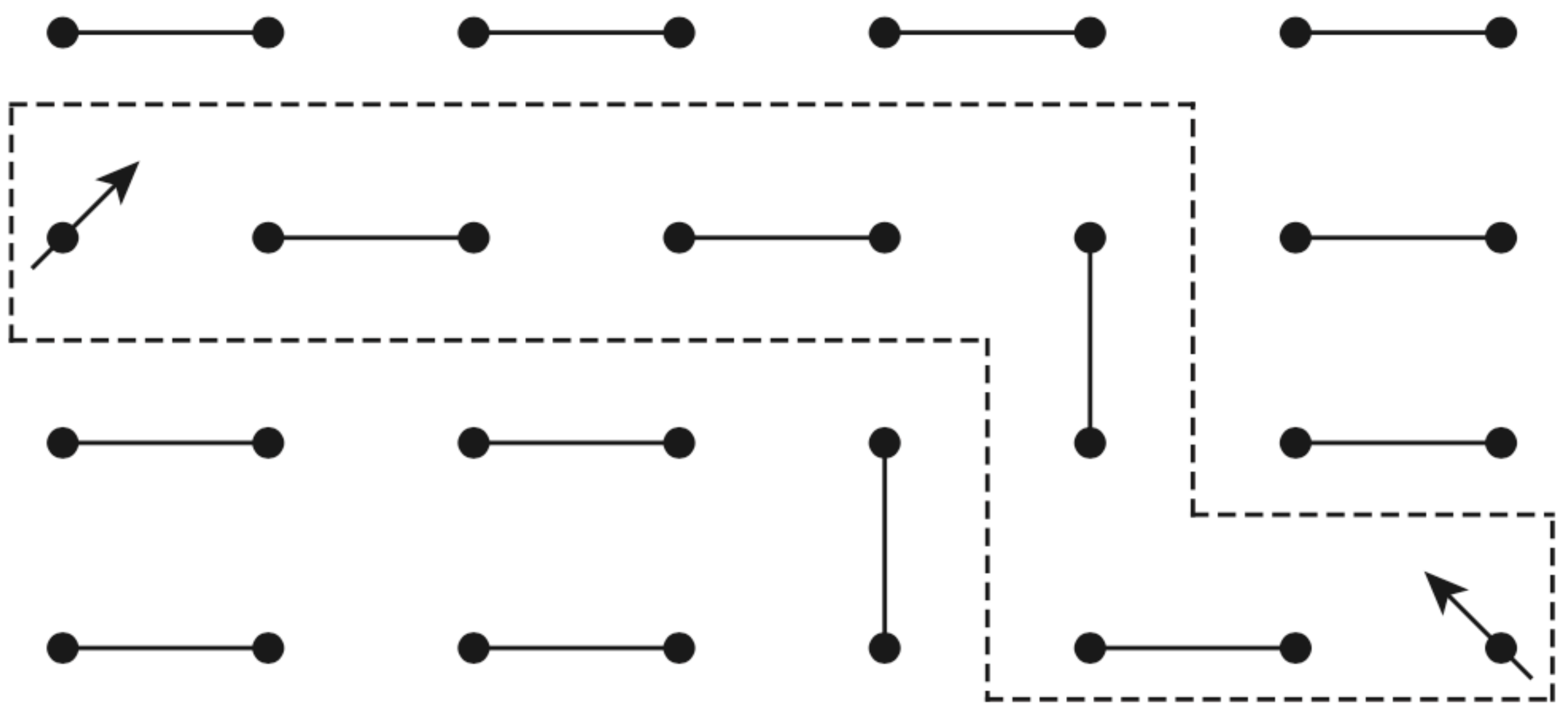
Assign two independent Z2 quantum numbers.

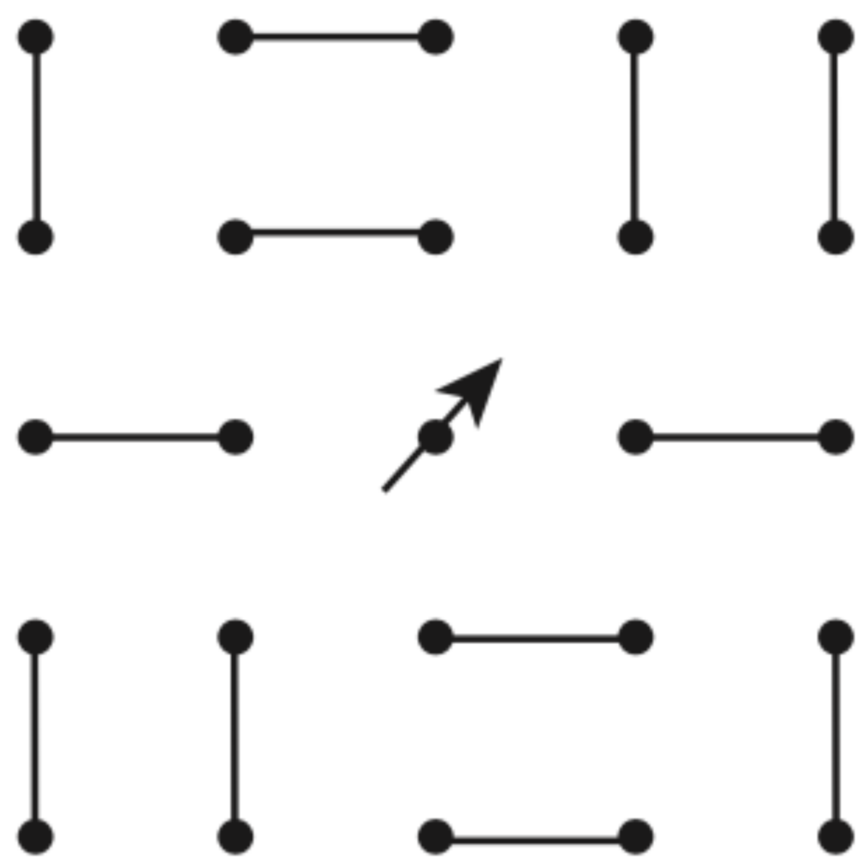
Valence-Bond-Solid & Resonating-Valence-Bond

Excitations in VBS

Excitations in 1D VBS state are deconfined spin 1/2 spinons, but does not occur in higher dimensions

