Ph.D. viva-voce presentation
June 26, 2024

Studies on interrelation among nonclassical features and applications to quantum communication

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(2017PHZ8375)

Supervisor
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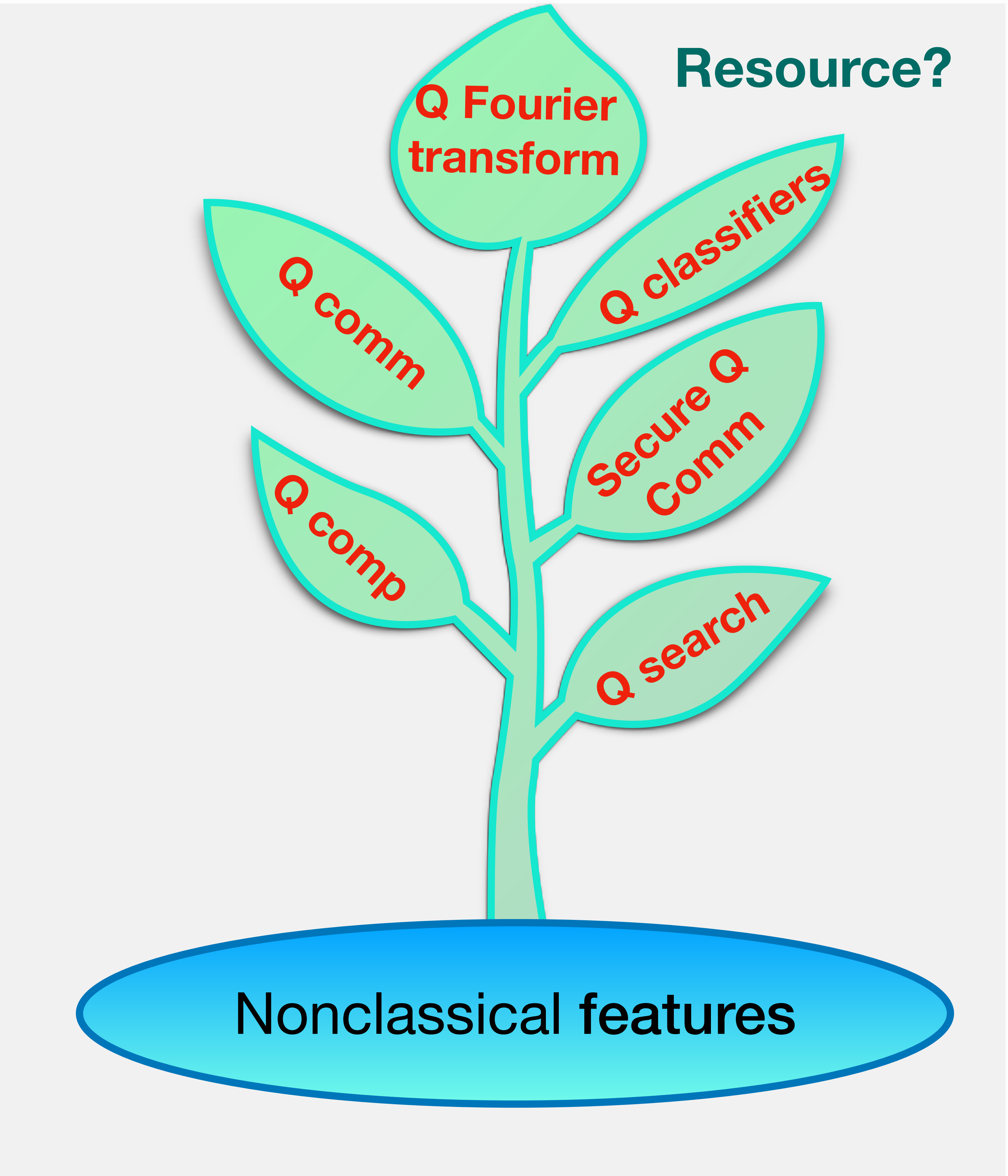

Overview

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Introduction

PART A: Interrelation among nonclassical features

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Interrelation of nonclassicality features through logical quantum systems

2

Weak measurements, nonclassicality and negative probability

3

Nonlocalities and entanglements through pseudoprojections

PART B: Quantum information processing with minimal resources

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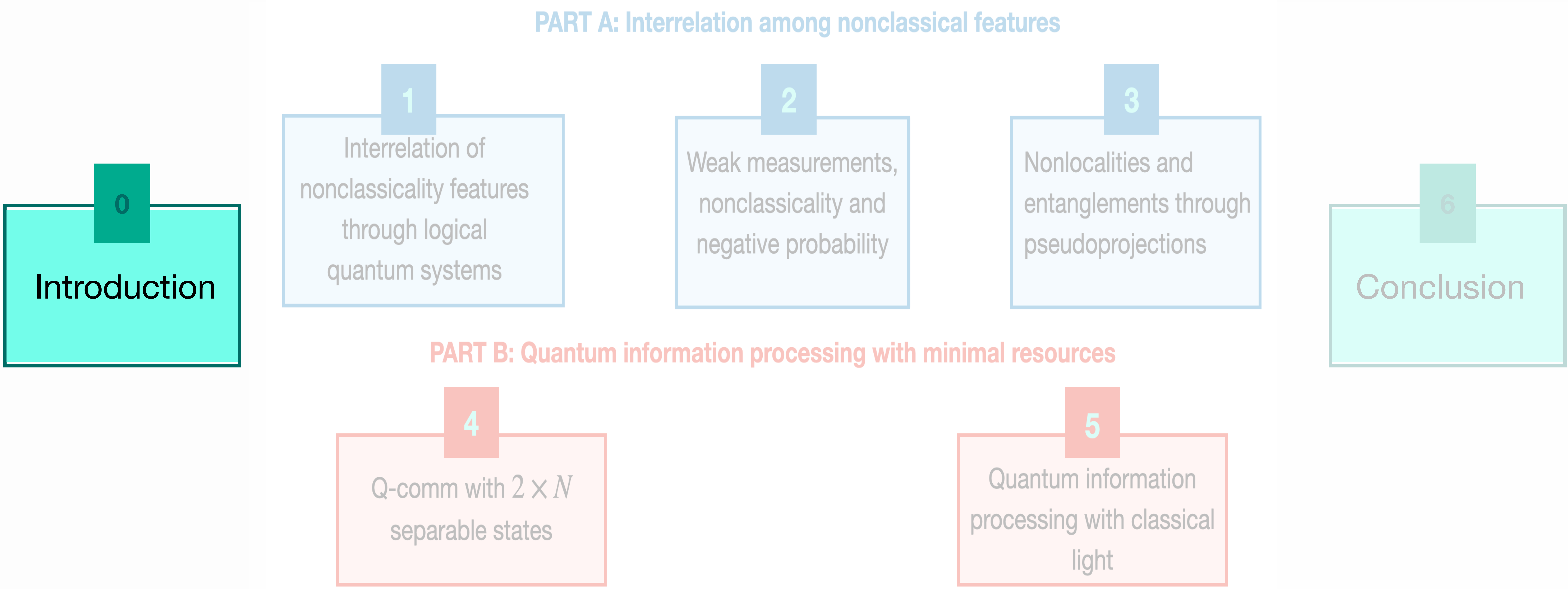
Q-comm with $2 \times N$ separable states

5

Quantum information processing with classical light

6

Conclusion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Nonclassicality features
in quantum mechanics

(i) Nonclassical probability

(ii) Entanglement

(iii) Nonlocality

(iv) Quantum coherence

⋮

Nonclassicality features
in quantum mechanics

(i) Nonclassical probability

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(iv) Quantum coherence

⋮

$$P(x,p) < 0$$

No joint probability scheme for noncommuting observables

$$\rho_{12} \neq \sum_i p_i \rho_{1i} \otimes \rho_{2i}$$

Inseparability

$$P(A=a_i, B=b_j) \neq \int d\lambda \rho(\lambda) P(A=a_i|\lambda) P(B=b_j|\lambda)$$

Non-existence of LHV model

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Principle of superposition

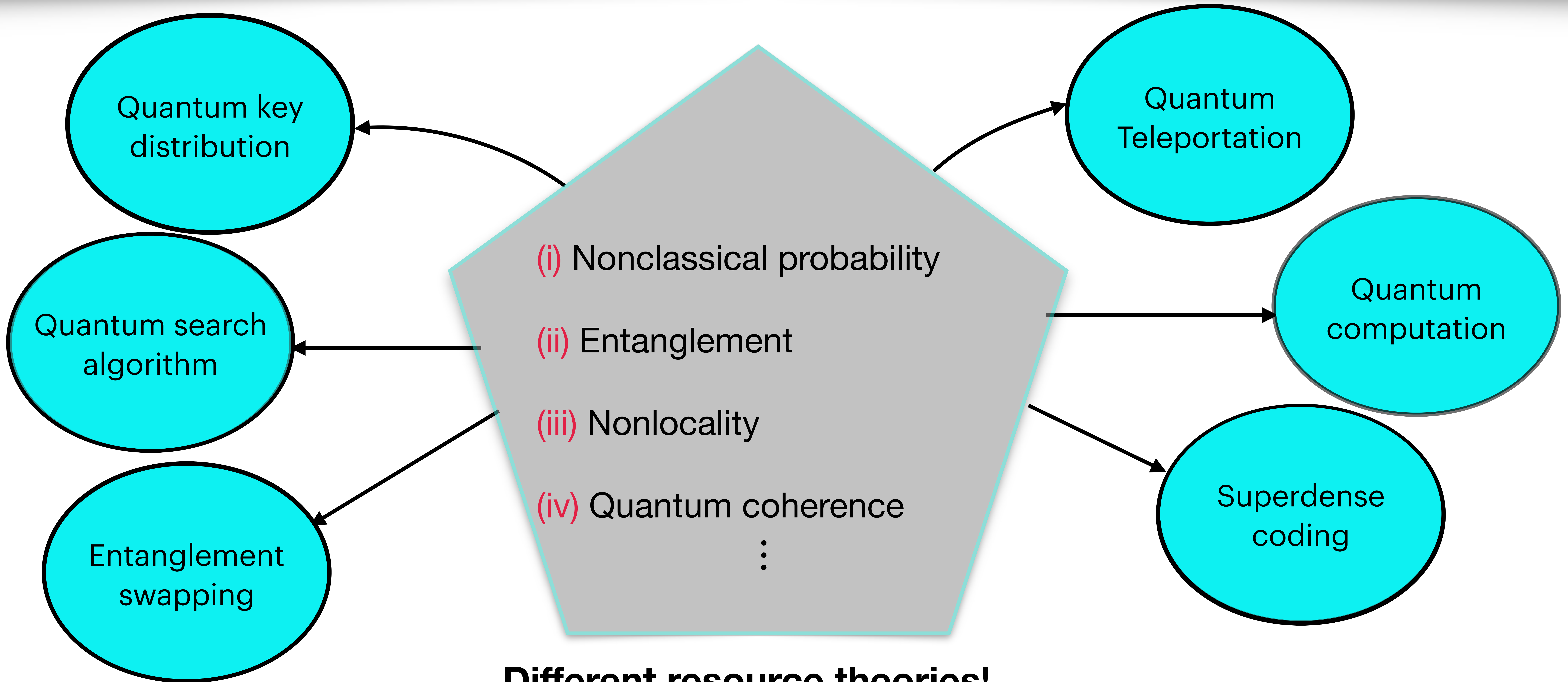
N. Brunner et al. *Reviews of modern physics* 86.2 (2014): 419.

R. Horodecki et al. *Reviews of modern physics* 81.2 (2009): 865.

A. Streltsov, G. Adesso, and M. B. Plenio. *Reviews of Modern Physics* 89.4 (2017): 041003.

Nonclassicality features

As resources in different tasks



Different resource theories!

N. Brunner et al. *Reviews of modern physics* 86.2 (2014): 419.

R. Horodecki et al. *Reviews of modern physics* 81.2 (2009): 865.

A. Streltsov, G. Adesso, and M. B. Plenio. *Reviews of Modern Physics* 89.4 (2017): 041003.

Nonclassicality features in quantum domain

(i) Nonclassical probability

(ii) Entanglement

(iii) Nonlocality

(iv) Quantum coherence

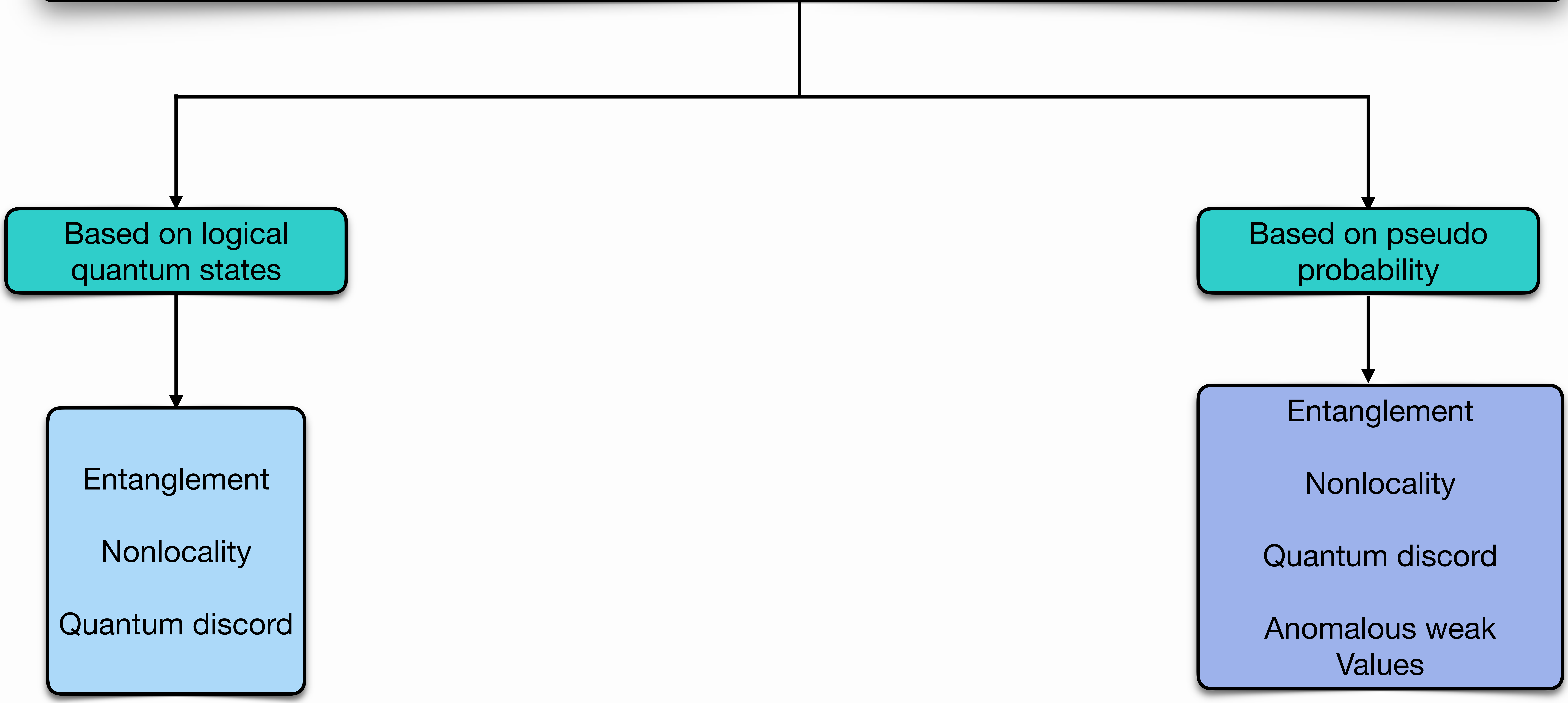
⋮

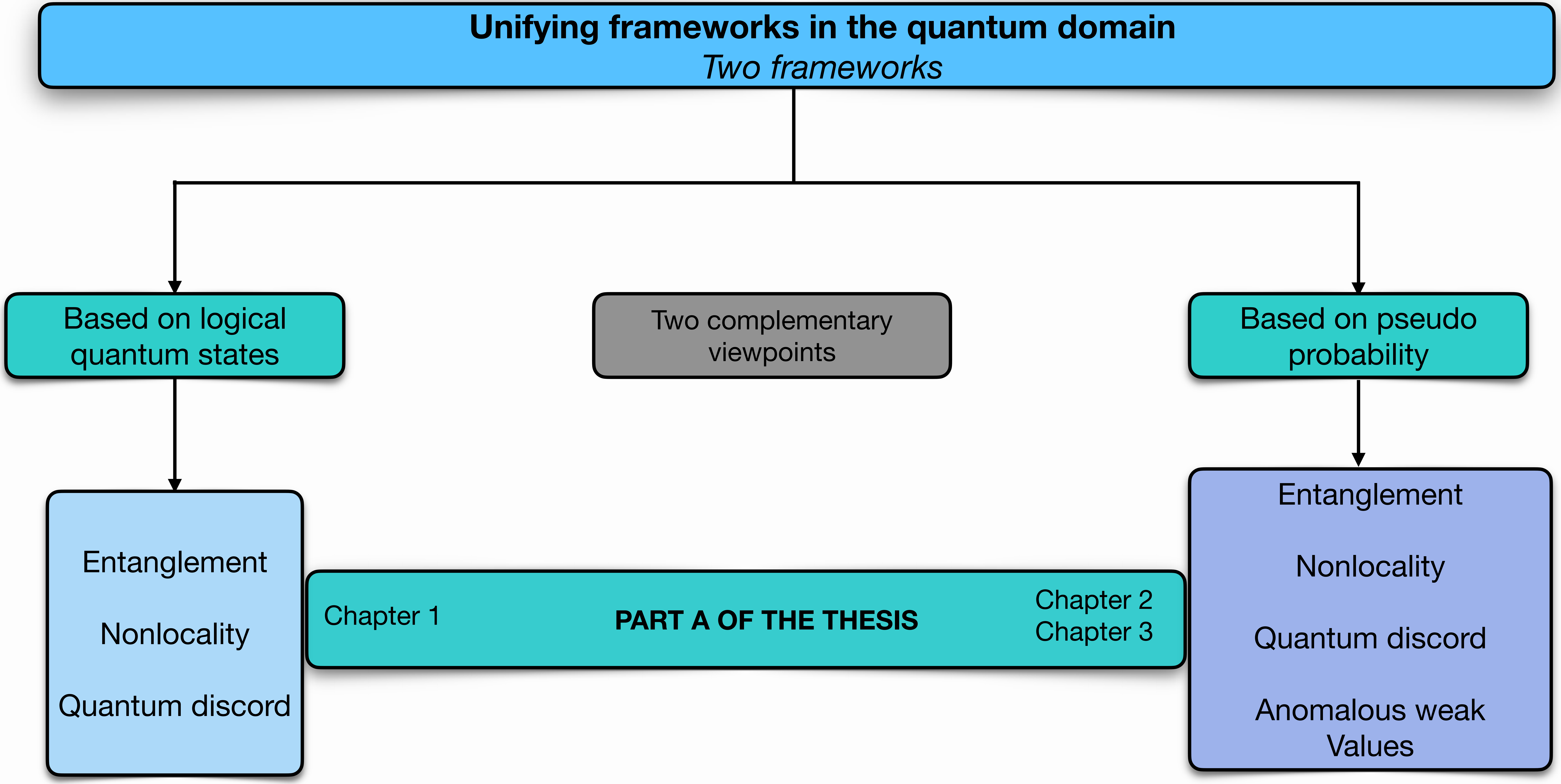
Is there a *unifying* framework for nonclassicality **in the quantum domain**?

YES!

Unifying frameworks in the quantum domain

Two frameworks





Unifying frameworks in the quantum domain
Two frameworks
Based on logical quantum states
Two complementary viewpoints
Based on pseudo probability
Entanglement
Nonlocality
Quantum discord
Chapter 1
PART A OF THE THESIS
Chapter 2
Chapter 3
Entanglement
Nonlocality
Quantum discord
Anomalous weak Values

What is "quantum" in quantum information processing?

Numerous notions of classicality

Application of classical simulability[1] in QIP

Lower dimensional **entangled** states $\cong$ High-dimensional **separable** states (Pol-OAM)

Information-transfer protocols with **high-dimensional separable** states

Chapter 4 (PART B OF THE THESIS)

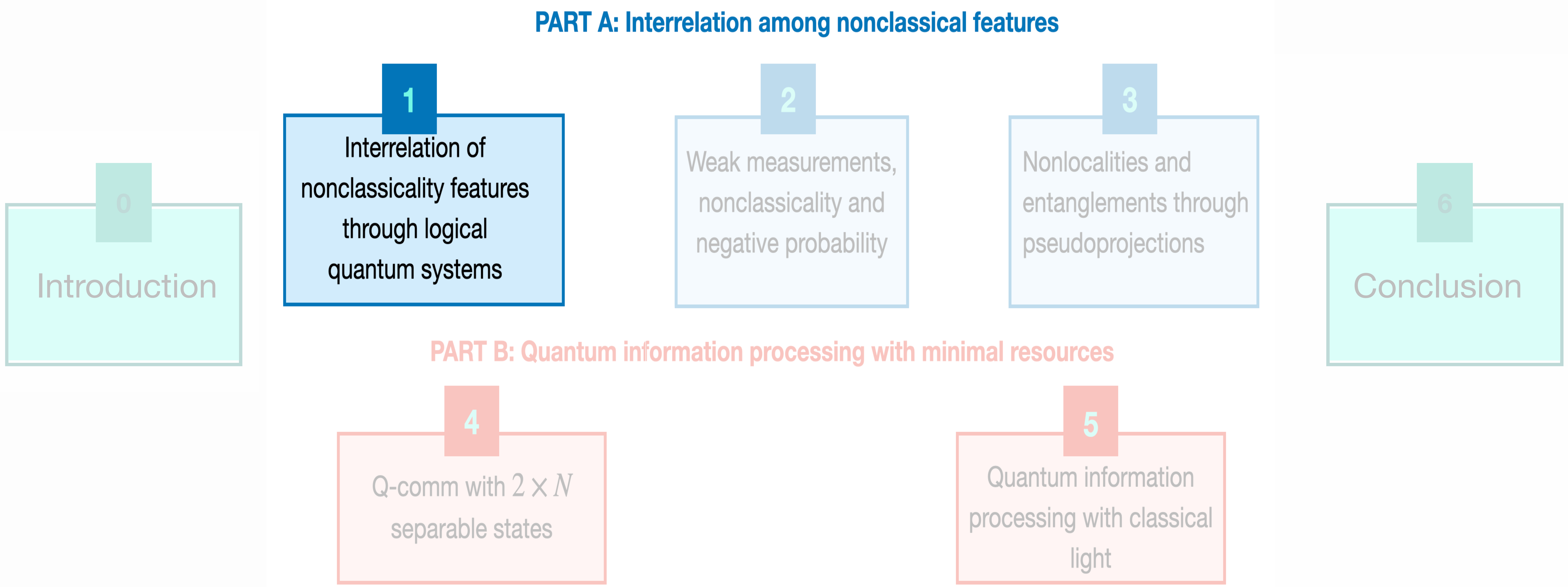
H. M. Bharath, and V. Ravishankar. *Phys. Rev. A* 89.6 (2014): 062110.

Classical-quantum interplay

Second approach: include classical waves

Resource	Implementability with classical light
Parallelism (Q-computation, q-search)	✓
Entanglement at the same location (Q Fourier transform)	✓
Nonlocality/ single-photon statistics (Q communication, QKD)	✗

Chapter 5 (PART B OF THE THESIS)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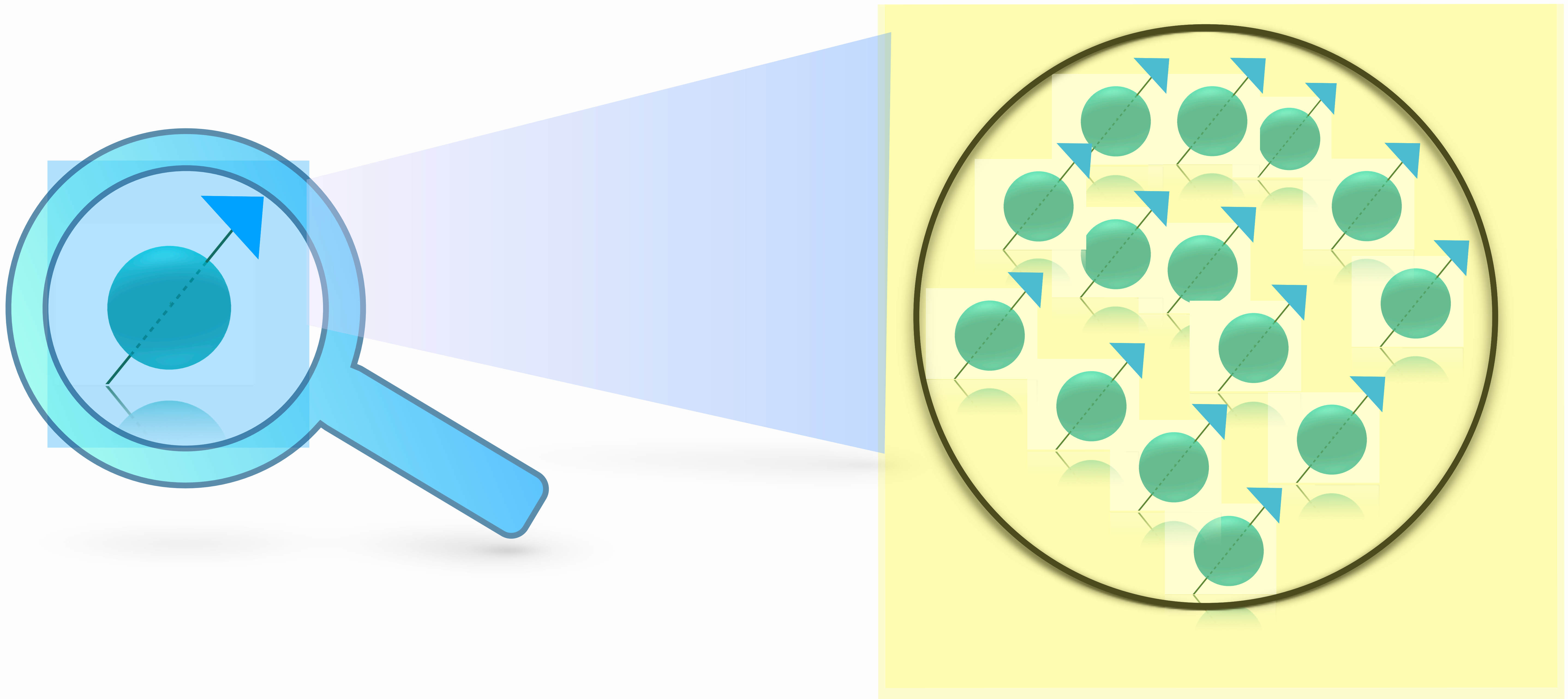
What is the state of a quantum system?

Depends on the experimental observation.

What is the state of a quantum system?

Depends on the experimental observation.



Example: Logical qubits and physical qubits

$$|0\rangle_L \equiv |000\rangle; \quad |1\rangle_L \equiv |111\rangle$$

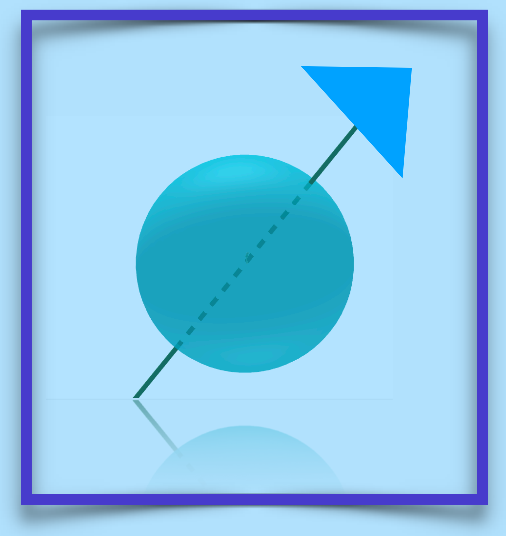
$$\alpha|0\rangle_L + \beta|1\rangle_L \equiv \alpha|000\rangle + \beta|111\rangle$$

A state with coherence
$\alpha \neq 0, \quad \beta \neq 0$

Entangled/ nonlocal state
$\alpha \neq 0, \quad \beta \neq 0$

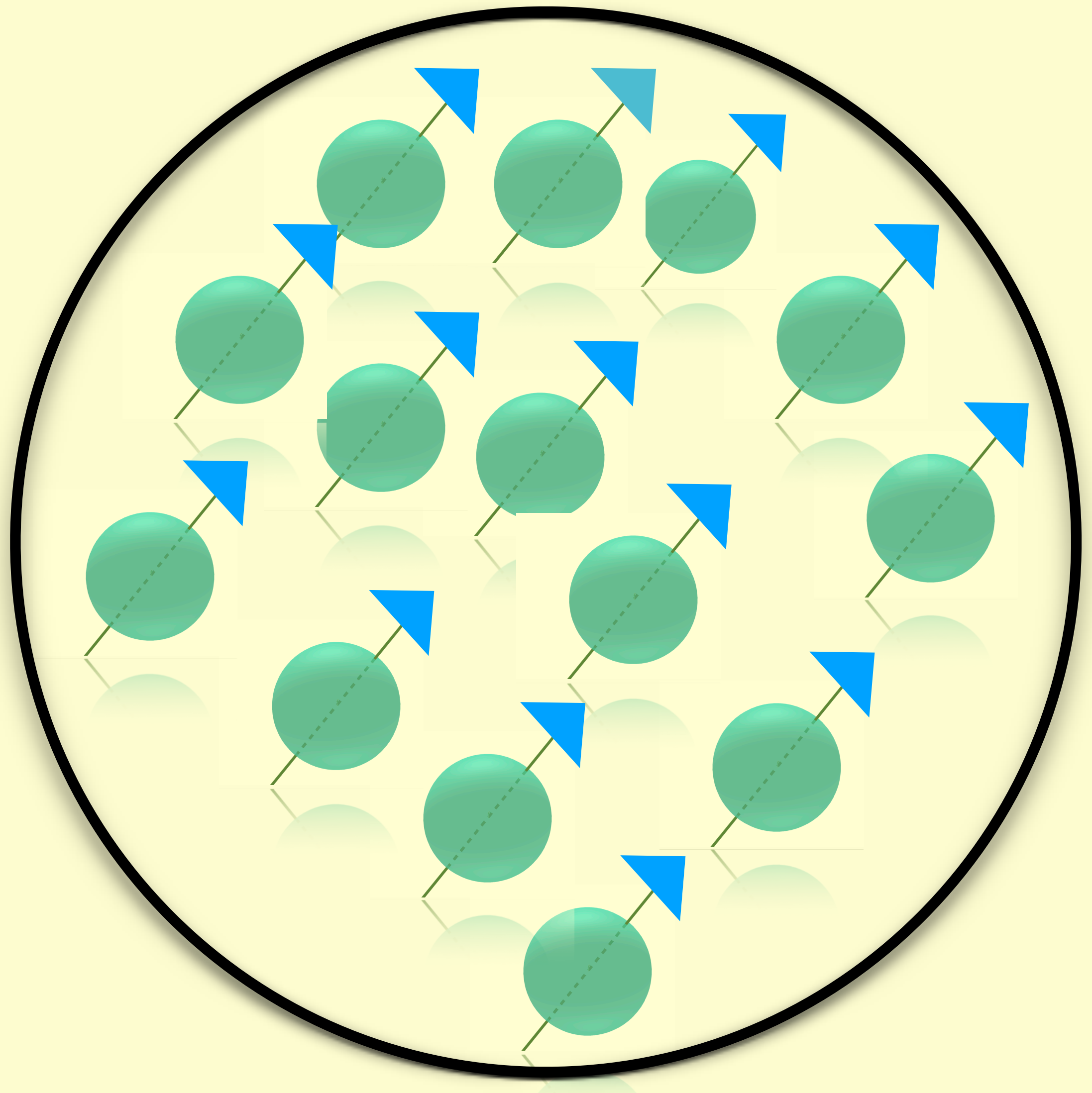
Logical bits: No coherence,
Logical qubits: have coherence. *(In general)*

Question of Interest



A logical state

$\equiv$



Underlying Physical state

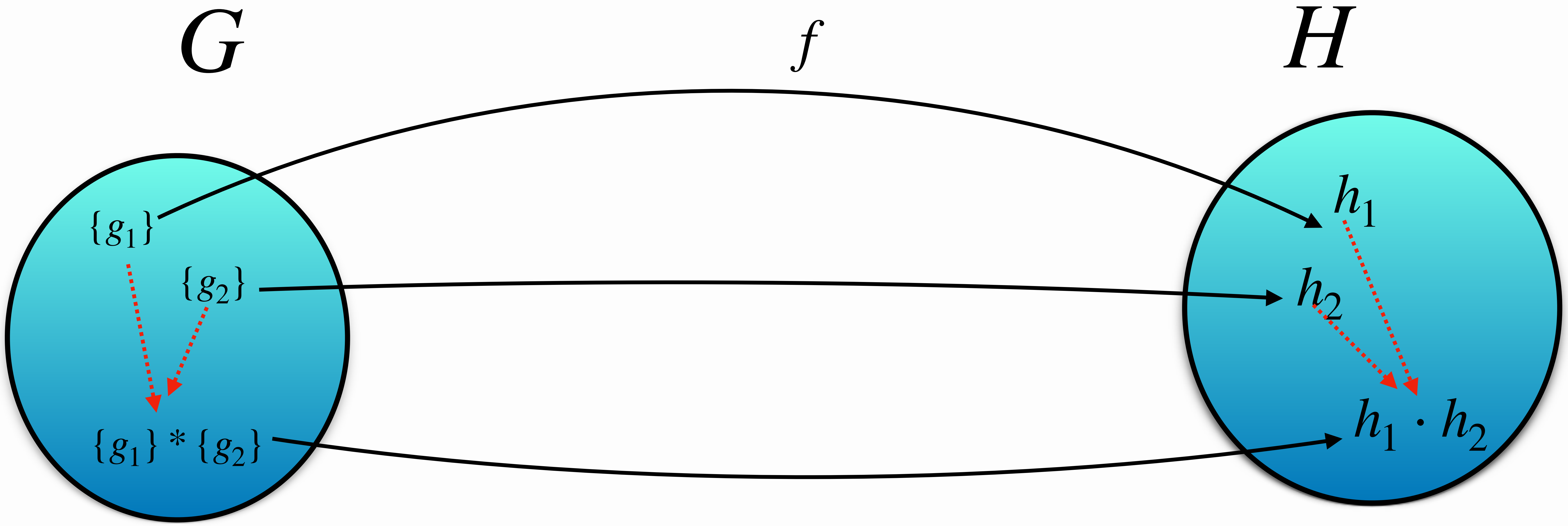
$\mathcal{N}_L$: Nonclassicality condition for logical qubits

$\rightleftarrows$?

$\mathcal{N}_{\text{phys}}$: Nonclassicality condition for physical qubits

Tools: Homomorphism between stabiliser groups

Homomorphism: A product-preserving map



$$f(g_1 \star g_2) = f(g_1) \cdot f(g_2)$$

Example

$G : \{\mathbb{Z}, +\}$

$H : \{0,1; \oplus_2\}$

$\{2n\} \to 0,\ \{2n+1\} \to 1$

Wu-Ki Tung. *Group theory in physics*. Vol. 1. World Scientific, 1985.

Stabiliser group

Stabilisers: Operators which have a state as eigenstate with eigenvalue $+1$.

