



# Alignment and Image Comparison

Erik Learned-Miller  
University of Massachusetts,  
Amherst



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# Lecture I

- Introduction to alignment
- A case study: mutual information alignment

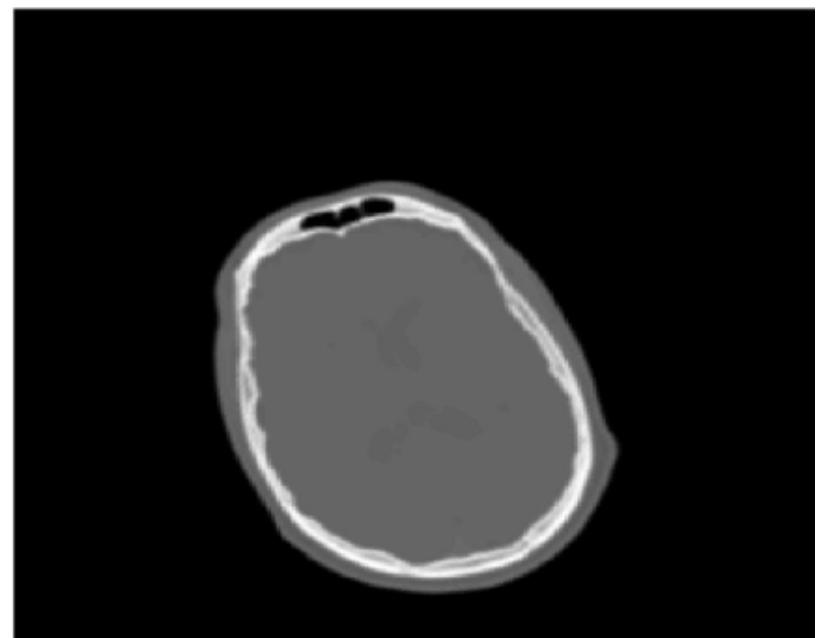
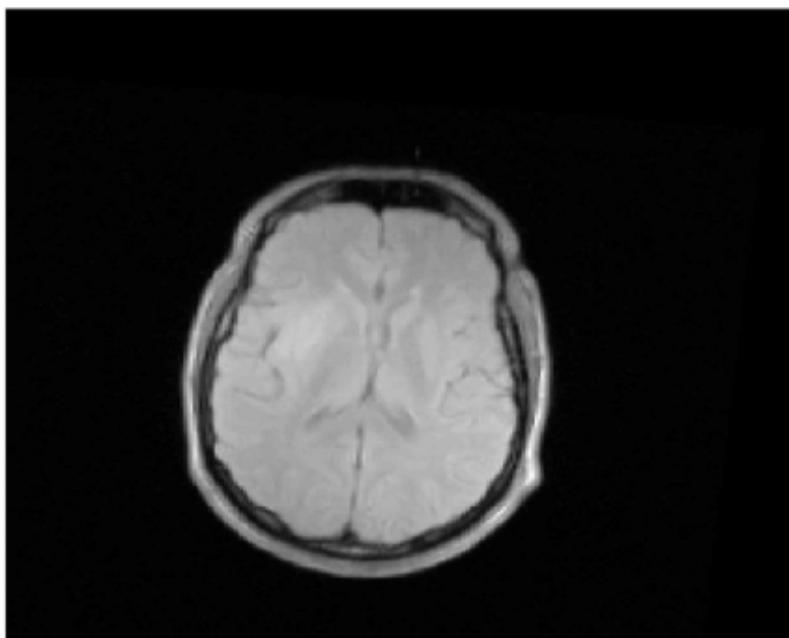
# Lecture I

- **Introduction to alignment**
- A case study: mutual information alignment

# Examples of Alignment

- Medical image registration
- Face alignment
- Tracking
- Joint alignment (model building)

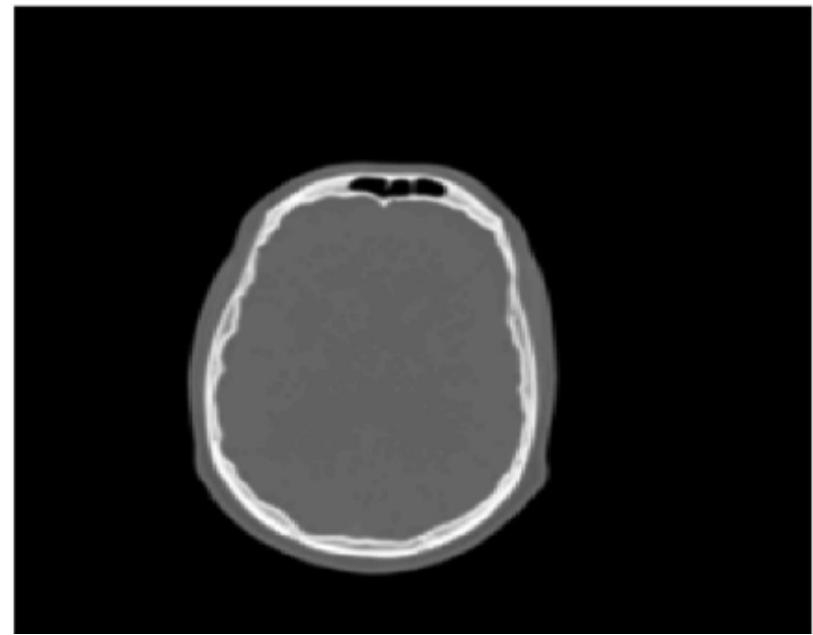
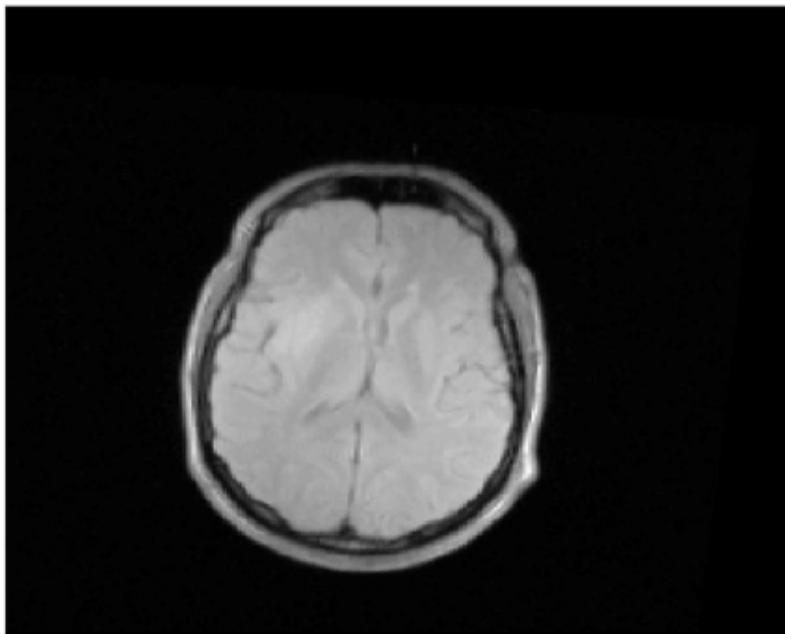
# Medical Image Registration



Lilla Zöllei and William Wells. Course materials for HST.582J / 6.555J / 16.456J, Biomedical Signal and Image Processing, Spring 2007.

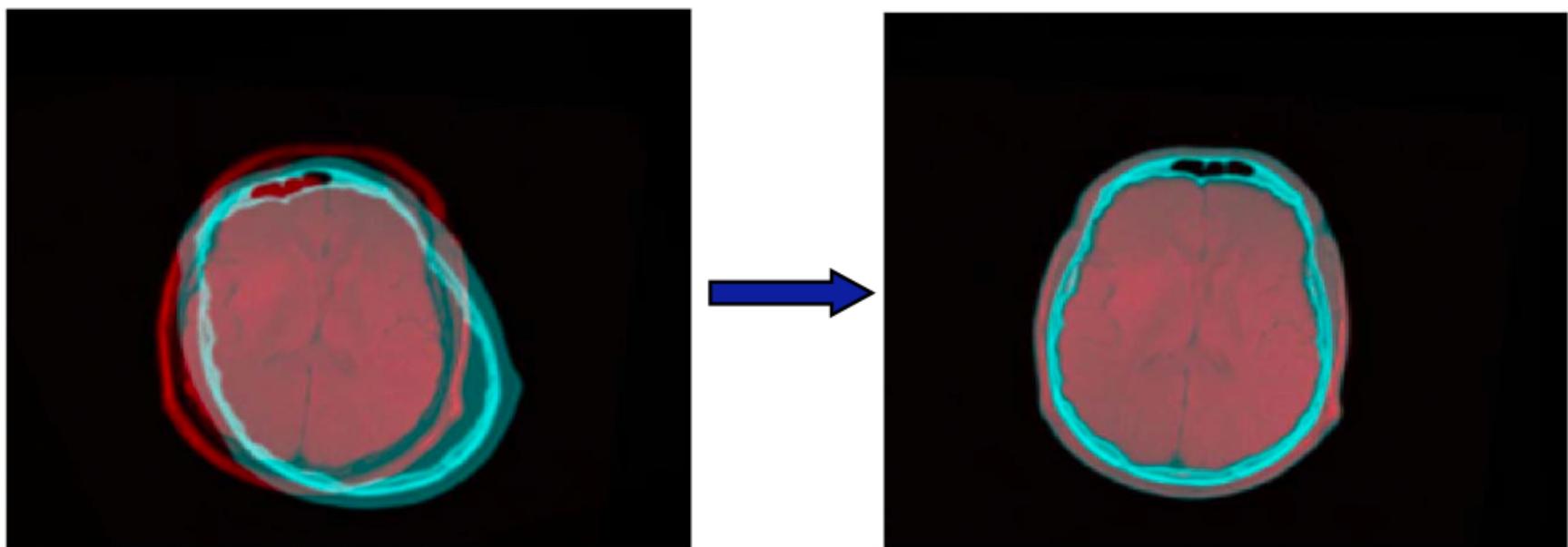
MIT OpenCourseWare (<http://ocw.mit.edu>), Massachusetts Institute of Technology. Downloaded on [July 20, 2012].

