University of Zurich

Department of Banking and Finance

A VARIANCE CHAOS

Seminar Paper

in

MSc Quantitative Finance

Student Name:	Silvia Forcina Barrero, Sophia Glaeser, Stefano Nicoli
Student number:	19-756-204,19-762-046, 19-762-384
Programme:	MSc Quantitative Finance
Semester:	Spring Term 2020
Examinor:	Prof. Dr. Markus Leippold
Supervisor:	Dr. Nikola Vasiljevix
Date:	2020-03-02

Contents

List of Tables

1 Introduction

The purpose of our project is to replicate and extend the analysis of the 'Downside Variance Risk Premium' study by *B. Fenou, M.R. Jahan-Parvar and C. Okou [2015]*. To this end, more recent data has been added and various methodologies have been implemented.
The rest of the project is structured as follows. In Section 2 we present an overview of 'Downside Variance Risk Premium' *[Fenou 2015]*. In Section 3 we present our construction methods for the various measures. In particular, we present the decomposition of the VRP and the construction of risk-neutral and realized semi-variances. Section 4 entails the data used in the analysis and the respective cleaning procedures. The main findings are presented in Section 5. In Section 6, we discuss the results and finally, Section 7 concludes.

2 Overview of 'Downside Variance Risk Premium'

In 2015, *B. Fenou, M.R. Jahan-Parvar and C. Okou* published 'Downside Variance Risk Premium' where they propose a new decomposition of the variance risk premium in terms of upside and downside variance risk premia (VRP^D and VRP^U respectively) and define their difference as skewness risk premium (SRP). Furthermore they evaluate the effectiveness of these measures as equity market returns predictors and find that VRP^D is the main driver of the variance risk premium.

The main advantage of this approach is that it relies on two facts that have been proven useful in predicting equity returns: distinguishing between positive and negative returns and including option-implied measures in the analysis.

The study is in line with the current asset pricing research that accepts the long term predictability of equity market returns in contrast to the traditional efficient-market hypothesis of market returns unpredictability. Furthermore, the paper confirms the results of *Bollerslev et al. [2009] (henceforth referred to as BTZ)* that suggest the presence of short term equity returns predictors, such as the variance risk premium (VRP), defined as the difference between option-implied and realized variance. It is in fact found that the VRP yields superior forecasts for stock market returns over shorter, within-year horizons (typically one quarter ahead). The authors decided to extend this finding and explore the impact of VRP^D and VRP^U by drawing on the intuition that investors favour good uncertainty over bad uncertainty, as the former increases the potential of substantial gains while the latter increases the likelihood of severe losses.

The principal finding of the study is that the VRP^D is the main component of the variance risk premium, making VRP^U's contribution only marginal. Moreover it is found that the skewness risk premia is a priced factor with significant prediction power for aggregate excess returns. In fact, a positive and significant relation is found between the VRP^D and the equity premium, as well as between the skewness risk premia and the equity premium. On average over 80% of the VRP is compensation for bearing changes in downside risk. This result is similar to the ones obtained by *Kozhan et al. [2014]*. Hence also the empirical regularities found in the VRP such as the hump-shaped R^2 and slope parameter patterns are explained by its downside component.

The empirical investigation first confirms that skewness, measured as the difference between upside and downside variances, is a priced factor and provides new evidence that the SRP, measured as the difference between the risk neutral and historical expectations of skewness, is both priced and has superior predictive power. The empirical investigation further highlights the fact that the SRP fills the time gap between the traditional long term predictors of excess returns, such as price-dividend or price-earning ratios, and the short term VRP predictor. There is a contribution of the SRP to the predictability of returns which takes effect beyond the one-quarter-ahead window documented by BTZ. Therefore, the model proposed in the paper closes the horizon gap between short term models such as BTZ and long-horizon predictive models such as *Fama and French [1988], Campbell and Shiller [1988], Cochrane [1991], and Lettau and Ludvigson [2001].*

Furthermore, it is shown that the prediction power of VRP^D and SRP increases over the term structure of equity returns. This result is robust to the inclusion of a wide variety of common pricing factors, resulting in the independence of the in-sample predictability of aggregate returns by VRP^D and SRP from the other commonly used pricing ratios (eg. price-dividend ratio, price-earning ratio, or default spread). In order to address data-mining concerns, out-of-sample forecasting exercises are conducted to establish that these predictive variables perform at least as well as the other common pricing variables in forecasting excess returns. The out-of-sample forecast ability comparison then confirms that other common predictors do not have a superior forecast ability in comparison with VRP^D and SRP.

The study furthermore tackles the links that macroeconomic announcements and events share with the VRP^D and SRP, as seen in *Amengual and Xiu [2014].* The correlations between VRP components and 124 macroeconomic and financial indicators are documented in a robustness study comparable to *Ludvigson et al. [2009]* and *Feunou et al. [2014].* A much lower correlation of the considered macroeconomic and financial indicators is found with VRP^D and SRP; compared to the one found with the VRP, as well as with the VRP^U.

In addition, the study is also relevant to the recent macro-finance literature which emphasizes the importance of higher-order risk attitudes in the determination of equilibrium asset prices. In particular, among these higher-order risk attitudes, we find prudence, which is a precautionary behavior which characterizes the aversion towards downside risk. For example, *Dionne et al. [2014]* show that consumption second-degree expectation dependence risk is a fundamental source of the macroeconomic risk driving asset prices through the use of a standard consumption-based capital asset pricing model . This consumption second-degree expectation dependence risk is a proxy for downside risk, that as already mentioned accounts for nearly 80% of the equity premium.

Finally, the authors *B. Fenou, M.R. Jahan-Parvar and C. Okou* support theoretically their empirical findings through the construction of an equilibrium consumption-based asset pricing model, where the representative agent is endowed with *Epstein and Zin [1989]* preferences, and where the consumption growth process is affected by distinct upside and downside shocks. It is shown that under common distributional assumptions for shocks to the economy, the equity risk premium, upside and downside variance risk premium and skewness risk premium can be derives in closed form.

It is important to notice that the model presented by the authors is not an alternative to jump-tail risk concerns or rare disaster models. The developed framework has as objective to address the asymmetries that are observed in 'normal times', however, it is also well-suited and capable of addressing regularities that emerge from a rare disaster happening, such as the Great Recession of 2007-2009.

