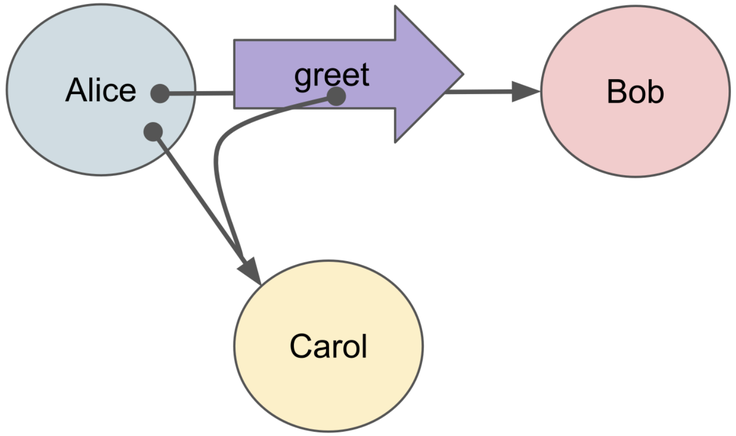
Reasoning about External Calls using Object Capabilities

Sophia Drossopoulou, Imperial College London

WG 2.3 Athens, 19th May 2025



James Noble

Susan Eisenbach

Julian Mackay

The Problem

External and Internal Code is tightly intertwined.

Object Capabilities used to control External Effects

Our Remit

Reason about Object Capabilities controling Effects:
1) Specification, 2) Verification

Internal (trusted) and External (untrusted) objects are intertwined.

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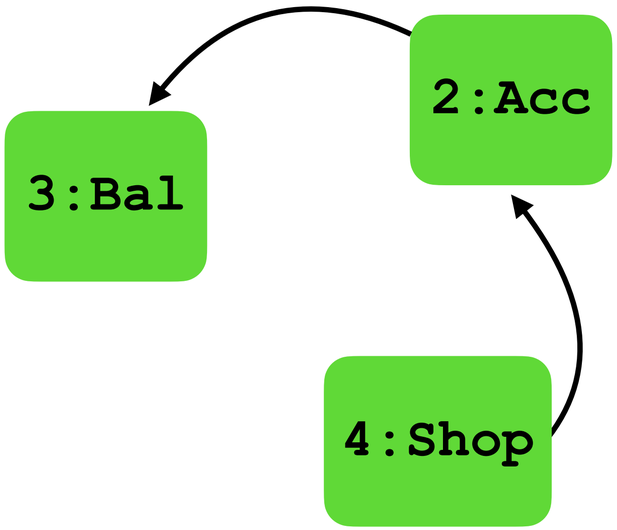

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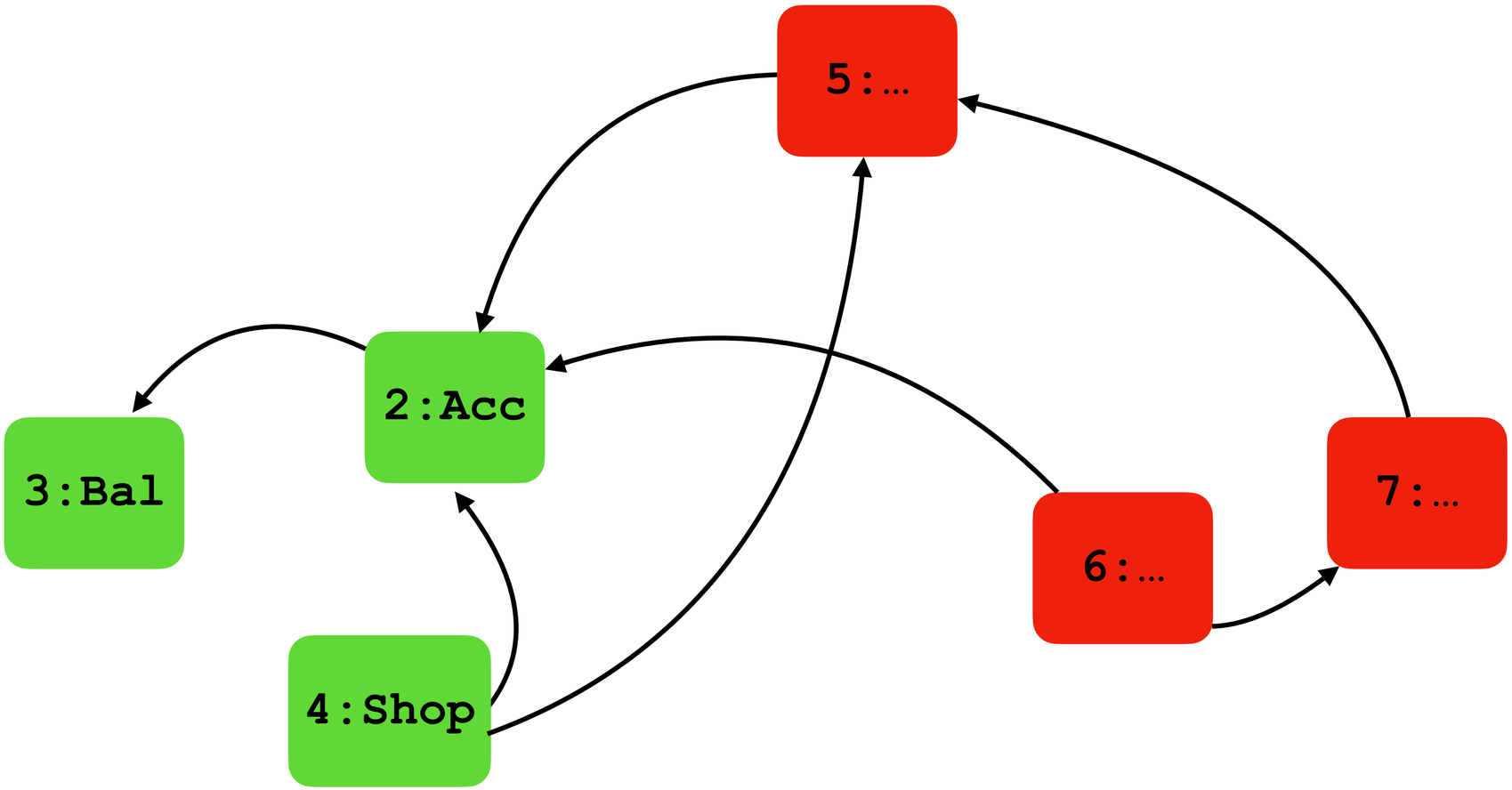
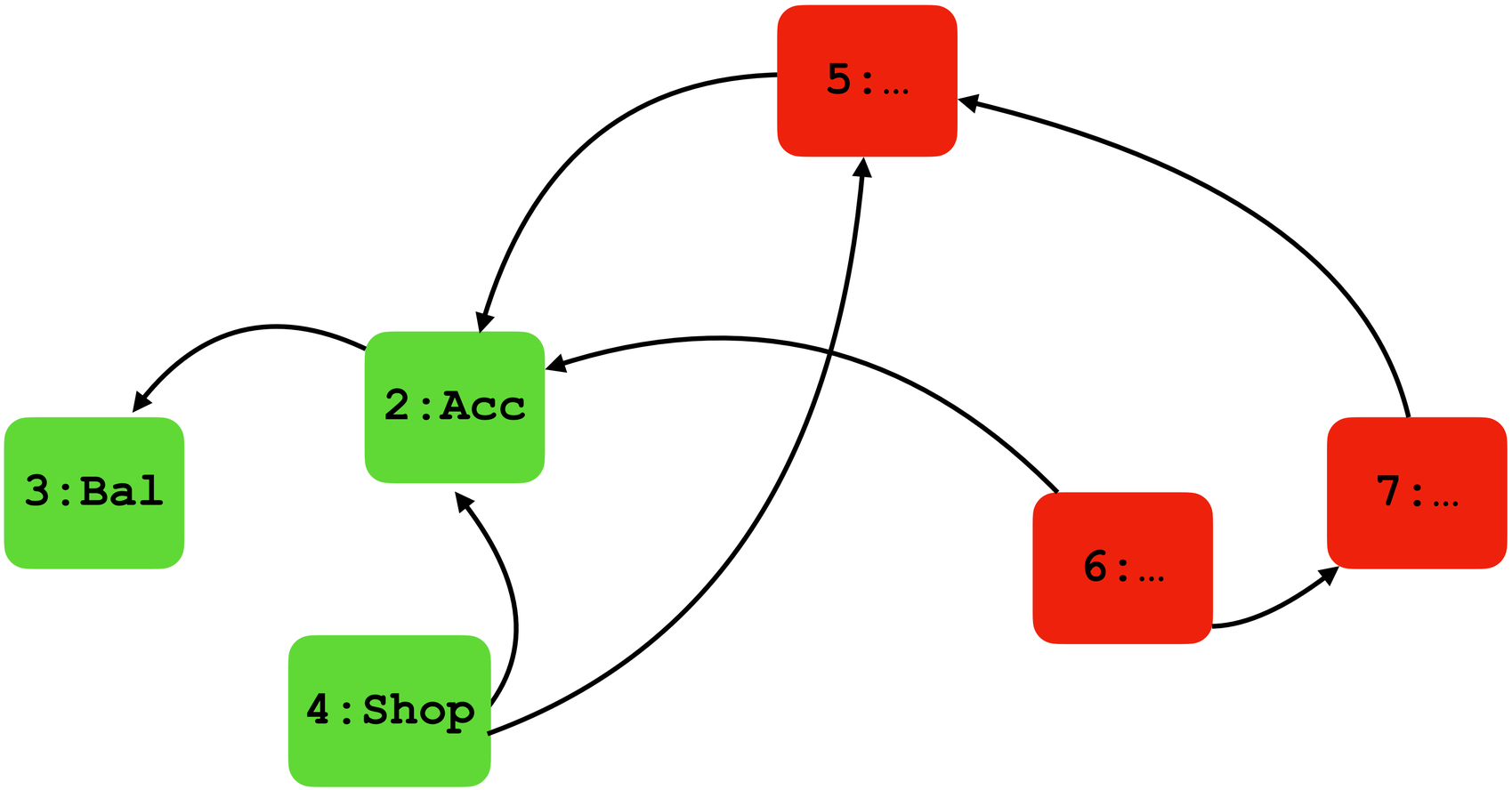


Internal (trusted) and External (untrusted) objects are intertwined.

Capability objects (ocap) **are necessary** for certain effects.



