Quantum Mechanics
PHL555

Problem Set 6

One dimensional problems

1. Consider the standard harmonic oscillator potential.

   (a) Show that the expectation values of momentum and position vanish if the state is a linear combination of either even parity or odd parity states.

   (b) Let $|\psi\rangle = a|n\rangle + b|n+1\rangle$. Determine the expectation values of position and momentum. Note that the coefficients are complex.

   (c) Suppose you add a term $E_0 x$ to the potential. Show that the problem is exactly solvable.

   In that case, how much is the work done on the system when the extra term is switched on?

   What happens to the uncertainties in x and p? Evaluate them explicitly.

2. Consider the potential $V(x) = \frac{1}{2}m\omega^2 x^2 \theta(x)$ where θ is the standard step function.

   (a) Determine the eigenfunctions and eigenvalues

   (b) Determine Δx, Δp and their product for the nth excited state.

   (c) What would be the experimental signature for this potential in a physical system?

3. Now consider the asymmetric potential. The frequency is $\omega_{1,2}$ for $x > 0$ and $x < 0$ respectively.

   (a) How do you find the eigenvalues and eigenfunctions? Can you solve it exactly?

   (b) If not, solve the classical problem and determine the solutions. Carefully evaluate the limits special cases $\omega_1 = \omega_2$ and $\omega_2 \to \infty$. You should get the correct answers.

(c) Employ Bohr-Sommerfeld quantization to determine the energy levels.

(d) Now re-attempt the quantum mechanical problem.

4. Consider a smooth well behaved potential $V(x)$. Let $\psi_E(x)$ be an eigenstate of energy E. Show that, in general, the wave function has to cross the x axis whenever it hits zero. Equivalently, show that it is linear around where it becomes zero.

5. *Reading assignment*: An extension of the above result is that the nth excited state has n nodes and the wave function oscillates from positive to negative or otherwise when it hits zero.

6. Also show that the bound state wavefunction can always be taken to be real.

7. Show that any attractive potential will support atleast one bound state. **Hint**: Remember square well potential and make intelligent use of the results.