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Section - C
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Assignment - 2

Q1 Evaluate $\Delta^4(1-x)(1-2x)(1-3x)(1-4x)$ if $h=1$

A1

$$(1-x)(1-2x)(1-3x)(1-4x)$$
$$= (1+2x^2-3x)(1+12x^2-7x)$$
$$= 24x^4 - 38x^3 + 35x^2 - 10x + 1$$
$$\Delta^4(24x^4) - \Delta^4(38x^3) + \Delta^4(35x^2) - \Delta^4(10x) + \Delta^4(1)$$
$$= 4! \times 24 = 576 \text{ (Ans)}$$

Q2 Evaluate $\Delta^6(1-ax)(1-bx^2)(1-cx^3)$ if $h=1$

A2

$$(1-ax)(1-bx^2)(1-cx^3)$$
$$(1-bx^2-ax+abx^3)(1-cx^3)$$
$$= -abcx^6 + \cdots$$
$$-\Delta^6(-abcx^6) = 720abc$$

Q3 Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

A3 $\Delta = E - 1$ & $\nabla = 1 - E^{-1}$

Taking RHS

$$\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \frac{E-1}{1-E^{-1}} - \frac{1-E^{-1}}{E-1} = \frac{\cancel{E-1} \times E}{\cancel{E-1}} - \frac{\cancel{E-1}}{\cancel{E-1} \times E}$$
$$= E - \frac{1}{E} = (E-1) + (1-E^{-1})$$
$$= \Delta + \nabla = \text{LHS}$$

H/P

Q4 Express the function $3x^4 - 4x^3 + x^2 - x - 9$ in factorial notation.

Ans 4

$$f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$$

1	3	-4	1	1	$-9 = E$
	↓	3	-1	0	
2	3	-1	0	$1 = D$	
	↓	6	10		
3	3	5	$10 = C$		
	↓	9			
	3	$14 = B$			
	$3 = A$				

$$f(x) = 3x^4 + 14x^3 + 10x^2 + x - 9$$

Q5 Find the missing values of following data is given

x	0	1	2	3	4	5
$f(x)$	1	–	5	21	28	35

Ans 5 As the interval size is equally spaced

So, we make Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	1	$y_1 - 1$	$6 - 2y_1$	$3y_1 + 5$	$-25 - 4y_1$	$54 + 5y_1$
1	y_1	$5 - y_1$	$11 + y_1$	$-20 - y_1$	$29 + y_1$	
2	5	16	-9	9		
3	21	7	0			
4	28	7				
5	35					

As there are 5 entries. y_0, y_2, y_3, y_4, y_5 are given, the y f^n can be represented as 4th degree polynomial.

$$\Delta^5 y_0 = 0.$$

$$5y_1 = -54$$

$$\boxed{y_1 = -10.8}$$

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Q6 Find the missing values of given that

x	0	2	4	6	8	10
f(x)	1	-	-	30	38	78

A6

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1	$y_1 - 1$	$y_2 - 2y_1 + 1$	$29 - 3y_2 + 3y_1$	$-81 + 6y_2 - 4y_1$
2	y_1	$y_2 - y_1$	$30 - 2y_2 + y_1$	$-52 + 3y_2 - y_1$	$106 - 4y_2 + y_1$
4	y_2	$30 - y_2$	$-22 + y_2$	$54 - y_2$	
6	30	8	32		
8	38	40			
10	78				

$6y_2 - 4y_1 = 81$

$16y_2 - 4y_1 = 424$

$\quad - \quad + \quad -$

$-10y_2 = -343 \Rightarrow \boxed{y_2 = 34.3}$

$-4y_2 + y_1 = -106$

$y_1 = 4 \times 34.3 - 106$

$y_1 = 137.2 - 106$

$\boxed{y_1 = 31.2}$

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Q7

Construct a forward difference table for the given data below.

x	2	3	4	5	6	7
y	-5	1	12	17	24	56

A7

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
2	-5	6.	5	-11	19	-4
3	1	11	-6	8	15	
4	12	5	2	23		
5	17	7	25			
6	24	32				
7	56					

Q8

If $u_0 = 2$, $u_1 = 13$, $u_2 = 111$, $u_3 = 213$, $u_4 = 400$. Calculate $\Delta^4 u_0$.

