PYL127 -2020

Problem Set 5
Central forces and small oscillations

1 Central Forces

1. Consider the linealy confining potential $V(r) = V_0 r$. Set up the lagrangian and obtain the equations of motion. Identify all the conserved quantities and set up the equation for $r(t)$.

2. Let $V(r) = -\lambda \frac{1}{r^3}$ be an attractive potential. Sketch the effective potential carefully and identify the equilibrium position. Argue, again carefully, if it is stable or unstable.

3. Let a test mass be in a parabolic orbit in a gravitational potential produced by another point mass. Exlicitly determine $r(t), \phi(t)$ by integration.
   hint: You may also use parabolic coordinates

4. Let the Sun have a small ellipticity in its mass distribution. That is, assume there is a uniform mass density distributed within the spheroid given by the equation

$$\frac{x^2 + y^2}{R^2} + \frac{z^2}{R^2 + c^2} = 1; \quad c^2 \ll R^2$$

   Obtain the correction for the gravitational potential in the leading order. Identify all the symmetries. And set up equations which give corrections to the usual orbits. Assume that the orbit lies outside the mass distribution.

5. Can this potential admit radial motion?

2 Oscillations

1. Consider a mass m connected to two springs on either side which are, in turn rigidly clamped to walls. Suppse the mass executed transverse vibrations. Determine its frequency.

2. Extend the argument to a system of N spring mass systems (N masses and $N + 1$ springs.

3. You are given a spring mass system (two masses and three springs) all in a straight line. By looking at the eigenfrequencies how do you distinguish whether the oscillations are longitudinal or transverse?

4. Consdier a chain of N simple pendula where each is connected to the bob of the prevous one. If there are small displacements of all the bobs in a given plane, set up the lagrangian and obtain equations of motion. *Hint*: $N = 2$ is dicussed in books.

5. Consider the special cases $N = 2, 3$. Obtain all the normal modes and hence obtain the most general solution for each of the pendula.

6. Consider a aquare mesh of two dimensional springs, all with the same spring constant and all masses the same. Extend the results obtained in the class and hence

   (a) Set up the lagrangian

   (b) Set up the eigenvalue equation for the normal modes. As a warm up, you first set up the eigenvalue equation for N spring amss systems in a linear chain

   (c) Take the continuum limit and derive the corresponding field equation.