# Medical Image Registration



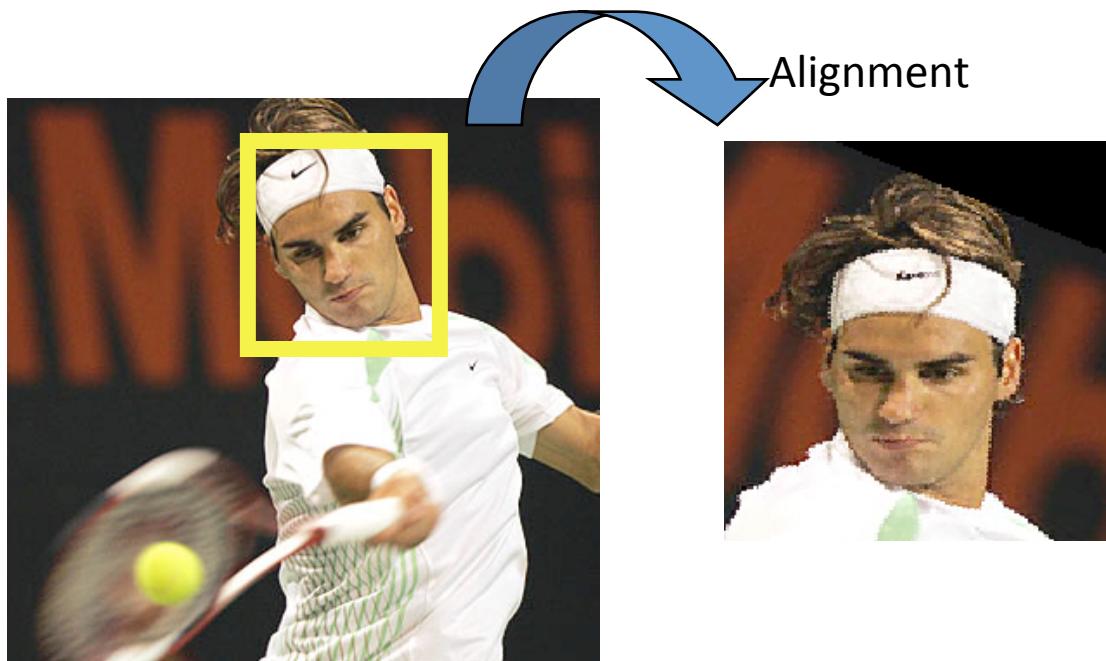
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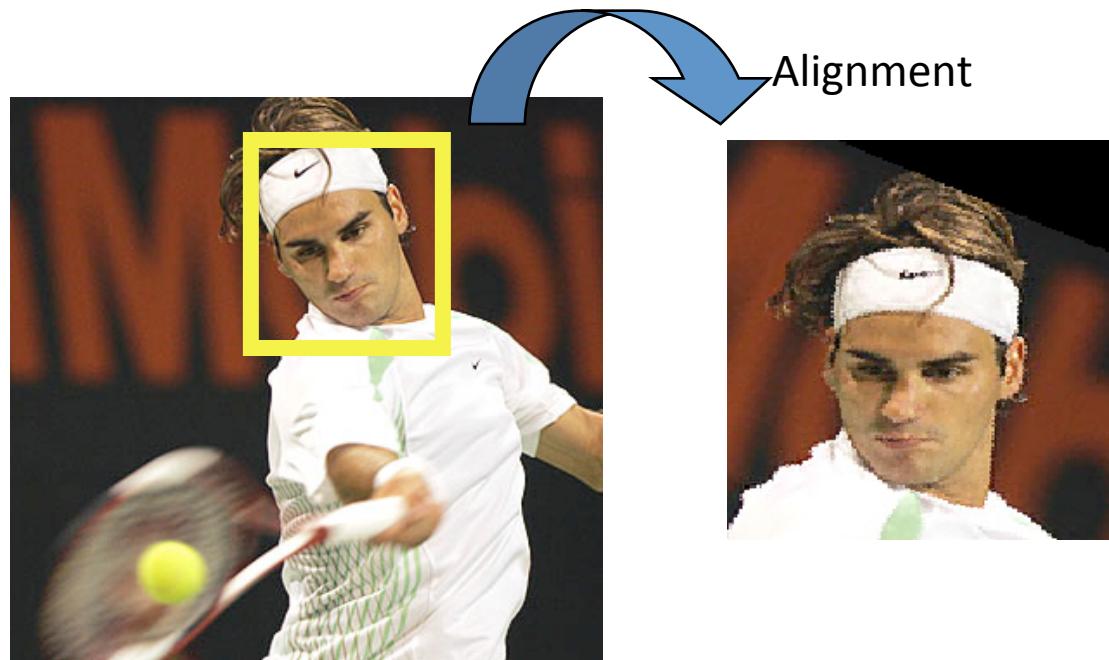


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[MIT OpenCourseWare](http://ocw.mit.edu) (<http://ocw.mit.edu>), Massachusetts Institute of  
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# Face Alignment



# Face Alignment



- *Surprisingly important for recognition algorithms...*

# Face Alignment

Original pictures...



# Face Alignment

After detection...



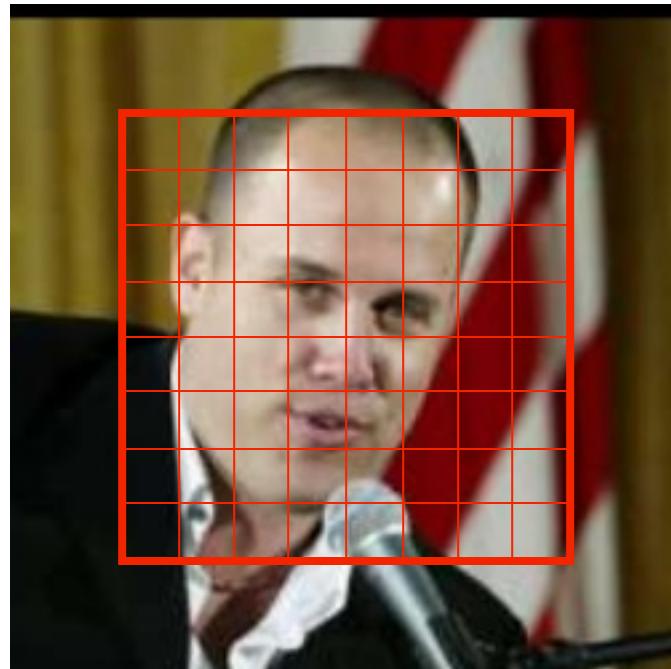
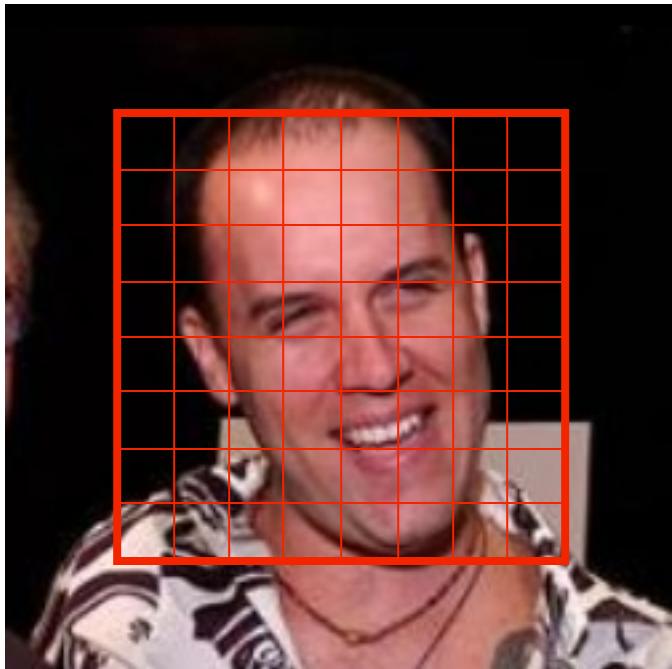
# Face Alignment

Cropping...



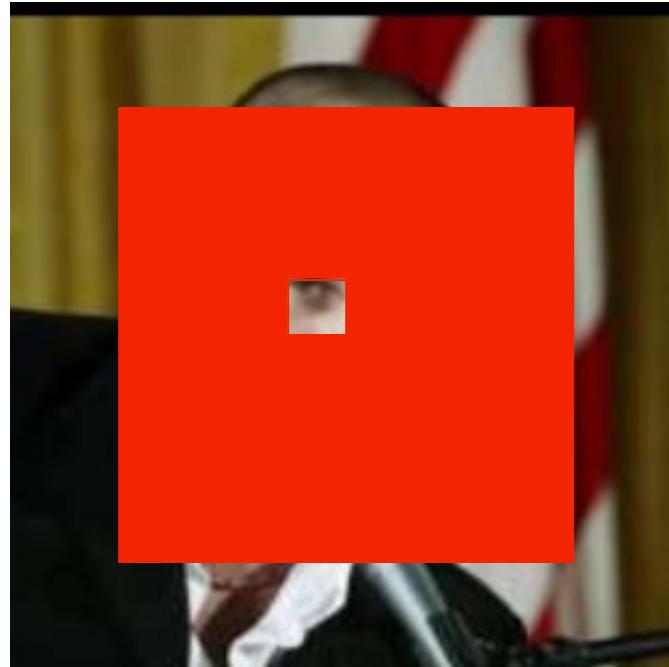
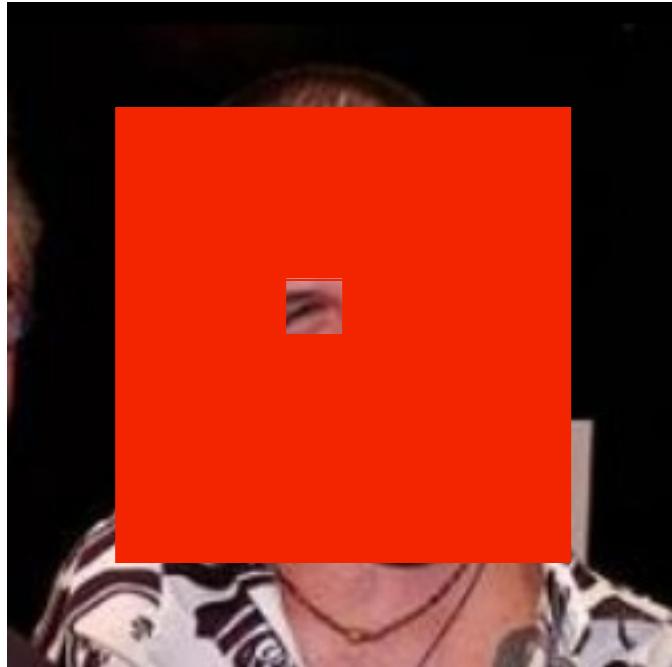
# Face Alignment

Patchwise comparison...



# Face Alignment

Differences are too large for successful recognition



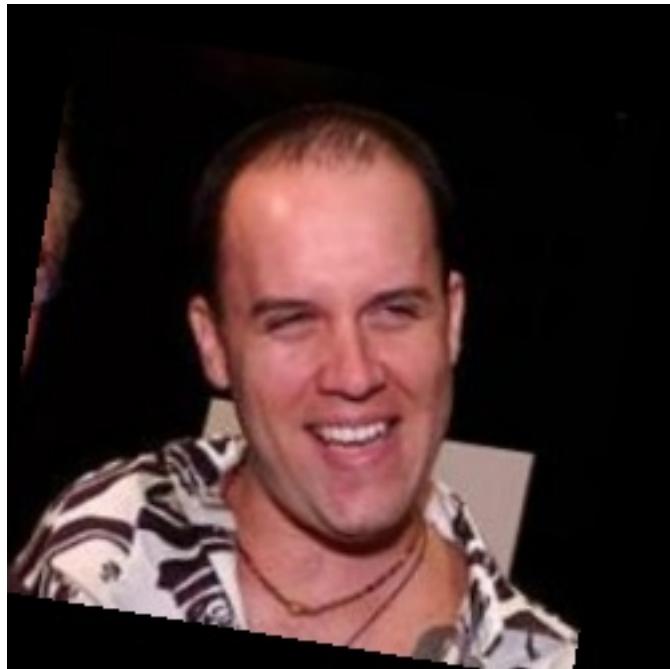
# Face Alignment

Cropping...

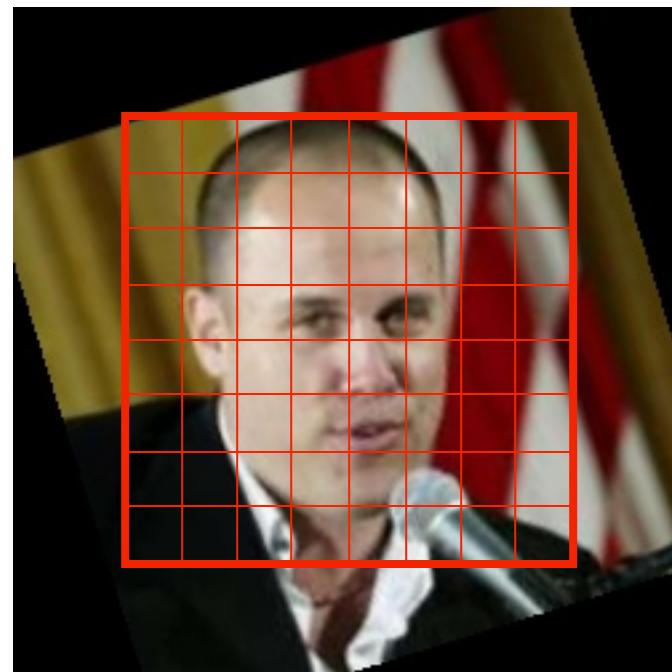
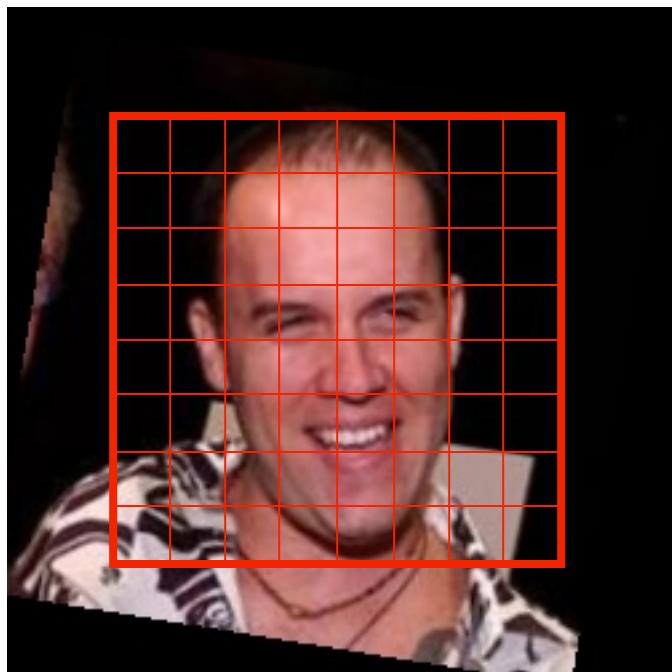


