MORPHOLOGICAL ENHANCEMENT OF MEDICAL IMAGES IN A LOGARITHMIC IMAGE ENVIRONMENT TOOLBOX

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ABSTRACT

The logarithmic image processing (LIP) theory is a mathematical framework that provides a set of specific algebraic and functional operations and structures that are well adapted to the representation and processing of non-linear images, and more generally of non-linear signals, valued in a bounded intensity range. This very well structured theory determined us to use the logarithmic image representation in our approach for defining a new set of mathematical morphology operators based upon structuring elements with a variable geometrical shape or adaptative structuring elements. The purpose of this paper is to define and to analyze the new multiplicative logarithmic morphological operators used in medical image enhancement. Finally, the experimental results reveal that this method has wide potential areas of impact which may include: Digital X-ray, Digital Mammography, Computer Tomography Scans, Nuclear Magnetic Resonance Imagery and Telemedicine Applications.

1. INTRODUCTION

The use of an adaptative structuring element which could change geometrical shape for each image pixel was proposed by Salembier since 1994, [3]. The criteria selected for adapting the structuring element is the minimization of mean squared error calculated from the output image and the desired signal. The total pixel number of such an adaptative structuring element varies in a bounded interval for a given neighborhood. In this paper we will present several possibilities of defining structuring elements with a variable geometrical shape within the logarithmic image processing theory, i.e. using only vector-oriented operations with images represented as gray-tone functions operands.

In fact, we can demonstrate that all the classical mathematical morphology operators are solely a particular case of the logarithmic mathematical morphology operators, based upon the definition of the structuring elements with a variable geometrical shape or adaptative structuring elements.

At least three categories of mathematical morphology operators can be defined within the context of a logarithmic image representation:

1. multiplicative logarithmic morphological operators;
2. additive logarithmic morphological operators;
3. additive-multiplicative logarithmic morphological operators;

In this paper we will only analyze the properties of the first category of derived mathematical morphology operators, i.e. multiplicative logarithmic morphological operators.

From now on we will associate both the image and the structuring element with gray-tone functions defined as follows:

$$F: D \to E,\ D \subset \mathbb{R}^2,\ E=[0, M) \text{ or } E=(-M, M),\ M>0$$

Also, we will denote by *I(D,E)* the *set of gray-tone functions*, defined on a spatial support $D \subset \mathbb{R}^2$, and taking values within a gray-tone interval $E=[0, M)$ or $E=(-M,M)$.

In the first place, we will use the definition of the specific operations in the *logarithmic image processing theory* as presented in [1] and [2]. In the set of gray levels E we will define the *logarithmic addition* $\oplus$:

$$\forall u, v \in E, \qquad u \oplus v = \frac{u+v}{1+\frac{u \cdot v}{M^2}}$$

where the operations in the right side are meant in $\mathbb{R}$.

In the same set of gray levels E we will define the *real scalar multiplication* $\otimes$.

For $\forall \lambda \in \mathbb{R}$, $\forall u \in E$, we define the *logarithmic product* between λ and u by:

$$\forall u \in E, \forall \lambda \in \mathbb{R} \qquad \lambda \otimes u = M \cdot \frac{(M+u)^{\lambda} - (M-u)^{\lambda}}{(M+u)^{\lambda} + (M-u)^{\lambda}}$$

where again the operations in the right hand side of the equality are meant in $\mathbb{R}$. The two operations, addition $\oplus$ and scalar multiplication $\otimes$ establish on E a real vector space structure [2].

