# How to compute the option implied volatility

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## 1 Context

There are 2 volatilities to be computed: the expectation under P and the expectation under Q.

**Expectation under P**
This is the expectation calculated using historic return data.

The authors use a random walk model

$$\mathbb{E}_t^{\mathbb{P}}\left[RV_{t,h}^{U/D}(\kappa)\right] = RV_{t-h,h}^{U/D}(\kappa) \tag{1}$$

where the realized variance is calculated per day, and summed up over a time horizon h

$$RV_t^{U/D}(\kappa) = \sum_{j=1}^{n_t} r_{j,t}^2 [\text{indicator above/below } \kappa] \tag{2}$$

$$RV_{t,h}^{U/D} = \sum_{j=1}^{h} RV_{t+j}^{U/D}(\kappa) \tag{3}$$

Also the authors briefly mention, that this formula converges to the instantaneous variance, assuming that the stock price follows a jump process. However, we only have to compute the sums above. So I don't think we have to deal much with jumps.

**Expectation under Q**
This is the expectation calculated using option data. The authors use so called model-free/corrior implied volatility, hence they do not make assumptions regarding the dynamics of the underlying.

$$IV_{t,h}^{U} = \mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,h}^{U}(\kappa)\right] \approx 2\int_{F_t epx(\kappa_F}^{\infty} \frac{M_0(S)}{S^2}\,\mathrm{d}S, \tag{4}$$

$$IV_{t,h}^{D} = \mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,h}^{D}(\kappa)\right] \approx 2\int_{0}^{F_t epx(\kappa_F} \frac{M_0(S)}{S^2}\,\mathrm{d}S \tag{5}$$

with

$$M_0(S) = min(P_t(S), C_t(S))$$

and with $\kappa_F = (\kappa - r_t^f)$ to get consistent thresholds when computing realized and option-implied volatility .

For the intuition and derivation of this formula: see other QExp document.

## 2 Information from online appendix

Description of procedure

1. preprocessing
   (a) sort call and put options by maturity and strike price
   (b) average bid-ask quotes for each contract
   (c) preprocess: apply filters as in Chang, Christoffersen, Jacobs (2013) (see document Stefano)

(d) only consider OTM contracts
(e) discard call with moneyness < 0.97, put with moneyness > 1.03

2. construct continuous grid

   (a) for each day and each maturity: interpolate implied volatilities over moneyness domain, using cubic spline (use only days with at least 2 OTM calls and 2 OTM puts)
   (b) outside of observed moneyness domain: extrapolate
   (c) obtain: 1,000 IV between moneyness of 0.01% and 300%.

3. construct moments

   (a) map IV to call and put prices: call for moneyness > 1, put for moneyness < 1
   (b) approxmiate integrals: Lobatto quadrature
   (c) linear interpolation for corresponding moments

Other things to consider

- need comparable numbers of OTM puts and calls in longer-horizon maturities

How to construct IV?

- threshold: $\kappa = 0$, process to risk-free: $b = F_t exp(\kappa)$
- model-free corridor risk-neutral volatility as in Andersen, Bondarenko, Gonzalez-Perez (2015), Andersen and Bondarenko (2007), Carr, Madan (1999)

# 3 What are the acutal steps and computations?

**P-Expectation**

$\mathbb{E}_t^{\mathbb{P}}\left[RV_{t,h}^{U/D}(\kappa)\right]$: compute the sum as in equation 2.

see document: Realized Variance for cleaning and other steps.

**Q-Expectation**

$\mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,h}^{U/D}(\kappa)\right]$. try to find a way to compute the integral from equation 4.

1. preprocess the data (step 1 above/Stefanos document)
2. compute the integral above
3. interpolate and extrapolate (step 2 above)

# 4 Remaining Questions

(all references of sections and step refer to this document)

1. section 2, step 2: So this they do after they constructed the IV? Sounds to me as if they skip the most important step.
2. section 2, step 3: Why to they map IV to put and call prices, what integrals do they compute? If they already have the IV, there are no more integrals to compute, or?
3. section 3, step 2: How will we compute the integrals? They mention Lobatto quadrature, Nikola refered us to the computation of the VIX.