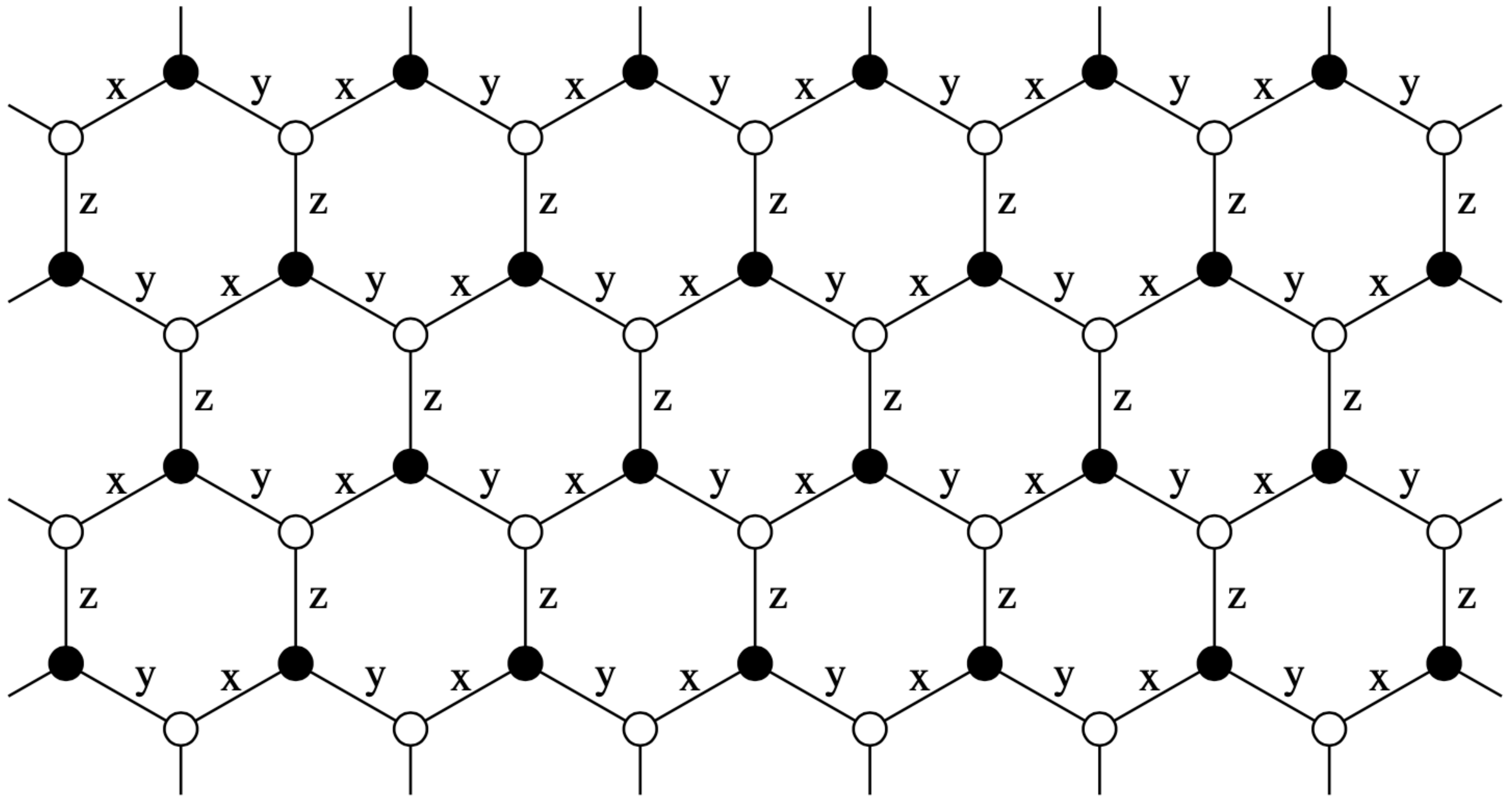
String has energy grows linearly with length, the elementary excitations are gapped spin-1 magnon.





Spinons are deconfined and survive as elementary excitations in the RVB spin liquid.

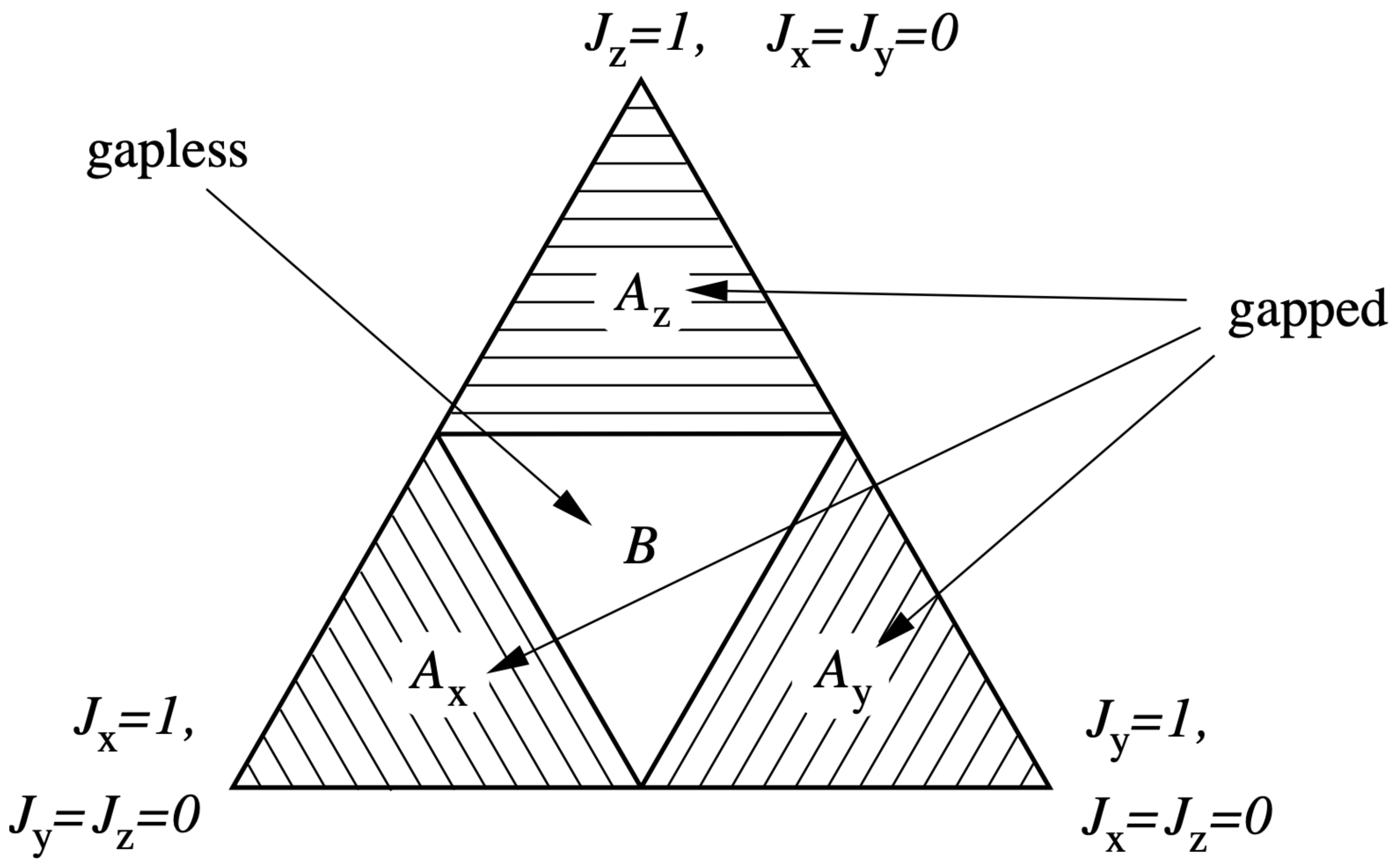
Kitaev's Honeycomb Model



$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z,$$

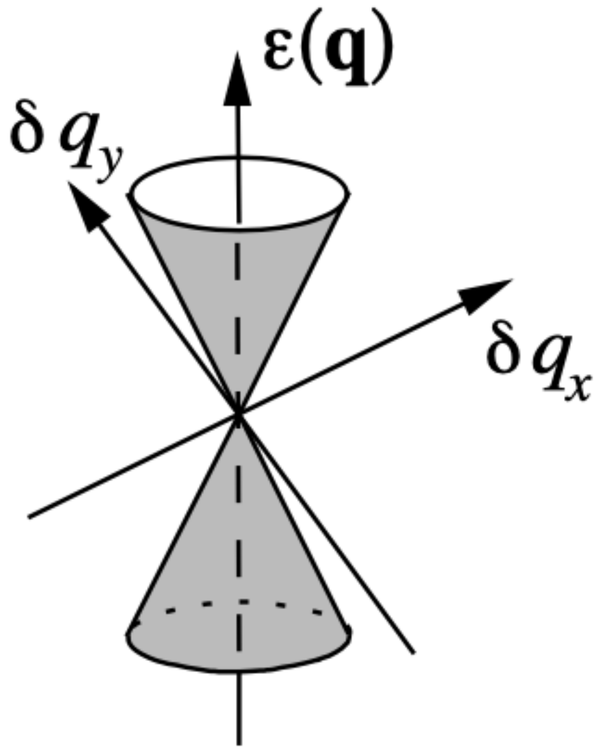
$$K_{jk} = \begin{cases} \sigma_j^x \sigma_k^x, & \text{if } (j,k) \text{ is an } x\text{-link;} \\ \sigma_j^x \sigma_k^y, & \text{if } (j,k) \text{ is an } y\text{-link;} \\ \sigma_j^x \sigma_k^z, & \text{if } (j,k) \text{ is an } z\text{-link.} \end{cases}$$