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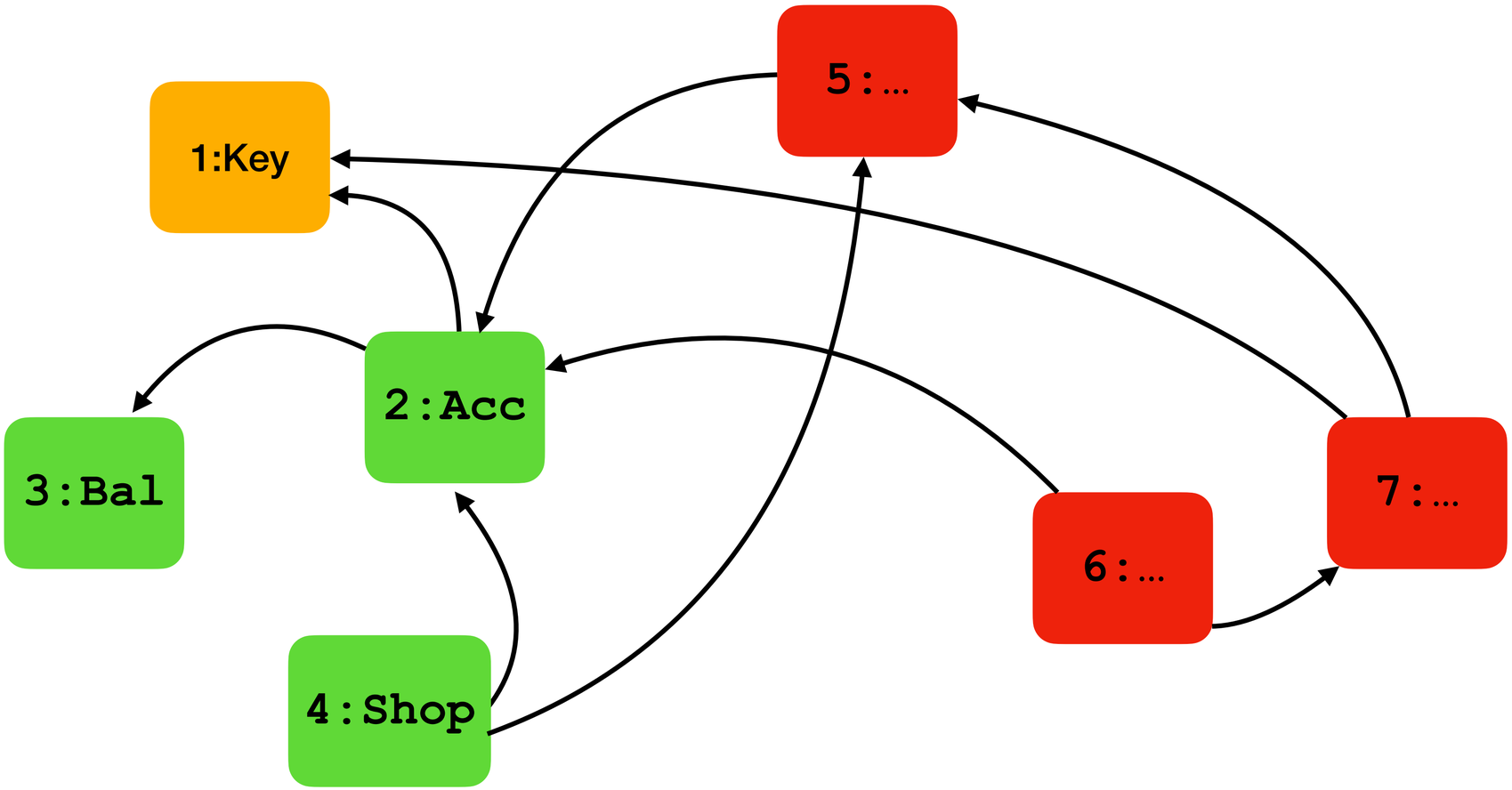
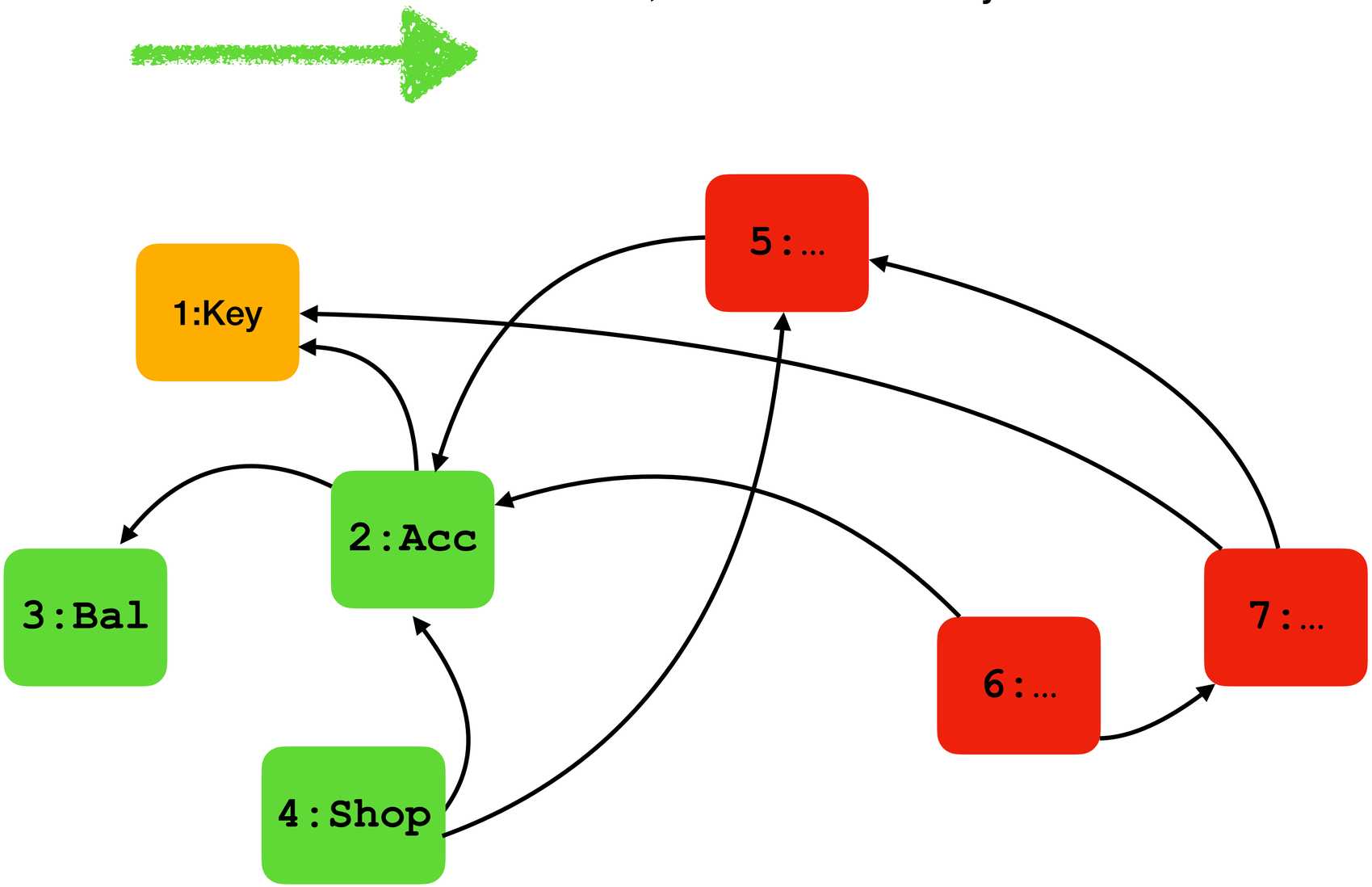


Internal Calls: Internal, or external objects call methods on internal objects.



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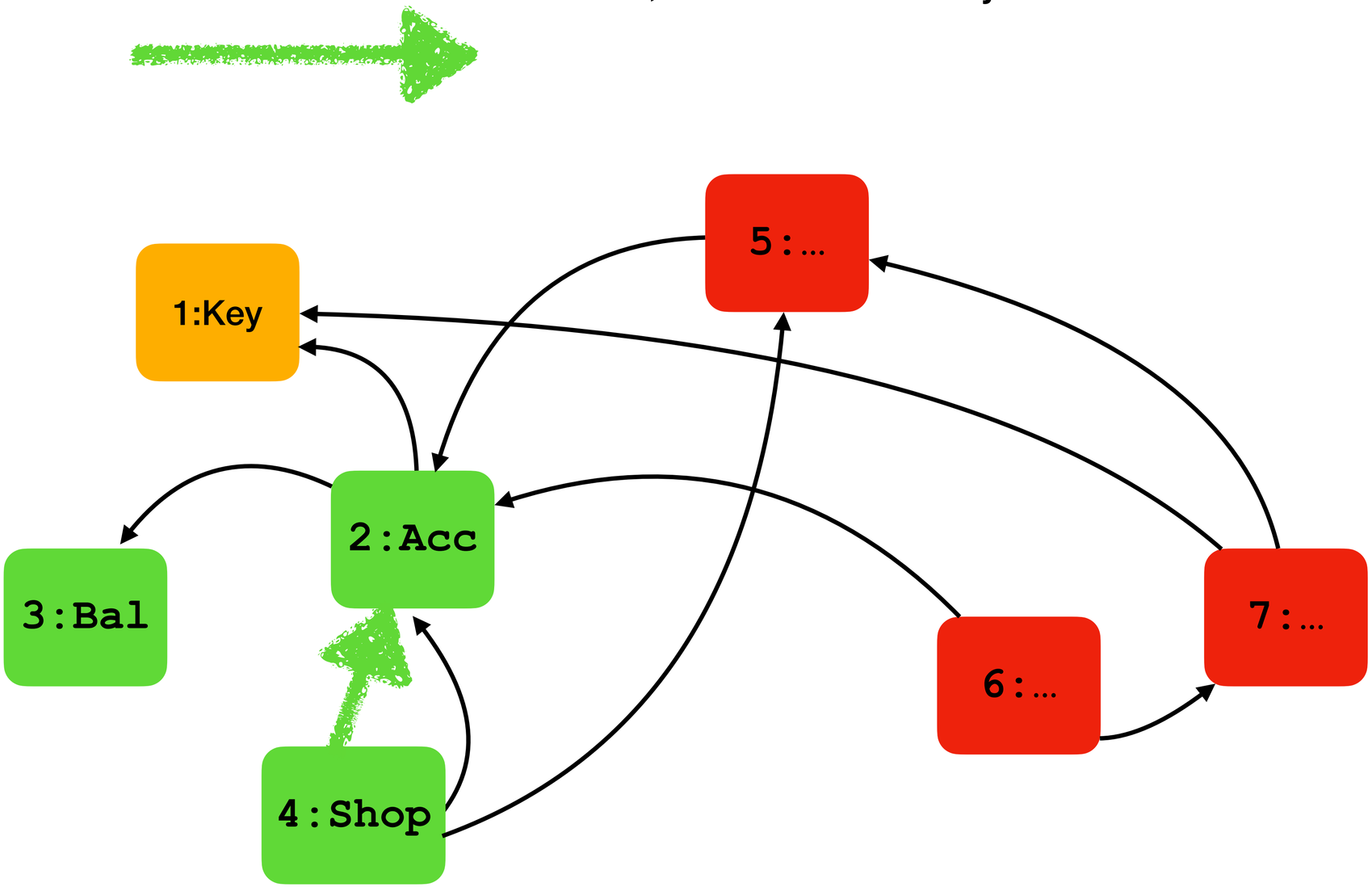

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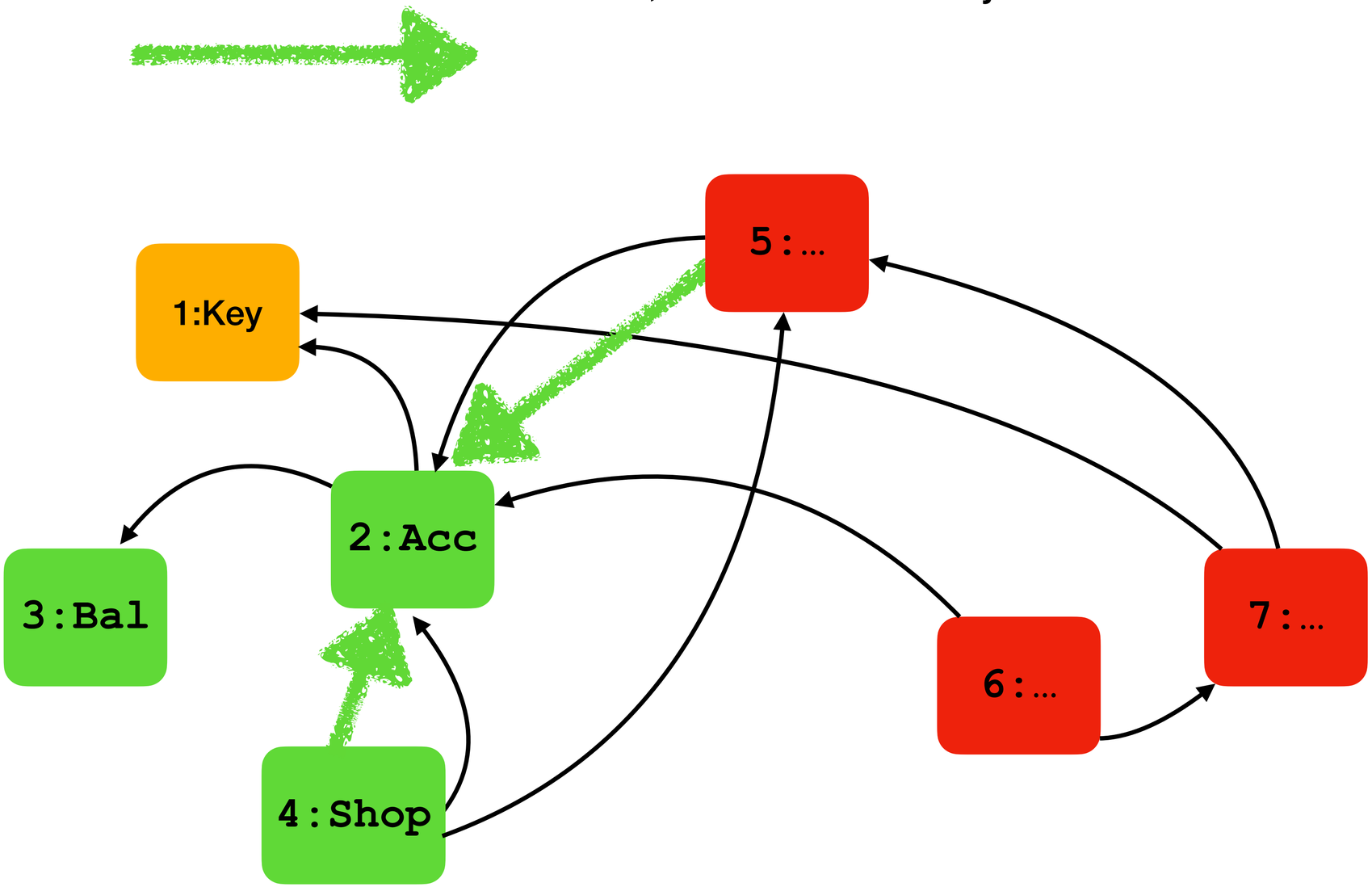
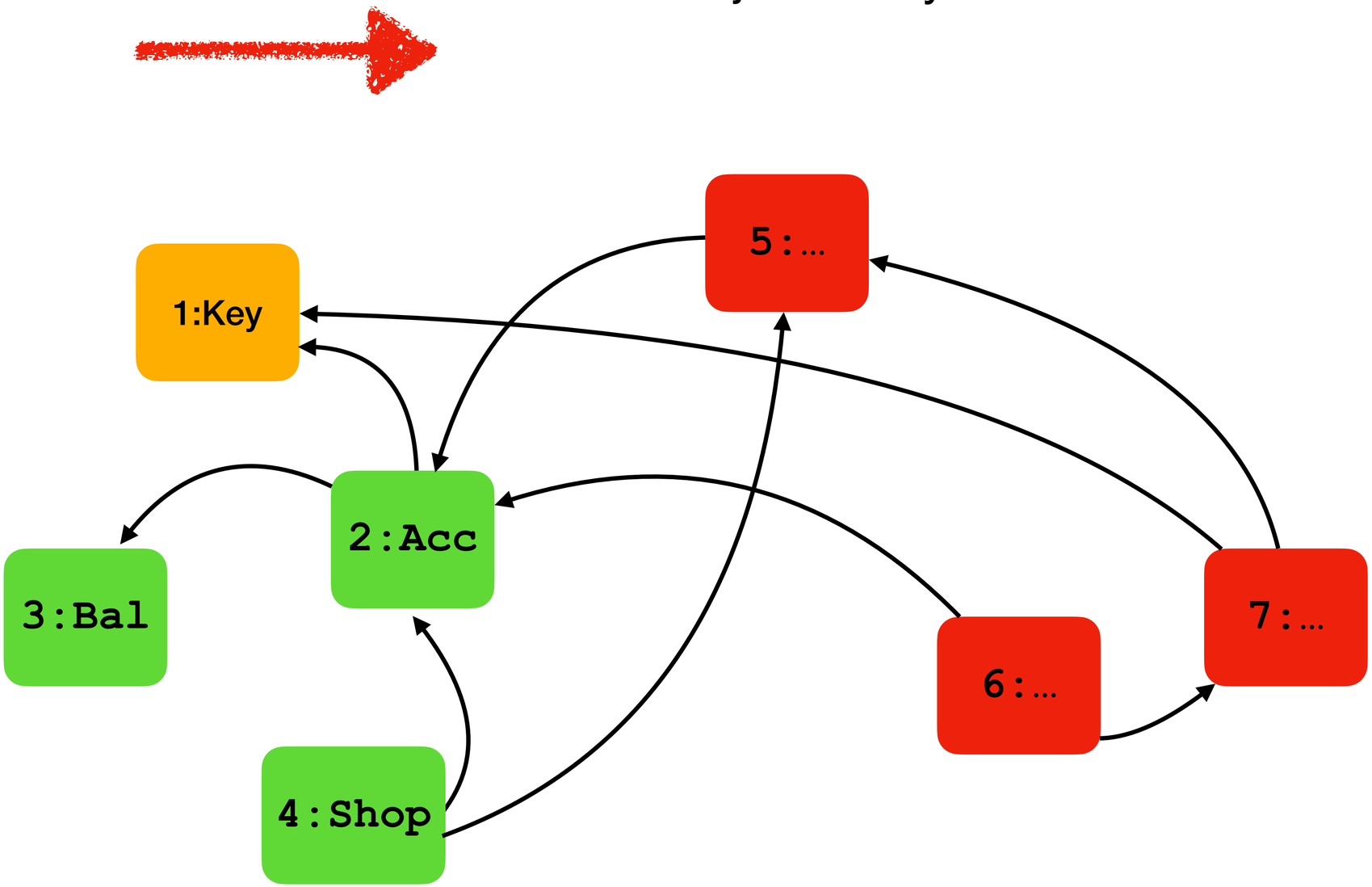


