Relation of nonclassical features through logical qubits

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(In collaboration with Prof. V. Ravishankar)

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Overview of the talk

1. Introduction

2. Preliminaries: Groups, Stabiliser Groups and homomorphism between stabiliser groups

3. Coherence in a logical qubit$\rightarrow$ Entanglement in a physical two-qubit system

4. Conclusion

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What is the state of a quantum system?

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Depends on the experimental observation.

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Example

$$|0\rangle$$

What is the state of a quantum system?

Depends on the experimental observation.

Example

$$|0\rangle = \frac{1}{\sqrt{2}}\Big(|01\rangle - |10\rangle\Big)$$

Multielectron atoms

Another example: Logical qubits and physical qubits

$$|0\rangle_L \equiv |000\rangle; \quad |1\rangle_L \equiv |111\rangle$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle_L + |1\rangle_L\right)$$

Another example: Logical qubits and physical qubits

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Another example: Logical qubits and physical qubits

$$|0\rangle_L \equiv |000\rangle; \quad |1\rangle_L \equiv |111\rangle$$

$$\alpha|0\rangle_L + \beta|1\rangle_L \equiv \alpha|000\rangle + \beta|111\rangle$$

Coherent state

$\alpha \neq 0, \quad \beta \neq 0$

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Coherent state

$\alpha \neq 0, \quad \beta \neq 0$

Entangled/ nonlocal state

$\alpha \neq 0, \quad \beta \neq 0$

How the condition for non classicality (coherence) in logical systems related to entanglement/ nonlocality in underlying physical systems?

Tools: Homomorphism between stabiliser groups

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Groups

A set $G = \{g\}$ is said to form a group under operation $*$ if the following properties are satisfied:

1. Closure: $\forall g_1, g_2 \in G, \quad g_1 * g_2 \in G\,.$

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4. Associativity: $\forall g_1, g_2, g_3 \in G, g_1 * (g_2 * g_3) = (g_1 * g_2) * g_3$

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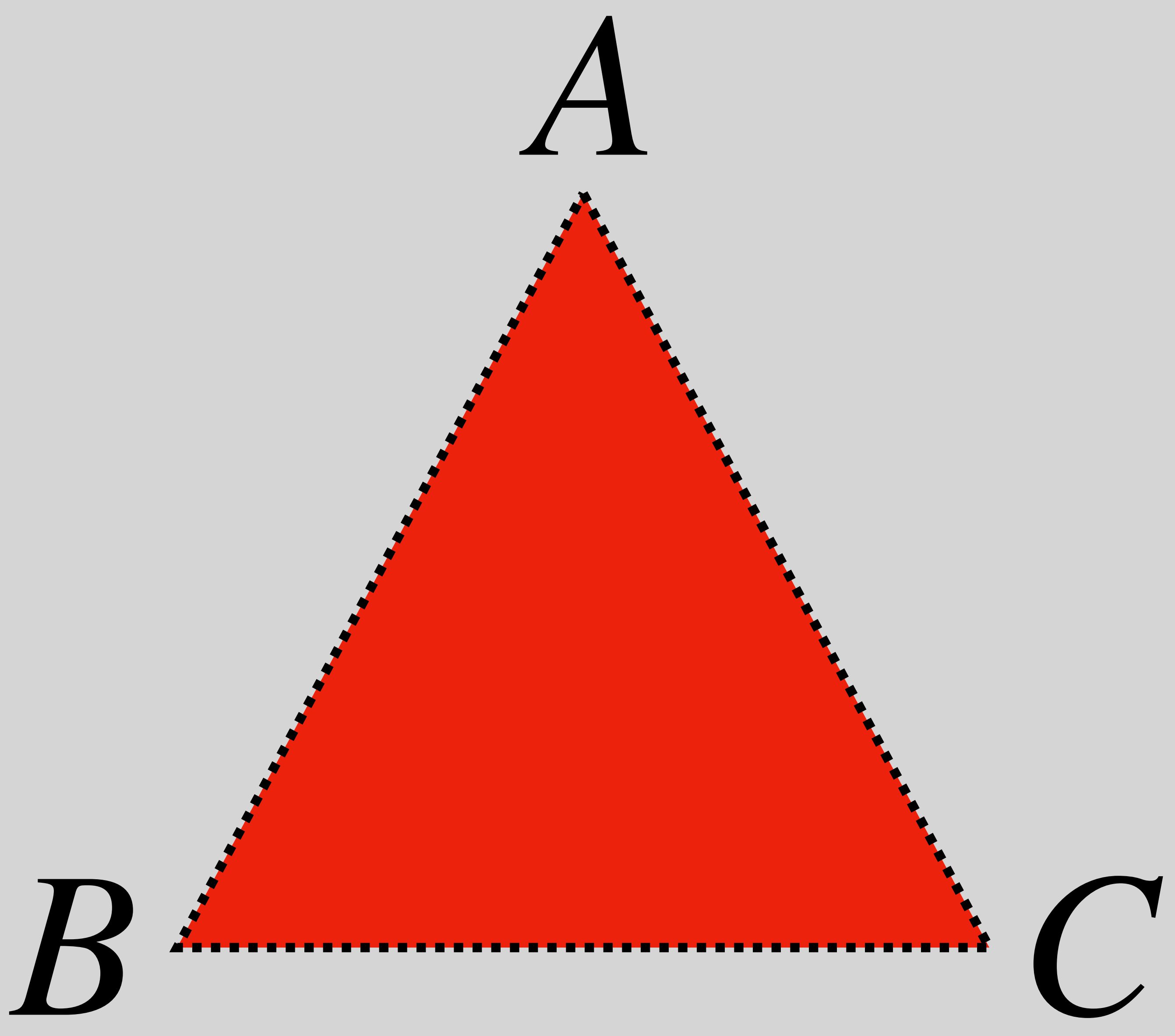
Example: Symmetry group of an equilateral triangle