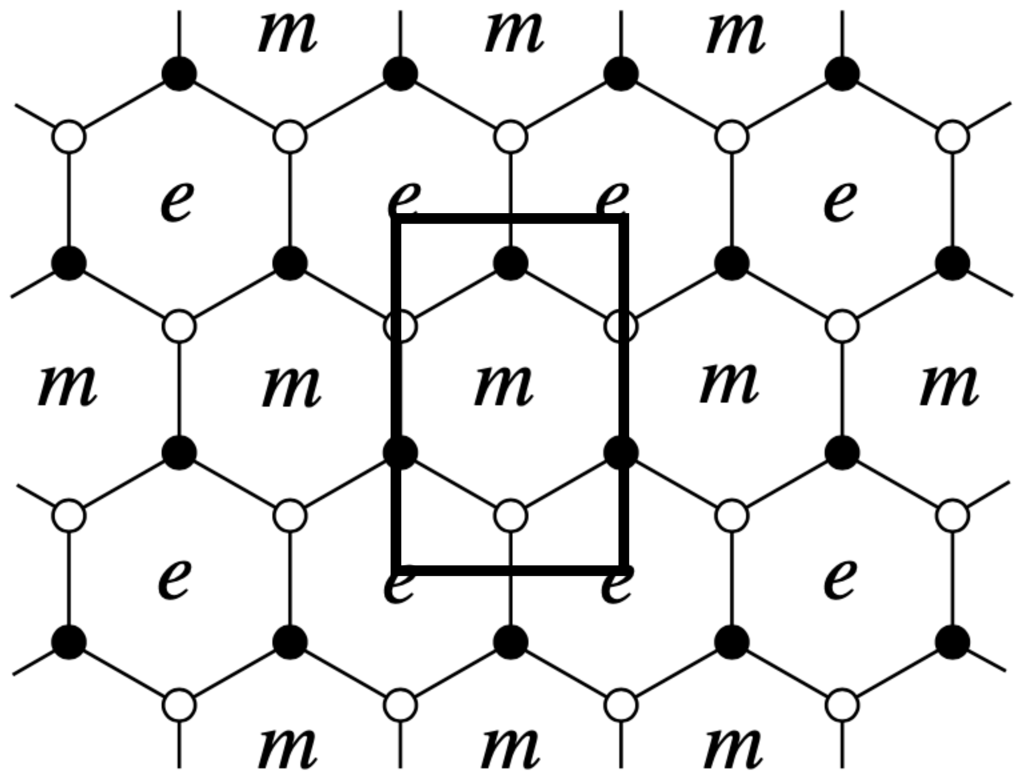
$$\tilde{\sigma}^x = ib^x c, \qquad \tilde{\sigma}^y = ib^y c, \qquad \tilde{\sigma}^z = ib^z c.$$



$$e \times e = m \times m = \varepsilon \times \varepsilon = 1,$$

$$e \times m = \varepsilon, \quad e \times \varepsilon = m, \quad m \times \varepsilon = e.$$

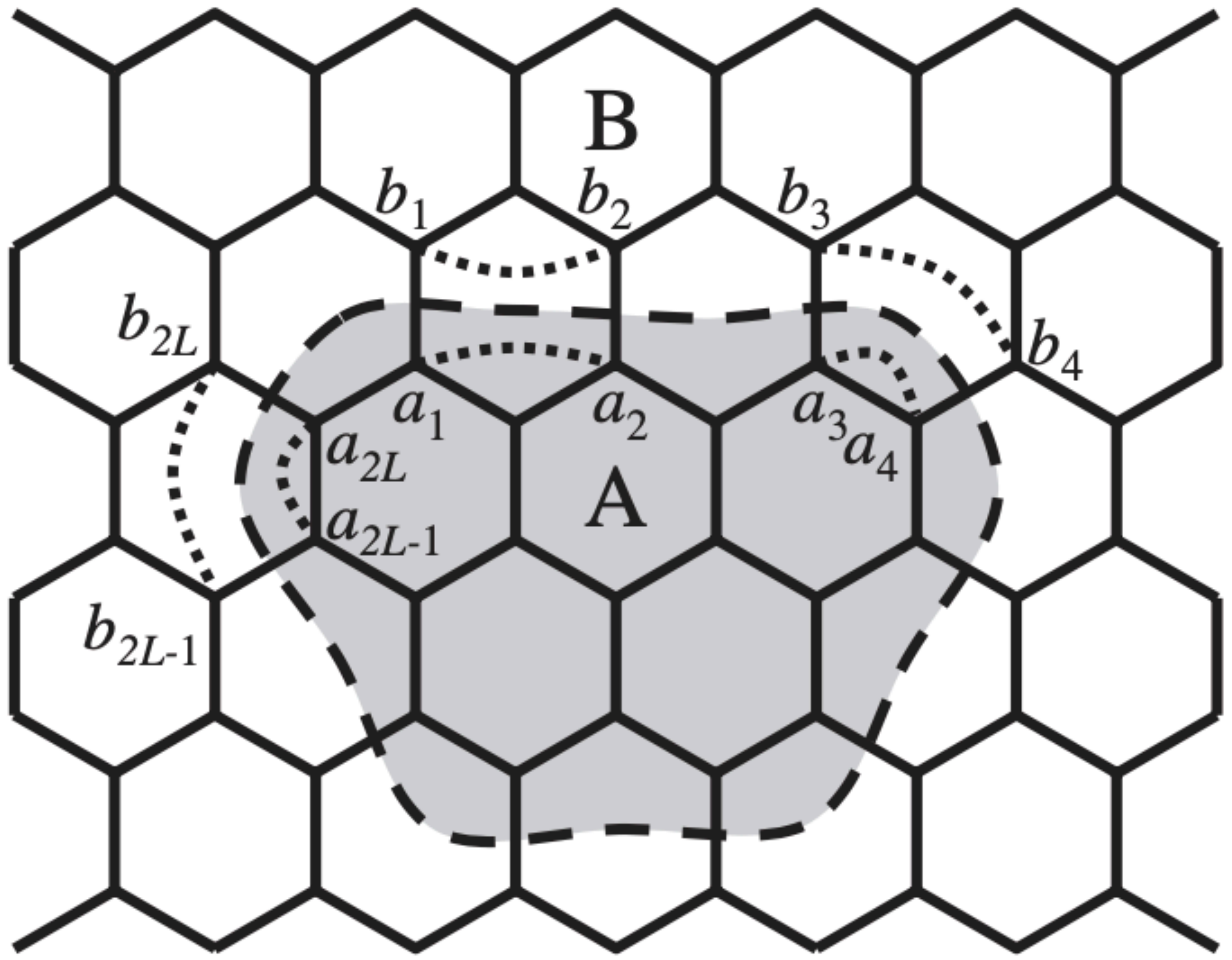




Kitaev, 2006

Kitaev's Honeycomb Model

Entanglement Entropy and Entanglement Spectrum



Direct product and projection

$$|\Psi\rangle = \frac{1}{\sqrt{2^{N+1}}} \sum_g D_g |u\rangle \otimes |\phi(u)\rangle,$$