# Face Alignment

Improved alignment

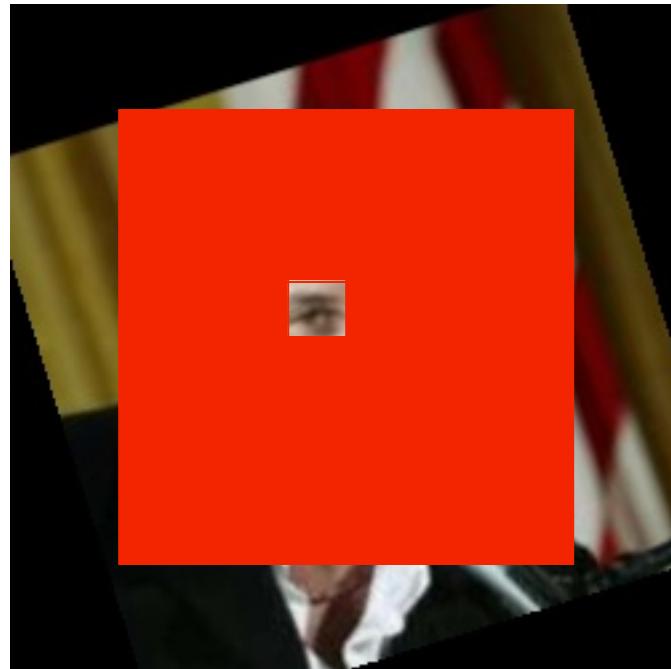
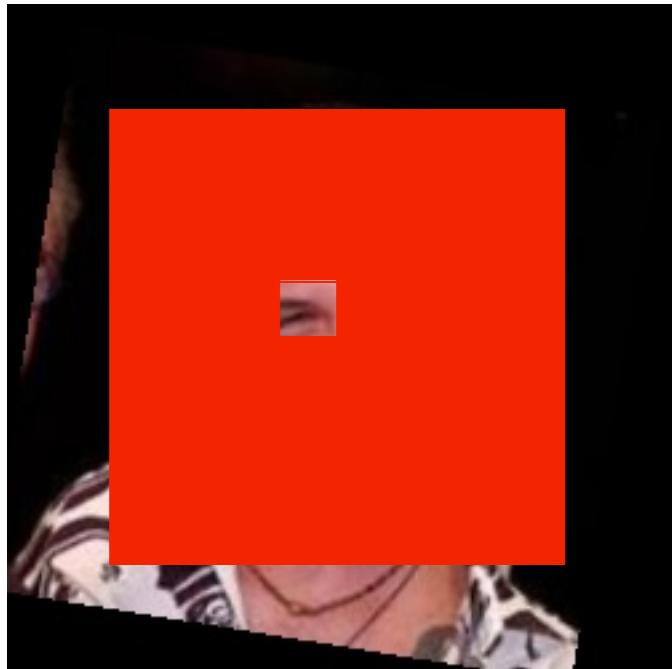


# Face Alignment

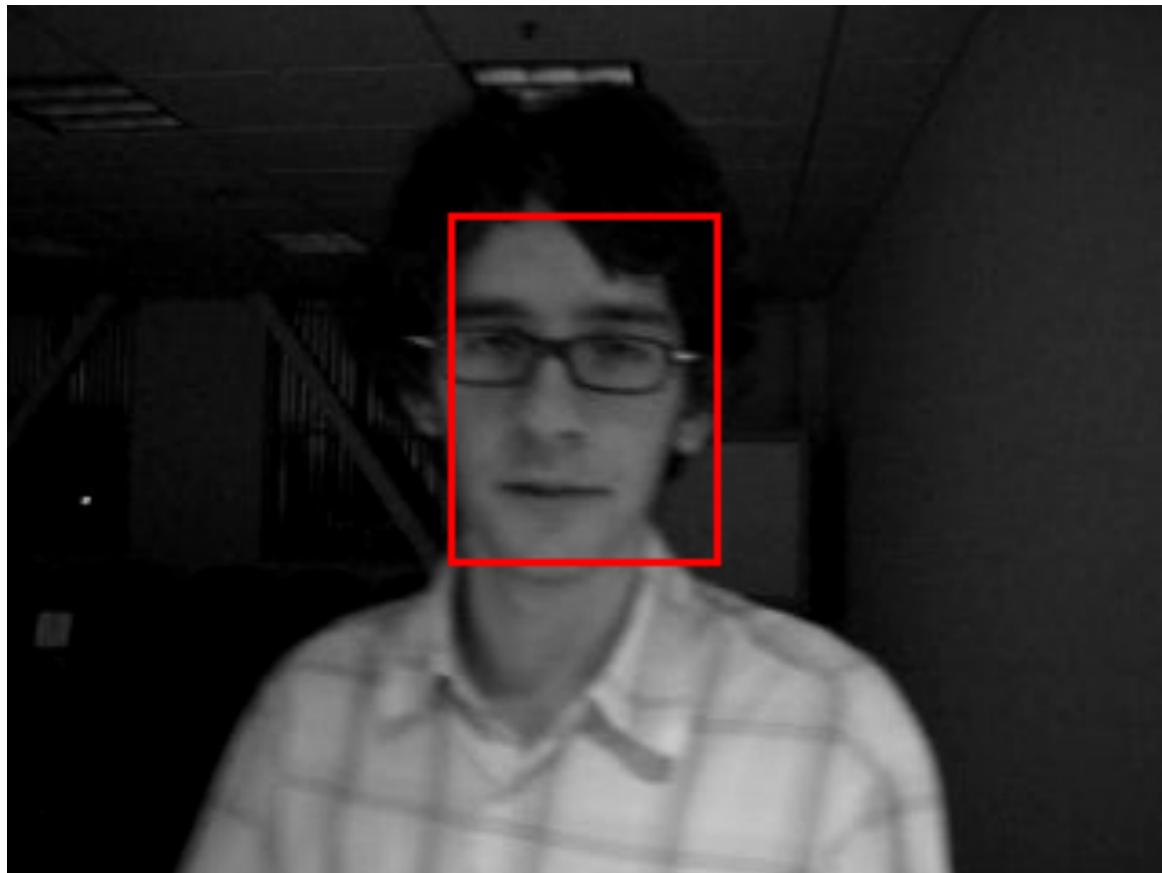


# Face Alignment

Recognition greatly improved...



# Alignment for Tracking



— DF

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# Alignment for Tracking

Frame T



Frame T+d



# Alignment for Tracking

Frame T



Frame T+d

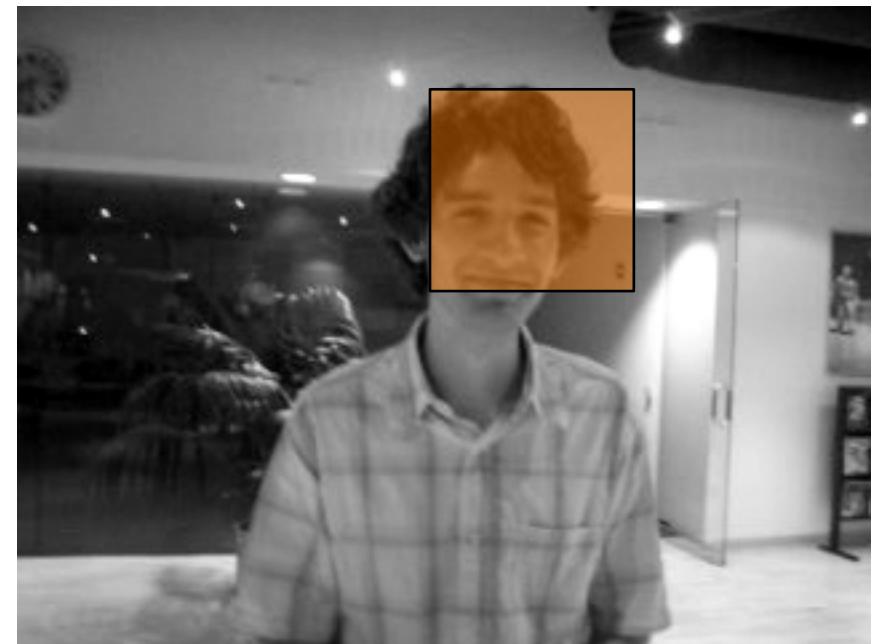


# Alignment for Tracking

Frame T



Frame T+d



# Alignment for Tracking

Frame T



Frame T+d



# Alignment for Tracking

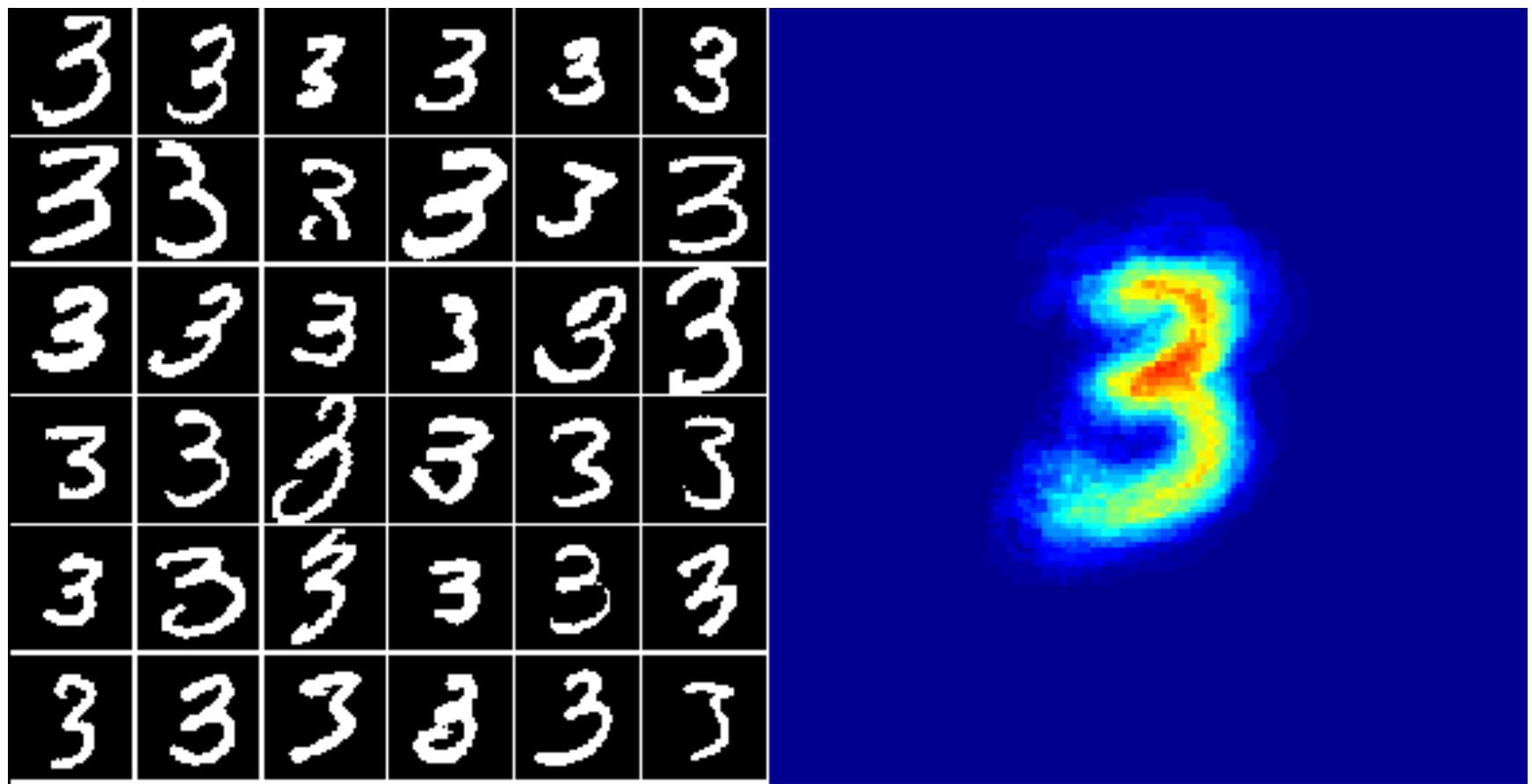
Find best match of patch I to image J,  
*for some set of transformations.*

image J

patch I



# Joint Alignment



# Joint Alignment



# Examples of Alignment

- Medical image registration
- Face alignment
- Tracking
- Joint alignment (model building)

# Questions for Thought

- How should we define alignment?
- What is the purpose of alignment?
- Is alignment a well-posed problem?
- Does a meaningful alignment always exist?
- How can human recognition be so robust to the alignments of objects?... How does the human visual system solve the alignment problem?