2.1 Construction of VRP and SRP

In this section we will briefly report the construction methodology for VRP and SRP used in *Fenou et al. [2015]* . VRP^D and SRP are two natural and linked components in the framework developed by *B. Fenou, M.R. Jahan-Parvar and C. Okou* and used as short-medium term predictors of the equity market returns.

To study the VRP^D, nonparametric measures of up and downside realized and risk-neutral semi-variances are constructed by drawing on the vast existing literature of realized and risk-neutral volatility. For this purpose, the downside and upside variance are defined as the realized variance of the negative and positive market returns respectively. It follows that the downside and upside variance risk premium (VRP^D and VRP^U) are defined as the difference between option-implied and realized downside and upside variance extracted from high frequency data. As in *Chang, Christoffersen, and Jacobs [2013]*, the construction approach avoids the traditional trade-off problem with estimates of higher moments from historical returns data needing long windows to increase precision but short windows to obtain conditional instead of unconditional estimates.

Moreover, a nonparametric measure of skewness is constructed using the difference between upside and downside variances, also called the relative upside variance. The difference between option-implied and realized relative upside variances is then considered a measure of skewness risk premium (SRP). The SRP, as already mentioned, is a priced factor hat enhances the predictive power of the VRP to horizons beyond one quarter ahead, filling the gap between the VRP and common long-term equity returns predictors.

Thus, in order to construct VRP and SRP, reliable measures for realized and risk-neutral variance and skewness are needed. The construction of realized downside and upside volatilities (also known as realized semi-variances) is addressed in *Barndorff-Nielsen [2010]* and the consensus in the literature about construction of these measures is followed. Similarly, risk-neutral measures of volatility are constructed following the existing literature such as in the studies of *Carr and Madan [1998, 1999, 2001]* and *Bakshi et al.[2003].* The construction of option-implied downside and upside volatilities is addressed in *Andersen et al. [2014].* On the other hand, traditional measures of skewness have well-documented empirical problems *Kim et al [2004]* such as the excessive sensitivity to outliers. These problems are overcome by using alternative and more robust measures such as by using Pearson and Bowley's skewness measures *Feunou et al. [2013] and Ghysels et al. [2011]* or imposing autoregressive structures to explore time variation in conditional skewness *Harvey and Siddique [1999, 2000].*

3 Methodology

In this section we describe the methodology followed to construct the relevant measures used as predictors for markets excess returns. For what concerns the construction of the relevant variables for the analysis, we have closely followed *Fenou's [2015]* approach, which draws on the vast existing literature of realized and risk-neutral volatility. For what concerns the models, in addition to the replication of the models proposed by *Fenou [2015]*, we implemented new models that would potentially allow us to gain new insights. A more detailed description is given below.

First, we decompose equity price changes into positive and negative log-returns with respect to an appropriately chosen threshold. Given the nature of the analysis we set this threshold to zero, however its value can be changed to better suit other types of analysis. In particular, for κ suitably chosen it is possible to investigate the tail behavior of returns.

3.1 Excess Returns

We denote the total equity returns at time t as $R_t^e = \frac{S_t + D_t - S_{t-1}}{S_{t-1}}$ where D_t is the dividend paid out in the following period. However, we can consider D_t equal to zero for high enough sampling frequencies. The log of prices is denoted bt $s_t = ln(S_t)$ and consequently log-returns are defined as $r_t = s_t - s_{t-1}$ and excess log returns as $r_t^e = r_t - r_t^f$, where r_t^f indicates the risk-free rate observed at $t-1$. Finally, cumulative excess returns are obtained by summing one-period excess returns, $r_{t \mapsto t+k} = \sum_{j=0}^{k} r_{t+j}^e$ where k is the prediction horizon. Once the positive returns have been separated from the negative, we can proceed to construct our non-parametric measures of upside and downside variances, and skewness.

3.2 Construction of the VRP

Following *Fenou [2015]*, we construct the variance risk premium as the difference between option-implied (IV) and realized variances (RV). These two components are the variances under the risk-neutral and physical measures respectively. Hence we need to build the upside and downside variances both under the physical and risk-neutral measure.

We construct the total realized variance of returns for a given trading day t as:

$$RV_t = \sum_{j=1}^{n_t} r_{j,t}^2 \tag{1}$$

where $r_{j,t}^2$ is the j^{th} intraday squared log return on day t and n_t is the number of intraday returns observed that day. Next, we add to the total realized variance, the squared overnight log-return, that is, the difference in log price between when the market opens at t and when it closes at $t-1$. We assume that the market opens at 9:30 AM and closes at 16:00 PM.

$$r_{overnight} = log(p_{open,t}) - log(p_{close,t-1}) \tag{2}$$

Once we obtain the total realized variance series, we ensure that the sample average realized variance equals the sample variance of daily log returns through scaling.

Following *Barndorff-Nielsen et al. [2010]* we decompose the realized variance into upside and downside realized variances:

$$RV_t^U(\kappa) = \sum_{j=1}^{n_t} r_{j,t}^2 \mathbb{I}_{[r_{j,t}>\kappa]} \tag{3}$$

$$RV_t^D(\kappa) = \sum_{j=1}^{n_t} r_{j,t}^2 \mathbb{I}_{[r_{j,t}\leq\kappa]} \tag{4}$$

where κ is the aforementioned threshold set equal to zero.

Once again, we add the squared overnight returns to the upside and downside realized variances. In particular, we add the log-return to the upside realized variance if this is 'positive', that is, if it exceeds the threshold, and to the downside realized variance if it is 'negative'. The sum of these two measures corresponds to the daily total realized variance, and hence it is necessary to apply the same scaling to both components. In particular, we multiply both components by the ratio of the sample variance of daily log-returns over the sample average of the (pre-scaled) realized variance.

$$RV_t = \tilde{RV}_t \frac{1}{\sigma_r} \tag{5}$$

$$RV_t^{U/D} = \tilde{RV}_t^{U/D} \frac{\sigma_r}{\mu_{RV}} \tag{6}$$

where σ_r is the sample variance of the daily log returns, μ_{RV} is sample average of the (pre-scaled) realized variance, $\tilde{RV}_t^{U/D}$ is the unscaled realized variance.

