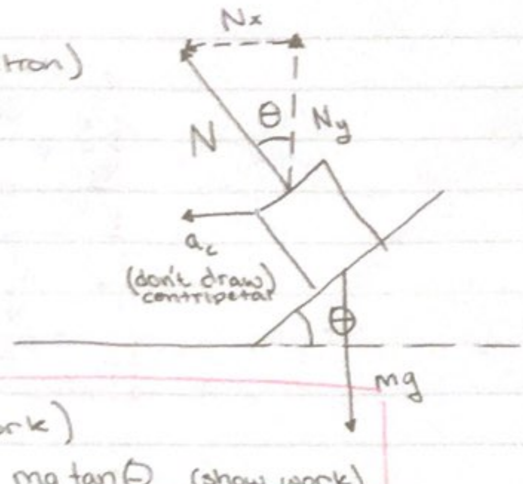
Circular Motion and Gravity

Banked Curve: (no friction)



$$\boxed{\begin{aligned} N_x &= F_c \\ N_y &= mg \\ N_x &= N \cdot \sin\theta \\ N_y &= N \cdot \cos\theta \end{aligned}}$$

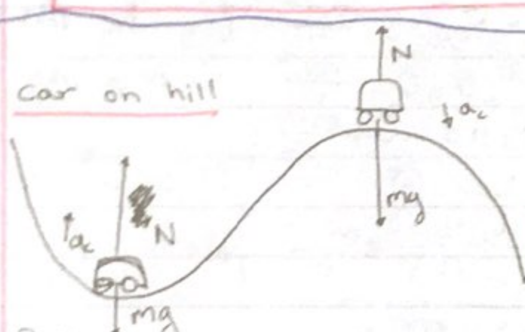
$N = \frac{mg}{\cos\theta}$ (show work)

$F_c = N_x = \frac{mg}{\cos\theta} \cdot \sin\theta = mg\tan\theta$ (show work)

$$a_c = \frac{F_c}{m}$$

$$v = \sqrt{\frac{F_c \cdot r}{m}} = \sqrt{a_c \cdot r}$$

Car on hill



Top

$\Sigma F_y = mg - F_N$

$ma = mg - F_N$

$F_N = mg - ma$

$$\boxed{F_N = mg - m\frac{v^2}{r}}$$

Bottom

$\Sigma F_y = F_N - mg$

$ma = F_N - mg$

$m\frac{v^2}{r} = F_N - mg$

$$\boxed{F_N = \frac{mv^2}{r} + mg}$$

Flies off: $F_N = 0$

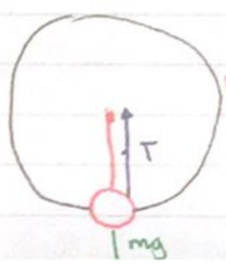
$$mg - m\frac{v^2}{r} = 0$$

Vertical circle:

$$\boxed{\text{speed} = \frac{2\pi r}{\text{Time}}}$$

Top:

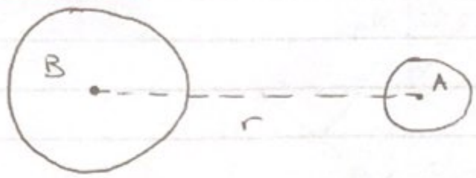
$$\boxed{T = \frac{mv^2}{r} - mg}$$



Bottom:

$$\boxed{T = \frac{mv^2}{r} + mg}$$

Two Planets



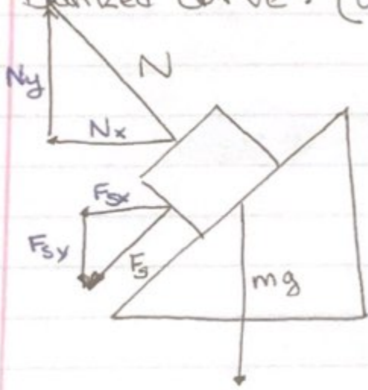
$$\boxed{F_c = G\frac{mM}{r^2} = 6.67\times10^{-11}\cdot\frac{m_B\cdot m_A}{r^2}}$$

$$\boxed{\frac{mv^2}{r} = \frac{GmM}{r^2}}$$

$$\boxed{v = \sqrt{\frac{rF_c}{m}} = \sqrt{\frac{GM}{r}}}$$

$$\boxed{T = \frac{2\pi r}{v}} \qquad \boxed{v = \frac{2\pi r}{T}}$$

Banked Curve: (with friction)



$$\sum F_x = N_x + F_{sx} = \frac{mv^2}{r}$$

$$N\sin\theta + F_s\cos\theta = \frac{mv^2}{r}$$

$$N\sin\theta + N\mu\cos\theta = \frac{mv^2}{r}$$

$$N(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{r}$$

$$\frac{\cancel{m}g(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{\frac{rg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}}$$

$$\sum F_y = N_y - mg - F_{sy}$$

$$N\cos\theta - mg - N\mu_s\sin\theta = 0$$

$$N(\cos\theta - \mu\sin\theta) = mg$$

$$N = \frac{mg}{\cos\theta - \mu\sin\theta}$$

$$\boxed{v = \sqrt{rg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}}$$