Replica trick

$$S = -\mathrm{Tr}_A[\rho_A \log \rho_A] = -\frac{\partial}{\partial n} \mathrm{Tr}_A[\rho_A^n]|_{n=1}.$$

$$\mathrm{Tr}_A[\rho_A^n] = \mathrm{Tr}_{A,G}[\rho_{A,G}^n] \cdot \mathrm{Tr}_{A,F}[\rho_{A,F}^n],$$

TEE $S_{topo} = -\log 2$

$$S = S_G + S_F = (\alpha + \log 2)L - \log 2 \qquad S_G = (L-1)\log 2 \qquad S_F = \alpha L + o(1)$$

Valid for all phases of the Kitaev Model

H. Yao and X.L. Qi, PRL 105, 080501 (2010)

Kitaev's Honeycomb Model

Entanglement Entropy and Entanglement Spectrum

$$S_F = -\frac{1}{2}\sum_{n,k_y}[\lambda_n \log\lambda_n + (1-\lambda_n)\log(1-\lambda_n)](k_y),$$

- $\lambda_n(k_y)$ are the eigenvalues of $C_{xx'}(k_y) = \langle \eta_x^\dagger(k_y)\eta_{x'}(k_y)\rangle$

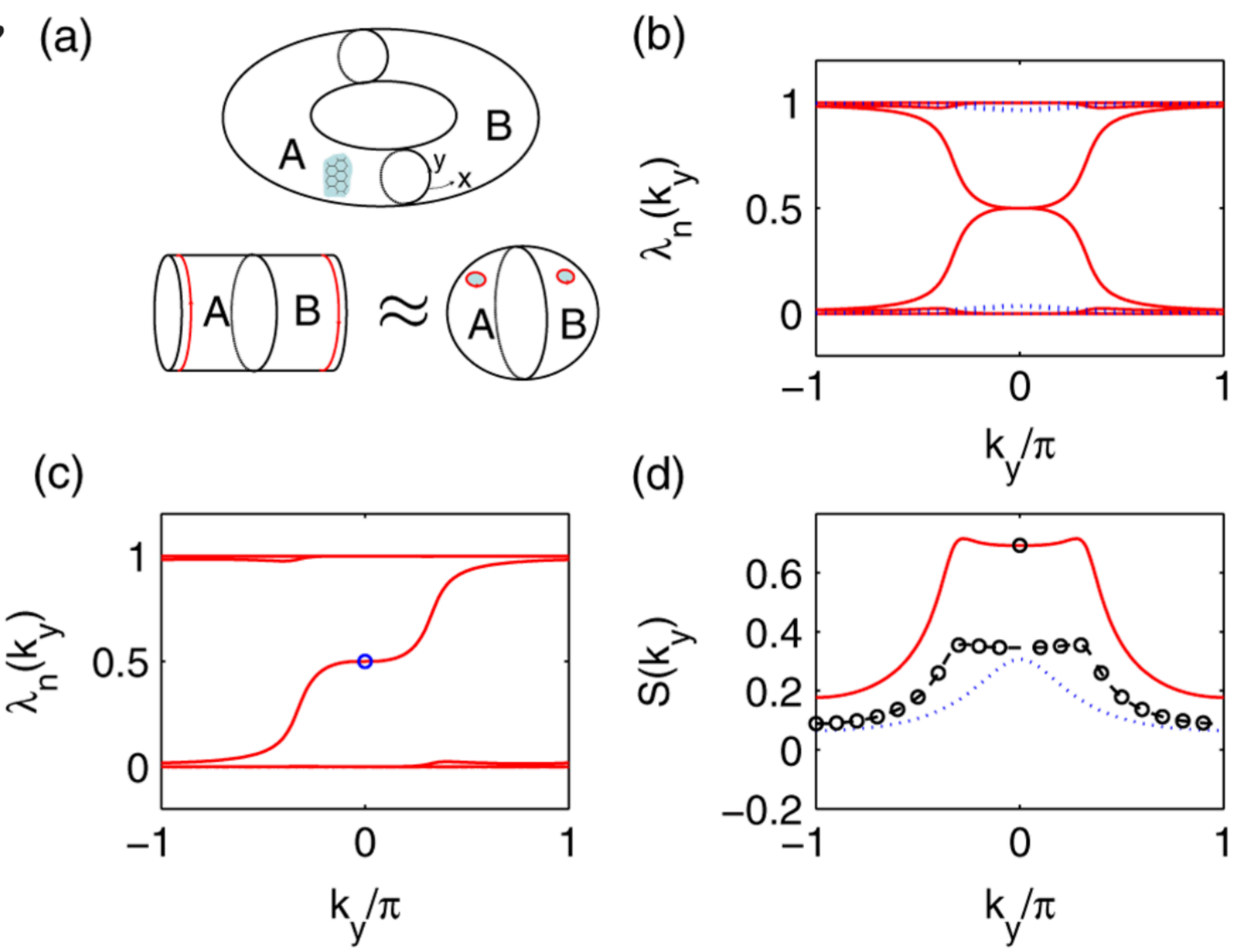
Gapped in the Abelian phase and gapless in the non-Abelian phase.

(b) torus divided into A and B regions

- Two gapless modes in the ES are from the boundaries between A B.
- Continuum limit $S_F = \sum_{k_y} S(k_y) \simeq L\int S(k_y)\frac{dk_y}{2\pi}$ satisfies the area law.
- Gap exists between the edge and bulk states with $\lambda_n(k_y)$ close to 0 or 1.

(c) cylinder with PBC

- $\lambda = 1/2$ blue circle is due to the nonlocal entanglement between the two MZMs at the open boundary.
- In the non-Abelian phase a cylinder with PBC for fermions is topologically equivalent to a sphere with two non-Abelian quasiparticles (σ particles).



H. Yao and X.L. Qi, PRL 105, 080501 (2010)

Kitaev's Honeycomb Model

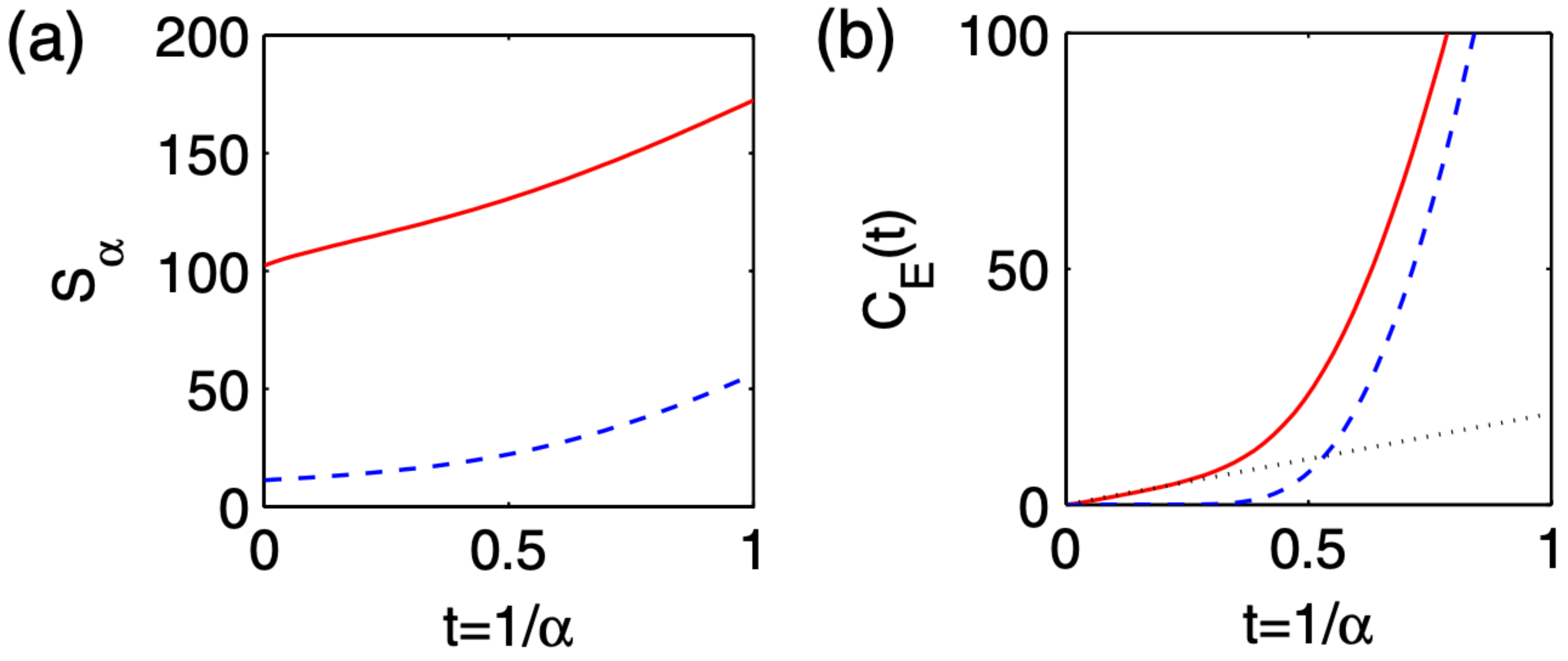
Entanglement Entropy and Entanglement Spectrum