For a given horizon h, the cumulative realized quantities are obtained by summing the one-day realized quantities over h non-overlapping periods:

$$RV_{t,h}(\kappa) = \sum_{j=1}^{h} RV_{t+j}(\kappa) \tag{7}$$

$$RV_{t,h}^{U}(\kappa) = \sum_{j=1}^{h} RV_{t+j}^{U}(\kappa) \tag{8}$$

$$RV_{t,h}^{D}(\kappa) = \sum_{j=1}^{h} RV_{t+j}^{D}(\kappa) \tag{9}$$

we use $h = 1, 2, 3, 6, 9, 12$ months.

By construction, the sum of the cumulative realized semivariances corresponds to the cumulative realized total variance:

$$RV_{t,h}(\kappa) := RV_{t,h}^{U}(\kappa) + RV_{t,h}^{D}(\kappa) \tag{10}$$

Finally, we can define the variance risk premia (VRP) as the premia resulting from bearing upside and downside variance risks:

$$VRP_{t,h} = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}] - \mathbb{E}_t^{\mathbb{P}}[RV_{t,h}] = (\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^{U}] - \mathbb{E}_t^{\mathbb{P}}[RV_{t,h}^{U}]) + (\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^{D}] - \mathbb{E}_t^{\mathbb{P}}[RV_{t,h}^{D}]) \tag{11}$$

Hence we can decompose the VRP in its respective upside and downside counterparts:

$$VRP_{t,h} := VRP_{t,h}^{U}(k) + VRP_{t,h}^{D}(k) \tag{12}$$

3.2.1 Construction of the P-Expectation

In order to evaluate equation (11) we need to define $\mathbb{E}_t^{\mathbb{P}}[RV_{t,h}^U]$ and $\mathbb{E}_t^{\mathbb{P}}[RV_{t,h}^D]$. To this end, we characterise the P-expectation through a Random Walk specification:

$$\mathbb{E}_t^{\mathbb{P}}[RV_{t,h}^{U/D}(\kappa)] = RV_{t-h,h}^{U/D}(\kappa) \tag{13}$$

Hence we estimate the P-Expectation for period h as the realized variance observed in the the past period.

3.2.2 Construction of the Q-Expectation

The risk-neutral expectation of $RV_{t,h}$ is built following the methodology of *Andersen and Bondarenko [2007]*. We denote these model-free risk-neutral semivariances as implied semivariances or $IV_{t,h}^{U/D}$.

We define the implied semivariances as:

$$IV_{t,h}^U = \mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,h}^U(\kappa)\right] \approx 2\int_{F_t(h)exp(\kappa_F)}^{\infty} \frac{M_0(k)}{k^2}dk \tag{14}$$

$$IV_{t,h}^D = \mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,h}^D(\kappa)\right] \approx 2\int_0^{F_t(h)exp(\kappa_F)} \frac{M_0(k)}{k^2}dk \tag{15}$$

with:

$$M_0(k) = min(P_t(k,h), C_t(k,h)), \text{ where} \tag{16}$$

$P_t(k,h)$ and $C_t(k,h)$ are put and call option prices respectively with strike price k and time-to-maturity h. $F_t(h)$ is the forward price at time t and tenor h. κ_F is defined as $\kappa_F := (\kappa - r_t^f)h$. In particular, here we assume $\kappa = 0$. We observe that we assume that κ here is equal to zero.

Even though this equation gives us a direction to follow to compute the implied semivariances, we have to handle the fact that their precise calculation requires, for every target maturity h, a fine grid (with respect to the strike price) of OTM option prices. However, in order to obtain this fine grid, a thorough process of data cleaning is necessary. In section 3 we present the preliminary data cleaning treatments applied, while in this section we present the methodology used in the more thorough part of the data cleaning process.

The construction of the implied semivariances relies on the following steps:

Dividend Issue and Implied Forward Since the computation of the BS call and put prices involves the dividend yield, which is hard to properly estimate, we decided to infer forward prices from the forward put-call parity and to apply the BS formula in terms of the forward instead of spot and dividend:

$$C_t(K,h,\sigma) = e^{-\int_t^{t+h} r_u du}(F_t(h)\mathcal{N}(d_1) - K\mathcal{N}(d_2)) \tag{17}$$

$$d_1 = \frac{\ln(\frac{F_t(h)}{K}) + \frac{\sigma^2}{2}h}{\sigma\sqrt{h}} \text{ and } d_2 = d_1 - \sigma\sqrt{h}, \tag{18}$$

where $\mathcal{N}$ is the cdf of the standard normal distribution.

In order to get the implied forwards, we recall the forward put-call parity:

$$C_t(K,h) + Ke^{-\int_t^{t+h} r_u du} = P_t(K,h) + F_t(h)e^{-\int_t^{t+h} r_u du}. \tag{19}$$

To calculate with as much accuracy as possible the forward rate from this equation, we have to care mainly for two problems:

1. We need a pair of call and put options with the same strike and the same maturity;

2. We need that both prices of this pair are reliable. Therefore, we need that both the put and the call are not ITM (or at least not deep ITM). The only way to solve this issue is to find a pair $(C_t(K,h), P_t(K,h))$ 'as much ATM as possible'. Hence, it satisfies $K \approx F_t(h)$.

The only reason why we downloaded the dividends is because for some pair (K,h) it happens that a couple as in 1. does not exist, so we have to compute the forward with spot and dividends. In order to address 2., once we gathered all the puts and the calls with the same strike and time-to-maturity, we took into consideration the pair $(\hat{C}_t(K,h), \hat{P}_t(K,h))$ that minimizes $|C_t(K,h) - P_t(K,h)|$. So, for every pair (K,h) we computed the implied forward as:

$$F_t(h) = K + e^{\int_t^{t+h} r_u du}(\hat{C}_t(K,h) - \hat{P}_t(K,h))$$

Now that we have used the ITM options for the computation of the implied forwards, we can continue our cleaning process deleting:

- call options with forward-moneyness below 1;
- put options with forward-moneyness above 1.

Furthermore, as suggested by *Chang et al.* we decided not to take into consideration the days without at least two OTM calls and two OTM puts. Fortunately, this is not the case in our time period.

Having dealt with the dividend issue, we finally have everything necessary to compute the implied volatility, using the BS formula. In particular, we compute it through the MatLab function blsimpv.

Creation of a fine arbibtrage-free grid

The most subtle step to have enough data to compute the implied semi-variances is the following one. For every single day and for every time-to-maturity present in the filtered options of that day, we want to create a fine grid with respect to the forward-moneyness $\frac{K}{F_t(h)}$ such that no static arbitrage opportunity arises from it. To do this, we followed the algorithm proposed by *Fengler [2009]*, adapting to our purpose the *MatLab*[1] code.