External Calls: Internal objects may call methods on external objects.



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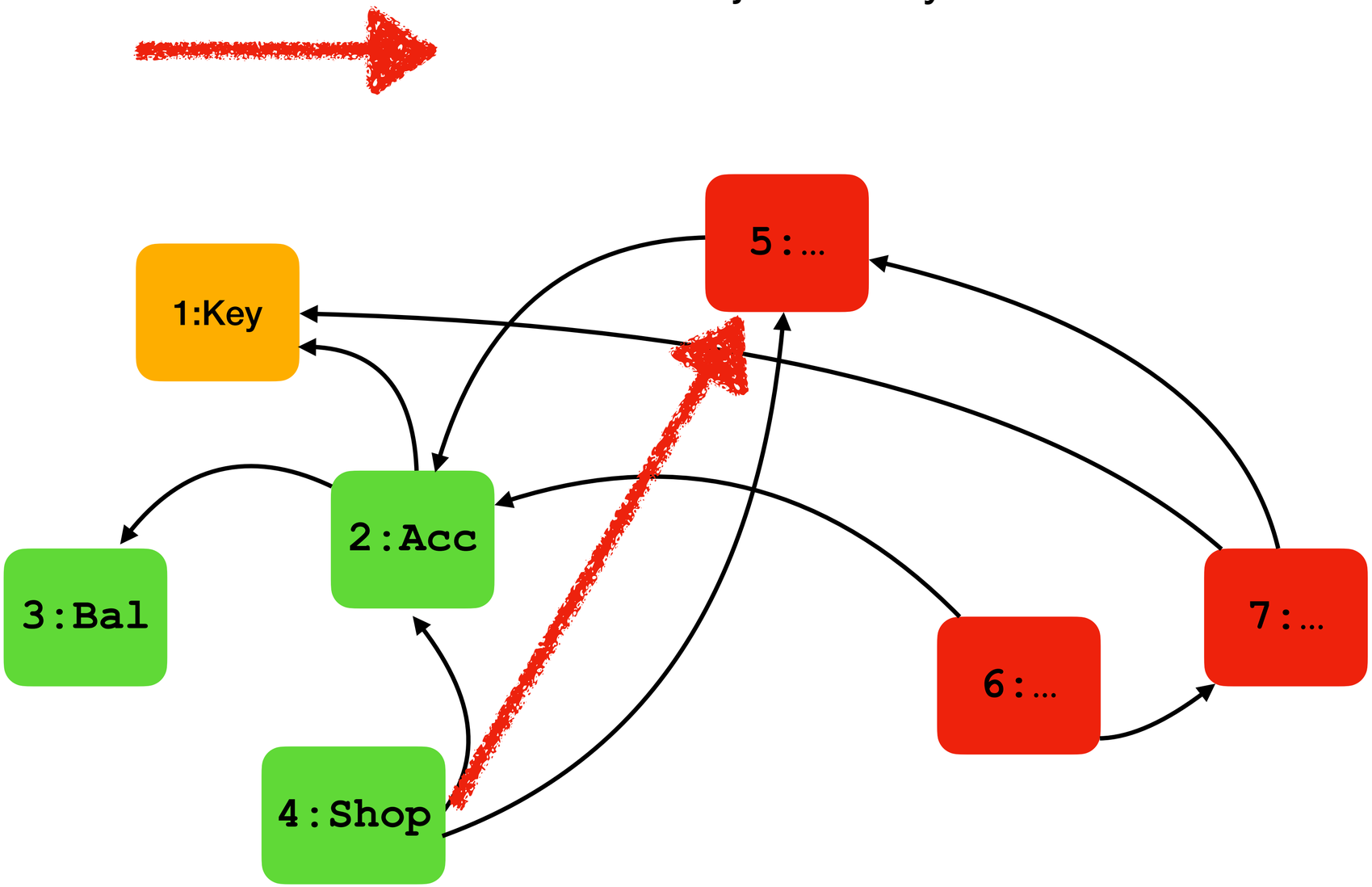

```
module M_intl
  method m1 ..
    ...  trusted code ...
  method m2(untrst:external)
    ... trusted code ...
     untrst.unkn(this)
    ... trusted code ...
```

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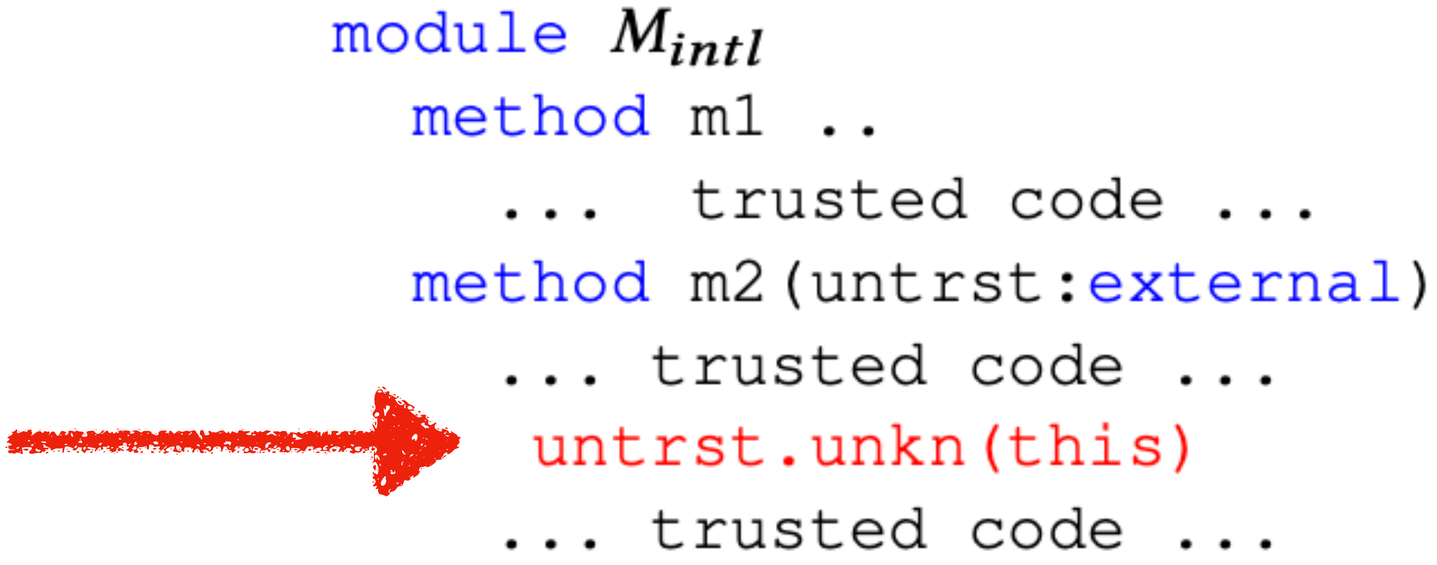
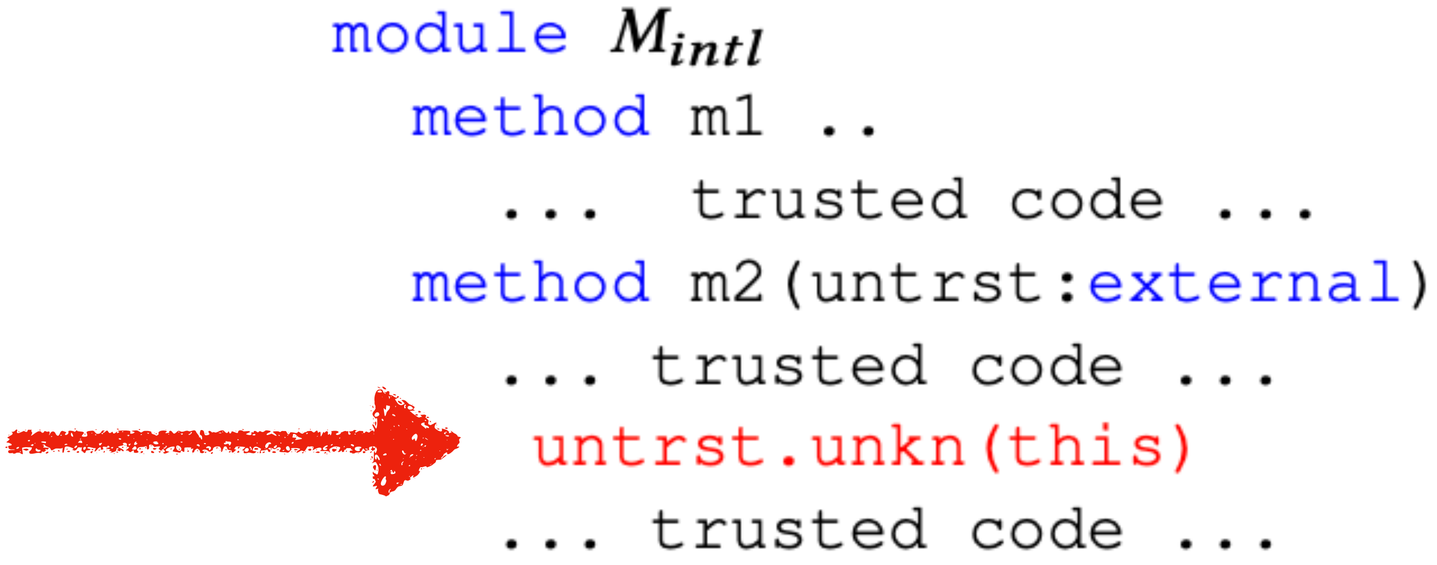