$$S_\alpha = \frac{1}{1-\alpha} \log \mathrm{Tr}\, \rho^\alpha$$

- Renyi entropy reduces to the EE at $\alpha \to 1$

$$C_E(t) = -t \frac{\partial^2}{\partial t^2}[(1-t)S_{1/t}], t = 1/\alpha$$

The capacity of entanglement is the analog of heat capacity C_v in a thermal system.



$t \to 0$, $C_E(t)$ vanishes esponentially for the Abelian phase but linearly for the non-Abelian phase, since the latter has a gapless entanglement spectrum with constant dos.

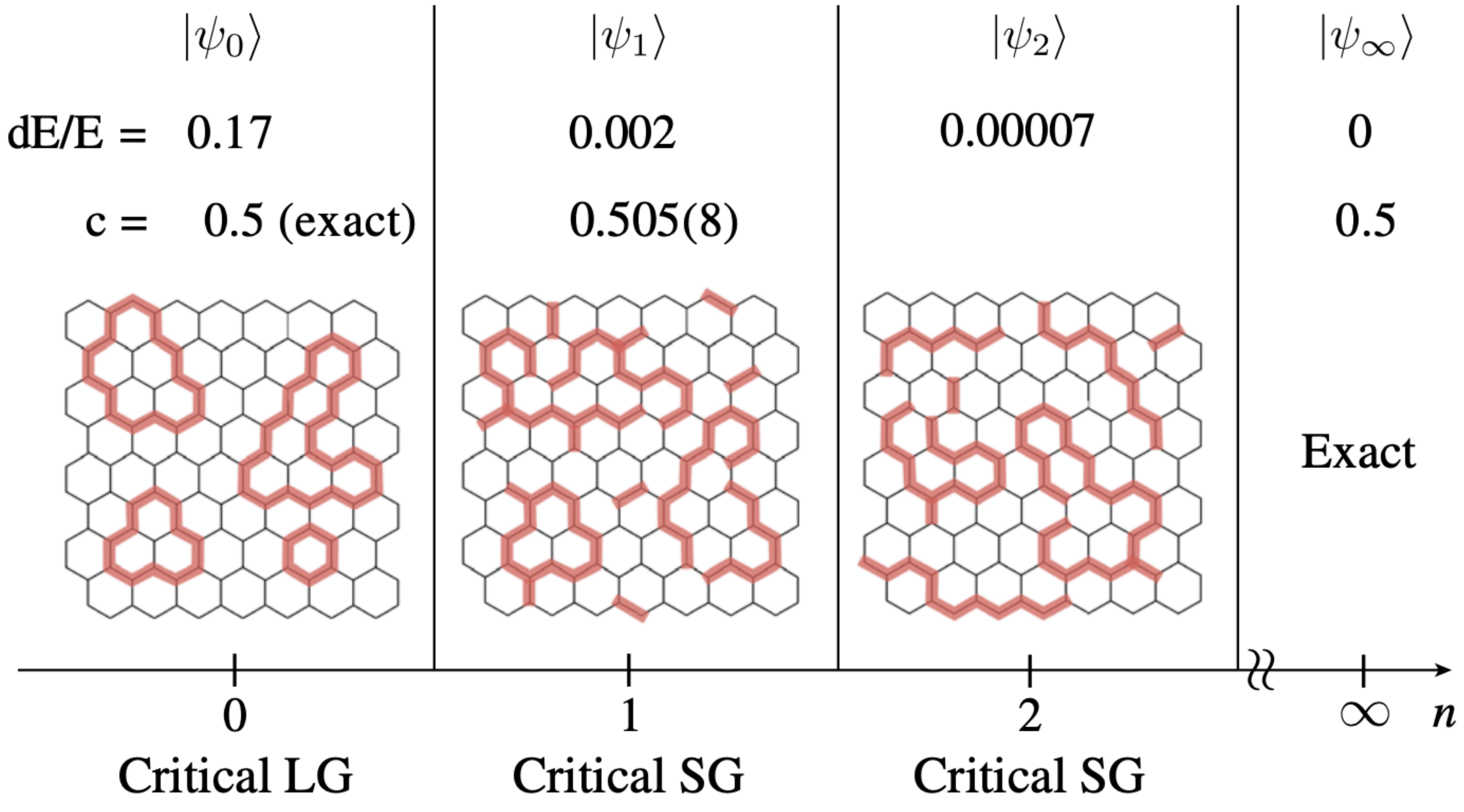
H. Yao and X.L. Qi, PRL 105, 080501 (2010)

Kitaev's Honeycomb Model

Tensor network representation

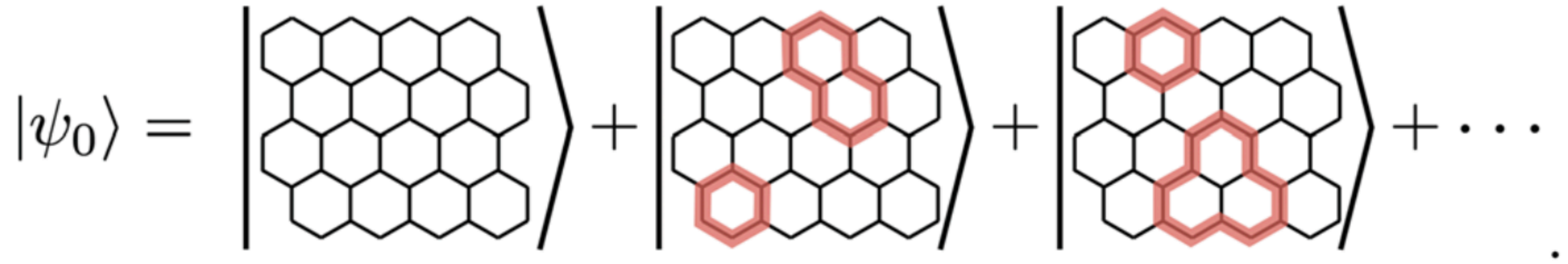
Reveals a hidden string gas structure of the KSL, without Majorana.

