Modeling *De Facto* Segregation

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Abstract

Segregation is the process of the separation of people or groups based on characteristics such as race, ethnicity, religion, gender, or social status. In particular, one element of segregation that has been particularly relevant in the past century is residential segregation, both through active policies like redlining and Jim Crows laws, as well as implicit processes such as gentrification, economic inequality, and group preferences. In 1969, Nobel Prize-winning economist Thomas Schelling outlined a cellular automaton model for modeling *de facto*, or non-intentional segregation of households with a binary classification in a community. In this paper, we extend Schelling's model in three ways. Firstly, we consider an extension of a 2-dimensional model to 3 dimensions with the same iteration method by adjusting the neighborhood calculation method. Next, we consider the case where household classifications are non-binary, such as household income or wealth. We extend this idea by considering a household's satisfaction with their distance from the city center, which is based on their wealth class. We find that the resulting residential dynamics closely resemble the demographic "Concentric Zone model" as outlined by Ernest Burgess in 1925. Lastly, given that individuals often exhibit a stronger preference for groups that are more similar to them, we examine the intersections of multiple classifications endowed to each individual household. We also consider various levels of attention that households pay to different standards. Based on the improved model, we propose a real-world application of campus dormitory allocation. Finally, we point out the applicability of these models to policy-making and education and make recommendations on how to further develop these models in future work.

1 Background

1.1 Segregation

Segregation is a sociological process by which people are separated into groups based on characteristics such as race, religion, gender, and socioeconomic status. The study of segregation has a long history, and is at the forefront of academic research today because of it's implications for social justice and cohesion, urban planning, health and well-being, and much more. For example, segregated communities, such as predominately African American neighborhoods in the United States, often have unequal access to resources, opportunities, and services, leading to disparities in education, healthcare, employment, and overall quality of life. This is evident in, for example, the COVID-19 mortality rate, which was twice as high for black Americans as it was for white Americans.(3). By better understanding the principles of segregation, we can work to enact policy that counteracts such social disparities and work for a more equitable society generally.

Processes of segregation are usually divided into two classes, *de jure* and *de facto*. *De jure* segregation refers to the enforced separation of people by law. For example, in the year 779, the imperial Tang Dynasty in China issued a policy that forbade Uighurs from pretending to be Chinese, marrying Chinese females, and enforcing the wearing of ethnic clothes.(4) By contrast, *de jure* segregation is a process of segregation that exists despite the fact that perhaps it is not intended. For example, in the United States, segregation of African American and white people occurs due to factors such as preference for white people to be among white people and African Americans to be among African Americans, linguistic differences, and the inability of African Americans to afford to live in areas which are predominately white, as these communities tend to be more affluent.

1.2 Assumptions

Here are some general assumptions that we make in order to abstract the process of segregation into our model.

1. The given area is an isolated zone; no one enters or exits.
2. Each household prefers a higher degree of similarity; preferences do not change over time.
3. Households have enough information before they move, which means they move to the nearest place where they feel happy.
4. If there are no neighbors around, the household is also considered satisfied.
5. If there is no vacant space that can make a household happy, they will remain in their current location.

2 Model Construction

2.1 One-Dimensional Model

In 1969, Nobel Prize-winning economist Thomas Schelling published a paper entitled *Models of Segregation* in which he outlined a model of *de facto* segregation among two binary groups.(7)

He assumed that there is an equal proportion of white and black households arranged in a one-dimensional array, and the households initially are randomly distributed among each other. This situation can be represented by a line of + and 0, where a + represents a white household and a 0 represents a black household. For example,

$$0\ \dot{+}\ 0\ 0\ 0\ \dot{+}\ \dot{+}\ 0\ +\ \dot{0}\ \dot{0}\ +\ +\ +\ \dot{0}\ +\ +\ \dot{0}\ +\ +\ \dot{0}\ 0\ +\ +\ 0\ 0\ \dot{+}\ \dot{+}\ 0\ 0$$

Figure 1: Initial distribution of white and black households in an array. All unhappy households are marked with a dot. In this example, there are 15 white households, 15 black households, 5 unhappy white households, and 6 unhappy black households.

Next, assume that the "neighborhood" of each household as extending four neighbors on either side of the household. We assume that a household is "happy" if at least half of their neighbors are in the same group as them. To update the model, going from left to right, we assume that each unhappy household inserts itself to the nearest location in which it will be happy, where nearness is measured as the number of households passed. For example, the next stage of the example in Figure 1 will be,

$$0\ 0\ 0\ 0\ \dot{+}\ 0\ 0\ 0\ 0\ 0\ 0\ +\ +\ +\ +\ +\ +\ +\ +\ +\ +\ \dot{0}\ +\ +\ +\ \dot{+}\ \dot{0}\ 0\ 0\ 0$$

Figure 2: Second state of white and black households in an array. Now there are just 4 unhappy households.

After one more iteration, we can see that all the households are happy, and the community is almost segregated into 3 distinct neighborhoods.

$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ +\ +\ +\ +\ +\ +\ +\ +\ +\ +\ +\ +\ +\ +\ +\ \dot{0}\ \dot{0}\ 0$$

Figure 3: Third state of white and black households in an array. Now there are just 2 unhappy households.

This model demonstrates one of the main points of Schelling's paper, namely, even though each individual is not in favor of segregation, the group is. To be specific, even though each household only wanted half of their neighbors to be like them, we ended up with large, homogeneous neighborhoods, where the average number of similar neighbors is 81.8% after just 3 iterations of moving.

2.2 Two-Dimensional Model

Schelling's original model can easily be generalized to two dimensions.(8) Continuing with each unit being one household, the two-dimensional version of Schelling's model models a map of a community of households. The model is defined as follows:

Assume we have a $n \times n$ grid with an equal number of black and white households. There may (and should) be vacant spots on the grid, hence, the number of each type of household is $\leq \frac{1}{2}n^2$. A neighborhood of a household (h_i) on the grid, denoted as $U_k(h_i)$, is defined as all of the houses within a $k \times k$ square centered at h, not including h itself, where k is an odd number with $k \leq n$. We assign a tolerance $0 \leq \tau_+ \leq 1$ for the white households and tolerance $0 \leq \tau_0 \leq 1$ for the black households. The tolerance parameters define the minimum proportion of neighbors that a household of each type requires in order to be happy.

