Quantum Mechanics
PHL555

Problem Set 9

Spin problems

1. Show that $[\sigma_i, \sigma_j] = 2\epsilon_{ijk}\sigma_k$.
2. Show the anticommutator $\{\sigma_i, \sigma_j\} = 2\delta_{ij}I_2$ where I_2 is the identity matrix.
3. Hence verify that $\vec{\sigma} \cdot \vec{a}\vec{\sigma} \cdot \vec{b} = \vec{a} \cdot \vec{b}I_2 + i\vec{\sigma} \cdot \vec{a} \times \vec{b}$.
4. State explicitly the conditions under which the above equation is valid.
5. Hence determine $\vec{\sigma} \cdot \vec{L}\vec{\sigma} \cdot \vec{L}$ where $\vec{L}$ are the angular momentum operators.
6. Determine explicitly the spin operators for $j = 3/2$.
7. Verify explicitly that they satisfy the relation $S_i^3 = S_i$.
8. A spin 1/2 particle of mass m and charge q is in a uniform magnetic field $B\hat{k}$. Set up the Pauli equation and solve the landau level problem. Determine all the energy levels and the eigenfunctions carefully.
9. Show that $U \equiv \exp(i\vec{\sigma} \cdot \hat{n}\frac{\theta}{2})$ is a unitary operator.
10. Evaluate U explicitly and show that $U = \cos\frac{\theta}{2} + i\vec{\sigma} \cdot \hat{n}\sin\frac{\theta}{2}$.