ROTATIONAL KINEMATICS

Equations

$\omega = \frac{\Delta\theta}{\Delta t}$ $\alpha = \frac{\Delta\omega}{\Delta t}$ $\omega_f^2 = \omega_0^2 + 2\alpha\theta$ $\omega_f = \omega_0 + \alpha t$

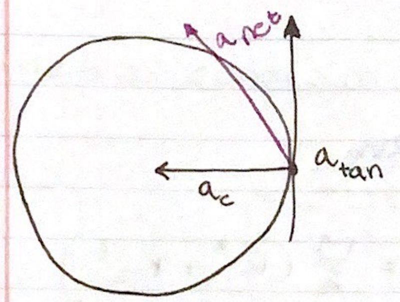
$v = r\omega$ $a = r\alpha$ $a_c = \frac{v^2}{r} = r\omega^2$ $\theta = \omega_f t - \frac{1}{2} a t^2$

$\tau = Fr\sin\theta$
$= mar\sin\theta$
$= m\alpha r^2 \sin\theta$

$\tau_{net} = I\alpha$

$I_{sphere} = \frac{2}{5} mr^2$

NET LINEAR ACCELERATION

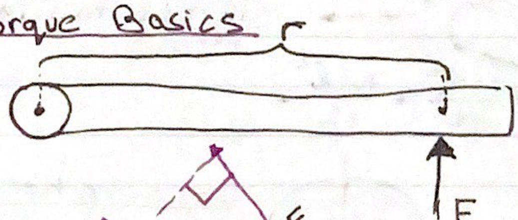


$a_c = \frac{v^2}{r} = r\omega^2$

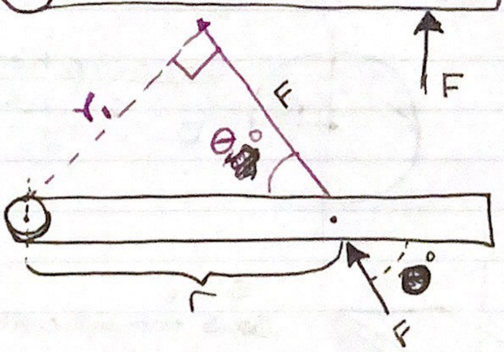
$a_{tan} = \alpha R$

$a_{net} = \sqrt{a_c^2 + a_{tan}^2}$

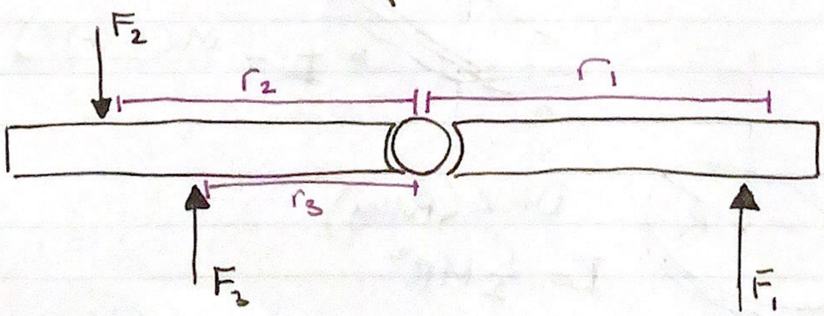
Torque Basics



$\tau = F \cdot r$

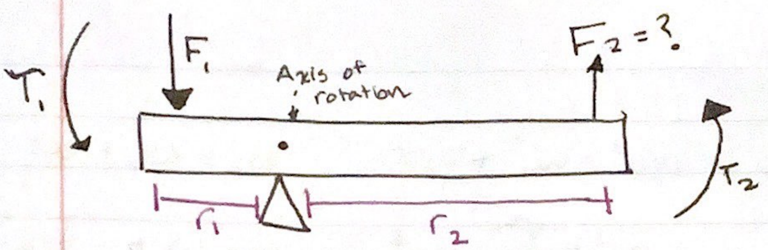


$r_1 = r \cdot \sin\theta$
$\tau = F \cdot r\sin\theta = F \cdot r_1$



$\tau_{net} = \tau_1 + \tau_2 + \tau_3$
$= F_1 r_1 + F_2 r_2 + F_3$

(remember direction



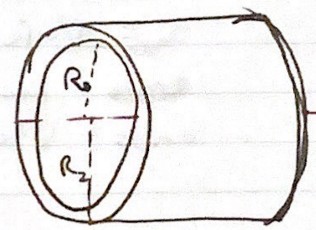
$$\tau_1 = \tau_2$$
$$F_1 r_1 = F_2 r_2$$
$$\tau_2 = \tau_1 = F_1 \cdot r_1$$
$$F_2 = \frac{\tau_2}{r_2}$$