An iteration of the model is defined as follows. In the order of left to right, top to bottom, each unhappy household moves to the vacant spot in the grid that is the closest place in which they are happy. If there is no such place, the household stays in-place. If there are multiple such places, pick one at random. The model runs for a fixed amount of iterations determined beforehand by the user or until there is a 100% happiness rate among all households.

$$\begin{vmatrix} & \dot{+} & & 0 \\ \dot{0} & & \dot{0} & 0 \\ + & + & & \dot{+} \\ + & \dot{0} & + & \dot{0} \end{vmatrix} \longrightarrow \begin{vmatrix} & & 0 & 0 \\ + & + & 0 & 0 \\ + & + & 0 & \\ + & & \dot{+} & 0 \end{vmatrix} \longrightarrow \begin{vmatrix} & & 0 & 0 \\ + & + & 0 & 0 \\ + & + & 0 & \\ + & + & & 0 \end{vmatrix}$$

Figure 4: 4×4 grid populated randomly with 6 white and 6 black households. We take $\tau_+ = \tau_0 = 0.5$ and $k = 3$. Households marked with a dot overhead are unhappy.

As we can see from Figure 4, the two-dimensional version of Schelling's model follows the same trend as the one-dimensional version. Namely, even though each household does not require a majority of their neighbors to be the same as them, the community ends up segregated.

There are several metrics one can define to measure the level of segregation in a population in order to analyze this model. For example, one can consider the proportion of households whose neighbors are all similar to them. For example, in the simple example in Figure 4, after 4 iterations, the number of households satisfying this requirement is $8/12 = 67\%$.

2.3 Analysis of Two-Dimensional Model

To analyze this segregation under different circumstances and visualize the outcome, we write Python code to simulate the whole procedure. The program flow is the following:

1. Initialize the following parameters:
   - width, length: the size of our map
   - empty rate: the empty rate of households on the map, i.e., how many empty houses we should have on the map
   - similarity threshold: the minimum number of neighborhoods with the same race as the household holder that the household holder will be satisfied, i.e. the intolerance of other races
   - number of iterations: the maximum number of iterations
   - number of races: the number of races

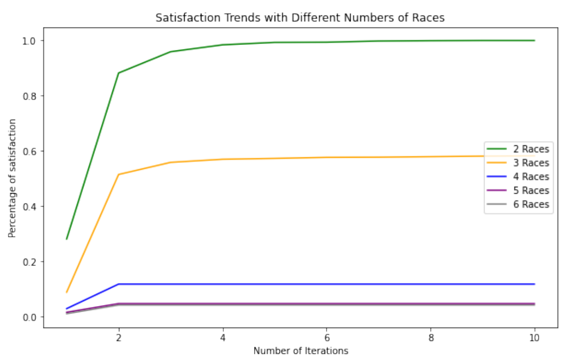
2. Generate a grid of positions according to width and length. Then it will randomly assign each position as either an empty house or an occupied house with a randomly assigned race.

3. Define a function as follows to check whether the given position is satisfied or not.

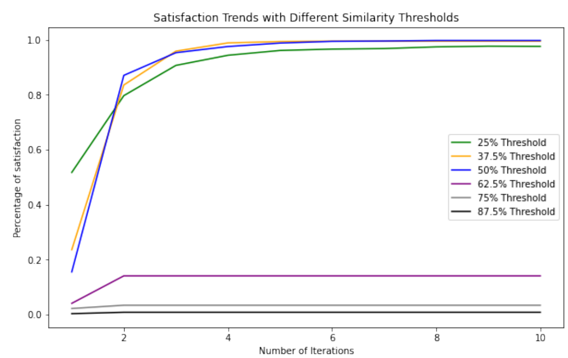
   (a) Determine the neighbors of the position
   (b) If there are no neighbors, we assume the agent is satisfied
   (c) Count the number of neighbors of the same race
   (d) Count the total number of neighbors
   (e) The agent is satisfied if the proportion of same-race neighbors to total neighbors is bigger than the similarity threshold.

4. A loop starts that runs for the number of iterations specified during initialization. Within each iteration:

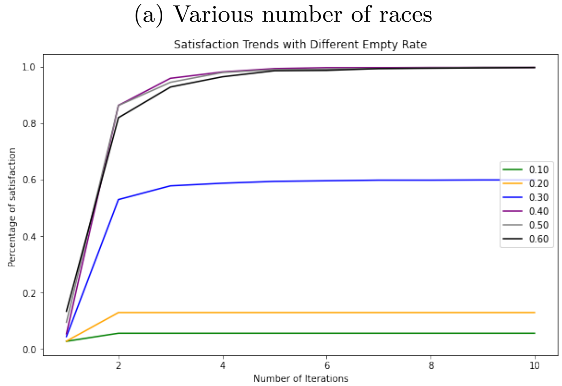
   (a) Find all unsatisfied agents using the algorithm above
   (b) For each unsatisfied agent, find the nearest place that can make it satisfied and let it move to it. If we can't find a satisfactory space for it, we just keep them in the original space. Also, we need to update the map after dealing with each unsatisfied agent.
   (c) Calculate the proportion of satisfied households to the total population.