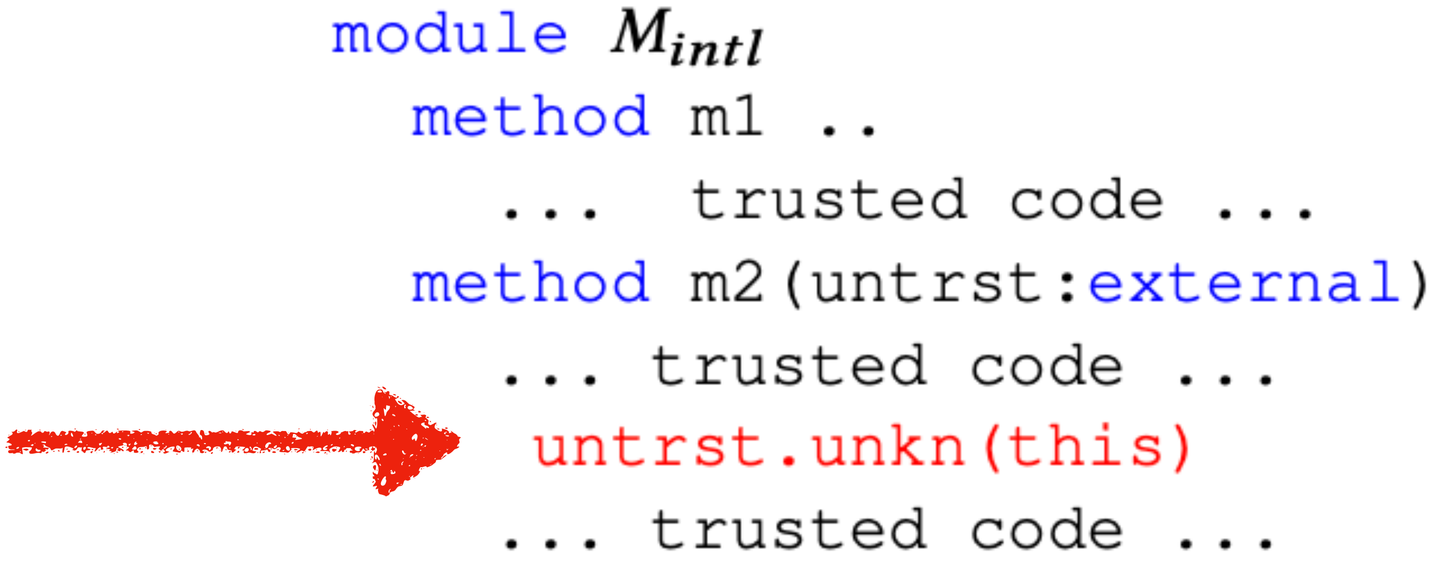
What are the possible effects at the external call?



What are the possible effects at the external call?

1) Some effects will **never** happen

2) Some effects **may** happen — *but only if* external access to certain capabilities.



What are the possible effects at the external call?

1) Some effects will **never** happen cf. object invariants

2) Some effects **may** happen — *but only if* external access to certain capabilities.

Our Work

```
class Shop

  field accnt:Account, invntry:Inventory, clients:external

  public method buy(buyer:external, anItem:Item)
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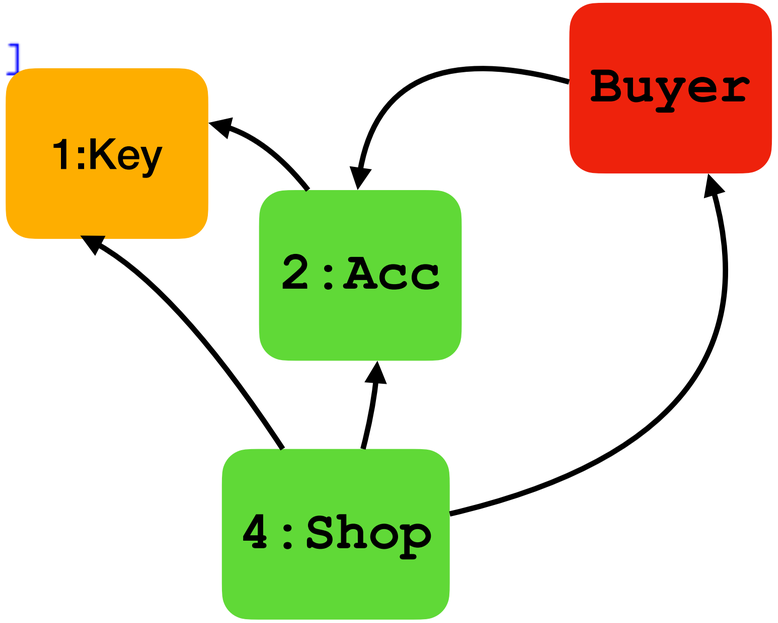

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  public method buy(buyer:external, anItem:Item)
     int price = anItem.price
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     buyer.pay(this.accnt, price)
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        this.send(buyer,anItem)
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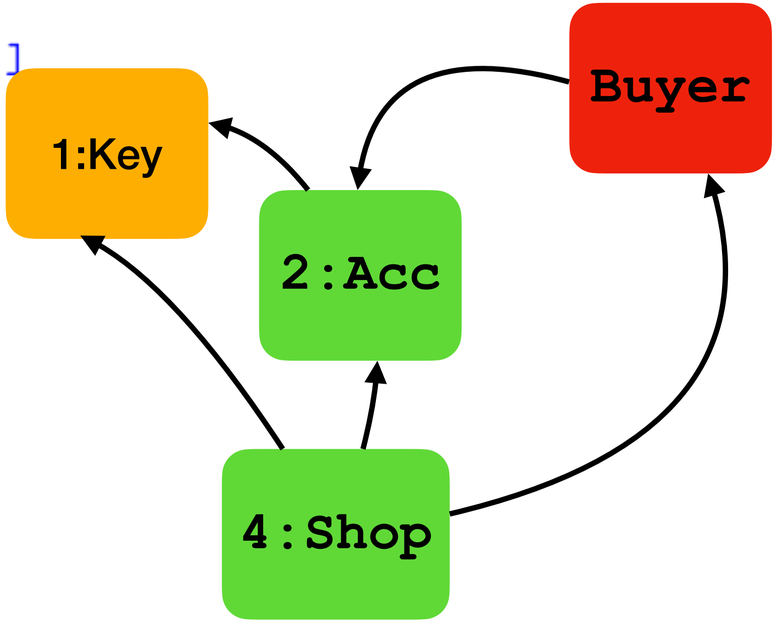

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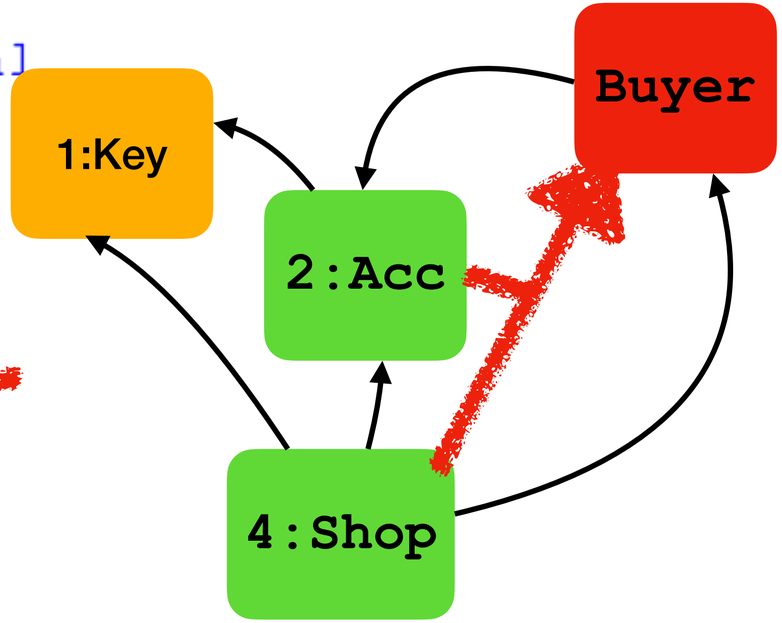

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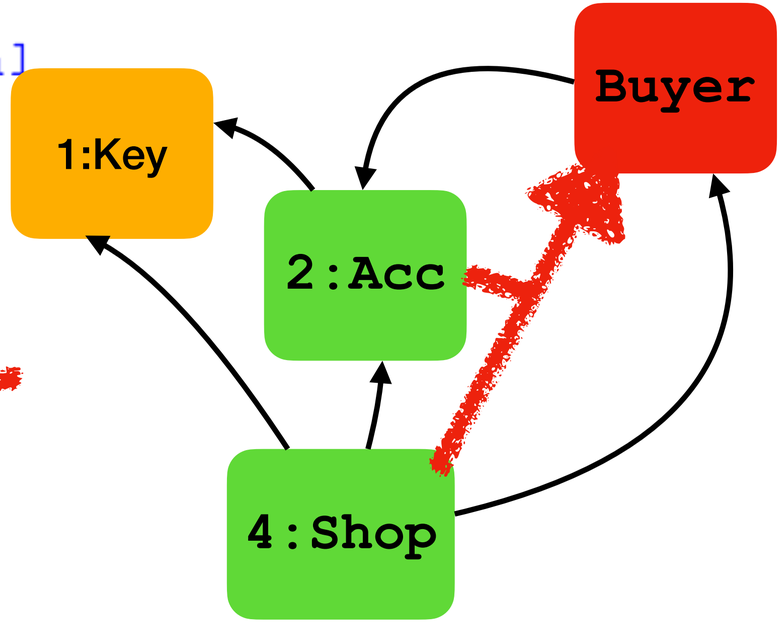

What are the possible effects at the external call?

Q Can buyer steal money from the shop's account?

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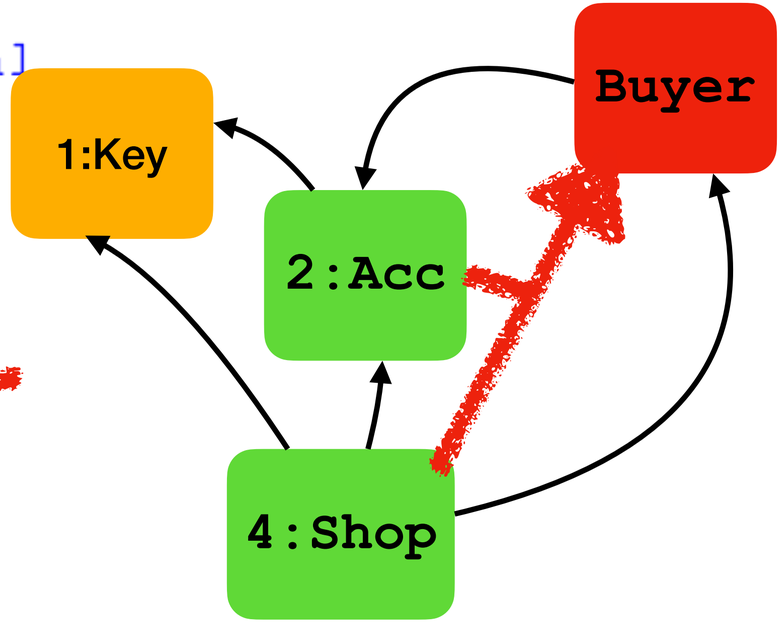

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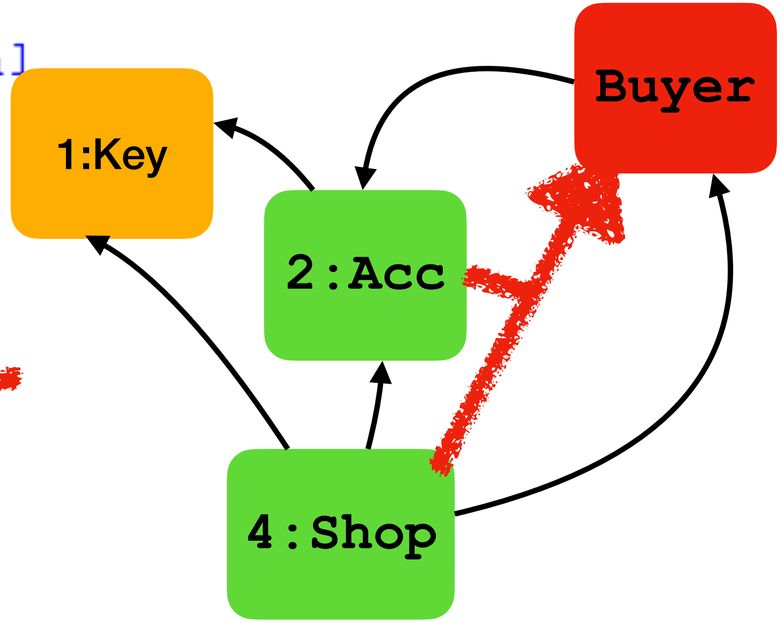

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A No, if a) Money not reduced unless external access to account's key

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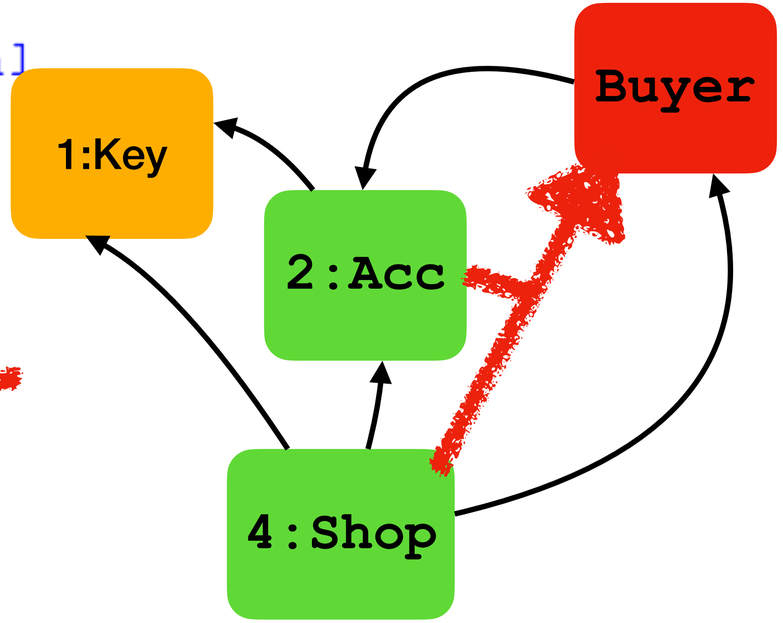

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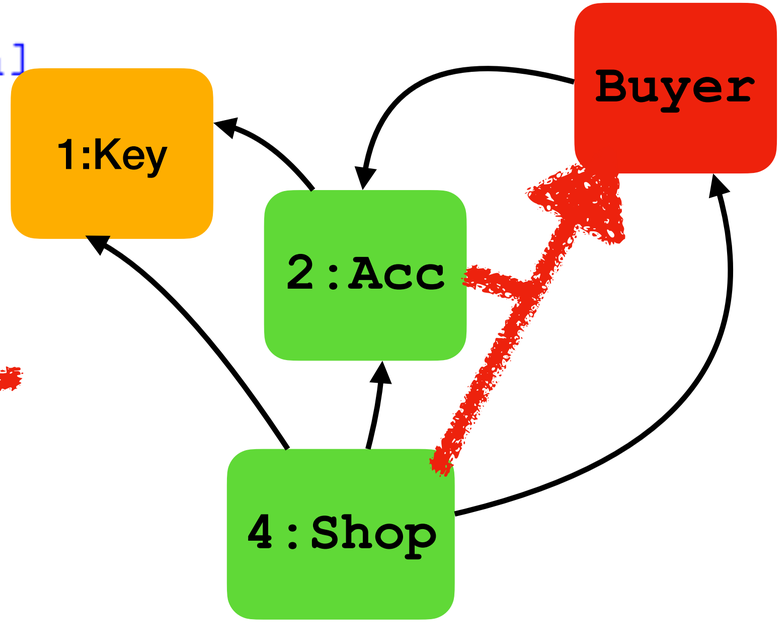

What are the possible effects at the external call?

Q Can buyer steal money from the shop's account?

A No, if a) Money not reduced unless external access to account's key

b) Buyer has no access to account's key

c) Module does not leak the account's key

The Account example in Code

The Account example in Code

(fields are private)

Three Modules