Example 1: $|\psi\rangle_L \equiv \frac{1}{\sqrt{2}}\left(|0\rangle_L + |1\rangle_L\right)$

$$G_L \equiv \{\mathbf{1}_L, X_L\}$$

Isomorphic to $\mathbb{Z}_2$

Example 2: Stabiliser group of a Bell state

$$|\psi\rangle_L \equiv \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$G_B \equiv \{\mathbf{1}_4, X_1X_2, -Y_1Y_2, Z_1Z_2\}$$

Isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$

Homomorphism between stabiliser groups: Example

$\{\mathbf{1}_L, X_L\}$ $\quad |\psi\rangle_L \equiv |0\rangle_L + |1\rangle_L$

$\{\mathbf{1}_4, X_1X_2, -Y_1Y_2, Z_1Z_2\}$ $\quad |\psi\rangle_L \equiv |00\rangle + |11\rangle$

$$\{\mathbf{1}_4, Z_1Z_2\} \rightarrow \mathbf{1}_L$$

$$\{X_1X_2, -Y_1Y_2\} \rightarrow X_L$$

Globally commuting but locally non commuting

$$[X_1X_2, Y_1Y_2] = 0$$
$$[X_i, Y_i] \neq 0$$

Detects coherence in the computational basis of logical states

Homomorphism between stabiliser groups: Example

$\{\mathbf{1}_L, X_L\}$ $\qquad |\psi\rangle_L \equiv |0\rangle_L + |1\rangle_L$

$\{\mathbf{1}_4, X_1X_2, -Y_1Y_2, Z_1Z_2\}$ $\qquad |\psi\rangle_L \equiv |00\rangle + |11\rangle$

$$X_1X_2|\psi\rangle_L = |\psi\rangle_L$$
$$-Y_1Y_2|\psi\rangle_L = |\psi\rangle_L$$

$\{\mathbf{1}_4, Z_1Z_2\} \to \mathbf{1}_L$

$\{X_1X_2, -Y_1Y_2\} \to X_L$

$$\langle X_L\rangle > c, \quad c \in [0,1)\,.$$
$$\langle 0_L|X_L|0_L\rangle = 0$$
$$\langle 1_L|X_L|1_L\rangle = 0$$
$$\langle \psi_L|X_L|\psi_L\rangle = 1$$

Globally commuting but locally non commuting

$$[X_1X_2, Y_1Y_2] = 0$$
$$[X_i, Y_i] \neq 0$$

Detects coherence in the computational basis of logical states

S. Asthana, *New Journal of Physics* 24.5 (2022): 053026.

Coherence in a logical qubit → entanglement in a two-qubit system

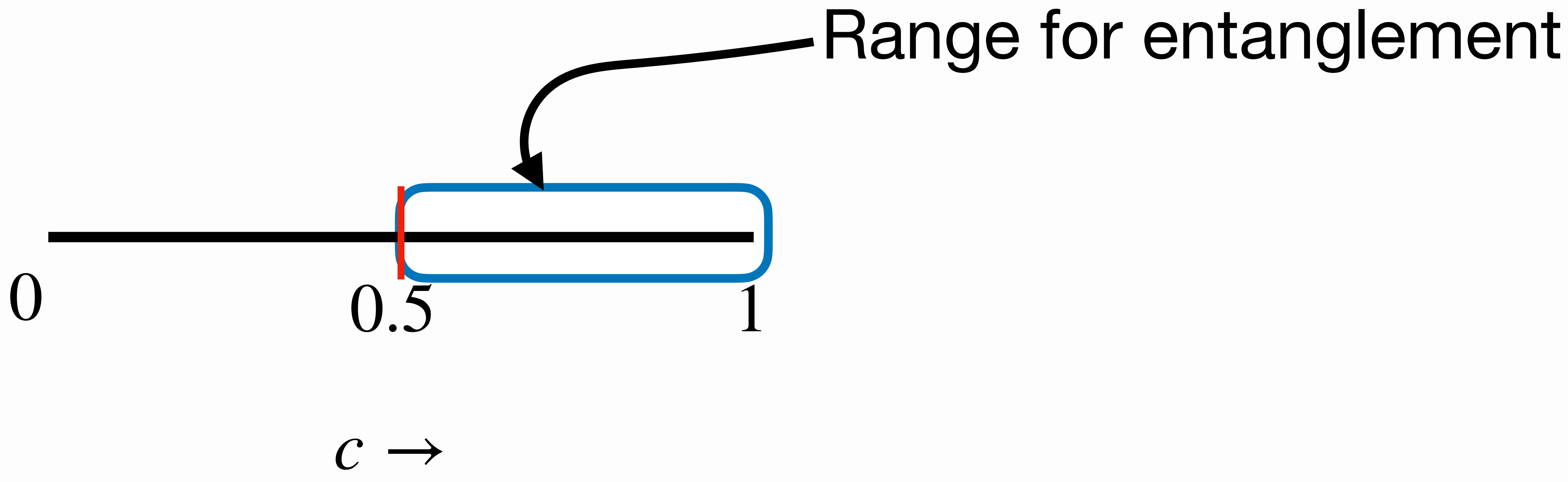
Condition for coherence in a logical qubit → $\boxed{\langle X_L \rangle > c}$

Coherence in a logical qubit → entanglement in a two-qubit system

Condition for coherence in a logical qubit → $\langle X_L \rangle > c$ $\Longrightarrow$ $\left\langle \frac{1}{2}(X_1X_2 - Y_1Y_2) \right\rangle > c\,.$ ← Condition for entanglement in a physical two-qubit system

$$c \in \left[\frac{1}{2}, 1\right)$$

Use the homomorphic images



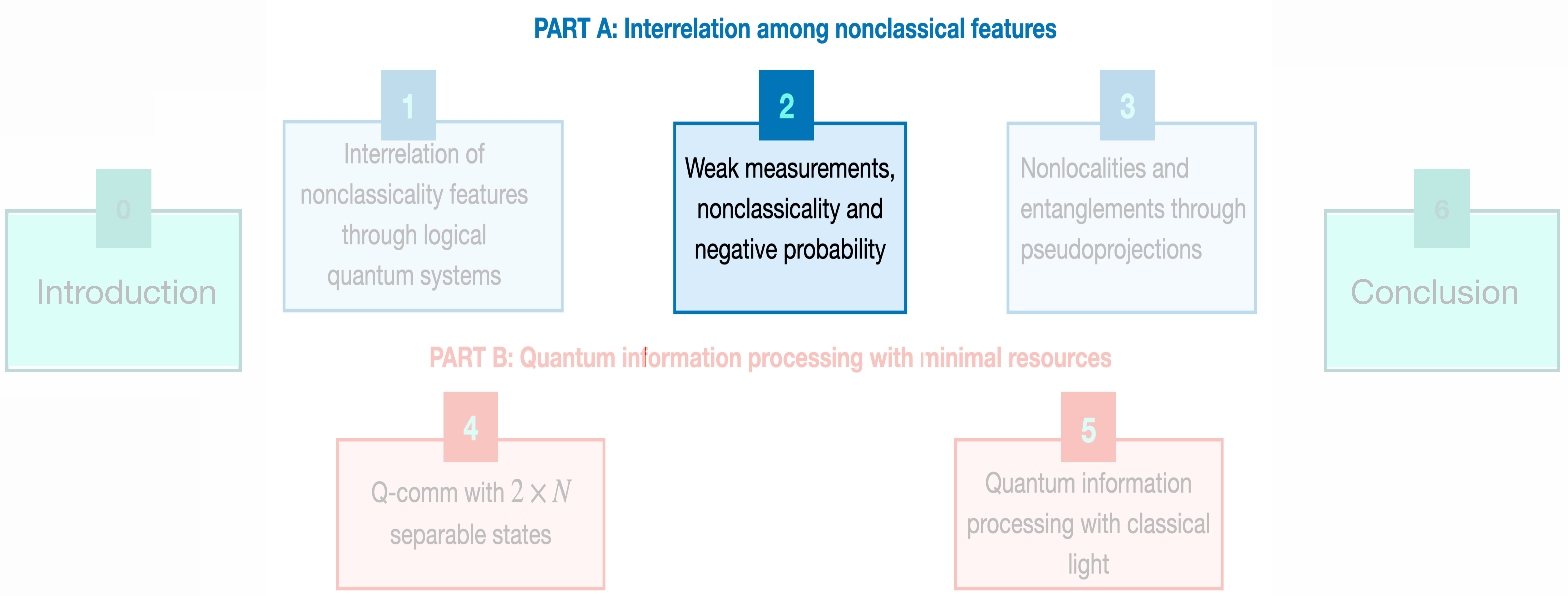
Conclusions

Coherence in logical qubits $\rightleftharpoons$ Nonclassical correlations in multiqubit systems

$|i\rangle_L \equiv |iii\rangle$

Mathematical tool: Stabiliser group homomorphism

- Generalisable to qudits as well.
- Interrelation between monotones of different resource theories.
- Other choices of logical qudits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Motivation

- QM: New theory of probability.
- Conditions for various manifests of nonclassicality: violation of classical probability rule

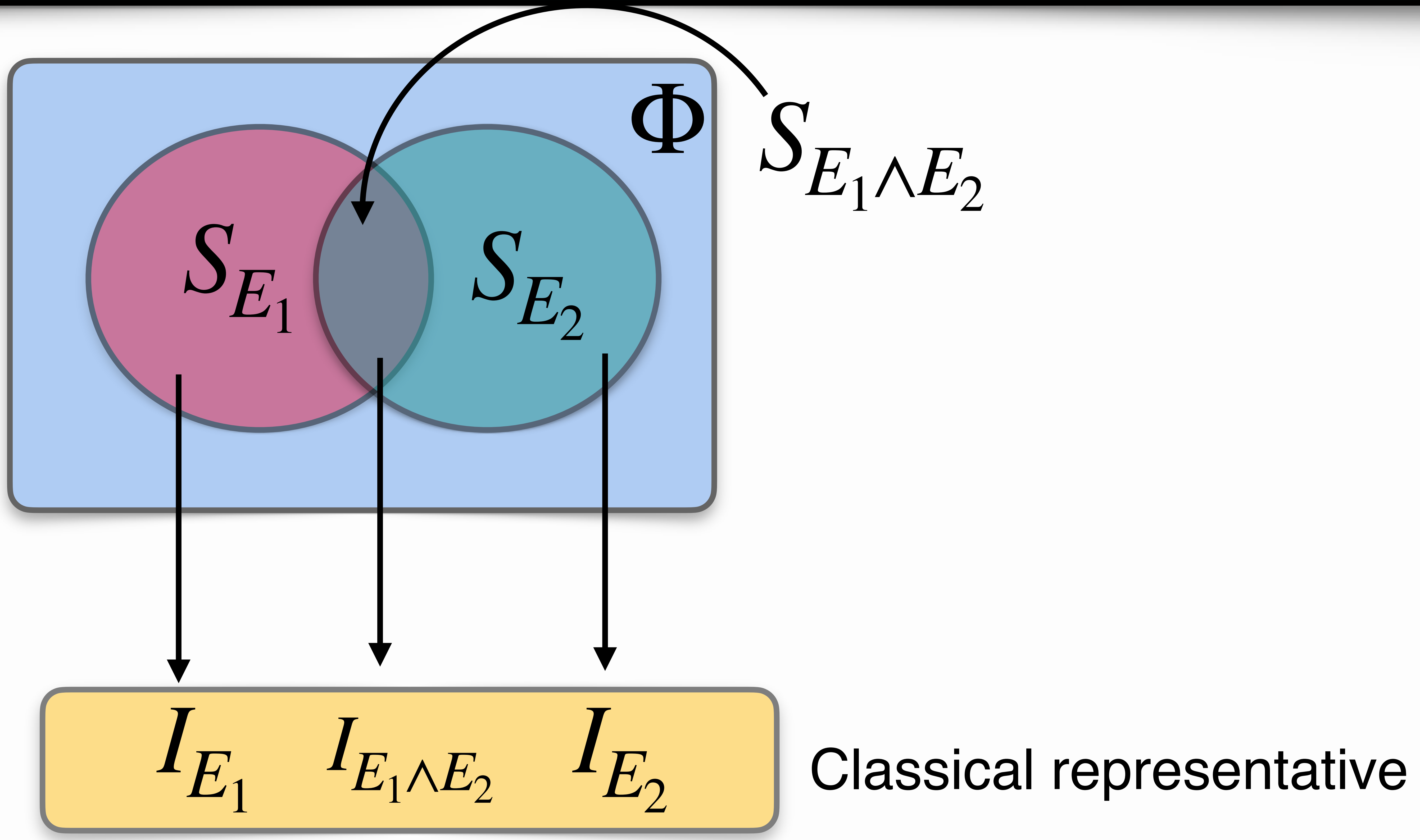
Question of interest

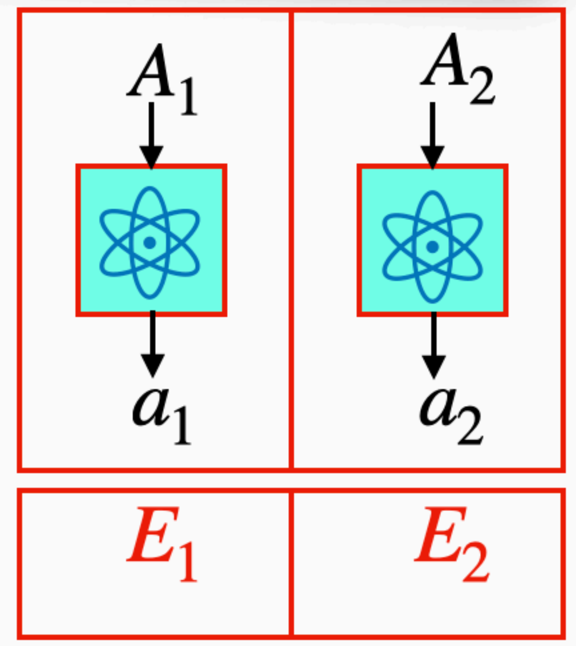
Quantum state ρ $\xrightarrow{\text{Conditions ?}}$ **Classical description** $0 \leq P_\rho(a, b, c, \cdots) \leq 1$

Tool

☞ Quantum representatives of classical indicator functions

Quantum representative of classical indicator functions

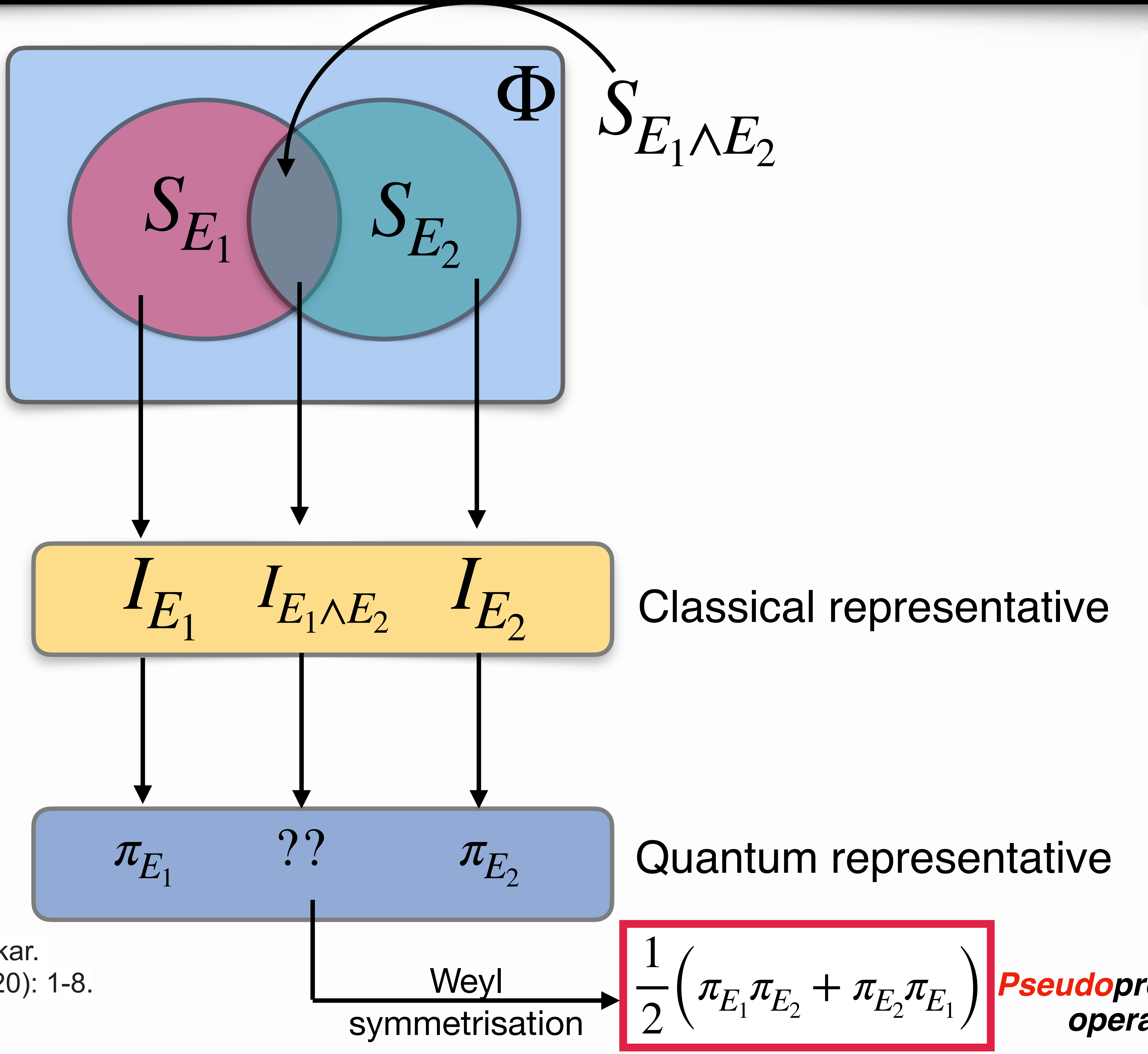


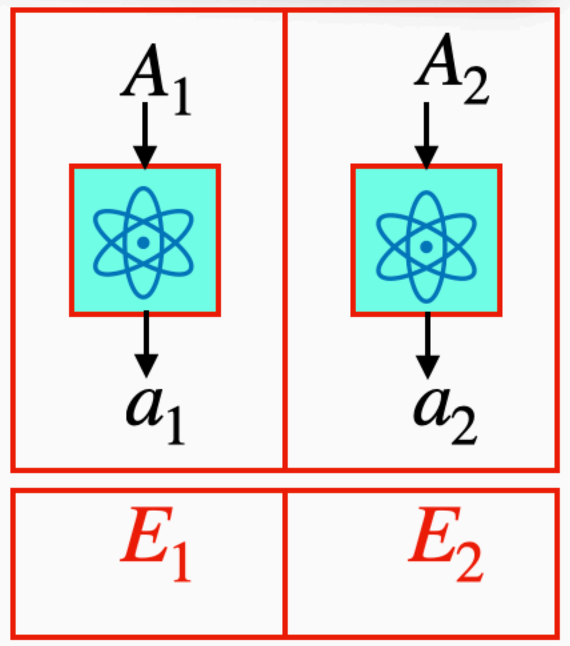


S. Adhikary, S. Asthana, and V. Ravishankar.
The European Physical Journal D 74 (2020): 1-8.

The Framework

Quantum representative of classical indicator functions





S. Adhikary, S. Asthana, and V. Ravishankar. *The European Physical Journal D* 74 (2020): 1-8.

Pseudoprojection operator

$$I^{A_1A_2}_{a_1a_2} \rightarrow \mathbf{\Pi}^{A_1A_2}_{a_1a_2} := \frac{1}{2}\left\{\pi^{A_1}_{a_1}, \pi^{A_2}_{a_2}\right\}$$

Properties

Hermitian ✓ Idempotent ✗ Nonnegative operator ✗

Pseudoprobability

$$\boxed{\mathscr{P}^{A_1A_2}_{a_1a_2} = \langle \mathbf{\Pi}^{A_1A_2}_{a_1a_2} \rangle}$$

S. Adhikary, S. Asthana, and V. Ravishankar. *The European Physical Journal D* 74 (2020): 1-8.

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Pseudoprobability

$$\boxed{\mathscr{P}^{A_1A_2}_{a_1a_2} = \langle \mathbf{\Pi}^{A_1A_2}_{a_1a_2} \rangle}$$

Signature of nonclassicality

$$\mathscr{P}^{A_1A_2}_{a_1a_2} = \langle \mathbf{\Pi}^{A_1A_2}_{a_1a_2} \rangle < 0$$

- How to obtain conditions for nonclassical resources, e.g., nonlocality and entanglement?

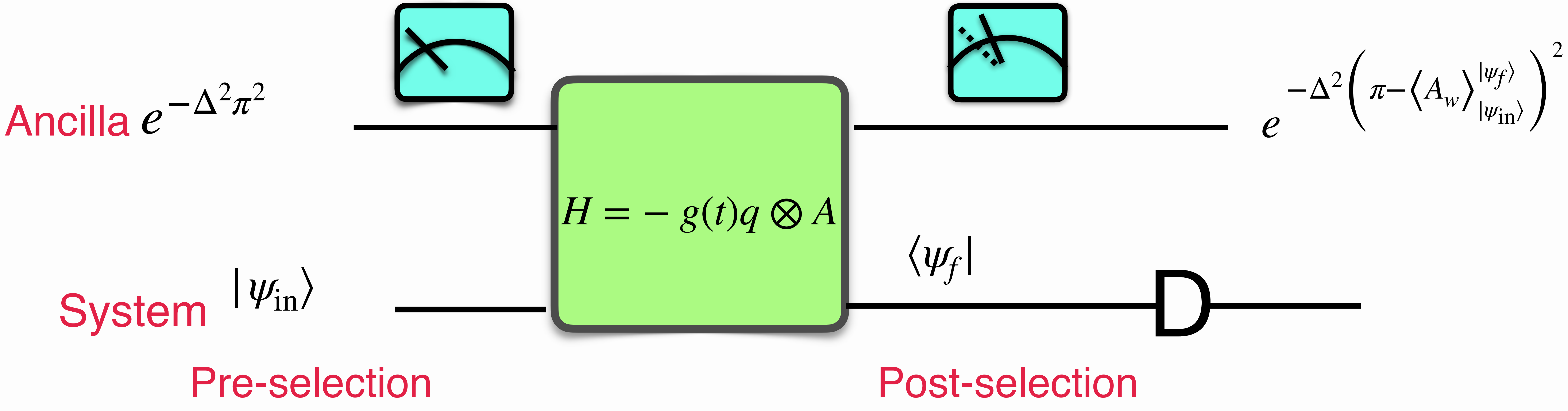
AND

- How to observe them experimentally?

- **Tools:** Weak measurements

S. Adhikary, S. Asthana, and V. Ravishankar. *The European Physical Journal D* 74 (2020): 1-8.

$$\left\langle A_w \right\rangle_{|\psi_{in}\rangle}^{|\psi_f\rangle} = \frac{\langle \psi_f | A | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$



Aharonov et al., Phys. Rev. Lett. 60, 1351

Weak value

- Weak value of an observable A on a pre-selected state $|\psi\rangle$ and a post-selected state $|\phi\rangle$

$$\boxed{\langle A_w\rangle_{|\psi\rangle}^{|\phi\rangle} := \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}\,.}$$

- If $\langle A_w\rangle_{|\psi\rangle}^{|\phi\rangle} \notin \mathrm{range(A)} \implies$ **anomalous weak value**
- Weak value of an observable A in the mixed pre-selected state ρ and post-selected state ρ_1 is:

$$\boxed{\langle A_w\rangle_{\rho}^{\rho_1} = \frac{\mathrm{Tr}(\rho_1 A\rho)}{\mathrm{Tr}(\rho_1\rho)}}$$

Aharonov et al., Phys. Rev. Lett. 60, 1351

H. M. Wiseman, Phys. Rev. A 65, 032111

Relation of Pseudoprobability with weak values

$$\mathbf{\Pi}_{a_1a_2} = \frac{1}{2}\left(\pi_{a_1}\pi_{a_2} + \mathrm{h.c.}\right)$$

$$\boxed{\langle \mathbf{\Pi}_{a_1a_2}\rangle_\rho} = \boxed{\mathrm{Tr}(\rho\pi_{a_1})\mathrm{Re}\langle \pi_{a_2}\rangle_\rho^{\rho_{a_1}}}; \quad \rho_{a_1} = \frac{\pi_{a_1}}{d}$$

Theoretical construct

Experimentally determinable quantity

$$\boxed{\langle \mathbf{\Pi}_{a_1a_2}\rangle < 0 \iff \mathrm{Re}\langle \pi_{a_2}\rangle_\rho^{\rho_{a_1}} < 0}$$

Negative pseudoprobability ≅ Anomalous weak value

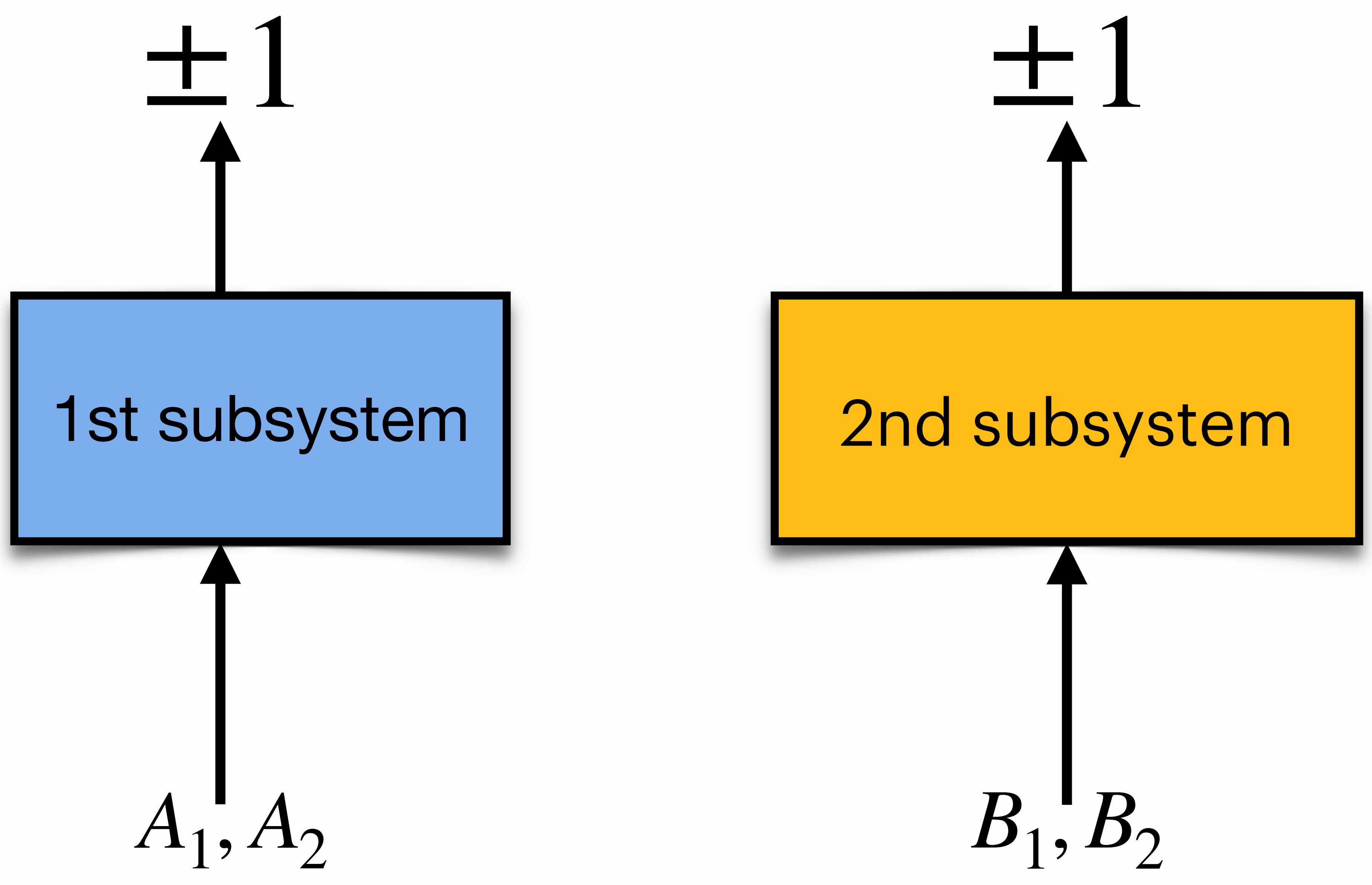
S Asthana and V. Ravishankar. *Quantum Information Processing* 20.10 (2021): 1-39

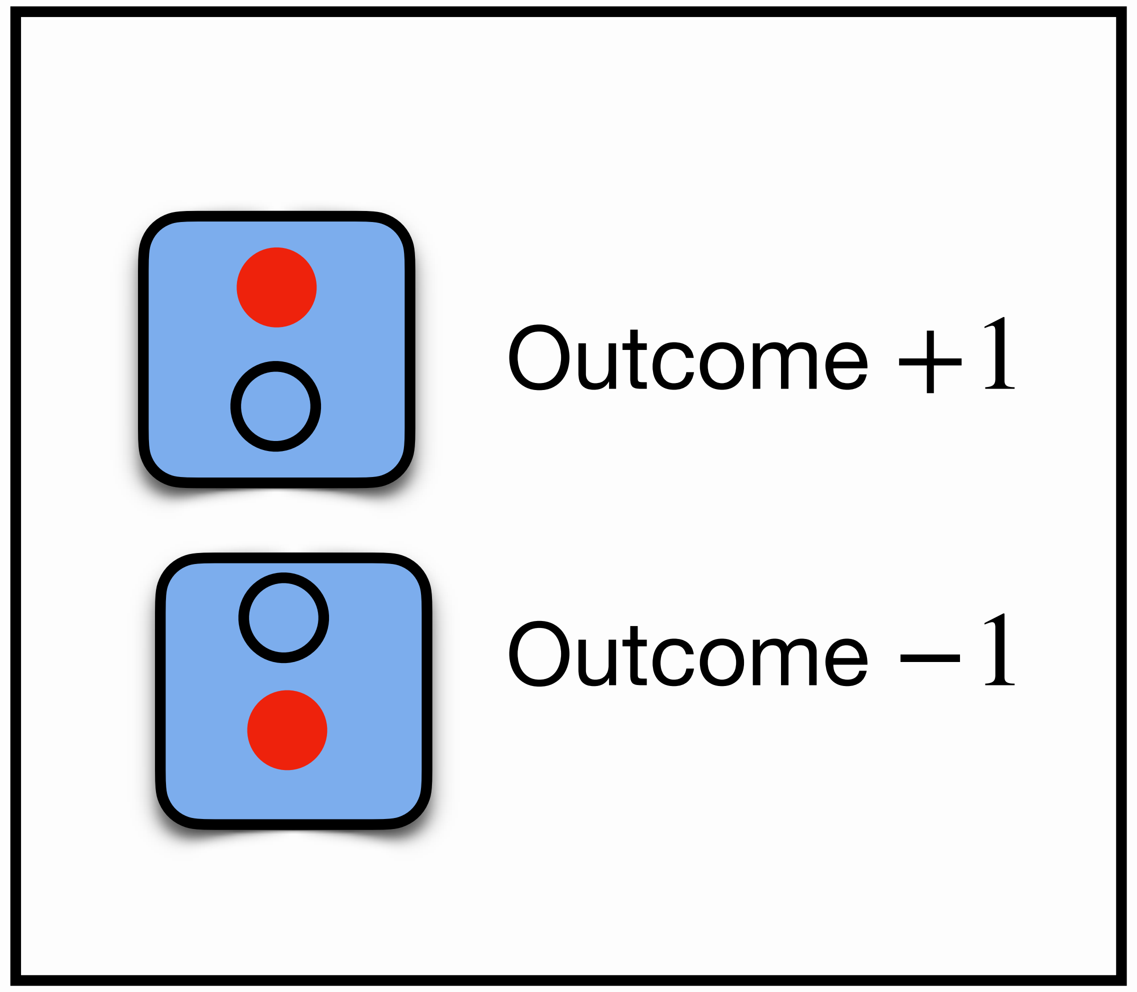
Anomalous weak values

?

CHSH nonlocality

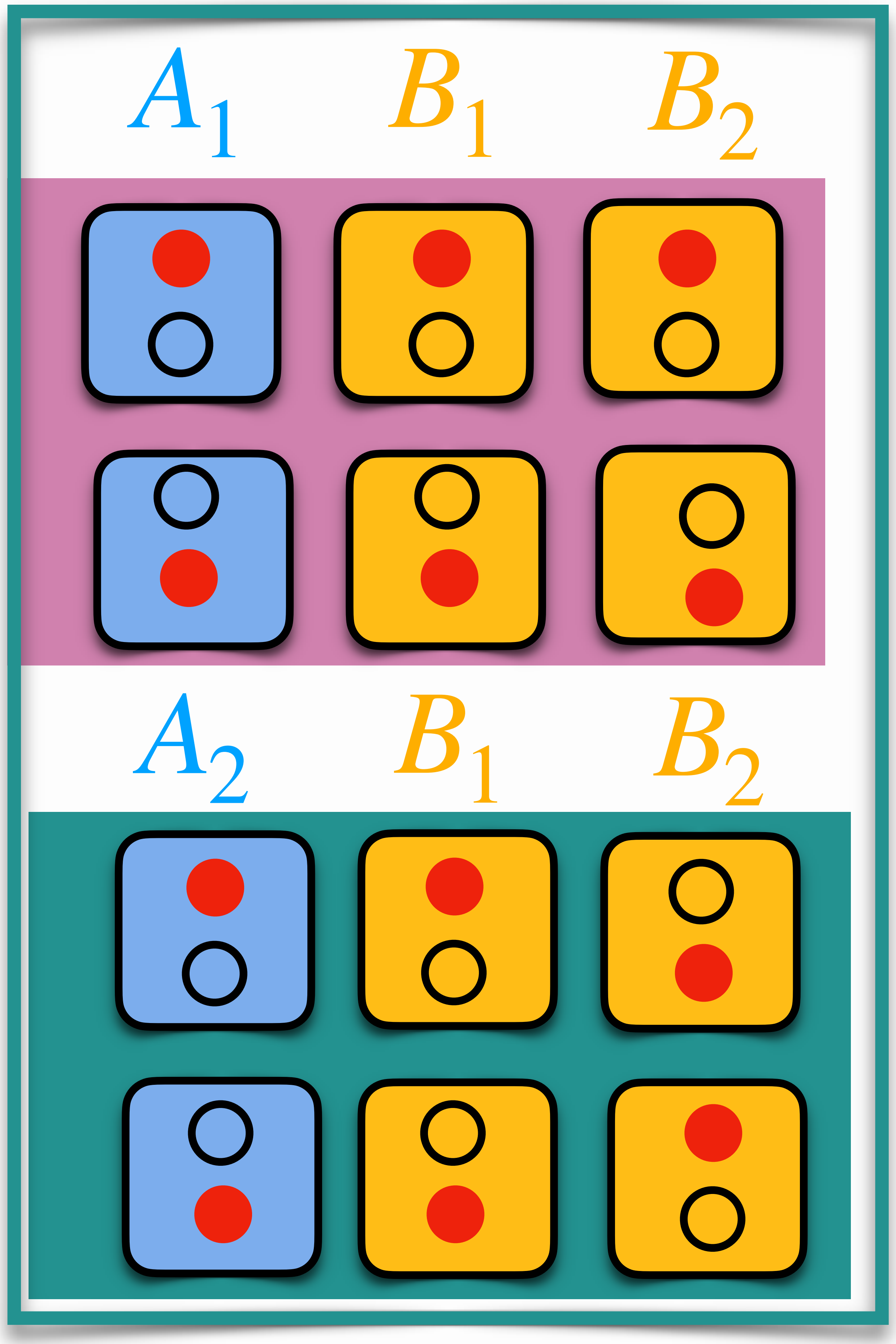
CHSH NL and weak values





CHSH NL and weak values

$$\mathscr{P}_{\mathrm{NL}} = \mathscr{P}(A_1 = B_1 = B_2) + \mathscr{P}(A_2 = B_1 = \overline{B}_2)\,.$$



S. Adhikary et al. *The European Physical Journal D* 74 (2020): 1-8.

S Asthana and V. Ravishankar. *Quantum Information Processing* 20.10 (2021): 1-39

Local terms cancel, correlations survive.

$$\mathscr{P}_{\rm NL} = \mathscr{P}(A_1 = B_1 = B_2) + \mathscr{P}(A_2 = B_1 = \overline{B}_2)\,.$$

$$\mathscr{P}_{\rm NL} < 0 \implies \boxed{\big\langle A_1(B_1 + B_2) + A_2(B_1 - B_2)\big\rangle > -2.}$$

What would be the corresponding weak measurements?

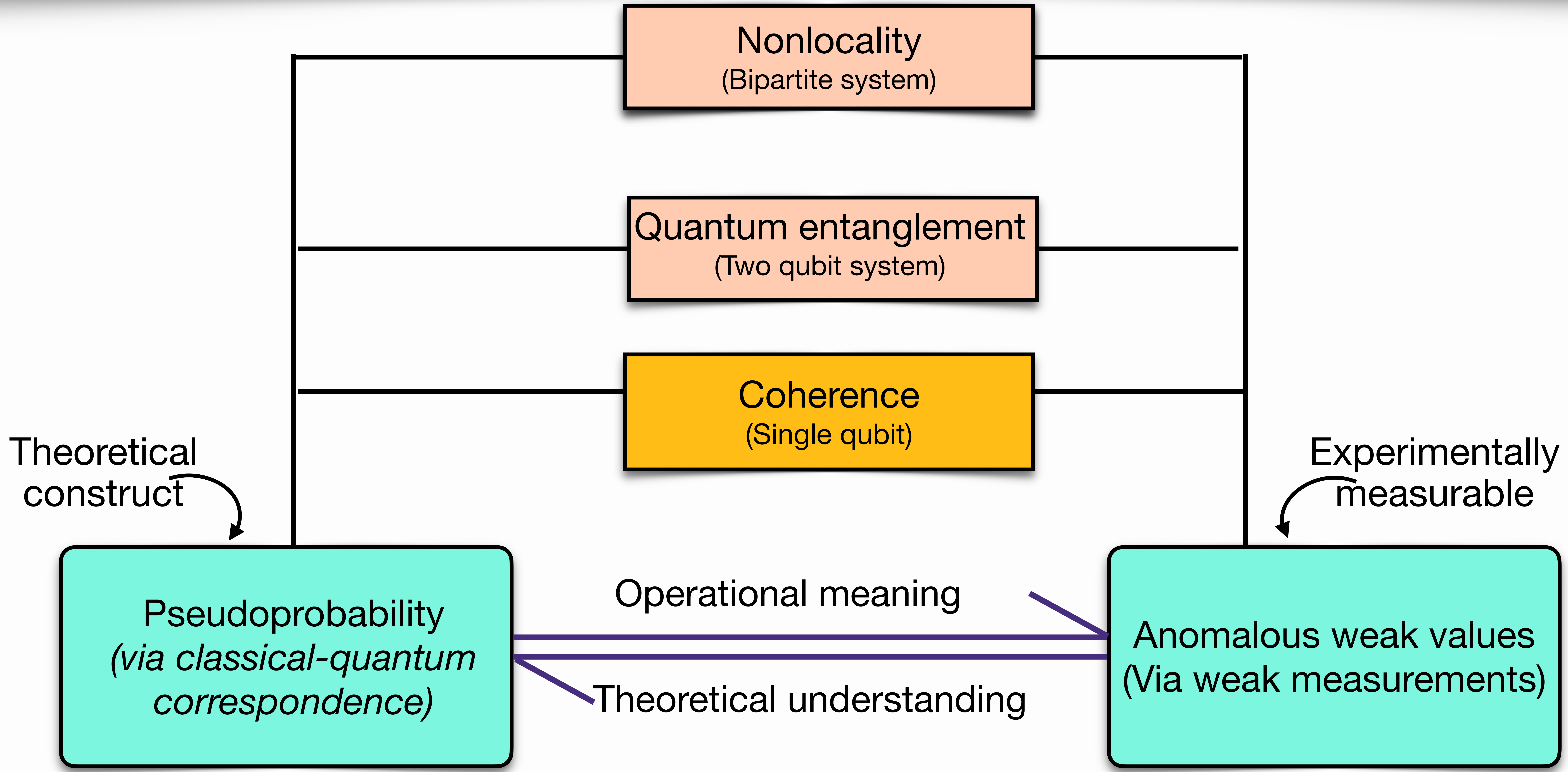
$$\mathscr{P}_{\rm NL} = \langle \pi_{A_1}\pi_{B_1}\rangle \mathrm{Re}\langle \pi_{B_2}\rangle_{\rho}^{\rho_{A_1B_1}} + \langle \pi_{\bar{A}_1}\pi_{\bar{B}_1}\rangle \mathrm{Re}\langle \pi_{\bar{B}_2}\rangle_{\rho}^{\rho_{\bar{A}_1\bar{B}_1}}$$
$$+\langle \pi_{A_2}\pi_{B_1}\rangle \mathrm{Re}\langle \pi_{\bar{B}_2}\rangle_{\rho}^{\rho_{A_2B_1}} + \langle \pi_{\bar{A}_2}\pi_{\bar{B}_1}\rangle \mathrm{Re}\langle \pi_{B_2}\rangle_{\rho}^{\rho_{\bar{A}_2\bar{B}_1}}$$

☞ For two qubit nonlocal Werner states, it is necessary that all the four weak values be negative.

