Quantum Mechanics
PHL555

Problem Set 8

Two dimensional and miscellaneous one dimensional problems

1. Show that any attractive potential in $1-D$ admits atleast one bound state.
   Hint: Use the result that a finite square well potential has this property.

2. Consider the attractive delta function potential $V(x) = -\lambda\delta(x)$.

   (i) Verify that it has exactly one bound state.
   Hint: The wave function is of the form $\exp(-\alpha|x|)$.

   (ii) Show that this can be obtained as a limiting case of a square well potential with the width $a \to 0$ and the height $V_0 \to \infty$ in a suitable fashion.

3. A particle of mass m is confined to lie ina circle of radius R. Fined the first four energy eigenfunctions and the corresponding eigenvalues.

4. Obtain the eigenstates of the radial momentum operator in $D = 2$.

5. Verify explicitly that $p_\phi = \vec{p} \cdot \hat{\phi} = \hat{\phi} \cdot \vec{p}$.

6. Find the connection between the angular momentum L_z and p_ϕ.

7. Start with the classical expression for the hamiltonian (energy) in terms of $p_r,\ p_\phi$ and use the operators constructed above to construct the quantum mechanical hamiltonian. Compare it with what you get by transforming the two dimensional Laplacian from cartesian to polar coordinates.

8. Obtain the solutions for a two dimensional oscillator of frequency ω in *polar* coordinates. Carefully enumerate the degeneracy of each energy eigenvalue.

9. Consider the first three energy levels. Write the relationship between the solutions in cartesian and polar coordinates for each level.

10. How would you experimentally verify the degeneracy formula for the oscillator?

11. You are given the attractive delta potential in two dimensions: $V(x, y) = -\lambda\delta(x)\delta(y)$. Analyse carefully if this admits a bound state.