```
module M_good
  class Shop    ... as earlier ...
  class Account
    field blnce:int
    field key:Key
    public method transfer(dest:Account, key':Key, amt:nat)
       if (this.key==key')  this.blnce-=amt; dest.blnce+=amt
    public method set(key':Key)
       if (this.key==null)  this.key=key'
```

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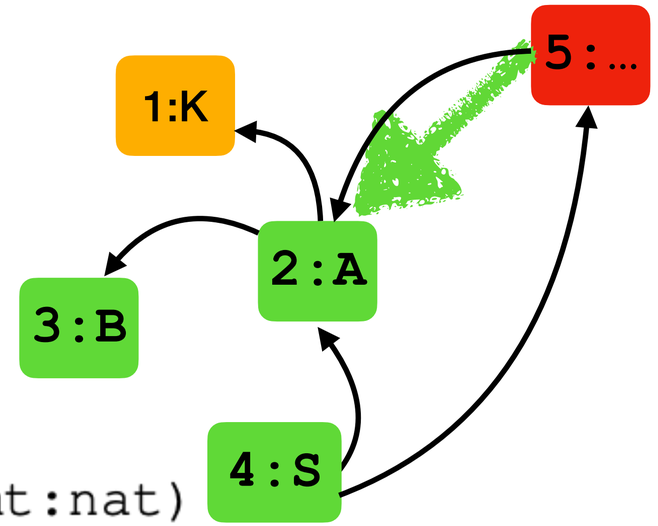

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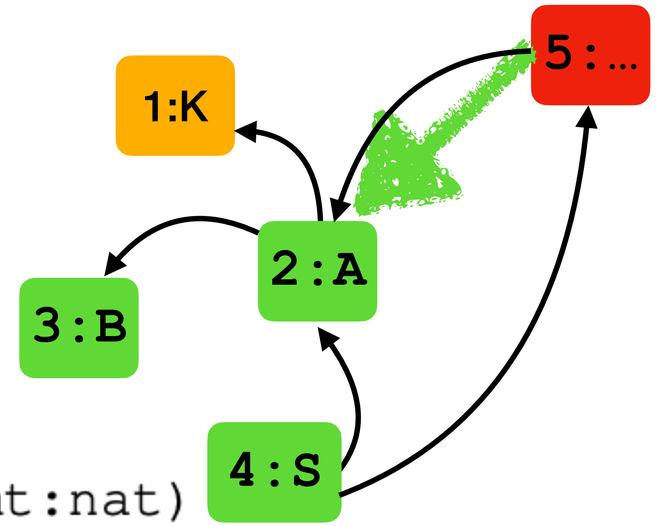


```
  module M_bad
    ... as earlier ...
    public method set(key':Key)
      this.key=key'
```

Three Modules

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module M_good
  class Shop    ... as earlier ...
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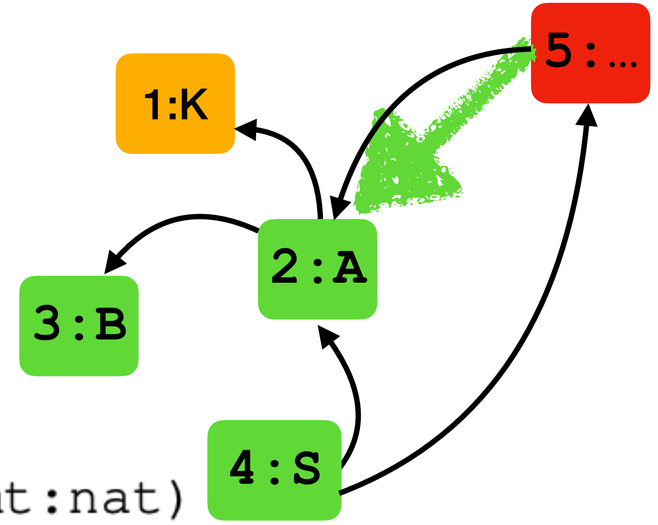


```
module M_bad
  ... as earlier ...
  public method set(key':Key)
    this.key=key'
```

```
module M_fine
  ... as earlier ...
  public method set(key',key'':Key)
    if (this.key==key') this.key=key''
```

Three Modules

```
module Mgood
  class Shop    ... as earlier ...
  class Account
    field blnc
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    public met                                    t:nat)
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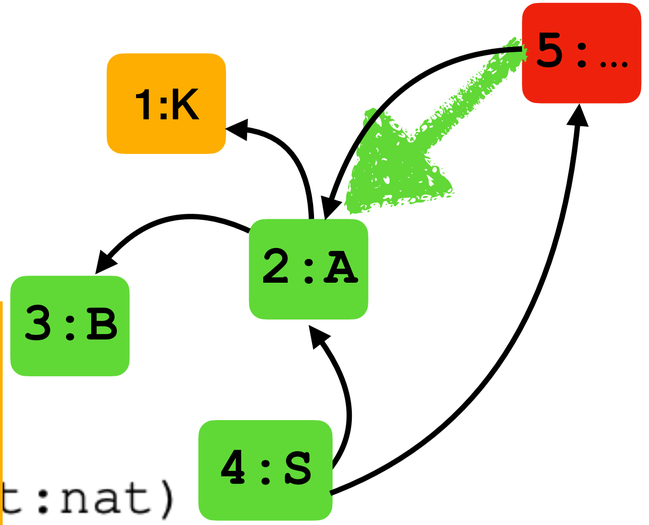

Remit_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{fine} \models S$

```
module Mbad
  ... as earlier ...
  public method set(key':Key)
    this.key=key'
```

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module Mfine
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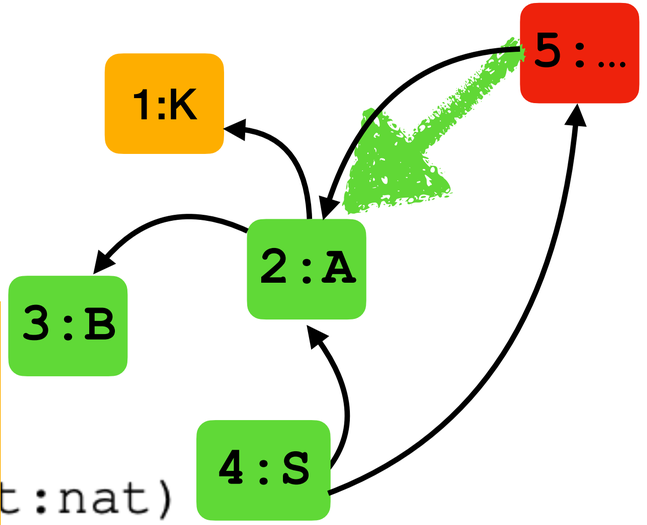

Remit_2: An inference system with which to prove

$M_{good} \vdash S$

$M_{bad} \nvdash S$

$M_{fine} \vdash S$

```
module Mbad
  ... as earlier ...
  public method set(key':Key)
    this.key=key'
```

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Three Modules

```
class Shop

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    buyer.pay(this.accnt, price)
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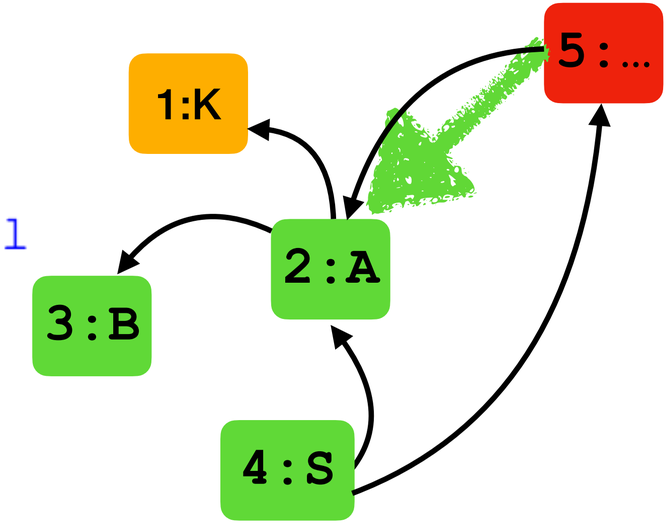

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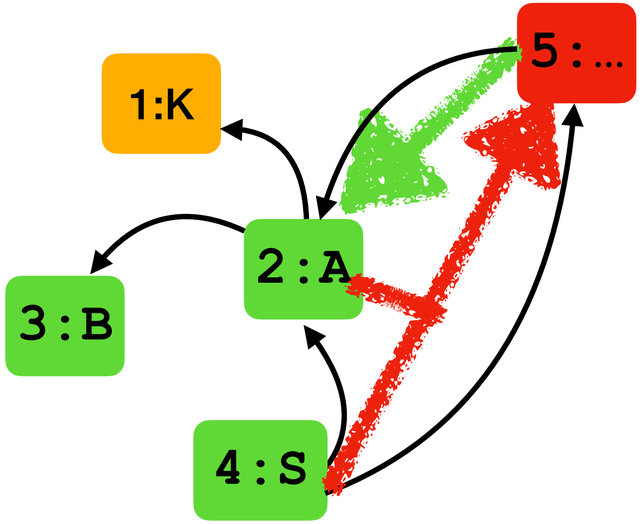

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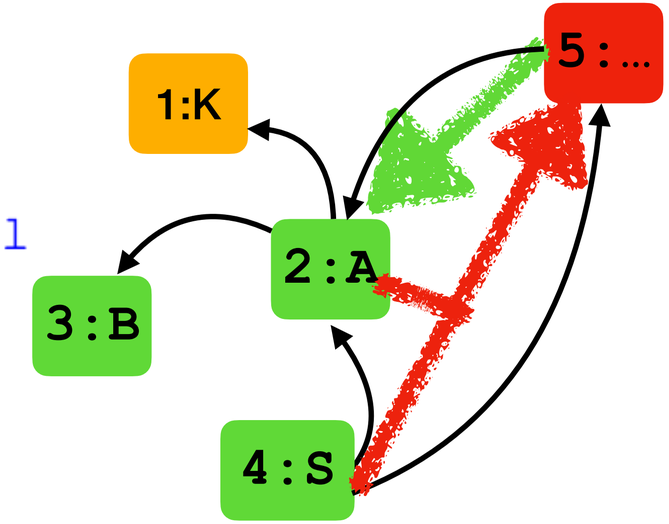


If a) `Account` comes from a “good” module, and
b) `buyer` has no “unprotected” access to `4.accnt.pwd`,
then
`buyer.payme(..)` will not decrease `4.accnt.blnce`,

Three Modules

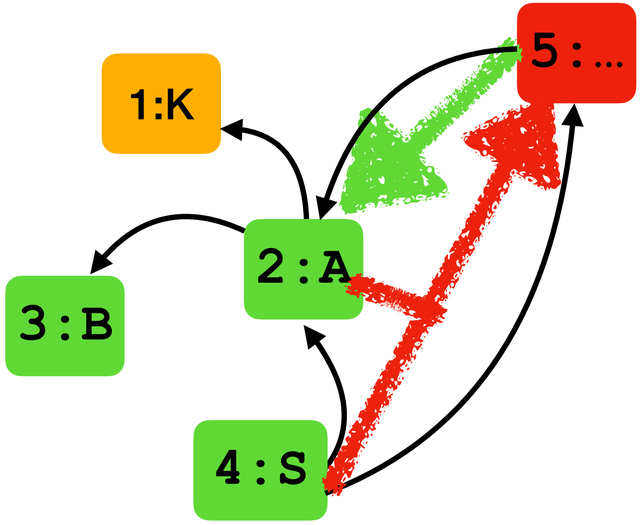
```
class Shop

