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Chapter 4 Reading Notes
Sections 4.1-4.4

The Solar System

- Here we are considering a system with as little frictional forces as possible, the Solar System.
- The motion of moons and planets is connected through mechanics.
- In this chapter we will study planetary motion and how we can use computational methods to solve these problems.
    - Covering a sun and one planet
    - Investigate some properties of the solar system.

4.1 Kepler's Laws

- In our first problem, we are considering the sun and the one planet, which is going to be Earth.
    - According to Newton's Law of Gravity, the magnitude of the force is given by: $F = GMsMe/r^2$. Ms and Me are masses of the sun and earth, r is the distance between them, and G is the gravitational contestant.
    - If the Sun's mass is sufficiently large then its motion can be neglected.
- Our goal is to be able to calculate the position of the Earth as a function of time.
- In Lab 06: Planetary Motion, we set this up by creating an array for x-position, y-position, x-velocity, and y-velocity. This was done in the initialize function.
- Following this we created the distance function which was going to calculate the distance r.
- We visualized this graphically to visualize the movement of the planet we are studying, which in this case is Earth.
- From our visualizations we can conclude that Earth has a circular orbit, however by changing Earth's velocity, it takes on a more elliptical orbit.
    - This is also due to eccentricity.
- In our example and lab, notice how we used velocity as 2*pi*r/1 (2*pi AU/Yr)
- Kepler's Observations:

1. All planets move in elliptical orbits, with the Sun at one focus.
2. The line joining a planet to the Sun sweeps out equal areas at equal times.
3. If T is the period and *a* is the semimajor axis of the orbit, then T^2/a^3 is a constant.

- The goal of our lab was to prove Kepler's third law, especially in our program.

4.2 The Inverse-Square Law and The Stability of Planetary Orbits

- For a two-body system, all three of Kepler's laws are consequences of the fact that the gravitational force follows the inverse-square law.
- The relative motion of a tow-body system where the interaction force depends only on the separation r can be studied as if it were a one-body system.
    - Only because one body will be at rest and the other body will be orbiting about it.

The moving body in this equivalent expression has a mass equal to the so-called reduced mass $\mu = m1m2 \div (m1 + m2)$ where m1 and m2 are the masses of the two original bodies.

- The position of the equivalent body is given by the relative displacement r=r2-r1.

- The orbital trajectory for a body of reduced mass (μ) is given in polar coordinates by:
$\frac{d2}{d\theta 2}(\frac{1}{r}) + \frac{1}{r} = -\mu r2/L\,F(r)$
    - The right hand side represents the angular momentum and F(r) is the force acting on the body.
- For this system we can express it in its inverse square form:

Since $F(r)$ has the inverse square form (i.e., $F(r) \propto 1/r^2$), (4.8) can be readily solved. For our solar system case, the solution can be expressed as

$$\frac{1}{r} = \left(\frac{\mu G M_S M_P}{L^2}\right)[1 - e\cos(\theta + \theta_0)] , \tag{4.9}$$

or, choosing $\theta_0 = 0$ (which defines the axes of the ellipse),

$$r = \left(\frac{L^2}{\mu G M_S M_P}\right)\frac{1}{1 - e\cos\theta} . \tag{4.10}$$

Although this result does not give us the position of the planet as a function of time, it gives us the shape of the orbital trajectory.

- If we think about the inverse square law from a different point of view, we can see how the inverse square law is a direct consequence of the field-line picture with the geometry of the Euclidean space in which we live.

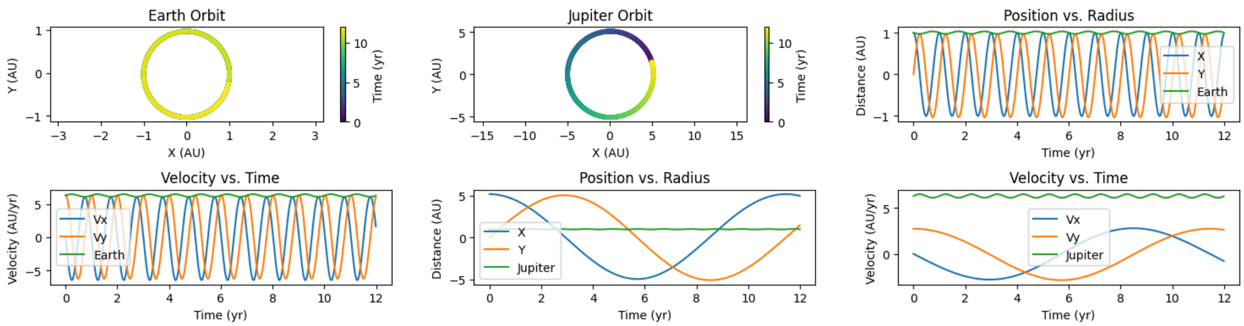
4.3 Precession of the Perihelion of Mercury

- We know planets have orbits that are very nearly circular, but Mercury and Pluto deviate the most from circular orbits.
    - The mention of another planet orbiting inside of Mercury or the possibility of a large amount of dust orbiting the Sun???
- The geometry of space views gravity in a much more complicated manner than our simple picture of Euclidean space and the inverse-square law.
- If the separation between two objects is small enough, then general relativity predicts deviations from the inverse-square law.
- The force law predicted by general relativity is: $Fg = \frac{GMsMm}{r^2}(1 + \frac{a}{r^2})$
    - Mm is the mass of Mercury and a=1.1x10^-8 AU^2.
- We used the same program as we did before, but now that we are incorporating a, it is now an adjustable parameter.
- To obtain the orbit of Mercury, we must determine initial conditions such as eccentricity, and orbit length.
- The precision of the perihelion of Mercury - the line of best fit from the graph in the textbook shows the slope of the line of best fit gives a precision rate for the value of a.

4.4 The Three-Body Problem and the Effect of Jupiter on Earth

- This section revolves around the three-body assignment we did in class last week.
- We adapted our notebook to model Earth's motion around the Sun to include the motion and gravitational influence of Jupiter.
- The code was written in a way where it could be adapted to other 2-body systems.

- The distance between the two bodies given x and y positions is denoted by r.



- In this figure, we are comparing Earth and Jupiter's orbit along with their position and velocity concerning time.
- When we increase the mass of Jupiter however, the orbit of Earth becomes more elliptical.