S. Adhikary et al. *The European Physical Journal D* 74 (2020): 1-8.

S Asthana and V. Ravishankar. *Quantum Information Processing* 20.10 (2021): 1-39

Conclusions



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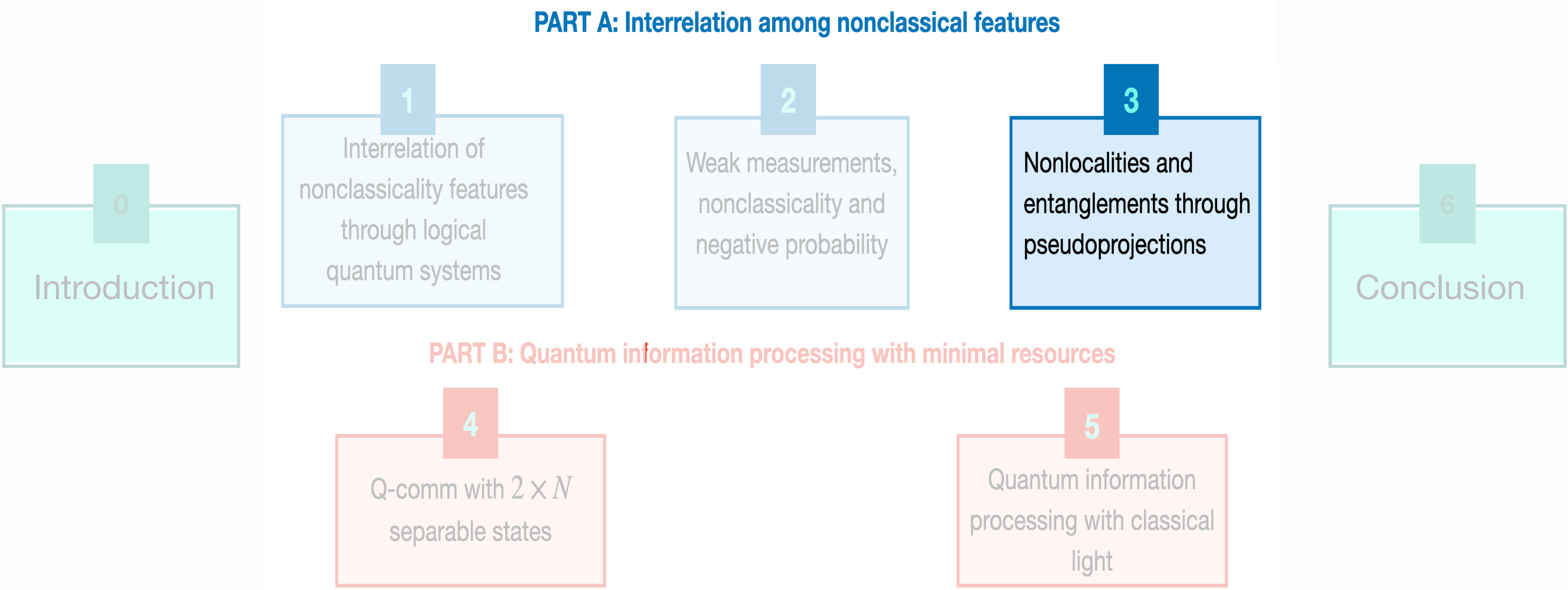
Aharonov et al., *Phys. Rev. Lett*. 60, 1351



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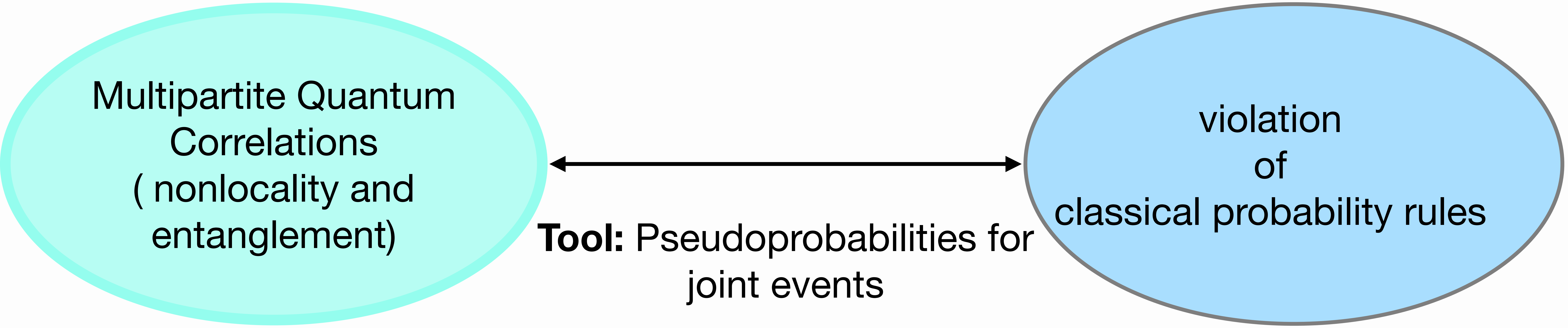
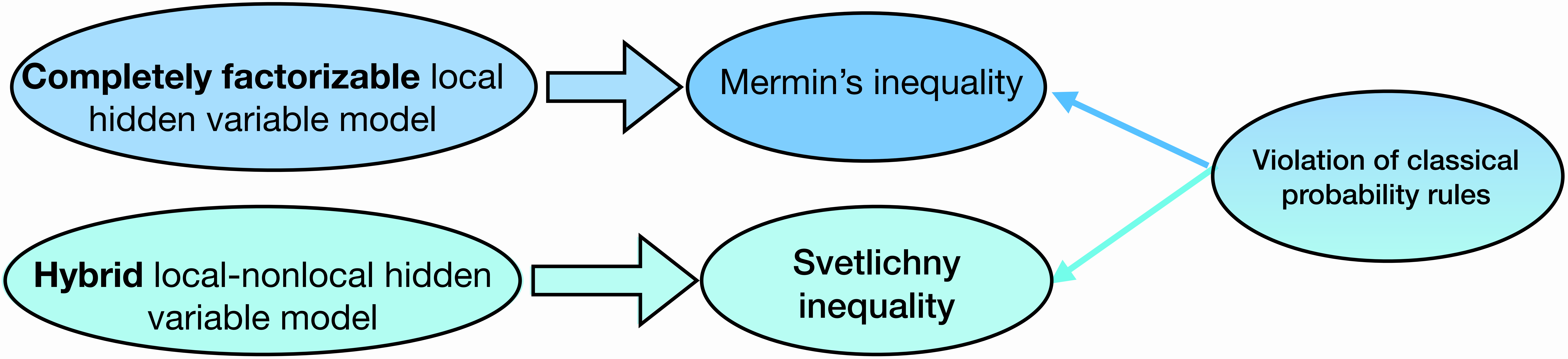
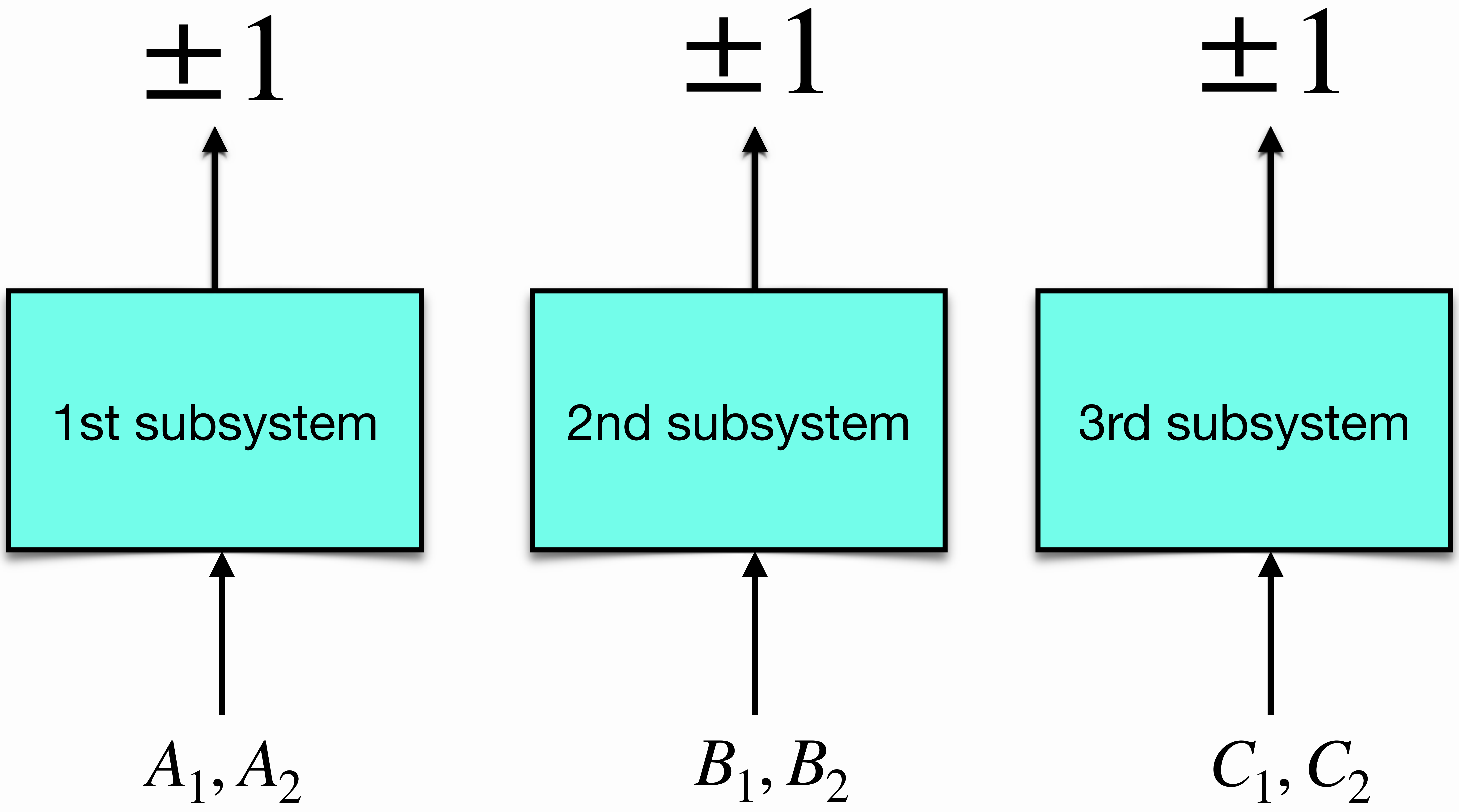
Question of Interest



An Example: Svetlichny and Mermin NL in Tripartite system



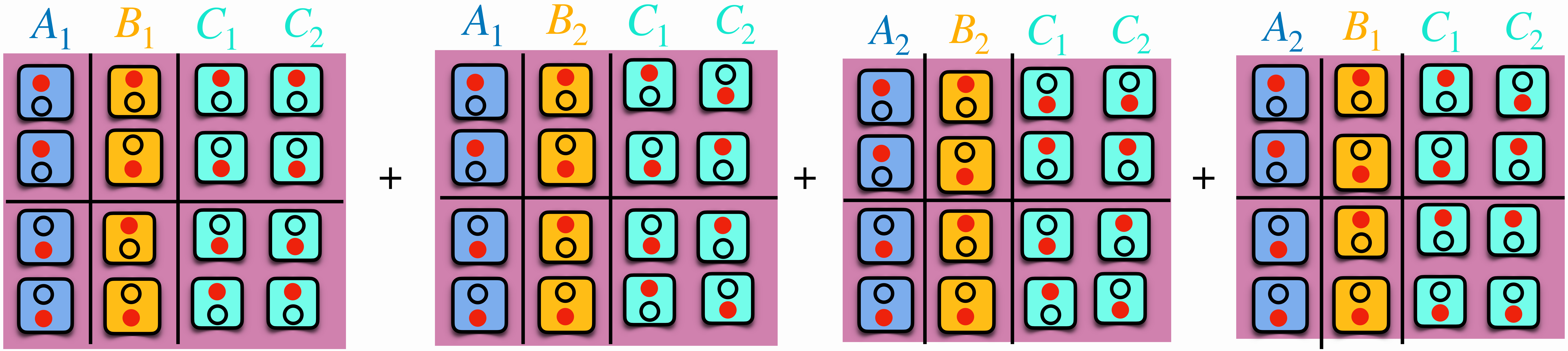
Nonlocality in a three-party system



Svetlichny Nonlocality in a three-party system

- Consider the sum of following eight joint probabilities.

$$\mathscr{P}^1(A_1; B_1 = C_1 = C_2) + \mathscr{P}^2(\overline{A_1}; B_1 = \overline{C_1} = \overline{C_2}) + \mathscr{P}^3(A_1; B_2 = C_1 = \overline{C_2}) + \mathscr{P}^4(\overline{A_1}; B_2 = \overline{C_1} = C_2) +$$
$$\mathscr{P}^5(A_2; B_2 = \overline{C_1} = \overline{C_2}) + \mathscr{P}^6(\overline{A_2}; B_1 = \overline{C_1} = C_2) + \mathscr{P}^7(A_2; B_1 = C_1 = \overline{C_2}) + \mathscr{P}^8(\overline{A_2}; B_2 = C_1 = C_2)$$



$\mathscr{P}^1, \mathscr{P}^2$ $\mathscr{P}^3, \mathscr{P}^4$ $\mathscr{P}^5, \mathscr{P}^6$ $\mathscr{P}^7, \mathscr{P}^8$

Svetlichny Nonlocality in a three-party system

- Consider the sum of following eight joint probabilities.

$$\mathscr{P}^1(A_1; B_1 = C_1 = C_2) + \mathscr{P}^2(\overline{A_1}; B_1 = \overline{C_1} = \overline{C_2}) + \mathscr{P}^3(A_1; B_2 = C_1 = \overline{C_2}) + \mathscr{P}^4(\overline{A_1}; B_2 = \overline{C_1} = C_2) +$$
$$\mathscr{P}^5(A_2; B_2 = \overline{C_1} = \overline{C_2}) + \mathscr{P}^6(\overline{A_2}; B_1 = \overline{C_1} = C_2) + \mathscr{P}^7(A_2; B_1 = C_1 = \overline{C_2}) + \mathscr{P}^8(\overline{A_2}; B_2 = C_1 = C_2)$$

Why this combination

Local terms cancel,
Two-body correlations cancel,
Three-body correlations survive.

Joint outcomes of two observables for the third party.

Svetlichny Nonlocality in a three-party system

- Consider the sum of following eight joint probabilities.

$$\mathscr{P}^1(A_1; B_1 = C_1 = C_2) + \mathscr{P}^2(\overline{A_1}; B_1 = \overline{C_1} = \overline{C_2}) + \mathscr{P}^3(A_1; B_2 = C_1 = \overline{C_2}) + \mathscr{P}^4(\overline{A_1}; B_2 = \overline{C_1} = C_2) +$$
$$\mathscr{P}^5(A_2; B_2 = \overline{C_1} = \overline{C_2}) + \mathscr{P}^6(\overline{A_2}; B_1 = \overline{C_1} = C_2) + \mathscr{P}^7(A_2; B_1 = C_1 = \overline{C_2}) + \mathscr{P}^8(\overline{A_2}; B_2 = C_1 = C_2)$$

- The joint probability $\mathscr{P} = \sum_{i=1}^{8} \mathscr{P}_i .$

- Imposing classicality condition: $0 \leq \mathscr{P} \leq 1$

$$|A_1B_1C_1 + A_1B_1C_2 + A_2B_1C_1 - A_2B_1C_2 + A_1B_2C_1 - A_1B_2C_2 - A_2B_2C_1 - A_2B_2C_2| \leq 4$$

Svetlichny inequality

This analysis holds for arbitrary finite dimensions of subsystems.

Increment in the number of observables in the joint probabilities?

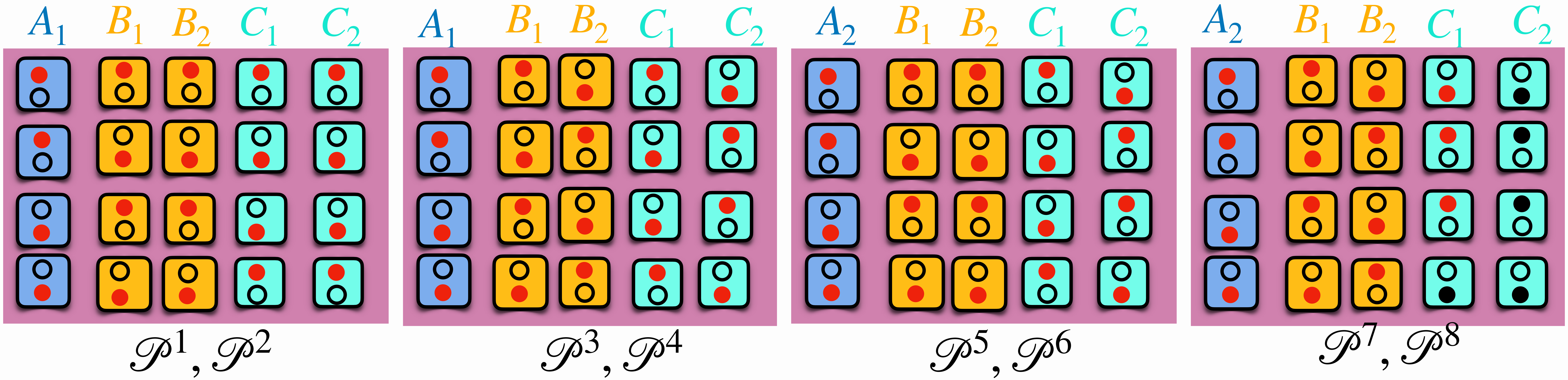
Mermin Nonlocality in a three-party system

$$\mathscr{P}^1(A_1; B_1 = B_2 = C_1 = C_2) + \mathscr{P}^2(\overline{A}_1; B_1 = B_2 = \overline{C}_1 = \overline{C}_2) +$$

$$\mathscr{P}^3(A_1; B_1 = \overline{B}_2 = C_1 = \overline{C}_2) + \mathscr{P}^4(\overline{A}_1; B_1 = \overline{B}_2 = \overline{C}_1 = C_2) +$$

$$\mathscr{P}^5(A_2; B_1 = B_2 = C_1 = \overline{C}_2) + \mathscr{P}^6(\overline{A}_2; B_1 = B_2 = \overline{C}_1 = C_2) +$$

$$\mathscr{P}^7(A_2; B_1 = \overline{B}_2 = \overline{C}_1 = \overline{C}_2) + \mathscr{P}^8(\overline{A}_2; B_1 = \overline{B}_2 = C_1 = C_2)$$



Mermin Nonlocality in a three-party system

$$\mathscr{P}^1(A_1; B_1 = B_2 = C_1 = C_2) + \mathscr{P}^2(\overline{A}_1; B_1 = B_2 = \overline{C}_1 = \overline{C}_2)+$$
$$\mathscr{P}^3(A_1; B_1 = \overline{B}_2 = C_1 = \overline{C}_2) + \mathscr{P}^4(\overline{A}_1; B_1 = \overline{B}_2 = \overline{C}_1 = C_2)+$$
$$\mathscr{P}^5(A_2; B_1 = B_2 = C_1 = \overline{C}_2) + \mathscr{P}^6(\overline{A}_2; B_1 = B_2 = \overline{C}_1 = C_2)+$$
$$\mathscr{P}^7(A_2; B_1 = \overline{B}_2 = \overline{C}_1 = \overline{C}_2) + \mathscr{P}^8(\overline{A}_2; B_1 = \overline{B}_2 = C_1 = C_2)$$

Joint outcomes of **two observables for the second and the third party.**

- The joint probability $$\mathscr{P} = \sum_{i=1}^{8} \mathscr{P}_i .$$
- Imposing classicality condition: $0 \leq \mathscr{P} \leq 1$

$$\boxed{|A_1(B_1C_1 + B_2C_2) + A_2(B_1C_2 - B_2C_1)| \leq 2}$$

N. D. Mermin, Phys. Rev. Lett. 65, 1838 (1990).

Conclusions and generalisations

Negative pseudo probability

N-party Svetlichny Nonlocality

N-party Mermin Nonlocality

Entanglement in N qubits

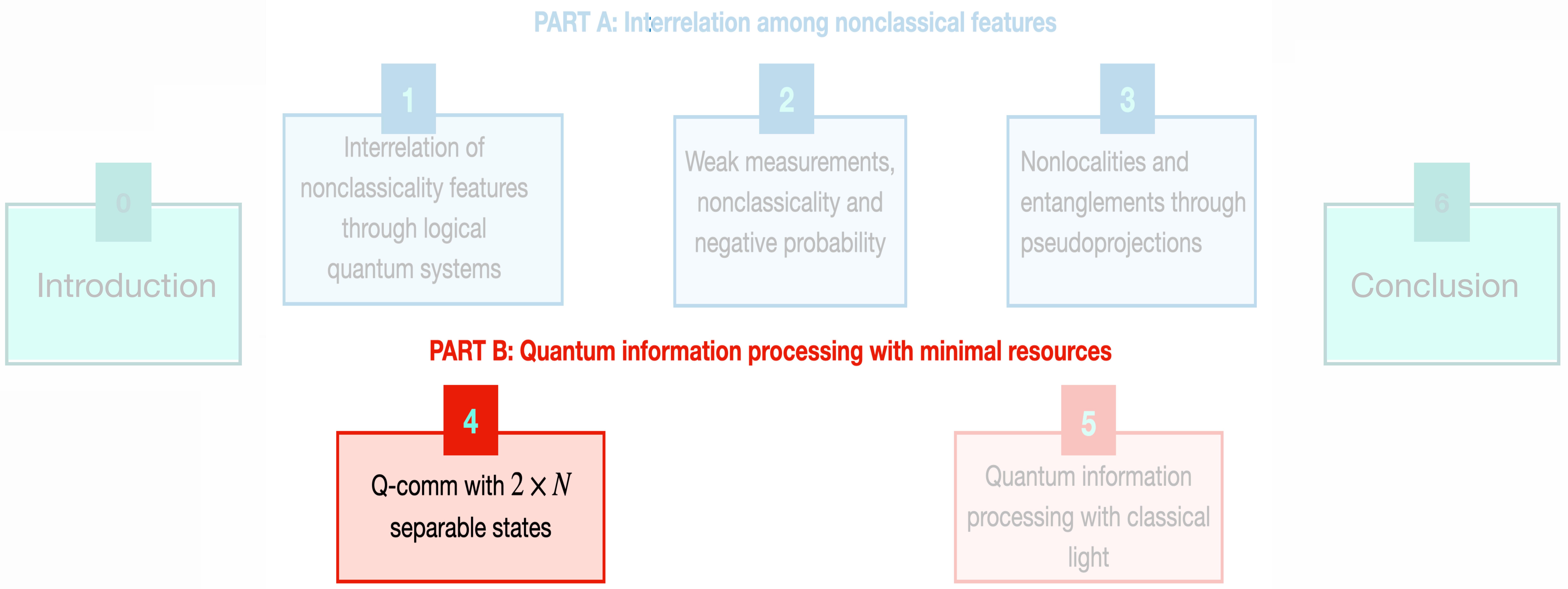
How does the difference show up?

Hybrid local nonlocal model $\rightarrow$ $(1,1,2)$ pseudoprobability

Fully factorisable LHV $\rightarrow$ $(1,2,2)$ pseudoprobability

Task: Identification of suitable events.
No mathematically involved technique

Generalisation to high-dimensional systems and multi input-output scenarios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Motivation

- **Entangled states:** Required for Q-comm, e.g., quantum teleportation.
- **Low conversion efficiency of** SPDC

Can we do remote transfer of information with separable states?

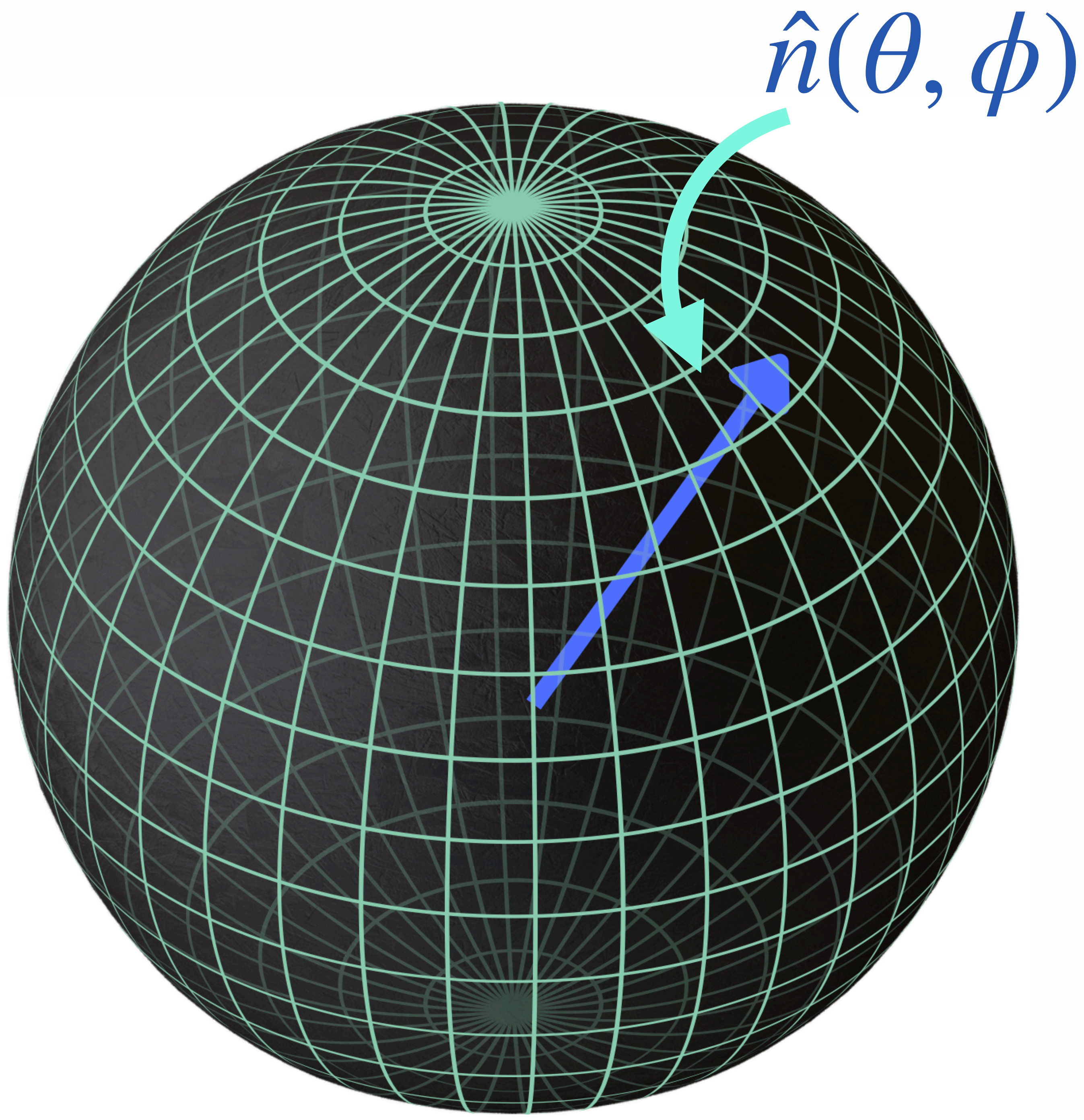
- **Special class of states:** $2 \times N$ dimensional separable states, identified as equivalent states[1].
- **Experimental generation:** Incoherent superpositions of $SU(2)$ coherent OAM states with appropriate polarisations[2].

---

1. Bharath and Ravishankar, *Phys. Rev. A* 89, 062110 (2014)
2. Tuan *et al.*, *IEEE J. Sel. Top. Quantum Electron.* 24, 1–9 (2018)

- **$SU(2)$ coherent states[1]:**

$$|\hat{n}(\theta,\phi)\rangle = e^{-iS_z\phi}e^{-iS_y\theta}|S_z=+S\rangle\,.$$



- The set of $SU(2)$ coherent states $\{|\hat{n}(\theta,\phi)\rangle\}$: over complete
- Number of states = number of points on the Bloch sphere
- **Q-Representation[1]:**

$$F(\hat{n}) = \frac{2S+1}{4\pi}\langle\hat{n}(\theta,\phi)|\rho|\hat{n}(\theta,\phi)\rangle$$

[1] J. M. Radcliffe, *J Phys A: Gen Phys* 4.3 (1971): 313.

Equivalent states

$\rho_1 \in \mathscr{H}_1 \cong \rho_2 \in \mathscr{H}_2$ if their Q-representations are the same.

Equivalent observables

$O_1 \cong O_2$ if $\mathrm{Tr}(\rho_1 O_1) = \mathrm{Tr}(\rho_2 O_2)$

Example

- Single qubit: $\rho^{\left[\frac{1}{2}\right]} = \frac{1}{2}(\mathbf{1} + \vec{\sigma}\cdot\hat{p})$
- Equivalent $2S+1$ dimensional state:

$$\rho^{[S]} = \frac{1}{2S+1}(\mathbf{1} + \hat{S}\cdot\hat{p}); \quad \hat{S} = \frac{\vec{S}}{S}$$

Example

- Observable for the qubit $\hat{O} = \vec{\sigma}\cdot\hat{n}$
- Equivalent observable:

$$\hat{O}' = \frac{3S}{S+1}\hat{S}\cdot\hat{n}$$

H. M. Bharath, and V. Ravishankar. *Phys. Rev. A* 89.6 (2014): 062110.

Extension to bipartite systems

$$\rho^{AB} \in \mathscr{H}^{A} \otimes \mathscr{H}^{B} \cong \rho'^{AB} \in \mathscr{H}'^{A} \otimes \mathscr{H}'^{B} \text{ if}$$

$$\langle \hat{m}, \hat{n} | \rho^{AB} | \hat{m}, \hat{n} \rangle = \langle \hat{m}, \hat{n} | \rho'^{AB} | \hat{m}, \hat{n} \rangle \ \forall \hat{m}, \hat{n}$$

Is it only a straightforward generalisation?

H. M. Bharath, and V. Ravishankar. *Phys. Rev. A* 89.6 (2014): 062110.

Setting up the stage

Extension to bipartite systems

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An apparent paradox

$$\begin{tikzcd}
 & \rho^{AB} \arrow[r, "\text{Coherent basis}"] & F(\hat{m}, \hat{n}) \arrow[d, "\text{Classical separability}"] \\
\sum_i p_i \rho_i^A \otimes \rho_i^B & & \sum_i p_i f_i(\hat{m}) g_i(\hat{n}) \arrow[ll, "\text{Retrieving the Quantum state}"]
\end{tikzcd}$$

H. M. Bharath, and V. Ravishankar. *Phys. Rev. A* 89.6 (2014): 062110.

Setting up the stage

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\sum_i p_i \rho_i^A \otimes \rho_i^B \arrow[u, "?"] & & \sum_i p_i f_i(\hat{m}) g_i(\hat{n}) \arrow[ll, "\text{Retrieving the Quantum state}"]
\end{tikzcd}$$

Classical simulation of entangled states

H. M. Bharath, and V. Ravishankar. *Phys. Rev. A* 89.6 (2014): 062110.

Example: classical simulation of entangled two qubit Werner states

$$\rho^{[2\otimes 2]} = \frac{1}{4}(\mathbf{1} - \alpha\vec{\sigma}_1 \cdot \vec{\sigma}_2)\ \alpha \in \left[-\frac{1}{3}, 1\right]$$

$$\rho^{[2\otimes(2S+1)]} = \frac{1}{2(2S+1)}\left(\mathbf{1} - \alpha\vec{\sigma}_1 \cdot \hat{S}_2\right)$$

Separable when $|\alpha| \leq \dfrac{S}{S+1}$.

- For a given α,

$$S_{\min} > \left[\left[\frac{|\alpha|}{1-|\alpha|}\right]\right]$$

$[[x]]$: smallest half-integer greater than or equal to x.

