CPSC 313 — Quiz #1

January 30, 2020

- No Aids Allowed.
- Answer all questions on the quiz sheet.
- Total marks: 25

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1. [3 marks] **True/False Questions**

   Answer *true* or *false* to each of the questions below. No explanations are necessary; just state your answer.

   (a) [1 mark] Every NFA accepts the empty string.

   false

   (b) [1 mark] The language $L = \{(01)^n \mid n \geq 0\}$ of binary strings comprised of an arbitrary number of concatenations of the string 01 is regular.

   true

   (c) [1 mark] The transition function δ of an NFA can take on the value $\emptyset$ (ε) (the empty set).

   ~~false~~ true

2. [4 marks] **Decision Problems**

(a) [2 marks] Give a formal description (in the form $L = \{\ldots \mid \ldots\}$) of the language for the following problem:

INPUT: a DFA M and a positive integer n
OUTPUT: "Yes" iff M has at least n states

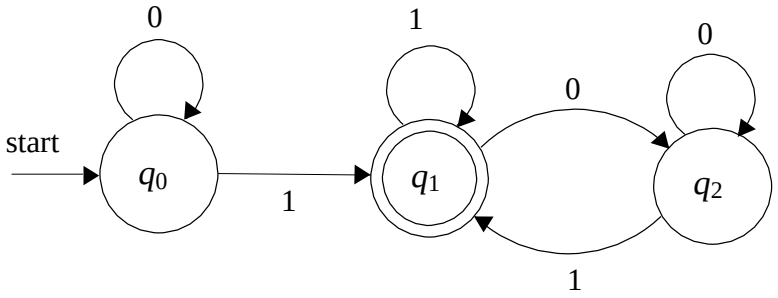
$L = \{M \text{ is DFA}, n \in \mathbb{Z}^{>0} \mid M \text{ has at least } n \text{ states}\}$

(b) [2 marks] State the decision problem for the language of all bit strings of length ≤ 10.

Input bit string w
Output: "Yes" iff $|w| \leq 10$

3. [10 marks] **DFAs**

Consider the DFA M, defined over the alphabet $\Sigma = \{0, 1\}$, with the following transition diagram:



(a) [3 marks] Give a formal description of M.

$M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

$F = \{q_1\}$

δ	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_1

(b) [2 marks] Write down the sequence of states that M assumes on input string $w = 0110$. Does M accept w?

q_0, q_0, q_1, q_1, q_2

no

(c) [2 marks] Write down the sequence of states that M assumes on input string $w = 01001$. Does M accept w?

$q_0, q_0, q_1, q_2, q_2, q_1$

yes

(d) [3 marks] Give a formal description (in the form $L = \{\ldots \mid \ldots\}$) of the language of M. Your description should be as simple as possible. State your answer only, no proof required.

$L = \{w \in \Sigma^* \mid w \text{ ends in } 1\}$

4. [8 marks] **NFAs**

Consider the language

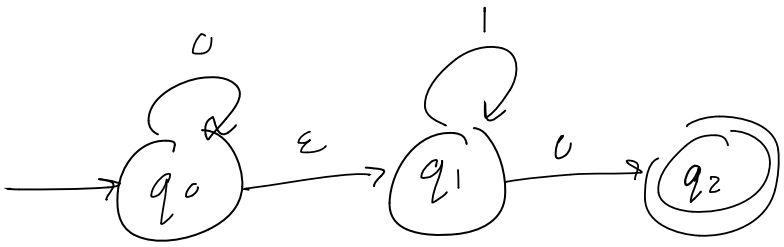
$$L = \{0^k 1^n 0 \mid k \geq 0, n \geq 0\}$$

over the alphabet $\Sigma = \{0, 1\}$.

(a) [2 marks] Is L regular? Why or why not?

yes, NFA can be constructed

(b) [3 marks] Give a state diagram of an NFA N with at most 3 states that accepts L.



(c) [3 marks] In this question, you will provide a partial proof of correctness of your NFA N of part (b). Prove that

$$L \subseteq L(N) .$$

Continue your proof on the next page if necessary. (You need not prove $L(N) \subseteq L$.)

Let $w \in L$, we need to prove $w \in L(N)$ Let $w = 0^k 1^n 0$ where $k, n \geq 0$

- First N will process 0 where q_0 only transitions back into q_0
- Next N process 1 where it will move from q_0 to q_1 and remain in q_1
- Lastly process 0, from q_1 to q_2 q_2 accepting state

Thus, $w \in L(N)$

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