Simple Harmonic Motion

Important Equations

$T = 2\pi\sqrt{\frac{m}{k}}$ $f = 1/T$ $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Delta x(t) = x_{max}\cos\frac{2\pi t}{T}$ $x = x_{max}\cos\omega t$ $v = -x_{max}\sin\omega t$

$v(t) = -v_{max}\sin\left(\frac{2\pi t}{T}\right)$ $a(t) = \frac{-kX_{max}}{m}\cos\frac{2\pi t}{T}$ $a_{max} = \frac{kx_{max}}{m}$

$F \approx -mg\theta$ $s = L\theta$ $\theta = \frac{s}{L}$ $F \approx -\frac{mg}{L}s$ $F = -kx$

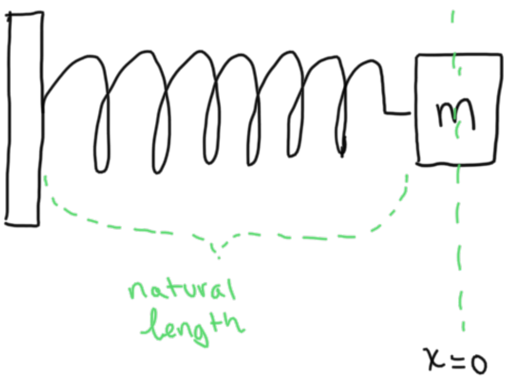
$T = 2\pi\sqrt{\frac{L}{g}}$ $PE = \frac{1}{2}kx^2$ $KE + PE = \text{constant}$

$KE = \frac{1}{2}mL^2\omega^2$ $PE = \frac{1}{2}mgL\theta^2$ $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_{max}^2$

$V_{max} = \sqrt{\frac{k}{m}}\,x_{max}$ $\omega_{max} = \sqrt{\frac{g}{L}}\,\theta_{max}$ $\omega = \frac{2\pi}{T} = 2\pi f$

$V_{max} = x_{max} \cdot \omega$ $a = -x_{max}\omega^2\sin\omega t$ $\omega = \sqrt{\frac{g}{L}}$

Basic Spring Mass



x_{max} = amplitude or max displacement

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{m}{k}}$

use these to solve for unknowns

$F_{\text{spring on mass}} = -k \cdot \Delta x$ ← to find force of spring on mass at a certain point

$V_{max} = \sqrt{\frac{k}{m}} \cdot x_{max}$ ← to find max velocity of the mass

Use energy to find velocity at a point

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_{max}^2$$

$$mv^2 + kx^2 = kx_{max}^2$$

$$v^2 = \frac{kx_{max}^2 - kx^2}{m}$$

$$v = \sqrt{\frac{k(x_{max}^2 - x^2)}{m}}$$

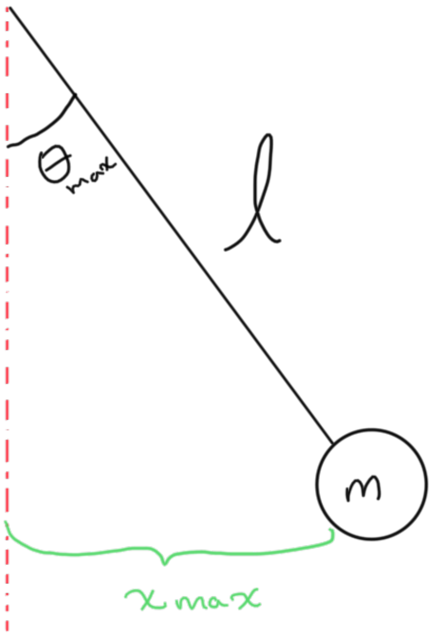
acceleration at a point:

use the $x = x_{max}\cos\frac{2\pi t}{T}$ equation to solve for t, then plug in t to $a(t) = \frac{-kX_{max}}{m}\cos\frac{2\pi t}{T}$ to find accel

—OR—

$$a = \frac{F_{net}}{m} = \frac{kx}{m}$$

Simple Pendulum



x_{max} = max x displacement

$T = 2\pi\sqrt{\frac{\ell}{g}}$ $\quad f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