A gray level image is a function defined on a bi-dimensional compact D from $\mathbb{R}^2$ taking the values in the gray level space E. We denote with *F(D,E)* the set of gray level images defined on D. The operations and the functions from gray level space E to gray level images *F(D,E)* can be extend in a very natural way (see [1], [2]|):

<u>*Logarithmic addition for gray level images:*</u>

$$\forall f_1, f_2 \in F(D,E),\ \forall (x,y) \in D,\ (f_1 \oplus f_2)(x,y) = f_1(x,y) \oplus f_2(x,y)$$

<u>*Logarithmic scalar multiplication for gray level images:*</u>

$$\forall \lambda \in \mathbb{R}, \forall f \in F(D,E),\ \forall (x,y) \in D,\ (\lambda \otimes f)(x,y) = \lambda \otimes f(x,y)$$

The two operations, addition $\oplus$ and scalar multiplication $\otimes$ establish on *F(D,E)* a real vector space structure [5].
For the morphological operators we will use the classical functional definition from [5] and [6], where the *morphological erosion* and *dilation* are defined as follows:

$$(f \ominus \breve{g})(x) = \inf\{f(y) - g(y-x) \mid y \in \mathbb{R}^n\}$$

$$(f \oplus \breve{g})(x) = \sup\{f(y) + g(y-x) \mid y \in \mathbb{R}^n\}$$

where f and g are semi-continuous functions from $\mathbb{R}^n$ to $\mathbb{R} \cup \{-\infty, +\infty\}$ and $\breve{g}$ is defined as: $\forall x \in \mathbb{R}^n$, $\breve{g}(x)=g(-x)$.

2. MULTIPLICATIVE LOGARITHMIC MORPHOLOGICAL OPERATORS

We will present in this section the definitions and the properties of *multiplicative logarithmic morphological operators* from the basic levels of their pyramid.

2.1 Multiplicative Logarithmic Morphological Erosion

Definition 1: *Multiplicative logarithmic morphological erosion* for the image f by structuring element g, represents the *gray-tone function* defined as follows:

$$(f \ominus_{ML} \breve{g})(x) = \inf\{k \otimes (f(y) - g(y-x)) \mid y \in \mathbb{R}^2\} \quad (1)$$

where $\breve{g}$ is defined as: $\forall x \in \mathbb{R}^n$, $\breve{g}(x)=g(-x)$ and $\otimes$ represents *LIP product of a gray-tone function with a real scalar.*

Observation 1: Scalar k can be constant for the entire image or can be, as well, another gray-tone function (an adaptative scalar multiplication) defined like this:

$$\forall x \in \mathbb{R}^2, \quad k(x) = \frac{f(x)}{M}$$

As shown in the last section, *"Experimental results"*, the structuring elements have a *variable geometrical shape* all over the definition domain, i.e. larger or wider when the derivative gray-tone function $f'(x)$ is low and narrower when $f'(x)$ high.

Within LIP model we will obtain the following definition:

$$(f \ominus_{ML} \breve{g})(x) =$$

$$=\inf\{M \cdot \frac{(M+(f(y)-g(y-x)))^k - (M-(f(y)-g(y-x)))^k}{(M+(f(y)-g(y-x)))^k + (M-(f(y)-g(y-x)))^k}$$

$$\mid y \in \mathbb{R}^2\}$$

2.2 Multiplicative Logarithmic Morphological Dilation

Definition 2: *Multiplicative logarithmic morphological dilation* for the image f by structuring element g, represents the *gray-tone function* defined as follows:

$$(f \oplus_{ML} \breve{g})(x) = \sup\{k \otimes (f(y) + g(y-x)) \mid y \in \mathbb{R}^2\} \quad (2)$$

where $\breve{g}$ is defined as: $\forall x \in \mathbb{R}^n$, $\breve{g}(x)=g(-x)$ and $\otimes$ represents *LIP product of a gray-tone function with a real scalar.*

The previous *Observation 1* also operates in this case. Again, by experimental results, we find the particular behavior of the *variable geometrical shape structuring element* all over the definition domain, i.e. more wide when the derivative function $f'(x)$ is low and more narrow when high.

