Quantum Mechanics
PHL555

Problem Set 6

One dimensional problems: more general cases

1. Let a particle of mass m be in a potential

$$V(x) = \left\{ \begin{array}{ccc} \infty & \text{if} & \infty < x < -L \\ 0 & \text{if} & -L < x < 0 \\ \frac{1}{2}m\omega^2 x^2 & \text{if} & x > 0 \end{array} \right\}$$

2. Determine the energy eigenvalues and eigenfunctions to the best of your ability.

3. Identify competing energy, momentum and length scales in the problem.

4. What would be the solutions in the limiting cases $L \to 0,\ \infty$

5. Similarly, study the limiting case when $\omega \to 0,\ \infty$.

6. Let a particle be confined by a potential $V(x) = \frac{V_0}{X^N}|x|^N$. Can you argue how the wave function behaves as $|x| \to \infty$ and get the energy levels?

7. You are given a potential $V(x) = \frac{1}{2}\alpha^2 x^2 \exp(-x^2/X^2)$.

   (a) Plot the function qualitatively and identify the minima and the maxima

   (b) If you wish to approximate the potential by an oscillator potential, what would be the effective frequency ω?

   (c) How many energy levels can this approximation yield with a reasonable accuracy?

8. Consider the Morse potential $V(x) = V_0\Big\{ \exp(-2ax) - 2\exp(-ax)\Big\}$.

   (a) Plot the potential and also the force.

   (b) Make the harmonic approximation and deduce the energies and wave functions of the first few energy levels.

(c) **Reading Assignment**: This potential is exactly solvable. Compare the approximate energies with the exact energies.

9. You are given the potential $V(x) = \sin 2\alpha x - 2\sin \alpha x$ in the interval $[-\pi, \pi]$ and it vanishes outside.

   (a) Construct some trial wave functions for the ground state and estimate the ground state energy. Clearly identify the variational parameters.

   (b) Find conditions on the parameter α for the first excited state to exist. It is enough to estimate intelligently.

   (c) How would you numerically solve this problem?

10. Consider the asymmetric square well potential: It has a height V_0 at $x = 0$ and a height V_1 at $x = L$. Solve the Schrödinger equation and find the ground state energy. Verify your answers when the two heights become equal.

11. Also study how binding energy vanishes as one of them, say $V_1 \to 0$.