# General Categories of Alignment

- Image to image
  - Align one image to another image as well as possible
    - Example: Medical images within patient MR to CT registration.
- Image to model
  - Align an image to a model for more precise evaluation
    - Example: Character recognition
- Joint alignment (congealing)
  - Align many images to each other simultaneously
    - Example: Build a face model from unaligned images.

# General Categories of Alignment

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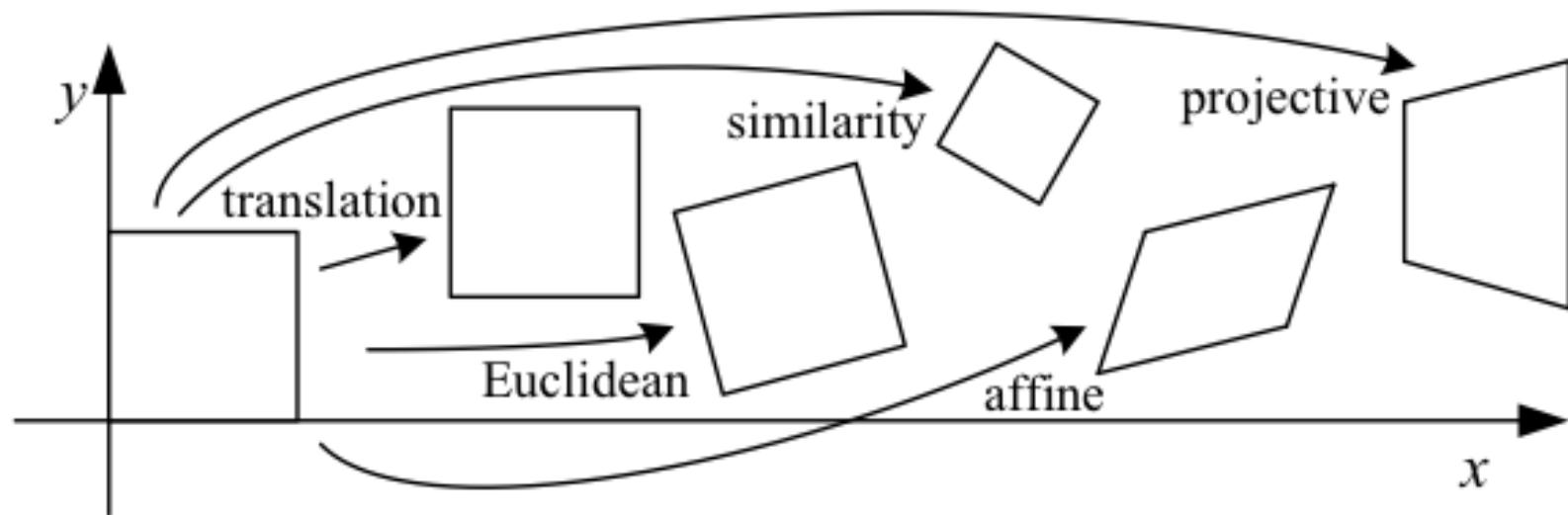
# Image to image alignment

- Basic elements:
  - Two images  $I$  and  $J$ .
  - A family of transformations.
  - An alignment criterion.
- Definition of image to image alignment:
  - Find the transformation of  $I$ ,  $T(I)$ , that optimizes the alignment criterion.

# Image to image alignment

- Basic elements:
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- Definition of image to image alignment:
  - Find the transformation of I,  $T(I)$ , that optimizes the alignment criterion.
- Note: there are many other possible definitions
  - Example: transform *both* images.

# Families of Transformations

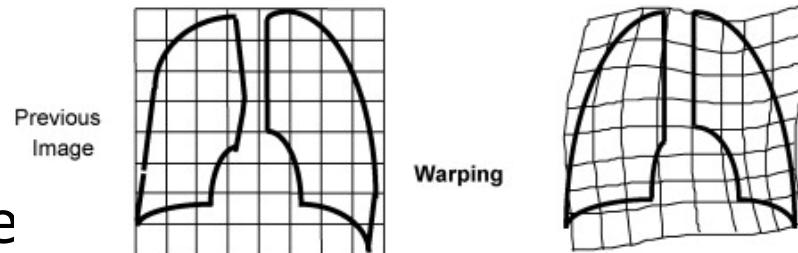


**Figure 2.4** Basic set of 2D planar transformations.

From Computer Vision: Algorithms and Applications, by Rick Szeliski

# Additional Transformation Families

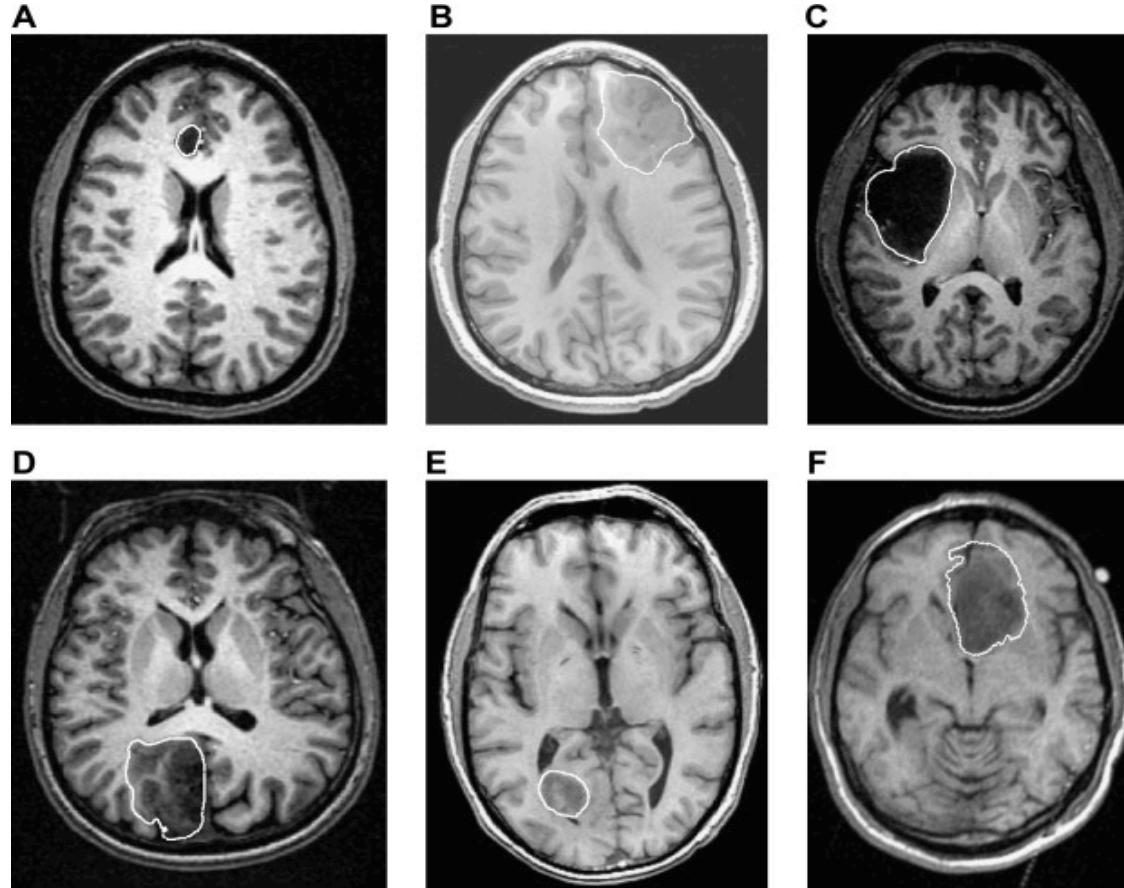
- “Warps”
  - Splines (e.g. cubic spline)
    - Polynomials with “control points”
  - Diffeomorphisms:
    - Arbitrary differentiable mappings of coordinate functions
- Discontinuous and non-differentiable mappings
  - Medical images often undergo non-differentiable mappings! Examples:
    - Growth of a brain tumor.
    - Surgical removal of a portion of the brain.



# Some Transformation Invariants and Relationships

Family	Family + Translation	Includes	Preserves
--	Discontinuous	Everything	Color?
--	Diffeomorphisms	All “warps” including perspective	Continuity and ...
--	Perspective	Affine	Straightness of lines and ..
Linear	Affine	Similarity, Shear, Linear, Non-uniform scaling	Parallel lines and ...
--	Similarity	Uniform scaling, rotation	angles and ...
--	Rigid (Euclidean)	Rotation	areas and lengths and ...
--	Translations		orientation.

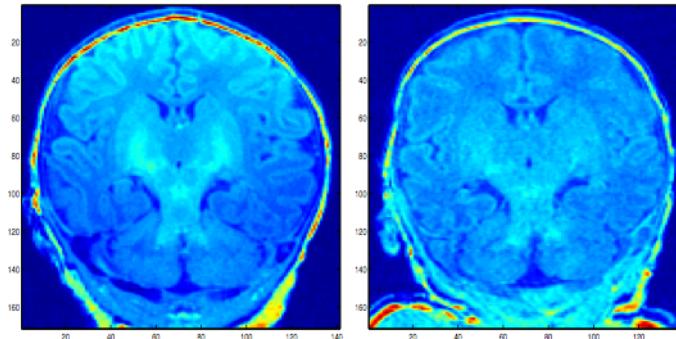
# Non-Diffeomorphic Transformations



Patient-specific non-linear finite element modelling for predicting soft organ deformation in real-time; Application to non-rigid neuroimage registration  
Adam Wittek<sup>a,,</sup>, Grand Joldes<sup>a</sup>, Mathieu Couton<sup>a, c, 1</sup>, Simon K. Warfield<sup>b</sup>, Karol Miller<sup>a</sup>

# Additional Transformation Families

- Brightness transformations:
  - Scaling of brightness
  - Brightness offsets
  - Smooth brightness changes
- Important in many applications
  - Example: Correction of MRI inhomogeneity bias



# Mechanics of Transformations

```
procedure forwardWarp( $f, h, \text{out } g$ ):
```

    For every pixel  $x$  in  $f(x)$

1. Compute the destination location  $x' = h(x)$ .
2. Copy the pixel  $f(x)$  to  $g(x')$ .

# Mechanics of Transformations

```
procedure forwardWarp( $f, h$ , out  $g$ ):
```

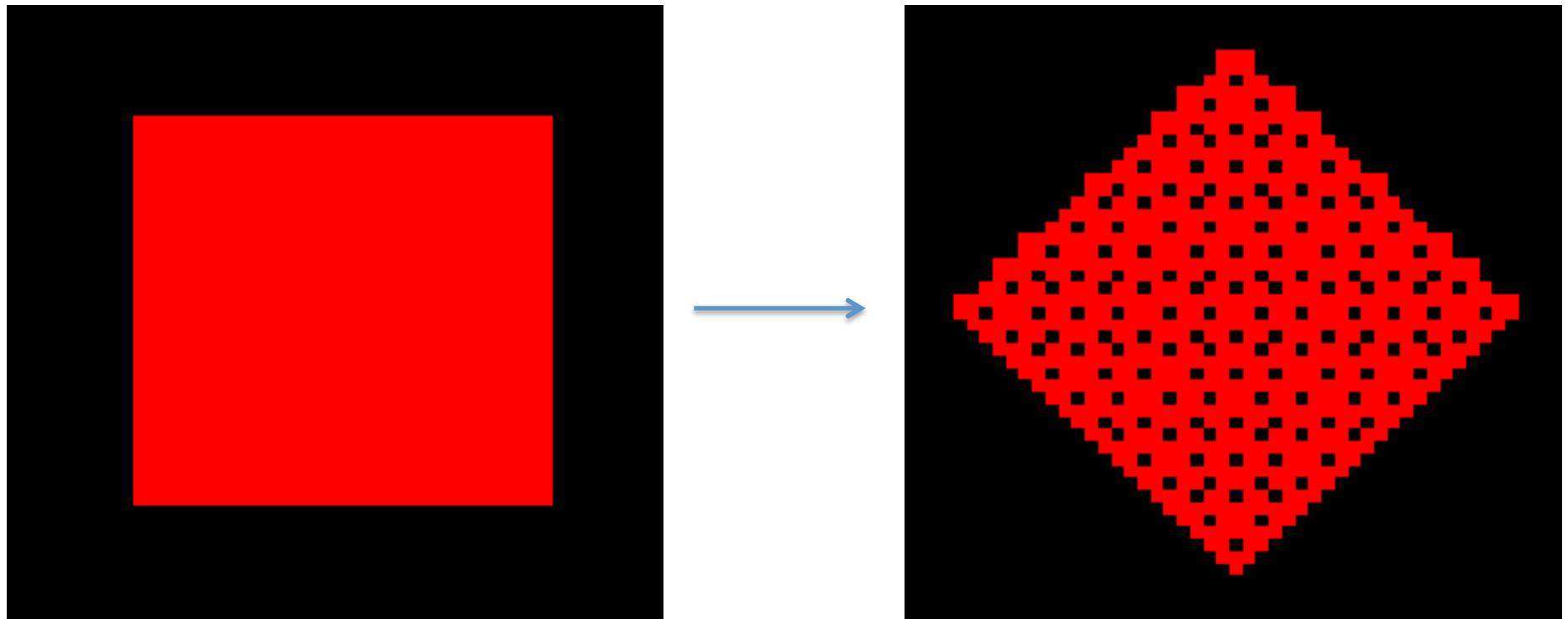
For every pixel  $x$  in  $f(x)$

1. Compute the destination location  $x' = h(x)$ .
2. Copy the pixel  $f(x)$  to  $g(x')$ .

Problems:

- Leaves gaps in destination image.
- Interpolation is less intuitive.