  field accnt:Account, invntry:Inventory, clients:external

  public method
    int price =
    int oldBlnc
    buyer.pay(t
    if (this.ac
       this.sen
    else
       buyer.te
```

Remit_3: An inference system with which to prove

external calls



If a) `Account` comes from a “good” module, and
b) `buyer` has no “unprotected” access to `4.accnt.pwd`,
then
`buyer.payme(..)` will not decrease `4.accnt.blnce`,

In Summary

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External Objects

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- may execute arbitrary code
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Our Specifications

- guarantee that certain effects happen only under certain condtic
- In genereal do *not* preclude these conditions

Remit_1: A specification language,

and module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{fine} \models S$

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- **only** by external objects calling methods on internal objects, and
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A ∧ “no external access to OCAP” is “invariant”

We need to determine

- “no external access to OCAP”
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We now determine

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1st Answer

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We now determine

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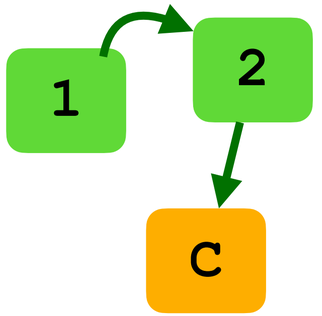

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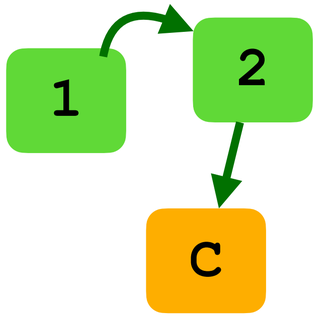

***Unsound*!**

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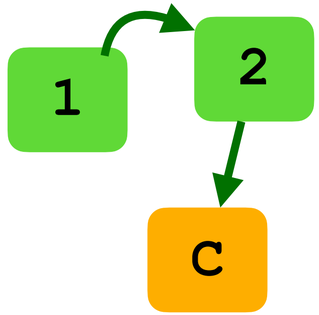

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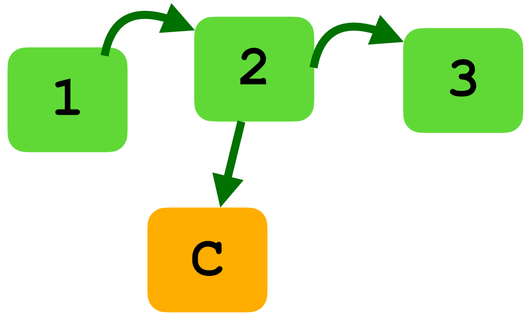

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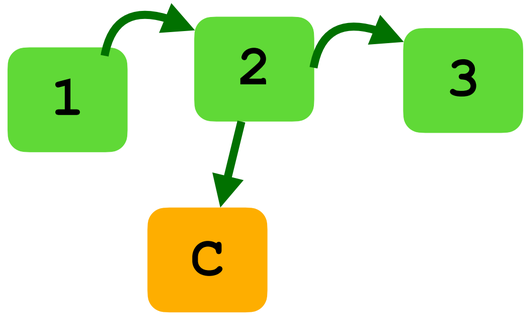

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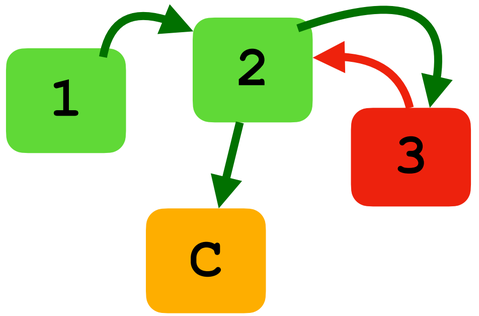

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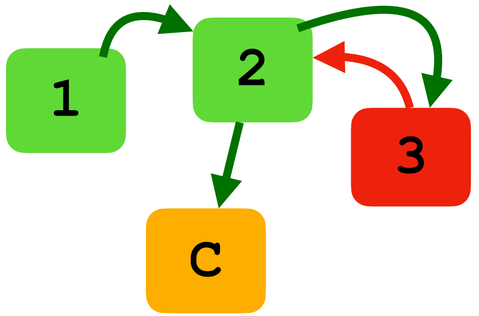

