Special Relativity
Electrodynamics and Michelson Morley Experiment

1 Preliminaries

We have so far studied strictly mechanical phenomena – gravitation and other newtonian potentials, coupled oscillators and their continuum limit, all within the framework of what we shall call Newtonian Relativity. By abuse of language it is usually called non relativistic mechanics. Recall that the phrase principle of relativity signifies that all inertial frames (IF) are equivalent to each other. Only relative velocities amongst them matter. In particular, an observer in an IF can only detect her velocity relative to other IF.

The transformation formulae relating one IF to another requires physical inputs. Newtonian Mechanics employs the observation, coming from mechanical phenomena, that (i) distances, (ii) clock rates, and (iii) mass are invariant, that is, remain the same in all IF. This leads to Galilean transformations which relate kinematical variables in different IF.

Two points need our attention here. Since the velocities add algebraically between any two IF, it follows that there is no upper limit on the relative velocty between two given IF. Correspondingly, the maximum signal velocity – that which conveys information of the interaction, is also not constrained to be finite. Another important consequence is that acclerations do not change (apart from possible rotations) under Galiliean transformations. This would be compatible Newton's second law if the forces also behave similarly. In particular, a velocity dependent *external* force would be inconsistent.

2 Electrodynamics and Galilean Transformations

Mechanical phenomena form a small subset of physical phenomena. The other important set of interactions are electromagnetic (EMI). They have the novel feature that forces can be velocity dependent. EMI also include optics as a special case. In fact EM theory is the culmination of uniification of three seemingly different phenomena: electric, magnetic and optical. The set of Maxwell's equations which describes all of them is the first example of what we would call a unified field theory.

2.1 Maxwell's equations (ME)

All of electrodynamic phenomena may be summarised by ME, together with the expression for the Lorentz force. In SI units, which w eshall quickly abandon, they are given by

$$\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\
\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \partial_t \vec{E} &= \mu_0 \vec{J} \\
\vec{\nabla} \cdot \vec{B} &= 0 \\
\vec{\nabla} \times \vec{E} + \partial_t \vec{B} &= 0 \qquad (1)
\end{aligned}$$

These equations tell us how the sources – charges and currents produce the fields. They are to be completed by describing the action of fields on the test charges and currents, which is given by the Lorentz force (LF) expressions

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad (2)$$

or, equivalently in terms of densities,

$$\frac{d\vec{\pi}}{dt} = \rho \vec{E} + \vec{J} \times \vec{B} \qquad (3)$$

where $\vec{\pi}$ is the momentum density (momentum per unit volume)of the test charge and current distributions. It is remarkable that Eqs 1-3 describe all known elecrodynamic phenomena, be it classical or quantum mechanical (by a suitable reinterpretation).

It behoves us to ask if electrodynamical phenomena are compatible with GT. More precisely, we must examine whether ME and LF are covariant under Lorentz transformations. This exercise becomes particularly simple and illuminating if we consider the fields in the absence of sources. As you are aware, this leads to the wave equations

$$\Box \vec{E} = 0; \ \ \Box \vec{B} = 0 \qquad (4)$$

where the symbol $\Box$ is a differential operator, the so called D'Alembertian, given by

$$\Box \equiv \vec{\nabla}^2 - \frac{1}{c^2} \partial_t^2; \ c^2 = \frac{1}{\epsilon_0 \mu_0} \qquad (5)$$

This equation is particularly significant for several reasons. In the class, I showed you that no electromagnetic phenomenon can determine the parameters ϵ_0, μ_0 independently. Only their product may be determined experimentally. Per se, therefore, it is not guaranteed that vacuum permittivity and permeability have independent physical significance. Eq 4, hoewever, raises the hope that they probebly have. That is because, it is essentially a wave equation with the speed of the wave being given by $c = \frac{1}{\sqrt{\epsilon_0 \mu_o}}$ which may be determined in the lab. Interestingly, its value agrees with the value obtained for the speed of light in free space by using optical techniques (and indirect observations involving planetary motion: Römer's experiment). That it is not a coincidence follows from the beautiful experiements of Hertz and J C Bose who independently produced EM waves in the lab by employing microwaves.