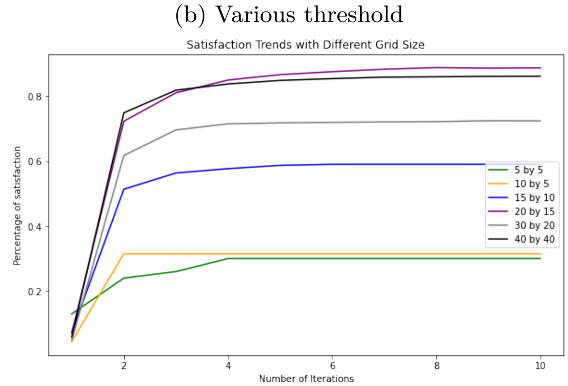
(a) Various number of races



(b) Various threshold



(c) Various empty rate



(d) Various map size

Figure 5: Satisfaction trend in 10 iterations with 1 changing variable

In our study, we examine satisfaction dynamics using five parameters: map size, empty rate, similarity threshold, number of iterations, and number of races. We conduct the simulation over 10 iterations. By fixing one parameter at a time, we explore how satisfaction manifests under varying conditions. In Figure 5's first graph, we configure a 20×20 grid with a 20% empty rate and set the similarity threshold at

0.625 (meaning an agent is satisfied if at least 5/8 of its neighbors are of the same race). We observe higher satisfaction with fewer races; however, with many races, not all agents achieve satisfaction, indicating challenges in forming homogeneous groups in a diverse setting.

In Figure 5's second graph, under the same map size and empty rate, increasing the similarity threshold paradoxically makes it more challenging to find similar neighbors due to the higher standards of similarity, often leading to dissatisfaction.

The third graph in Figure 5, with constant map size, the threshold at 62.5%, and two races, shows that a higher empty rate facilitates higher satisfaction because it offers more relocation opportunities for agents seeking same-race neighbors.

Lastly, Figure 5's final graph, with a 20% empty rate, a 50% threshold, and two races, demonstrates that increasing the grid size promotes racial clustering, as a larger grid provides more empty spaces, allowing races more freedom to be satisfied.

2.4 Three-Dimensional Model

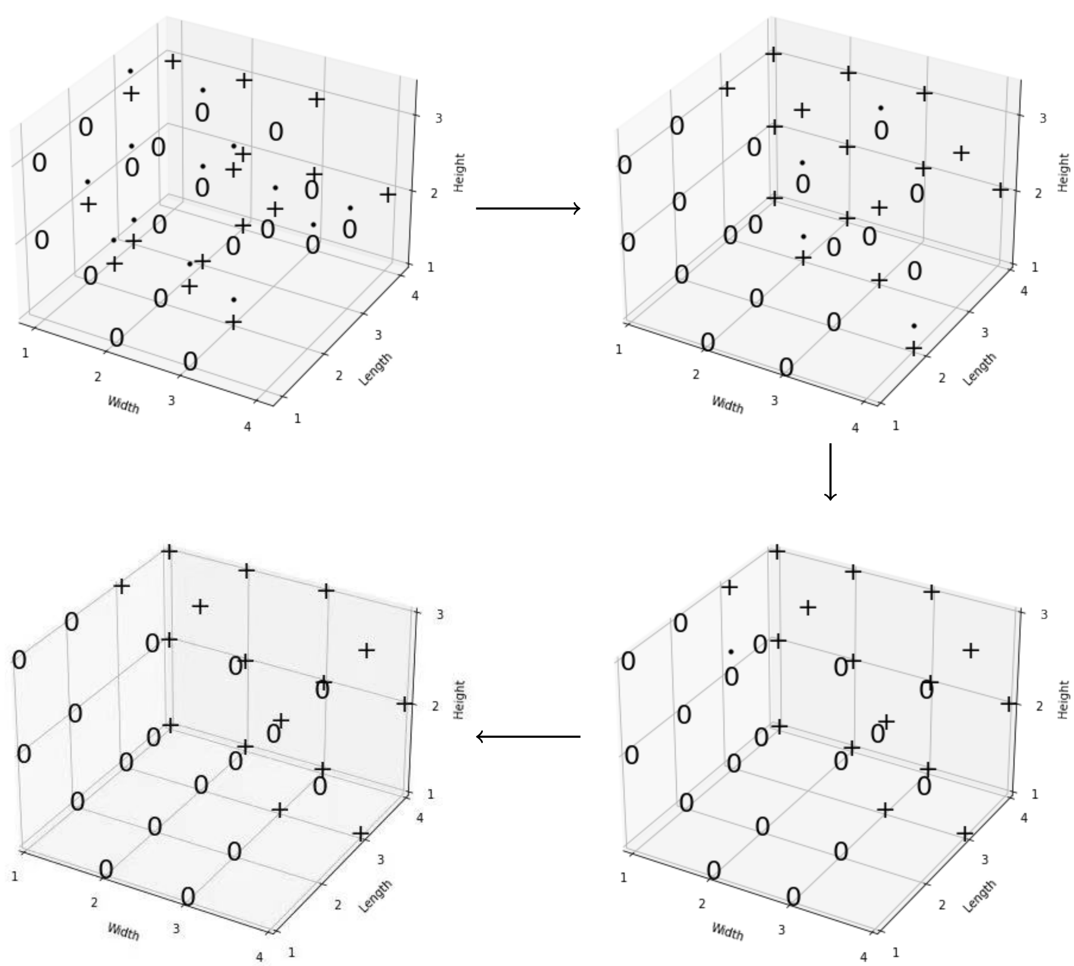


Figure 6: $4 \times 4 \times 3$ grid populated randomly with 18 white and 16 black households. We take $\tau_+ = \tau_0 = 0.5$ and $k = 3$. Households marked with a dot overhead are unhappy.

Based on the foundational work of Schelling's segregation models in one and two dimensions, the three-dimensional Schelling model introduces a more complex and nuanced simulation of social dynamics. This model represents a community within a three-dimensional space, such as high rise building with multiple floors, each containing multiple apartments.

In this 3D model, we consider a $l \times w \times h$ grid, where l, w, and h represent the length, width, and height of the grid, respectively. Each unit within this grid corresponds to a household, which can be either white or black, similar to the previous models. We ensure the grid is populated randomly at the start.

A neighborhood in this model is defined as the set of households within a cube of dimensions $k \times k \times k$, centered at a household (not including the household itself), where k is an odd number. Each household has a defined tolerance level, τ_+ for white households and τ_0 for black households, indicating the minimum proportion of same-type neighbors required to be "happy."