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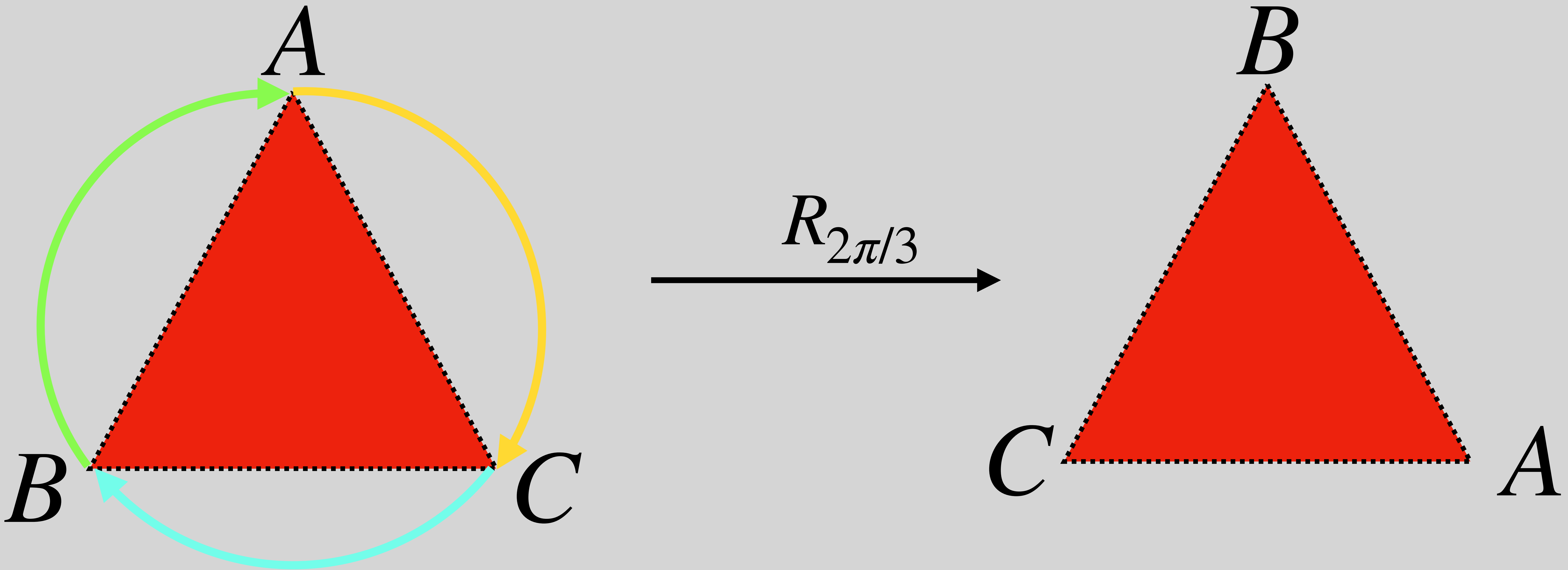

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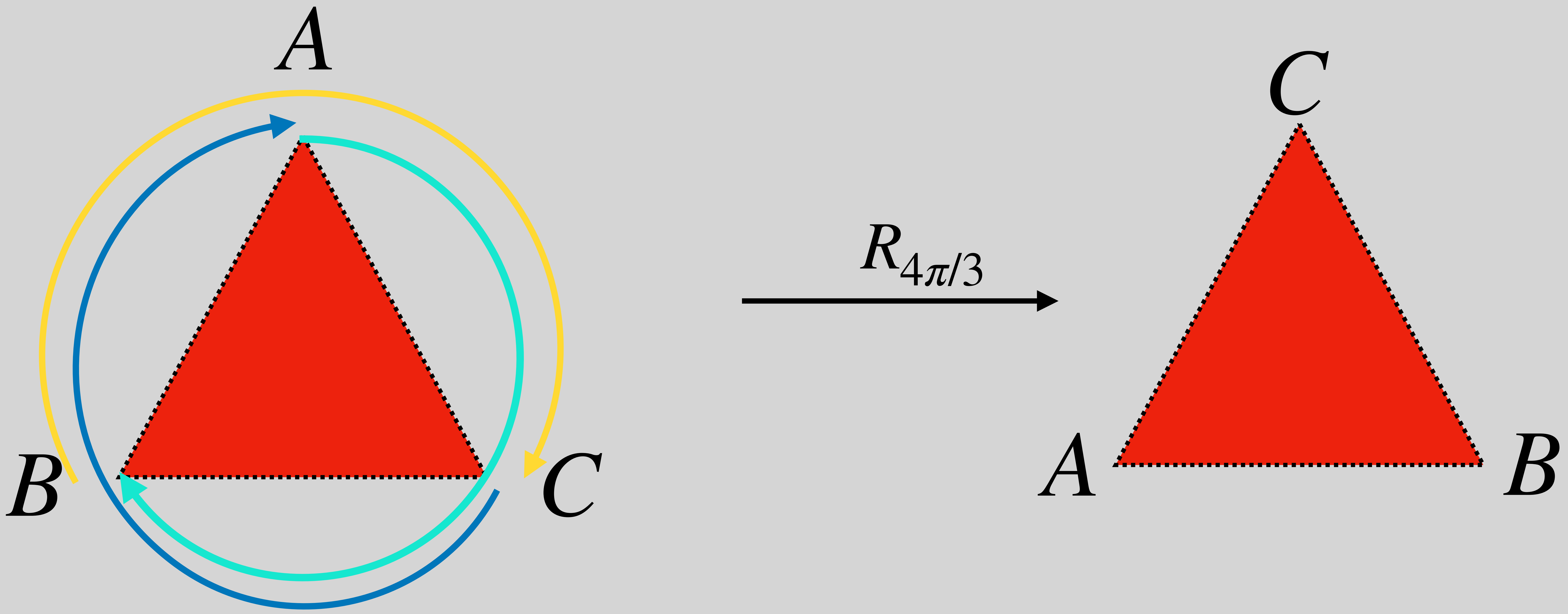