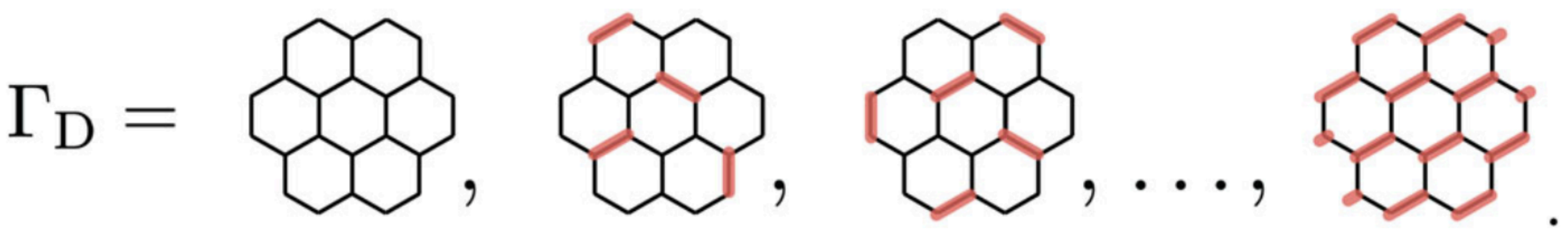
Loop gas, String gas states analogous to the AKLT and RVB state.



$$\hat{Q}_{\mathrm{LG}} = t\mathrm{Tr}\prod_\alpha Q^{ss'}_{i_\alpha j_\alpha k_\alpha}|s\rangle\langle s'|,$$

$$Q^{ss'}_{ijk} = \tau_{ijk}\left[(\hat{\sigma}^x)^{1-i}(\hat{\sigma}^y)^{1-j}(\hat{\sigma}^z)^{1-k}\right]_{ss'},$$





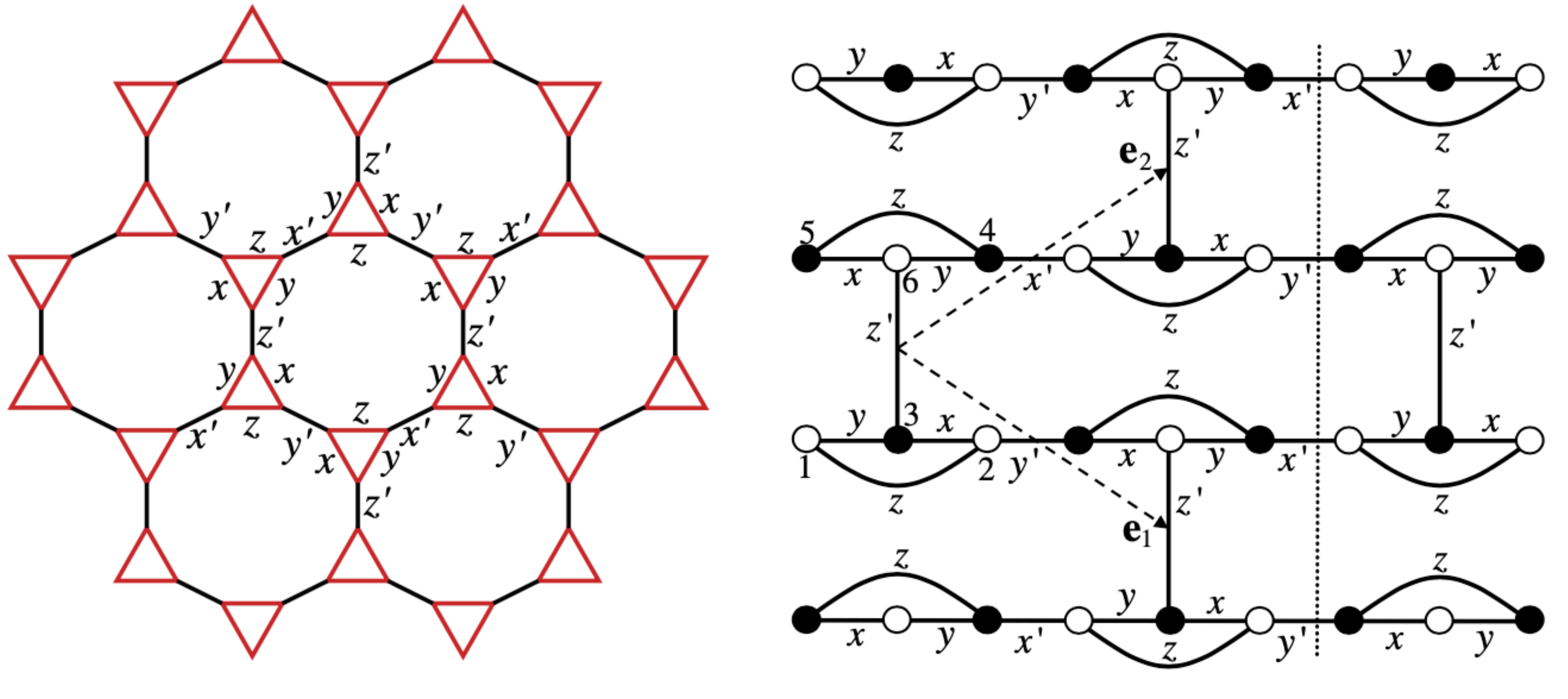
$$\hat{R}_{\mathrm{DG}} = t\mathrm{Tr}\prod_\alpha \hat{R}_{i_\alpha j_\alpha k_\alpha}$$

$$\hat{R}_{ijk} = \zeta_{ijk}(\hat{\sigma}^x)^i(\hat{\sigma}^y)^j(\hat{\sigma}^z)^k.$$

Hyun-Yong Lee, Ryui Kaneko, Tsuyoshi Okubo and Naoki Kawashima, PRL 123, 087203 (2019)

Kitaev's Model on a Decorated Honeycomb Lattice

- Chiral spin liquid as the exact ground state of the Kitaev model on a decorated honeycomb lattice.
- This CSL state spontaneously breaks time reversal symmetry but preserves other symmetries.
- Two topologically distinct CSL's separated by a quantum critical point.



(a) (b)

