PYL127 -2020

Problem Set 2 – Generalised coordinates and Forces

The problems on generalsed coordinates are in addition to the onees found in problem set 1. We also ask for generalused forces here. This also helps you to become familiar with various coordinate systems.

1. A particle of mass m is constrained to move on a parabola given by $y = \frac{1}{2}x^2$ in the xy plane. In addition, it is acted upon by a uniform gravitational field along the $-y$ direction.

   (a) Set up suitable genarlised coordinate(s)

   (b) Find generalised force(s)

   (c) Setup the Euler Lagrange equation.

   Repeat the exercise for

2. A free particle constrained to move on the surface of a paraboloid ($x^2 + y^2 = z$)

3. A double simple pendulum oscillating in a plane

4. A conical pendulum

5. Two particles of the same mass and charge constrained to move on a sphere

6. Two particles whose centre of mass is at rest, at the origin. They interact with each other according to Hooke's law.

7. Three particles of the same mass with the constraint that their centre of mass is at rest (hint: Jacobi coordinates). All of them carry the same charge Q.

8. A one dimensional rigid rod of length L, one end of which is fixed, and on which a particle of mass moves. The whole system is acted upon by the earth's field.

   Some exercises to help you with lengths and areas:

9. Find the expression for length of the segment of an ellipse between two coordinates θ_1 and θ_2. If possible, integrate.

10. Same for the curve $\sin x$ between $x = 0$ and $x = \pi/2$

11. Consider a closed curve $\mathcal{C}$ parametrised by the angle θ with respect to some origin, contained within in the curve. Let the distance – from the origin – of the point on the curve, at θ be $R(\theta)$. How do you write the expression for

    - The area contained within the curve?
    - The length of the curve.
      Note: If $R(\theta)$ is independent of θ, it is a circle of radius R.

    Work out the special cases when

    (a) $R(\theta) = R_0 + \epsilon \sin\theta;\ \epsilon \ll R_0$

    (b) $R(\theta) = R_0 + \epsilon \cos^2\theta;\ \epsilon \ll R_0$

    Additional exercises:

12. Set up the lagrangian for a free particle in three dimensions in

    (a) Cartesian coordinates

    (b) Cylindrical coordinates

    (c) spherical polar coordinates

13. Repeat exercise mentioned in (12) for an isotropic oscillator in three dimensions.