Even though we do not want to describe with details how the algorithm works, we want to stress that, differently from *Feunou et al. [2015]*, we did not extrapolate the implied volatility

[1] we thank the authors who made their code available on `https://github.com/marta-riva/duoGozzo/tree/master/IVS_Code`

for out-of-range levels of moneyness. Instead, for every day and every time-to-maturity, we created a grid that ranges from the minimum to the maximum level of moneyness given on that day, in light of *Andersen et. al [2015]*. We will show the reasoning and consequences of this choice through the following equations.

IV Computation

We consider:

$$\begin{aligned} IV^U_{t,h}\_approx &:= \frac{2}{F_t(h)} \int_{exp(-r^f_t h)}^{max\_fwd\_mon} \frac{M_0(Mon_t(h)F_t(h))}{Mon_t(h)^2} dMon_t(h) \\ &\leq \frac{2}{F_t(h)} \int_{exp(-r^f_t h)}^{\infty} \frac{M_0(Mon_t(h)F_t(h))}{Mon_t(h)^2} dMon_t(h) = IV^U_{t,h} \end{aligned} \tag{20}$$

$$\begin{aligned} IV^D_{t,h}\_approx &:= \frac{2}{F_t(h)} \int_{min\_fwd\_mon}^{exp(-r^f_t h)} \frac{M_0(Mon_t(h)F_t(h))}{Mon_t(h)^2} dMon_t(h) \\ &\leq \frac{2}{F_t(h)} \int_{0}^{exp(-r^f_t h)} \frac{M_0(Mon_t(h)F_t(h))}{Mon_t(h)^2} dMon_t(h) = IV^D_{t,h} \end{aligned} \tag{21}$$

where max_fwd_mon and min_fwd_mon are the maximum and minimum moneyness level present in the filtered options on that day, respectively. Without explaining step-by-step the representation of the implied semi-variances with respect to the forward moneyness $Mon_t(h) := \frac{K}{F_t(h)}$, we specify that we changed the variable of the integral in order to use our grid.

An immediate drawback of using $IV^U_{t,h}\_approx$ and $IV^D_{t,h}\_approx$ instead of $IV^U_{t,h}$ and $IV^D_{t,h}$, respectively, is that we are ignoring the tails of the integrals. On the other hand, a benefit of this method is that we avoid the extrapolation procedure, which would lead to not precise estimates of the prices with moneyness close to zero, risking to overestimate the value of that tail. Indeed, looking at $IV^D_{t,h}$, if the computation of the OTM put options is not of high quality, the risk is that the denominator $Mon_t(h)^2$ gives to the integral too high and not reliable values. Moreover, since we are not computing the implied semi-variances to do option pricing or other purposes that require an estimation as much precise as possible, but to see whether upside and downside variance risk premium can predict equity excess returns, we believe that underestimating both these variances does not compromise the predictive power of them.

The last step of the computation consists in interpolating and extrapolating the implied semi-variances to get $IV^{U}_{t,h}\_approx$ and $IV^{D}_{t,h}\_approx$ with h=1,2,3,6,9,12 months. The extrapolation is a necessary step since we want to compute these values for h=12 months, but in the cleaning procedure, we discarded all the options with time-to-maturity above one year.

As a last observation, we want to stress that, as stated by *Jiang et al [2005]*: "it (the BS model) is merely used as a tool to provide a one-to-one mapping between option prices and implied volatilities". Hence, we can claim that this computation of the implied semi-variances is model-free.

3.3 Construction of the SRP

It has been demonstrated that the difference between upside and downside variances (standardized by total variance) satisfies the criteria of a measure of skewness *[Groeneveld and Meeden 1984] [Feunou et al. 2014]*. That is, it is invariant to affine transformations of a random variable, is an odd function of a random variable, and assumes zero value for a symmetrically distributed random variable. This decomposition of the skewness its particularly useful as it relies only on the calculation of the second moment and can therefore be computed in instances with undefined third moments.

We build this nonparametric measure of skewness ($RSV_{t,h}$) by subtracting downside variance from upside semi-variance:

$$RSV_{t,h}(\kappa) = RV^{U}_{t,h}(\kappa) - RV^{D}_{t,h}(\kappa) \tag{22}$$

Thus we obtain a left-skewed distribution when $RSV_{t,h}(\kappa) < 0$ and a right-skewed distribution when $RSV_{t,h}(\kappa) > 0$.

Similarly to the variance risk premium (VRP), we can characterise a nonparametric and model-free notion of skewness risk premium ($SRP_{t,h}$), as the difference between risk neutral and objective expectations of the realized skewness, or equivalently the difference between the two components of the VRP.

$$SRP_{t,h} = \mathbb{E}_t^{\mathbb{Q}}[RSV_{t,h}] - \mathbb{E}_t^{\mathbb{P}}[RSV_{t,h}] = VRP_{t,h}^{U}(k) - VRP_{t,h}^{D}(k) \tag{23}$$

$SRP_{t,h}$ has a twofold interpretation, it can be viewed as a skewness premium when $RSV_{t,h}(\kappa) < 0$ and as skewness discount when $RSV_{t,h}(\kappa) > 0$. The former indicates the compensation for an agent who bears downside risk and the latter the amount that the agent is willing to pay to secure a positive return on an investment.

4 Data

By effectively isolating the systematic risk associated with the volatility-of-volatility, it can be shown that the difference between the current returns variation (approximated by RV) and the markets risk-neutral expectation of future returns variation (approximated by IV) is a useful predictor of the future returns *[BTZ 2009]*. Moreover, as shown above, we can construct the additional predictor SRP as a byproduct of these measures. We thus need reliable raw data to build the $RV^{U/D}$ and $IV^{U/D}$. The data used in this study is available through the Wharton Database. Specifically, we use the 3-month Treasury Bill rate ad our risk-free rate, the S&P 500 composite index as a proxy for the aggregate market portfolio, 5-minute intraday S&P 500 data and option data.

4.1 Excess Returns

In order to document the prediction power of $VRP^{U/D}$ and SRP for monthly excess equity market returns, we use the S&P 500 composite index as a proxy for the aggregate market portfolio. Next, we construct the excess returns by subtracting 3-month T-Bill rates from the annualised log-returns of the S&P 500 composite index.

$$r_t^e = ln(1 + R_t) \times 12 - ln(1 + R_t^f) \tag{24}$$

where $R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$ is the equity return and R_t^f is the risk-free rate.