The process of the model is as follows. Identify all "unhappy" households in the grid—those where less than the threshold of the neighbors within their defined neighborhood cube are of the same type. Then move each unhappy household to the closest vacant position within the grid where it would be happy, based on its tolerance level. This position must optimize the happiness of the household, considering the entire 3D neighborhood. If no such position exists, the household remains in place. The simulation proceeds iteratively, re-evaluating happiness and systematically moving unhappy households, until either a maximum number of iterations is reached or all households are happy.

Figure 6 shows an example of a 3D grid and shows the same trend with one-dimension and two-dimension. It still ends up in segregation and everyone get satisfied.

3 Models with Continuous Classes

3.1 Wealth Model

As Schelling originally developed his model to demonstrate racial segregation between white people and black people in the United States, the classifications of households are defined as a binary relation. However, there are other attributes which could lead to segregated neighborhoods for which it is not sufficient to consider a binary classification such as "black" and "white".

For example, levels of wealth are distributed on a continuous spectrum and should be modeled as such. There are several reasons why wealth is a major contributor to *de facto* segregation. Firstly, wealthier households can afford more expensive homes in desirable areas, while lower-income individuals are often limited to more affordable housing options, which may be concentrated in specific neighborhoods. Furthermore, wealthier neighborhoods often have better access to high-quality schools, healthcare facilities, parks, and recreational amenities. This attracts more affluent residents and can lead to a cycle where wealthier areas continue to improve and attract even more wealth, while less affluent areas struggle to attract investment. Also, as with the other cases, people often choose to live near others with similar socio-economic backgrounds, lifestyles, and values.(6)

We define a model which assigns each household in the grid a random number between 0 and 1 pulled from a uniform distribution, with 1 being the most wealthy to 0 being the most poor. Let a household's wealth be given by w_i and the neighborhood of household i be $U(h_i)$. We define a household as "happy" if the following inequality is satisfied,

$$\left| w_i - \frac{1}{|U(h_i)|} \sum_{j \in U(h_i)} w_j \right| \leq \tau$$

where τ is a tolerance parameter set by the user. Hence, a household is happy with any average neighborhood wealth that is within an interval of 2τ centered at the household's wealth.

We can clearly see a cluster of wealthy households and districts of poorer households emerge after several iterations of the model in Figure 7. We can quantify this segregation by looking at the average distribution

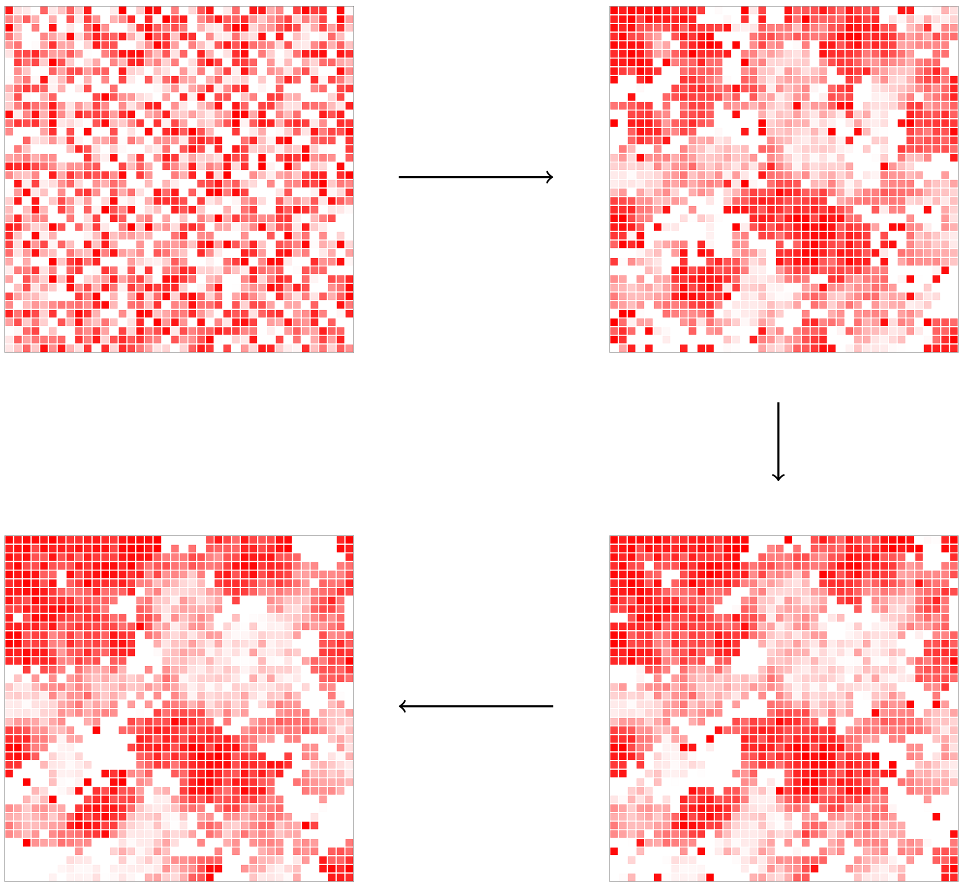


Figure 7: Continuous class wealth segregation model in a 40×40 grid with $\tau = 0.25$ and 20% empty properties. A neighborhood is defined as the eight (or fewer) households immediately surrounding a household. Darker households have a higher wealth and lighter households have a small wealth. The first image is the initial state, and all following images are 60 iterations after the previous one.

of wealth within each neighborhood. That is, we can see how much the average wealth of a neighborhood differs from that of an individual household in that neighborhood.

We find that the average difference between a household's wealth and their neighborhood after 10 trials of 200 iterations of the model is just 0.0856 with a standard deviation of 4.0×10^{-3}, indicating a very tight spread of the data around the mean. Furthermore, we can chart the value of this statistic versus the number of iterations as in Figure 8.