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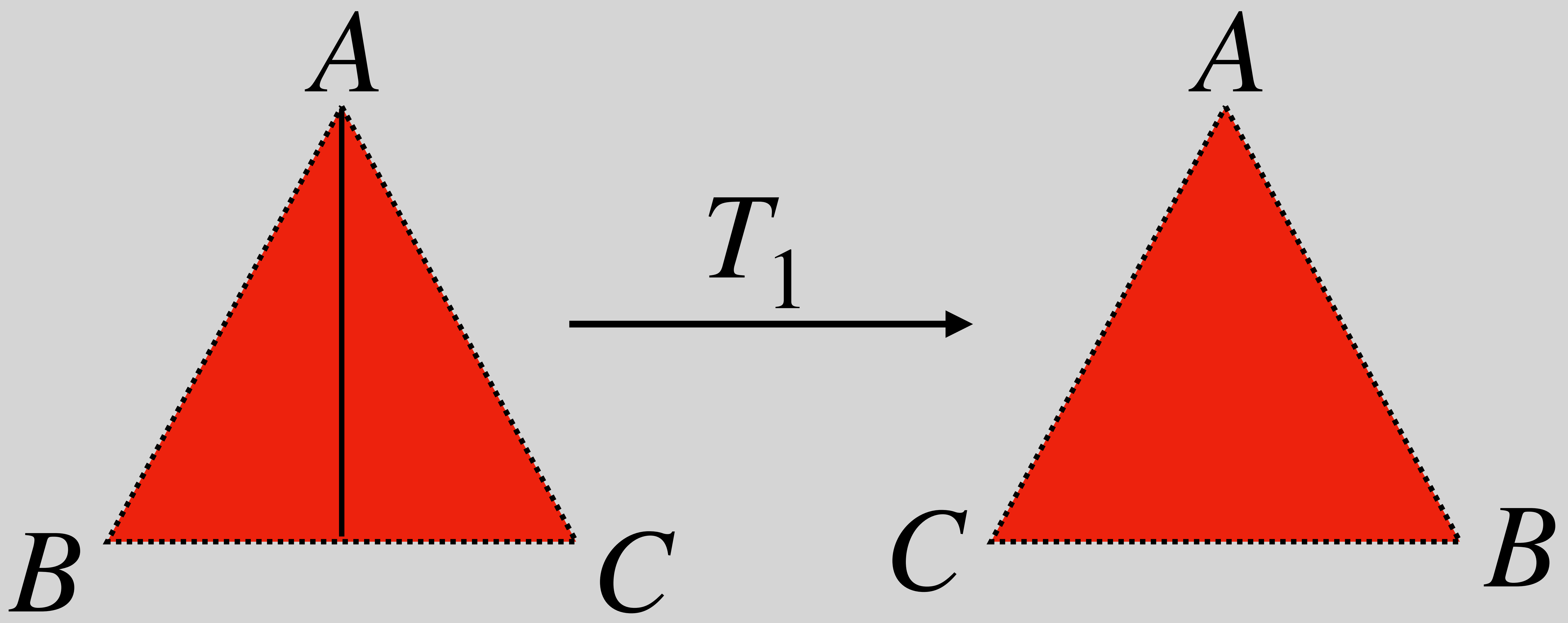