Both the S&P 500 composite index and the 3-month T-Bill rate series have been gathered from the CRSP database and we focus our analysis on the 2007-2017 sample period.

4.2 RV: High Frequency Data

To build the daily $RV^{U/D}$s series, we use 5-minute intraday S&P 500 data and, once again, we focus on the 2007-2017 sample period, which yields a total of **insert number** daily observations. The data is available through the Wharton Database.

For what concerns the 2007-2008 period, we have gathered tick data from the TAQ database, and subsequently proceeded to delete any duplicate and zero entries to obtain more consistent data. Finally, to obtain 5-minute data and overcome the issue of irregularly spaced data points we have created 5-minute bins by using the median. We have preferred to create these 5-minutes bins with the median rather than the mean value, as the former is more robust to outliers. For what concerns the 2008-2017 period we have obtained minute-data from the Wharton database. For both datasets, we averaged the market's bid and ask quotes in order to obtain an average S&P 500 level. Finally, we have merged these two datasets to create an unique 5-minute intraday dataset comprising the whole period. We have dealt with missing values by recurring to interpolation.

We obtain the total realized variances by adding up the sum of the 5-minute squared log-returns and the daily squared overnight returns. We then decompose the total realized variances into their upside and dowside components with respect to the threshold κ. By construction we have that total realized variance is obtained by adding the downside (RV^D) and the upside realized variance (RV^U). The three series are scaled such that the average total realized variance series matches the unconditional variance of the S&P 500 returns.

4.3 IV: Options Data

All of the data used for this computation is taken by OptionMetrics. From 'Option Prices' we downloaded the SPX European call and put options from 2007 to 2017 included. For the same period, we got zero curves from 'Zero Coupon Yield Curve' and, finally, from 'Index Dividend Yield' we downloaded the continuously compounded S&P500 dividend yield.

Since not all the options give us reliable information about the market, we decided to apply some preliminary standard filtering procedures to delete illiquid options. In particular:

- we computed the mid price as the average between bid price and ask price. Then we deleted the options with mid price below 3/8$;

- we deleted all the options with too long (>1 year) or too short (<7 days) time-to-maturity;
- we deleted all the options that have not been traded for more than three days in a row.

Since the tenors of the zero curves do not match the maturities of the options, we interpolated linearly the continuously compounded zero-coupon rates to solve this issue.

4.4 Summary Statistics

In the following we compare our data to *Fenou's [2015]* data through the use of the summary statistics presented by the original paper. Before going into detail we would like to mention that our analysis presents several limitations with respect to the dataset, reflected in the summary statistics. Firstly we did not have the same time period of data available, limiting our analysis to the years 2007 to 2017 compared to 1996 to 2015. Secondly, the daily variables are aggregated to periods between one month and one year for the models and unfortunately we were uncertain which aggregation period was presented in the reported summary statistics. Thirdly, we assumed the aggregation periods were constructed overlapping but can not be certain of this. Hence were not able to replicate the exact summary statistics that the authors presented and can not assume that our replication dataset matches that of the original paper. As a consequence, we limit ourselves to comparing the variables in their relative size to one another.

The total variance risk premium and downside variance risk premium are on average positive over our sample period. The upside variance risk premium however is on average negative. This aligns both with the authors theoretical and empirical findings, assuming that investors dislike bad uncertainty and are hence willing to pay a premium to be hedged against variation in bad uncertainty and vice versa. The realized variance is on average positive over the sample period both for the upside and downside measure, as is the implied volatility. Excess returns were on average 0.46% per day.

Table 1 presents the summary statistics of the data used in our models.

Table 1: summary statistics

	VRP	VRP^U	VRP^D	RV^U	RV^D	IV^U	IV^D	excess return
mean	0.067227	-0.034161	0.101388	0.147523	0.168599	0.112790	0.268409	0.460595
std	0.250356	0.135721	0.165122	0.246959	0.321733	0.155481	0.347752	4.809556
min	-3.344685	-1.812280	-1.532405	0.008506	0.004881	0.006662	0.043216	-35.874178
max	1.911116	0.397544	1.513572	2.346593	3.166236	1.732368	3.522595	20.273505

Notes: This table reports the summary statistic for the variables used in our model. For representative purposes only the aggregation period of one months is displayed. The variables reported are in percentage and non-annualized.

The graph below shows the time series plot of the variance risk premia and its up and downside decomposition.

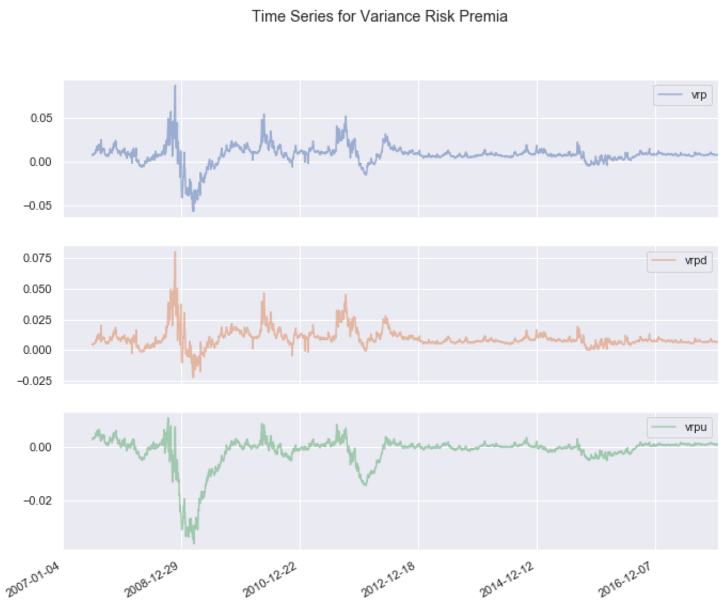

Figure 1: Time series plot of VRP, VRP^U and VRP^D.

