

CIS 399 – Homework 2

Sophia Trump
strump@brynmawr.edu

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Partners: Paul Brown, Amelia Earhart.

Problem 1

Show that $(1 - FNR) = \frac{A}{A+C}$.

From the definition of equality of false negative rates, we know that $(1 - FNR)$ can be rewritten as

1.

$$1 - \frac{C}{A+C}$$

Simplifying 1,

$$\begin{aligned} 1 - \frac{C}{A+C} &= \frac{1}{1} - \frac{C}{A+C} \\ &= \frac{1(A+C)}{1(A+C)} - \frac{C}{A+C} \\ &= \frac{A+C}{A+C} - \frac{C}{A+C} \\ &= \frac{A+C-C}{A+C} \\ &= \frac{A}{A+C} \end{aligned}$$

Thus, from the above, $(1 - FNR) = \frac{A}{A+C}$. QED.

Problem 2

Show that $(1 - PPV) = \frac{B}{A+B}$.

From the definition of equality of positive predictive value, we know that $(1 - PPV)$ can be rewritten as

1.

$$1 - \frac{A}{A+B}$$

Simplifying 1,

$$\begin{aligned}
 1 - \frac{A}{A+B} &= \frac{1}{1} - \frac{A}{A+B} \\
 &= \frac{1(A+B)}{1(A+B)} - \frac{A}{A+B} \\
 &= \frac{A+B}{A+B} - \frac{A}{A+B} \\
 &= \frac{A+B-A}{A+B} \\
 &= \frac{B}{A+B}
 \end{aligned}$$

Thus, from the above, $(1 - PPV) = \frac{B}{A+B}$. QED.

Problem 3

Show that $\frac{BR}{1-BR} = \frac{A+C}{B+D}$.

From the definition of the base rate of a population, BR can be rewritten as

1.

$$\frac{A+C}{|P|}$$

Substituting 1, $\frac{BR}{1-BR} = \frac{A+C}{B+D}$ becomes

2.

$$\frac{\frac{A+C}{|P|}}{1 - \frac{A+C}{|P|}}$$

From the definition of $|P|$, 2 can be further rewritten as

3.

$$\frac{\frac{A+C}{A+B+C+D}}{1 - \frac{A+C}{A+B+C+D}}$$

Simplifying 3,

$$\begin{aligned}
 \frac{\frac{A+C}{A+B+C+D}}{1 - \frac{A+C}{A+B+C+D}} &= \frac{\frac{(A+C)(A+B+C+D)}{A+B+C+D}}{1 - \frac{(A+C)(A+B+C+D)}{A+B+C+D}} \\
 &= \frac{A+C}{(A+B+C+D) - (A+C)} \\
 &= \frac{A+C}{B+D}
 \end{aligned}$$

Thus, from the above, $\frac{BR}{1-BR} = \frac{A+C}{B+D}$. QED.

Problem 4

Using the results from Problems 1, 2, and 3, show that:

$$FPR = \frac{BR}{1 - BR} \times \frac{1 - PPV}{PPV} \times (1 - FNR)$$

. From Problem 3, we know $\frac{BR}{1 - BR}$ can be rewritten as

1.

$$\frac{A + C}{B + D}$$

From Problem 2, we know $(1 - PPV)$ can be rewritten as

2.

$$\frac{B}{A + B}$$

From Problem 1, we know $(1 - FNR)$ can be rewritten as

3.

$$\frac{A}{A + C}$$

Substituting 1, 2, and 3, $\frac{BR}{1 - BR} \times \frac{1 - PPV}{PPV} \times (1 - FNR)$ becomes

4.

$$\frac{A + C}{B + D} \times \frac{\frac{B}{A + B}}{\frac{A}{A + B}} \times \frac{A}{A + C}$$

Simplifying 4,

$$\begin{aligned} \frac{A + C}{B + D} \times \frac{\frac{B}{A + B}}{\frac{A}{A + B}} \times \frac{A}{A + C} &= \frac{A + C}{B + D} \times \frac{\frac{B(A + D)}{A + B}}{\frac{A(A + B)}{A + B}} \times \frac{A}{A + C} \\ &= \frac{A + C}{B + D} \times \frac{B}{A} \times \frac{A}{A + C} \\ &= \frac{(A + C)BA}{(B + D)A(A + C)} \\ &= \frac{BAA + BAC}{(AB + AD)(A + C)} \\ &= \frac{BAA + BAC}{AAB + AAD + CAB + CAD} \\ &= \frac{BA(A + C)}{A(AB + AD + CB + CD)} \\ &= \frac{B(A + C)}{AB + AD + CB + CD} = \frac{B(A + C)}{(A + C)(B + D)} \\ &= \frac{B}{B + D} \end{aligned}$$

From the definition of equality of false positive rates, we know FPR can be rewritten as

$$\frac{B}{B + D}$$

This is equivalent to the simplified version of $\frac{BR}{1 - BR} \times \frac{1 - PPV}{PPV} \times (1 - FNR)$, as shown above. Thus, $FPR = \frac{BR}{1 - BR} \times \frac{1 - PPV}{PPV} \times (1 - FNR)$. QED.

Problem 5

The statement from Problem 4 holds for the entire population as well as for each group individually. Suppose that all three fairness notions are satisfied by our hypothesis, i.e. $FPR_1 = FPR_2$, $FNR_1 = FNR_2$, and $PPV_1 = PPV_2$. Further, assume that all of these values, as well as the base rates, are neither 0 nor 1. Show that this implies that the base rates of the groups must be equal.

Problem 6

Show that if our hypothesis makes no mistakes (i.e. $B_1 = B_2 = 0$ and $C_1 = C_2 = 0$), all three fairness notions will be satisfied, regardless of the base rates for each group.