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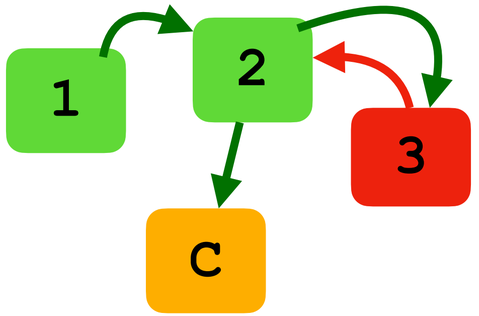

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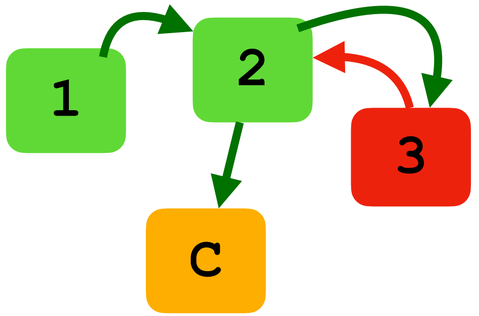

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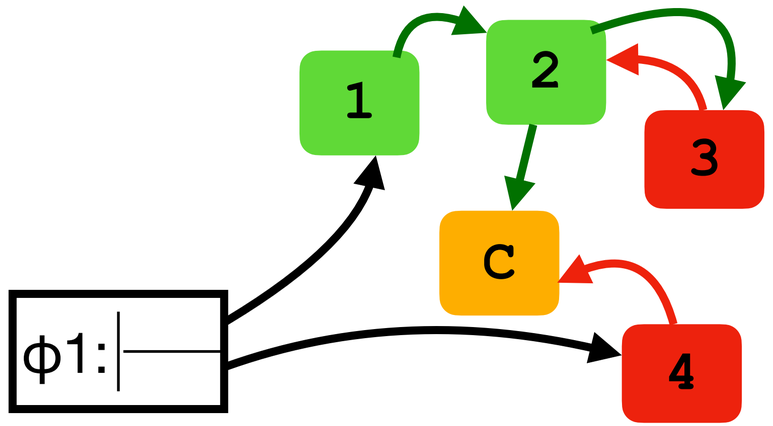

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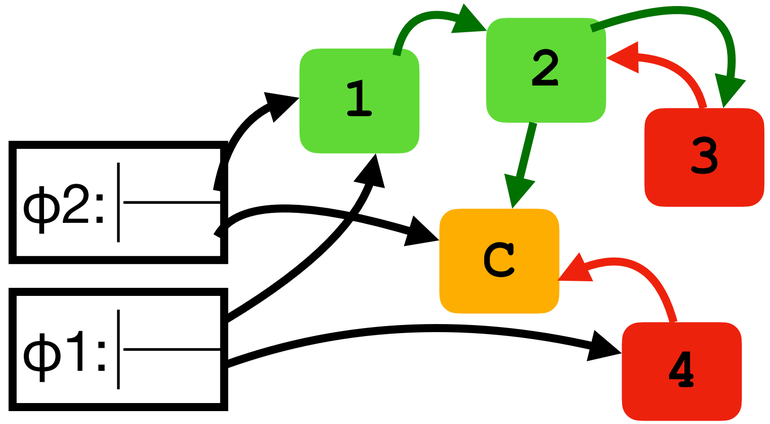

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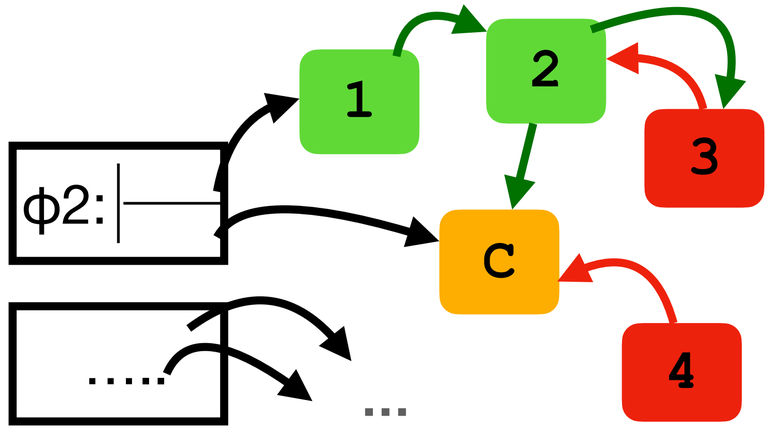

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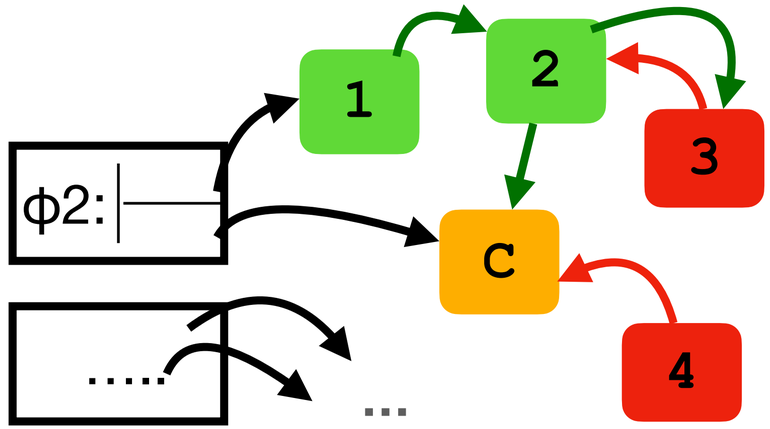

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invariant ≜ preserved in external states (`this` is external),
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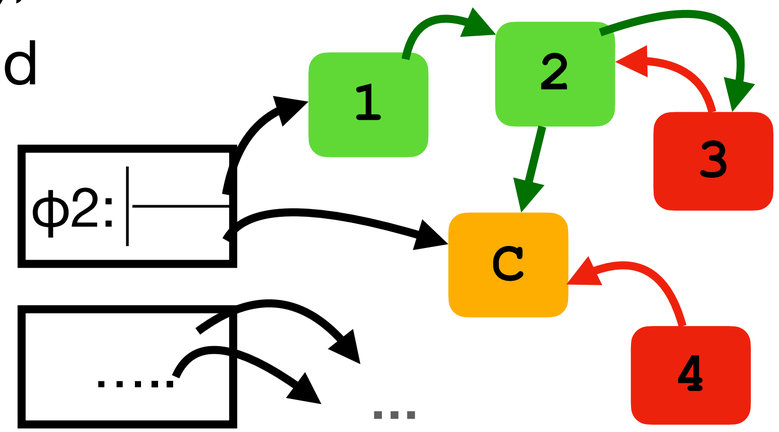

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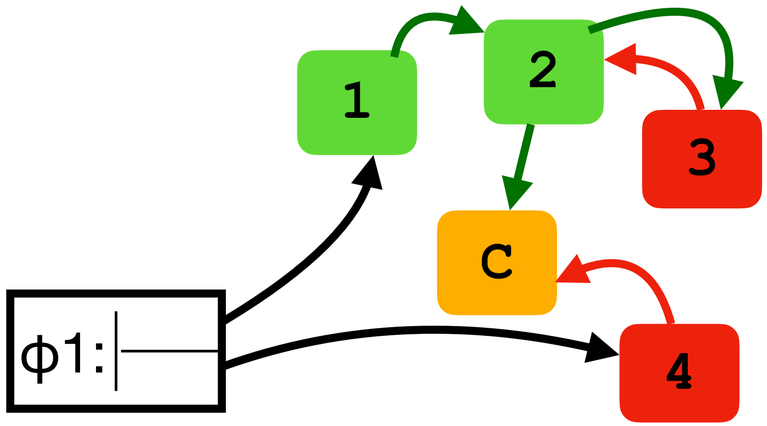

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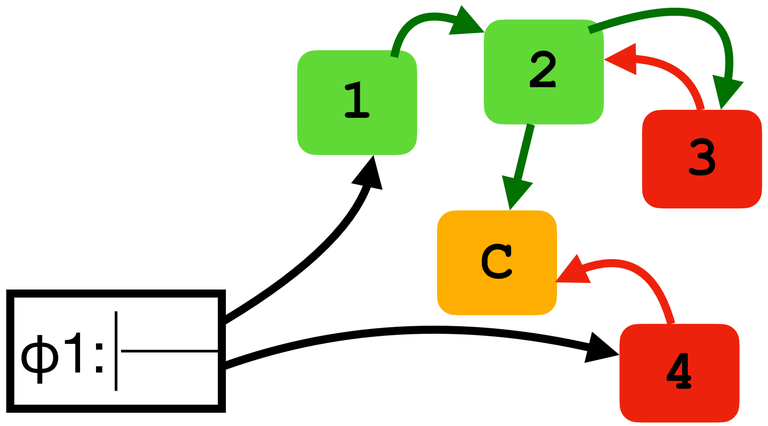
《o》



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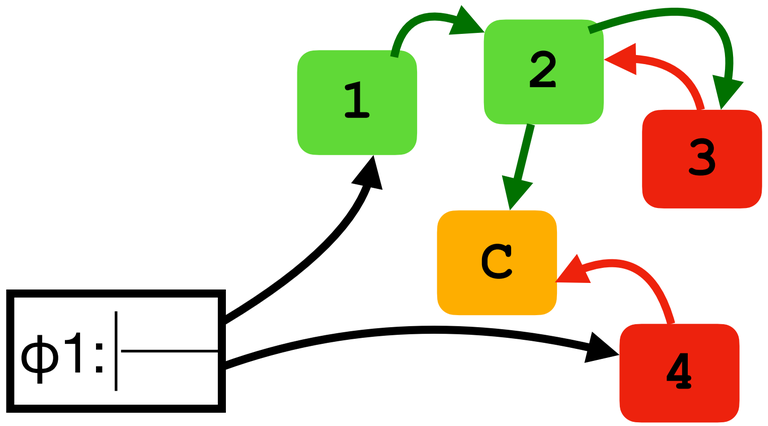

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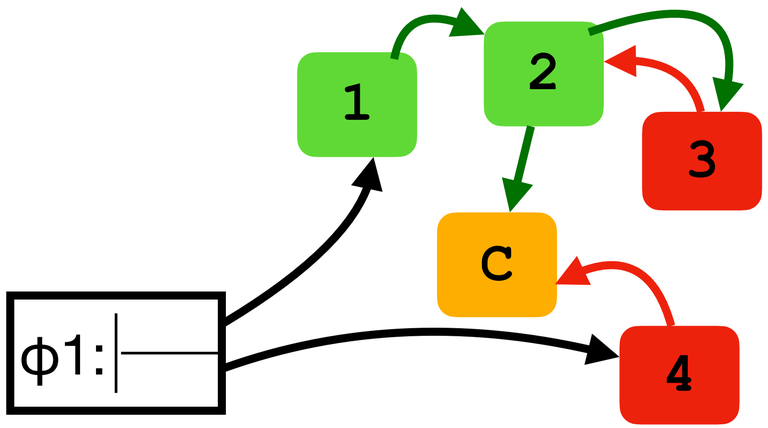

$\Phi_1 \not\models \langle\!\langle C \rangle\!\rangle$

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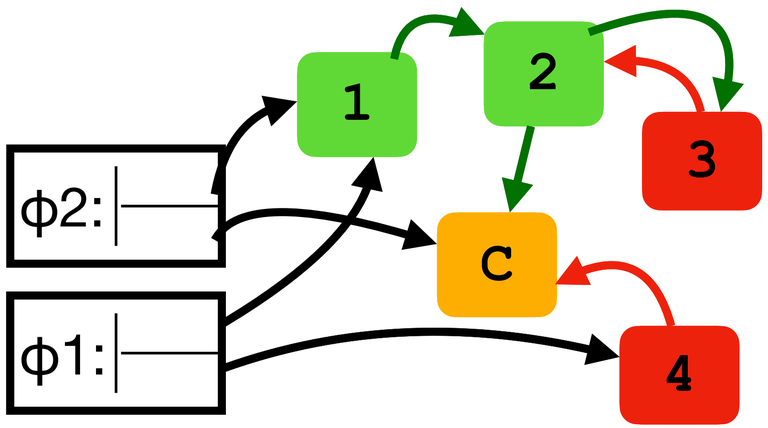

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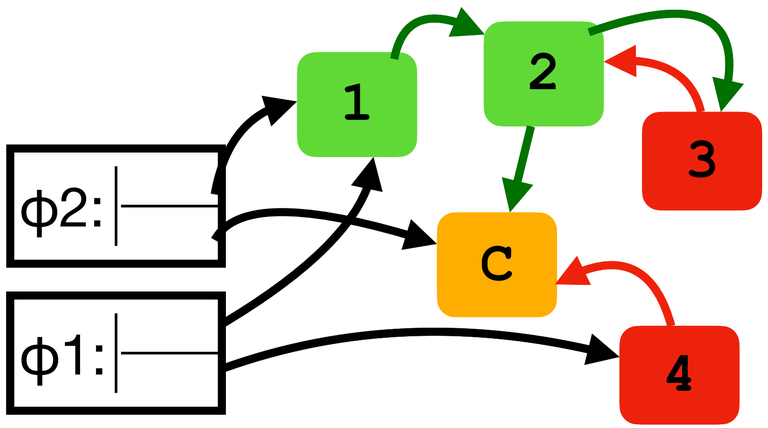

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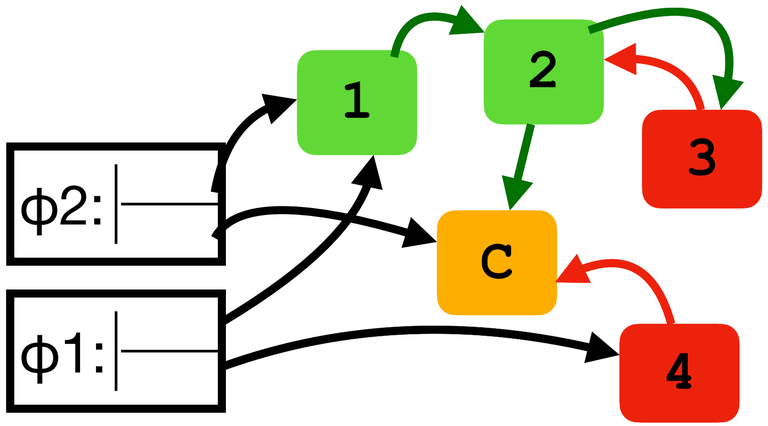

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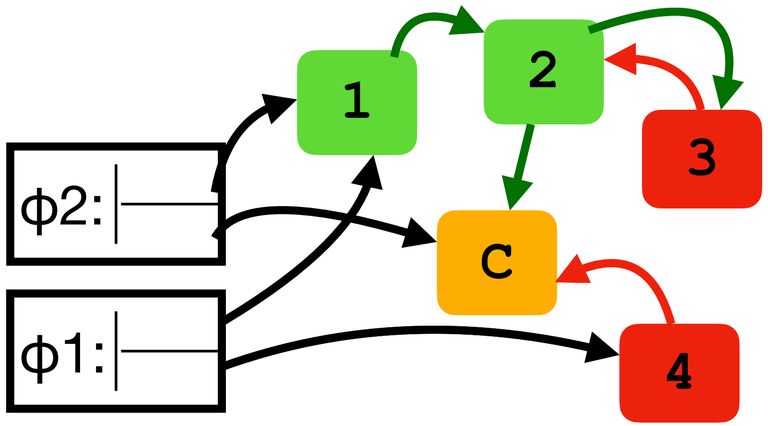

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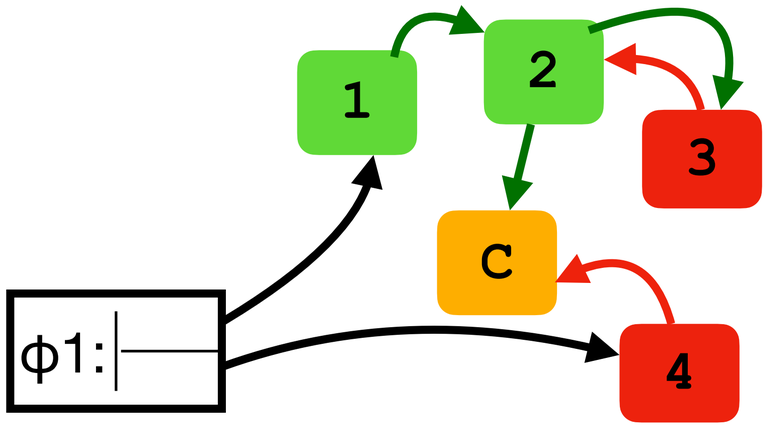
Protection is “relative” to top frame

We also define …



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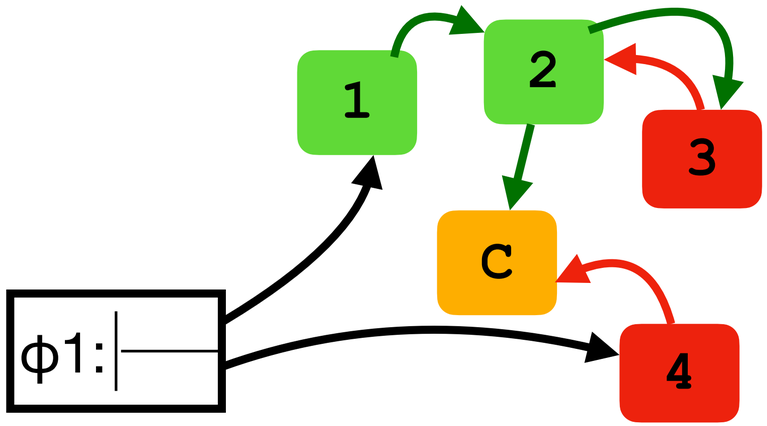

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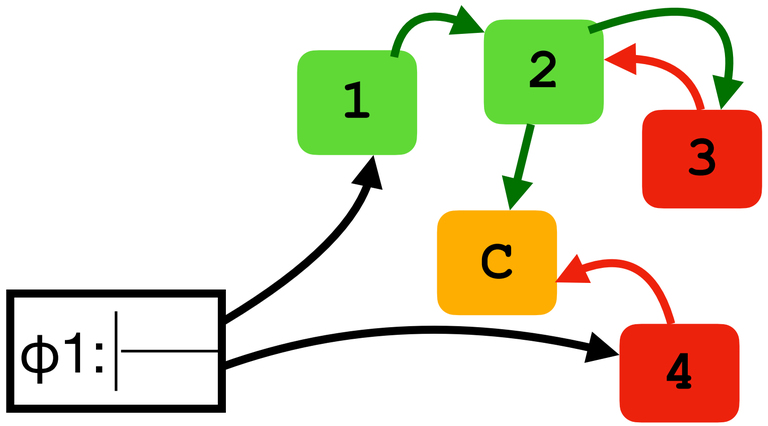

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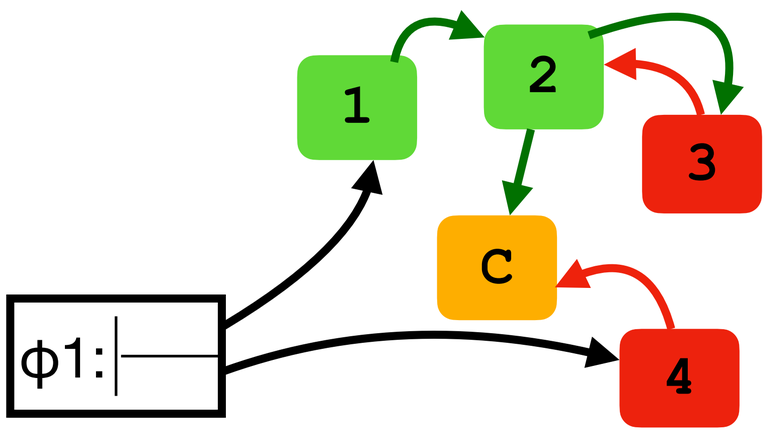

