Quantum Mechanics
PHL555

Problem Set 2

Wave functions and operators

1. You are given the probability density $\psi(x) = \exp(-|x|)$.

   Normalize the wave function to unit probebility.

   Determine the expectation values for the observables x^n.

   Show that if you know $\langle x^n \rangle$ for all n, show that the probabiliy density can be reconstructed.

2. Replace $\psi(x)$ by $\psi'(x) = \psi(x)\exp(ix/X)$. Determine $\langle x^n \rangle$ for all n again.

   Now determine $\langle p^n \rangle$ for both the states. What do you conclude?

3. Repeat the exercise in the momentum representation.

4. Repeat the exercise when

$$\psi(x) = \frac{A}{x^2 + X^2}.$$

   Determine A by the normalization condition, and proceed to complete all other exercises.

5. Show that the operator $\frac{\partial}{\partial x}$ has only imaginary eigenvalues.

6. Similarly show that $A = xp + px$ is a hermitian operator.

7. Determine the eigenvalues and eigenfunctions of A.

8. Find the fallacy in the argument: $\frac{\partial}{\partial x}$ is also hermitian with eigenvalues p with eigenstates $\exp(px)$.

9. Using the properties of the Dirac Delta, prove formally that $\int dk \exp ik(x_1 - x_2) = 2\pi\delta(x_1 - x_2)$

10. You are given a state $\psi(r, \phi) = f(r)\exp(i3\phi)$ in polar coordinates (in two dimensions). Determine expectation values of $\vec{r}, \vec{p}$ and $\vec{\mathrm{r}} \times \vec{\mathrm{p}}$? Be careful how you define the last quantity.