1 Regression

The following section aims to present the in-sample regression analysis examining the explanatory power of the variance risk premium (total, upside and downside) and the realized skewness for future excess returns. Following the original paper we aggregate the explanatory variables to the horizons h = 1,2,3,6,9,12 and the excess returns to the horizons k = 1,2,3,6,9,12. We implement two types of models, one regression including one variable at a time, and a second regression comparing for each the variance risk premium, the implied volatility and the realized volatility the respective upside and downside measure. The models then read

$$r^e_{t \to t+k} = \beta_0 + \beta_1 x_t(h) + \epsilon_{t \to t+k} \tag{1}$$

$$r^e_{t \to t+k} = \beta_0 + \beta_1 x^U_t(h) + \beta_2 x^D_t(h) + \epsilon_{t \to t+k} \tag{2}$$

where $r^e_{t \to t+k}$ is the cumulative excess return between time t and $t+k$, $x_t(h)$, is one of the predictors (variance risk premium total, variance risk premium upside, variance risk premium downside, realized skewness) and $x^U_t(h)$ and $x^D_t(h)$ distinguish between the upside and downside measures. As the regressions are replicated using using a rolling window the standard errors are based on a autocorrelation and heteroscedasticity robust covariance matrix. We evaluate our models using p-statistics and adjusted R-squareds.

The reference paper focuses the empirical analysis on two main points, showing that (i) using the aggregated VRP yields misleading results, because VRP^U and VRP^D have intrinsically different features and the VRP is mainly driven by VRP^D and (ii) predictability of equity returns is mainly driven by the risk-neutral expectation as opposed to the physical probability measure. For the single variable regressions the authors observe the highest explanatory power for the downside realized variance, whereas upside realized variance results are considerably weaker. They also observe the highest adjusted R-squared for a prediction horizon of k=3. For the two variable regressions the authors observe that together with the downside variance risk premium the upside variance risk premium strength is gradually lost, and that the predictability is driven stronger by the implied volatilities than the realized volatilities.

Our regression results for model 1 are presented in table 1 and our regression results for model 2 are presented in table 2.

For the single variable regression we find similar patterns as described above, however with weaker distinctive power. We find the highest explanatory power for the total and downside realized variance, whereas the upside realized variance has overall weaker results. Concerning the prediction horizon there is a weak hump to be seek at k=3. However, some results do not fit in the pattern, for example we observe high explanatory power for upside realized variance at high aggregation horizons h.

For the two variable regressions there is no clear pattern to be found when comparing the upside and downside realized variance. However we can confirm, that the predictive results are driven more by the risk-neutral measures than the realized measures.

Overall, we are only partly able to confirm our reference papers results. As mentioned previously we have a data availability different to that from the authors and not all of their data treatment was without questions for us. Hence this is a very likely reason why our regression results differ.

Table 1: regressions using one variable at a time

h	1		2		3		6		9		12	
	p-val	R^2	p-val	R^2	p-val	R^2	p-val	R^2	p-val	R^2	p-val	R^2
k	Panel A: Realized total variance											
1	0.0	0.035	0.026	0.006	0.0	0.033	0.682	-0.0	0.068	0.003	0.039	0.003
2	0.0	0.013	0.0	0.033	0.0	0.074	0.091	0.005	0.0	0.01	0.046	0.002
3	0.0	0.023	0.0	0.048	0.0	0.033	0.0	0.023	0.0	0.026	0.005	0.005
6	0.0	0.015	0.001	0.007	0.002	0.005	0.001	0.011	0.032	0.003	0.408	-0.0
9	0.001	0.01	0.064	0.002	0.478	-0.0	0.003	0.009	0.088	0.002	0.109	0.002
12	0.014	0.005	0.332	0.0	0.502	0.0	0.004	0.008	0.044	0.002	0.044	0.006
k	Panel B: Realized downside variance											
1	0.001	0.013	0.622	-0.0	0.0	0.019	0.911	-0.0	0.691	-0.0	0.532	-0.0
2	0.02	0.004	0.0	0.02	0.0	0.067	0.705	-0.0	0.473	0.0	0.319	0.0
3	0.008	0.008	0.0	0.034	0.0	0.035	0.034	0.004	0.084	0.003	0.544	-0.0
6	0.0	0.023	0.0	0.025	0.0	0.034	0.039	0.004	0.002	0.009	0.006	0.007
9	0.0	0.031	0.0	0.034	0.0	0.031	0.001	0.01	0.0	0.017	0.334	0.002
12	0.0	0.026	0.0	0.031	0.0	0.036	0.0	0.013	0.0	0.017	0.782	-0.0
k	Panel C: Realized upside variance											
1	0.0	0.043	0.003	0.011	0.0	0.025	0.436	0.001	0.002	0.012	0.0	0.014
2	0.0	0.018	0.0	0.022	0.0	0.031	0.015	0.013	0.0	0.033	0.0	0.031
3	0.0	0.03	0.0	0.028	0.001	0.01	0.0	0.041	0.0	0.068	0.0	0.052
6	0.071	0.002	0.138	0.001	0.0	0.006	0.0	0.076	0.0	0.068	0.0	0.05
9	0.194	0.0	0.0	0.012	0.0	0.026	0.0	0.088	0.0	0.076	0.0	0.051
12	0.001	0.004	0.0	0.02	0.0	0.03	0.0	0.094	0.0	0.079	0.0	0.057
k	Panel D: Realized Skewness											
1	0.165	0.003	0.075	0.005	0.864	-0.0	0.316	0.001	0.04	0.006	0.166	0.002
2	0.167	0.002	0.987	-0.0	0.006	0.007	0.013	0.007	0.0	0.018	0.0	0.015
3	0.146	0.002	0.519	0.0	0.055	0.006	0.014	0.01	0.0	0.025	0.0	0.02
6	0.0	0.012	0.0	0.026	0.0	0.052	0.0	0.087	0.0	0.119	0.0	0.047
9	0.0	0.038	0.0	0.06	0.0	0.083	0.0	0.122	0.0	0.157	0.019	0.031
12	0.0	0.044	0.0	0.069	0.0	0.095	0.0	0.136	0.0	0.162	0.076	0.022