Within LIP model we will obtain the following definition:

$$(f \oplus_{ML} \breve{g})(x) =$$

$$=\sup\{M \cdot \frac{(M+(f(y)+g(y-x)))^k - (M-(f(y)+g(y-x)))^k}{(M+(f(y)+g(y-x)))^k + (M-(f(y)+g(y-x)))^k}$$

$$\mid y \in \mathbb{R}^2\}$$

Because *multiplicative logarithmic morphological erosion* and *dilation* are dual transformations, but not reverse to each other, their functional composition allow us to generate the pyramid of the derivative transformations: *multiplicative logarithmic morphological opening* and *closing.*

2.3 Multiplicative Logarithmic Morphological Opening

Definition 3: *Multiplicative logarithmic morphological opening* for the image f by structuring element g, represents the *gray-tone function* denoted by ψ_g^{AL} and defined as follows:

$$\psi_g^{ML} = (f \ominus_{ML} \breve{g}) \oplus_{ML} g \quad (3)$$

where $\breve{g}$ is defined as: $\forall x \in \mathbb{R}^n$, $\breve{g}(x)=g(-x)$ and $\ominus_{ML}$ and $\oplus_{ML}$ represent *multiplicative logarithmic morphological erosion* and *dilation,* respectively, already defined above.

2.4 Multiplicative Logarithmic Morphological Closing

Definition 4: *Multiplicative logarithmic morphological closing* for the image f by structuring element g, represents the *gray-tone function* denoted by φ_g^{AL} and defined as follows:

$$\varphi_g^{ML} = (f \oplus_{ML} \breve{g}) \ominus_{ML} g \quad (4)$$

where $\breve{g}$ is defined as: $\forall x \in \mathbb{R}^n$, $\breve{g}(x)=g(-x)$ and $\ominus_{ML}$ and $\oplus_{ML}$ represent *multiplicative logarithmic morphological dilation* and *erosion,* respectively, already defined above.

3. LOGARITHMIC TOP-HAT TRANSFORMS

In the context of the transmitted signals through different optical mediums, the objects observed on a dark background generate smaller peaks in the associated 1D signal corresponding to segmentation threshold r, then the peaks generated in the case of a light background. This phenomenon occurs due to the non-linear physics laws for the absorption associated to the optical mediums. Using the logarithmic contrast *M.Jourlin* [1] and *Montard* (1997) have introduced the notion of *Logarithmic Top-Hat (LTH).* In this context we can introduce two new different logarithmic top-hat transforms:

Definition 5: *Logarithmic White Top-Hat Transform* of an image f represents the logarithmic contrast between the gray-tone function and its logarithmic opening:

$$LWTH(f)(x,y) = \left(\frac{f(x,y) - \delta_{\breve{g}}[\varepsilon_g(f(x,y))]}{1 - \frac{\delta_{\breve{g}}[\varepsilon_g(f(x,y))]}{M}} \right) \quad (5)$$

where $\varepsilon_g(f(x,y))=(f \ominus_{ML} g)(x)$ stands for the logarithmic morphological erosion and $\delta_{\breve{g}}(f(x,y))=(f \oplus_{ML} \breve{g})(x)$ stands for the logarithmic morphological dilation (i.e. $\psi_g^{ML}=\delta_{\breve{g}}[\varepsilon_g(f(x,y))]$

stands for the logarithmic morphological opening, obviously).

Like in the classic morphology, we can introduce its associated complementary transform:

Definition 6: *Logarithmic Black Top-Hat Transform* of an image f represents the logarithmic contrast between the logarithmic closing of a gray-tone function and function itself:

$$LBTH(f)(x,y) = \left(\frac{\varepsilon_{\breve{g}}[\delta_g(f(x,y))] - f(x,y)}{1 - \frac{f(x,y)}{M}} \right) \tag{6}$$

where $\varphi_g^{ML} = \varepsilon_{\breve{g}}\,[\delta_g\,(\,f(x,\,y))\,]$ stands for the logarithmic morphological closing.

These transforms are used in image segmentation by detecting the interesting objects associated with the peaks of the gray-tone function f (in the case of logarithmic white top-hat transform defined by opening) or by detecting the local minimums (in the case of logarithmic black top-hat transform defined by closing). Image segmentation is realized by selecting the pixels (x, y) in the spatial domain, satisfying the following criteria:

$$LWTH(f)(x,y) > r$$

The purpose of the structuring element g and the segmentation threshold r is to select the important peaks (higher and wider then the unimportant noise peaks).