A9

We can get $\Delta^4 u_0$ using Forward Difference table.

	u	Δu	$\Delta^2 u$	$\Delta^3 u$	$\Delta^4 u$
u_0	2	11	87	-83	164
u_1	13	98	4	81	
u_2	111	102	85		
u_3	213	187			
u_4	400				

Hence

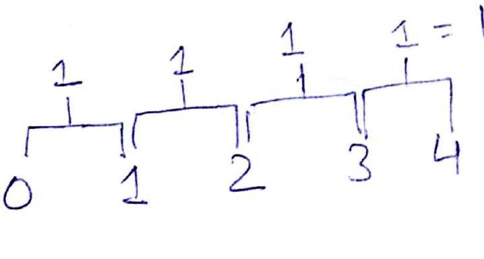
$$\boxed{\Delta^4 u_0 = 164}$$

Q9 Using Newton's Forward Interpolation formula, find the cubic polynomial & hence evaluate $f'(1.5)$ by the following data.

x	0	1	2	3	4
$f(x)$	-1	0	13	50	123

A9 As we can see the interval is equally spaced so we can use Forward Difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	-1	1	12	12	0
1	0	13	24	12	
2	13	37	36		
3	50	73			
4	123				



$1 = h \Rightarrow \boxed{h = 1}$

$$u = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$$

Newton's forward interpolation is given by

$$f(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0) + \dots$$

$$f(x) = -1 + x(1) + \frac{x(x-1)}{2!}(\cancel{12}^{6}) + \frac{x(x-1)(x-2)}{6!} \times \cancel{12}^{2} + 0$$

$$= -1 + x + (x^2 - x)6 + (x^3 - 3x^2 + 2x)2$$

$$= -1 + x + \cancel{6x^2} - 6x + 2x^3 - \cancel{6x^2} + 4x$$

$$= 2x^3 - x - 1$$

$$f'(x) = 6x^2 - 1$$

$$f'(1.5) = 6(1.5)^2 - 1 = 13.50 - 1.0$$

$$\boxed{f'(1.5) = 12.5}$$

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Q10 Given that

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$y=f(x)$	7.989	8.403	8.781	9.129	9.451	9.750	10.031

find $f(1.2)$:

A10 It is given in the table that

$$\boxed{f(1.2) = 8.781}$$

Q11 Evaluate $f(36)$ & $f(37)$ given that: $f(25) = 0.2707$, $f(30) = 0.3027$, $f(35) = 0.3386$ & $f(40) = 0.3794$.

A11 As all the intervals are equally spaced then we can make forward difference table. & $\boxed{h=5}$

x	y	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
25	0.2707	0.0320	0.039	0.082
30	0.3027	0.0359	0.0121	
35	0.3386	0.0480		
40	0.3794			

~~Case 2 :- x = 36~~

~~$u = \frac{36-30}{5} = \frac{6}{5} = 1.2$~~

~~$f(36) = 0.3027 + 1.2 \times 0.0359 + 1.2 \times 0.2 \times 0.0121$~~

~~$= 0.348224$~~

Case 1 :- $x = 37$

$$u = \frac{37-35}{5} = \frac{2}{5} = 0.4$$

$$f(37) = 0.3386 + 0.4 \times 0.0480$$

$$\boxed{f(37) = 0.3578}$$

(38) when $x = 36$

$$u = \frac{36-35}{5} = \frac{1}{5} = 0.2$$

$f(36) = 0.3386 + 0.2 \times 0.0480$

$$\boxed{f(36) = 0.3482}$$

values.

Q12 Evaluate $(46.24)^{1/3}$ given that

x	41	45	49	53
$y = x^{1/3}$	3.4482	3.5569	3.6593	3.7563

A12

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
41	3.4482	0.1087	−0.0063	0.0009
45	3.5569	0.1024	−0.0054	
49	3.6593	0.0970		
53	3.7563			

$x = 46.24$ & $h = 4$

$$u = \frac{46.24 - 45}{4} = \frac{1.24}{4} = 0.31$$

$f((46.24)^{1/3}) = 3.5569 + (0.31 \times .1024) + (0.31 \times (-0.69) \times (-0.0054))$

$f(46.24^{1/3}) = 3.5569 + 0.031744000 + 0.001155060$

$= \underline{3.5897}$ (Ans).