Notes: Regression results of the two variable model. The R^2s are adjusted R^2s, the standard errors based on a heteroscedasticity and autocorrelation robust covariance matrix (HAC) to account for the overlap in the regression.

Table 2: regressions comparing upside and downside

h	1			2			3			6			9			12		
	p-val		R^2	p-val		R^2	p-val		R^2	p-val		R^2	p-val		R^2	p-val		R^2
	up	down		up	down		up	down		up	down		up	down		up	down	
k	Panel A: Variance Risk Premium																	
1	0.0	0.238	0.044	0.005	0.76	0.011	0.001	0.006	0.033	0.353	0.503	0.001	0.002	0.124	0.015	0.0	0.208	0.016
2	0.001	0.604	0.017	0.0	0.0	0.032	0.0	0.0	0.078	0.009	0.172	0.014	0.0	0.005	0.043	0.0	0.0	0.044
3	0.0	0.408	0.03	0.0	0.0	0.048	0.191	0.0	0.037	0.0	0.662	0.041	0.0	0.01	0.079	0.0	0.0	0.07
6	0.633	0.0	0.023	0.001	0.0	0.031	0.0	0.0	0.054	0.0	0.0	0.114	0.0	0.0	0.156	0.0	0.0	0.094
9	0.0	0.0	0.041	0.0	0.0	0.061	0.0	0.0	0.082	0.0	0.0	0.148	0.0	0.0	0.196	0.0	0.016	0.079
12	0.0	0.0	0.044	0.0	0.0	0.069	0.0	0.0	0.094	0.0	0.0	0.163	0.0	0.0	0.203	0.0	0.076	0.076
k	Panel B: Risk-neutral measures																	
1	0.776	0.998	0.002	0.245	0.318	0.003	0.029	0.035	0.01	0.0	0.0	0.031	0.0	0.0	0.041	0.003	0.002	0.015
2	0.001	0.001	0.018	0.0	0.0	0.038	0.0	0.0	0.061	0.0	0.0	0.095	0.0	0.0	0.102	0.0	0.0	0.047
3	0.0	0.0	0.034	0.0	0.0	0.064	0.0	0.0	0.095	0.0	0.0	0.151	0.0	0.0	0.16	0.0	0.0	0.069
6	0.0	0.0	0.056	0.0	0.0	0.127	0.0	0.0	0.187	0.0	0.0	0.268	0.0	0.0	0.288	0.003	0.0	0.085
9	0.0	0.0	0.091	0.0	0.0	0.175	0.0	0.0	0.242	0.0	0.0	0.329	0.0	0.0	0.356	0.059	0.006	0.069
12	0.0	0.0	0.099	0.0	0.0	0.186	0.0	0.0	0.259	0.0	0.0	0.365	0.0	0.0	0.399	0.07	0.01	0.075
k	Panel A: Realized (physical) measures																	
1	0.012	0.103	0.026	0.0	0.0	0.024	0.0	0.001	0.024	0.0	0.0	0.014	0.015	0.008	0.006	0.015	0.006	0.006
2	0.0	0.001	0.026	0.0	0.0	0.033	0.006	0.039	0.02	0.0	0.0	0.033	0.0	0.0	0.017	0.006	0.001	0.013
3	0.0	0.002	0.034	0.001	0.029	0.024	0.103	0.33	0.011	0.0	0.0	0.028	0.0	0.0	0.029	0.02	0.003	0.017
6	0.019	0.004	0.006	0.0	0.0	0.021	0.0	0.0	0.021	0.0	0.0	0.065	0.0	0.0	0.055	0.007	0.0	0.041
9	0.857	0.28	0.007	0.283	0.049	0.018	0.095	0.008	0.028	0.003	0.0	0.057	0.06	0.0	0.052	0.483	0.028	0.042
12	0.869	0.272	0.014	0.721	0.162	0.022	0.669	0.121	0.026	0.294	0.008	0.053	0.306	0.012	0.052	0.602	0.29	0.053

Notes: Regression results of the two variable model. The R^2s are adjusted R^2s, the standard errors based on a heteroscedasticity and autocorrelation robust covariance matrix (HAC) to account for the overlap in the regression.