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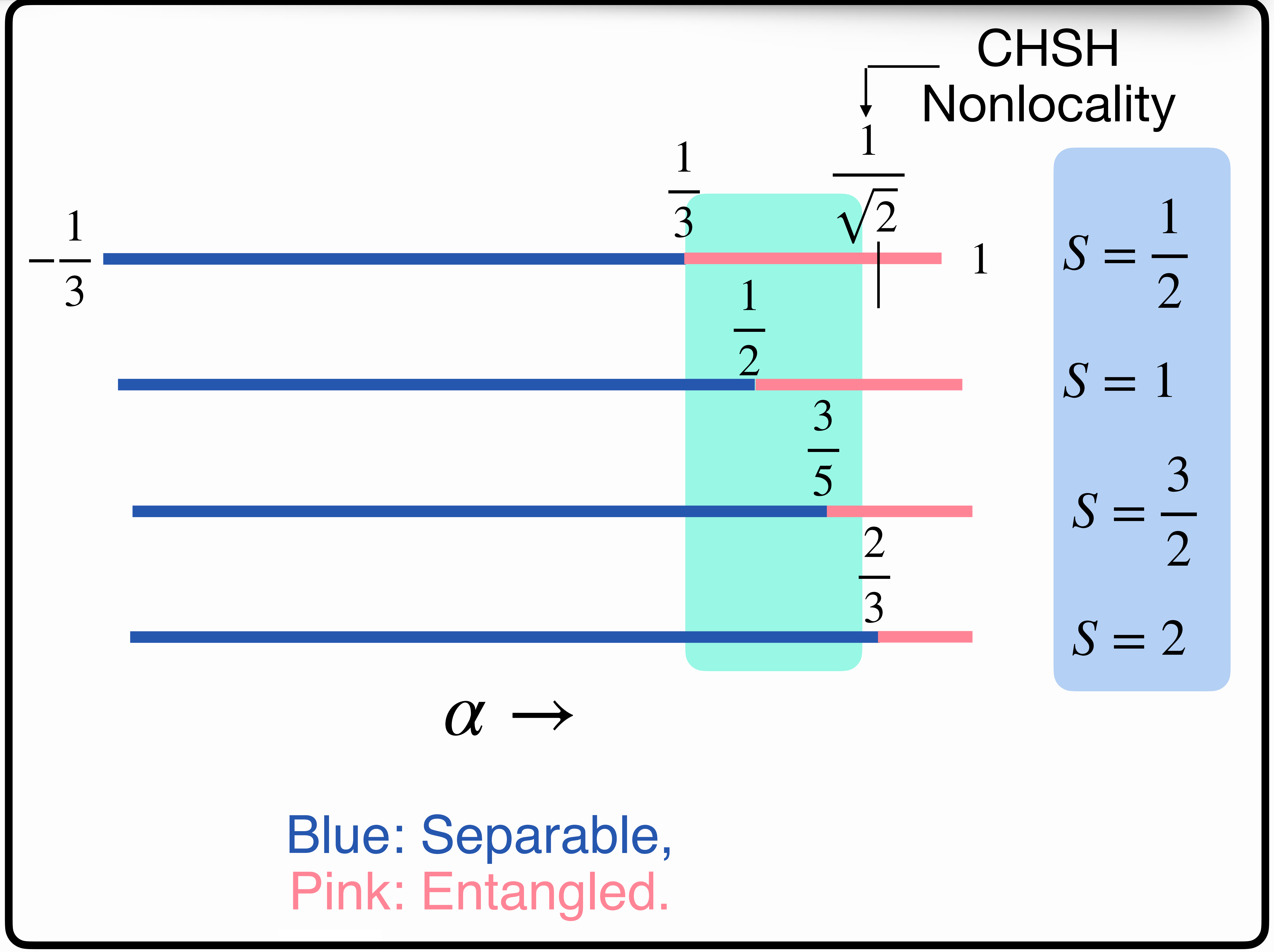

1. Range for separability increases with dimensions.
2. Higher ent, higher dim for sep equivalent.

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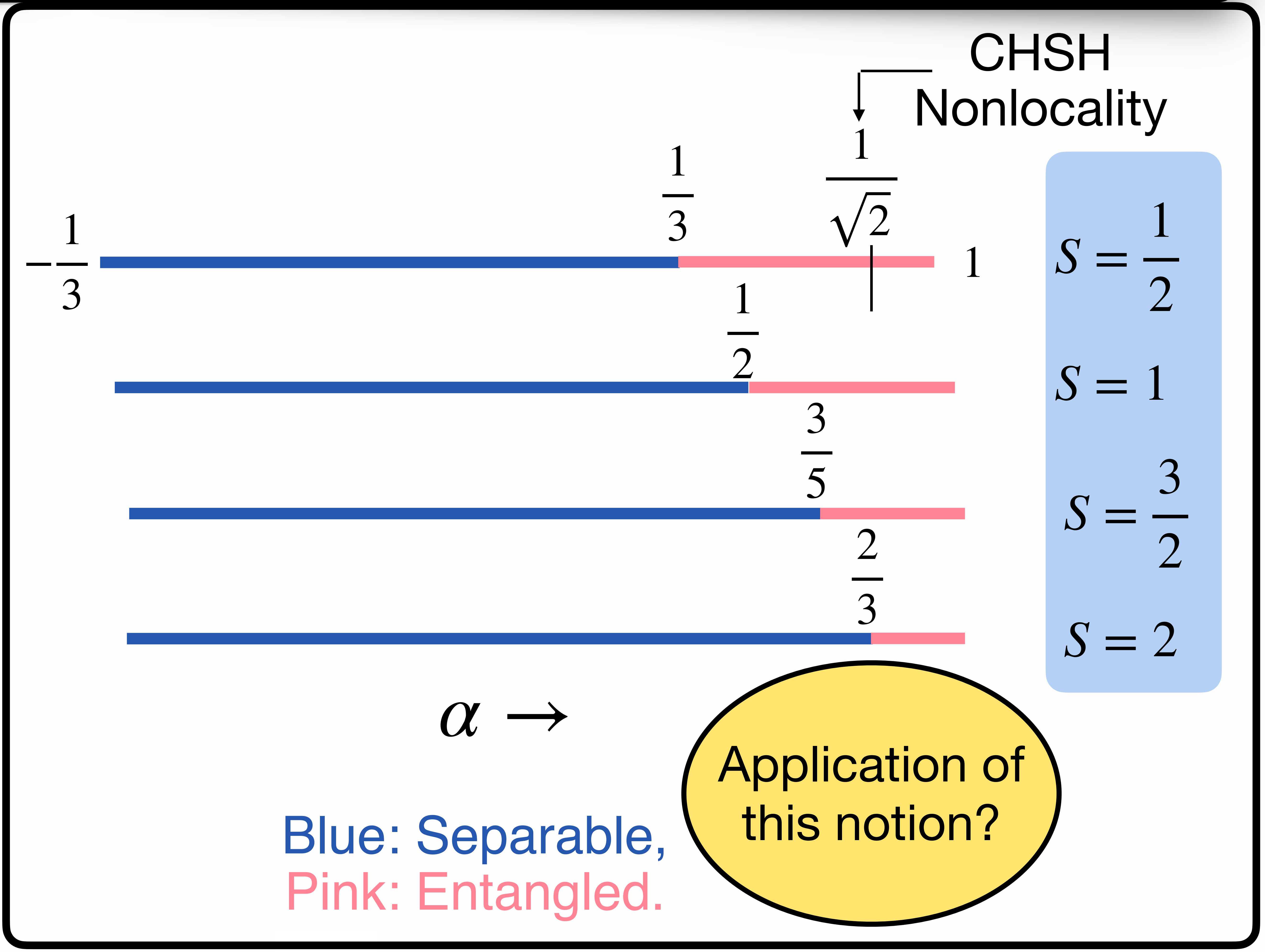
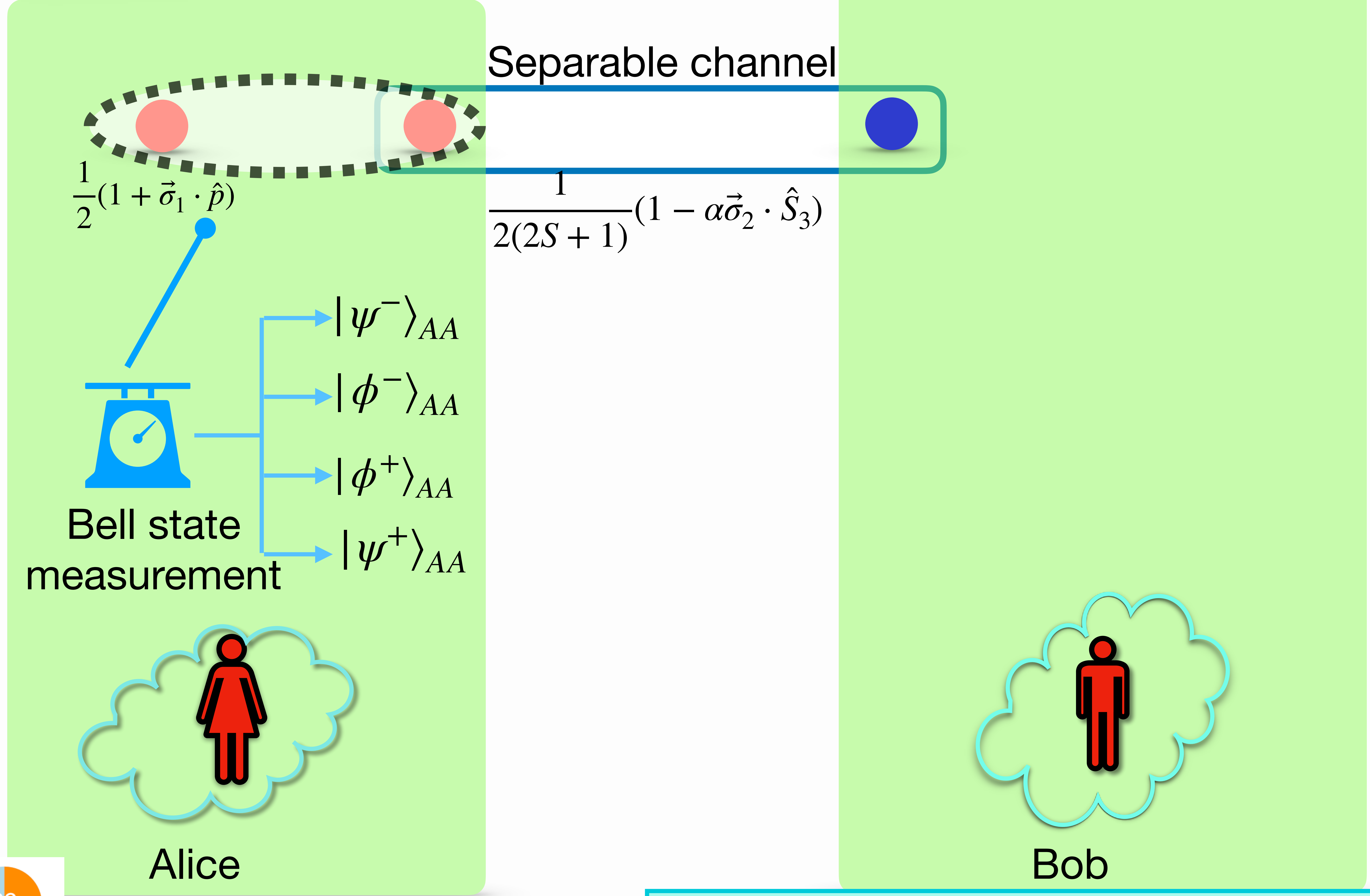


1. Range for separability increases with dimensions.
2. Higher ent, higher dim for sep equivalent.

Application: Transfer of information form an unknown qubit to a remote qudit



$$|\psi^{\pm}\rangle_{AA} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

$$|\phi^{\pm}\rangle_{AA} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

S. Asthana, R. Bala, V. Ravishankar *Quant Inf Proc.* 2022 Jan;21(1):1-24.

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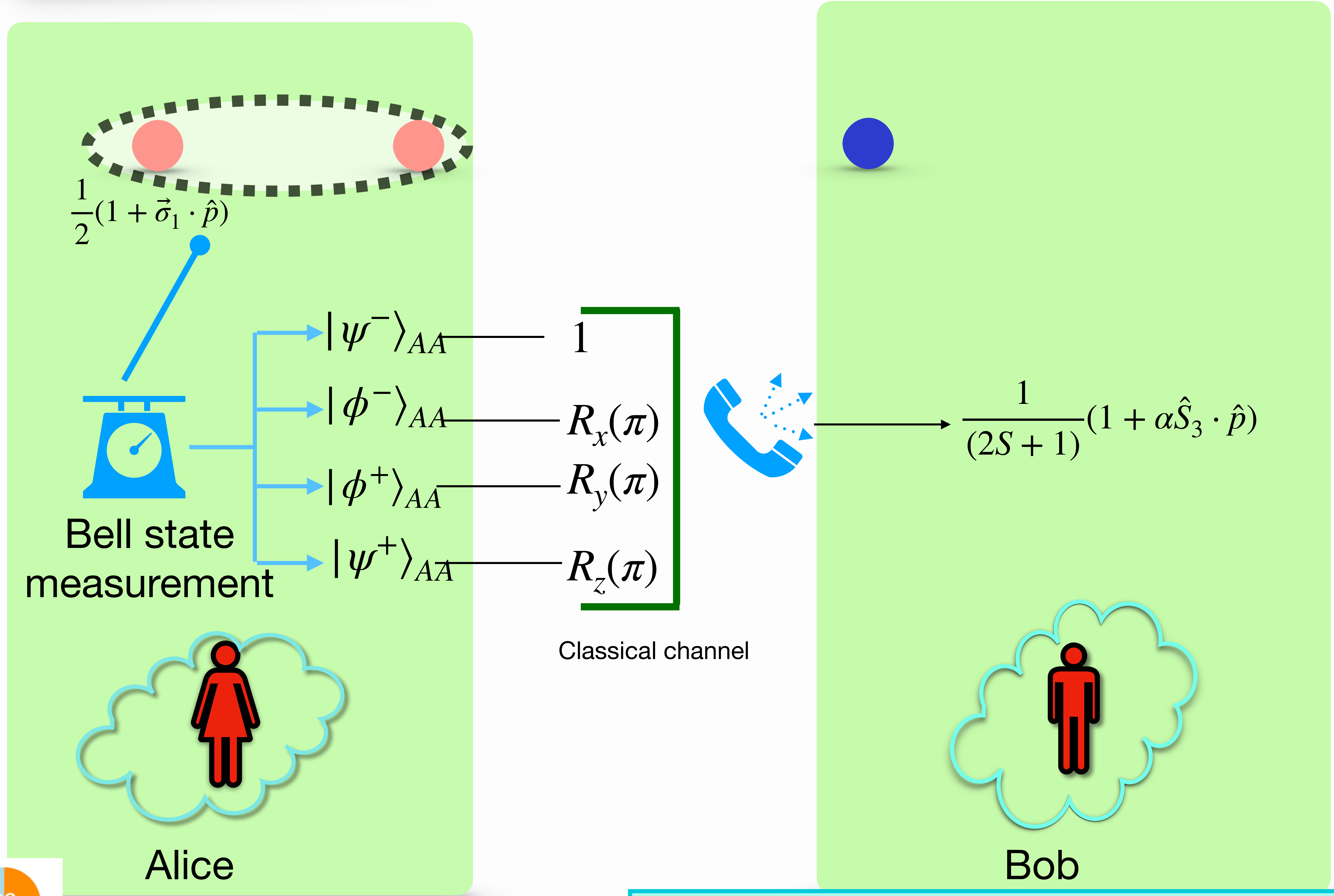

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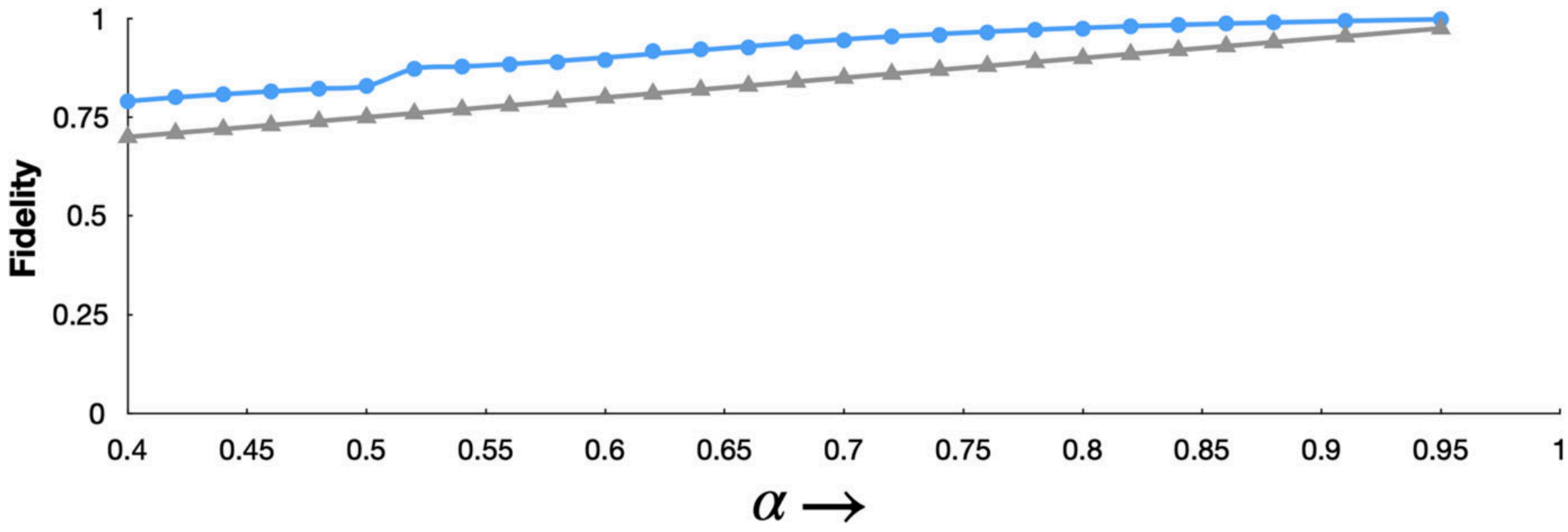
Performance of the protocol?

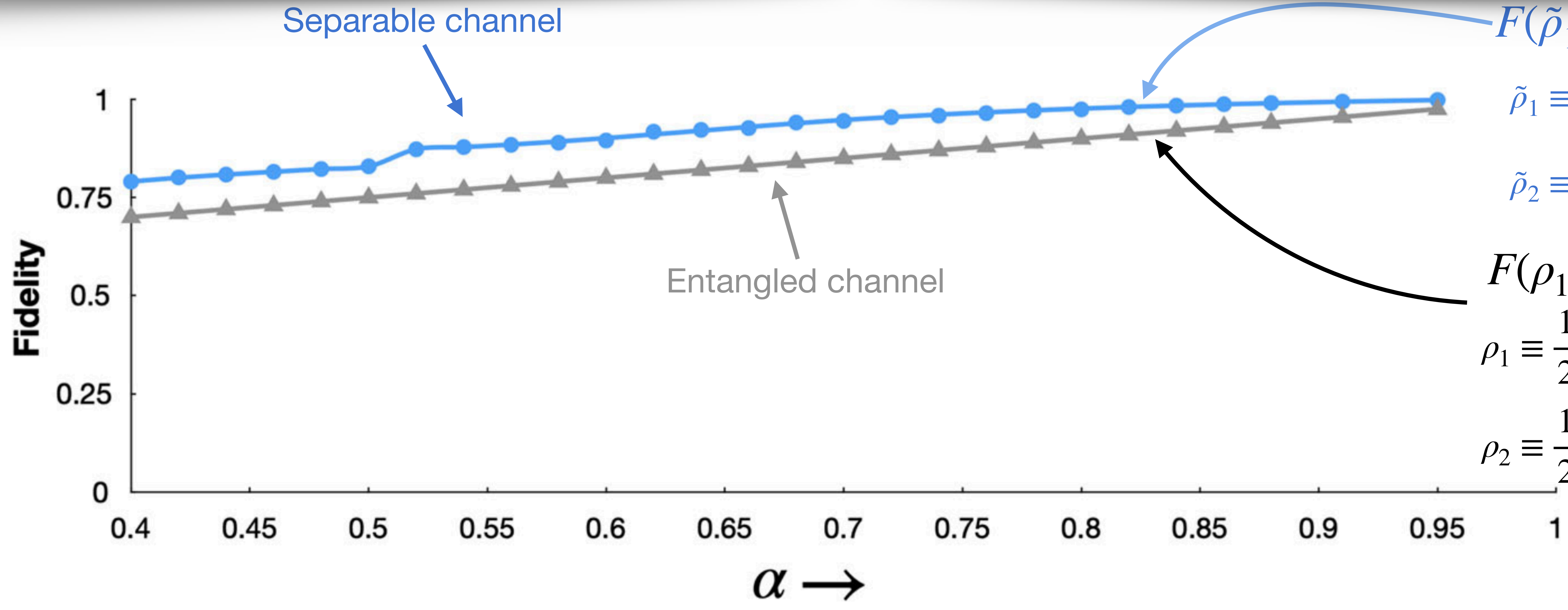
S. Asthana, R. Bala, V. Ravishankar
Quant Inf Proc. 2022 Jan;21(1):1-24.

Figure of merit: fidelity



Blue: Plot of $F(\tilde{\rho}_1, \tilde{\rho}_2)$ with respect to α for $S_{\min}$ when a noisy separable channel is used.
Grey: Plot of $F(\rho_1, \rho_2)$ with respect to α.

Figure of merit: fidelity



$F(\tilde{\rho}_1, \tilde{\rho}_2)$

$$\tilde{\rho}_1 \equiv \frac{1}{2S+1}(1 + \alpha \hat{S}_3 \cdot p)$$

$$\tilde{\rho}_2 \equiv \frac{1}{2S+1}(1 + \hat{S}_3 \cdot p)$$

$F(\rho_1, \rho_2)$

$$\rho_1 \equiv \frac{1}{2}(1 + \alpha \vec{\sigma}_3 \cdot p)$$

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Blue: Plot of $F(\tilde{\rho}_1, \tilde{\rho}_2)$ with respect to α for S_{min} when a noisy separable channel is used.

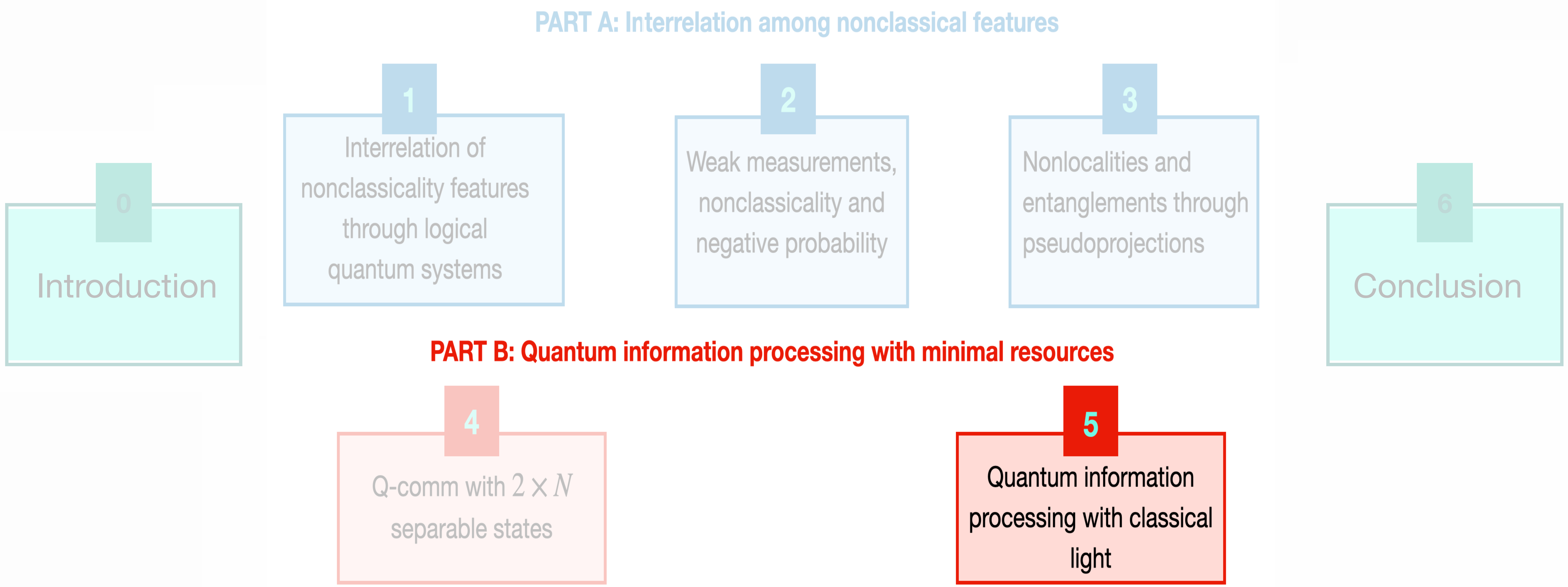
Grey: Plot of $F(\rho_1, \rho_2)$ with respect to α.

$$F(\rho_1, \rho_2) = \left(\mathrm{Tr} \sqrt{\sqrt{\rho_1} \rho_2 \sqrt{\rho_1}} \right)^2$$

Conclusion and future scope

- Quantum communication protocols using separable equivalent states.
- Transfer of information encoded in qutrits: protocols by employing equivalent states of two-qutrit entangled states[1].

S. Adhikary, I. K. Panda, and V. Ravishankar. *Annals of Physics* 377 (2017): 87-95.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Classical-quantum interplay

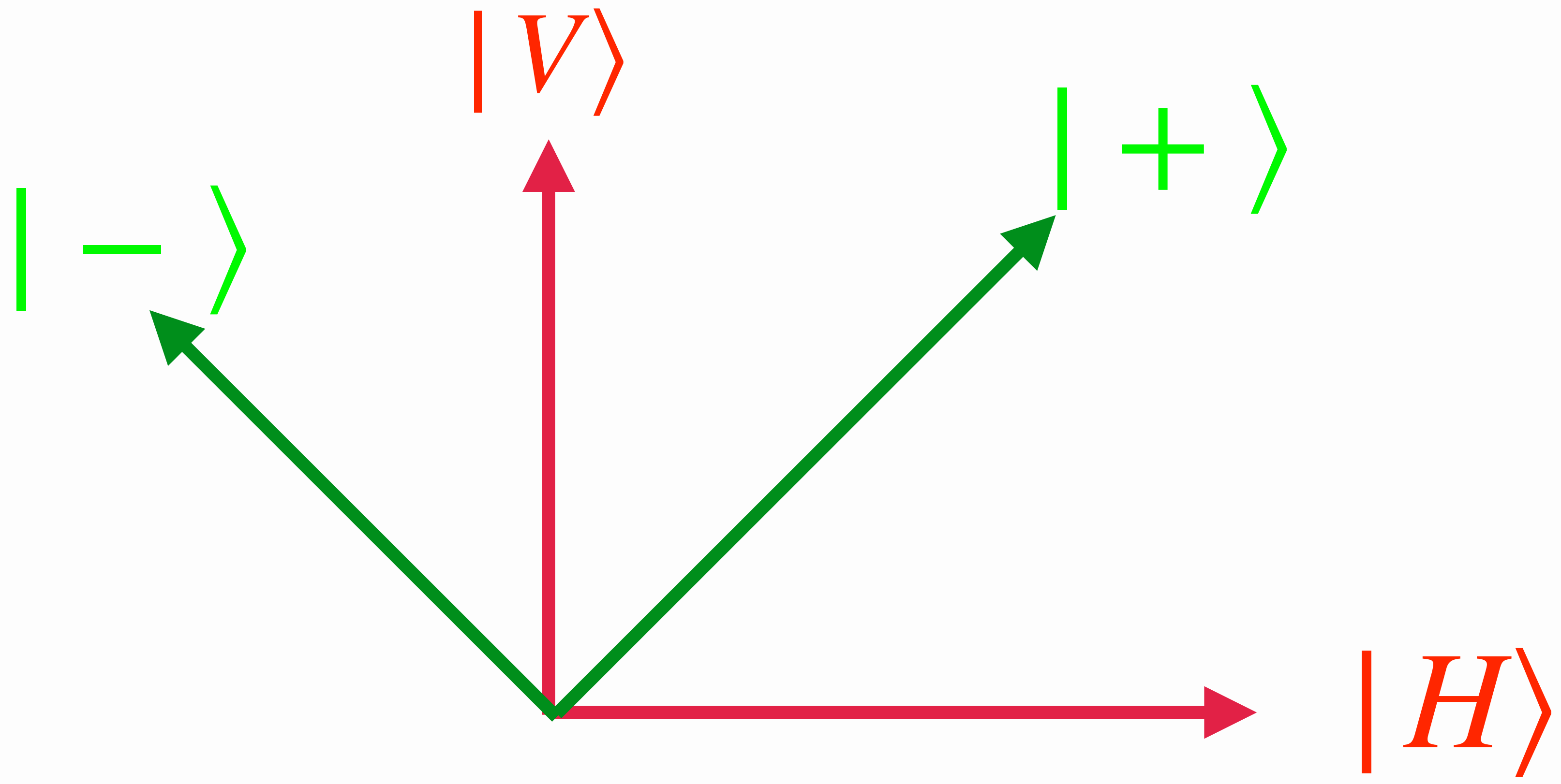
What is “quantum” in quantum information processing?
(Which “quantum” feature acts as a resource in a particular protocol?)

A re-look at “classical” and “quantum” resources **with classical waves included**.

Classical bits

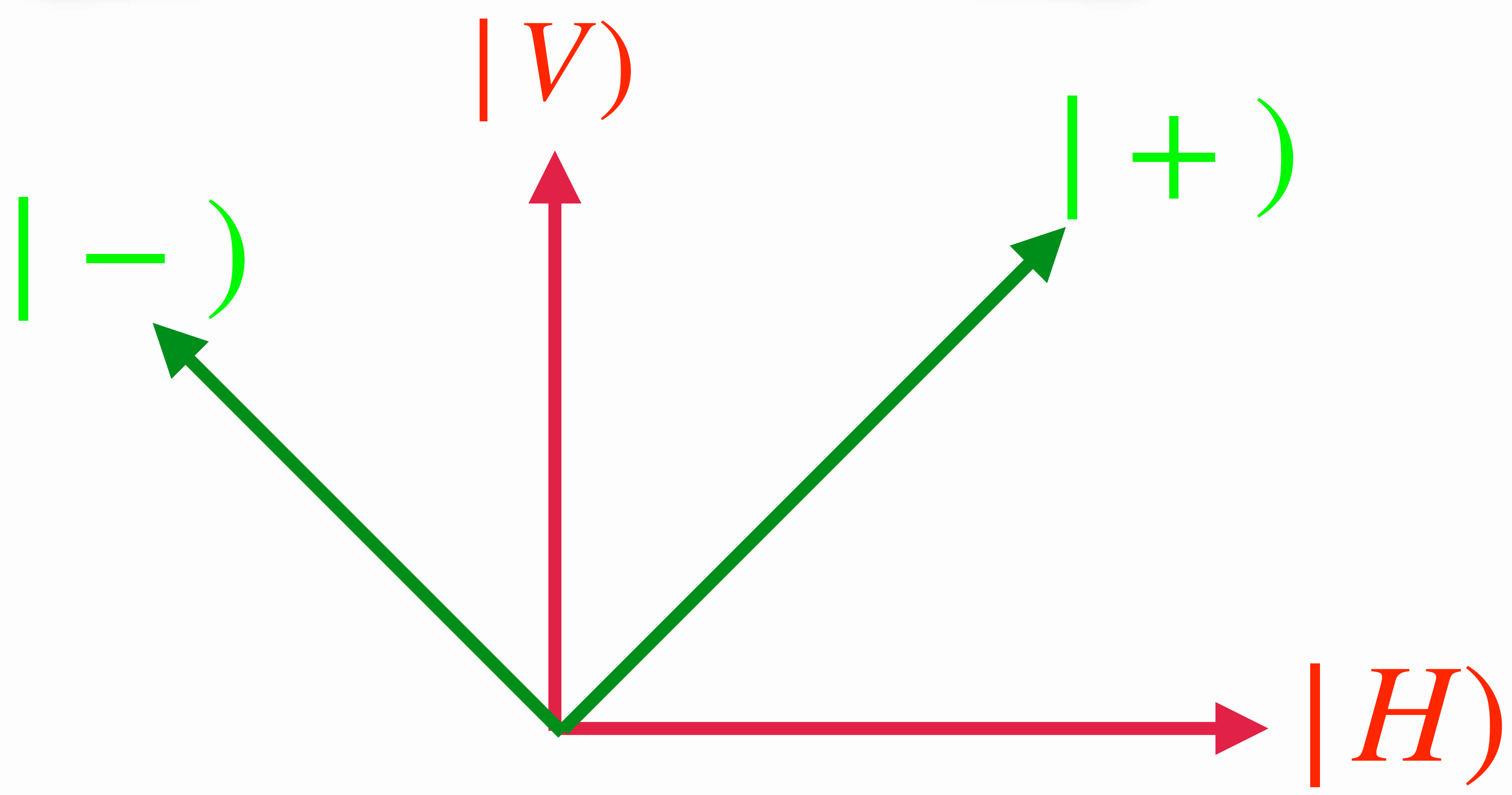
Does not allow for superposition

Quantum bits



Allows for superposition

Classical bits



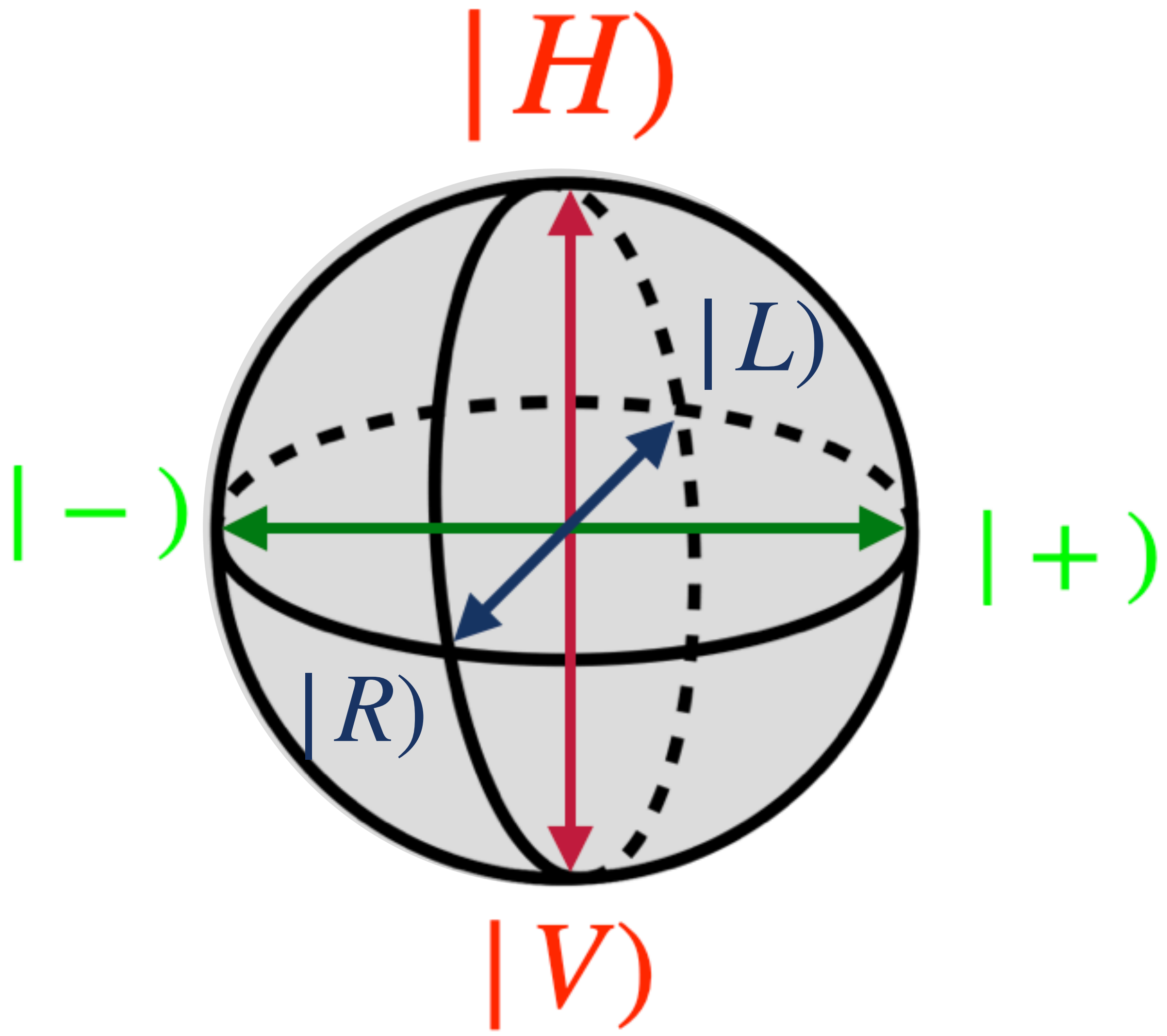
Classical waves

Allows for superposition

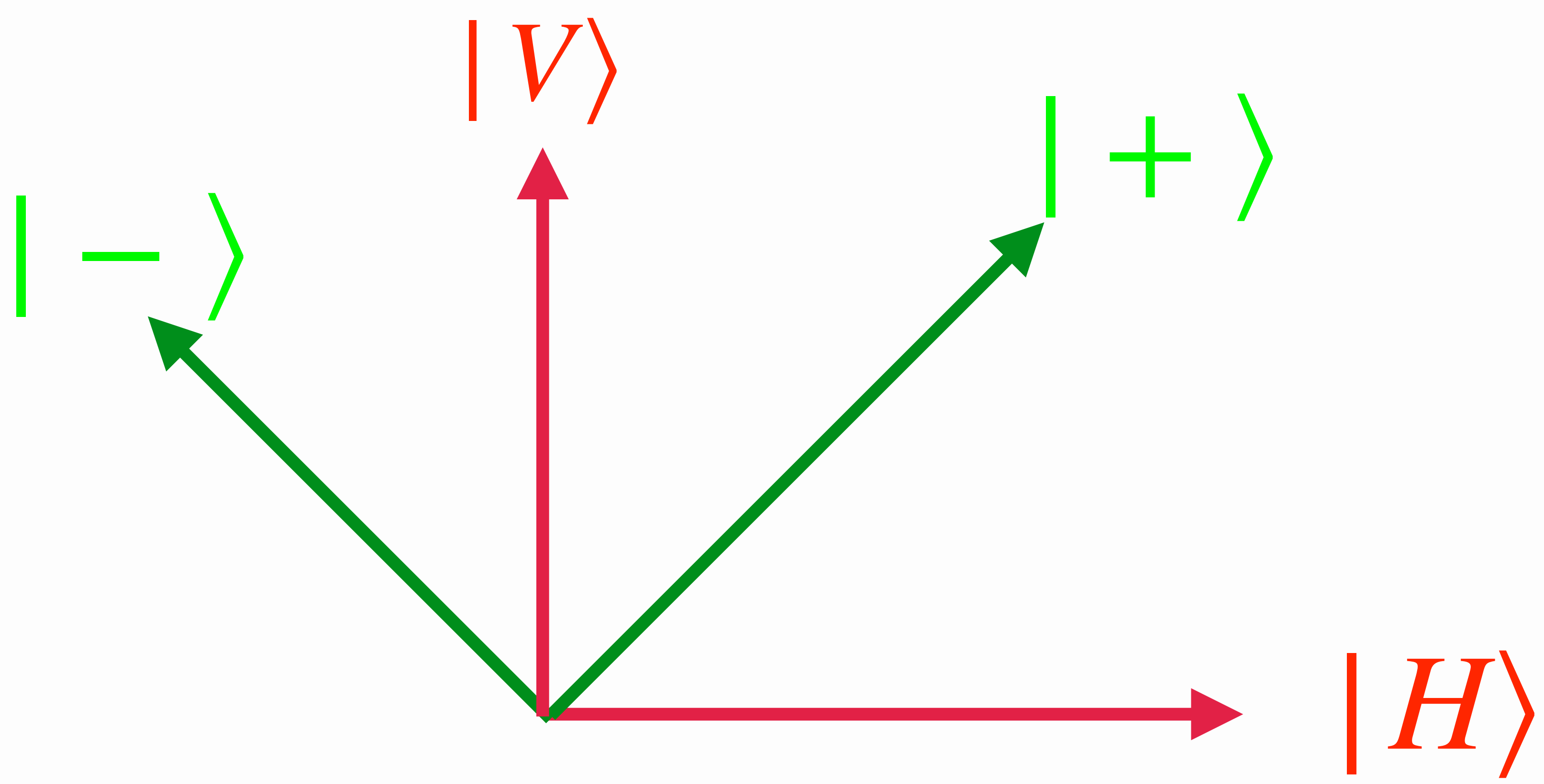
"Poincare sphere"

Orthonormal modes

Field amplitude



Quantum bits



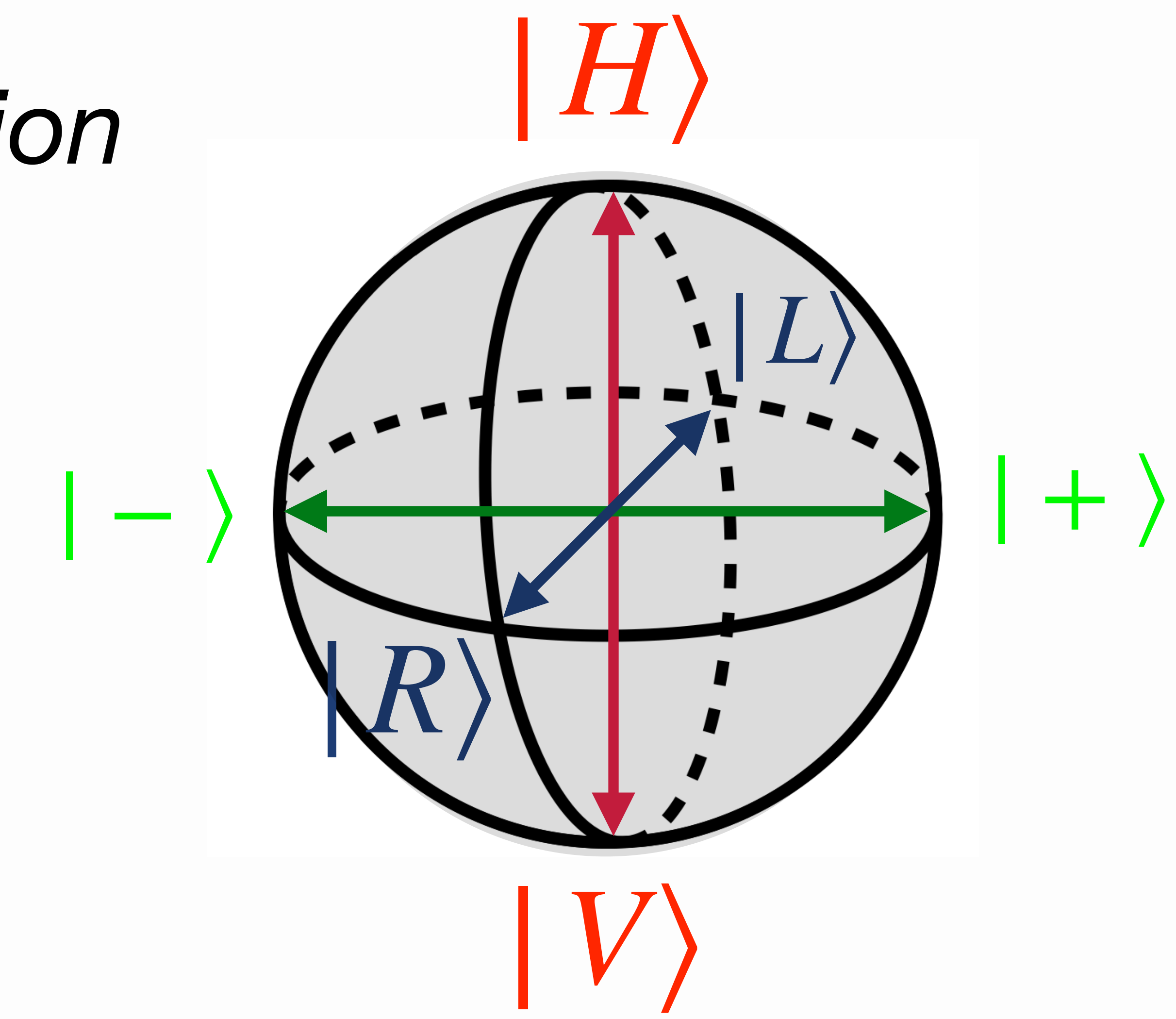
Quantum states

Allows for superposition

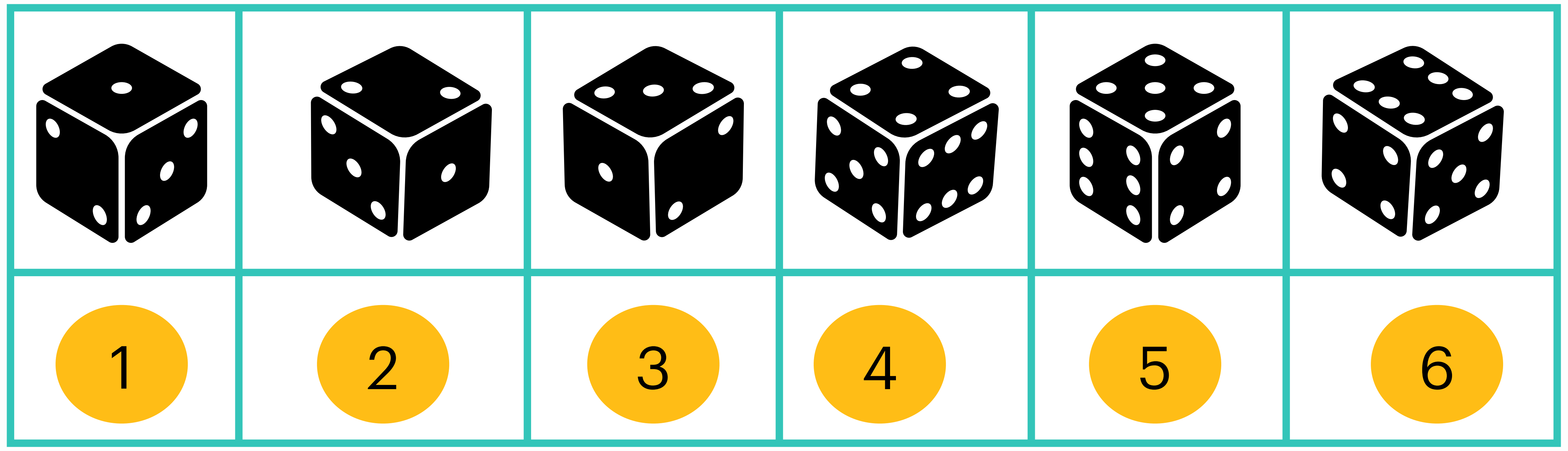
"Bloch sphere"

Orthonormal states

Probability amplitude



Classical dits



$$u(LG_{p,l}) \propto \left(\frac{r\sqrt{2}}{w}\right)^{l} L_p^l\left(\frac{2r^2}{w^2}\right) f_{p,l}(r,z)\ e^{-il\phi}$$

$$f_{p,l} = (-1)^p\ e^{\frac{-ikr^2z}{2(z^2+z_r^2)}}\ e^{\frac{-r^2}{w^2}}\ e^{-i(2p+l+1)\tan^{-1}\left(\frac{z}{z_r}\right)}$$

p : radial mode index,
l : OAM mode index,
w : Beam waist,
z_r : Rayleigh range,
L_p^l : Associated Laguerre polynomial

$$w = w_0\sqrt{1+\frac{z^2}{z_r^2}},\quad z_r = \frac{\pi w_0^2}{\lambda}$$

Willner et al., Apps. Phys. Rev. 8, 041312, (2021).

