CIS 399 – Homework 2

Sophia Trump
strump@brynmawr.edu

February 17, 2019

Partners: Paul Brown, Amelia Earhart.

Problem 1

Show that $(1 - FNR) = \frac{A}{A+C}$.

From the definition of equality of false negative rates, we know that $(1 - FNR)$ can be rewritten as

$$1 - \frac{C}{A+C}$$

Simplifying the above,

$$\begin{aligned} 1 - \frac{C}{A+C} &= \frac{1}{1} - \frac{C}{A+C} \\ &= \frac{1(A+C)}{1(A+C)} - \frac{C}{A+C} \\ &= \frac{A+C}{A+C} - \frac{C}{A+C} \\ &= \frac{A+C-C}{A+C} \\ &= \frac{A}{A+C} \end{aligned}$$

Thus, from the above, $(1 - FNR) = \frac{A}{A+C}$. QED.

Problem 2

Show that $(1 - PPV) = \frac{B}{A+B}$.

From the definition of equality of positive predictive value, we know that $(1 - PPV)$ can be rewritten as

$$1 - \frac{A}{A+B}$$

Simplifying the above,

$$\begin{aligned} 1 - \frac{A}{A+B} &= \frac{1}{1} - \frac{A}{A+B} \\ &= \frac{1(A+B)}{1(A+B} - \frac{A}{A+B} \\ &= \frac{A+B}{A+B} - \frac{A}{A+B} \\ &= \frac{A+B-A}{A+B} \\ &= \frac{B}{A+B} \end{aligned}$$

Thus, from the above, $(1 - PPV) = \frac{B}{A+B}$. QED.

Problem 3

Show that $\frac{BR}{1-BR} = \frac{A+C}{B+D}$.

From the definition of the base rate of a population, BR can be rewritten as

$$\frac{A+C}{|P|}$$

Substituting the above, $\frac{BR}{1-BR} = \frac{A+C}{B+D}$ becomes

$$\frac{\frac{A+C}{|P|}}{1 - \frac{A+C}{|P|}}$$

From the definition of $|P|$, the above can be further rewritten as

$$\frac{\frac{A+C}{A+B+C+D}}{1 - \frac{A+C}{A+B+C+D}}$$

Simplifying the above,

$$\begin{aligned} \frac{\frac{A+C}{A+B+C+D}}{1 - \frac{A+C}{A+B+C+D}} &= \frac{\frac{(A+C)(A+B+C+D)}{A+B+C+D}}{1 - \frac{(A+C)(A+B+C+D)}{A+B+C+D}} \\ &= \frac{A+C}{(A+B+C+D) - (A+C)} \\ &= \frac{A+C}{B+D} \end{aligned}$$

Thus, from the above, $\frac{BR}{1-BR} = \frac{A+C}{B+D}$. QED.

Problem 4

Using the results from Problems 1, 2, and 3, show that:

$$FPR = \frac{BR}{1-BR} \times \frac{1-PPV}{PPV} \times (1 - FNR)$$

.

Problem 5

The statement from Problem 4 holds for the entire population as well as for each group individually. Suppose that all three fairness notions are satisfied by our hypothesis, i.e. $FPR_1 = FPR_2$, $FNR_1 = FNR_2$, and $PPV_1 = PPV_2$. Further, assume that all of these values, as well as the base rates, are neither 0 nor 1. Show that this implies that the base rates of the groups must be equal.

Problem 6

Show that if our hypothesis makes no mistakes (i.e. $B_1 = B_2 = 0$ and $C_1 = C_2 = 0$), all three fairness notions will be satisfied, regardless of the base rates for each group.