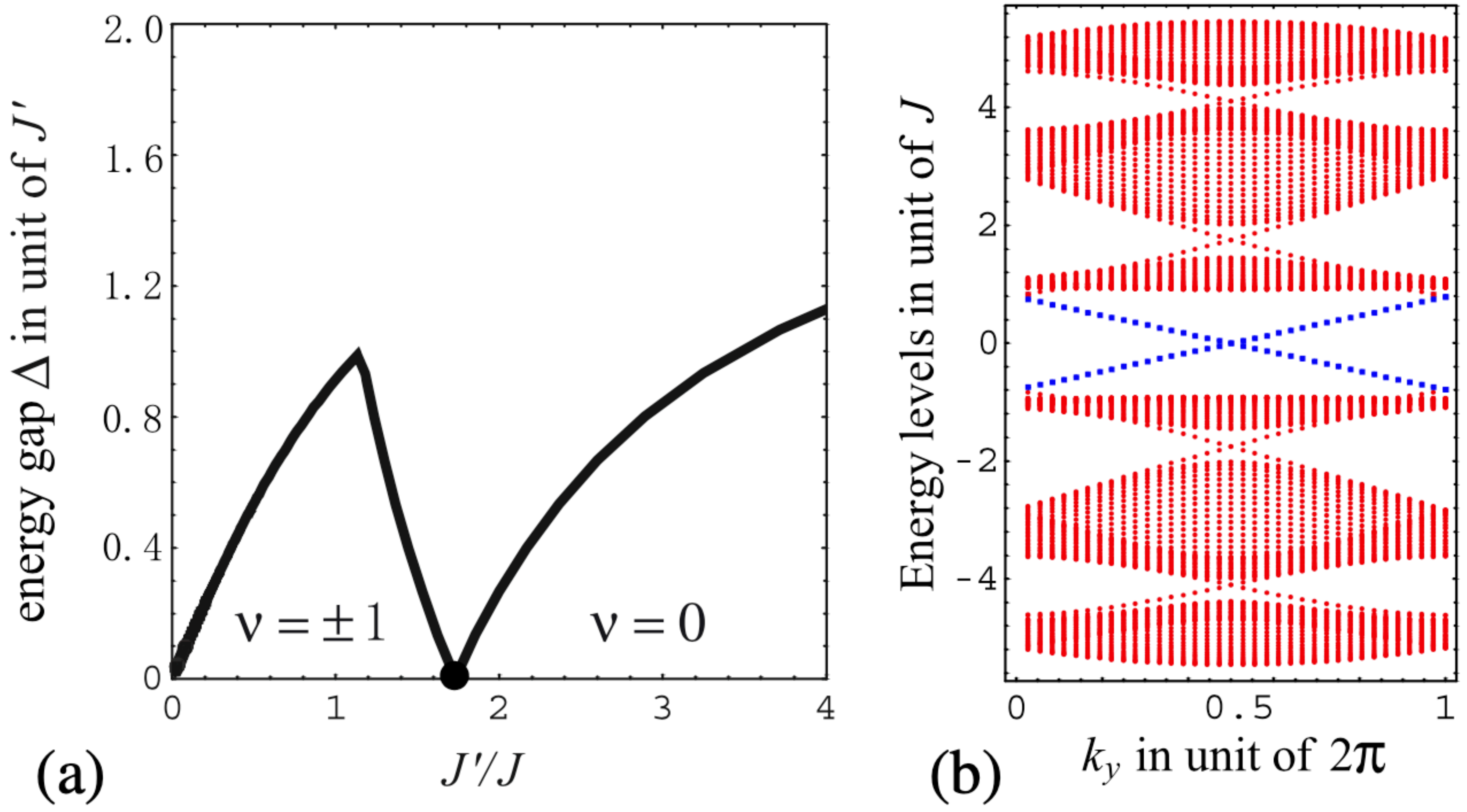
$$\mathcal{H} = \sum_{x-\text{link}} J_x \sigma_i^x \sigma_j^x + \sum_{y-\text{link}} J_y \sigma_i^y \sigma_j^y + \sum_{z-\text{link}} J_z \sigma_i^z \sigma_j^z + \sum_{x'-\text{link}} J'_x \sigma_i^x \sigma_j^x + \sum_{y'-\text{link}} J'_y \sigma_i^y \sigma_j^y + \sum_{z'-\text{link}} J'_z \sigma_i^z \sigma_j^z,$$

H. Yao and S. A. Kivelson, PRL 99, 247203 (2007)

Kitaev's Model on a Decorated Honeycomb Lattice

- Symmetric case $J_\alpha \equiv J, J'_\alpha \equiv J'$.
- Odd Chern number obey non-Abelian statistics.
- $\nu = 0$ phase has Abelian statistics.
- One edge state crossing with zero energy, $\nu = \pm 1$.

为什么ν = 0称为CSL?



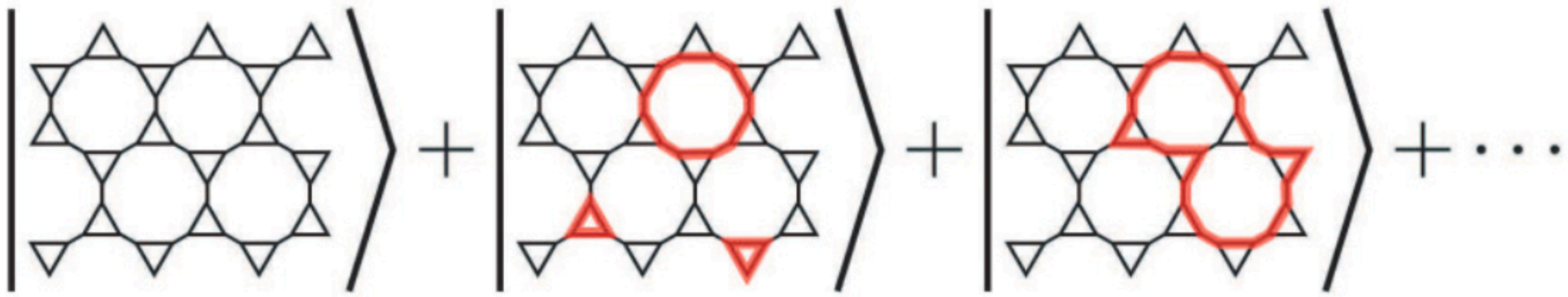
Discussion

- CSL's TRS breaking signatures : zero-field Kerr effect and thermal Hall conductance.
- Low energy effective theory of non-Abelian phase : $SU(2)_2$ Chern-Simons theory.

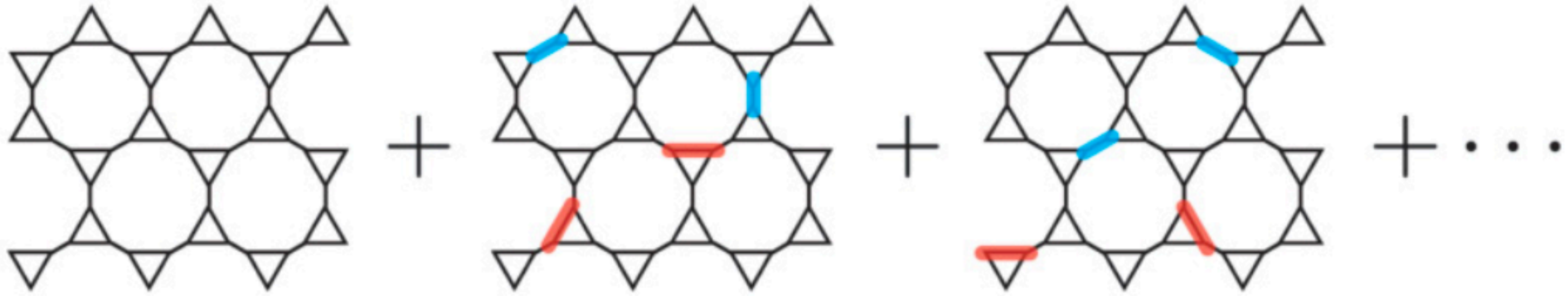
H. Yao and S. A. Kivelson, PRL 99, 247203 (2007)