5 Results

5.1 Regression

The following section aims to present the in-sample regression analysis examining the explanatory power of the variance risk premium (total, upside and downside) and the realized skewness for future excess returns. Following the original paper we aggregate the explanatory variables to the horizons h = 1,2,3,6,9,12 and the excess returns to the horizons k = 1,2,3,6,9,12. We implement two types of models, one regression including one variable at a time, and a second regression comparing for each the variance risk premium, the implied volatility and the realized volatility the respective upside and downside measure. The models then read

$$r^e_{t\to t+k} = \beta_0 + \beta_1 x_t(h) + \epsilon_{t\to t+k} \tag{25}$$

$$r^e_{t\to t+k} = \beta_0 + \beta_1 x^U_t(h) + \beta_2 x^D_t(h) + \epsilon_{t\to t+k} \tag{26}$$

where $r^e_{t\to t+k}$ is the cumulative excess return between time t and $t+k$, $x_t(h)$ is one of the predictors (total variance risk premium, upside variance risk premium upside, downside variance risk premium, realized skewness) and $x^U_t(h)$ and $x^D_t(h)$ distinguish between the upside and downside measures. As the regressions are replicated using a rolling window the standard errors are based on a autocorrelation and heteroscedasticity robust covariance matrix. We evaluate our models using p-statistics and adjusted R-squareds.

The reference paper focuses the empirical analysis on two main points, showing that (i) using the aggregated VRP yields misleading results, because VRP^U and VRP^D have intrinsically different features and the VRP is mainly driven by VRP^D and (ii) predictability of equity returns is mainly driven by the risk-neutral expectation as opposed to the physical probability measure. For the single variable regressions the authors observe the highest explanatory power for the downside realized variance, whereas upside realized variance results are considerably weaker. They also observe the highest adjusted R-squared for a prediction horizon of k=3. For the two variable regressions the authors observe that together with the downside variance risk premium the upside variance risk premium strength is gradually lost, and that the predictability is driven stronger by the implied volatilities than the realized volatilities.

Our regression results for model 25 are presented in table 2 and our regression results for model 26 are presented in table 3.

For the single variable regression we find similar patterns as described above, however with weaker distinctive power. We find the highest explanatory power for the total and downside realized variance, whereas the upside realized variance has overall weaker results. Concerning the prediction horizon there is a weak hump to be seek at k=3. However, some results do not fit in the pattern, for example we observe high explanatory power for upside realized variance at high aggregation horizons h.

For the two-variable regressions there is no clear pattern to be found when comparing the upside and downside realized variance. However we can confirm, that the predictive results are driven more by the risk-neutral measures than the realized measures.

Overall, we are only partly able to confirm our reference papers results. As mentioned previously we have a data availability different to that from the authors and not all of their data treatment was without questions for us. Hence this is a very likely reason why our regression results differ.

Table 2: regressions using one variable at a time

h	1		2		3		6		9		12	
	p-val	R^2	p-val	R^2	p-val	R^2	p-val	R^2	p-val	R^2	p-val	R^2
k	Panel A: Realized total variance											
1	0.0	0.035	0.026	0.006	0.0	0.033	0.682	-0.0	0.068	0.003	0.039	0.003
2	0.0	0.013	0.0	0.033	0.0	0.074	0.091	0.005	0.0	0.01	0.046	0.002
3	0.0	0.023	0.0	0.048	0.0	0.033	0.0	0.023	0.0	0.026	0.005	0.005
6	0.0	0.015	0.001	0.007	0.002	0.005	0.001	0.011	0.032	0.003	0.408	-0.0
9	0.001	0.01	0.064	0.002	0.478	-0.0	0.003	0.009	0.088	0.002	0.109	0.002
12	0.014	0.005	0.332	0.0	0.502	0.0	0.004	0.008	0.044	0.002	0.044	0.006
k	Panel B: Realized downside variance											
1	0.001	0.013	0.622	-0.0	0.0	0.019	0.911	-0.0	0.691	-0.0	0.532	-0.0
2	0.02	0.004	0.0	0.02	0.0	0.067	0.705	-0.0	0.473	0.0	0.319	0.0
3	0.008	0.008	0.0	0.034	0.0	0.035	0.034	0.004	0.084	0.003	0.544	-0.0
6	0.0	0.023	0.0	0.025	0.0	0.034	0.039	0.004	0.002	0.009	0.006	0.007
9	0.0	0.031	0.0	0.034	0.0	0.031	0.001	0.01	0.0	0.017	0.334	0.002
12	0.0	0.026	0.0	0.031	0.0	0.036	0.0	0.013	0.0	0.017	0.782	-0.0
k	Panel C: Realized upside variance											
1	0.0	0.043	0.003	0.011	0.0	0.025	0.436	0.001	0.002	0.012	0.0	0.014
2	0.0	0.018	0.0	0.022	0.0	0.031	0.015	0.013	0.0	0.033	0.0	0.031
3	0.0	0.03	0.0	0.028	0.001	0.01	0.0	0.041	0.0	0.068	0.0	0.052
6	0.071	0.002	0.138	0.001	0.0	0.006	0.0	0.076	0.0	0.068	0.0	0.05
9	0.194	0.0	0.0	0.012	0.0	0.026	0.0	0.088	0.0	0.076	0.0	0.051
12	0.001	0.004	0.0	0.02	0.0	0.03	0.0	0.094	0.0	0.079	0.0	0.057
k	Panel D: Realized Skewness											
1	0.165	0.003	0.075	0.005	0.864	-0.0	0.316	0.001	0.04	0.006	0.166	0.002
2	0.167	0.002	0.987	-0.0	0.006	0.007	0.013	0.007	0.0	0.018	0.0	0.015
3	0.146	0.002	0.519	0.0	0.055	0.006	0.014	0.01	0.0	0.025	0.0	0.02
6	0.0	0.012	0.0	0.026	0.0	0.052	0.0	0.087	0.0	0.119	0.0	0.047
9	0.0	0.038	0.0	0.06	0.0	0.083	0.0	0.122	0.0	0.157	0.019	0.031
12	0.0	0.044	0.0	0.069	0.0	0.095	0.0	0.136	0.0	0.162	0.076	0.022

Notes: Regression results of the two variable model. The R^2s are adjusted R^2s, the standard errors based on a heteroscedasticity and autocorrelation robust covariance matrix (HAC) to account for the overlap in the regression.