3.2 Wealth-Distance Model

One of the major demographic shifts in the past century was the process of "suburban sprawl". With the advent of automobiles and improved transportation, wealthier individuals began moving to the suburbs to escape the congestion, noise, and pollution of the cities. This trend was further fueled by government policies that subsidized suburban development and home ownership. Also, suburbs often offer larger homes and more space, which are attractive to wealthier individuals who can afford the higher property prices and taxes. In contrast, cities tend to have more affordable housing options, such as apartments and smaller homes, which are more accessible to lower-income individuals. Furthermore, there can be a self-reinforcing cycle where

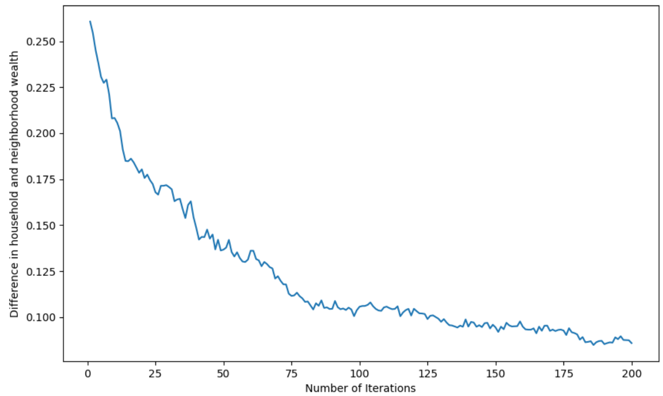


Figure 8: Graph displaying the convergence of the difference in household and neighborhood wealth in the wealth model for 200 iterations. We set the parameters the same as for Figure 7. The low level of disparity in neighborhood wealth indicates a wealth-segregated population.

affluent individuals move to certain areas, driving up property values and attracting more affluent residents, while lower-income individuals are pushed to less desirable areas due to rising costs.(2) This can lead to economic segregation.

We can incorporate this element of *de facto* segregation into our continuous wealth model by introducing a positional term that contributes to the overall satisfaction of a household. We assume that the most desirable place in the suburbs is the circle centered at the city center $(0, 0)$ with a radius of four-fifths of the city border (B). This is because the prime properties are both reasonably close to the economic hub of the city while being far-away enough to be secluded. Furthermore, we assume that the affordability of properties decreases linearly with distance from the ideal circle, which determines the ideal location for people with varying levels of wealth.

In specific, we assume the ideal circles for a given household's wealth w_i have radii $\frac{4B}{5}w_i$ and $\frac{4B}{5}(2-w_i)$ and are centered at $(0, 0)$. In order to construct a function which models how well a given household conforms to this assumption, we can construct two Gaussian functions with maximums at the radii of the ideal circles, namely,

$$f(z) = \begin{cases} \exp\left(-\frac{(z-\frac{4B}{5}w_i)^2}{2\sigma^2}\right) & \text{if } 0 \leq z \leq \frac{4B}{5} \\ \exp\left(-\frac{(z-\frac{4B}{5}(2-w_i))^2}{2\sigma^2}\right) & \text{if } \frac{4B}{5} \leq z \leq B \end{cases}$$

where z is the modulus of the household position vector $h_i = \langle x_i, y_i \rangle$ with $|x_i|, |y_i| \leq B$ which measures it's distance from the origin.

Furthermore, we introduce a parameter $0 \leq \varepsilon \leq 1$, which describes the minimum satisfaction a household needs from the distance from their ideal circle in order to be happy. Hence, a household is "happy" if both the inequalities,

$$\left| w_i - \frac{1}{|U(h_i)|} \sum_{j \in U(h_i)} w_j \right| \leq \tau \text{ and } 1 - f(|h_i|) \leq \varepsilon$$

are satisfied.

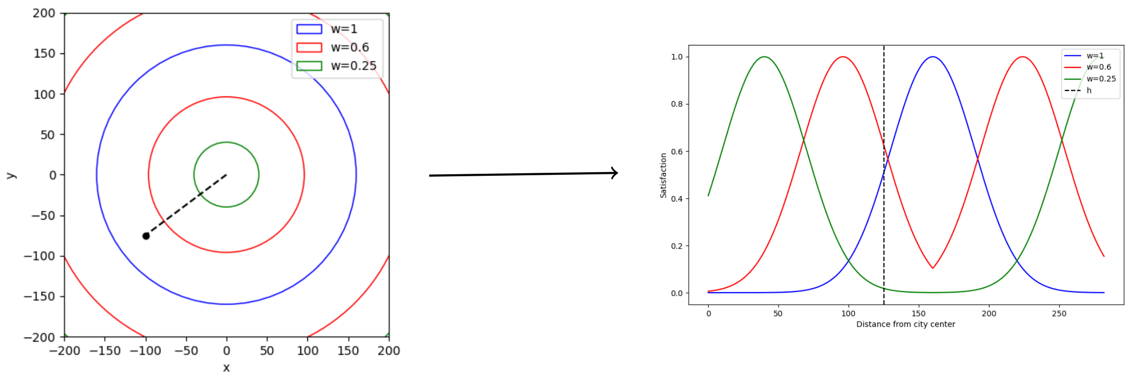


Figure 9: The left hand represents the spatial layout of the city, and properties that lie on the blue circle are the most ideal. From this, we define the satisfaction metric on the right-hand side. There are two peaks for lower wealth because households are most satisfied a certain distance away from either side of the ideal circle. Averaged over values of w with $\sigma = B/10$, households are satisfied with 50% of the total state space for about 50.2% of the domain of the satisfaction function, therefore we take $\varepsilon = 1 - 0.502 = 0.498$.