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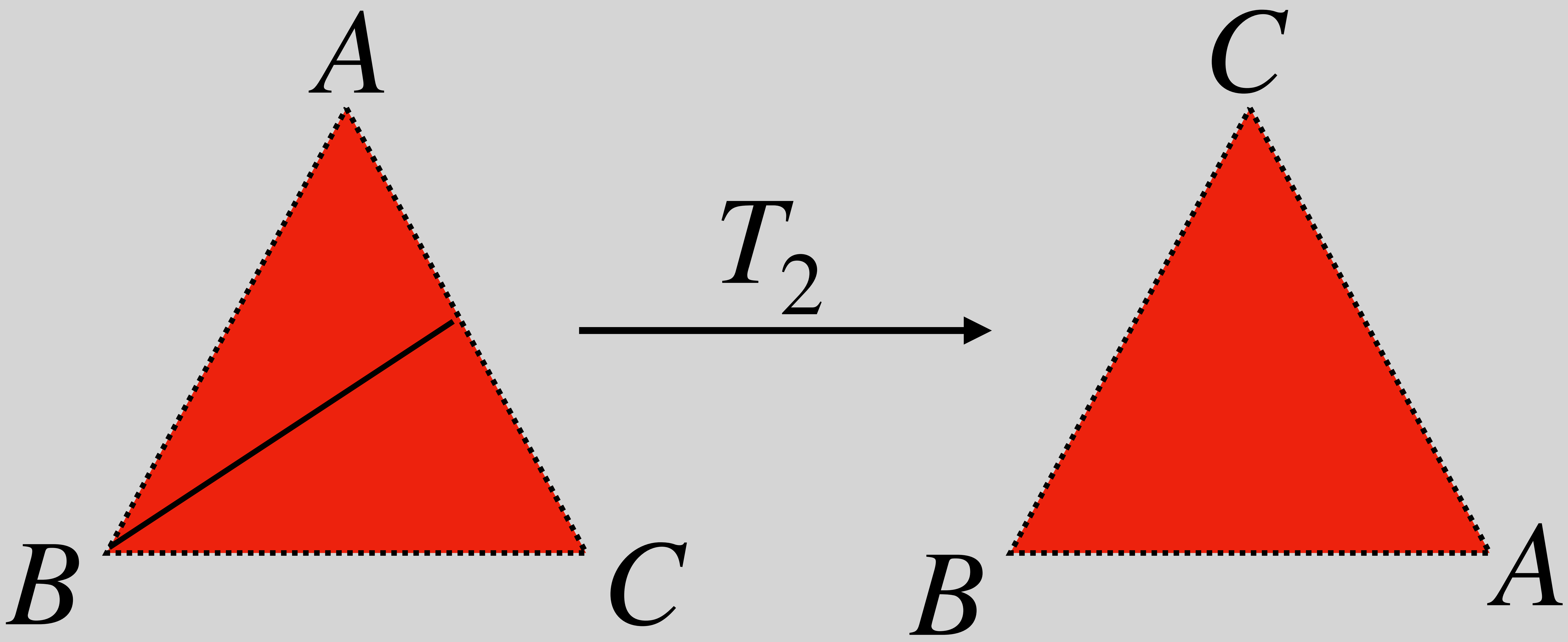

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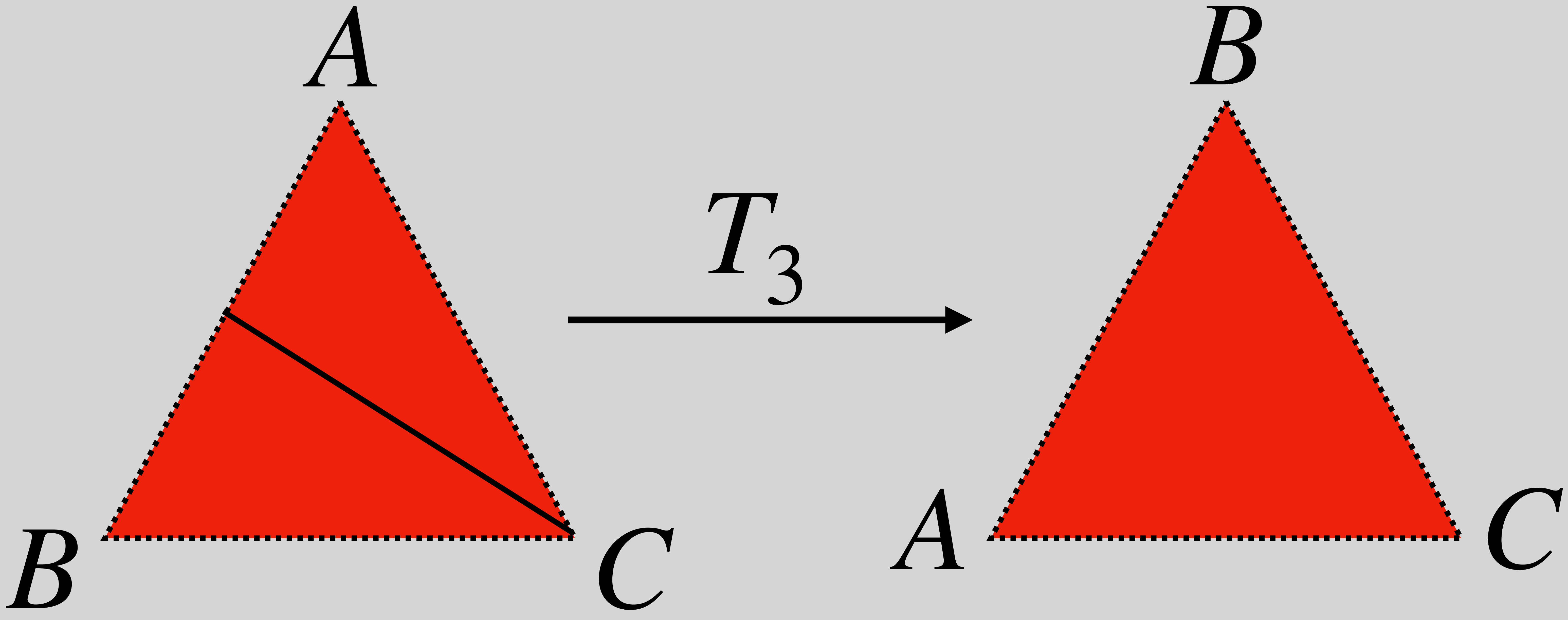