Classical dits

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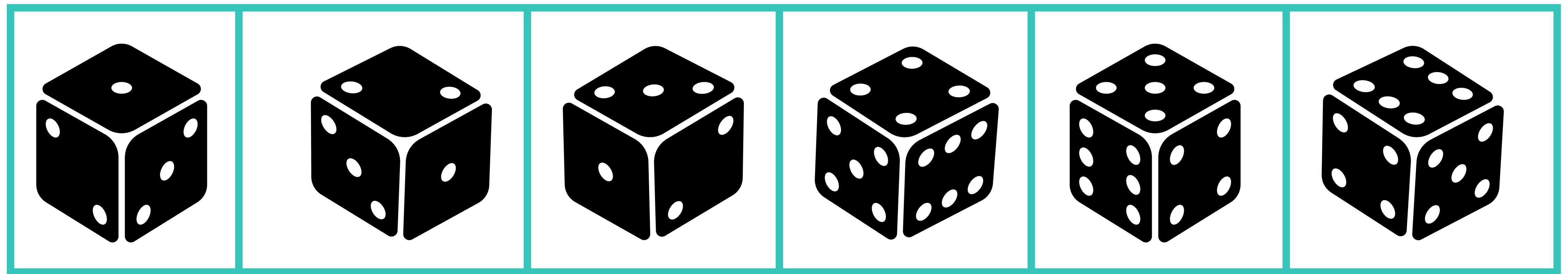
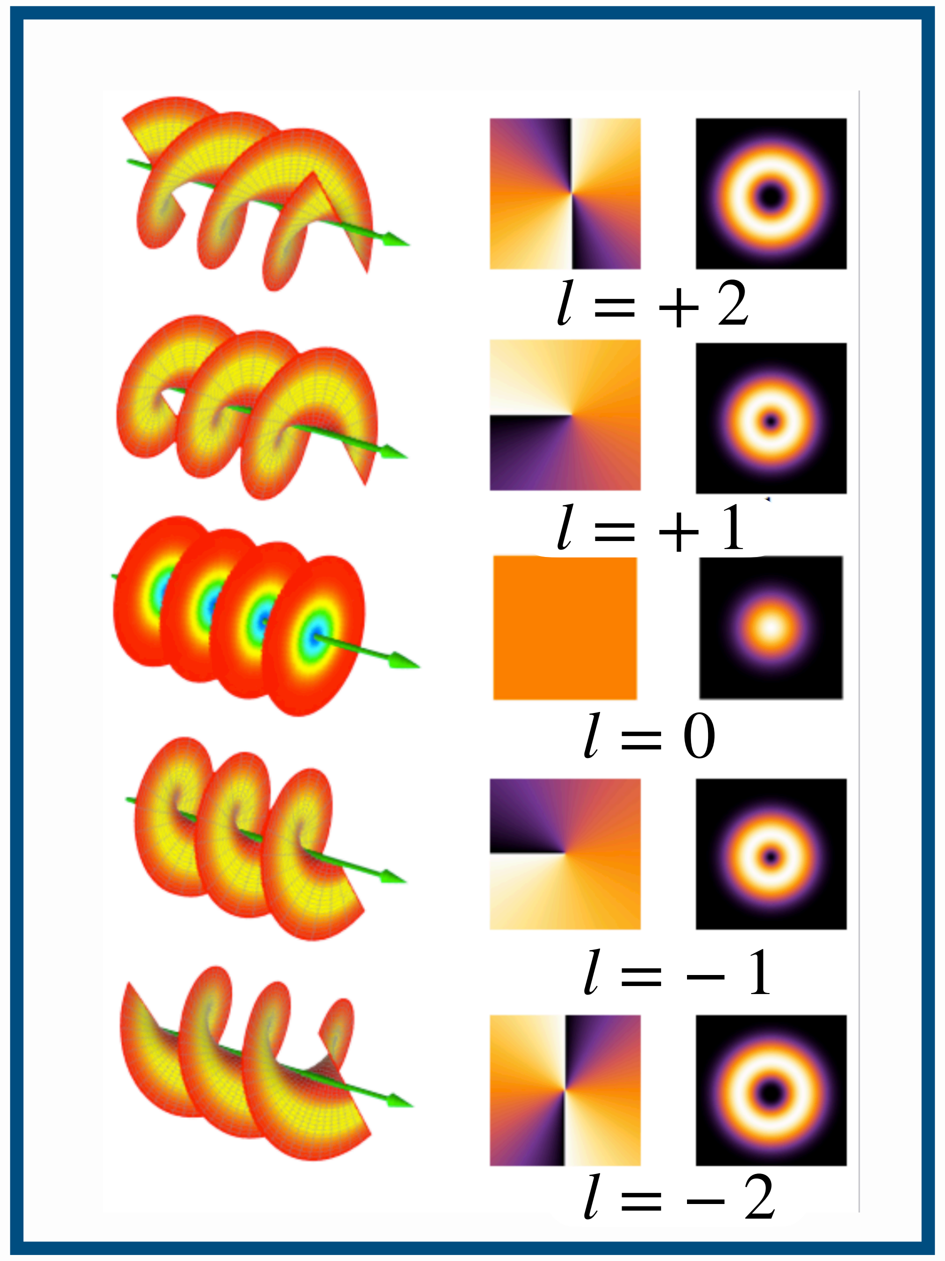


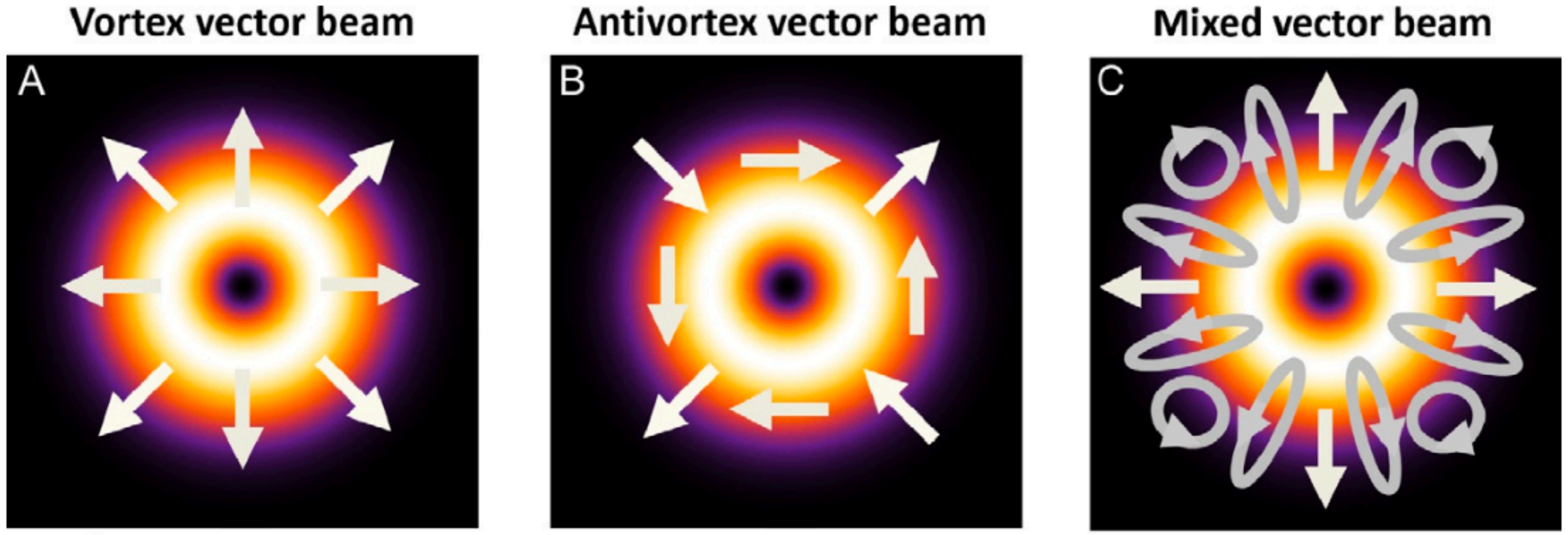
1	2	3	4	5	6



- Advancements in their generation, manipulation, and measurements.

Willner et al., Apps. Phys. Rev. 8, 041312, (2021).

Classically entangled light



$\frac{1}{\sqrt{2}}(LG_{01},\hat{R} + LG_{0-1},\hat{L})$ $\frac{1}{\sqrt{2}}(LG_{0-1},\hat{R} - LG_{01},\hat{L})$ $\frac{1}{\sqrt{2}}(LG_{01},\hat{D} - LG_{0-1},\hat{A})$

Forbes, Andrew, Andrea Aiello, and Bienvenu Ndagano. "Classically entangled light." *Progress in Optics*. Vol. 64. Elsevier, 2019. 99-153.

Equivalent states $\xrightarrow[??]{\text{Classical analogue}}$ Equivalent beams

Mathematically the same, physically different.

$SU(2)$ coherent optical beams

$$\boxed{|\hat{n}(\alpha,\beta)) = e^{-iS_3\beta}e^{-iS_2\alpha}|S_3 = +S)}$$

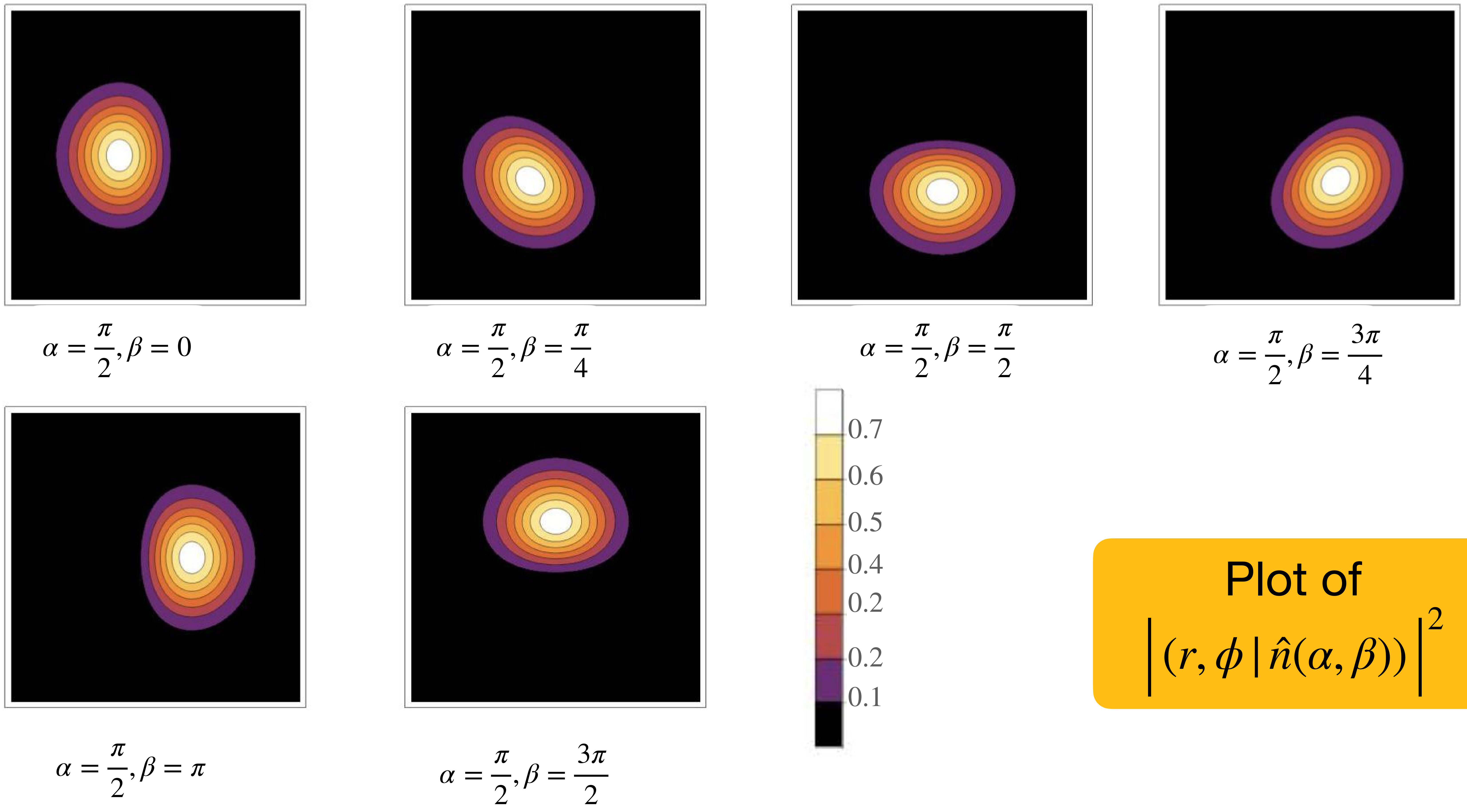
$\hat{n} = \sin\alpha\cos\beta\ \hat{e}_1 + \sin\alpha\sin\beta\ \hat{e}_2 + \cos\alpha\ \hat{e}_3$ where $\hat{e}_1 \perp \hat{e}_2 \perp \hat{e}_3\,.$

Illustration

- Identify eigenmodes of S_3 with three Laguerre Gauss modes $|\mathrm{LG}_{0-1}), |\mathrm{LG}_{00}), |\mathrm{LG}_{01})$

$$\boxed{|\hat{n}(\alpha,\beta)) = e^{-iS_3\beta}e^{-iS_2\alpha}|\mathrm{LG}_{01}) = \frac{1}{2}e^{i\beta}(1-\cos\alpha)|\mathrm{LG}_{0-1}) - \frac{\sin\alpha}{\sqrt{2}}|\mathrm{LG}_{00}) + \frac{1}{2}e^{-i\beta}(1+\cos\alpha)|\mathrm{LG}_{01})}$$

$$|\hat{n}(\alpha,\beta)) = e^{-iS_3\beta}e^{-iS_2\alpha}|\mathrm{LG}_{01}) = \frac{1}{2}e^{i\beta}(1-\cos\alpha)|\mathrm{LG}_{0-1}) - \frac{\sin\alpha}{\sqrt{2}}|\mathrm{LG}_{00}) + \frac{1}{2}e^{-i\beta}(1+\cos\alpha)|\mathrm{LG}_{01})$$



Plot of

$$\left|(r,\phi|\hat{n}(\alpha,\beta))\right|^2$$

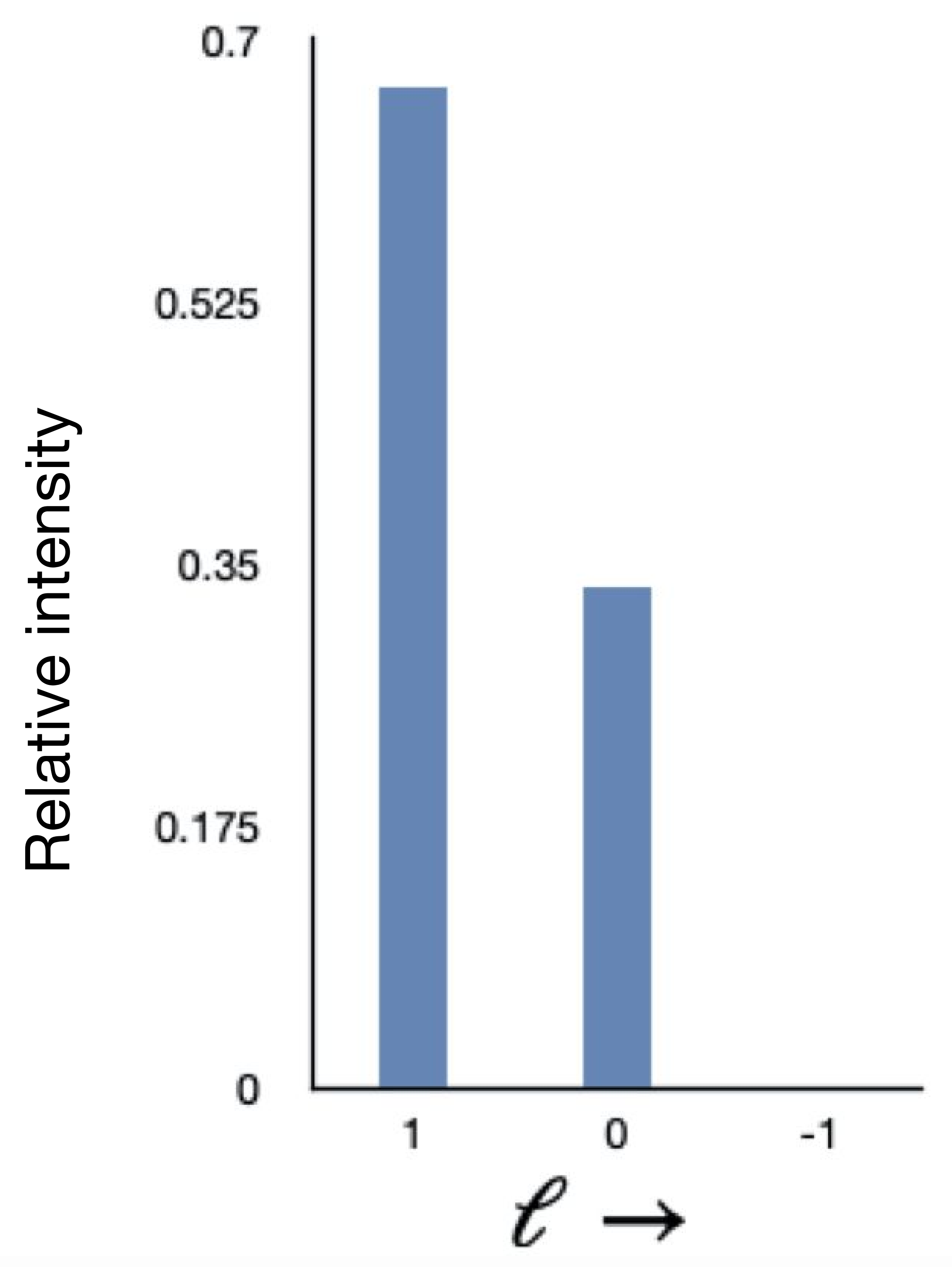
Q-representation of an optical beam

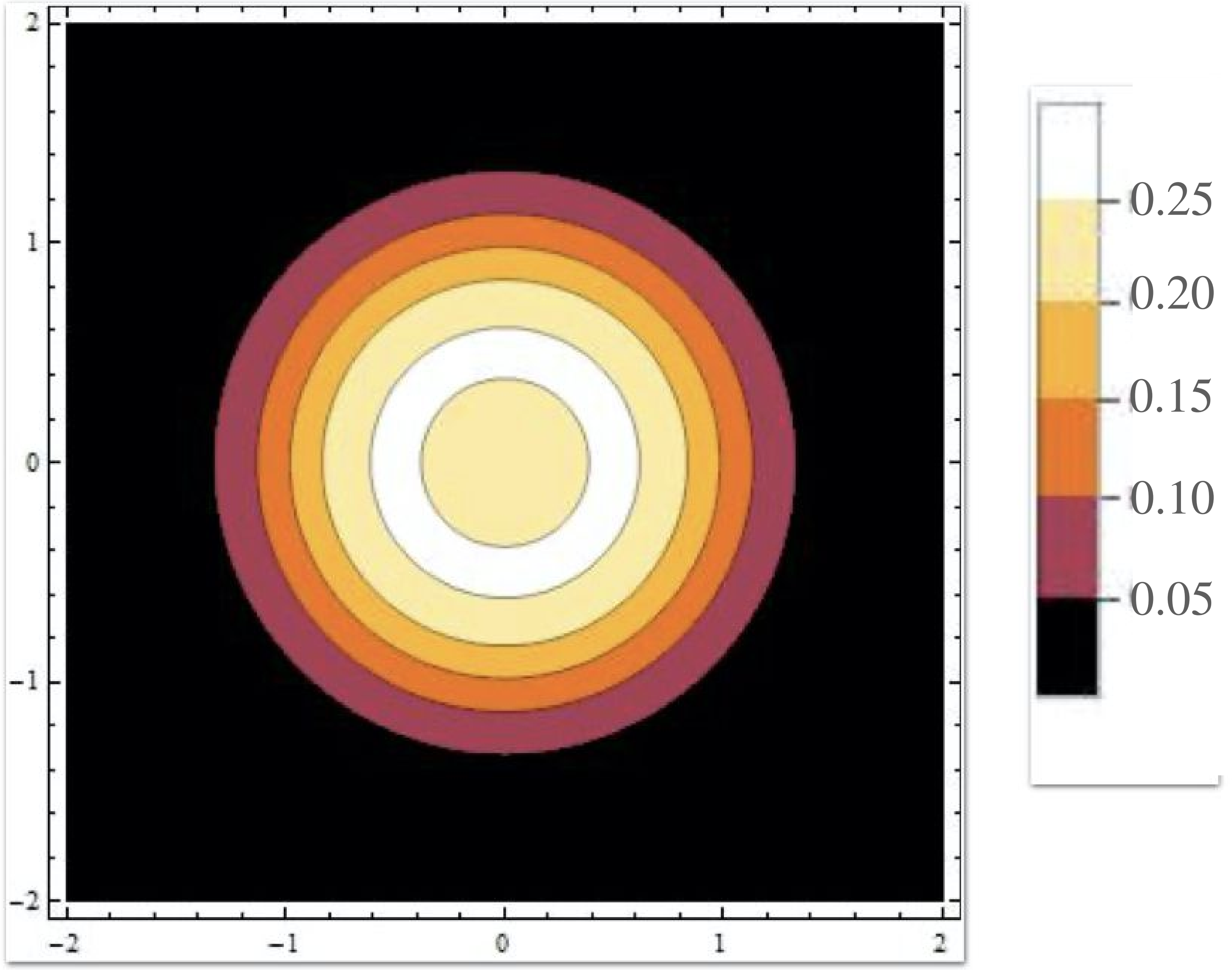
- The set of $SU(2)$ coherent beams $\{\,|\hat{n}(\theta,\phi))\}$: over complete.
- J : coherent mode representation (CMR) of an optical beam
- Q-representation of an optical beam:

$$F(\hat{n}) = \frac{2S+1}{4\pi}(\hat{n}(\theta,\phi)\,|\,J\,|\,\hat{n}(\theta,\phi))$$

- **Equivalent beams:** Two beams $J_1 \in \mathscr{H}^{d_1}$ and $J_2 \in \mathscr{H}^{d_2}$ sharing the same Q$-$ representation.

Intensity profile of a three-dimensional beam equivalent to a pure two-dimensional beam $\frac{1}{2}(\mathbf{1} + \sigma_z)$





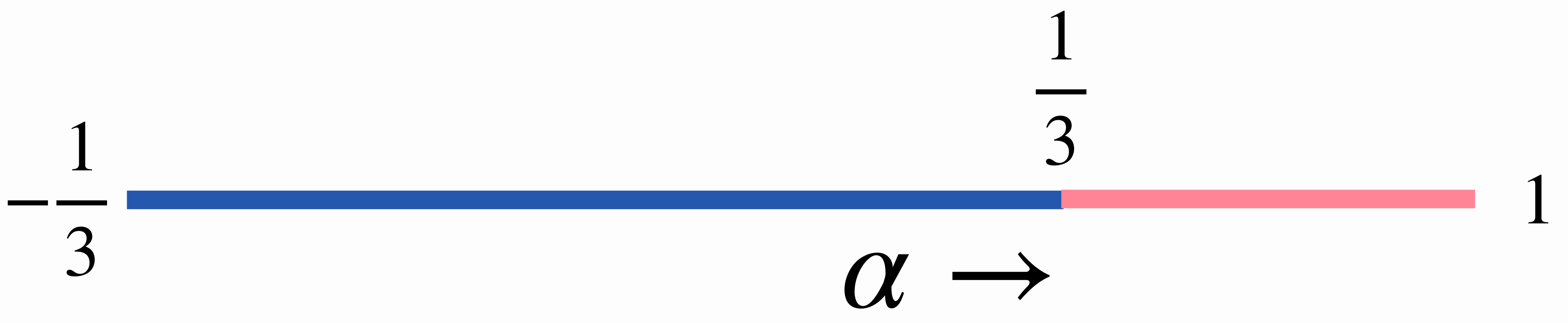
Three dimensional equivalent beam
$0.67\,|\mathrm{LG}_{01})(\mathrm{LG}_{01}| + 0.33\,|\mathrm{LG}_{00})(\mathrm{LG}_{00}|$

Separable equivalents of classically entangled Werner beams

A : polarisation
B :OAM

Werner beams: the beam with the same CMR as Werner states in QM, i.e.,

$$J_W^{[2,2]}[\alpha] = \frac{1}{4}(\mathbf{1} - \alpha\vec{\sigma}^A \cdot \vec{\sigma}^B)\,.$$



Blue: Classically separable,
Pink: Classically entangled.

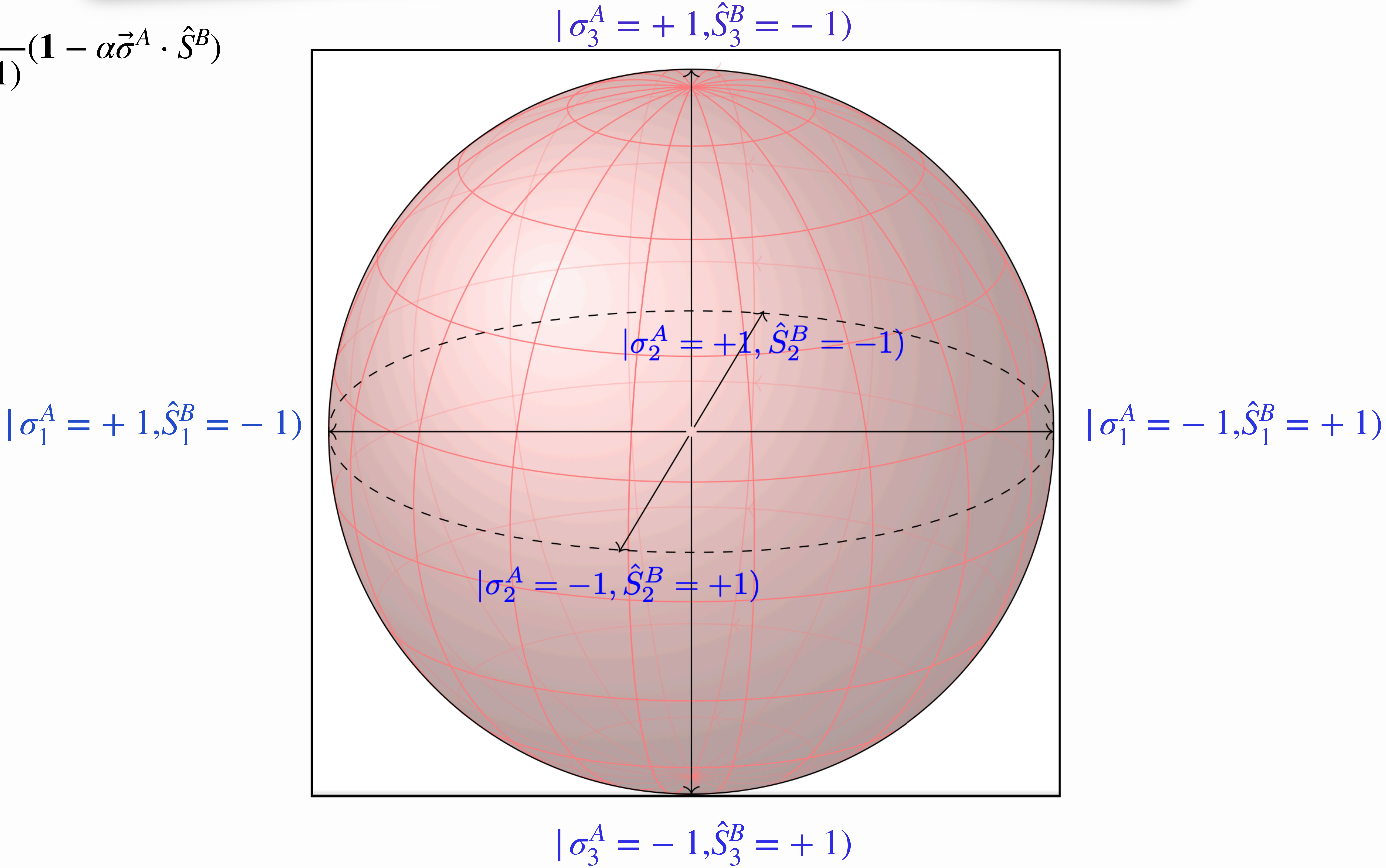
The CMR of the equivalent $2 \otimes (2S+1)$ beams:

$$J_W^{[2,2S+1]}[\alpha] = \frac{1}{2(2S+1)}(\mathbf{1} - \alpha\vec{\sigma}^A \cdot \hat{S}^B), \qquad \alpha \in \left[-\frac{S}{S+1}, 1\right]$$

How to prepare them experimentally?: **Rotational invariance**

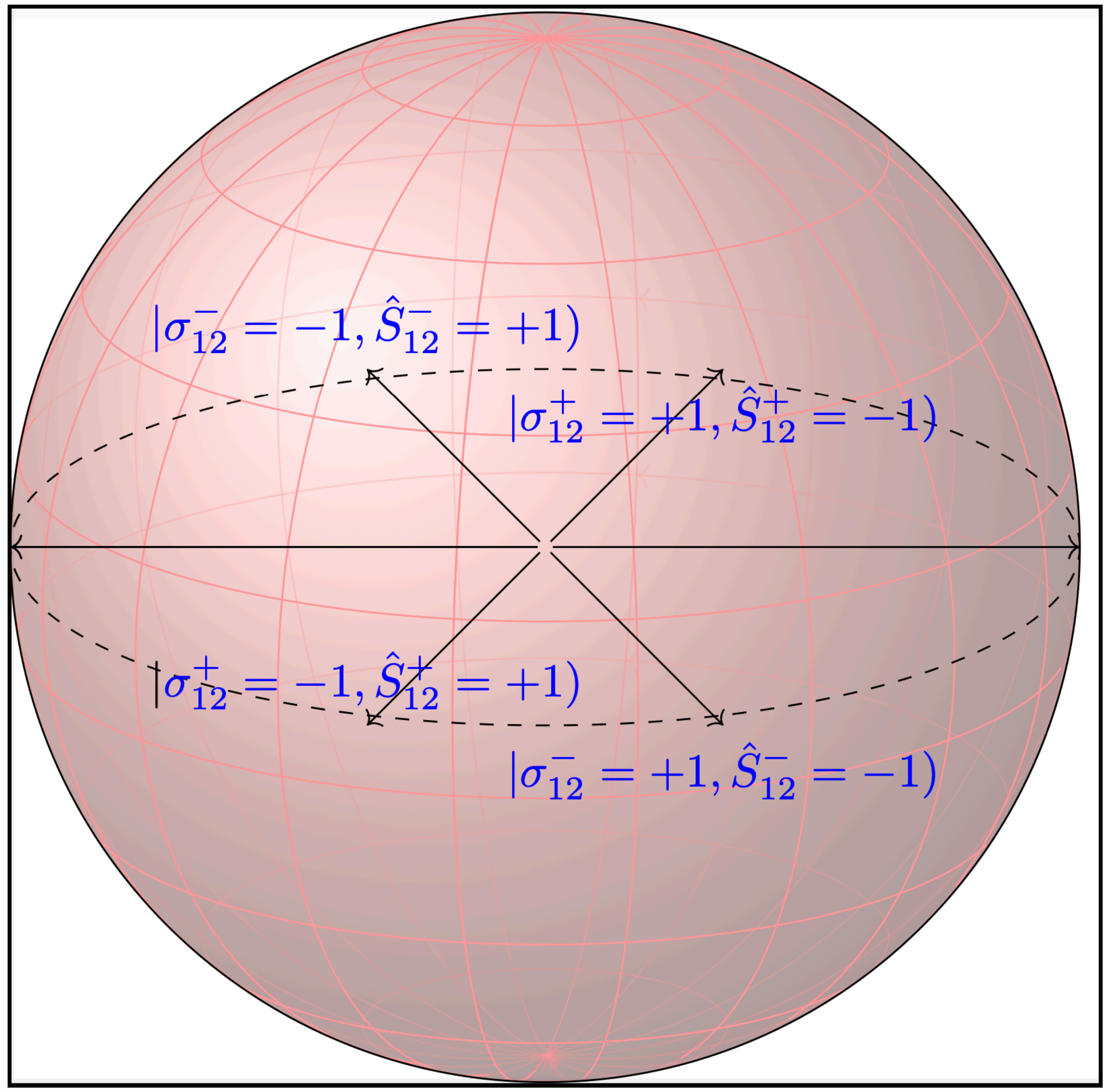
Generation of separable equivalent of Werner beams

$$J_W^{[2,2S+1]}[\alpha] = \frac{1}{2(2S+1)}(\mathbf{1} - \alpha\vec{\sigma}^A \cdot \hat{S}^B)$$