3 The aether hypothesis

Let us pause to recall that waves, as we have understood from our study of coupled oscillators and physical observations, require a medium for production and propagation. Its speed is a property of the medium. In the case which we discussed in class, its speed is given by

$$c = \sqrt{\frac{\sigma}{\rho}}; Z = \sqrt{\sigma\rho} \tag{6}$$

in terms of the spring tension and the mass per unit length, and Z is its impedance. Let us compare these expressions with the corresponding ones for EM wave:

$$c = \sqrt{\frac{1}{\epsilon_0\mu_0}};\ Z = \sqrt{\frac{\mu_0}{\epsilon_0}} \tag{7}$$

Comparing Eq 6, 7 we can arrive at a mechanistic interpretation for permittivity and permeability: where we identify $\frac{1}{\mu_0}$ to be the equivalent of spring tension and permittivity, of the mass density. This raises the hope that future phenomena may allow us to determine their values separately too.

All astronomical observations and conclusions drawn from them are consistent with the assumption that the speed of llight is independent of the speed of its source (Example: Aberration). This is compeletely consistent with the wave theory which identifies it with the proerties of the medium in which the wave propagates. One may thus postulate a universally existing medium, Aether in which all bodies move, and in which light also propagates.

An observer moving with respect to the medium would measure a different speed of light. Thus, ME as written above can be valid only in the frame at respect to the medium, called aether. They would require modification in other inertial frames. That is, ME would not appear the same in all IF.

An all pervading medium offers a naturally preferred frame of reference – the one that is at rest in Aether. For, only in this frame would the speed of light be given by Eq 7. In all other frames, by Galilean transformation, the speed would not be the same. Thus we can always determine whether we are at rest with repsect to Aether or not by careful measurements of speed of light.

The concept of Aether offers other aesthetic explanations: As to how forces can be felt between bodies which are spatially separated. It also provides a material basis for the beautiful concept of fields which Faraday introduced in his studies. It would, perhaps also describe that frame of reference with respect to which the distant stars were all supposed to be at res (in the static infinite model for the universe). So a detection of the Earth's motion to aether would be of paramount interest. This quest was undertaken by Michelson and Morley, at the end of the nineteenth century.

4 Michelson Morley (MM) Experiments

The experiment performed b MM and repeated several times by them employed interference of light produced by the interferometer which Michelson himself had invented. The interferometer consists of two perpendicular arms at the ends of which are two mirrors, a half silvered mirror (at the centre) that directs the light impinging on it to both of them, a source and finally to an eye piece

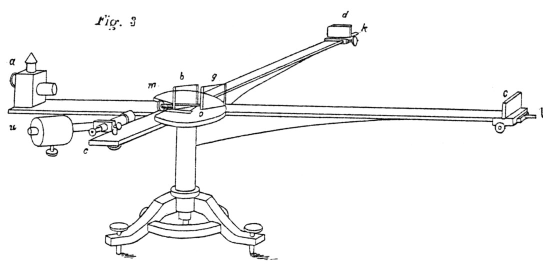

Figure 1: Credits: Wiki.

Schematically, it may be represented as in figure 2. Let the earth be moving

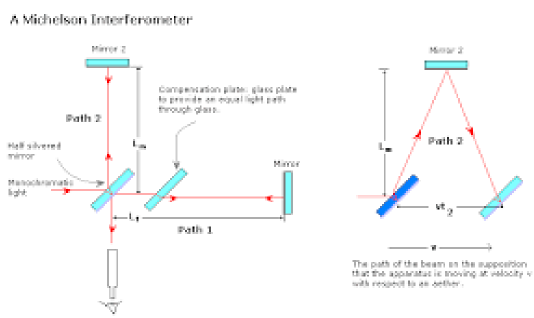

Figure 2: Credits: Wiki.

with a speed v with respect to aether. And, for simplicity, take the horizontal arm to be aligned along the direction of motion. As worked out in the class, and briefly repeated below, one expects a shift in interference fringes caused by the relative motion.