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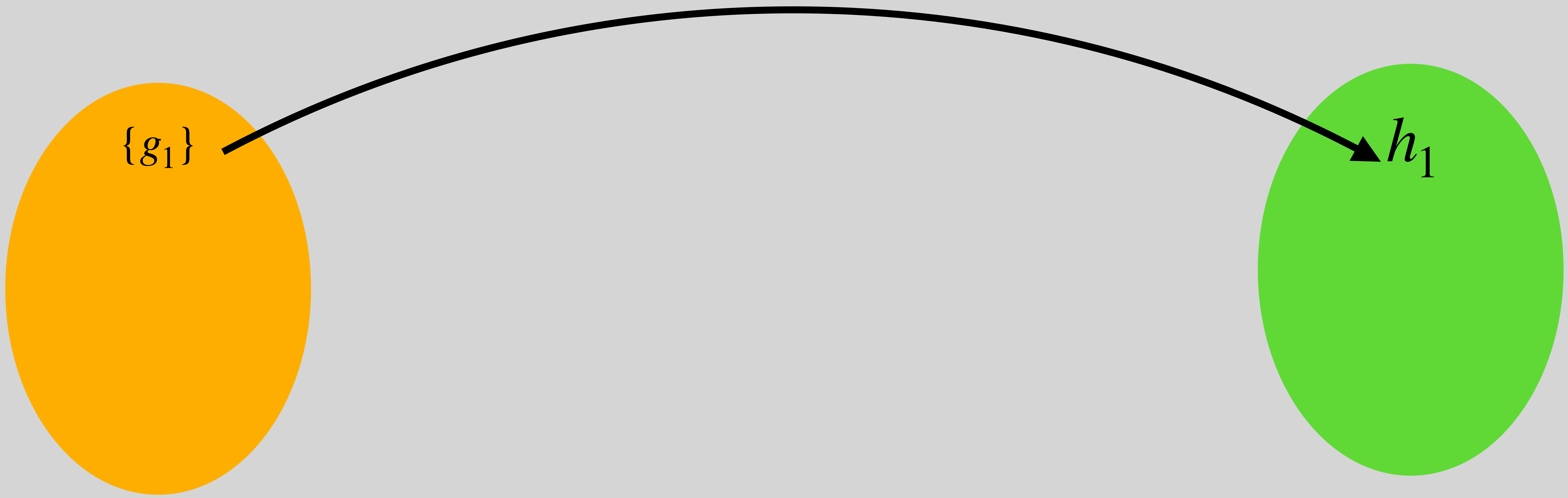
Symmetry group of an equilateral triangle: Group multiplication table

*	$\mathbf{1}$	$R_{2\pi/3}$	$R_{4\pi/3}$	T_1	T_2	T_3
$\mathbf{1}$	$\mathbf{1}$	$R_{2\pi/3}$	$R_{4\pi/3}$	T_1	T_2	T_3
$R_{2\pi/3}$	$R_{2\pi/3}$	$R_{4\pi/3}$	$\mathbf{1}$	T_2	T_3	T_1
$R_{4\pi/3}$	$R_{4\pi/3}$	$\mathbf{1}$	$R_{2\pi/3}$	T_3	T_1	T_2
T_1	T_1	T_3	T_2	$\mathbf{1}$	$R_{4\pi/3}$	$R_{2\pi/3}$
T_2	T_2	T_1	T_3	$R_{2\pi/3}$	$\mathbf{1}$	$R_{4\pi/3}$
T_3	T_3	T_2	T_1	$R_{4\pi/3}$	$R_{2\pi/3}$	$\mathbf{1}$

Tools: Homomorphism of stabiliser groups



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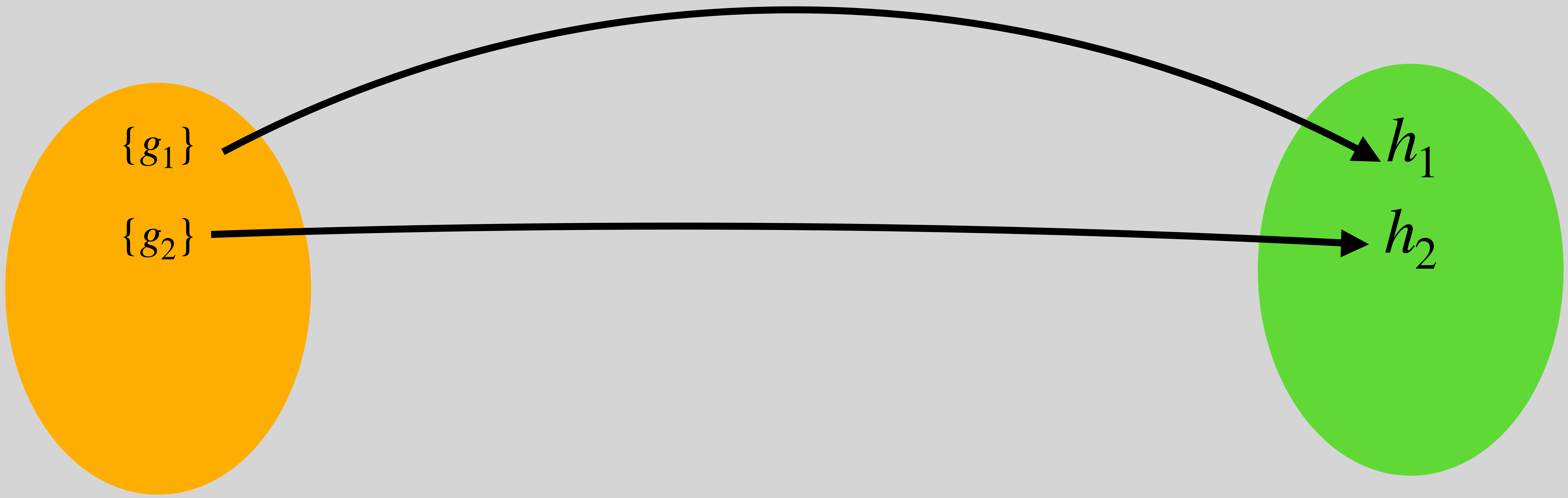
Homomorphism