# Example of Forward Warp (rotation by 45 degrees)



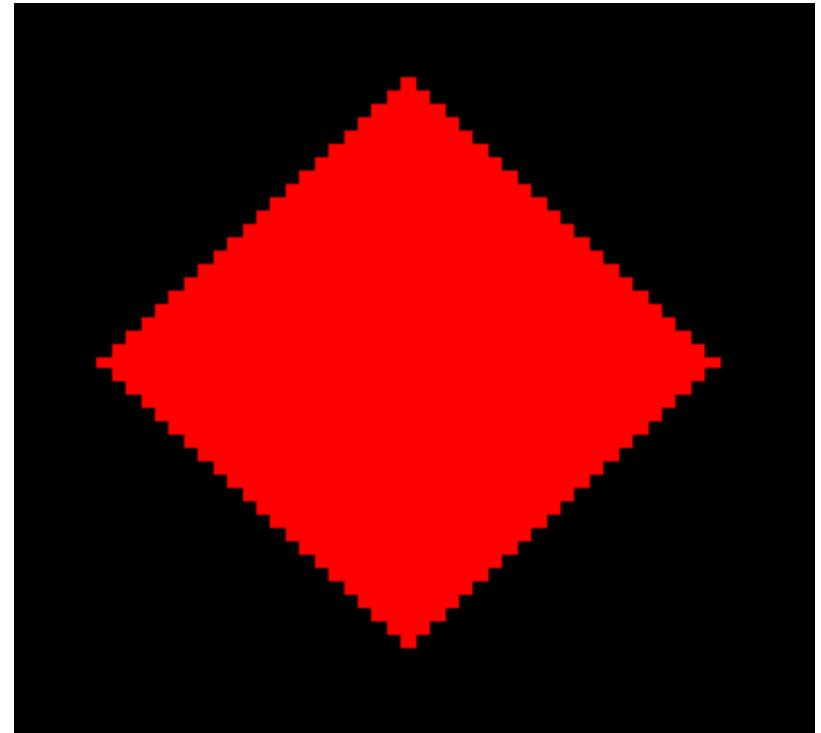
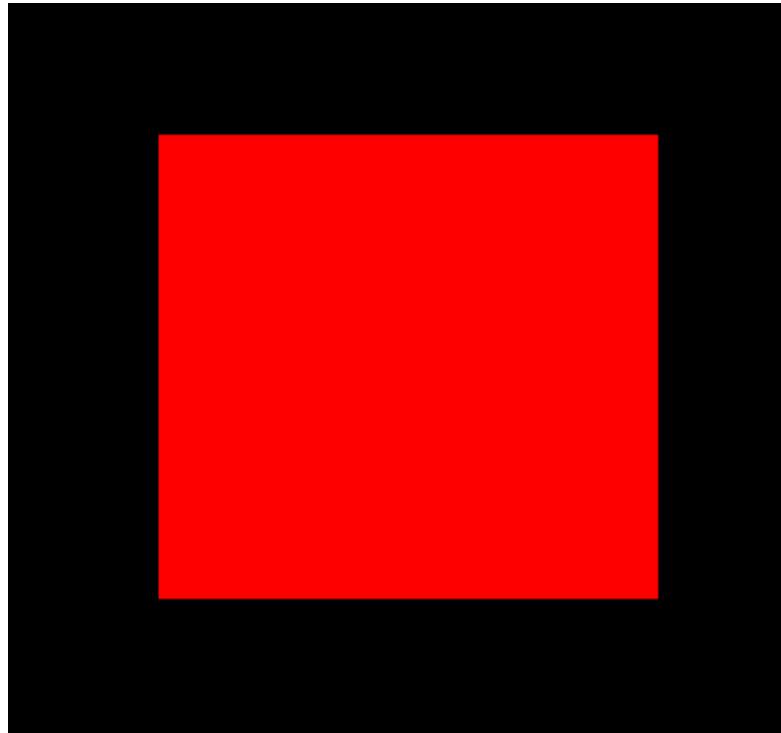
# Mechanics of Transformations

```
procedure inverseWarp( $f$ ,  $\mathbf{h}$ , out  $g$ ):
```

For every pixel  $\mathbf{x}'$  in  $g(\mathbf{x}')$

1. Compute the source location  $\mathbf{x} = \hat{\mathbf{h}}(\mathbf{x}')$
2. Resample  $f(\mathbf{x})$  at location  $\mathbf{x}$  and copy to  $g(\mathbf{x}')$

# Example of Reverse Warp



# Alignment Criteria

- How to “score” an alignment.
- What should we compare at each location?
  - Pixel colors?
  - Edge features?
  - Complex features?
- Given what we are comparing, what function should we use to compare those things?
- This is an open question!

# Alignment Criteria

- Alignment criteria clearly depend upon the *image representation*:
  - A gray value at each pixel location (grayscale image).
  - A red-blue-green triple at each pixel location (standard color image).
  - An edge strength and orientation at each pixel.
  - Color histograms
  - Histograms of oriented gradients (HOG features).
  - Many other possible representations.

# Alignment Criteria

- Some simple criteria:
  - Sum of squared differences of feature values at each pixel (L2 difference):

$$f(I, J) = \sum_{i=1}^N (I_i - J_i)^2 \quad \text{or} \quad \sqrt{\sum_{i=1}^N (I_i - J_i)^2}$$

- Sum of absolute differences (L1 difference).
- Normalized correlation
  - Usually used with gray scale representation.

# How do we choose an alignment criterion?

- How do we judge whether an alignment criterion is good or bad?
  - Should it match human judgments?
  - Should it have a simple mathematical formulation?
  - What representation is a “good” representation?
    - It may depend upon the task.
- We will address this question in more detail in later lectures.

# Definition of alignment

- Formal definition of alignment for images I and J:

$J_T$ : transformation of image  $J$  by transform  $T$

$\mathcal{T}$ : a set of transformations

$$T^* = \operatorname{argmin}_{T \in \mathcal{T}} f(I, J_T)$$

- How should we perform this optimization?

# Optimizing the Alignment Criterion

- Exhaustive search
  - Try all possible image transformations!
  - Gets extremely expensive as the family of transformations gets larger.
- Search for “keypoints” and align the keypoints.
  - SIFT based alignment
  - See Szeliski book for excellent treatment.
- Gradient descent (“local search”)
  - Slowly change the transformation to improve the alignment score.
    - Depends strongly on “landscape” of alignment function.

# Summary of Intro

- Categories of alignment
  - Image to image
  - Image to model
  - Joint image alignment
- Definition of image to image alignment
  - Choose a representation for images
  - Choose a family of transformations
  - Choose a criterion of alignment
  - Optimize alignment over family of transformations

# Lecture I

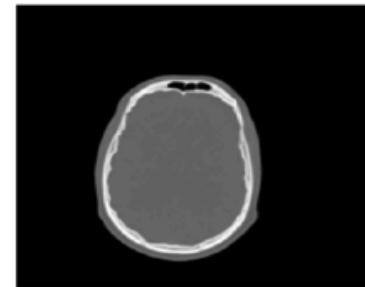
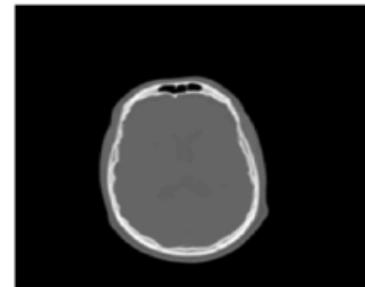
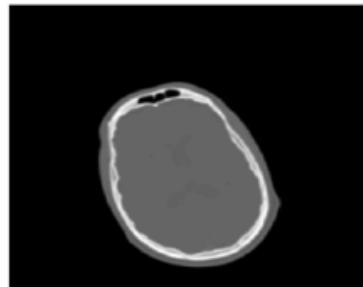
- Introduction to alignment
- **A case study: mutual information alignment**

# Alignment by the Maximization of Mutual Information

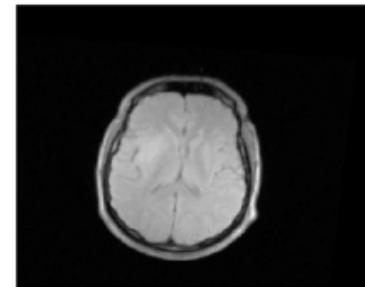
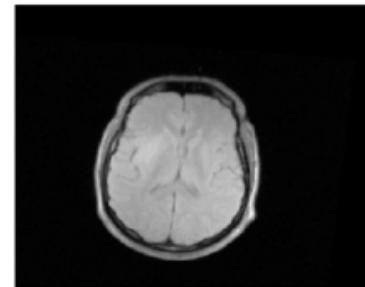
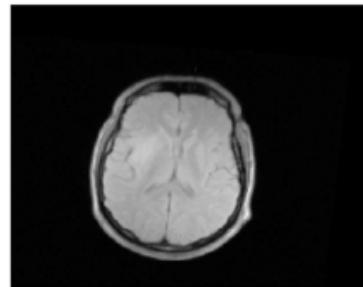
- Classic example of M.I. alignment:
  - Aligning medical images from different modalities
    - magnetic resonance images
    - computed tomography images
  - Magnetic resonance images
    - Measures proton density (in some cases) of tissue
  - Computed tomography
    - Measures X-ray transparency
- Original work by Viola and Wells, and also by Collignon et al.