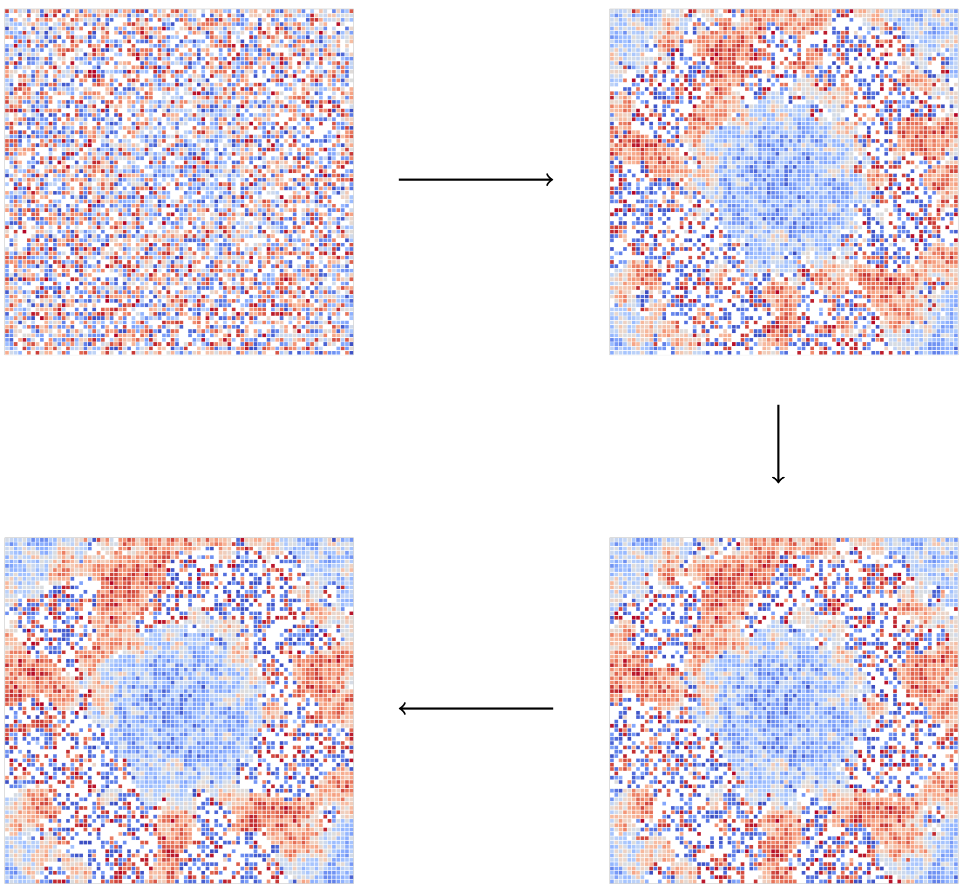


Figure 10: Wealth-distance model in a 80×80 grid, with $\tau = 0.25$ and $\varepsilon = 0.498$ and 20% vacant properties. In this way, we ensure that each individual household is open to non-segregated communities both in wealth and distance from the city center. We take the ideal circle to have radius $\frac{4B}{5}$. Warmer colors have a higher wealth, and cooler colors have a smaller wealth. [180 iterations].

In Figure 10, we see a clear pattern in the distribution of households in the city which follows the "Concentric Zone (CBD) Model" of a city as developed by Ernest Burgess in 1925,(1) as shown in Figure 11. Namely, we see a clustering of the poorest people in the city center, which corresponds to the transition zone and blue-collar residential area. Immediately around this cluster, there is a ring of beige households, representing moderate wealth and defining the middle-income residential area in the CBD model. Finally, there is a clear ring of wealthy neighborhoods in the commuter residential area that appears in our model.

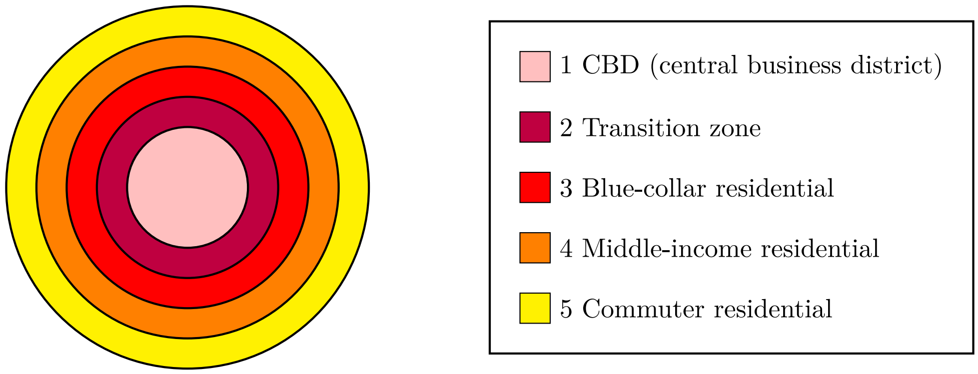


Figure 11: Ernest Burgess's Concentric Zone model of a city, as developed in 1925. This model was developed to explain the demographics in the city of Chicago. The CBD is the economic center of the city. The transition zone consists of recent immigrant groups, deteriorated housing, factories, and abandoned buildings. Then, the three residential areas increase in quality up to the commuter residential district, which consists of people wealthy enough to afford larger and more expensive housing as well as transportation to and from entertainment and work.

Clearly, there is extreme clustering in the center of the city, which doesn't leave enough affordable housing for the less wealthy people to live in, leaving them to flee far outside the city center, past the commuter zone. This mimics a real phenomenon known as "residential displacement." Residential displacement occurs when people are forced to move out of their homes or neighborhoods due to factors such as rising housing costs, gentrification, and redevelopment. As a result, low-income individuals and families may be pushed to the outskirts of the city or to areas with fewer resources and opportunities, exacerbating issues of inequality and segregation.(5) Furthermore, we notice that there is a lower density of housing in the commuter zone, which is a key feature of suburban sprawl. Namely, buildings usually have "fewer stories and are spaced farther apart, separated by lawns, landscaping, roads or parking lots."(9)

This is supported by the fact that the mean wealth of dissatisfied people after 10 trials of 200 iterations of our model is 0.3871 with a standard deviation of just 0.0076. This indicates that the less-wealthy people are more dissatisfied with their household configuration while the wealthier people are more satisfied. Furthermore, we conjecture that this number is inflated artificially because our model has finite bounds on the city's area, forcing poor people to live among wealthy people in the commuter zone, whereas in reality households can move arbitrarily far-away from a city. Therefore, we predict that the average wealth of dissatisfied people after the model converges is even lower in reality.