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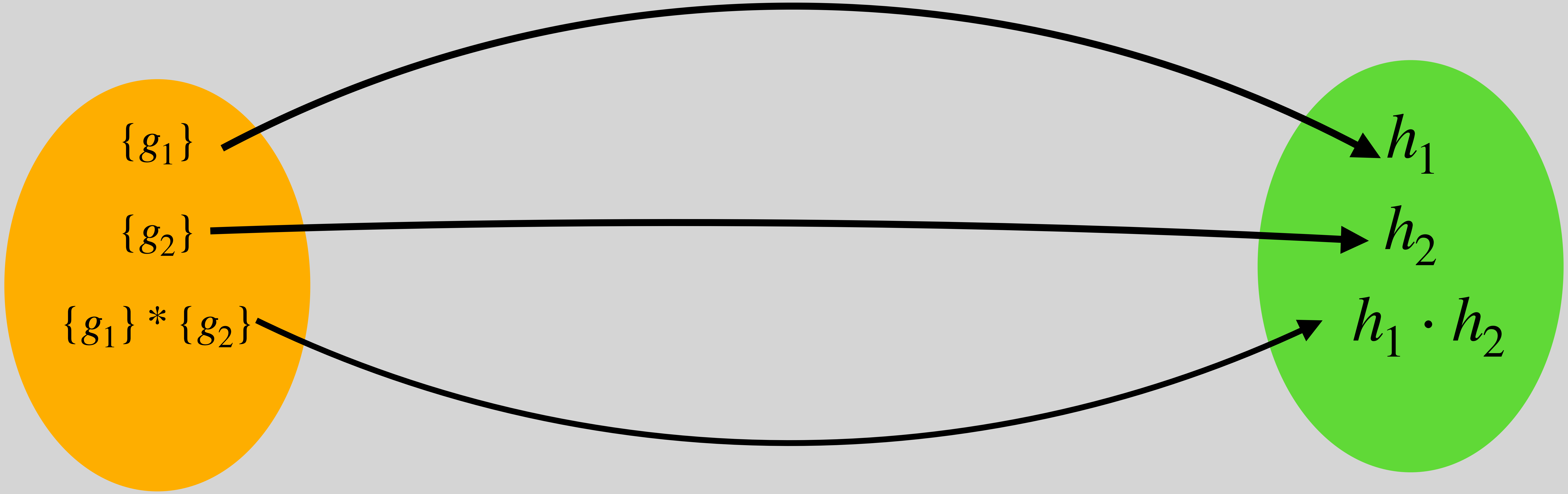

A product preserving map

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Stabiliser group

Stabilisers: Operators which have a state as their eigenstates with eigenvalue $+1$.

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Example: $$|\psi\rangle_L \equiv \frac{1}{\sqrt{2}}\Big(|0\rangle_L + |1\rangle_L\Big)$$

$$\mathbf{1}_L|\psi\rangle_L = |\psi\rangle_L$$

$$X_L|\psi\rangle_L = |\psi\rangle_L$$

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$$X_L|\psi\rangle_L = |\psi\rangle_L$$

$\cdot$	$\mathbf{1}_L$	X_L
$\mathbf{1}_L$	$\mathbf{1}_L$	X_L
X_L	X_L	$\mathbf{1}_L$

Instrumental in quantum error correction

Stabiliser group of a Bell state

$$|\psi\rangle_L \equiv \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)$$

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$$\mathbf{1}_4|\psi\rangle_L = |\psi\rangle_L$$

$$X_1X_2|\psi\rangle_L = |\psi\rangle_L$$

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$\bullet$	$\mathbf{1}_4$	X_1X_2	$-Y_1Y_2$	Z_1Z_2
$\mathbf{1}_4$	$\mathbf{1}_4$	X_1X_2	$-Y_1Y_2$	Z_1Z_2
X_1X_2	X_1X_2	$\mathbf{1}_4$	Z_1Z_2	$-Y_1Y_2$
$-Y_1Y_2$	$-Y_1Y_2$	Z_1Z_2	$\mathbf{1}_4$	X_1X_2
Z_1Z_2	Z_1Z_2	$-Y_1Y_2$	X_1X_2	$\mathbf{1}_4$

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Homomorphism between stabiliser groups

$$\{\mathbf{1}_L, X_L\} \qquad |\psi\rangle_L \equiv |0\rangle_L + |1\rangle_L$$

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$$\{\mathbf{1}_4, Z_1Z_2\} \rightarrow \mathbf{1}_L$$

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Globally commuting but locally non commuting

Detects coherence in the logical space

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Stabiliser group of a GHZ state

$$|\psi\rangle_L \equiv \frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big)$$

$$\{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_3Z_1, X_1X_2X_3, -X_1Y_2Y_3, -Y_1X_2Y_3, -Y_1Y_2X_3, -Y_1Y_2Y_3\}$$

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Homomorphism between stabiliser groups

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$$\{\mathbf{1}, Z_1Z_2, Z_2Z_3, Z_1Z_3\} \rightarrow \mathbf{1}_L$$