4.1 Theoretical preliminaries

Since the apparatus id moving as a whole with respect to aether, light takes different times to cover the distance between reaching the horizontal mirror and returning to the beam splitter. If the arm length is L_1, the time taken is

$$t_1 = L_1\left\{\frac{1}{c+v} + \frac{1}{c-v}\right\} = \frac{2L_1}{c}\frac{1}{1-v^2/c^2} \equiv \frac{2L_1}{c}\gamma^2 \qquad (8)$$

Similarly, fir the vertical arm of length L_2, the time taken will be

$$t_2 = \frac{2L_2}{c}\gamma \tag{9}$$

The time difference, which gives rise to interference is given by

$$\Delta t = t_1 - t_2 = \frac{2\gamma}{c}(L_1\gamma - L_2) \tag{10}$$

$$\equiv \Delta t' = t_1' - t_2' = \frac{2\gamma}{c}(L_1 - L_2\gamma) \neq \Delta t \tag{11}$$

which causes a shift in the fringe which MM wished to measure.

It is reasonable to use the fact that the quantity $\beta = v/c \ll 1$. Why? because much of astronomy assumes ignores the change in light speed because of earth's motion and yet gives very reliable results, in agreement with observations. Thus, expanding binomially, we get the final experssion, for the difference $\Delta\tau = \Delta t - \Delta t'$,

$$\Delta\tau = \frac{L_1 + L_2}{c}\beta^2 \tag{12}$$

which, in turn, gives rise to a fringe shift. The number that gets shifted is obtained by multiplying the above equation by the frequency of light, whence

$$\Delta N = \frac{L_1 + L_2}{\lambda}\beta^2 \tag{13}$$

The experiment had additional features which is not apparent in the figures above. By skilful employment of compensators, it was possible to increase the effective length by observing interference only after multiple reflections. Thus, the effective total length could be made 11 mm.

5 Results and discussion

5.1 Experimental sensitivity

We may arrive at estimates by starting with the snesitivity of the appratus and errors in the known values of observables. The minimum ΔN they could observe was 10^{-2}. The uncertainty in the value of c was about 5%. For simplicity, choose sodium yellow line which has a wavelength $\lambda = 580$nm. If we take $L_1 + L_2 = 20m$, we get $10^{-2} = 3.4 \times 10^7\beta^2$ which means that the minimum speed that can be detected is of the order of a few kilometers per second. MM experiment could the detect speed upto 20 km/s. For comparison, note that the eearth moves around the Sun with a speed of $30km/s$. Interestingly, this value gives us the estimate that $\Delta\tau \sim 10^{-17}$s which is a remarkably small interval even by today's standards. MM were able to bring down the upper limit on the speed of the earth that could be estimated, to 3.5 km s^{-1} by further refining the experiments.

5.2 Experimental results

MM consistently reported that their results were null, in the sense that they could not detect any fringe shift that was consistent with the minimum resolution that the appratus had. This is not necessarily convincing. It is legitimate to object that $v \le 3.5\ kms^{-1}$ cannot be interpreted as null. This must be understood in the sense that barring an extra ordinary coincidence, after we take into account that our solar system itself is revolving around the galaxy with a speed about seven times that of the earth around the sun, it is unlikely that it can have such a low velocity with respect to aether for all times.

5.2.1 Day night effect

The day night effect on the fringes were also studied by MM. I made an incorrect statement in the class that it can be used to declare the null result. I was wrong. To see that, just notice that the rotational speed of the earth about its axis is just 0.46 km per second which is roughly eight times smaller than the sensitivity. However, MM used this to rule out a hypthesis called aether drag. I shall not get into it here.

5.2.2 Current status

A large number of experiments have been done subsequently, the latest one involving optics, being in 1973. This places an upper limit of a tiny value of 2.5 cm s^{-1} which is remarkable. One of the most recent experiments done in 2015 looks at variation of speed of light in different directions in the earth's frame. Galilean velocity addition does give different values. It is found that $\frac{\Delta c}{c} = 2\frac{v}{c} \le 10^{-17}$ which justifies that the speed of the earth cannot be detected with respect to aether.