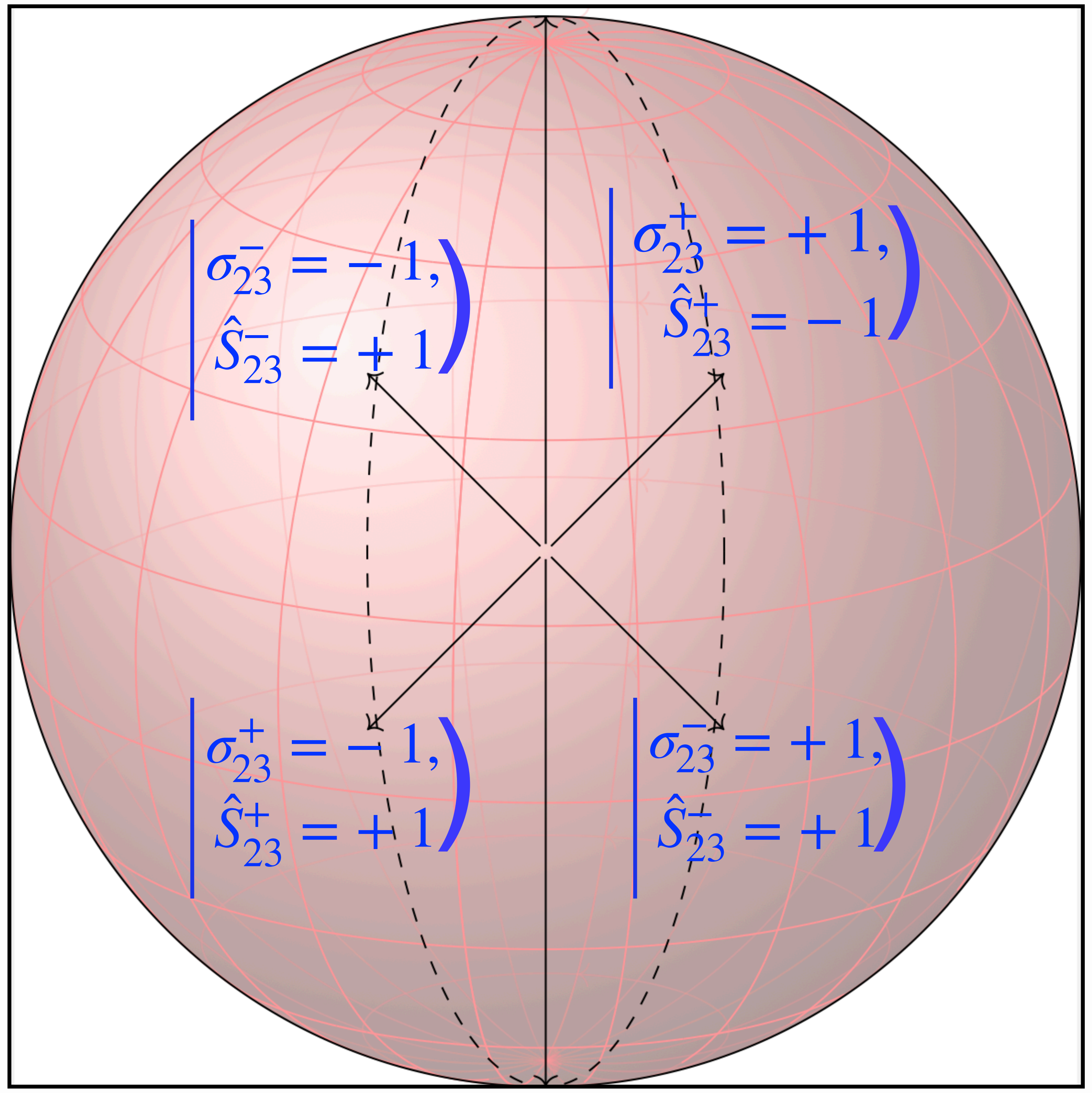


Generation of separable equivalent of Werner beams

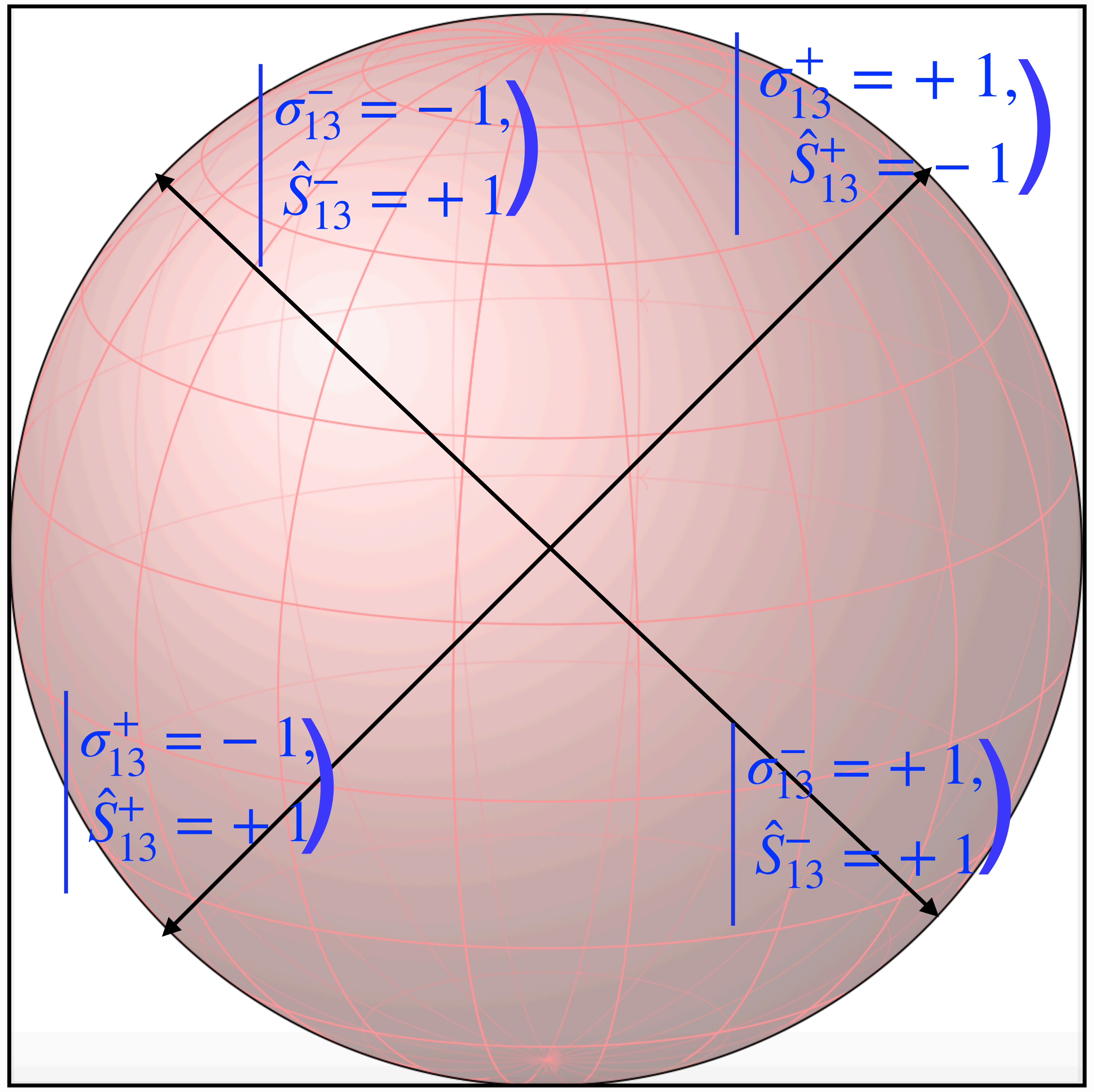
(12) plane



(23) plane



(13) plane



$$\sigma^{(\pm)}_{12} = \frac{1}{\sqrt{2}}(\sigma_1 \pm \sigma_2)$$

$$\hat{S}^{(\pm)}_{12} = \frac{1}{\sqrt{2}}(\hat{S}_1 \pm \hat{S}_2)$$

Efficiency of generation of separable equivalent of Werner beams

Increase in Entanglement/ nonlocality

Dimension	$\lvert \alpha_{\max} \rvert$	Six beams	Twelve beams	Eighteen beams
2×3	0.5	100%	100%	100%
2×4	0.6	91%	98%	99%
2×5	0.66	86%	96%	98%
2×6	0.71	82%	95%	97%
2×7	0.75	79%	94%	97%
2×8	0.78	76%	92%	96%
2×9	0.8	74%	91%	95%
2×11	0.83	71%	88%	94%
2×13	0.86	70%	86%	92%
2×15	0.875	68%	86%	91%

Entangled

Nonlocal

Efficiency of generation of separable equivalent of Werner beams

Dimension	$\lvert \alpha_{\max} \rvert$	Six beams	Twelve beams	Eighteen beams
2×3	0.5	100%	100%	100%
2×4	0.6	91%	98%	99%
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2×9	0.8	74%	91%	95%
2×11	0.83	71%	88%	94%
2×13	0.86	70%	86%	92%
2×15	0.875	68%	86%	91%

Dominance of anisotropic terms

Entangled

Nonlocal

Efficiency of generation of separable equivalent of Werner beams

Dimension	$\lvert\alpha_{\max}\rvert$	Six beams	Twelve beams	Eighteen beams
2×3	0.5	100%	100%	100%
2×4	0.6	91%	98%	99%
2×5	0.66	86%	96%	98%
2×6	0.71	82%	95%	97%
2×7	0.75	79%	94%	97%
2×8	0.78	76%	92%	96%
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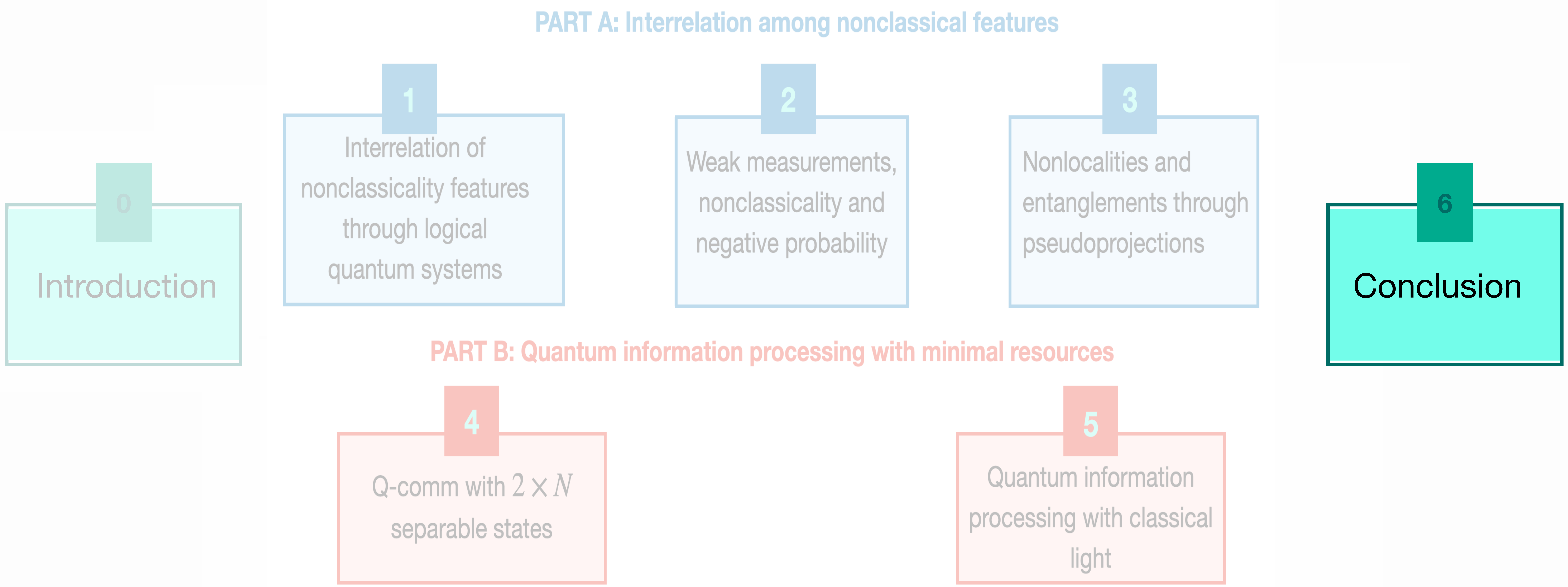
2×11	0.83	71%	88%	94%
2×13	0.86	70%	86%	92%
2×15	0.875	68%	86%	91%

Entangled

Nonlocal

Conclusions

- Protocols requiring parallelism and local entanglement can be implemented with classical light.
- Implications on classical-quantum conundrum.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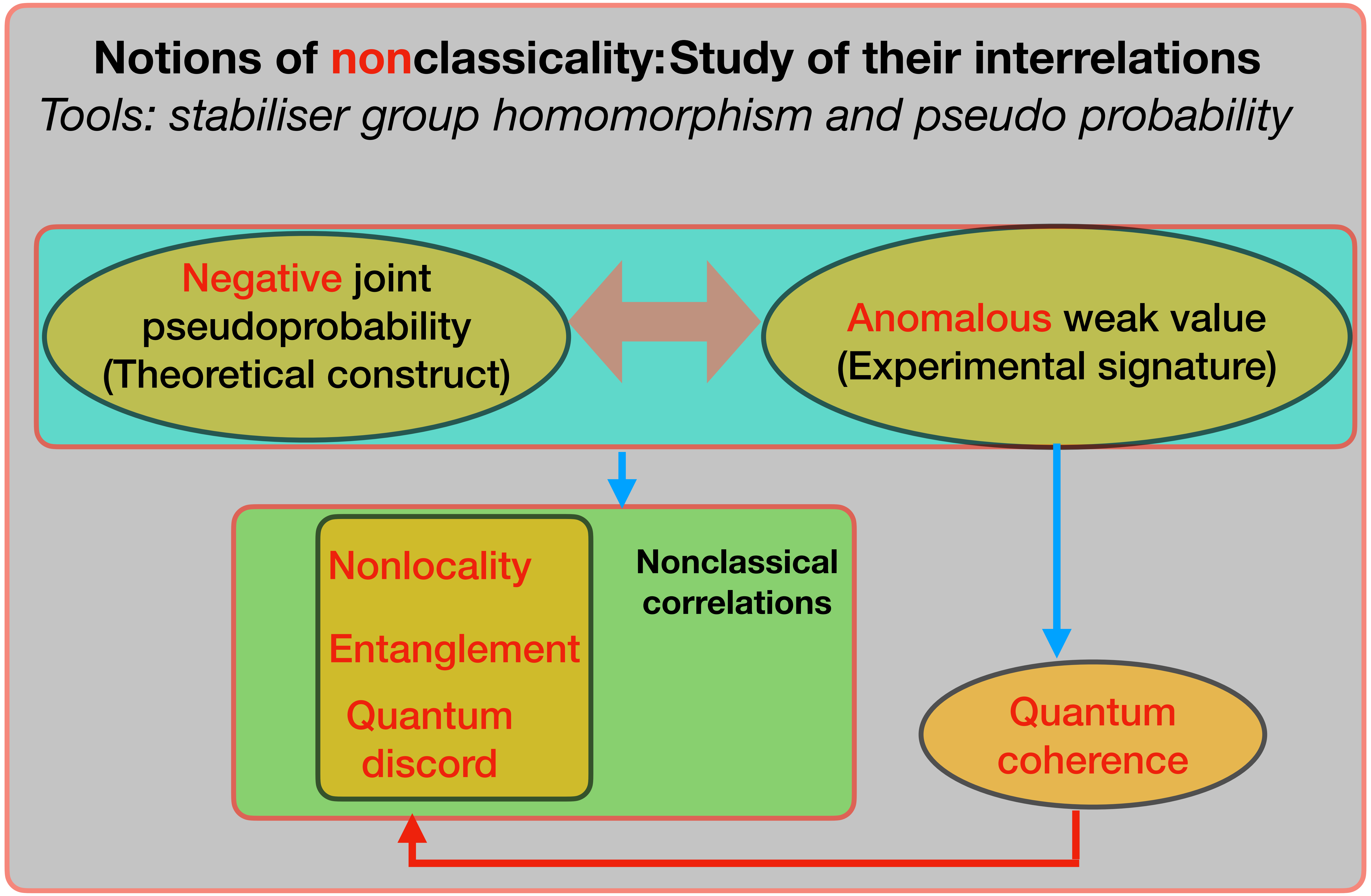
Conclusions and future directions



1. Interrelation of nonclassicality of **quantum channels** through Pseudoprobability
2. Interrelation among **monotones** of different resource theories

Conclusions and future directions

QIP with minimal resources

Guiding principle: Information retrieval through expectation values

Information encoding
Through equivalent states → Information transfer through separable states

Quantum resource

Quantum coherence

Quantum entanglement

Classical resemblances

Coherence in classical waves

Classical entanglement
(Vector vortex beams)

What is exactly a quantum resource?

Single-photon statistics/ nonlocality

Acknowledgements

- Prof. V. Ravishankar
- Prof. P. Senthilkumaran, Prof. Bhaskar Kanseri, Prof. Shravan Kumar
- Prof. R. Srikanth and Prof. Andrew Forbes
- Soumik Adhikary and Rajni Bala
- All the members of Einstein Research Scholar Lab
- Friends and family.
- CSIR for funding.

Publications: Included in thesis

“Non-locality and entanglement in multi-qubit systems from a unified framework.”
S. Asthana, S. Adhikary, and V. Ravishankar.
Quantum Information Processing **20.1** (2021) 1-33.

‘Weak measurements, non-classicality and negative probability.”
S. Asthana and V. Ravishankar.
Quantum Information Processing **20.10** (2021) 1-39.

“Quantum communication with SU (2) invariant separable $2 \times N$ level systems.”
S. Asthana, R. Bala, and V. Ravishankar.
Quantum Information Processing **21.1** (2022) 1-24.

“State transfer with separable optical beams and variational quantum algorithms with classical light.”
S. Asthana, and V. Ravishankar.
JOSA B **39.1** (2022) 388-400.

“Interrelation of nonclassicality conditions through stabiliser group homomorphism.”
S. Asthana.
New Journal of Physics **24** 053026 (2022).

“Interrelation of nonclassicality features in higher-dimensional systems through logical operators.”
S. Asthana, and V. Ravishankar.
arXiv preprint arXiv:2203.06635 (2022).

Publications: not included in thesis

"Contextuality-based quantum conferencing."
R. Bala, S. Asthana, and V. Ravishankar.
Quantum Information Processing 20.10 (2021): 1-27.

"Combating errors in propagation of orbital angular momentum modes of light in turbulent media".
R Bala, S Asthana and V Ravishankar.
International Journal of Theroetical Physics 61 263 (2022): 1-19.

"Combating errors in quantum communication: an integrated approach"
R. Bala, S. Asthana and V. Ravishankar.
Scientific Reports 13.1 (2023): 1-10.

"Quantum and semi-quantum key distribution in networks"
R. Bala, S. Asthana, and V. Ravishankar.
International Journal of Theoretical Physics, 62 104 (2023): 1-25.

"Integrated semiquantum secure communication."
R. Bala, S. Asthana, and V. Ravishankar.
IET Quantum Communication (2023).

"Bell-CHSH non-locality and entanglement from a unified framework."
S Ashikary, S Asthana, and V. Ravishankar.
The European Physical Journal D 74 (2020): 1-8.

Conferences attended

Talks ( )

- "Young researchers' meet QIQT 2023"

  Interrelation of nonclassical features

- "Young Quantum 2023 ($\langle You \,|\, Qu \rangle$)" held at HRI, Prayagraj

  Interrelation of nonclassical features

- "Progress in Quantum Science and Technologies" held at IIT Madras (23-27 January, 2023)

  Q-comm with $SU(2)$ invariant separable states.

- "YouQu2020" (in Online mode), held at HRI, Prayagraj (October 12-15, 2020).

  Nonlocality and entanglement in multiparty systems from a unified framework

- "QIQT 2021" held online (Organising institute: IISER Kolkata).

  Q-comm with $SU(2)$ invariant separable states.

- "KOBIT-5" held online (April 13-14, 2021).

  Q-comm with $SU(2)$ invariant separable states.

- "KOBIT-6" held online (February 3-4, 2022).

  Interrelation of nonclassical features through logical qudits

Conferences attended

Poster Presentations

- “Quantum Information Processing 2023 (QIP 2023)” held at Ghent University, Belgium (4-10 February, 2023).
- “Quantum resources: from mathematical foundations to operational characterisation” held in Singapore December 2022.
- “Departmental symposium on advances in Physics, 2019” held at IIT Delhi.
- “Conference on Quantum Information and Many-Body Theory” held at IIT BHU (March 1-3, 2019).
- “3rd International Symposium on Single Photon based Quantum Technologies”, held online (September 15-17, 2020).
- “Q-Turn, 2020” held online (November 23-27, 2020).
- “Quantum Foundations, Technology and Applications 2020 (QFTA2020)” (December 4-9, 2020).
- “Departmental symposium on advances in Physics, 2020” held at IIT Delhi.

Attended

- “ICQIF-2022” held online (February 14-24, 2022).
- “ISQIT 2019” held at Pune.

Conference Proceedings

"Quantum communication with SU(2) invariant separable polarisation-OAM states of light."

Sooryansh Asthana, Rajni Bala and V. Ravishankar.

Frontiers in Optics. (pp. JW5A-77) Optica Publishing Group, 2022.

" Implementation of variational quantum classifier with structured light."

Sooryansh Asthana and V. Ravishankar.

Frontiers in Optics. (pp. JW5A-93) Optica Publishing Group, 2022.

"Collaborative error-immune information transfer with OAM modes of light."

Rajni Bala, Sooryansh Asthana, and V. Ravishankar.

Frontiers in Optics (pp. JTu4B-71). Optica Publishing Group.

"Semi-quantum key distribution in networks with OAM states of light"

Rajni Bala, Sooryansh Asthana, and V. Ravishankar.

Frontiers in Optics (pp. JTu4B-65). Optica Publishing Group.

Thank you

Points to ponder

(i) In justifying the inclusion of classical light in the study, the thesis has argued that quantum information processing tasks solely requiring parallelism can also be simulated with classical light. Here **I wonder if the term "emulate" is preferable to "simulate", in the sense that the former is a stronger term**, indicative of an equivalence of the classical state to its quantum counterpart. The reason is that all quantum phenomena can in principle be classically simulated, though not efficiently. That is, certain quantum processes performed with polynomial quantum resources (in time or space) are believed to require exponential classical resources.
In this light, the student is invited to consider how his results relate to **the Knill-Laflamme theorem.**

Gotteman-Knill theorem

Clifford circuits may be simulated classically.

A quantum circuit using only the following elements can be simulated efficiently on a classical computer:

1. Preparation of qubits in the computational basis states,
2. Clifford gates (Hadamard gates, CNOT gates, phase gate $\mathscr{S}$), and
3. Measurements in the computational basis.

Our work

Protocols requiring parallelism and/ or entanglement at a single location may be implemented with classical light.

We used ‘simulate’ because distributed quantum computing cannot be implemented with classical light.

(2) In a generalized probability theory, entanglement is characterized by the given state falling outside the minimal tensor product, i.e., by its inexpressibility as a probabilistic mixture of product states. However it can still be represented as a linear combination of product states (with possible negative weights). **Does the idea of quasiprobability evoke the same sort of mathematical structure?**

Pseudoprojection operators: hermitian operators with negative eigenvalues.
Any operator may be expressed as a convex sum of pseudoprojection operators with nonnegative weights.
The “negative weight” gets incorporated in the eigenvalue of pseudoprojection operator.

Example

Separable states: $\sum_i p_i \rho_{1i} \otimes \rho_{2i} \quad 0 \leq p_i \leq 1$

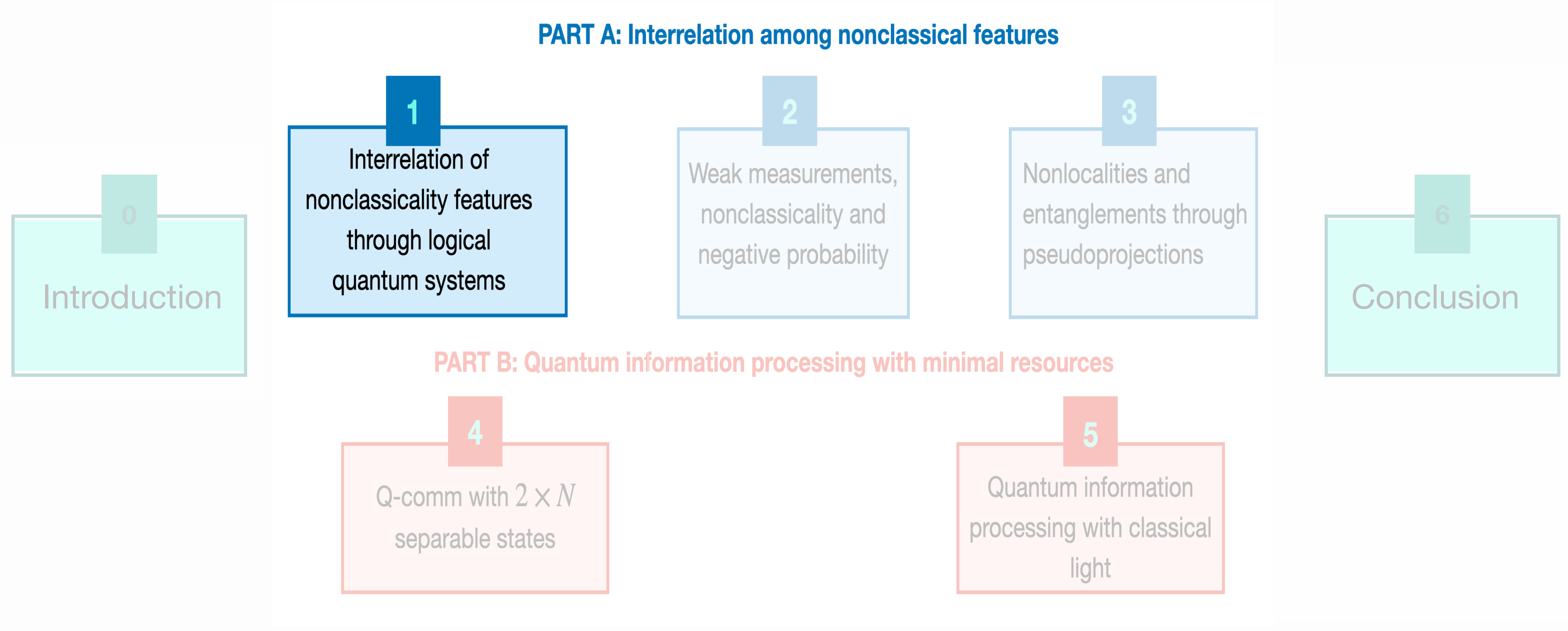
$$\boldsymbol{\Pi}_{xyz}^{+++} \otimes \boldsymbol{\Pi}_{xyz}^{---} + \boldsymbol{\Pi}_{xyz}^{+--} \otimes \boldsymbol{\Pi}_{xyz}^{-++} + \boldsymbol{\Pi}_{xyz}^{+-+} \otimes \boldsymbol{\Pi}_{xyz}^{-+-} + \boldsymbol{\Pi}_{xyz}^{++-} \otimes \boldsymbol{\Pi}_{xyz}^{--+}$$

$$= \frac{1}{8^2}\Big\{(\mathbf{1} + \sigma_{1x} + \sigma_{1y} + \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} - \sigma_{2y} - \sigma_{2z}) + (\mathbf{1} + \sigma_{1x} - \sigma_{1y} - \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} + \sigma_{2y} + \sigma_{2z}) + (\mathbf{1} + \sigma_{1x} - \sigma_{1y} + \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} + \sigma_{2y} - \sigma_{2z}) + (\mathbf{1} + \sigma_{1x} + \sigma_{1y} - \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} - \sigma_{2y} + \sigma_{2z})\Big\}$$

$$= \frac{1}{16}\Big\{(\mathbf{1} + \sigma_{1x} + \sigma_{1y} + \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} - \sigma_{2y} - \sigma_{2z}) + (\mathbf{1} + \sigma_{1x} - \sigma_{1y} - \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} + \sigma_{2y} + \sigma_{2z}) + (\mathbf{1} + \sigma_{1x} - \sigma_{1y} + \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} + \sigma_{2y} - \sigma_{2z}) + (\mathbf{1} + \sigma_{1x} + \sigma_{1y} - \sigma_{1z}) \otimes (\mathbf{1} - \sigma_{2x} - \sigma_{2y} + \sigma_{2z})\Big\}$$

$$\propto \frac{1}{4}(1 - \sigma_{1x}\sigma_{2x} - \sigma_{1y}\sigma_{2y} - \sigma_{1z}\sigma_{2z})$$

Backup slides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Groups

A set $G = \{g\}$ is said to form a group under operation $*$ if the following properties are satisfied:

1. Closure: $\forall g_1, g_2 \in G, \quad g_1 * g_2 \in G$.

2. Existence of identity: There exists an element e such that $\forall g \in G, g * e = e * g = g$.

3. Existence of inverse: $\forall g \in G,$ there exists an element g^{-1} such that $g * g^{-1} = g^{-1} * g = e$.

4. Associativity: $\forall g_1, g_2, g_3 \in G, g_1 * (g_2 * g_3) = (g_1 * g_2) * g_3$

Tung, Wu-Ki. *Group theory in physics*. Vol. 1. World Scientific, 1985.

Stabiliser group of a GHZ state

$$|\psi\rangle_L \equiv \frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big)$$

$$G_{\mathrm{GHZ}} \equiv \{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_3Z_1, X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2X_3, -Y_1Y_2Y_3\}$$

Gottesman, D. (1997). *Stabilizer codes and quantum error correction*. California Institute of Technology.

Homomorphism between stabiliser groups: Example 2

$$G_{\mathrm{GHZ}} \equiv \{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_3Z_1, X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2X_3, -Y_1Y_2Y_3\}$$

$$\{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_1Z_3\} \to \mathbf{1}_L$$

$$\{X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2Y_3\} \to X_L$$

S. Asthana, *New Journal of Physics* 24.5 (2022): 053026.

Homomorphism between stabiliser groups: Example 2

$$G_{\mathrm{GHZ}} \equiv \{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_3Z_1, X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2X_3, -Y_1Y_2Y_3\}$$

$$\{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_1Z_3\} \rightarrow \mathbf{1}_L$$

$$\boxed{\{X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2Y_3\}} \rightarrow X_L$$

Used for construction of Mermin's NL inequality

Detects coherence in the computational basis of logical states

S. Asthana, *New Journal of Physics* 24.5 (2022): 053026.

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Used for construction of Mermin's NL inequality

Detects coherence in the computational basis of logical states

$$\langle X_L \rangle > c, \quad c \in [0,1)\,.$$

$$\langle 0_L | X_L | 0_L \rangle = 0$$
$$\langle 1_L | X_L | 1_L \rangle = 0$$
$$\langle \psi_L | X_L | \psi_L \rangle = 1$$

S. Asthana, *New Journal of Physics* 24.5 (2022): 053026.

Homomorphism between stabiliser groups: Example 2

$$G_{\mathrm{GHZ}} \equiv \{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_3Z_1, X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2X_3, -Y_1Y_2Y_3\}$$

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Used for construction of Mermin's NL inequality

Detects coherence in the computational basis of logical states

$$(x_1x_2x_3)(-x_1y_2y_3)(-y_1x_2y_3)(-y_1y_2x_3) = -1$$

$$\boxed{\left| \left\langle X_1X_2X_3 - X_1Y_2Y_3 - Y_1X_2Y_3 - Y_1Y_2X_3 \right\rangle \right| \leq 2}$$

Homomorphism between stabiliser groups: Example 2

$$\{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_3Z_1, X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2X_3, -Y_1Y_2Y_3\}$$

$$\{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_1Z_3\} \to \mathbf{1}_L$$

$$\{X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2Y_3\} \to X_L$$

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Detects coherence in the computational basis of logical states

$$\langle X_L \rangle > c, \quad c \in [0,1)\,.$$

$$\langle 0_L | X_L | 0_L \rangle = 0$$
$$\langle 1_L | X_L | 1_L \rangle = 0$$
$$\langle \psi_L | X_L | \psi_L \rangle = 1$$

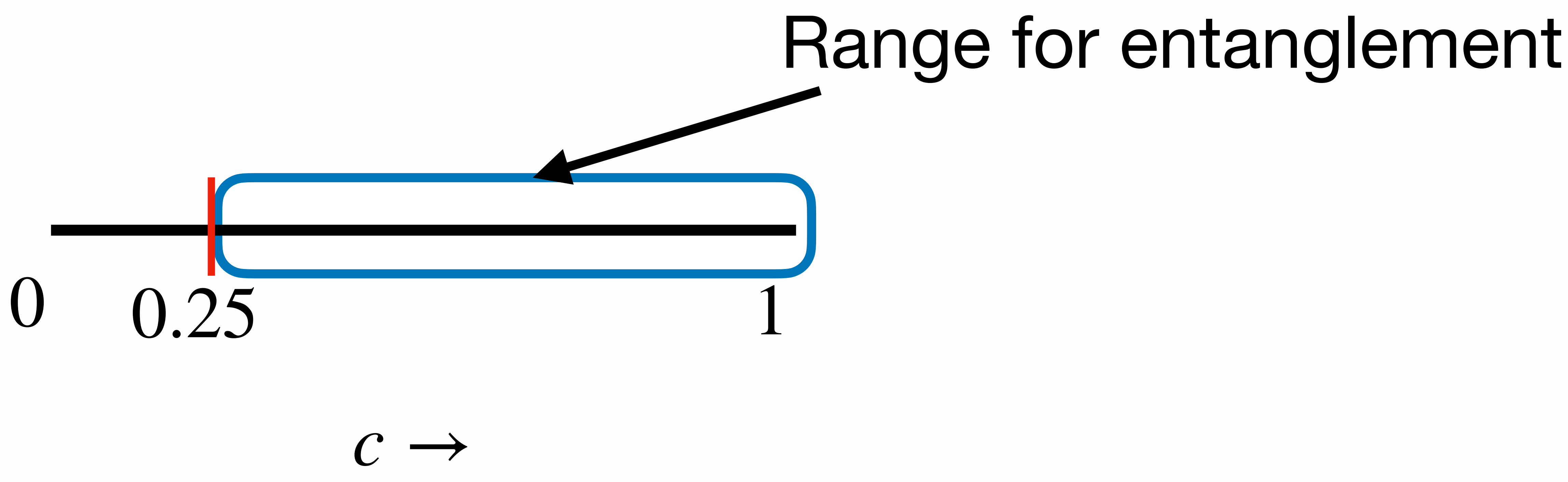
$$|\psi\rangle_L \equiv \frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle \Big)$$

$$\Big| \Big\langle X_1X_2X_3 - X_1Y_2Y_3 - Y_1X_2Y_3 - Y_1Y_2X_3 \Big\rangle \Big| = 4 > 2$$

Coherence in a logical qubit → nonlocality in a three-qubit system

Condition for coherence in a logical qubit → $\boxed{\langle X_L \rangle > c}$ $\Longrightarrow$ $\boxed{\left\langle \frac{1}{4}(X_1X_2X_3 - X_1Y_2Y_3 - Y_1X_2Y_3 - Y_1Y_2X_3) \right\rangle > c\,.}$