Table 3: regressions comparing upside and downside

h	1			2			3			6			9			12		
	p-val		R^2	p-val		R^2	p-val		R^2	p-val		R^2	p-val		R^2	p-val		R^2
	up	down		up	down		up	down		up	down		up	down		up	down	
k	Panel A: Variance Risk Premium																	
1	0.0	0.238	0.044	0.005	0.76	0.011	0.001	0.006	0.033	0.353	0.503	0.001	0.002	0.124	0.015	0.0	0.208	0.016
2	0.001	0.604	0.017	0.0	0.0	0.032	0.0	0.0	0.078	0.009	0.172	0.014	0.0	0.005	0.043	0.0	0.0	0.044
3	0.0	0.408	0.03	0.0	0.0	0.048	0.191	0.0	0.037	0.0	0.662	0.041	0.0	0.01	0.079	0.0	0.0	0.07
6	0.633	0.0	0.023	0.001	0.0	0.031	0.0	0.0	0.054	0.0	0.0	0.114	0.0	0.0	0.156	0.0	0.0	0.094
9	0.0	0.0	0.041	0.0	0.0	0.061	0.0	0.0	0.082	0.0	0.0	0.148	0.0	0.0	0.196	0.0	0.016	0.079
12	0.0	0.0	0.044	0.0	0.0	0.069	0.0	0.0	0.094	0.0	0.0	0.163	0.0	0.0	0.203	0.0	0.076	0.076
k	Panel B: Risk-neutral measures																	
1	0.776	0.998	0.002	0.245	0.318	0.003	0.029	0.035	0.01	0.0	0.0	0.031	0.0	0.0	0.041	0.003	0.002	0.015
2	0.001	0.001	0.018	0.0	0.0	0.038	0.0	0.0	0.061	0.0	0.0	0.095	0.0	0.0	0.102	0.0	0.0	0.047
3	0.0	0.0	0.034	0.0	0.0	0.064	0.0	0.0	0.095	0.0	0.0	0.151	0.0	0.0	0.16	0.0	0.0	0.069
6	0.0	0.0	0.056	0.0	0.0	0.127	0.0	0.0	0.187	0.0	0.0	0.268	0.0	0.0	0.288	0.003	0.0	0.085
9	0.0	0.0	0.091	0.0	0.0	0.175	0.0	0.0	0.242	0.0	0.0	0.329	0.0	0.0	0.356	0.059	0.006	0.069
12	0.0	0.0	0.099	0.0	0.0	0.186	0.0	0.0	0.259	0.0	0.0	0.365	0.0	0.0	0.399	0.07	0.01	0.075
k	Panel A: Realized (physical) measures																	
1	0.012	0.103	0.026	0.0	0.0	0.024	0.0	0.001	0.024	0.0	0.0	0.014	0.015	0.008	0.006	0.015	0.006	0.006
2	0.0	0.001	0.026	0.0	0.0	0.033	0.006	0.039	0.02	0.0	0.0	0.033	0.0	0.0	0.017	0.006	0.001	0.013
3	0.0	0.002	0.034	0.001	0.029	0.024	0.103	0.33	0.011	0.0	0.0	0.028	0.0	0.0	0.029	0.02	0.003	0.017
6	0.019	0.004	0.006	0.0	0.0	0.021	0.0	0.0	0.021	0.0	0.0	0.065	0.0	0.0	0.055	0.007	0.0	0.041
9	0.857	0.28	0.007	0.283	0.049	0.018	0.095	0.008	0.028	0.003	0.0	0.057	0.06	0.0	0.052	0.483	0.028	0.042
12	0.869	0.272	0.014	0.721	0.162	0.022	0.669	0.121	0.026	0.294	0.008	0.053	0.306	0.012	0.052	0.602	0.29	0.053

Notes: Regression results of the two variable model. The R^2s are adjusted R^2s, the standard errors based on a heteroscedasticity and autocorrelation robust covariance matrix (HAC) to account for the overlap in the regression.

5.2 Out of Sample Analysis

In this section we evaluate the forecast ability of downside variance risk premia and its upside and downside decomposition. In particular, we want to demonstrate that our in-sample univariate regressions do not lose predictive ability once they are used for forecasting purposes.

To this end, we follow the literature on predictive accuracy tests and perform recursive expanding window regressions, through the use of the R package *'rollRegres'*. To generate one-period out-of-sample predictions $y_{t+1|t}$ for y_{t+1}, we split the total sample of T observations into in-sample and out-of-sample portions. More precisely, we use half of the total sample for the initial in-sample estimation ($[\frac{T}{2}]$) and the second half for the initial forecast evaluation. We then proceed to estimate the regression coefficients recursively with the last in-sample observation ranging from $t = [\frac{T}{2}]$ to $t = T - 1$, and for each t we compute the one-step ahead forecast $t + 1$.

We evaluate the prediction accuracy of the models through the use of the root mean squared error (RMSE). We define the forecast errors as the difference between the observed values and their forecasting quantities:

$$e_{t+1|t} = y_{t+1} - y_{t+1|t} \tag{27}$$

and the RSME as:

$$RMSE = \sqrt{\sum_{i=1}^{T} \frac{1}{T} e_{t+1|t}^2} \tag{28}$$

In the following table we report the main results of our out of sample analysis, in particular, we report the RMSE and average R^2 of the out of sample regressions for different construction horizons ($h = 1, 2, 3, 6, 12$). The regressors considered are the variance risk premia and its upside and downside decomposition.

	Average R^2 (%)	RMSE (%)
h=1		
VRP	4.82	0.29
VRP^D	4.15	0.24
VRP^U	2.47	0.13
h=2		
VRP	5.49	0.31
VRP^D	6.33	0.24
VRP^U	1.50	0.11
h=3		
VRP	3.81	0.40
VRP^D	4.68	0.34
VRP^U	0.96	0.12
h=6		
VRP	2.66	0.82
VRP^D	2.99	0.77
VRP^U	0.98	0.15
h=12		
VRP	2.01	1.99
VRP^D	1.87	1.84
VRP^U	1.03	0.27

From the table above we can see that for all construction horizons the RMSE is low. We can therefore conclude that our regressions possess good predictive ability. On the other hand, the average R^2 of the regressions is extremely low. We can partly blame this on the inclusion of the Great Recession period in our data sample.

In general we can observe how the best performance is given by the downside variance risk premia across all construction horizons h. On the other hand, the upside variance risk premia exhibits the worst performance. This is in line with *Fenou' s [2015]* results. We also observe that performance decreases with the construction horizon. In particular, we have a peak at $h = 2$.

