Quantum Mechanics
PHL555

Problem Set 2

Hermitian Operators

1. You are given the matrices

$$(i)\ A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 3 & 5 & 0 \end{pmatrix}; \qquad (ii)\ B = \begin{pmatrix} 0 & 1 & 5 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{pmatrix}$$

$$(iii)\ C = \begin{pmatrix} 0 & -i & 2i \\ i & 0 & 2+3i \\ -2i & 2-3i & 5 \end{pmatrix}; \qquad (iv)\ D = \begin{pmatrix} 1 & 0 & 6i \\ 0 & -1 & 3 \\ -6i & 3 & 4i \end{pmatrix}$$

   (a) Identify the matrices which are hermitian.

   (b) Determine the eigenvalues of all the matrices.

   (c) Determine the eigenstates for each Hermitian operator.

   (d) Construct an eigenbasis for each hermitian operator.

   (e) Pick out any two hermitian operators. Expand the eigenstates of the first operator in the eigenbasis of the other.

2. Consider two matrices

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}: \qquad B = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

   (a) Verify that the matrices $A,\ B$ commute.

   (b) Determine the eigenvalues of A and B and their degeneracies.

   (c) Determine a common eigenbasis for both the matrices.

   (d) Can you express one of them as a function of the other?

3. Operator A, B have their eigenvalues and eigenstates as given below:

3.2	u_1	1.3	u_5
3.2	u_2	3.4	u_4
4.5	u_3	3.4	$u_1 + 2u_2$
4.8	u_3	3.4	u_4
1.0	u_4	3.8	$u_1 - 2u_2$
4.5	u_5	3.0	u_3

(a) Do the two operators commute?

(b) If so, find a common eigenbasis.

(c) Is the eigenbasis unoque? If so, why?

(d) Suppose that $v = 3a_1 + 4a_4 + 3a_3$. What are the probabilities for the allowed values of A, B.

(e) Find the expectation values of A, B.

(f) Find $\Delta A \Delta B$ for the state ψ.

(g) Are A, B simultaneously measurable?