Sophia Pavia
Chapter 3 Oscillatory Motion and Chaos

Overview/General Idea

Oscillatory phenomena occur in atoms, the behavior of currents and voltages in electronics, and planetary orbits.
Simple Harmonic Motion can best be visualized as a pendulum.

3.1 Simple Harmonic Motion

- If we imagine a simple pendulum, we have mass m, a massless string, and theta is the angle the string makes with the vertical.
    - We also only assume that only two forces are present here, gravity, and the string's tension.
- $F(\theta) = -mgsin(\theta)$
- Force is always opposite to the displacement from the vertical
- By using Newton's second law, we know that force is equal to the mass times the acceleration of the particle along the circular arc which is the particle trajectory.
- $d^2(\theta)/dt^2 =- g/l * (\theta)$
- The motion of a simple pendulum is sinusoidal
- We can use the Euler method to show how energy increases over time.
- We use smaller timesteps to produce less error
- The Euler method is more stable than numerical methods and takes far less time to complete

3.2 Making the Pendulum More Interesting: Adding Dissipation, Nonlinearity, And A Driving Force

- In this section, we are making a pendulum more realistic
- Adding damping is the manner in which friction becomes a component.
- In most cases, the damping force is proportional to the velocity
- The frictional force can be represented by $-q(d\theta/dt)$
- The equation of the damped pendulum is now $d^2(\theta)/dt^2 =- g/l * (\theta) -q(d\theta/dt)$
- From the damped pendulum, we can also add an external driving force that pumps energy either into or out of the system. The equation to represent this is:
- $d^2(\theta)/dt^2 =- g/l * (\theta) -q(d\theta/dt)+ F * sin(\Omega t)$
- So now our pendulum has mass, and an electric charge that is supplying the external force in the form of an oscillating electric field.

3.3 Chaos In The Driven Nonlinear Pendulum

- Now that a numerical method is created and working, we have added three new components: small angle approximation, friction, and a sinusoidal driving force.
- So now our model $d^2(\theta)/dt^2 =- g/l * (\theta) -q(d\theta/dt)+ F * sin(\Omega t)$ is a nonlinear, damped, and driven pendulum.

- We modeled this in the lab but with a different problem, but we still used the Euler_Cromer_Calculate method for approximation.
- We had to estimate the period of oscillations from our graphs, and we had to take note of our initial positions, and initial velocity of our sinusoidal graphs.
- If our driving force behavior never repeats, we call this chaotic behavior.

The Lyapunov Exponent ?

- A system can be both deterministic and unpredictable, which is also how we describe chaos.
- The behavior in the chaos is not completely random, however, we describe it as behaving strangely in phase space.