Quantum Mechanics
PHL555

Problem Set 2

Periodic boundary conditions

1. Construct the normalized eigenstates of momentum for a particle restricted to an interval $[0, L]$ with periodic boundary conditions. And hence determine the allowed energies.

2. Consider $O_1 = \cos(\alpha x)$. Find the values of α for which O_1 is a legitimate operator in the space of allowed functions.

3. Determine the uncertainty ΔO_1 in the eigenstates of momentum.

4. Repeat the exercise for $O_2 = \sin(\alpha x)$.

5. Find the states for which uncertainties $\Delta O_{1,2}$ are a minimum.

6. Construct a wave function ψ, by taking suitable superpositions, which satisfies $\langle p \rangle_\psi = 0$ and $\Delta E_\psi \neq 0$.

7. You are given two superposed states $\phi_{1,2}(x) = \psi_1(x) + \exp(-i\alpha)\psi_3(x)$ of momenta $p_{1,3}$ respectively.

   (a) Determine Δp and ΔE for the two states.

   (b) How will you experimentally distinguish the two states?

   (c) Sketch the probability densities for $\phi_{1,2}$.

8. Let $\phi(x) = A \sum_{n=-\infty}^{\infty} \frac{1}{n} \psi_n(x)$ expanded in terms of the eigenstates of momentum, except at $n = 0$.

   (a) Determine A by normalising the state to unit probability.

   (b) Determine the expectation value of energy in this state.

   (c) Repeat the exercise when the expansion coefficients are given by $\frac{1}{n^2}$.