These results look promising, and could further be improved by performing multi-step ahead forecasts for $y_{t+k|t}$ with $k = 2, 3, 6, 12$ in order to evaluate for which forecast horizon (short, medium or long term) our proposed predictors reach their peak performance.

It has to be observed that the use of serial correlation and heteroskedasticity robust standard errors (eg. Newey-West) is necessary to obtain consistent results. In this way the overlap in the regression is explicitly taken into account.

6 Discussions

Overall, the paper presents a very comprehensive and innovative investigation of the predictability of excess returns. The authors fill the gap to investigate, why the well-known relationship between the variance risk premium and excess return that holds in the long-run can sometimes not be observed in the short run. The authors also combine decomposing returns and using option-implied measures, and they implement a comprehensive list of econometric models and construct an economic equilibrium to shed light on the aforementioned question.

Having mentioned the above, we however also found some aspects that complicated the replication for us and that we would hence like to point out. Our main challenge was that part of their analysis was not described in great detail. This concerned not only data treatment but also the implementation of the models. As it is the case with using option data for risk-neutral expectation, a main restriction of the analysis lies in the data treatment, hence this was a significant challenge to our replication. Moreover, we were surprised that they calculated their models using overlapping data, given the fact that their data availability stretched a period from 1996 to 2015.

To give an outlook on further models that could be implemented, it might be promising to implement their models using non-overlapping data, hence reducing the problem of large correlation in the variables though it would possibly require limiting the analysis to an aggregation and forecasting horizon of a quarter at the most. Moreover, an investigation of different thresholds κ could yield informative results.

As a larger extension of their approach, it would be interesting to use machine-learning techniques to the question investigated. In the robustness section of the paper, the authors add other well-known predictors of the equity return to their model. In these models particularly it would be interesting to use model-selection techniques that put a penalty on new parameters added, such as lasso or ridge regression. Moreover, we thought about whether it would be possible to use unsupervised learning techniques to separate the variance risk premium, not assuming that upside and downside is the most informative separation for predicting equity returns.

7 Conclusion

We analyzed the paper 'Downside Variance Risk Premia' by *B. Fenou, M.R. Jahan-Parvar and C. Okou [2015]*, we discussed their methodology and investigated whether their results hold in a different time period (2007-2017). We computed the physical expectation of the realized variance using historical intraday returns and the risk-free one working on European call and put option data observed in the market. We analyzed the predictability of excess returns using the variance risk premium, realized variance, and implied volatility. Overall, downside variance risk premium has a higher explanatory power for future excess returns and risk-neutral expectations contribute stronger to the predictability than realized measures. Finally, we proceeded to evaluate the one-step ahead prediction accuracy of our models through expanding window regressions. We have found low RMSE values across all construction horizons with peak performance for $h = 2$. The best results where obtained when using the downside variance risk premia as predictor.

References

Amengual, D. and Xiu, D. (2014). *Resoution of policy uncertainty and sudden declines in volatility*. Tech. rep. Working Paper, CEMFI and Chicago Booth.

Bakshi, G., Kapadia, N. and Madan, D. (2003). *Stock return characteristics, skew laws and the differential pricing of individual equity options*. Tech. rep. 16:101–43. Review of Financial Studies.

Barndorff-Nielsen, O. E., Kinnebrock, S. and Shephard, N. (2010). *Volatility and time series econometrics: Essays in honor of robert f. engle, chap. Measuring downside risk: realised semivariance*. Tech. rep. 117–36. Oxford University Press.

Bollerslev, T., Tauchen, G. and Zhou, H. (2009). *Expected stock returns and variance risk premia*. Tech. rep. 22:4463–92. Review of Financial Studies.

Campbell, J. Y. and Shiller, R. J. (1988). *The dividend-price ratio and expectations of future dividends and discount factors*. Tech. rep. 1:195–228. Review of Financial Studies.

Carr, P. and D., Madan. (1998). *Volatility, chap. Towards a Theory of Volatility Tradings*. Tech. rep. 417–27. Risk Publications.

Chang, B., Christoffersen, P. and Jacobs, K. (2013). *Market skewness risk and the cross-section of stock returns*. Tech. rep. 107:46–68. Journal of Financial Economics.

Cochrane, J. H. (1991). *Production-based asset pricing and the link between stock returns and economic fluctuations*. Tech. rep. 46:209–37. Journal of Finance.

Dionne, G., Li, J. and Okou, C. (2014). *An extension of the consumption-based CAPM model*. Tech. rep. Working Paper, HEC Montreal, Lingnan University, and UQAM.

Epstein, L. G. and Zin, S. E. (1989). *Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework*. Tech. rep. 57:937–69. Econometrica.

Fama, E. F. and French, K. R. (1988). *Dividend yields and expected stock returns*. Tech. rep. 22:3–25. Journal of Financial Economics.

Feunou, B., Fontaine, J.-S et al. (2014). *Risk premium, variance premium, and the maturity structure of uncertainty*. Tech. rep. 18:219–69. Review of Finance.

Feunou, B., J.P.R., Mohammad and Okou, C. (2015). *Downside Variance Risk Premium.* Tech. rep. Finance and Economics Discussion Series 2015-020. Washington: Board of Governors of the Federal Reserve System.

Ghysels, E., Plazzi, A. and Valkanov, R. (2011). *Conditional Skewness of Stock Market Returns in Developed and Emerging Markets and its Economic Fundamentals.* Tech. rep. Working Paper, Kenan- Flagler Business School-UNC, and Rady School of Business-UCSD.

Groeneveld, R. and Meeden, G. (1984). *Measuring skewness and kurtosis.* Tech. rep. 33:391–9. The Statistician.

Harvey, C. R. and Siddique, A. (1999). *Autoregressive Conditional Skewness.* Tech. rep. 34:465–88. Journal of Financial and Quantitative Analysis.

Kim, T.H. and White, H. (2004). *On More Robust Estimation of Skewness and Kurtosis.* Tech. rep. 1:56–73. Finance Research Letters.

Kozhan, R., Neuberger, A. and Schneider, P. (2014). *The skew risk premium in the equity index market.* Tech. rep. 26:2174–203. Review of Financial Studies.

Lettau, M. and Ludvigson, S. (2001). *Consumption, aggregate wealth, and expected stock returns.* Tech. rep. 56:815–50. Journal of Finance.

Ludvigson, S.C. and Ng, S. (2009). *Macro factors in bond risk premia.* Tech. rep. 22:5027–67. Review of Financial Studies.