PYL127 -2020

Problem Set 3 – lagrangian and symmetries

1. Construct the lagrangian for a conical pendulum in a suitable coordinate system. Determine the equation(s) of motion and identify the conserved quantities.

2. A simple harmonic oscillator in two dimensions($D = 2$) experiences a restoring force $\vec{F} = -k\vec{r}$. Construct the lagrangian in cartesian, polar and parabolic coordinates.

3. Repeat the exercise in $D = 3$, in cartesian, cylindrical and spherical polar coordinates.

4. A simple pendulum is hung from a pivot which executes a circular motion in the horizontal plane. Identify the generalised coordinates, set up the lagrangian, and obtain the equations of motion.

5. A particle is free to move on a rod of length L, one end of which is fixed to the ground. The rod is otherwise free. Again, get the generalised coordinates, set up the lagrangian and the equations of motion.

6. Two particles interact with each other with the following potentials ($r = \vec{r}_1 - \vec{r}_2$)

   - $V(\vec{r}_1, \vec{r}_2) = \frac{\vec{r}\cdot\hat{n}}{r^3}$
   - $V(\vec{r}_1, \vec{r}_2) = V(r)$
   - $V(\vec{r}_1, \vec{r}_2) = \frac{\hat{r}_1\cdot\hat{r}_2}{r^3}$
   - $V(\vec{r}_1, \vec{r}_2) = V(x, y, z_1, z_2)$

   In each of the cases, identify the conserved quantities (through ignorable coordinates)

7. Consider a test particle moving in the gravitational field of another mass with uniform mass density distributed over (i) a sphere, (ii) a spheroid and (iii) an ellipsoid. They are far apart. Deduce the force acting on the particle by keeping the first two nonvainishing expressions in the multipole expansion of the potential. Set up the lagrangian and identify the conserved quantities.

8. *Reading exercise*: Verify that apart from angular momentum, Runge-Lens vector is also conserved in an $\frac{1}{r}$ potential.

9. Verify explicitly that if the lagrangian $L \propto v^n; n > 2$, galilean invariance will be lost.

10. Consider two polar coordinates in a plane where the two origins do not coincide. How are their lagrangians related? Obtain the solution by explicit construction.