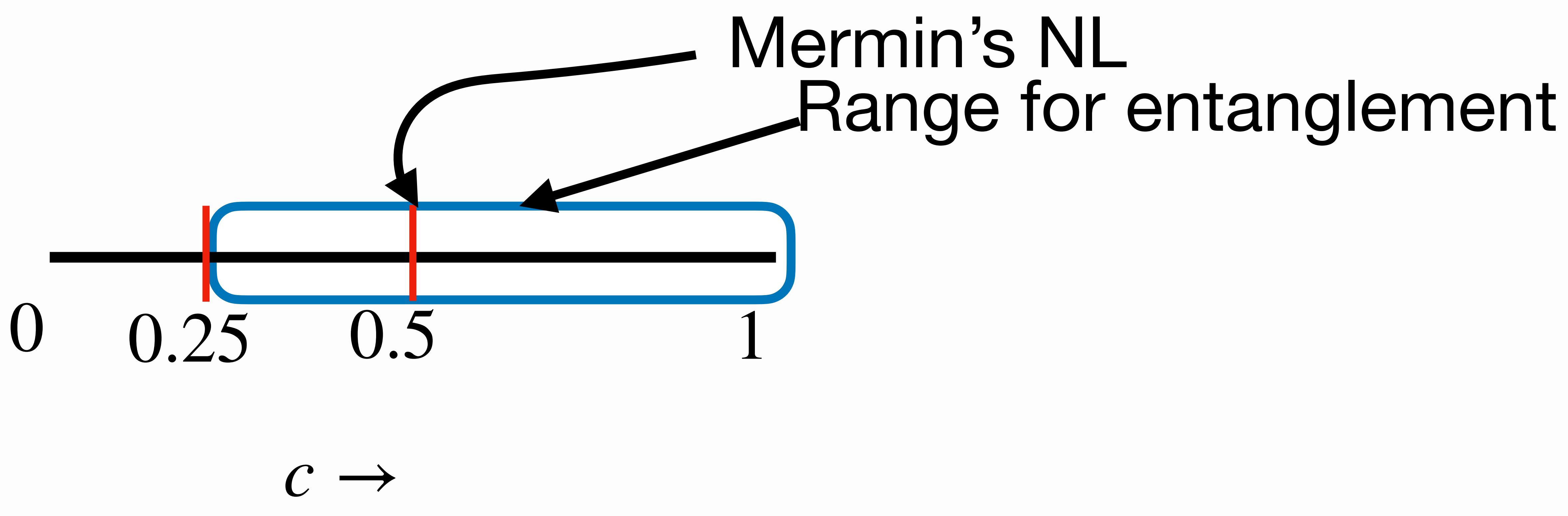
Use the homomorphic images

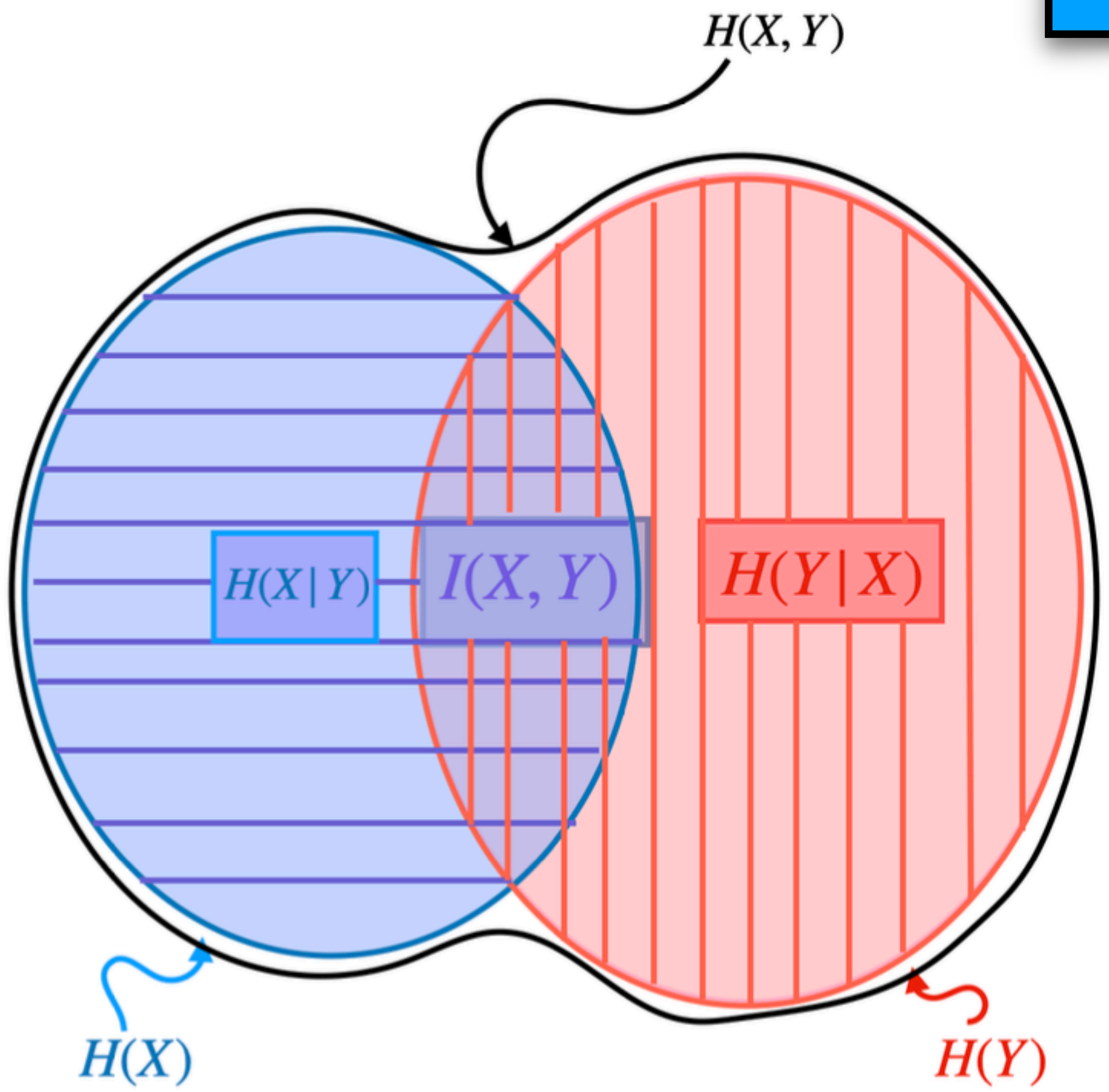


Coherence in a logical qubit → nonlocality in a three-qubit system

Condition for coherence in a logical qubit → $\langle X_L \rangle > c$ $\Longrightarrow$ $\left\langle \frac{1}{4}(X_1X_2X_3 - X_1Y_2Y_3 - Y_1X_2Y_3 - Y_1Y_2X_3) \right\rangle > c\,.$

Use the homomorphic images



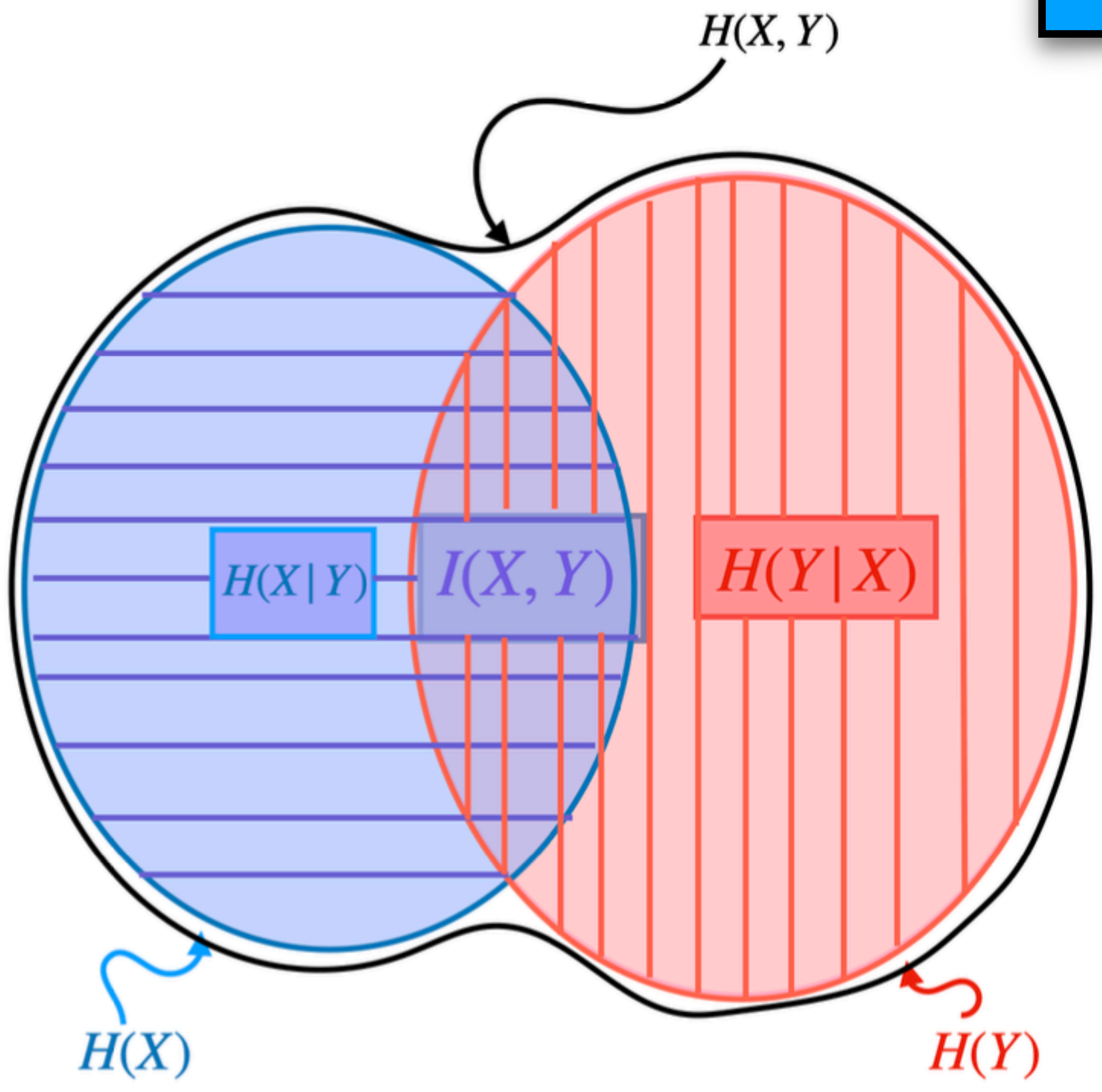


Quantum discord

$$I(A:B) = H(A) + H(B) - H(A:B)$$

$$J(A:B) = H(A) - H(B|A).$$

Quantum discord



$$I(A:B) = H(A) + H(B) - H(A:B)$$

$$J(A:B) = H(A) - H(B|A).$$

$$D(\rho) = I(A:B) - J(A:B)$$

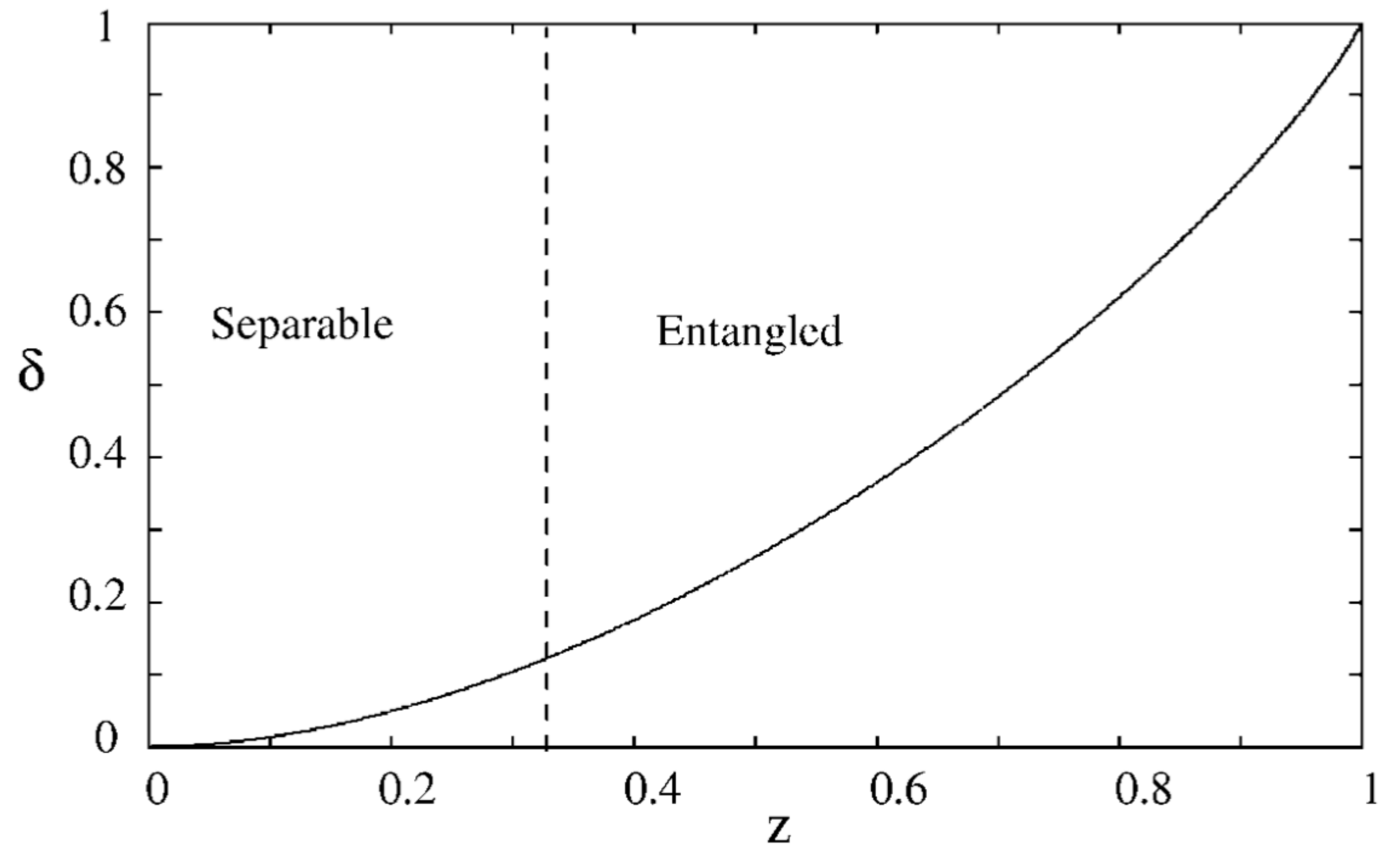


FIG. 2. Value of the discord for Werner states $\frac{1-z}{4}\mathbf{1} + z|\psi\rangle\langle\psi|$, with $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Discord does not depend on the basis of measurement in this case because both $\mathbf{1}$ and $|\psi\rangle$ are invariant under local rotations.

Ollivier, H., & Zurek, W. H. (2001). Quantum discord: a measure of the quantumness of correlations. *Physical review letters*, *88*(1), 017901.

PART A: Interrelation among nonclassical features

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Weak measurement

$$P_\epsilon = |\langle f|\hat{U}(\epsilon)|i\rangle|^2 = |\langle f|(1 - i\epsilon\hat{A} + \cdots)|i\rangle|^2$$

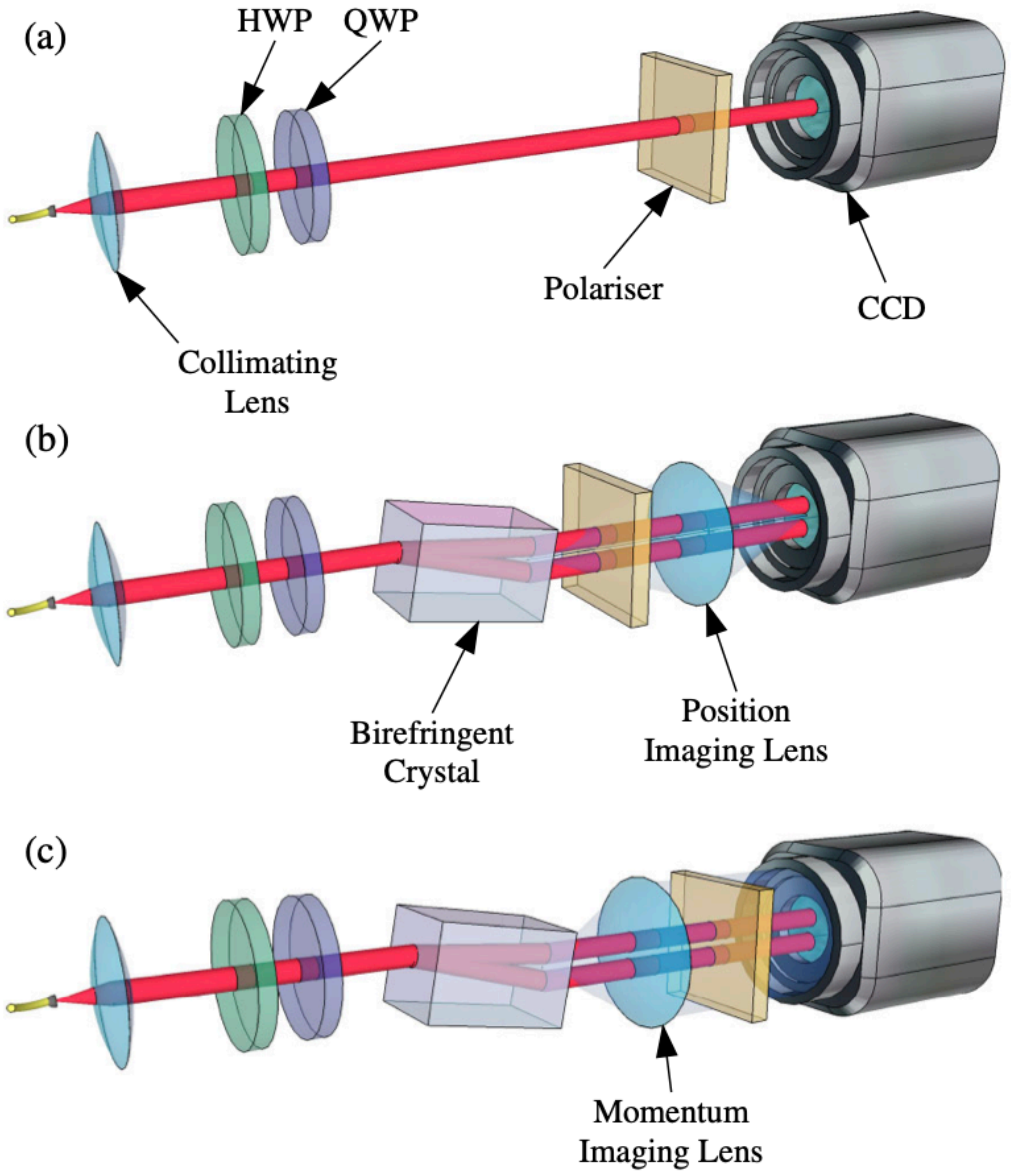
$$= P + 2\epsilon\mathrm{Im}\langle i|f\rangle\langle f|A|i\rangle + o(\epsilon^2)$$

$$\langle i|f\rangle \neq 0$$

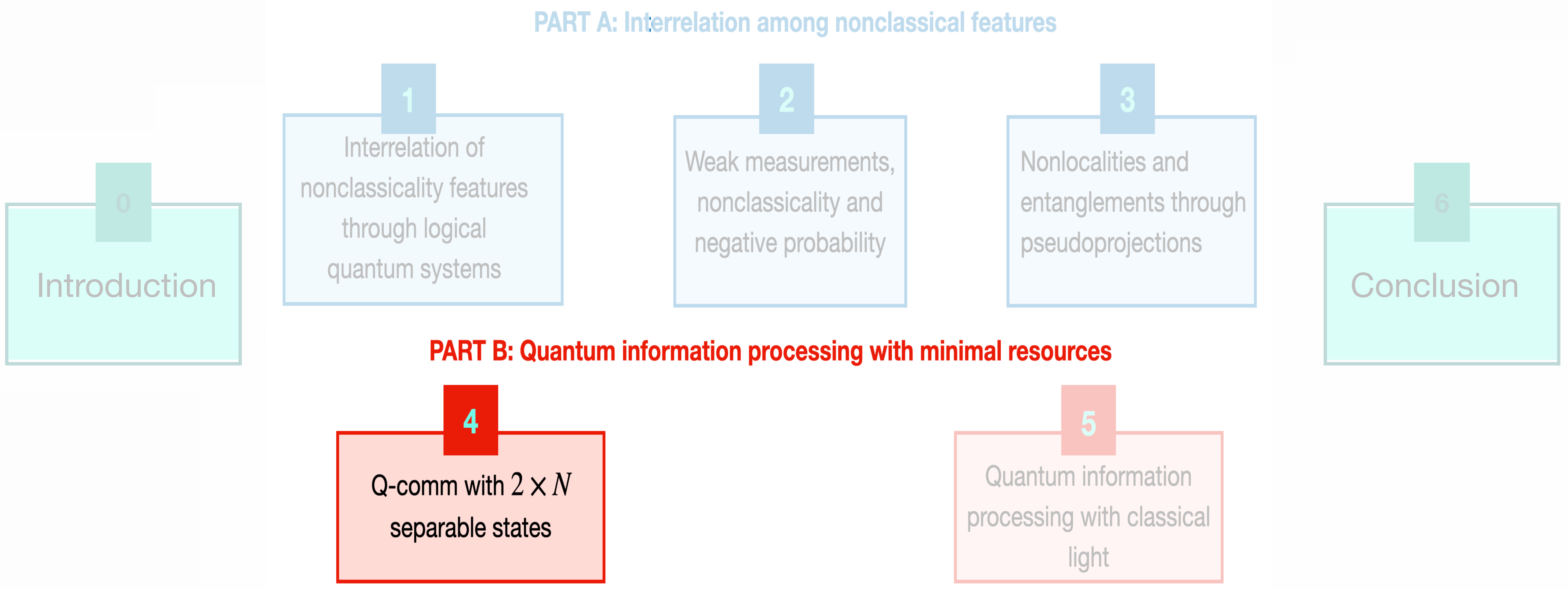
$$\frac{P_\epsilon}{P} = 1 + 2\epsilon\mathrm{Im}\ A_w - \epsilon^2[\mathrm{Re}A_w^2 - |A_w|^2] + o(\epsilon^3)$$

Dressel, Justin, et al. "Colloquium: Understanding quantum weak values: Basics and applications." *Reviews of Modern Physics* 86.1 (2014): 307.

Weak measurement



Dressel, Justin, et al. "Colloquium: Understanding quantum weak values: Basics and applications." *Reviews of Modern Physics* 86.1 (2014): 307.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Type of states	Typical yield
multidimensional entangled OAM states	~ $10^{-3} - 10^{-1}$ Hz
Separable OAM states	$10 - 10^{3}$ Hz
Heralded single-photon generation (in the polarization domain) from crystals	~ $10^{3} - 10^{5}$ per second

Yield of entangled states

State	Count	Fidelity
GHZ state $\lvert 000\rangle + \lvert 111\rangle$	~mHz	$\sim 85\,\%$
$\left(\lvert 000\rangle + \lvert 111\rangle + \lvert 220\rangle\right)$	15 mHz	$80.1\,\%$
$\left(\lvert 000\rangle + \lvert 111\rangle + \lvert 220\rangle + \lvert 331\rangle\right)$	0.66 Hz	$85.4\,\%$

M. Erhard *et al*., *Nature Reviews Physics*, 2(7):365-381, (2020).
X. Hu *et al*., *npj Quantum Information*, 6(1): 1-5, (2020).

Classical-quantum interplay

What is "quantum" in quantum information processing?

Numerous notions of classicality within the quantum domain

Separability $\langle O_1^A O_2^B \rangle = \sum_i p_i \langle O_1^A \rangle_{\rho_i^A} \langle O_2^B \rangle_{\rho_i^B}$

Locality $P(\hat{a}, \hat{b}) = \int d\lambda \rho(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda), \ |A(\vec{a}, \lambda)| \leq 1, \ |B(\vec{b}, \lambda)| \leq 1.$

Brunner, N., et al. *Reviews of modern physics* 86.2 (2014): 419.
Horodecki, R., et al. *Reviews of modern physics* 81.2 (2009): 865.

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H. M. Bharath & V. Ravishankar *Phys. Rev. A* 89, 062110 (2014).

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3. H. M. Bharath & V. Ravishankar *Phys. Rev. A* 89, 062110 (2014).

Comparison

Quantum teleportation protocol[1]

- Initial state of Alice's system: $\frac{1}{2}(\mathbf{1} + \vec{\sigma}_1 \cdot \hat{p})$
- Channel (**entangled**): $\frac{1}{4}(\mathbf{1} - \alpha\vec{\sigma}_2 \cdot \vec{\sigma}_3)$
- State of Bob's system at the end of the protocol:

$$\frac{1}{2}(\mathbf{1} + \alpha\vec{\sigma}_3 \cdot \hat{p})$$

Information transfer protocol

- Initial state of Alice's system: $\frac{1}{2}(\mathbf{1} + \vec{\sigma}_1 \cdot \hat{p})$
- Channel (**separable**): $\frac{1}{2(2S+1)}(\mathbf{1} - \alpha\vec{\sigma}_2 \cdot \hat{S}_3)$
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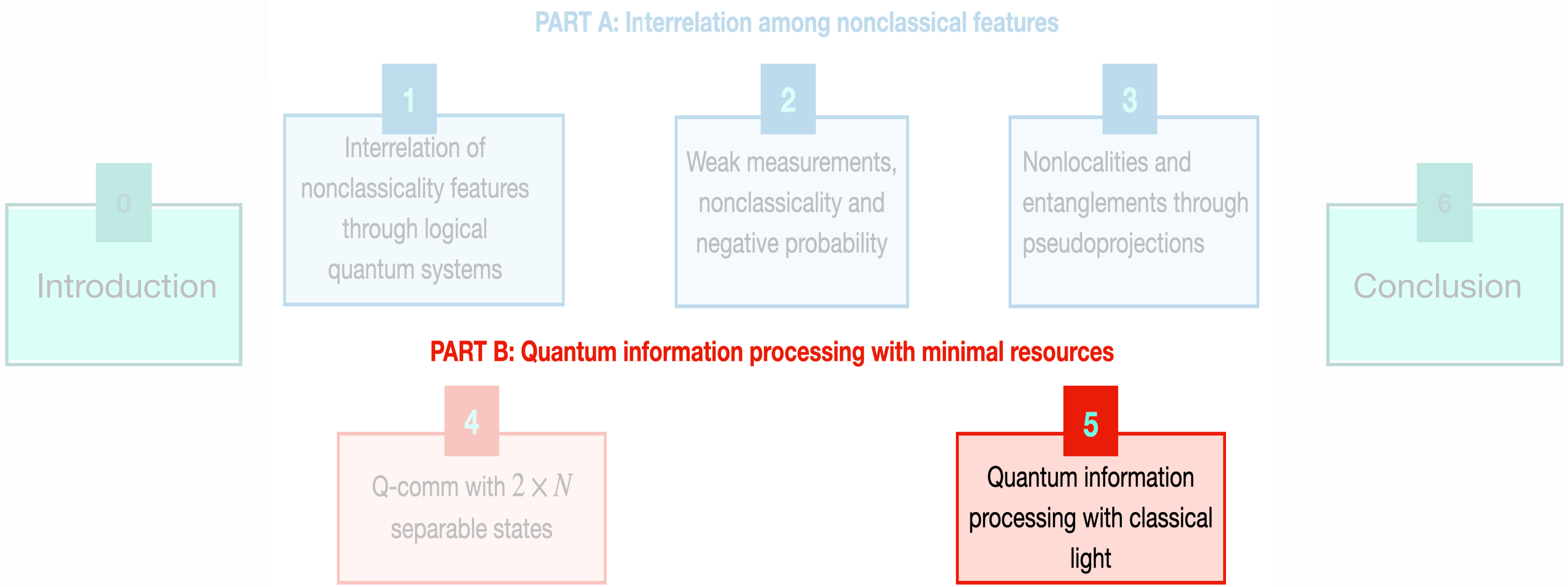
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Orbital angular momentum (OAM) of light

Paraxial Helmholtz equation

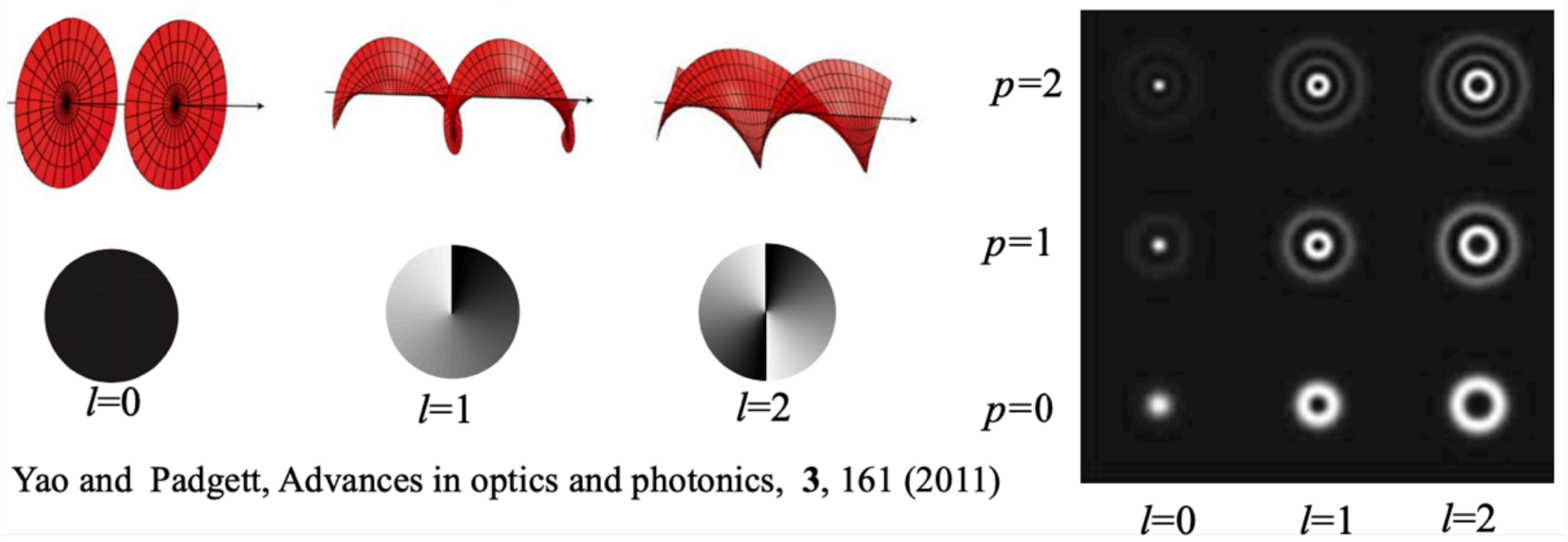
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik\frac{\partial}{\partial z}\right)E(x,y,z) = 0 \rightarrow \left(\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2} + 2ik\frac{\partial}{\partial z}\right)E(\rho,\phi,z) = 0$$

Solutions are of the form: $E(\rho,\phi,z) = \psi(\rho,\phi,z)e^{ikz}$

Laguerre Gaussian modes are solutions to paraxial Helmholtz equation

$$LG_p^l(\rho,\phi,z) = \frac{C}{1+z^2/z_R^2}\exp\left[i(2p+l+1)\tan^{-1}\frac{z}{z_R}\right]\left[\frac{\rho/2}{w(z)}\right]^l L_p^l\left[\frac{2\rho^2}{w^2(z)}\right]\exp\left(-\frac{\rho^2}{w(z)^2}\right)\exp\left[-\frac{ik^2\rho^2 z}{2(z^2+z_R^2)}\right]e^{-il\phi}$$

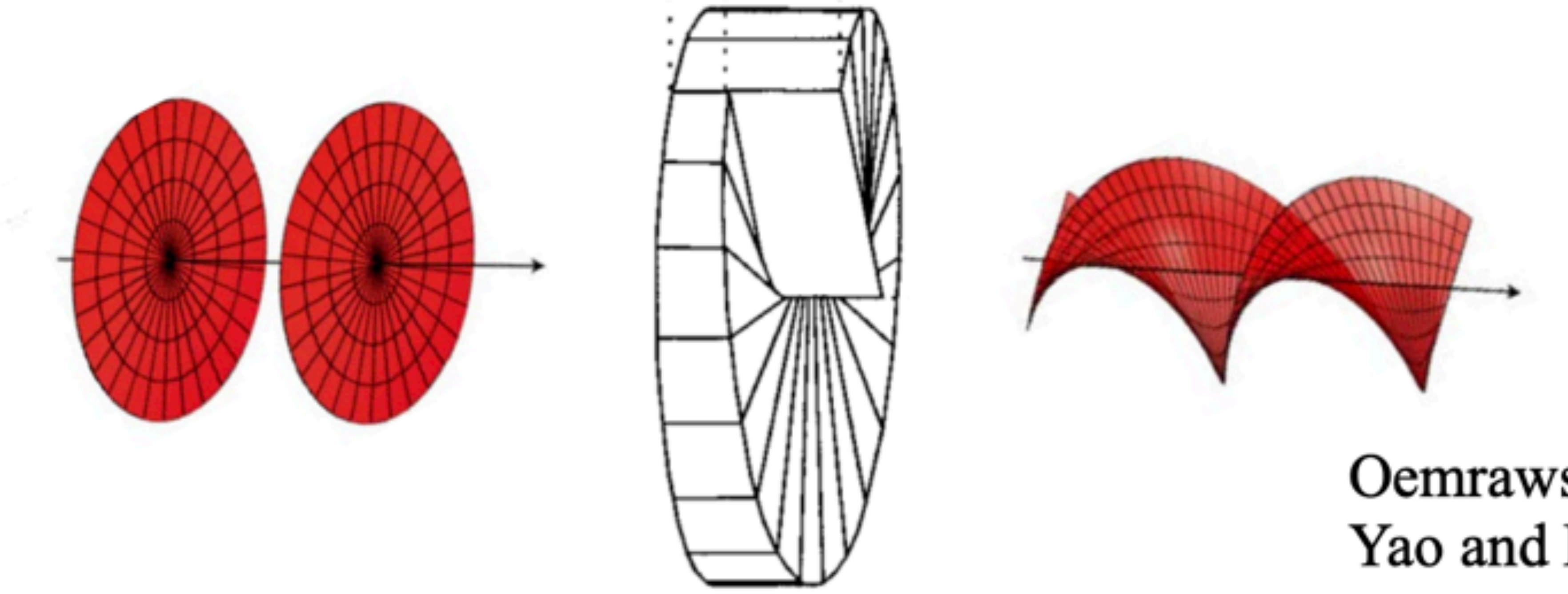
Phase of the LG mode with $p = 0$



Yao and Padgett, Advances in optics and photonics, **3**, 161 (2011)

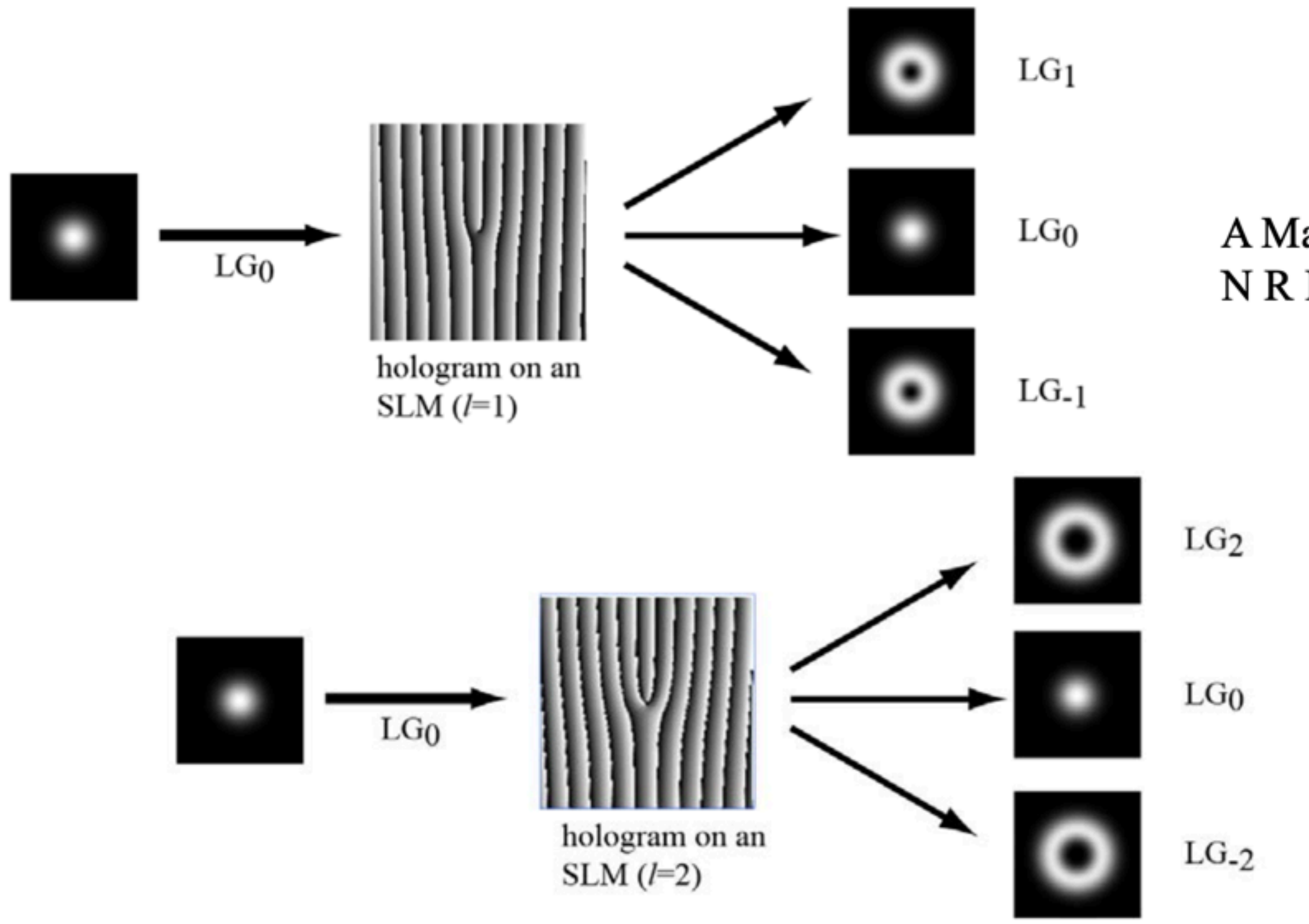
How to generate OAM modes

1. Using a spiral phase plate



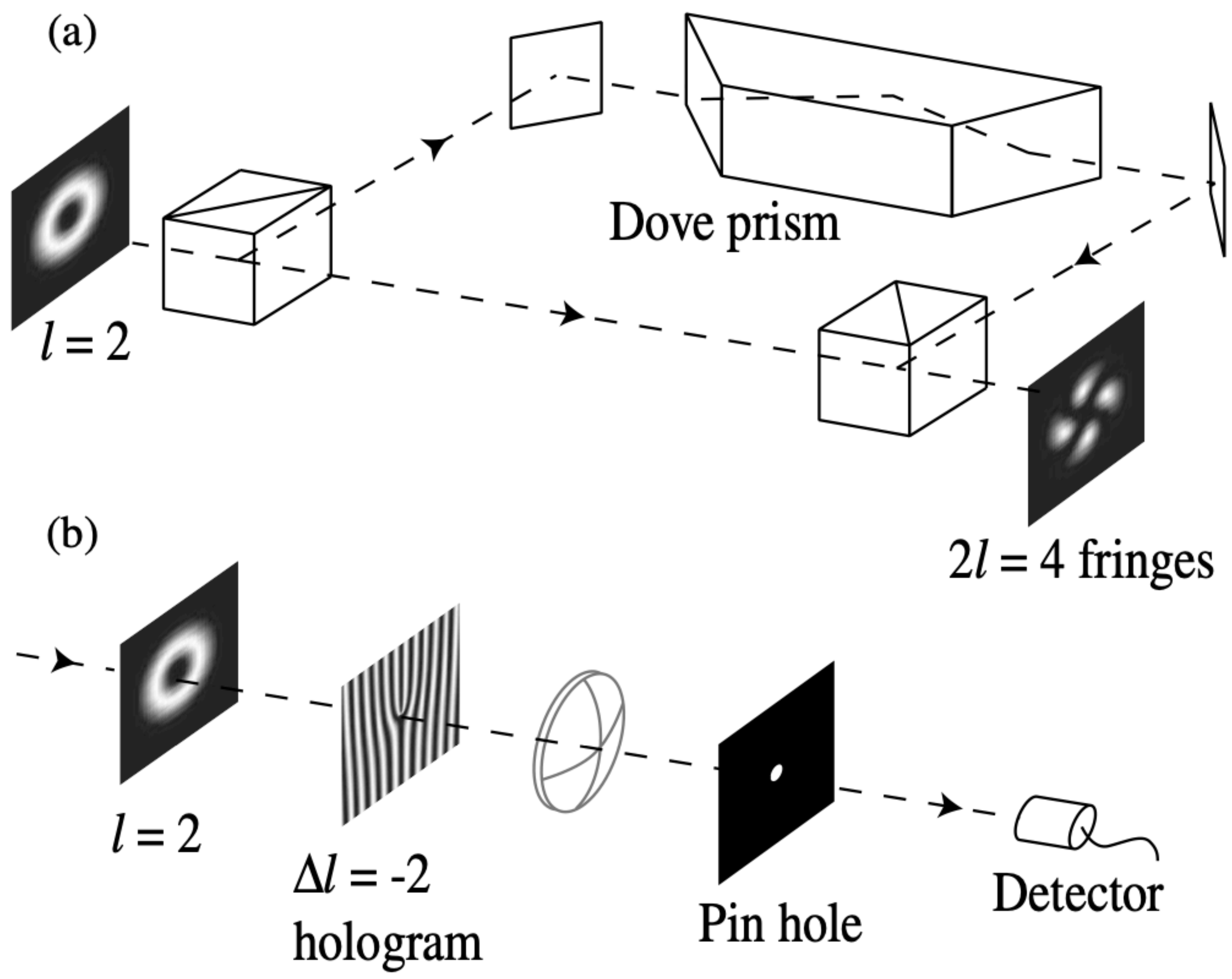
Oemrawsingh et al., Applied Optics, 43, 688 (2004)
Yao and Padgett, Advances in optics and photonics, **3**, 161 (2011)

2. Using a SLM



A Mair *et al.* Nature 412 **313** (2001)
N R Heckenberg *et al.* Opt. Lett. **17**, 221 (1992)

Detection of OAM modes



Leach, Jonathan, et al. "Measuring the orbital angular momentum of a single photon." *Physical review letters* 88.25 (2002): 257901.