Definition 7: *Multiplicative Logarithmic White Top-Hat Transform* of an image f represents the logarithmic difference between the gray-tone function and its multiplicative logarithmic opening:

$$WTH_{ML}(f)(x,y)=f\ominus\psi_g^{ML}=f\ominus((f\ominus_{ML}\breve{g})\oplus_{ML}g) \tag{7}$$

Definition 8: *Multiplicative Logarithmic Black Top-Hat Transform* of an image f represents the logarithmic difference between multiplicative logarithmic closing of the gray-tone function and function itself:

$$BTH_{ML}(f)(x,y)=\varphi_g^{ML}\ominus f=((f\oplus_{ML}\breve{g})\ominus_{ML}g)\ominus f \tag{8}$$

4. MULTIPLICATIVE LOGARITHMIC CONTRAST

For the morphologically sensitive contrast enhancement of the images represented in a logarithmic environment the following definition of the *Multiplicative Logarithmic Contrast* was used:

Definition 9: *Multiplicative Logarithmic Contrast* of an image f represents the logarithmic addition between the *gray-tone function* and its *multiplicative logarithmic white top-hat transform* followed by the logarithmic difference with its *multiplicative logarithmic black top-hat transform*:

$$Contrast_{ML}(f)=f\oplus WTH_{ML}(f)\ominus BTH_{ML}(f) \tag{9}$$

5. EXPERIMENTAL RESULTS

The experimental results, subsequently presented in this paper, used the multiplicative version of the logarithmic top-hat transforms and the multiplicative logarithmic contrast as defined above:

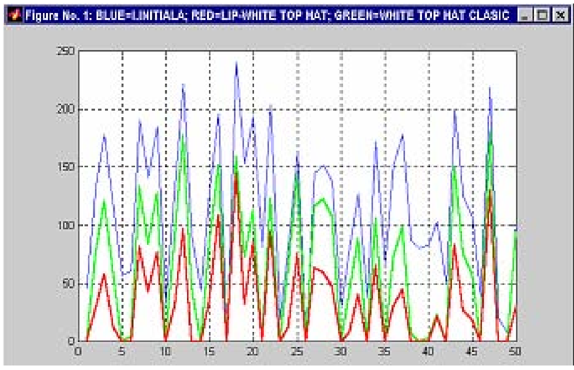


Figure 1 - Multiplicative logarithmic white top-hat transform

Blue=gray-tone function; Red= multiplicative logarithmic white top-hat; Green= classical morphological white top-hat

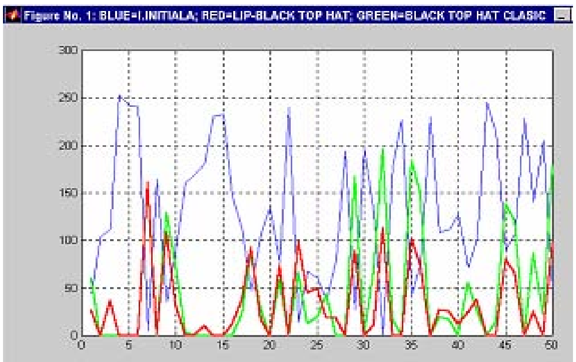


Figure 2 -Multiplicative logarithmic black top-hat transform

Blue=gray-tone function; Red= multiplicative logarithmic black top-hat; Green= classical morphological black top-hat

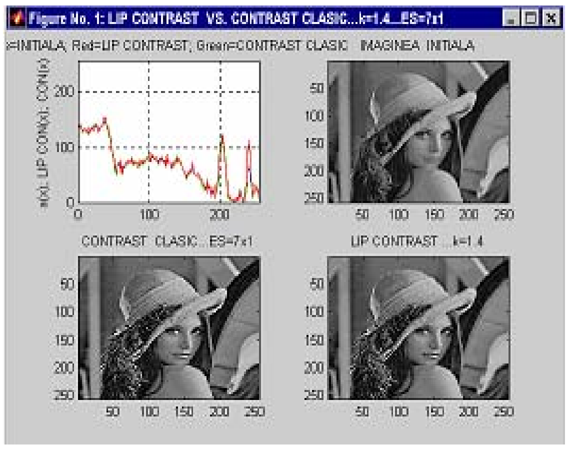


Figure 3 -Multiplicative logarithmic morphological contrast

Blue=one single row from the original image represented as a 1D gray-tone function; Red=multiplicative logarithmic contrast (real scalar parameter used k=1.4; plane structuring element 7x1); Green=classical morphological contrast with the same plane structuring element 7x1; Logarithmic contrast works better for the very narrow peaks in the original signal or gray-tone function as the structuring element has a variable geometric shape.

