

## CIS 399 – Homework 2

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February 17, 2019

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### Problem 1

Show that  $(1 - FNR) = \frac{A}{A+C}$ .

From the definition of equality of false negative rates, we know that  $(1 - FNR)$  can be rewritten as

$$1 - \frac{C}{A+C}$$

Simplifying the above,

$$\begin{aligned} 1 - \frac{C}{A+C} &= \frac{1}{1} - \frac{C}{A+C} \\ &= \frac{1(A+C)}{1(A+C)} - \frac{C}{A+C} \\ &= \frac{A+C}{A+C} - \frac{C}{A+C} \\ &= \frac{A+C-C}{A+C} \\ &= \frac{A}{A+C} \end{aligned}$$

Thus, from the above,  $(1 - FNR) = \frac{A}{A+C}$ . QED.

### Problem 2

Show that  $(1 - PPV) = \frac{B}{A+B}$ .

### Problem 3

Show that  $\frac{BR}{1-BR} = \frac{A+C}{B+D}$ .

### Problem 4

Using the results from Problems 1, 2, and 3, show that:

$$FPR = \frac{BR}{1-BR} \times \frac{1-PPV}{PPV} \times (1-FNR)$$

.

**Problem 5**

The statement from Problem 4 holds for the entire population as well as for each group individually. Suppose that all three fairness notions are satisfied by our hypothesis, i.e.  $FPR_1 = FPR_2$ ,  $FNR_1 = FNR_2$ , and  $PPV_1 = PPV_2$ . Further, assume that all of these values, as well as the base rates, are neither 0 nor 1. Show that this implies that the base rates of the groups must be equal.

**Problem 6**

Show that if our hypothesis makes no mistakes (i.e.  $B_1 = B_2 = 0$  and  $C_1 = C_2 = 0$ ), all three fairness notions will be satisfied, regardless of the base rates for each group.