~~Rotational~~ Inertia (Rotational)

Depends on Mass but also how the mass is distributed

Hoop

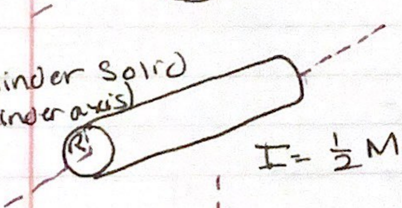
$I = MR^2$



Ring

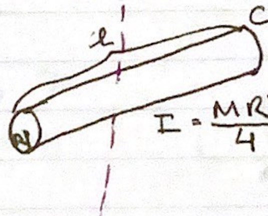
$$I = \frac{M}{2}(R_1^2 + R_2^2)$$

Cylinder Solid (cylinder axis)



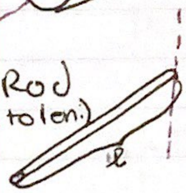
$I = \frac{1}{2}MR^2$

Cylinder Solid (central diameter axis) → (if rod is thin, ignore first term)



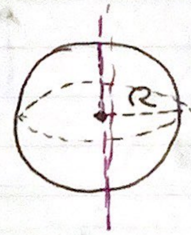
$$I = \frac{MR^2}{4} + \frac{M\ell^2}{12}$$

Thin Rod (axis ⊥ to len.)



$$I = \frac{M\ell^2}{3}$$

Solid Sphere

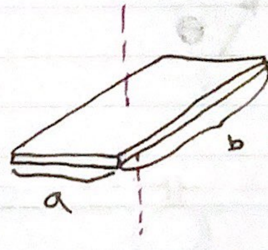


$$I = \frac{2MR^2}{5}$$

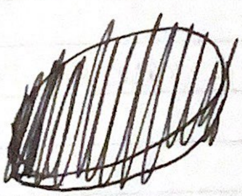
Thin Shell Sphere

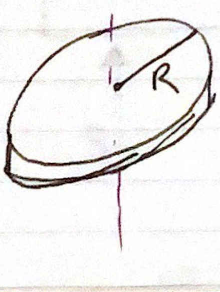
$$I = \frac{2MR^2}{3}$$

Slab (axis through center)



$$I = \frac{M(a^2 + b^2)}{2}$$





Disk (Pulley)

$I = \frac{1}{2}MR^2$

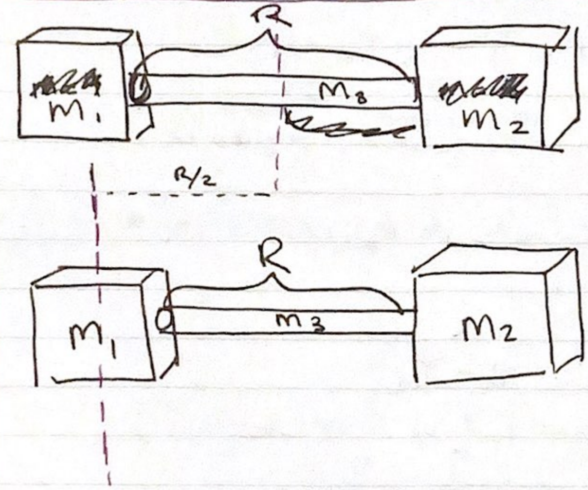
$\tau = I\alpha$ $\qquad$ $I_{(Basic)} = m \cdot r^2$

