Lecture 1

1 Scope of Classical Mechanics (CM)

This is the first topic that we started the course with. There are two domains in classical mechanics: Newtonian mechanics (NM) and Einsteinian (EM). They are usually referred to as non-relativistic and relativistic, respectively. This is not strictly accurate since NM is consistent with the principle of relativity under Galilean transformations. EM, of which NM is a limiting case, employs Lorentz transformations. But we do not quibble any more. Quantum mechanics (QM) is that which is not classical, in the sense that it is intrinsically, a probability theory. It also obeys the principle of relativity. When Galilean transformations are employed, it is called non-relativistic (NRQM). When Lorentz transformatiosn are employed, it is relativistic (RQM). Again, the former is a limiting case of the latter.

It is also generally understood that CM is an approximate to QM. QM is then the most fundamental theory. However, the limit QM $\rightarrow$ CM is delicate. There is no such difficulty with EM $\rightarrow$ NM.

1.1 Scales and domains of applicability

It is possible to fix the domains of applicability of CM, if only roughly, by considering two important fundamental physical scales provided by nature: (i) the topmost speed that can be attained by any physical signal, which is exemplified by the speed of light c, and (ii) the fundamental unit of angular momentum, $\hbar$.

Carving out the domain of NM from EM is simple. We require that
$v/c \ll 1$. Exact quantification depends on the accuracy with which speeds can be measured and the precision tests that one needs to perform. Problems in your assignment give you an idea.

With QM, it gets trickier. Partly because the classical limit of QM is not easy to work out. You will appreciate that more in one of your QM courses, hopefully.

Yet, We can still make some rudimentary analyses.

Planck's constant, $\hbar$ has the dimension of angular momentum, the product of position and momentum. Hence, so does the product $\Delta x \Delta p$ which is the uncertainty. You can use all these features to analyse a given situation.

1.1.1 Example 1

Let us say that we have measured the position and momentum of a particle with respective standard deviations ΔX and ΔP. You may take them to be least counts of the measuring instruments. If their product is much larger than $\hbar$, we need not worry about quantum corrections. On the other hand, consider the Rutherford model for Hydrogen atom. Combined with other data, we have an idea of the size of the atom and also the momentum of the electron. You compare it against $\hbar$ and you conclude that it is unsafe to use CM.

1.1.2 Example 2

Let us go to very low temperatures. Thermal energies, and hence the thermal momenta become very small. If the de Broglie wavelength becomes comparable to interparticle distance (which you get from the number density) you know CM is unreliable.

1.1.3 Final Example

Let us take a gas of hydrogen atoms to a temperature of $10^{10}K$. The thermal energy is comparable to the threshold energy for e^+e^- production, which is quintessentially a quantum phenomenon in the relativistic domain.

2 Space and Time

2.1 Properties of space

Several assumptions are to be made, based on experience and observations made on properties of distances amongst particles. Consider properties of space first

1. We first assume that space is homogeneous. By that mean there is no privileged position in space. As a corollary, what is physical is only mutual distances. Alternately put, if two particles are displaced by the same extent, the distance between them remains unchanged. A more technical and better term is that there is translational invariance.

2. We assume that there is no preferred direction in space. This is called isotropy. More technically, it is rotational invariance. What is physical is only the mutual angle between say, two rods.

3. Space is Euclidean. In a two dimensional world, e.g, both an infinite plane and a two sphere are homogeneous and isotropic. There is no preferred point on a sphere, and no preferred direction either. Such constructions are possible in the physical three dimensional space too. When we say that space is Euclidean, we have in mind the standard geometric properties which you sudied in school:

   (a) Parallel lines do not meet.

   (b) Sum of three angles of a triangle add up to π.

2.2 Properties of time

The basic postulate is that there is no preferred event. What matters is only the time interval. In other words, watches are of great convenience, but only stop watches give physically meaningful information.

Once we make these postulates, only those forces and interactions that are consistent with them are allowed. Speaking experimentally, no observation reveals the existence of a preferred primordial origin or direction or event. Once formulated fully mathematically, they lead to important conservation laws. They will be discussed later.

3 Coordinate systems

Is there a coordinate system that captures all the properties of space naturally? Yes, and that is the dear old rectangular coordinate system(RCS) . It is characterized by (i) mutually orthogonal axes, and (ii) basis vectors which are not dependent on position. the distances and angles are given by the familiar geometric formulas. How are homogeneity and isotropy expressed? Start with an RCS. You can obtain an infinite number of them by arbitrarily shifting the origin and rotating the coordinates (in any order). The assertion is that they are all equivalent. The distance between points, and angles between lines are unchanged by these operations. Such quantities are called invariants.

Euclidean nature of space lies at the heart of almost all applications, with the exception of a few consequences from general relativity. Confirmation of trajectories (including missiles), planetary motion, estimation of distances in astronomy, all of civil engineering, space exploration, propagation of light in vacuum, $\cdots$. The list goes on. The agreement with observations is spectacular. So spectacular that there were mathematicians and philosophers who believed that no other geometry was conceivable.

4 Other coordinate systems

At times, it is convenient to introduce other coordinate systems, Oblique (OCS) and curvilinear (CCS). In OCS, the axes are not mutually orthogonal, but they are straight lines. In CCS, the basis vectors depend on the location. Example: Polar coordinates. The basis vectors $\hat{r}, \hat{\phi}$ keep changing their directions as we move in space. But they remain mutually orthogonal, at any given point. More generally, they need not even be locally orthogonal.

Real life problems require that we be familiar with some of CCS. The most important ones are the spherical polar and the cylindrical coordinate systems. Parabolic coordinates also have an interesting role to play.

Combine this writeup with lectures and problem sets. That should take care of may of your concerns. The slides will be uploaded separately.