Lecture 2: Newton's Laws

This lecture contains material that you are already familiar with. It merely arranges the concepts in a certain way for recapitulation, clarity and continuity.

1 The First Law

We introduced coordinate systems (CS) in the last lecture. In considering the transformations, we did not include coordinate systems which are in motion relative to each other. The interesting question is, of course, if there is a coordinate system, or a class of coordinate systems which is/are preferred over all others which are moving with respect to it.

The resolution to this question begins with observation and insight. The observation refers to (i) concept of smooth surfaces, and (ii) idea of impulsive forces. The insight is in (i) the intuitive notion that physical forces have their origin in the sorrounding matter, and that they get weaker as they move away.

First we accept that it ispossible to know, beforehand, whether forces are acting on a body or not, indeendent of the state of motion of the body. Tjis goes back to the experience that we do not have to look out to know if a force is acting on us. Examples: A weight pressing, a muscle pulling, the car in which you are sitting is accelerating (say turning). In all these cases it is not necessary for you to look out to find whether or not you are acclerating.

Conversely, one knows – by experience, that if an object is accelerating with respect to you, the relative motion is caused by a force acting on you or the other body (could be both). You have relative accleration with respect to a train which is gaining speed (you are on the platform), but you correctly conclude that the force is acting on the train and not on you.

I have been mentioning acceleration and not velocity. The reason for that comes from experience with smooth surfaces and impulsive forces. If a surface is sufficiently smooth, and you apply an impulsive force on a body on that surface, it suffers an instantaneous accleration. Thereafter, it moves with uniform velocity – though no force is acting. Ther eare other observational evidences. A person in boat moving in a calm lake with uniform velocity comcludes that teh person at the bank is moving with equal and opposite velocity. That is, if one did not look out, there would be no emans of knowing the state of ones motion (Aryabhatta). And of course the motion of the tran in the enxt platform. You have to look at pillars and shops to confirm that you are actually at rest.

These observations, coupled with insight, led to the first law: The Galilean law of inertia. We identify a special set of coordinate systems, all with uniform motion with respect to each other. These are characterised by the common property that a body which is free does not accelerate in *any* of the coordinate systems. This set is given a special name: The class of inertial frames.

The first law is essentially an assertion of the existence of inertial frames (IF). But given the fact that bodies act on each otehr without necessarily being in contact (all forces are not contact forces), how do we ensure that no force is acting on a body? The answer is in the experience that force diminsihes with distance. The law presumes that we can have CS which, to a sufficient degree of approximation, and over sufficiently large regions in space and time intervals, behave as IF. As an example, we may refer to frames fixed to te Earth, the centre of mass of the Eart-Sun system, the centre of mass of the solar system, that of our galaxy, and so on. In this chain, a CS is closer to the ideal IF than its preceding CS.

This has been dramatised by the famous experiment of Galileo which I discussed in the class.

2 The Second Law

The second law quantifies the action of a force mathematically. This quantification is applicable only in IF. In other words, the second law does not stand independent of te first.

2.1 Momentum

Two fundamental concepts required are (i) inertia or mass, and (ii) momentum. Experience tells us that as the 'quantity' of matter increases its response to applied forces decreases. And forces must, necessarily, change the velocity. The first definition that we have is of momentum.
Definition The momentum of a body is defined by

$$\vec{p} = m(v)\vec{v} \tag{1}$$

where $m(V)$ is the inertia, or the mass of the body. Note that it is allowed to depend on the speed but not on the velocity of the body. This follows from isotropy of space.
The Second Law: We are now in a position to state the second law. Let $\vec{F}_{app}$ be the force applied on a particle. Then, in any inertial farme, the particle responds to the force by changing its momentum:

$$\frac{d\vec{p}}{dt} = \vec{F}_{app} \tag{2}$$

Note that the RHS is the cause and the LHS, effect.

In Newtonan mechanics it is assumed that the inertia (mass) is indepndent of the state of motion of the body. In that case, the law assumes the form

$$m\frac{d\vec{v}}{dt} = \vec{F}_{app} \tag{3}$$

This equation is meaningful only if $m \neq 0$.

2.2 Some examples of applied forces

You are all familiar with purely position dependent forces

1. Hooke's Law:$\vec{F} = -k(\vec{r} - \vec{R})$ where $\vec{R}$ is the equilibrium position

2. Gravitational Law: $\vec{F} = -\frac{GMm}{r^2}\hat{r}$ where $\hat{r}$ is the unit vector directing from tha mass M towards the mass m. The force is acting on m

3. Coulomb law, a similar version with charges

4. An electric dipole acting on a charge q:

$$\vec{F} = q\vec{E} \equiv q\frac{3(\hat{r}.\vec{d})\vec{r} - \vec{d}}{r^3}$$

where $\vec{d}$ is the electric dipole moment and $\vec{r}$ is the radius vector from the dipole to the test charge.

3 Principle of relativity

3.1 Euclidean transformations

The forces listed above come with different strengths and different ranges, distance dependence. The first one is valid for only small displacements while gravitational law is universal, and has no such restrictions. Yet, they all share one common characterestic: That they transform *exactly* in the same way as the RHS of the law does, namely, accelerations, with respect to the Eucldean transformations – translations and rotations. When there is such an agreement, we say that the equations are covariant. The LHS and the RHS behave in the same way. Physically this means that all IF, related to each other by Euclidean transformations are equivalent. There is no way to distinguish one from the other physically.

3.1.1 Equivalence of IF

We may ask, does the principle of relativity extend to all inertial frames even when they are in relative uniform motion? The examples given above answer the question in the affirmative in the newtonian realm. That is, if we agree that mass, distances and time intervals do *not* change from one IF to another. In fact, the invariant character of these quantities leads to the Galilean transformations which you are all familiar with.

What would be the most general Galilean transformation? Remember that given an IF, w ecan construct an infinite number of coordinate systems (we consider only RCC) by Euclidean transformations. Over and above that, we can perform pure velocity transformations, technically called boosts.

Thus, if we denote by X, the column vector consisting of the ordered entries $(t, \vec{r})$, the effect of a Galilean transformation on X would be written, in some order, say as

$$X' = B(\vec{V}) R_{\hat{n}}(\theta) T(\vec{A}) T(\tau) X$$

which indicates successive operations, time translation $t' = t + \tau$, space translation $\vec{r}\,' = \vec{r} + \vec{A}$, rotation by an angle θ about an axis $\hat{n}$ and, finally, a velocity transformation to a new IF by velocity $\vec{V}$. The order is not sancrosanct, but the final answer depends on the order which you employ.

- Write each of the above transformations as Matrices which act on the column vector X.

- Convince yourself that the total number of parameters is 10.

One should not hasten to conclude that everyday phenomena automatically favour relativity under Galilean transformations. Consider the equation for the speed of sound in air: $c = \frac{\gamma k T}{\rho}$. You know the meaning of the symbols from your class 12. In a moving frame, $c \to c \pm V$, but the formula has only constants which do not change from one IF to another. That means that the IF in which speed is indeed c is favoured. The principle of relativity is apparently violated.

However, it can be resurrected if we demand that under Galilean transformations, air also acquires a relative velocity withe respect to new frame.

The challenge to Galilean transformations would come from velocity dependent forces. We shall not worry about them until we study special relativity.