

UNIVERSITY OF TARTU  
Institute of Computer Science  
Computer Science Curriculum

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# Analysing information distribution in complex systems

Bachelor's Thesis (9 ECTS)

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# 1 Introduction

Complex systems

In Chapter 1, the basics of classical information theory and partial information decomposition are covered. The chapter ends with an overview of the numerical estimator for PID.

The subsequent 3 chapters each introduce a specific complex system and the results of measuring information distribution in it.

Chapter 2 - Elementary Cellular Automata Chapter 3 - Ising model Chapter 4 - Artificial Neural Networks

In the final, concluding chapter, a summary of the contributions of this thesis is given, alongside suggestions for further work.

## 2 Background

### 2.1 Classical information theory

#### 2.1.1 Entropy

Let  $X$  be a discrete random variable with a set of values from the set  $\{x_1, x_2, \dots, x_n\}$  and probability mass function  $p(x_i) = P\{X = x_i\}$  ( $i = 1, \dots, n$ ). Shannon [Sha48] defines the *entropy* of  $X$  as follows:

$$H(X) = - \sum_{i=1}^n p(x_i) \log_2 p(x_i).$$

#### 2.1.2 Joint and Entropy

#### 2.1.3 Kullback-Leibler distance

#### 2.1.4 Mutual information

### 2.2 Partial information decomposition

### 2.3 Numerical estimator

## 3 Elementary cellular automata

### 3.1 Problem description

### 3.2 Related work

### 3.3 Experimental setup

### 3.4 Results

### 3.5 Discussion

## 4 Ising model

### 4.1 Problem description

### 4.2 Related work

### 4.3 Experimental setup

### 4.4 Results

### 4.5 Discussion

## 5 Neural networks

### 5.1 Problem description

### 5.2 Related work

### 5.3 Experimental setup

### 5.4 Results

### 5.5 Discussion

## 6 Conclusion



## References

- [Sha48] C. E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27(3):379–423, 1948.

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