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$$\boxed{\begin{array}{l} \langle X_L \rangle > c, \quad c \in [0,1)\,. \\ \langle 0_L | X_L | 0_L \rangle = 0 \\ \langle 1_L | X_L | 1_L \rangle = 0 \\ \langle \psi_L | X_L | \psi_L \rangle = 1 \end{array}}$$

Used for construction of Mermin's inequality

Detects coherence in the logical space

Homomorphism between stabiliser groups

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$$(X_1X_2X_3)(-X_1Y_2Y_3)(-Y_1X_2Y_3)(-Y_1Y_2X_3) = -1$$

$$\boxed{\left| \left\langle X_1X_2X_3 - X_1Y_2Y_3 - Y_1X_2Y_3 - Y_1Y_2X_3 \right\rangle \right| \leq 2}$$

Homomorphism between stabiliser groups

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$$-Y_1X_2Y_3|\psi_L\rangle = |\psi_L\rangle$$

$$-Y_1Y_2X_3|\psi_L\rangle = |\psi_L\rangle$$

$$\left| \left\langle X_1X_2X_3 - X_1Y_2Y_3 - Y_1X_2Y_3 - Y_1Y_2X_3 \right\rangle_{|\psi_L\rangle} \right| = 4 > 2$$

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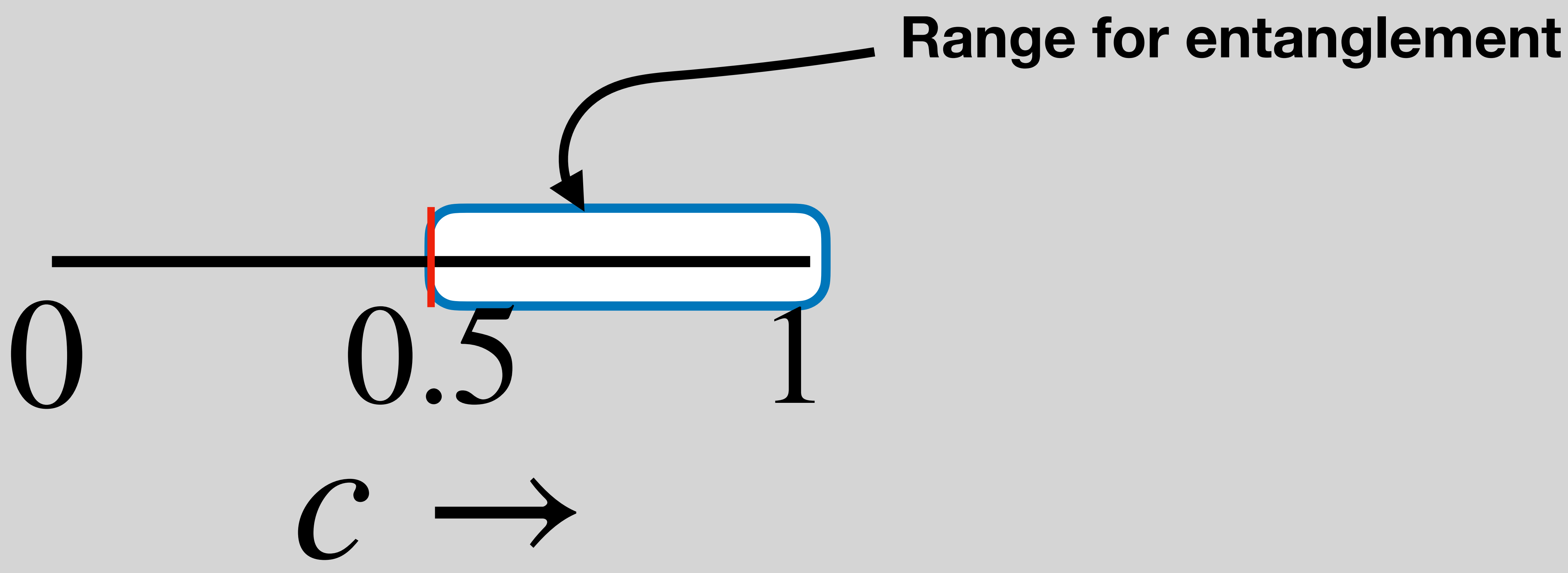
Coherence in a logical qubit → entanglement in a two-qubit system

Condition for coherence in a logical qubit → $\boxed{\langle X_L \rangle > c}$ $\Longrightarrow$ $\boxed{\langle X_1 X_2 - Y_1 Y_2 \rangle > 2c\,.}$ ← Condition for entanglement in a physical two-qubit qubit

$c \in \left[\frac{1}{2}, 1\right)$

Use the homomorphic images



Coherence in a logical qubit → nonlocality in a three-qubit system

Condition for coherence in a logical qubit

$$\boxed{\langle X_L \rangle > c} \implies \boxed{\langle X_1X_2X_3 - X_1Y_2Y_3 - Y_1X_2Y_3 - Y_1Y_2X_3 - Y_1Y_2Y_3 \rangle > 4c\,.}$$

