PYL555: Quantum Mechanics
(Major – Outline of Solutions)

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Instructions:

1. Read all the questions carefully before you start writing your answers.
2. Ensure that the answers for a given question are in one place and not scattered.
3. Mention explicitly if you are using $\hbar = 1$.

Question 1. (a) (5 marks) State if the following statements are true or false, with a short but clearly stated reason:

- The eigenstates of momentum in three dimensions admit all possible values of angular momenta. **Ans**: Correct. the impact parameter can be indefinitely large for a given energy. Equivalently, expamsion in terms of spherical harmonics involves all values of l.
- The asymptotic form of the wave function for a particle in the ground state of the one dimensional potential $V(x) \propto \frac{1}{x^2+X^2}$, when $|x| \to \infty$, is $\psi(x) \sim \exp(-x^2/a^2)$. **Ans**: Incorrect. Only an absolutely confining potential like oscillator will allow that. Even Coulomb gives exponentially falling wave function.
- Every attractive potential in one dimension admits at least one bound state. **Ans**: Correct. Because every square well potential has one, and can always be embedded in the attractive potential.
- The oscillator potential $V \propto (x-X)^2$ in one dimensional has a parity symmetry. **Ans**: Correct. Reflelction about X.
- the degeneracy of the second excited state of a three dimensional isotropic oscillator is 10. **Ans**: Correct. Verify by Counting.

Question 2. A particle of mass m is constrained to lie within a sphere of radius R.

(a) (5 marks) Identify the quantum numbers associated with the ground state. **Ans**: $n = 0, l = 0, m = 0$.

(b) (10 marks) If it is in its first excited state, obtain its energy and its eigenfunctions with a clear mention of the degeneracy. You may leave the normalization factor free. **Hint.** *You may like to use the fact that*

$$j_n(x) = (-1)^n x^n \Big(\frac{1}{x}\frac{d}{dx}\Big)^n \frac{\sin(x)}{x}$$

Ans: *I suppose that it corresponds to* $n = 1, l = 1$ *Obtained by looking at the zeroes of the spherical Bessel. In that case, degeneracy will be three.*

Question 3. Consider a muonic atom which is a bound state of a proton and a muon, which has the same charge as the electron, but with a mass $m = 200m_e$. Recall that $m_p = 2000m_e$.

(a) (6 marks) Taking that *fully* into account, obtain the ratio of its energy eigenvalues with that of the hydrogen atom.
Ans: The ratio is simply the ratio of the *reduced masses*. Not just ordinary masses.

(b) (5 marks) Also deduce the ratio of the of expectation values of r^k for the two atoms.
Ans: You proceed as follows. Find the length scale. Suppose it depends on m^a. Then r^k depends on m^{ak}. The corresponding ratios will give the answer.

(c) (7 marks) If we include the spins in the atoms, find the total angular momentum in the ground state. Argue carefully for which of them would the spin-spin interaction energy be stronger.
Ans: Parallel magnetic moments attract, and hence reduce the energy. Therefore the singlet state has the smaller energy. The triplet state has higher energy. The magnetic moment is inversely proportional to the mass. Therefore, the spin-spin interaction will be stronger for hydrogen atom.

Question 4. (a) (5 marks) Construct the rotation matrix for $s = 1/2$ about the Y axis explicitly (write the full 2×2 matrix explicitly).
Ans: Straightforward. $d^{\frac{1}{2}}(\theta) = \cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\sigma_y$.

(b) (7 marks) Consider the addition of two spins, both $s = 1$. Find explicitly the state $|1,1\rangle$ in the uncoupled basis $\Big\{|m_1, m_2\rangle\Big\}$.
Hint. *Use a fact related to the maximum value of m, and proceed along the natural way*

Ans: Procedure. Identify $|11\rangle \equiv |2,2\rangle$. Operate S_- on this state to get the state $|2,1\rangle$. The orthogonal linear combination yields $|1,1\rangle$.

<u>The End</u>