ROTATIONAL ENERGY
ANGULAR MOMENTUM

Important Equations

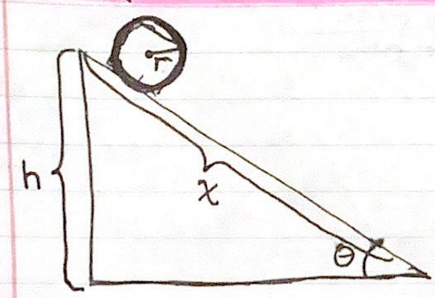
$I = mr^2$ (very general equation) $p = mv$ $\Delta p = m\Delta v$

$L = I\omega$ $\Delta L = I\Delta\omega$ $L = mvr\sin\theta$ $\Delta L = m\Delta v r$

$KE_{Linear} = \frac{1}{2}mv^2$ $KE_{rot} = \frac{1}{2}I\omega^2$ $KE_{rot} = \frac{1}{2}I\left(\frac{v}{r}\right)^2$

$v = r\omega$ $a = r\alpha$ $s = r\theta$ $\Delta p = F\Delta t$

Ramp



Uniform Mass

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + F_f \cdot x$$

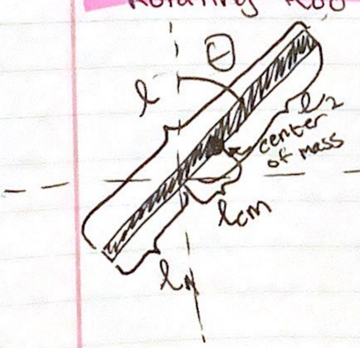
Find inertra from non-uniform Mass

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + F_f x$$

$$mgh = \frac{1}{2}I\cdot\left(\frac{v^2}{r^2}\right) + \frac{1}{2}mv^2 + F_f x$$

$$\frac{2(mgh - \frac{1}{2}mv^2 - F_f x)}{\left(\frac{v^2}{r^2}\right)} = I$$

Rotating Rod



$I = I_{cm} + md^2$

$= \frac{ml^2}{12} + m\cdot l_{cm}^2$

h = Total vertical distance covered by center of mass: $l_{cm}\cdot\cos(\theta) + l_{cm}$

angular Speed when vertical:

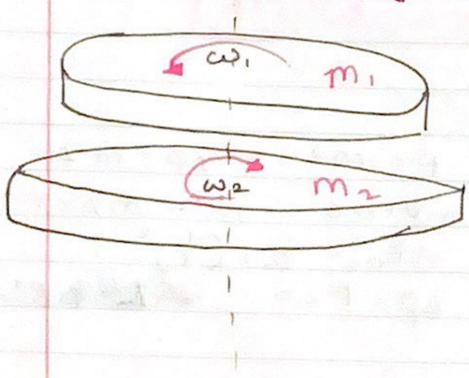
$$mgh = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{2mgh}{I}}$$

tangential velocity at a tip

$$v = r\omega$$

Two disks (stick together)



$$L = I_1\omega_1 + I_2\omega_2$$

$$I_1\omega_1 + I_2\omega_2 = I_1\omega_f + I_2\omega_f$$

$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$p = m_1 v_1 + m_2 v_2$$

$$m_1 v_1 + m_2 v_2 = m_1 v_f + m_2 v_f$$

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Total KE:

Initial KE $= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

Final KE $= \frac{1}{2}v_f(m_1 + m_2) + \frac{1}{2}\omega_f(I_1 + I_2)$