PYL555: Quantum Mechanics
(II Minor Solutions)

10 Oct, 2015   4 PM – 5 PM

Question 1. **All questions are in $D = 2$ spatial dimensions except when explictly stated.**

(a) (5 marks) A particle of mass m is constrained to the $X - Y$ plane. If it is in a potential $V(x, y) = \frac{1}{2}k_1 x^2 + \frac{1}{2}k_2 y^2$, $k_1 > k_2$, find the expectation value of $\vec{r}^2$ in the first excited state.

   Solution: The energy levels are given by $\hbar\left\{n_1\omega_1 + n_2\omega_2 + \frac{1}{2}(\omega_1 + \omega_2)\right\}$. The first excited state corresponds to $n_1 = 0$, $n_2 = 1$. Hence $\langle \vec{r}^2 \rangle = \frac{\hbar}{m}(\omega_1 + 3\omega_2)$; Note that $\omega^2 = \frac{k}{m}$.

(b) (5 marks) Construct the radial momentum operator in $D = 4$ spatial dimensions.

   Solution: Use the definition $p_r = \frac{1}{2}(\vec{p}\cdot\hat{r} + \hat{r}\cdot\vec{p})$. Use also that $\vec{\nabla}_D \cdot \vec{r} = D$. You will get $p_r = -i\hbar(\partial_r + \frac{3}{2r})$.

(c) (10 marks) A particle is in the second excited state of the Morse potential

$$V(r) = V_0\Big(\exp(-2\alpha r) - 2\exp(-\alpha r)\Big).$$

   Which of the following wave functions are allowed candidates for the wave function? All constants are positive. Identify the quantum numbers associated with the allowed states.

   - (1) $\psi(x, y) = (r - a)(r - b)\exp(-cr^2)$
   - (2) $\psi(x, y) = \exp(-ar)\exp(2i\phi)$
   - (3) $\psi(x, y) = (r - a)(r - b)\exp(-cr^2)\exp(-2i\phi)$
   - (4) $\psi(x, y) = \frac{1}{x^4 + a^4}\exp(-by^2)$

   Solution: Leading order potential around the minima is of an oscillator. The second excited state has two nodes in the radial wave function. Possible choices are (1) and (3). The first one is an s state and the next belongs to $m = -2$.

(d) (5 marks) A particle of mass m and charge q is constrained to the $X - Y$ plane, with a magnetic field $\vec{B}(\vec{r}) = B\theta(R - r)\hat{k}$. Construct a Hamiltonian for the charged particle.

   solution: I will leave this for you to figure out. Use polar coordinates and you get the naswer in no time. What you have to note is that $\vec{A}$ should be continuous at $r = R$.

<u>The End</u>