Kitaev’s Model on a Decorated Honeycomb Lattice

Tensor network representation

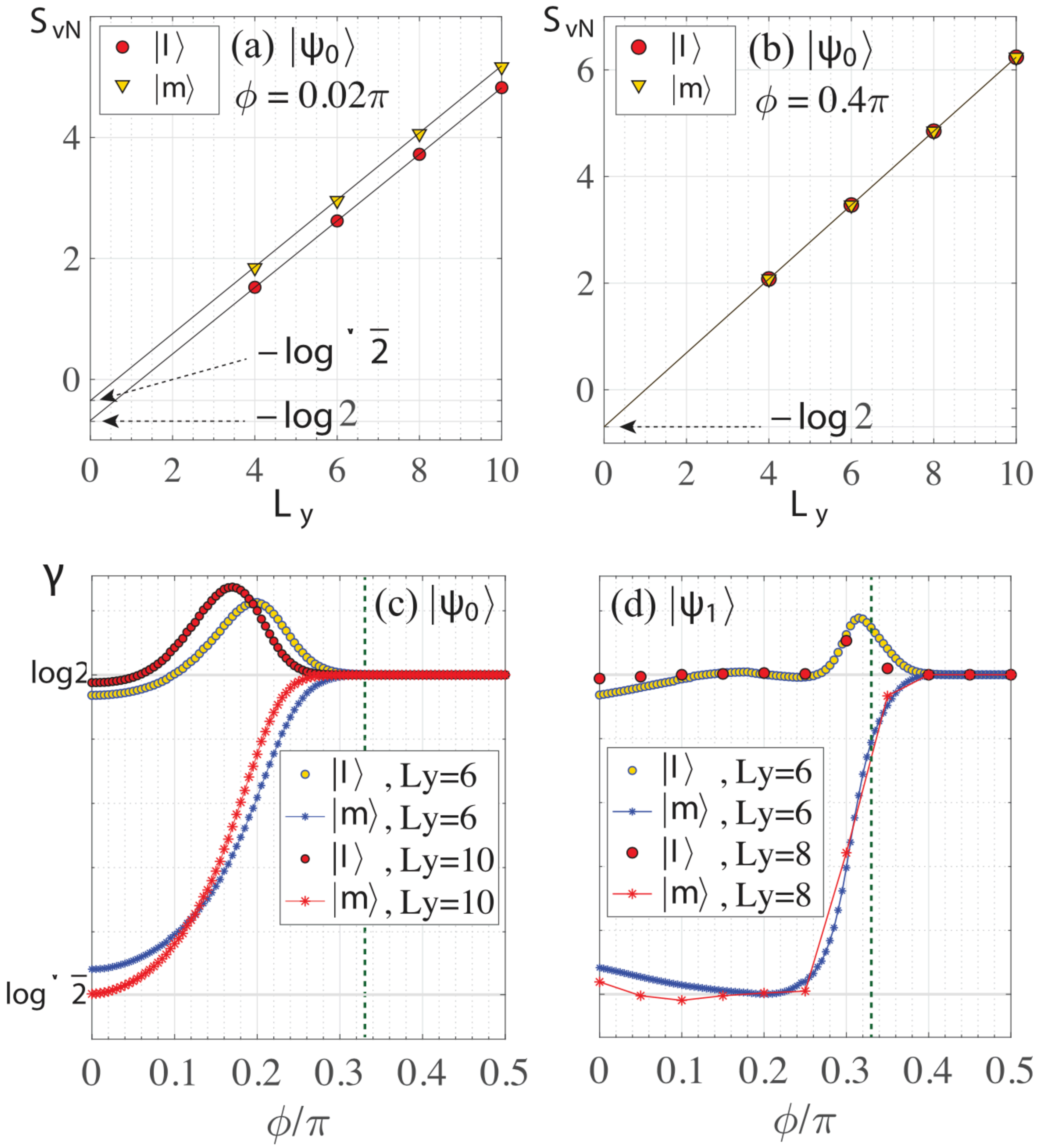
$|\psi_{\mathrm{LG}}\rangle =$ 

$$|\psi_{LG}(\theta)\rangle \equiv \hat{Q}_{LG}\,|\psi(\theta)\rangle$$

$\hat{R}_{\mathrm{DG}} =$ 

$$|\psi_{SG}(c_1, c_2)\rangle = \hat{Q}_{LG}\hat{R}_{DG}(c_1, c_2)\,|\psi_{(111)}\rangle$$

In non-Abelian phase and around the phase-transition point



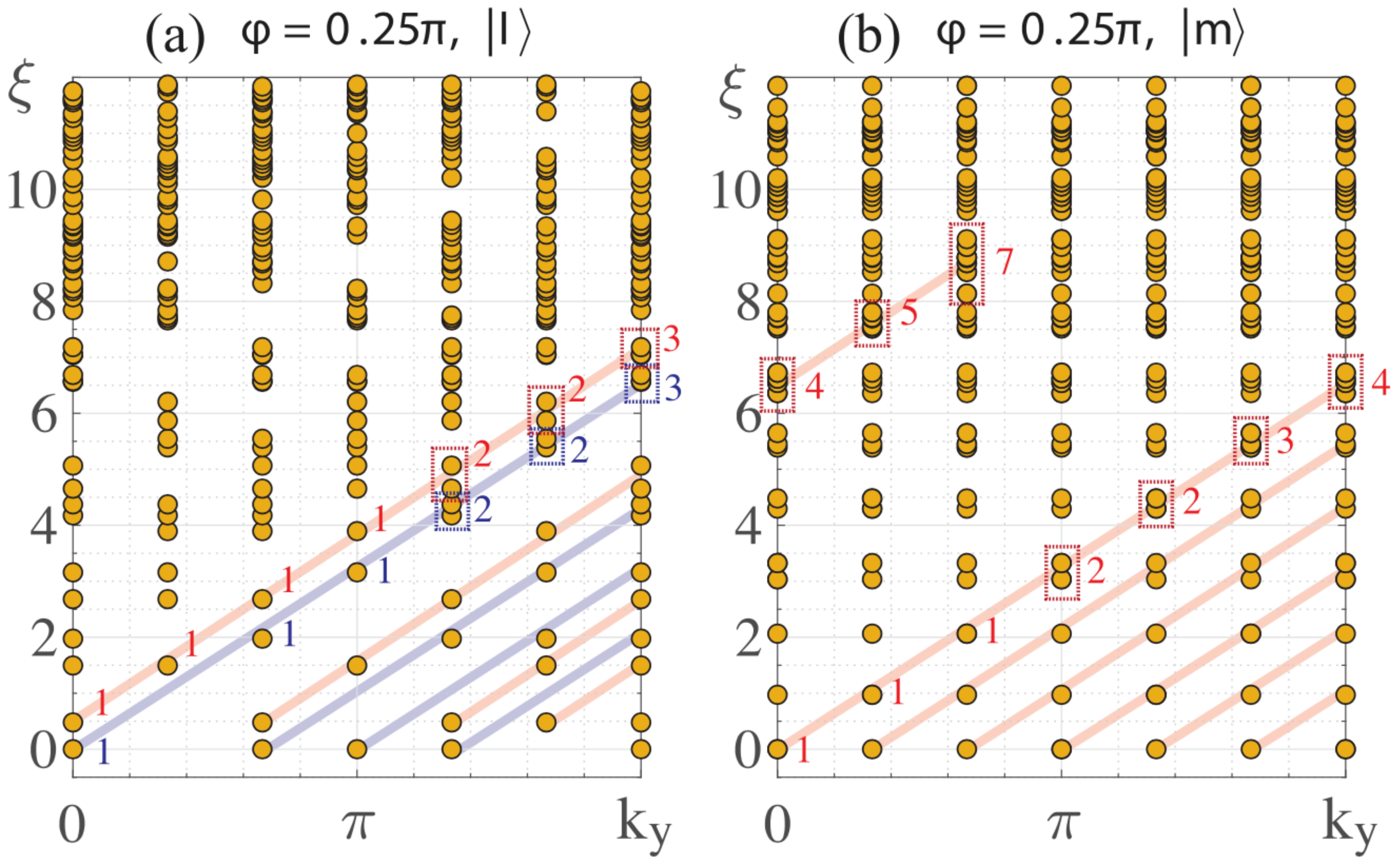
Hyun-Yong Lee, Ryui Kaneko, Tsuyoshi Okubo and Naoki Kawashima, PRB 101, 035140 (2020)

Kitaev's Model on a Decorated Honeycomb Lattice

Tensor network representation

The Abelian and non-Abelian CSL ground states of the KSM are well represented by the LG and SG states.

Identify the chiral edge modes in the non-Abelian phase with the Ising CFT.



Hyun-Yong Lee, Ryui Kaneko, Tsuyoshi Okubo and Naoki Kawashima, PRB 101, 035140 (2020)