First stage of OAM mode sorter

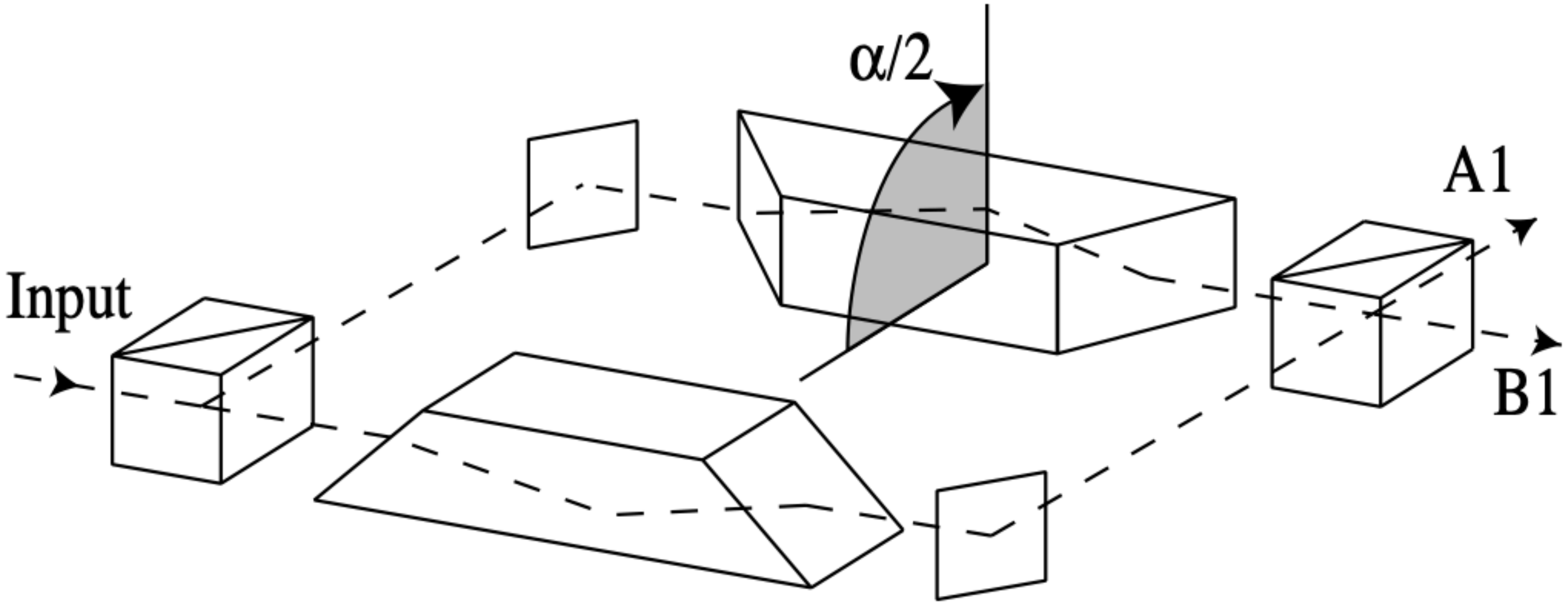
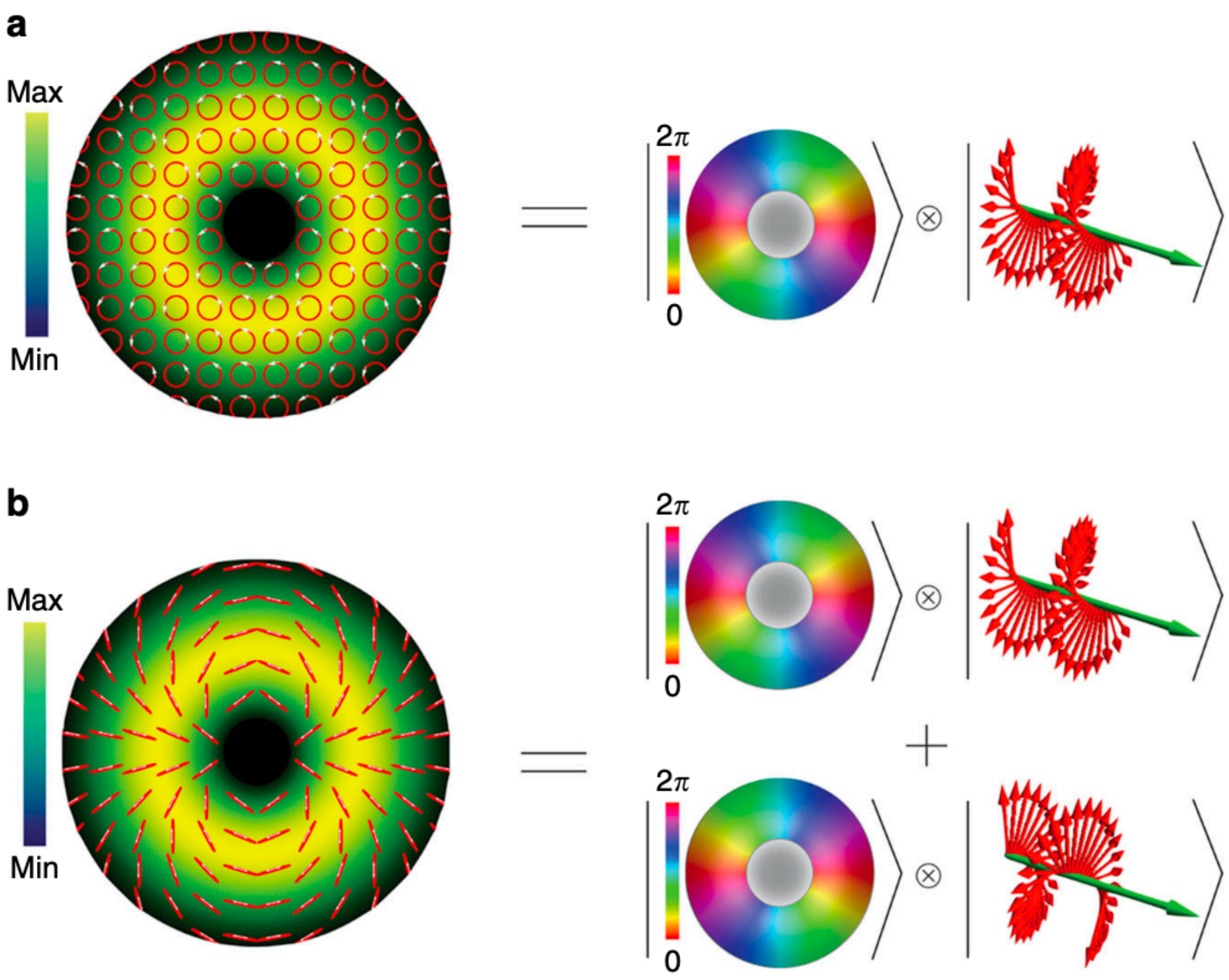


FIG. 3. First stage of our OAM sorter. A Mach-Zehnder interferometer with a Dove prism placed in each arm. The beams in the two arms are rotated with respect to each other through an angle α, where $\alpha/2$ is the relative angle between the dove prisms. In the example shown, $\alpha/2 = \pi/2$, this device sorts photons with even values of l into Port A1 and those with odd values of l into Port B1.

Leach, Jonathan, et al. "Measuring the orbital angular momentum of a single photon." *Physical review letters* 88.25 (2002): 257901.

Classical entanglement

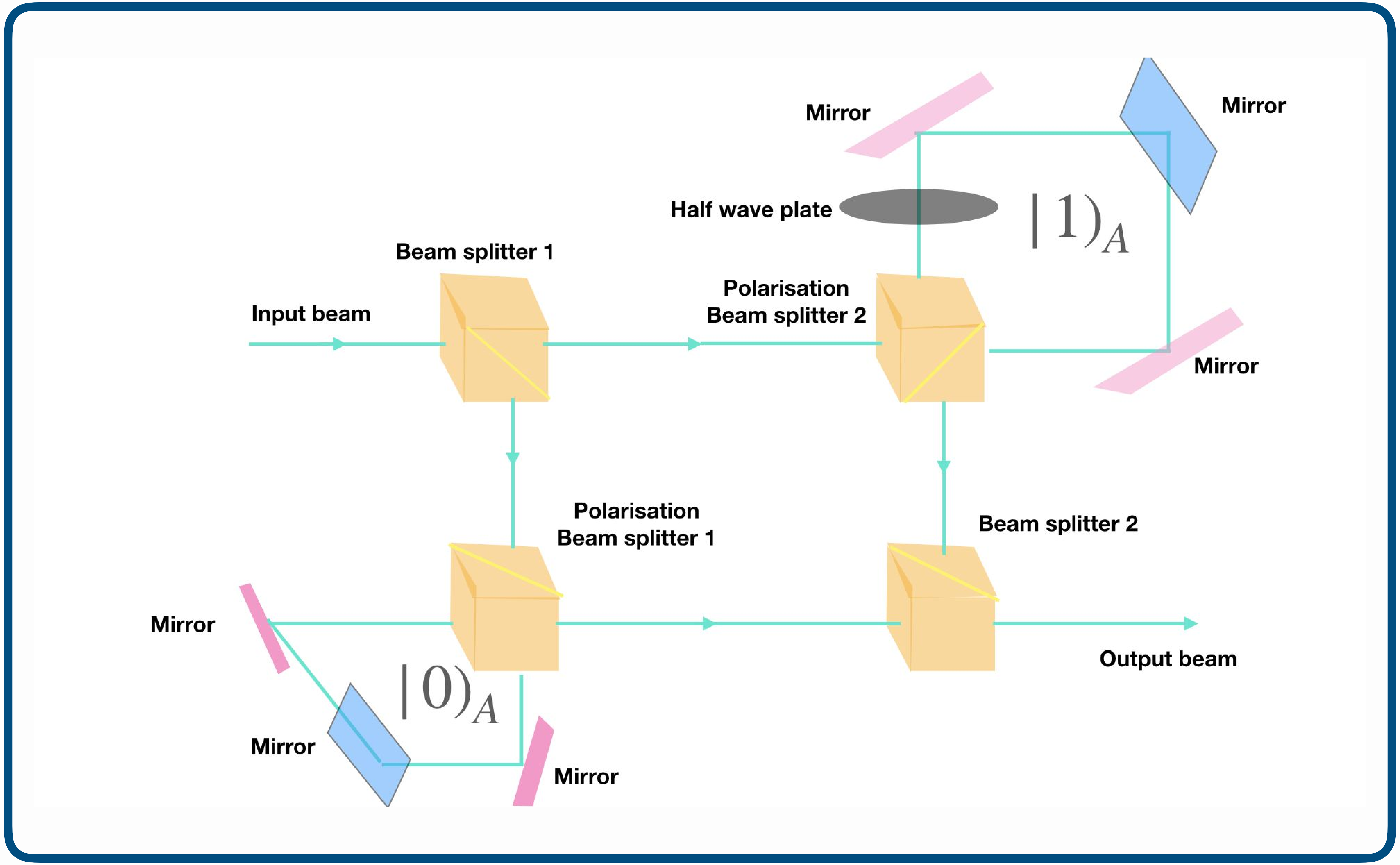


Uniform polarisation
Separable beam

Classically entangled

Konrad, Thomas, and Andrew Forbes. "Quantum mechanics and classical light." *Contemporary Physics* 60.1 (2019): 1-22.

CNOT Gate in path-polarisation degree of freedom



Generation of SEW $J^{\frac{1}{2},1}\left[\frac{1}{2}\right]$

- At. $\eta = \dfrac{1}{2}$, $J^{\frac{1}{2},\frac{1}{2}}[\eta]$ represents a classically entangled beam.
- Its equivalent beam, for $s = 1$, is however separable. In fact, its separable expansion is given by,

$$
\begin{aligned}
\frac{1}{6}\left[\mathbf{1} - \frac{1}{2}\vec{\sigma}^A \cdot \vec{S}^B\right] =& \frac{1}{6}\Big\{|V)(V| \otimes |\psi_1)(\psi_1| + |H)(H| \otimes |\psi_2)(\psi_2| \\
& + |-)(-| \otimes |\psi_{3+})(\psi_{3+}| + |+)(+| \otimes |\psi_{3-})(\psi_{3-}| \\
& + |R)(R| \otimes |\psi_{4+})(\psi_{4+}| + |L)(L| \otimes |\psi_{4-})(\psi_{4-}|\Big\},
\end{aligned}
$$

where, the symbols have the following meaning:

$$|\pm) = \frac{1}{\sqrt{2}}\{|H) \pm |V)\}, |R) = \frac{1}{\sqrt{2}}\{|H) + i|V)\}, |L) = \frac{1}{\sqrt{2}}\{|H) - i|V)\},$$

$$|\psi_1) = |\mathrm{LG}_{01}); \;\; |\psi_2) = |LG_{0-1})$$

$$|\psi_{3\pm}) = \frac{1}{2}|\mathrm{LG}_{0-1}) \pm \frac{1}{\sqrt{2}}|\mathrm{LG}_{00}) + \frac{1}{2}|\mathrm{LG}_{01})$$

$$|\psi_{4\pm}) = \frac{1}{2}|\mathrm{LG}_{0-1}) \pm \frac{i}{\sqrt{2}}|\mathrm{LG}_{00}) - \frac{1}{2}|\mathrm{LG}_{01}).$$

SU(2) coherent optical beams

- In order to realize SU(2) coherent state in a laser cavity, the eigenstates should fulfil the coherent-superposition condition of SU(2) wave-packet.
- Length of a plano- concave cavity= L
- Formed by a gain medium, a concave spherical mirror with the radius of curvature of R as the output coupler, and a plane mirror high-reflective for laser.

$$(\nabla^2 + k_{n,m,l}^2)\psi_{n,m,l}(x,y,z) = 0$$

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$$(\nabla^2 + k_{n,m,l}^2)\psi_{n,m,l}(x,y,z) = 0$$

Solutions:

$$\psi_{n,m,l}^{(\mathrm{HG})}(x,y,z) = \frac{1}{\sqrt{2^{m+n-1}\pi m!n!}}\frac{1}{w(z)}e^{-\frac{x^2+y^2}{w^2(z)}}H_n\left[\frac{\sqrt{2}x}{w(z)}\right]H_m\left[\frac{\sqrt{2}y}{w(z)}\right]e^{ik_{n,m,l}\tilde{z}-i(m+n+1)\tan^{-1}(z/z_R)}$$

$$\tilde{z} = z + \frac{(x^2+y^2)z}{2(z^2+z_R^2)} \qquad w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} \qquad z_R = \frac{\pi w_0^2}{\lambda}$$

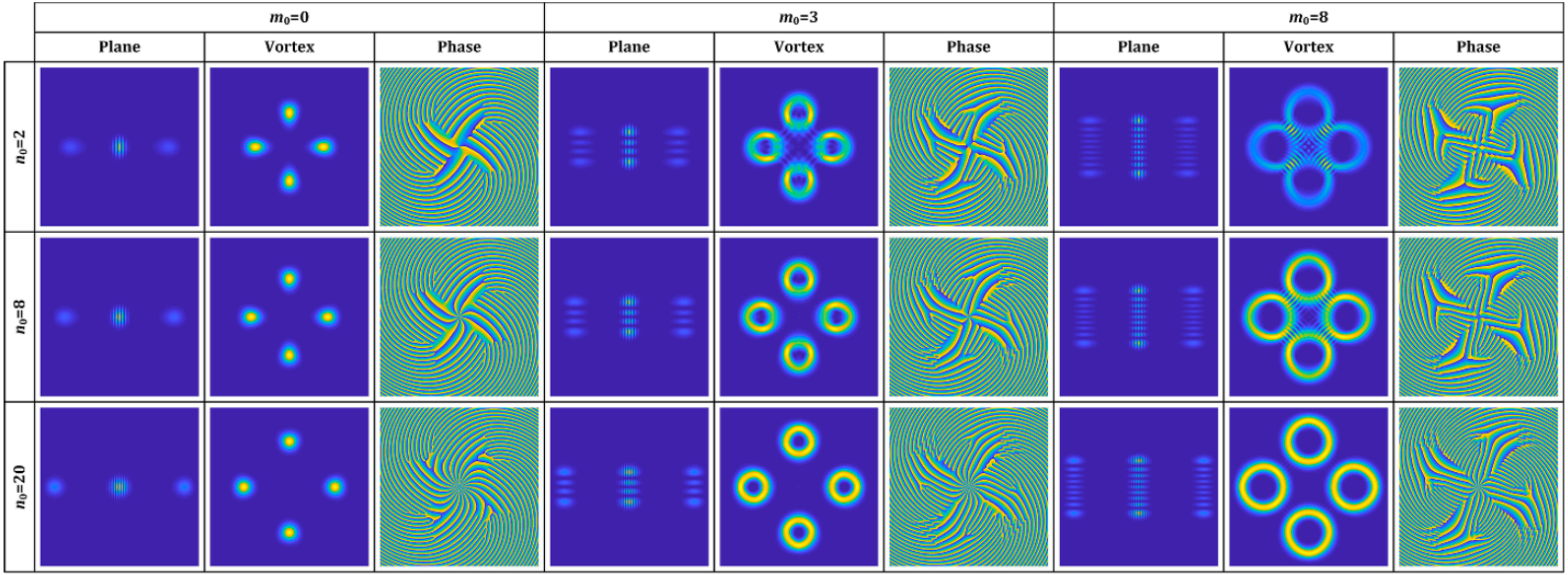
$$f_{n,m,l} = l\Delta f_L + \left(n+\frac{1}{2}\right)\Delta f_x + \left(m+\frac{1}{2}\right)\Delta f_y$$
$$= [l + (n+m+1)\Omega]\Delta f_L \qquad \Delta f_L = \frac{c}{2L}$$

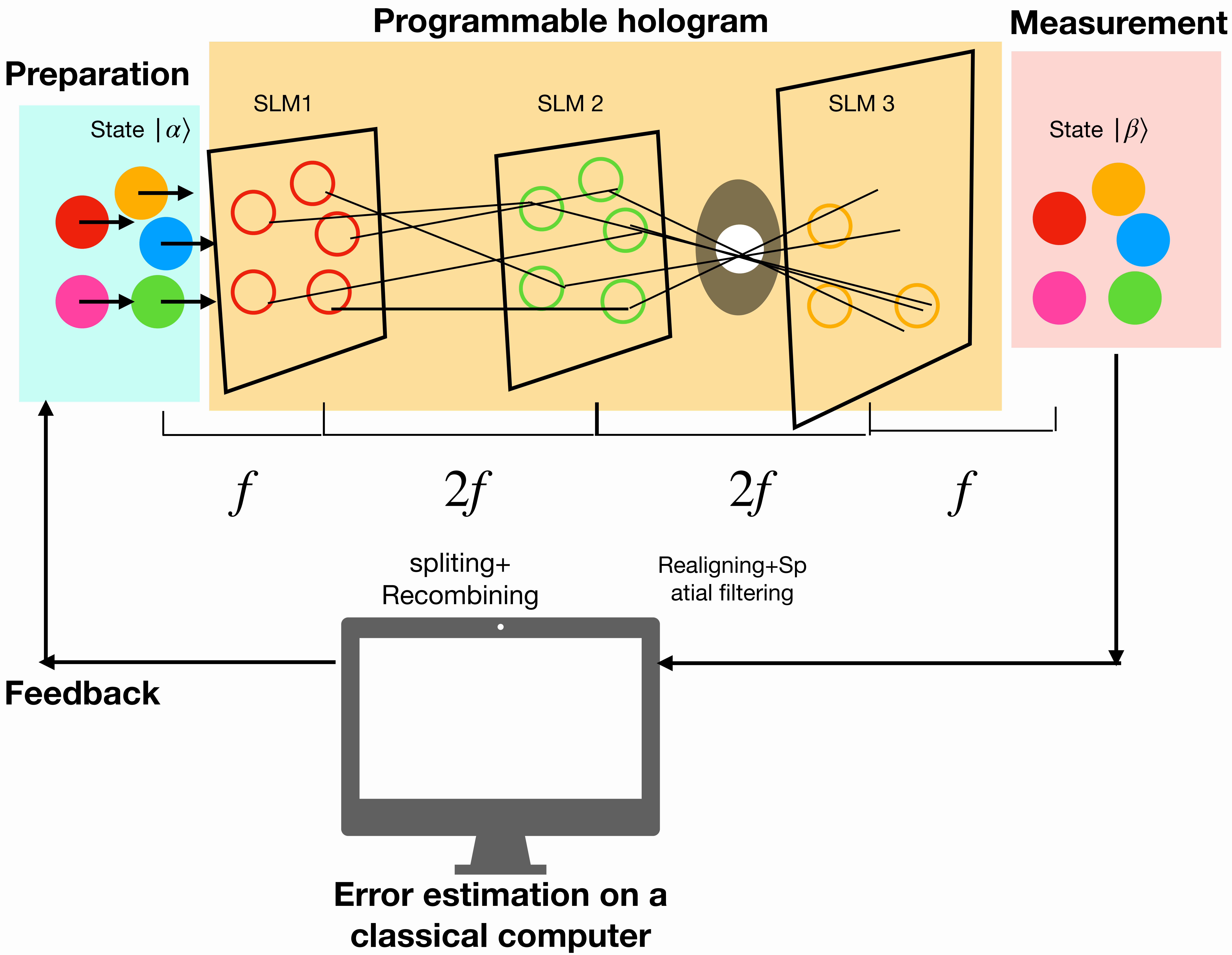
$$\Delta f_x = \Delta f_y = \Delta f_T = \Delta f_L \frac{1}{\pi} \tan^{-1}\left(\frac{L}{z_R}\right)$$

$$\Omega = \frac{P}{Q} = \frac{1}{\pi} \cos^{-1}\left(\sqrt{1 - \frac{L}{R}}\right)$$

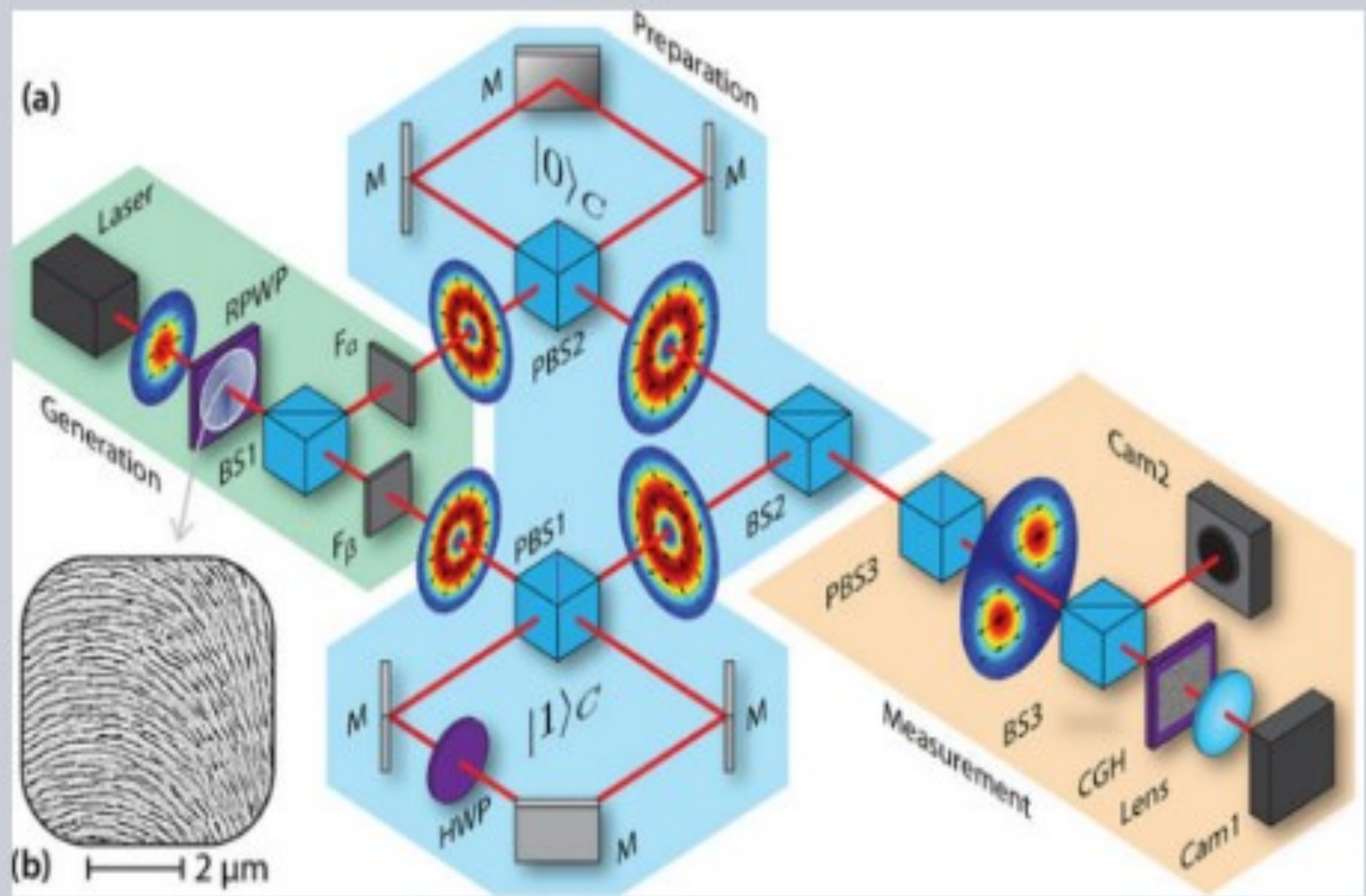
Can be changed from 0 to $1/2$ by changing the cavity length as $0 < L < R$.

$n + m + l = N \implies$ Frequency degenerate family of Hermite Gauss modes as the complete set.


m_0=0
m_0=3
m_0=8
Plane
Vortex
Phase
Plane
Vortex
Phase
Plane
Vortex
Phase
n_0=2
n_0=8
n_0=20


Programmable hologram
Measurement
Preparation
State $|\alpha\rangle$
SLM1
SLM 2
SLM 3
State $|\beta\rangle$
f
$2f$
$2f$
f
spliting+
Recombining
Realigning+Sp
atial filtering
Feedback
Error estimation on a
classical computer

Abstract Teleportation describes the transmission of information without transport of neither matter nor energy. For many years, however, it has been implicitly assumed that this scheme is of inherently nonlocal nature, and therefore exclusive to quantum systems. Here, we experimentally demonstrate that the concept of teleportation can be readily generalized beyond the quantum realm. We present an optical implementation of the teleportation protocol solely based on classical entanglement between spatial and modal degrees of freedom, entirely independent of nonlocality. Our findings could enable novel methods for distributing information between different transmission channels and may provide the means to leverage the advantages of both quantum and classical systems to create a robust hybrid communication infrastructure.



ORIGINAL PAPER

Demonstration of local teleportation using classical entanglement

Diego Guzman-Silva[1], Robert Brüning[2], Felix Zimmermann[1], Christian Vetter[1], Markus Gräfe[1], Matthias Heinrich[1], Stefan Nolte[1], Michael Duparré[2], Andrea Aiello[3,4], Marco Ornigotti[1,*], and Alexander Szameit[1]

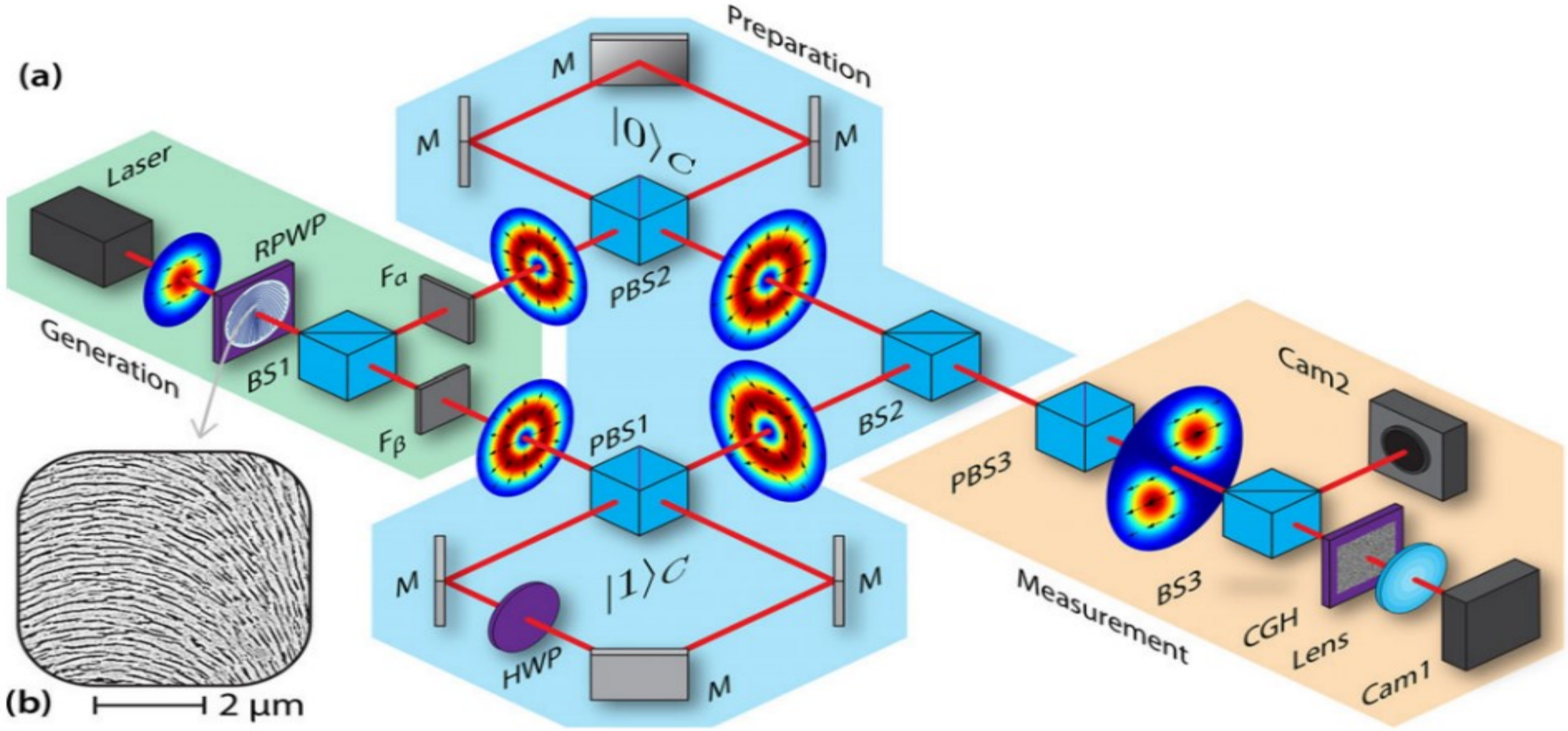
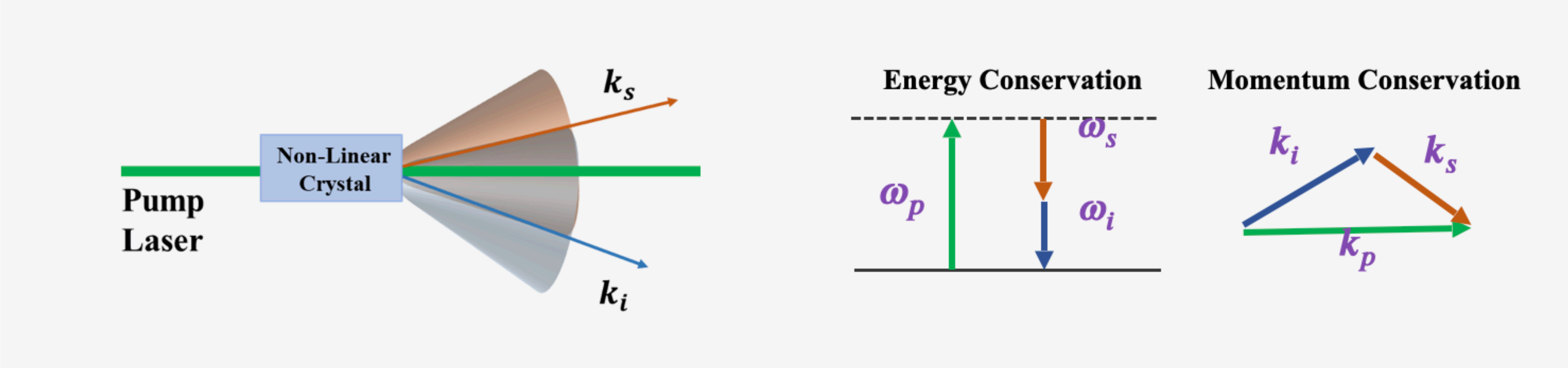


Figure 1 (a) Generation: A continuous-wave He-Ne laser is prepared in a classically entangled state with radial polarization by a rotating polarization wave plate (RPWP). The initial state $|\Psi\rangle$ is then obtained using a beam splitter (BS1). Two density filters (F_α and F_β) are used to encode the information in the cebit C. Preparation: The initial state $|\Psi\rangle$ is then sent through a C-NOT gate, realized using two Sagnac interferometers with a polarizing beam splitter. The lower interferometer, corresponding to $|1\rangle_C$ also contains a half-wave plate (HWP) to rotate the polarization. The two parts of the beam are then recombined through a second beam splitter (BS2), which implements a Hadamard operation for the cebit C. Measurement: The correct output state is selected by choosing the x-polarization (PBS3) of the lower output channel of BS2. The reflected beam from a third beam splitter (BS3) is sent to a CCD camera (Cam2) for direct acquisition of the intensity profile and angle measurement. The transmitted beam from BS3 is instead sent to the modal decomposition stage, consisting of a computer generated hologram (CGH) a lens and a second CCD camera (Cam1). (b) Exemplary scanning electron microscope image of a nanograting-based polarization rotating wave plate such as the RPWP used to generate the classically entangled beam with radial polarization.

Spontaneous Parametric Down Conversion (SPDC)



pm/V

$$\kappa = \frac{2}{3}\frac{\boxed{d_{\text{eff}}}}{\varepsilon_0 V}\sqrt{\frac{\omega_p\omega_s\omega_i}{2\varepsilon_0 V}}$$

$$P_i^{NL} = \varepsilon_0 \sum_j \sum_k \chi_{ijk}^{(2)}.E_j E_k$$

$$\hat{H}_{\text{SPDC}} = i\hbar\kappa(\hat{a}_1\hat{a}_2\hat{a}_3^{\dagger}e^{i\Delta\vec{k}.\vec{r}-i\Delta\omega.t} + \hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{a}_3 e^{-i\Delta\vec{k}.\vec{r}+i\Delta\omega.t})$$
$$= i\hbar\kappa(\hat{a}_i\hat{a}_s\hat{a}_p^{\dagger}e^{i\Delta\vec{k}.\vec{r}-i\Delta\omega.t} + \hat{a}_i^{\dagger}\hat{a}_s^{\dagger}\hat{a}_p e^{-i\Delta\vec{k}.\vec{r}+i\Delta\omega.t})$$

$$P_3(z,t) = \wp_3.e^{i(k_3 z-\omega_3 t)} + c.c$$

$$\wp_3 = 4\varepsilon_0 d_{\text{eff}} E_1 E_2.$$

$$k_1 + k_2 = k_3 = \frac{n_3\omega_3}{c} = \frac{n_1\omega_1}{c} + \frac{n_2\omega_2}{c}$$

Momentum conservation

$$e \mapsto o + o \text{ type I},$$

$$e \mapsto e + o \text{ type II},$$

$$e \mapsto o + e \text{ type II}.$$