# CT and MR images

CT



MR

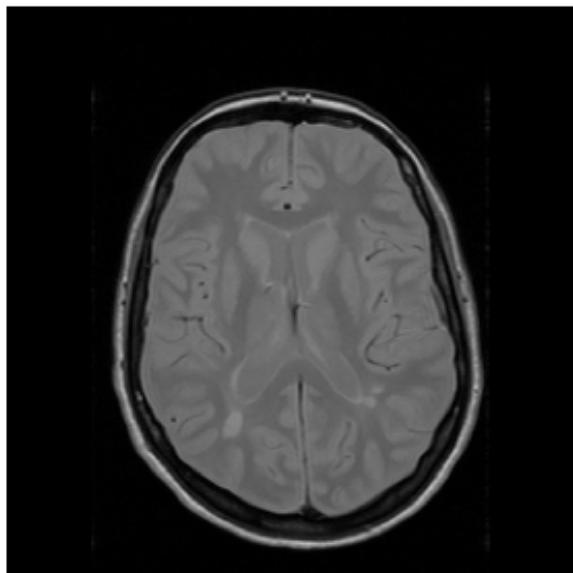


misaligned

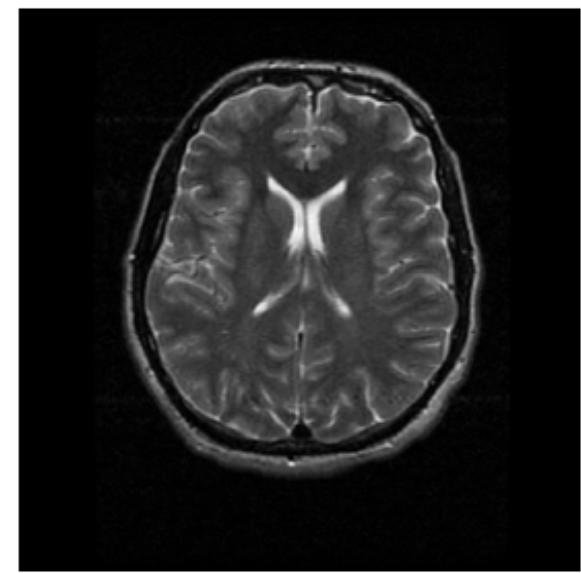
slightly misaligned

aligned

# Two Different MR modalities



Same anatomy but  
left is  $T_1$  weighted,  
right is  $T_2$  weighted



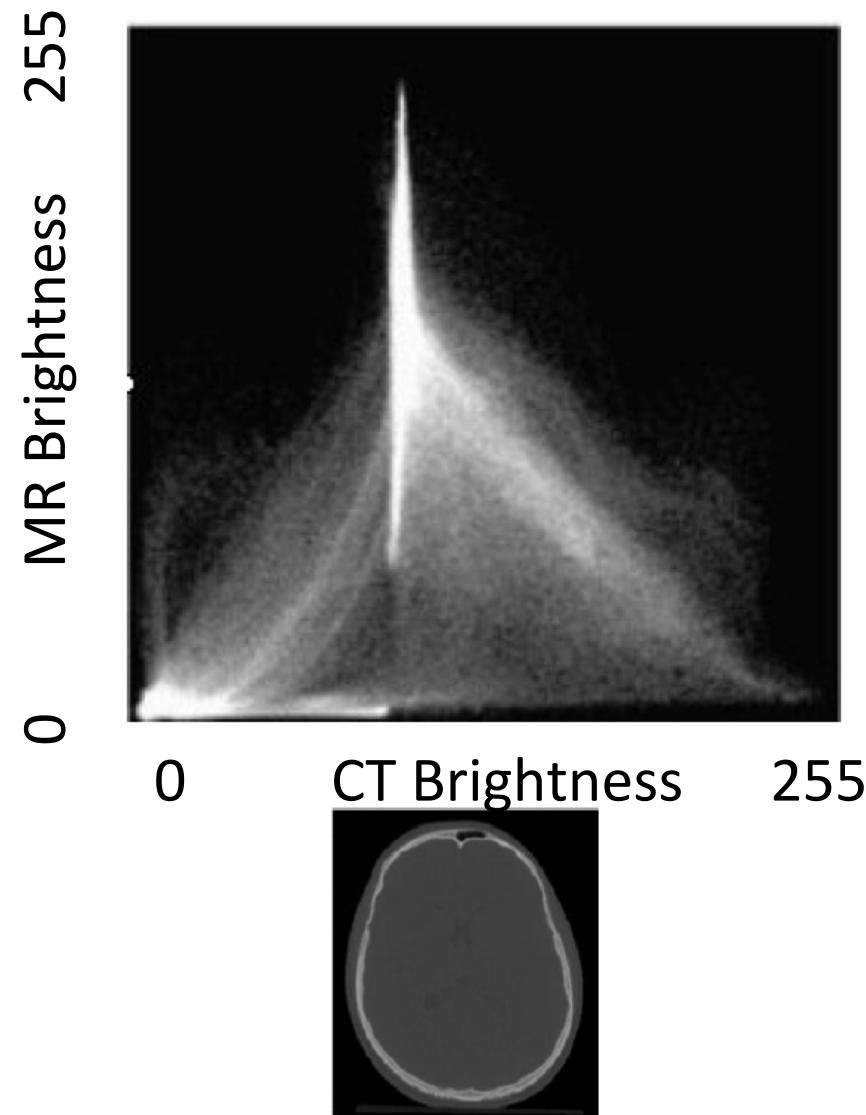
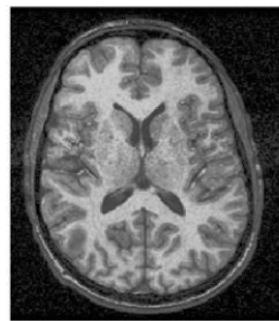
# Traditional alignment criteria fail

- When images are registered:
  - L2 error is not low
    - Same tissue has different value in MR and CT
  - Correlation score is not high
    - MR and CT are not linearly related
  - Many other criteria also fail
  - Need a different criterion....

# The Random Variable View

- Consider two images  $I$  and  $J$  of different modalities. Assume for the moment that they are aligned.
- Consider a random pixel location  $X$ .
  - Let  $X_i$  be the brightness value in image  $I$  at  $X$ .
  - Let  $X_j$  be the brightness value in image  $J$  at  $X$ .
- $X_i$  and  $X_j$  are random variables.
- What can we say about  $X_i$  and  $X_j$ ?

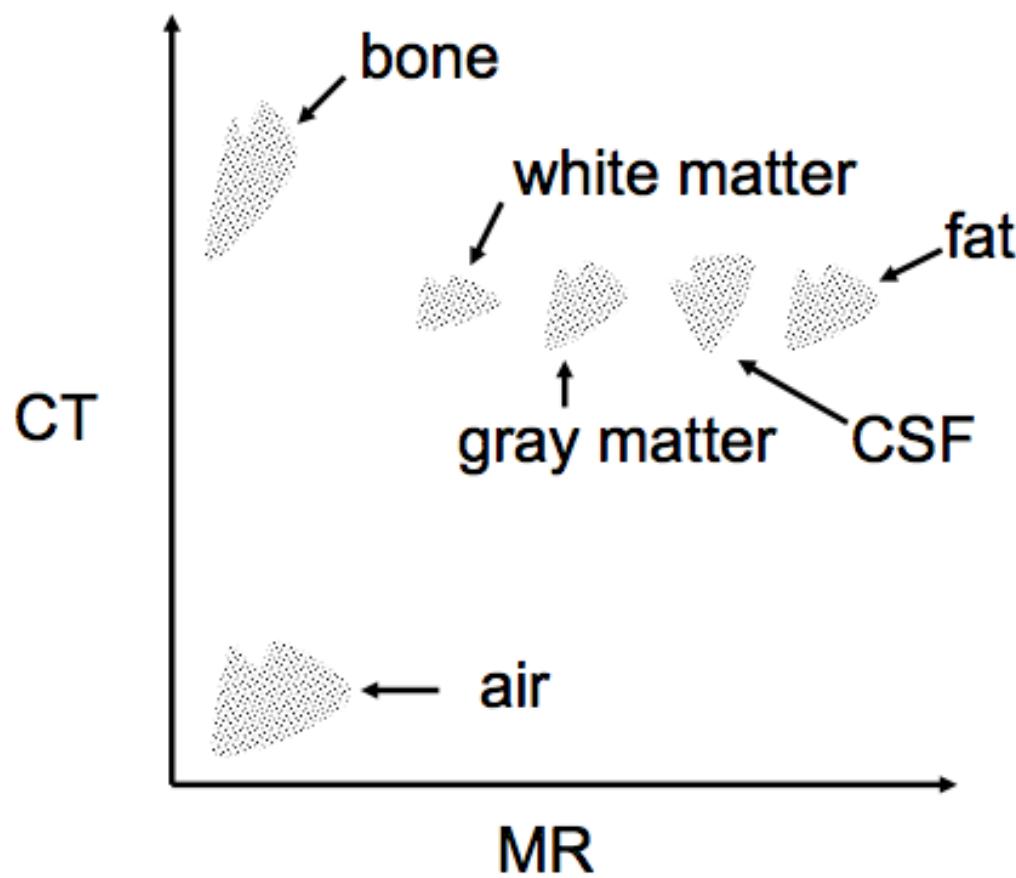
# Relationship Between $X_i$ and $X_j$



Joint Brightness  
Histogram

Figures from  
Michael Brady  
Oxford University

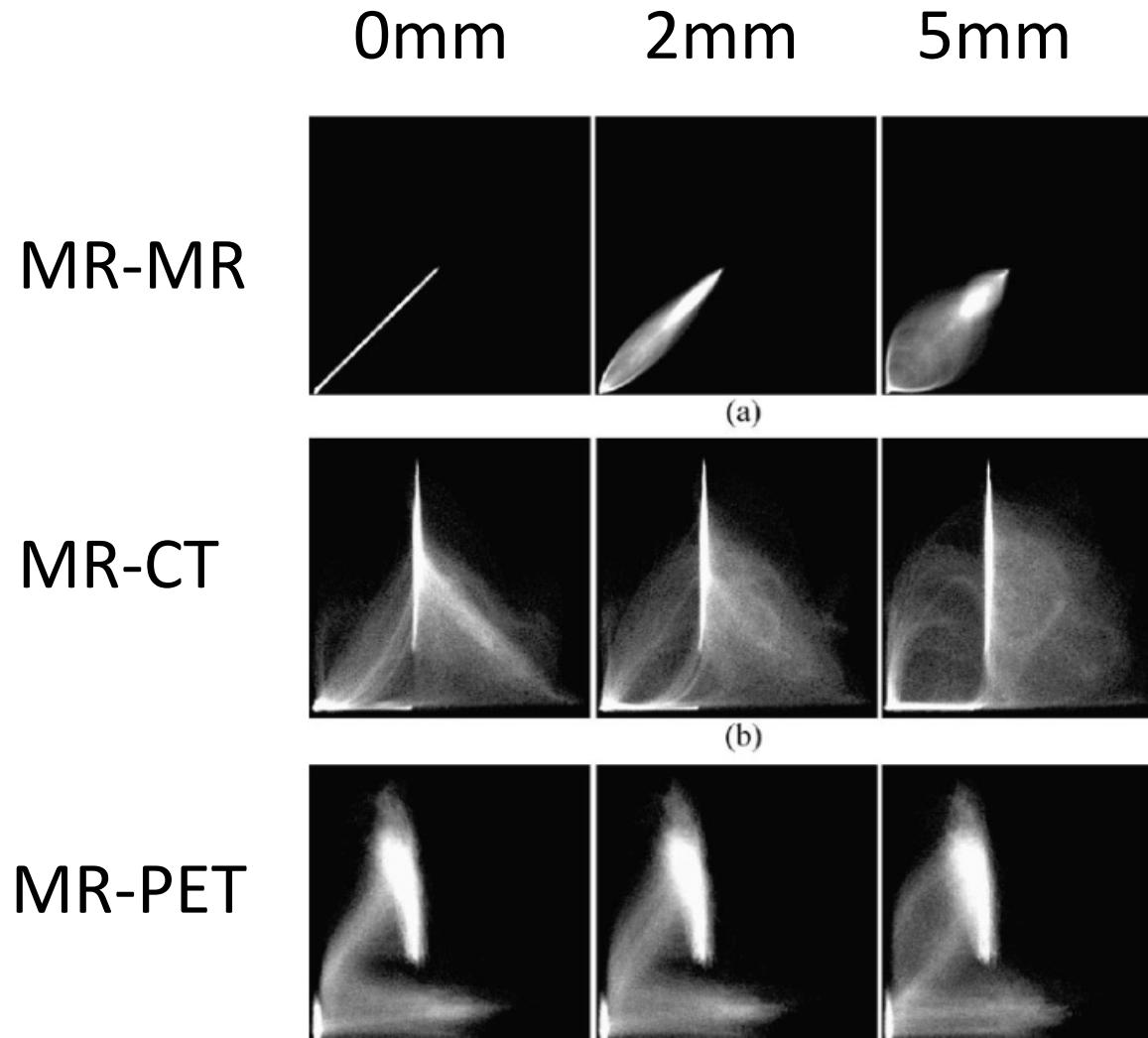
# MR and CT images



# Statistical Dependence

- MR and CT values are NOT linearly related
- MR and CT are not FUNCTIONALLY related
  - There is no function that maps one to another.
- However, they have strong *statistical dependence*.
- When MR and CT pixels are unaligned, the dependence drops.
  - Basic idea: move images around to *maximize statistical dependence*