Parallel Axis Theorem

$$I_{NEW} = I_{CM} + M \cdot d^2$$

new axis — Inertia at center of Mass — distance from center of mass

EXAMPLE INERTIA PROBLEMS



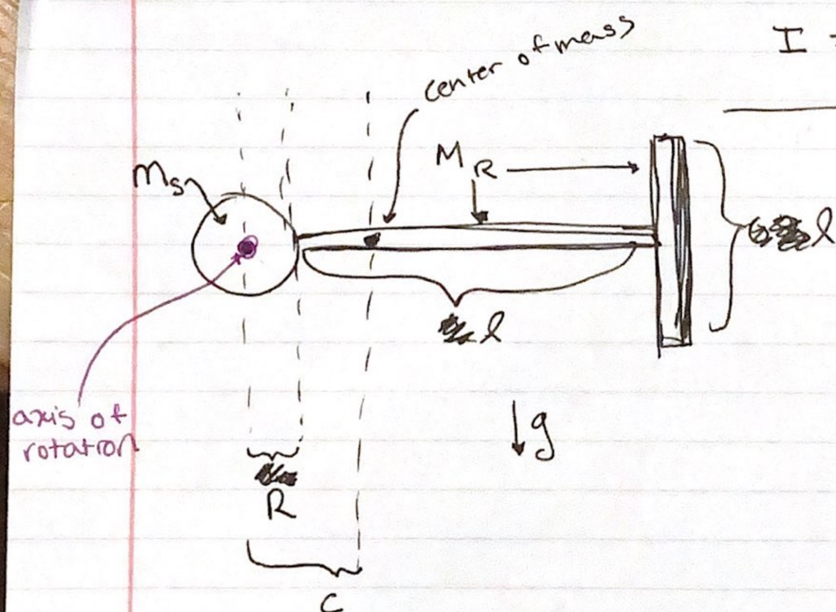
$$I_{net} = M_1\left(\frac{R}{2}\right)^2 + m_2\left(\frac{R}{2}\right)^2 + \frac{M_3R^2}{12}$$

if $m_1 = m_2$ + then center of Rod is center of Mass.

$$I_{net} = M_1 \times 0 + M_2 \times R^2 + \frac{m_3 R^2}{3}$$

OR (using Parallel axis Theorem)

$$I = I_{CM} + M_{total} \cdot \left(R/2\right)^2$$



$$\tau_{net} = (m_R + M_R + M_S) \cdot g \cdot c$$

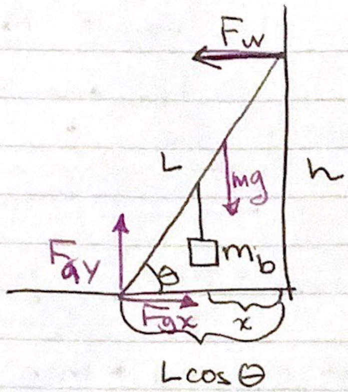
$$I = (I_s + I_{R_1} + I_{R_2})$$

$$= \frac{2}{5} \cdot m_S \cdot R^2 + \frac{M_R + (R+\ell)^2}{3}$$

$$+ \left[\frac{M_R}{12} \cdot (R+\ell)^2 + (R+\ell)^2 \cdot M_R\right.$$

$$\alpha = \frac{\tau_{net}}{I}$$

$$a = c \cdot \alpha$$



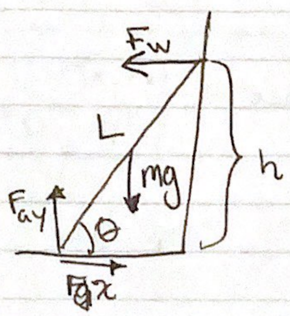
$F_{ay} = F_N$ $\quad$ $F_{ax} = F_s = (mg + m_b g)\cdot\mu$

$F_{ay} = (mg + m_b g)$ $\quad$ $F_{ax} = F_W$

$\sum \tau = \tau_1 + \tau_2 + \tau_3 = 0$

Top $\quad$ bottom by perso $\quad$ botton by ladder

$0 = F_W(h) - mg(L\cos\Theta - x) - mg(L\cos\theta/2)$



$\sum \tau = \tau_1 - \tau_2$

$= F_W(h) - mg\left(\frac{L\cos\Theta}{2}\right)$

Pulley

$-m_2 a + m_2 g = \frac{1}{2} m a$