6. LOGARITHMIC MORPHOLOGICAL TRANSFORMS APPLIED IN MEDICAL IMAGERY

Medical images represent, in most cases, a noisy image environment due, primarily, to the limitations placed on X-ray dose, for example. In a large number of situations the medical expert must inspect, visually segment and analyze various objects within a medical image in order to diagnose diseases. In order to reduce the amount of inconsistencies in the diagnostic conclusions it is necessary to use computer enhanced images whenever medical imagery is required for correct decision.

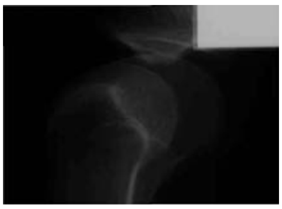
The logarithmic morphological transforms presented above are useful to any medical imaging application where automatic contrast enhancement and sharpening is needed. Potential areas of impact may include:

- Digital X-ray
- Digital Mammography
- Computer Tomography Scans
- Nuclear Magnetic Resonance Imagery
- Telemedicine Applications

Particularly, a potential bottleneck can be found in telemedicine applications, where there is a limited bandwidth data channel between doctor and patient. This logarithmic morphological model reduces the high input dynamic range, potentially reducing the high bandwidth requirement.

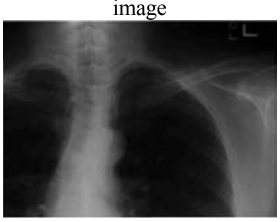
The most important performances of this logarithmic morphological model are:

- Sharpening: i.e., compensation for the blurring introduced into the image by the image formation process. This allows fine details to be seen more easily than before.
- Dynamic range compression: i.e., the ability to represent large input dynamic range into relatively small output dynamic range.
- Color constancy: i.e., the ability to remove the effects of the illumination from the subject. This allows consistency of output as illumination changes.

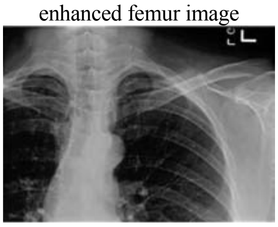


a) Original digital X-ray femur image

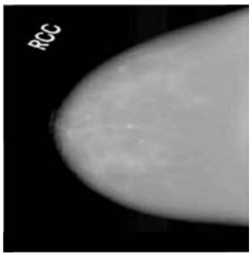
b) Logarithmic morphological enhanced femur image



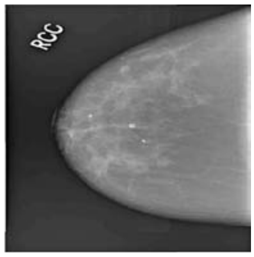
c) Original digital X-ray lungs image



d) Logarithmic morphological enhanced lungs image

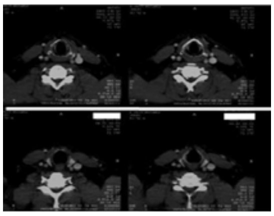


e) Original digital mammography image for breast cancer diagnose

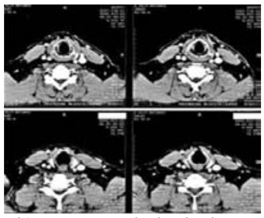


f) Enhanced mammography revealing calcium formation

Figure 4 - X-ray enhancement



a) Original computer tomography scan



b) Log. morphological enhanced computer tomography image

c) Original digital eye image

d) Contrast and sharpness enhancement using logarithmic morphological transforms

Figure 5 - Tomography scan and digital medical images logarithmic morphological enhancement

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