# Joint Distribution as a Function of Displacement



Figures from  
Michael Brady  
Oxford University

# Mutual Information

Two random variables  $X$  and  $Y$  are *statistically independent* if and only if

$$P(X, Y) = P(X)P(Y)$$

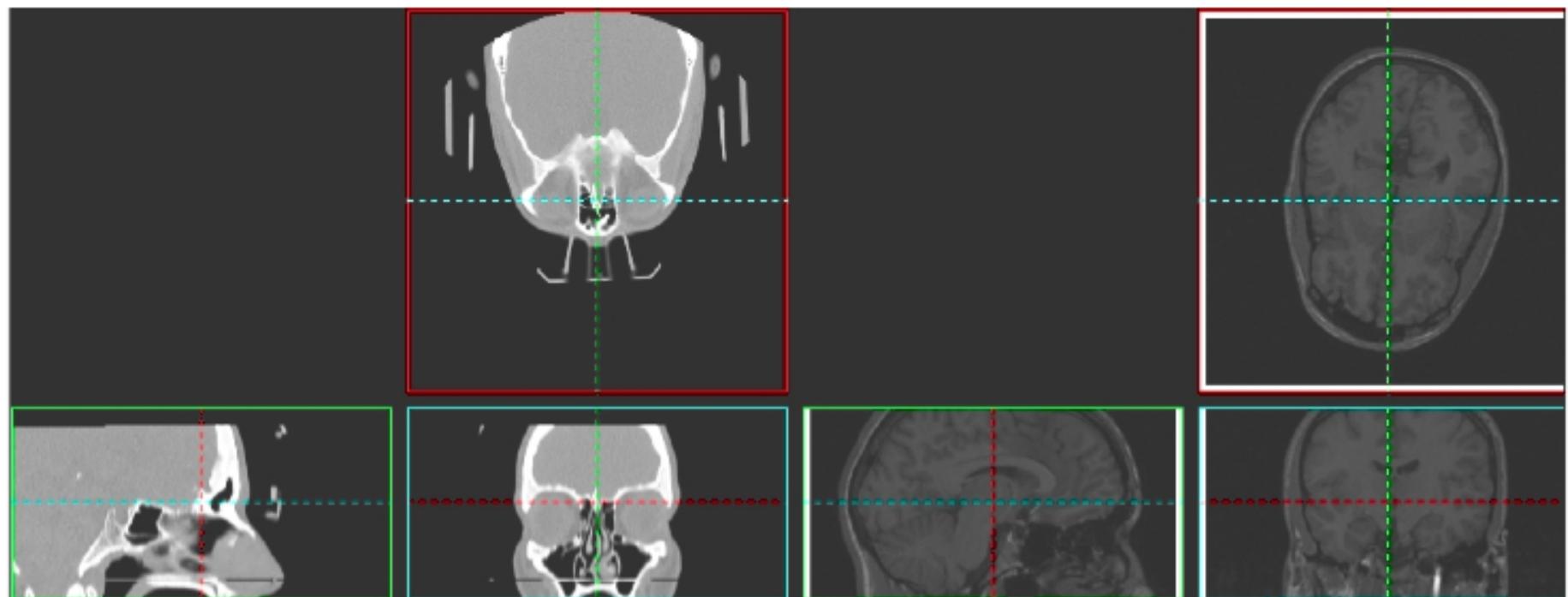
The *mutual information* between two random variables  $X$  and  $Y$  is

$$I(X, Y) = \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} P(x, y) \log \frac{P(x, y)}{P(x)P(y)}.$$

# Mutual Information

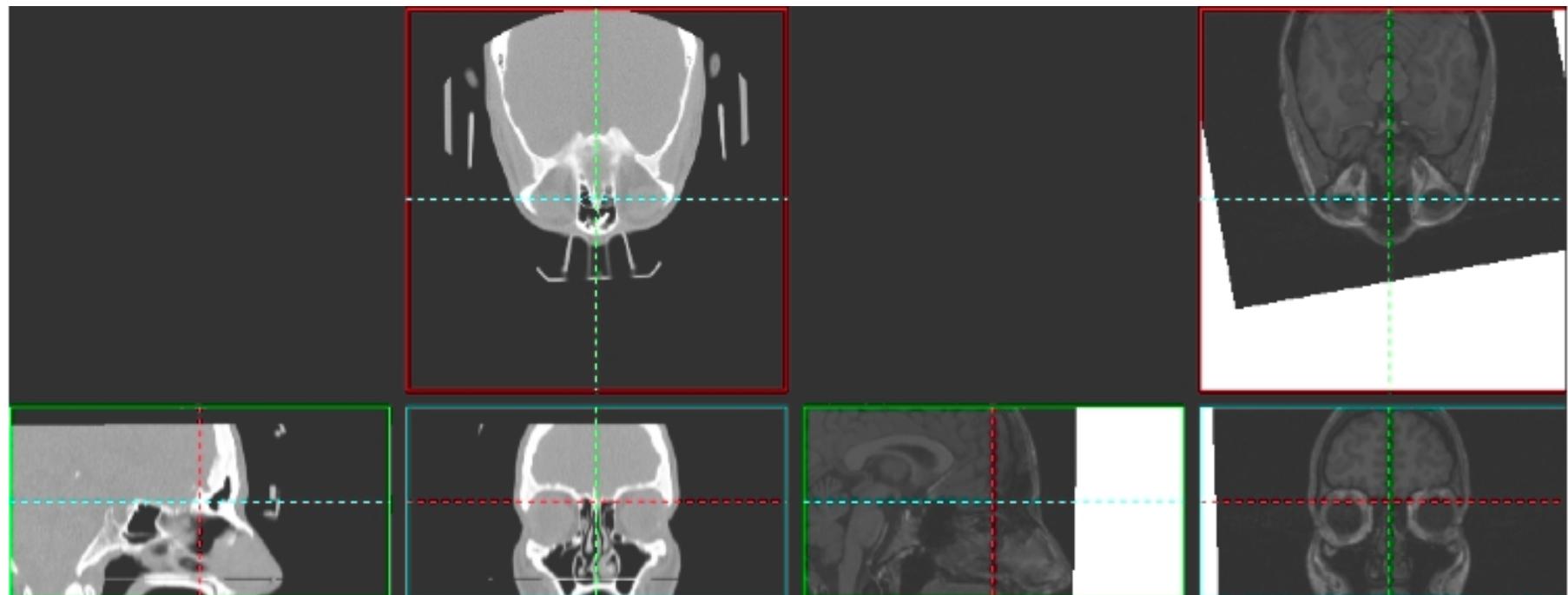
- Is 0 only when two variables are independent.
- Goes up as variables become more dependent.
  - Goal: maximize dependence so maximize mutual information.

# Example on real data



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# Example on real data



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# Summary

- Basic elements of alignment
  - Representation
  - Alignment criterion
  - Method of optimization
- Mutual information alignment
  - Criterion address problems of aligning images from different modalities
  - Why not always use mutual information alignment?
    - Chess board example.

# The Time Traveler's Camera



# The Time Traveler's Camera



# The Time Traveler's Camera



# The Time Traveler's Camera

- As it turns out...
  - *Displaying* holograms is very difficult, BUT...
  - *Recording them* is EASY

# The Time Traveler's Camera

- As it turns out...
  - *Displaying* holograms is very difficult, BUT...
  - *Recording them* is EASY



# Sergey Prokudin-Gorsky

- Russian chemist and photographer
- 1909-1915: toured Russia for the czar to record pictures of Russia
- Took color “negatives” even though there were only extremely crude ways to reproduce them!

# Some dude with a big sword

actually the Emir of Bukhara



# Some dude with a big sword



# Some dude with a big sword



!!!





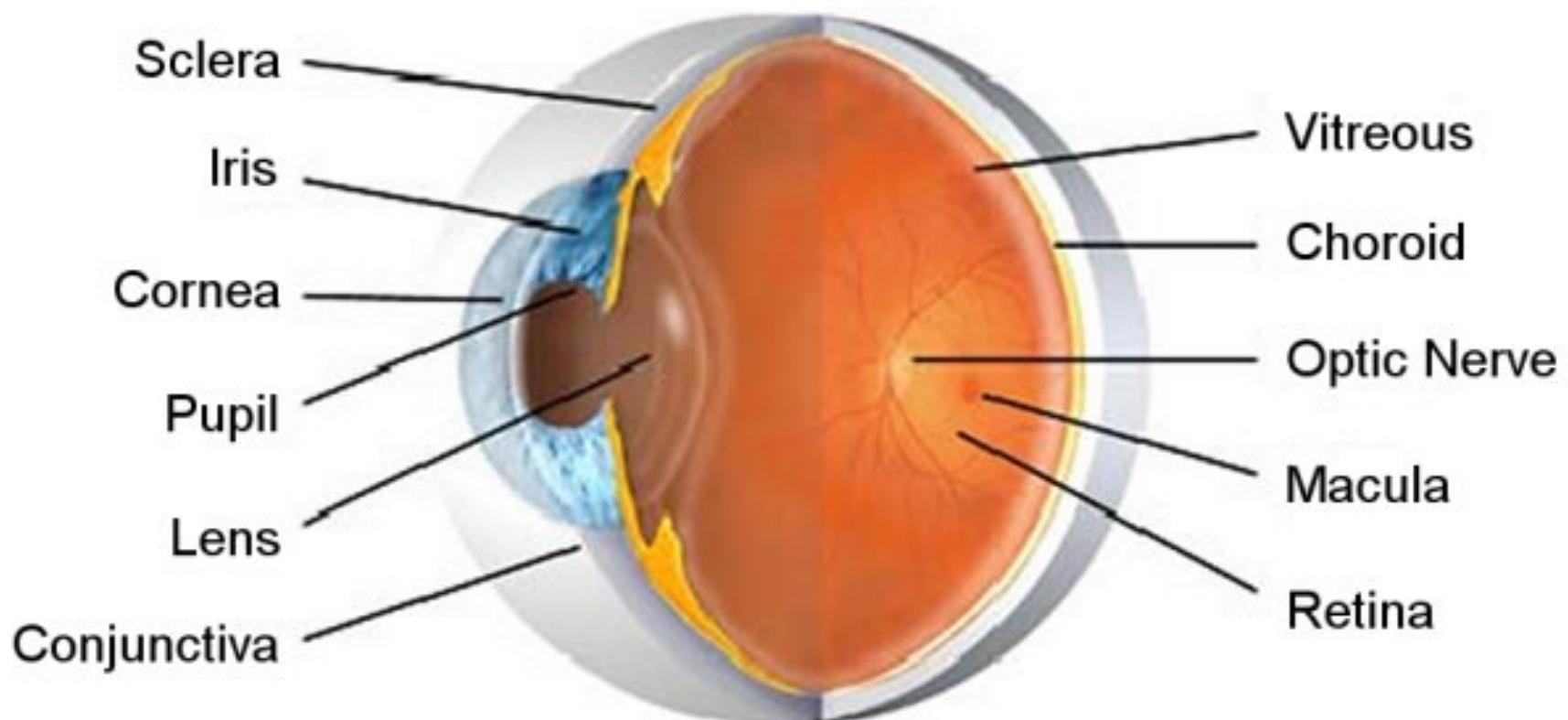
*“The sad irony of the technique of the Three-Color Photography was that Prokudin-Gorsky himself never saw the entire collection of his images in color. Most of the surviving images are preserved in the form of triple black and white negatives and only rarely as historic color prints or slides which in any case are no match to present day color photographs.”*