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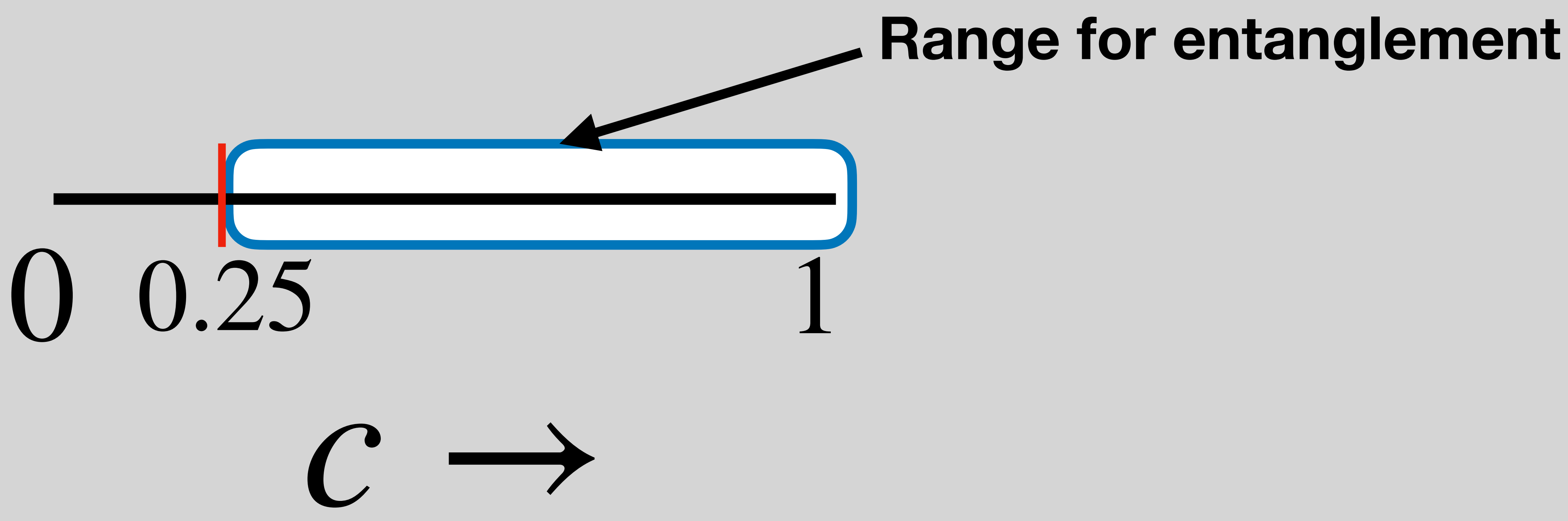

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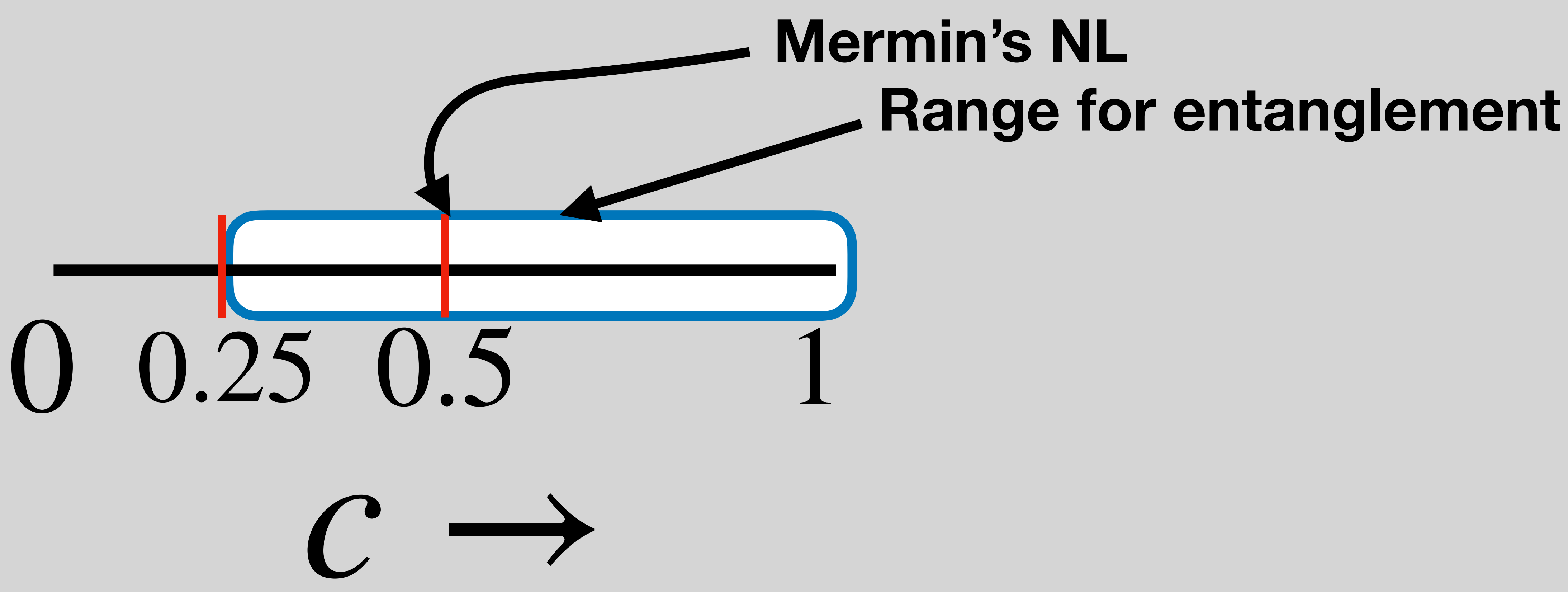

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Coherence in logical qubits is a manifestation of nonclassical correlation in the underlying physical multiqubit systems.

Generalisable to qudits as well.

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What about monotones?

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THANK YOU!!