4 Graduated Preference Model with Randomness

4.1 Model Construction

Taking into account the diversity of cultural backgrounds, educational attainments, and personal values, individuals exhibit varying levels of tolerance towards their out-groups. This suggests that the similarity threshold for each person should be dynamically varied, rather than fixed. Similarly, everyone has their own criteria and degree of emphasis for different aspects, which will also affect the similarity assessment.

Moreover, the demarcation between groups extends beyond mere binary categorization; for instance, considering the existence of biracial individuals between white and black communities, or the nuanced distinctions within socioeconomic strata, exemplified by the middle class between the extremes of wealth and poverty. Drawing insights from relevant data and personal experiences, we employ random value generation to gauge individuals' preferences for similarity across different attributes. Additionally, we assume that individuals derive greater satisfaction with out-groups possessing higher similarity, while their satisfaction diminishes with out-groups showing greater disparities.

By recognizing and incorporating such complexities, we endeavor to refine the segregation model to simulate real-world dynamics better, thereby advancing its application in social analyses.

In this model, we assume that there are four races and four wealth statuses in the group. The process of the model is as follows:

1. Initialize the following parameters: width, length, empty rate, number of iterations, number of races, and number of wealth statuses.
2. Generate a grid of positions according to width and length. Then it will randomly assign each position as either an empty house or an occupied house with randomly assigned race, wealth status, the weight of attention to race, the weight of attention to wealth, and similarity threshold.
3. Define a function to check whether the given position is satisfied or not.
   (a) Determine the neighbors of the position
   (b) If there are no neighbors, we assume the agent is satisfied
   (c) Define the similarity is calculated by

   $$1 - \frac{1}{\text{total races(wealth statuses)}} \times \text{ race(wealth) differences}$$

   For example, if four wealth statuses are defined as Low, Lower-Middle, Middle, and High, then the wealth status difference is calculated as follows: there is a difference of 1 between Low and Lower-Middle and a difference of 2 between Low and Middle.
   (d) The satisfaction between the agent and neighbors is calculated by

   $$\text{race similarity} \times \text{the weight of attention to race} + \text{wealth similarity} \times \text{the weight of attention to wealth}$$

   (e) The agent is satisfied if the satisfaction is greater than its similarity threshold.
4. A loop starts that runs for the number of iterations specified during initialization. Within each iteration:
   (a) Find all unsatisfied agents using the function above
   (b) For each unsatisfied agent, find the nearest place that can make it satisfied and let it move to it. If we can't find a satisfactory space for it, we just keep them in the original space. Also, we need to update the map after dealing with each unsatisfied agent.
   (c) Calculate the average satisfaction rate among all people

In Figure 12, we simulate the distribution pattern after 30 iterations with code. The four races are represented by red, green, blue, and cyan, respectively, and the four wealth statuses are represented by square, circle, diamond, and triangle, respectively. Upon comparing the distributions before and after the iteration, we observe the formation of segregation among the households of the same race or wealth status on a small scale, while noting a transitional and interlocking distribution among households with few differences.

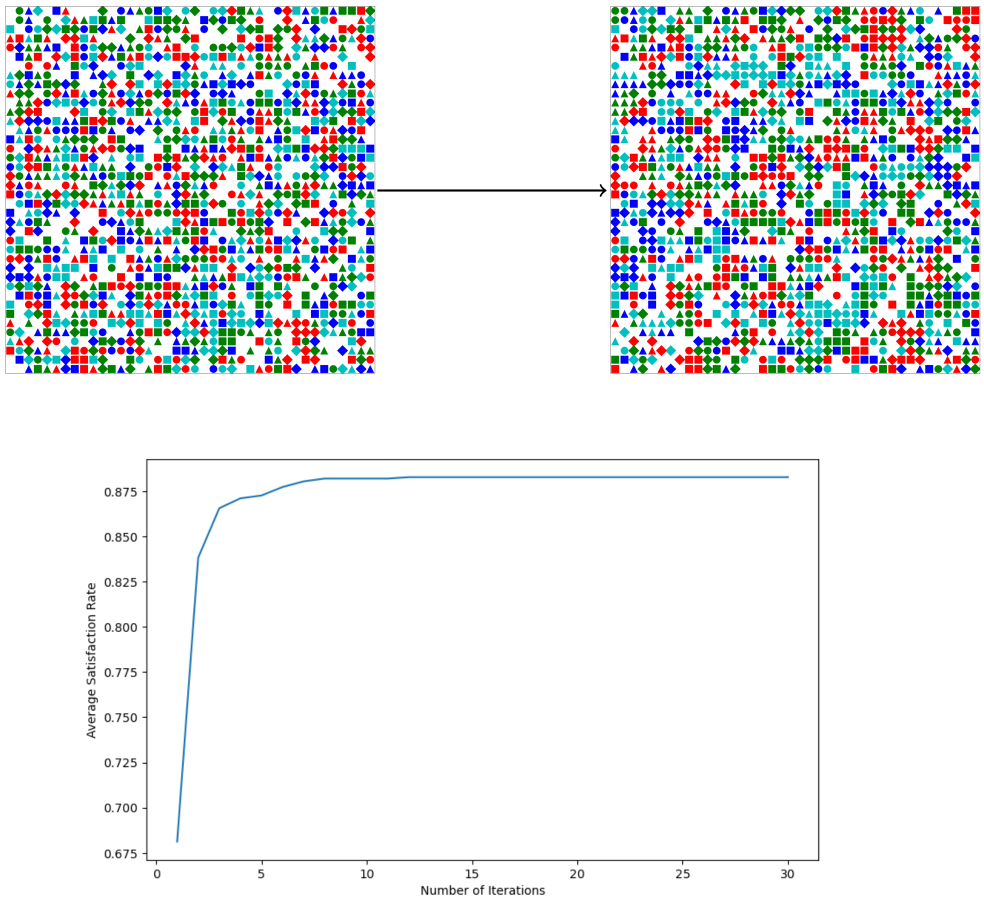


Figure 12: When employing the graduated preference model with randomness, we set up a 40×40 area with an empty rate of 0.2. The average satisfaction rate stabilizes at approximately 0.875 within 15 iterations.