$\omega_{max} = \sqrt{\frac{g}{l}}\,\theta_{max}$ $\quad \omega = 2\pi f$

$V_{max} = x_{max} \cdot \omega$

$F = -mg\theta$

$a = \frac{F}{m} = -g\theta$

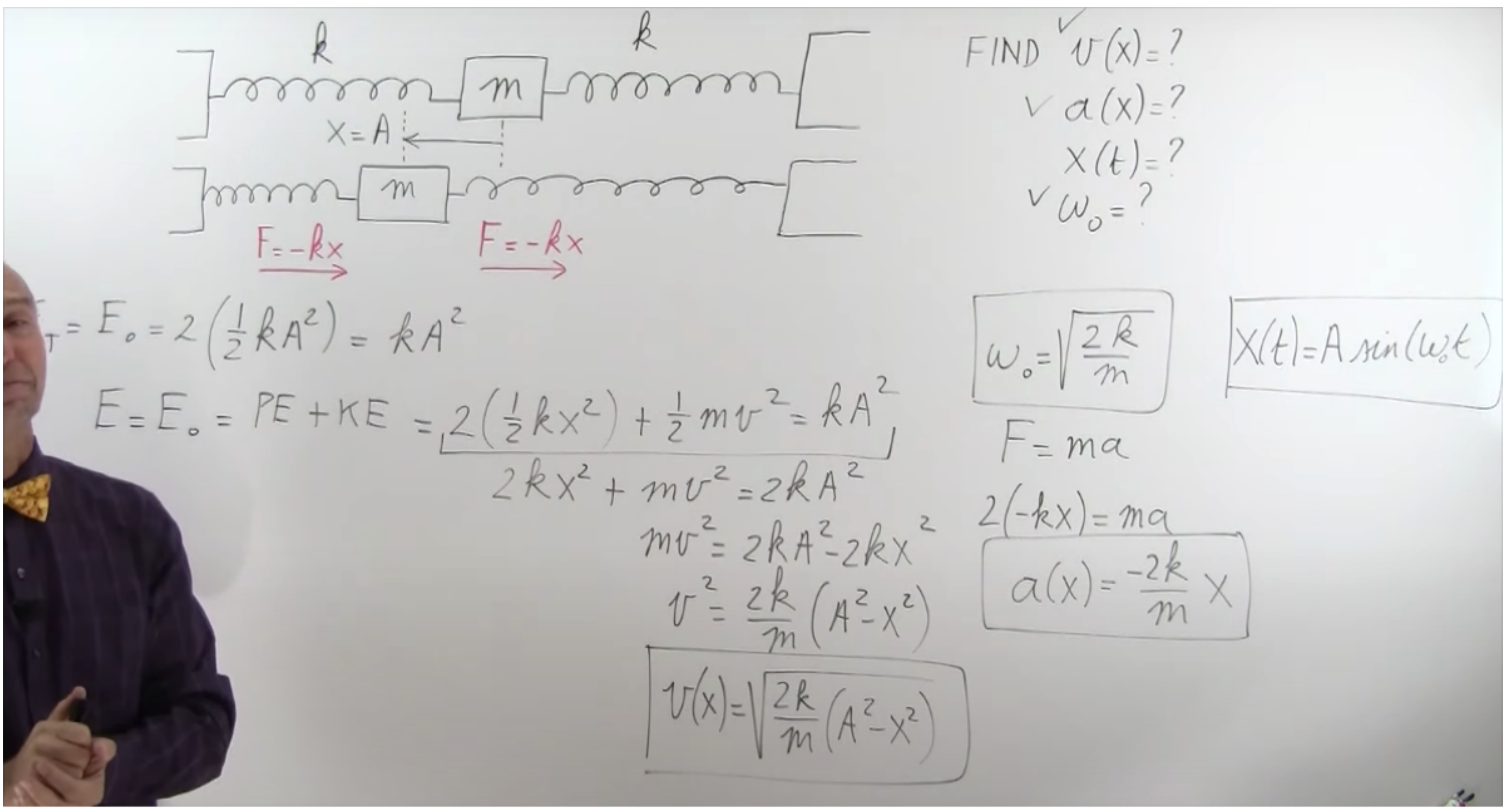
Use energy to find velocity at a point

$$\frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 = \frac{1}{2}mgL\theta_{max}^2$$

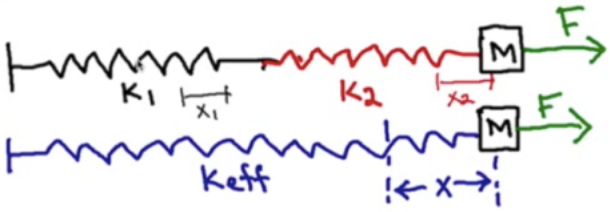
$$L\omega^2 + g\theta^2 = g\theta_{max}^2$$

$$\omega = \sqrt{\frac{g(\theta_{max}^2 - \theta^2)}{L}}$$

Two Springs on Either side



Two springs on one side



$$F = k_2 x_2 = k_1 x_1$$

$$F = k_{eff}\, x$$

$$x = x_1 + x_2$$

$$\frac{F}{k_{eff}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$$

$$w = \sqrt{\frac{k_{eff}}{M}}$$

$$\boxed{w = \sqrt{\frac{k_1 k_2}{M(k_1 + k_2)}}}$$