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Q13 Evaluate $\Delta^2 \log 30$ & $\nabla^4 \log 10$ for the following data given below.

x	10	20	30	40	50
$y = \log x$	1	1.3010	1.4771	1.6021	1.6990

A13 As all intervals are equally spaced so we can use forward / backward difference table to calculate the asked values

Forward Difference table :-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
					-0.0508
		0.3010	-0.1249	0.0738	
10	1	0.1761	-0.0511	0.0230	
20	1.3010				
		0.1250	-0.0281		
30	1.4771				
40	1.6021	0.0969			
50	1.6990				

Now $\boxed{\Delta^2 \log 30 = -0.0281}$

$\boxed{\Delta^4 \log 10 = -0.0508}$

$\nabla^4 \log 10 = ?$

Q114 Use Stirling's formula to find y_{28}, given $y_{20} = 8321$, $y_{25} = 7816$, $y_{30} = 7536$, $y_{35} = 7126$ & $y_{40} = 6706$

A114

x	20	25	30	35	40
y	8321	7816	7536	7126	6706

Construct a Forward Difference table,

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-1	20	8321	-505	225	-355	475
0	25	7816	-280	-130	120	
1	30	7536	-410	-10		
2	35	7126	-420			
3	40	6706				

$x = 28$, $x_0 = 25$ & $h = 5$.

$$u = \frac{x - x_0}{h} = \frac{28-25}{5} = \frac{3}{5} = 0.6$$

$$y = y_0 + u\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u^2-1)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right)$$
$$+ \frac{u^2(u^2-1^2)}{4!}\Delta^4 y_{-2} + \ldots$$

$$= 7816 + 0.6\left(\frac{-280-505}{2}\right) + \frac{0.36}{2} \times 225 + \frac{0.6(-0.64)}{6} \times \left(\frac{-355}{2}\right)$$

$$\boxed{y = 7632.36}$$

Q16

Using Lagrange's interpolation formula, find $f(8)$ from the following table

x	4	7	9	10
$f(x)$	14	15	21	27

A16

$x_0 = 4$ $\quad x_1 = 7$ $\quad x_2 = 9$ $\quad x_3 = 10$

$y_0 = 14$ $\quad y_1 = 15$ $\quad y_2 = 21$ $\quad y_3 = 27$

$$P(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$= \frac{(x-7)(x-9)(x-10)}{(4-7)(4-9)(4-10)} \times 14 + \frac{(x-4)(x-9)(x-10)}{(7-4)(7-9)(7-10)} \times 15$$

$$+ \frac{(x-4)(x-7)(x-10)}{(9-4)(9-7)(9-10)} \times 21 + \frac{(x-4)(x-7)\cancel{(x-10)}(x-9)}{(10-4)(10-7)(10-9)} \times 27$$

$$= \frac{(x^2-16x+63)(x-10)}{(-3)(-5)(-6)} \times 14 + \frac{(x^2-13x+36)(x-10)}{(3)(-2)(-3)} \times 15$$

$$+ \frac{(x^2-11x+28)(x-10)}{5 \times 2 \times -1} \times 21 + \frac{(x^2-11x+28)(x-9)}{6 \times 3 \times 1} \times 27$$

$$= \frac{x^3 - 16x^2 + 63x - 10x^2 + 160x - 630}{-90} \times 14$$

$$+ \frac{x^3 - 13x^2 + 36x - 10x^2 + 130x - 360}{18} \times 15$$

$$+ \frac{x^3 - 11x^2 + 28x - 10x^2 + 110x - 280}{-10} \times 21$$

$$+ \frac{x^3 - 11x^2 + 28x - 9x^2 + 99x - 252}{18} \times 27$$

$$= \frac{x^3 - 26x^2 + 223x - 630}{-90} \times 14 + \frac{x^3 - 23x^2 + 166x - 360}{18} \times 15$$

$$+ \frac{x^3 - 21x^2 + 138x - 280}{-10} \times 21 + \frac{x^3 - 20x^2 + 127x - 252}{18} \times 27$$