4.2 Application Scenario — Campus Dormitory Allocation

Let's assume for the Fall semester of 2024, DKU Residence Life randomly allocates students across a four-story dormitory, and each room is occupied by one student. Given the diversity in students' daily schedules, noise tolerance levels, and cleanliness standards, students dissatisfied with their neighbors have the opportunity to move to the nearest suitable room every two weeks. The graduated preference model with randomness can be applied in this scenario using a simplified three-dimensional model.

Here are the newly added assumptions:

1. Each room is occupied by one student.
2. For each floor, there are totally of 100 rooms (5×20).
3. Neighbors are considered as the cubic of length 3 centered by the individual.
4. Students assess their similarities only based on their quietness, cleanliness, and daily schedules, including bedtimes and wake-up times.
5. Each preference is categorized into four levels as what we did in the graduated preference model.

The process of the model is as follows:

1. Initialize the following parameters: width, length, empty rate, number of iterations, number of levels of quietness and cleanliness, and types of daily schedules.

2. Generate a grid of positions according to width and length. Then, it will randomly assign each position as either an empty room or an occupied room with randomly assigned attributes and similarity threshold, as well as the weights of attention to these three attributes.

3. Define a function to check whether the given position is satisfied or not.

   (a) Determine the neighbors of the position
   (b) If there are no neighbors, we assume the student is satisfied
   (c) Define the similarity is calculated by

$$1 - \frac{1}{\text{total levels(types)}} \times \text{ attribute differences}$$

   (d) The satisfaction between the agent and neighbors is calculated by

$$\text{total satisfaction} = \sum_{\text{all attributes}} (\text{ attribute similarity} \times \text{the weight of attention to specific attribute })$$

   (e) Students are satisfied if the satisfaction is greater than their similarity threshold

4. A loop starts that runs for the number of iterations specified during initialization. Within each iteration:

   (a) Find all unsatisfied students using the function above
   (b) For each unsatisfied student, find the nearest place that can make it satisfied and let it move to it. If we can't find a satisfactory space for it, we just keep them in the original space. Also, we need to update the map after dealing with each unsatisfied student.
   (c) Calculate the satisfaction rate for all people

After conducting a simulation with an empty rate of 0.1 over 10 iterations (5 months), we plot the heat map and we find almost every students will be satisfied with their current room.

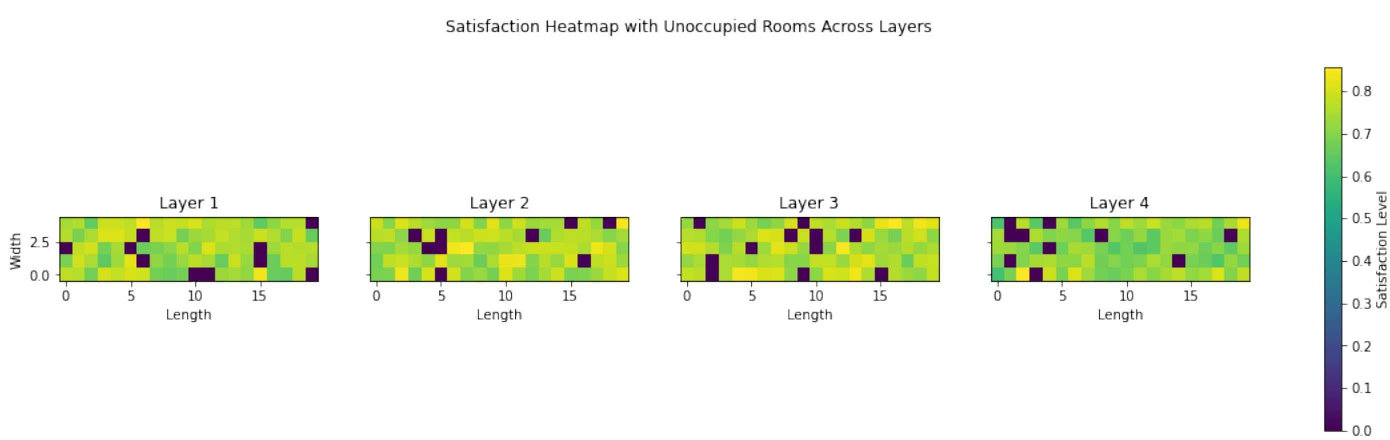


Figure 13

5 Conclusion

In this paper, we have examined Thomas Schelling's cellular autonomon models for residential segregation and proposed three novel extensions which add to the complexity of his original model. Namely, we added a third spatial dimension to the model, considered characterizations of households that are not discrete, and considered the intersection of multiple different characterizations.

This study and modeling of *de facto* segregation also serve a distinct pedagogical purpose. Namely, it is clear from examining these models that aggregate individual preferences can sometimes lead to paradoxical group preferences. For example, looking back at the one-dimensional model, we note that an arrangement such as,

$$0 \dot{+} 0 \dot{+} 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ldots 0 \ + \ 0 \dot{+} 0 \dot{+} 0$$

Figure 14: Array of households with arbitrarily high satisfaction and complete desegregation.

has a fixed number of 4 unhappy households, therefore it has an arbitrarily high average satisfaction rate. That said, the households are completely desegregated. The existence of such a counterexample shows that individuals do not prefer segregation, even though this is the result of running the model every time. This simple model conveys the point that individual tolerance is not enough to combat the forces of segregation, and that legislative interventions are crucial.

We believe that much more work can be done to explore the dynamics of segregation through cellular autonomon models. For example, our "wealth-distance" model considers two major determinants of segregation, but leaves-out other major factors for simplicity such as race, distinct elements of the city such as parks, economic centers, healthcare centers, and schools, and religion. Furthermore, we only consider **de facto** segregation, and more work can be done to incorporate the effects of government policies such as redlining and diversity programs into the model. Also, another further avenue could be to tailor this model based on real-life demographic data such as people's preferences to live among certain groups and spatial layouts of different groups.

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