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Suren Manvelyan

# Human Eye Anatomy



<http://www.freedomscientific.com/resources/vision-anatomy-eye.asp>

# Eye muscles

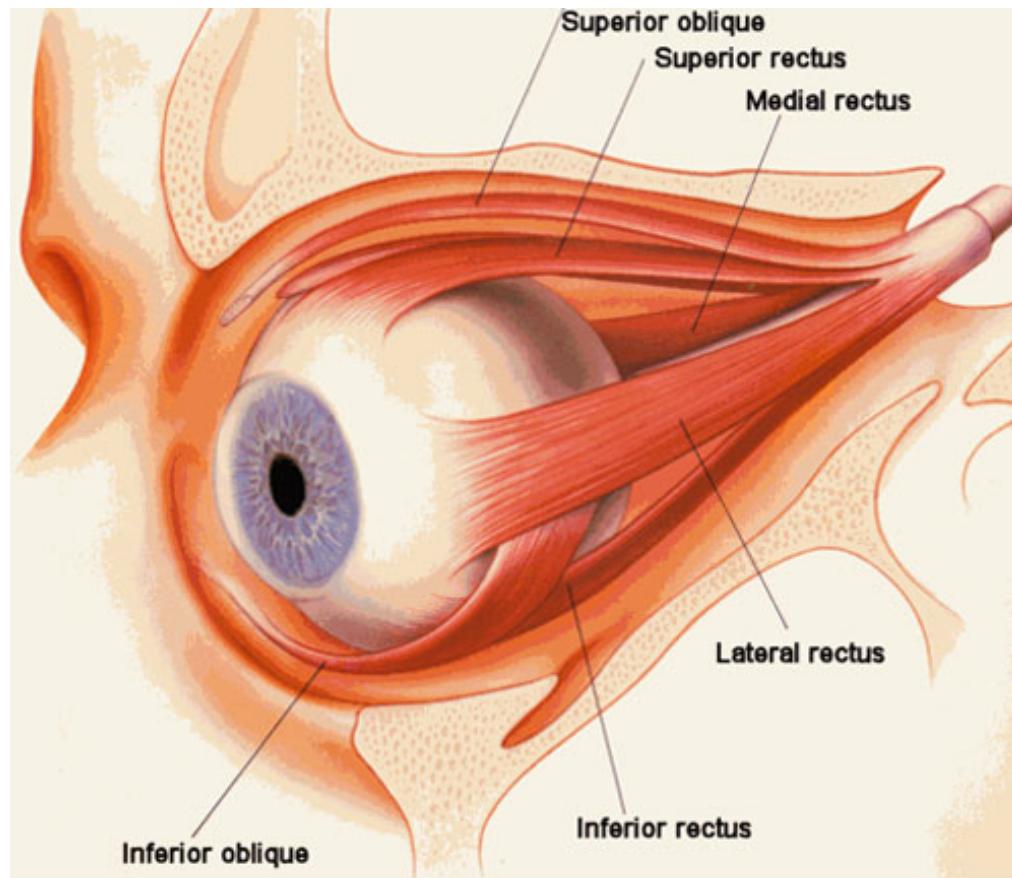


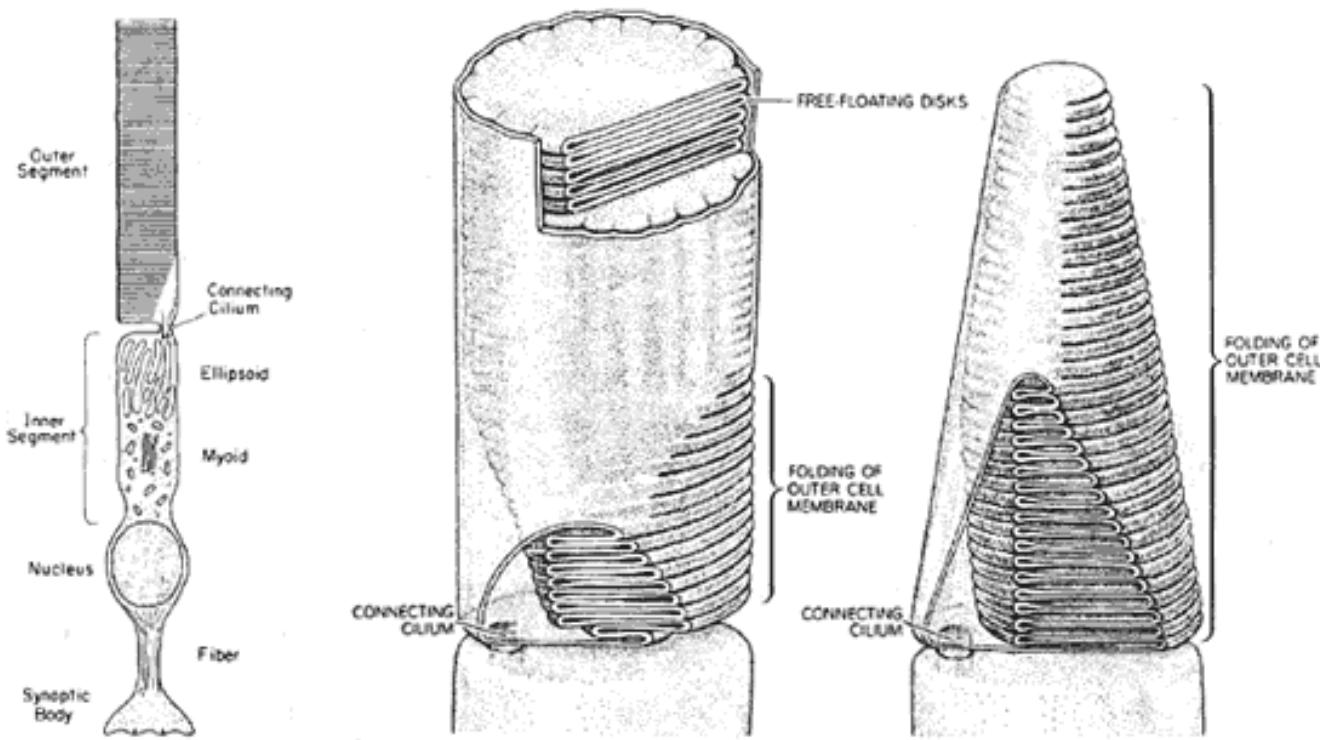
Fig. 1 Extraocular Muscle Anatomy

<http://www.aapos.org/terms/conditions/22>

# Rods and Cones on the Retina

- Rods: Sensitive to light at all visible wavelengths
- Cones: 3 types: sensitive to different parts of the spectrum.

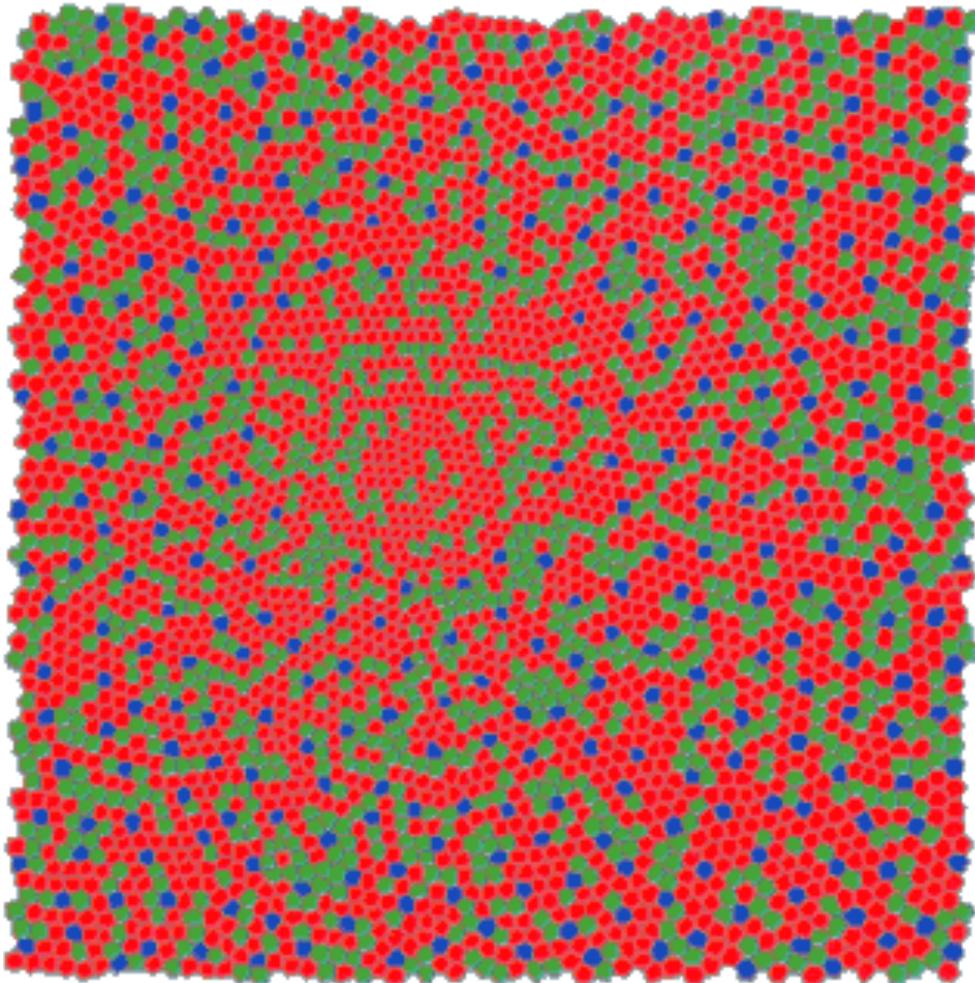
# Rods and Cones on the Retina



At the left is a generalized conception of the important structural features of a vertebrate photoreceptor cell. At the right are shown the differences between the structure of rod (left) and cone (right) outer segments. These diagrams are from Young (1970) and Young (1971).

# Cones on the retina

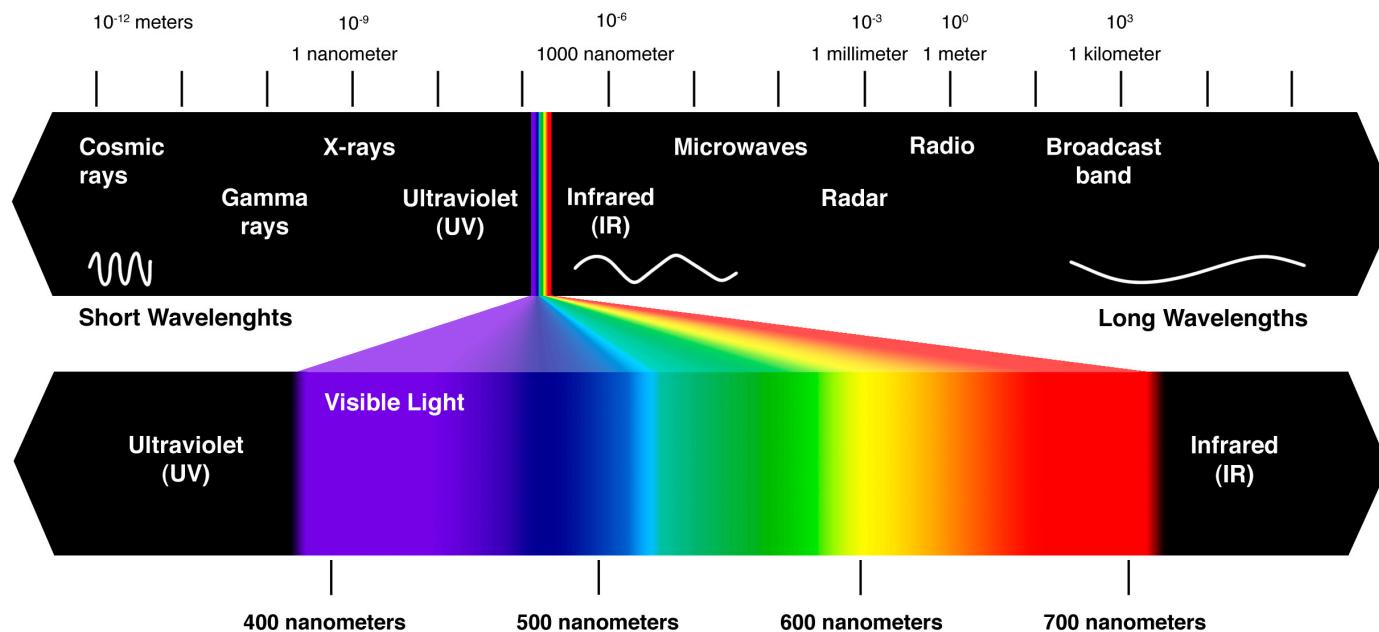
Cone mosaic



[http://www.cis.rit.edu/people/faculty/montag/vandplite/pages/chap\\_9/ch9p1.html](http://www.cis.rit.edu/people/faculty/montag/vandplite/pages/chap_9/ch9p1.html)

# Why do RGB images work?

- Tristimulus theory:
  - The human eye has three types of color receptors, or cone cells: short wavelength (“blue”), medium wavelength, and long wavelength.



# Tristimulus Theory

- Fundamental premise:
  - If we can produce an image B that generates the same responses on the retina as image A, then A and B will appear exactly the same.
  - Image A: the real world.
  - image B: a digital image of the real world
- Question: how to make a digital image of the real world when we only have a black and white camera?

# What your long cones see.

- Long cones, also known as “red” cones are most sensitive to light at the red end of the visible spectrum.
- What do they “see”?





Red cones response to these images is about the same.







Red cones response to these images is about the same.



Red cones response to these images is about the same.



There's only one problem...

# There's only one problem...























# Everybody's Favorite Author