$$= \cancel{x^3 - 20x^2 + 120x}$$

$$= \frac{14x^3 - 364x^2 + 3122x - 8820}{-90} + \frac{189x^3 - 3969x^2 + 26082x - 52920}{-90}$$

$$+ \frac{210x^3 - 4425x^2 + 29595x - 61020}{90}$$

$$= \frac{1}{90}\Big((-203 + 210)x^3 + (+4333 - 4425)x^2 + (-29204 + 29595)x$$

$$+ (61740 - 61020)\Big)$$

$$= \frac{1}{90}\left(7x^3 - 92x^2 + 391x + 720\right)$$

$$f(8) = \frac{1}{90}\left(3584 - 5888 + 3128 + 720\right)$$

$$= \frac{1}{90}(1544) =$$

$$\boxed{f(8) = 17.1556}$$

A17

$x_0 = 1$, $x_1 = 2$, $x_2 = 5$, $x_3 = 9$

$y_0 = 0$, $y_1 = 0$, $y_2 = -248$, $y_3 = 13104$

$$P(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x_3)$$

$$= \frac{(x-1)(x-2)(x-9)}{(5-1)(5-2)(5-9)} \times -248 + \frac{(x-1)(x-2)(x-5)}{(9-1)(9-2)(9-5)} \times 13104$$

$P(1.2) = 1.54752 + 4.7424$

$$\boxed{P(1.2) = 6.28992}$$

Q18 Construct dividend difference table for the arguments 3, 4, 6, 9 of the function $f(x) = x^3 - 3x + 1$

A18 $\Delta_{x_1} y_0 = \dfrac{y_1 - y_0}{x_1 - x_0}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
3	19	$\frac{53-19}{4-3} = 34$	$\frac{73-34}{6-3} = 13$	$\frac{19-13}{9-3} = 1$
4	53	$\frac{199-53}{6-4} = 73$	$\frac{168-73}{9-4} = 19$	
6	199	$\frac{703-199}{9-6} = 168$		
9	703			

Q19 Using Newton Dividend Difference formula, find $f(-3)$:-

x	-4	-1	1	2	6
$f(x)$	545	233	5	9	333

A19 Newton's Dividend difference formula is given by,

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0$$
$$+ (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \dots + (x-x_{n-1})\Delta^n y_0$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$.	$\Delta^4 y$.
-4	545	-104	$\frac{-114+104}{-1+4} = 3.33$	$\frac{59-3.33}{1+1} = 27.835$	$\frac{3-27.835}{6+4} = -2.4835$
-1	233	-114.	$\frac{4+114}{2} = 59$	$\frac{77-59}{2+4} = 3$.	
1	5	4	$\frac{81-4}{2-1} = 77$		
2	9	81			
6	333				

Using formula,

$$f(x) = 545 + (x+4)(-104) + (x+4)(x+1)\,3.33 + (x+4)(x+1)(x-1)\,27.835$$
$$+ (x+4)(x+1)(x-1)(x-2)(-2.4835).$$

$$f(-3) = 545 - 104 - 6.66 + 219 + 99.34$$

$$\boxed{f(-3) = 752.68}$$

Q20 Evaluate $f(7)$ by using Newton's dividend difference formula for the following data.

x	3	6	10	15	16
$f(x)$	12	92	142	436	502

A20

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
3	12	26.67	-4.723	4.07	-0.3780
6	92	12.5	11.575	-0.8445	
10	142	58.8	1.44		
15	436	66			
16	502				

$f(x)$ - Newton's Dividend Difference formula is given by,

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 y_0$$

$$f(x) = 12 + (x-3)26.67 + (x-3)(x-6)(-4.723) + (x-3)(x-6)(x-10) \times 4.07 + (x-3)(x-6)(x-10)(x-15)(-0.3780)$$

$$f(7) = 12 \times 4 \times 26.67 + 4 \times 1 \times -4.723 + 4 \times 1 \times -3 \times 4.07 + 4 \times 1 \times -3 \times -8 \times -0.3780$$

$$f(7) = 12 + 106.68 - 18.892 - 48.84 - 36.288$$

$$f(7) = 118.68 - 104.02$$

$$\boxed{f(7) = 14.66}$$