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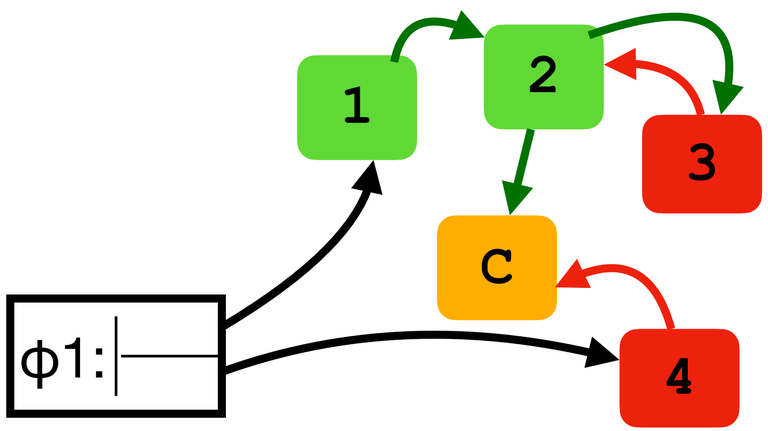

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$$M, \sigma \vDash \texttt{this}\texttt{:}\ \text{ext} \ \wedge\ \alpha_1 :C_1,\ldots\alpha_n{:}Cn\ \wedge A[\alpha_1,\ldots\alpha_n\ /\ x_1,..x_n]$$

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Remit_1_b : Express/meaning “preserved in external states, during (scoped) execution of current call

We propose “*scoped* invariants”, of the form $\forall x_1{:}C_1,\ldots x_n{:}C_n\ \{\ A\ \}$

where A is a normal assertion, and may talk of protection.

Eg $\forall\, \texttt{s:Shop}.\ \{\ \texttt{s.account} \neq \texttt{null} \rightarrow \langle\!\langle\ \texttt{s.account.key} \rangle\!\rangle\ \}$

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Any number of execution steps,
before returning from current frame

Definition

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An example:

Consider call graph below, with green disks for internal states (eg σ_2),
and pink disks for external states (eg σ_{28}).

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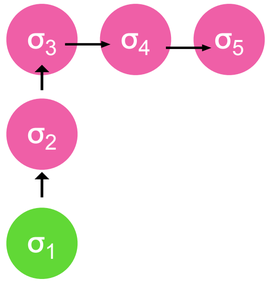

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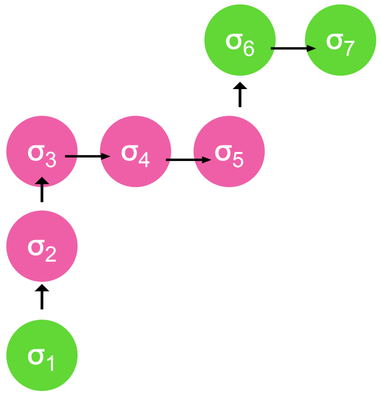

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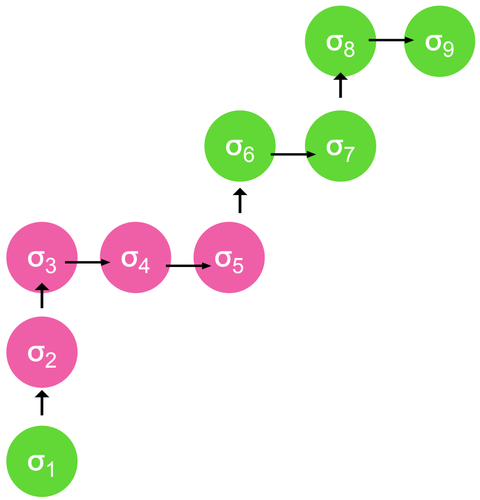

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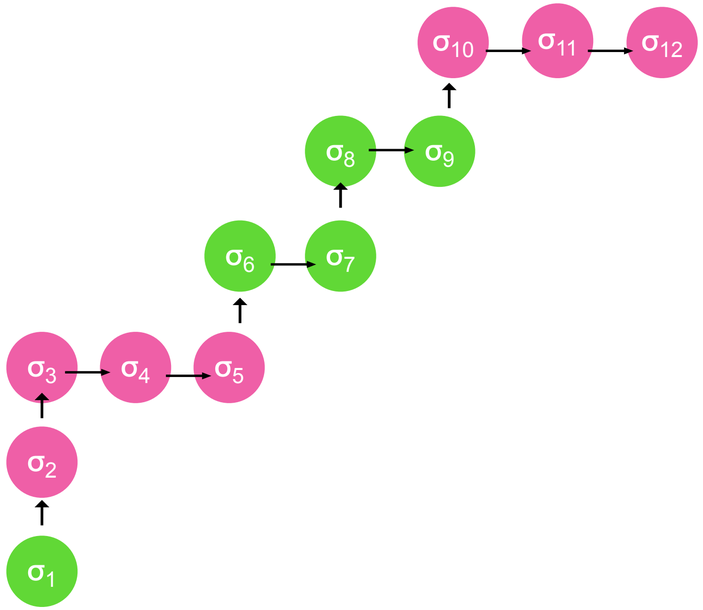

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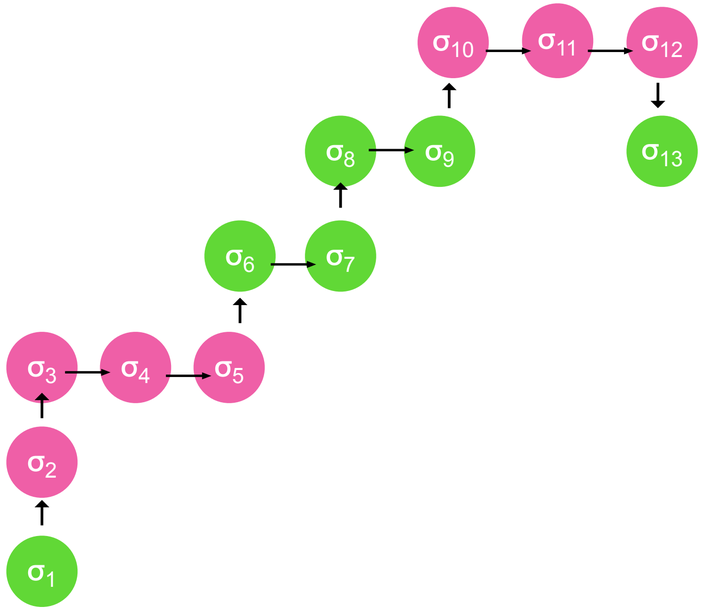

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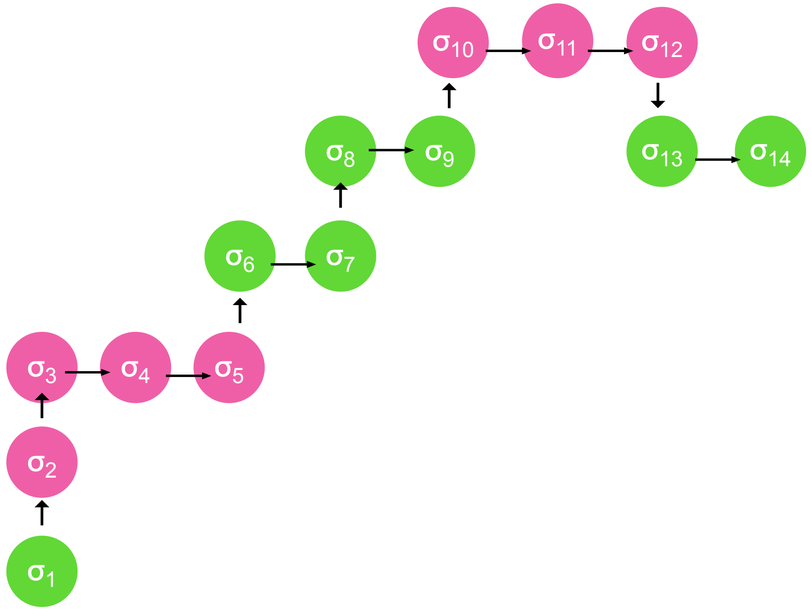

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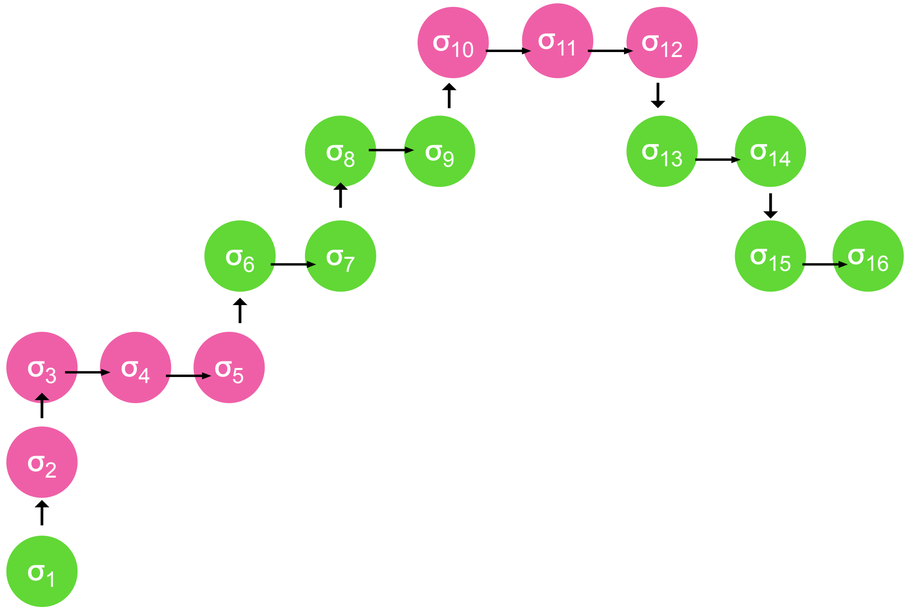

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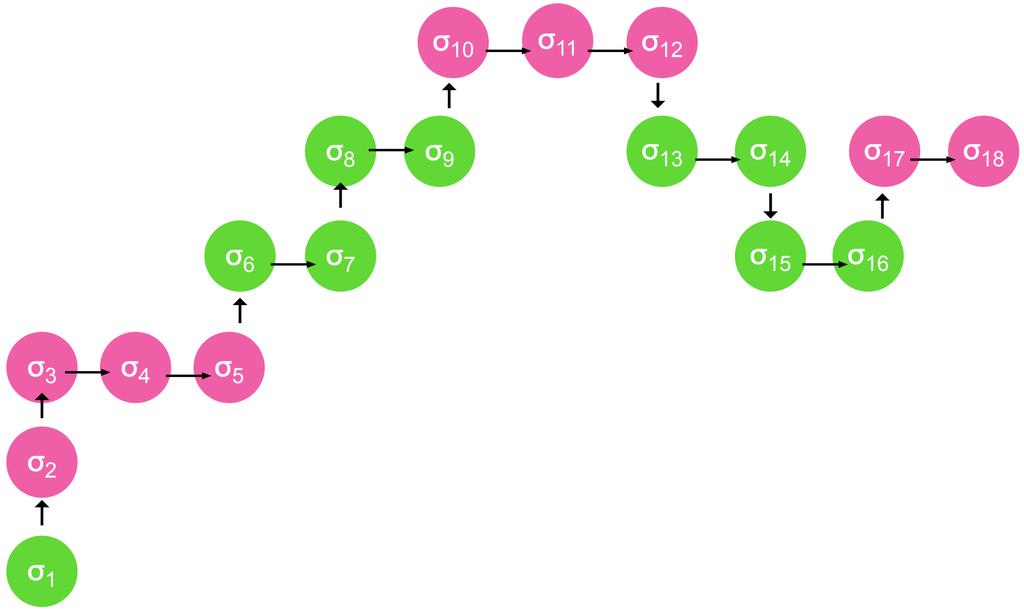

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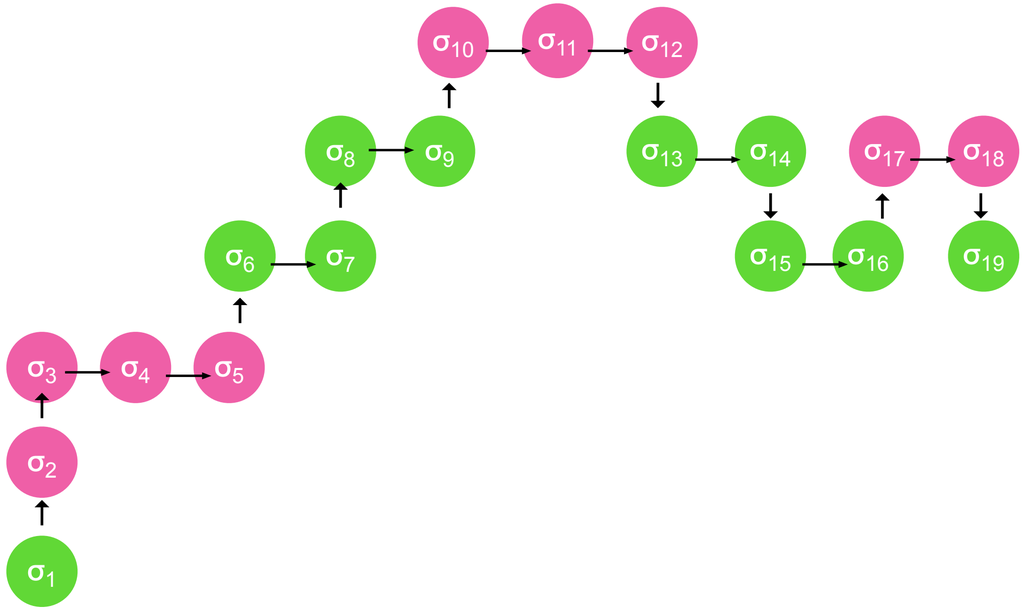

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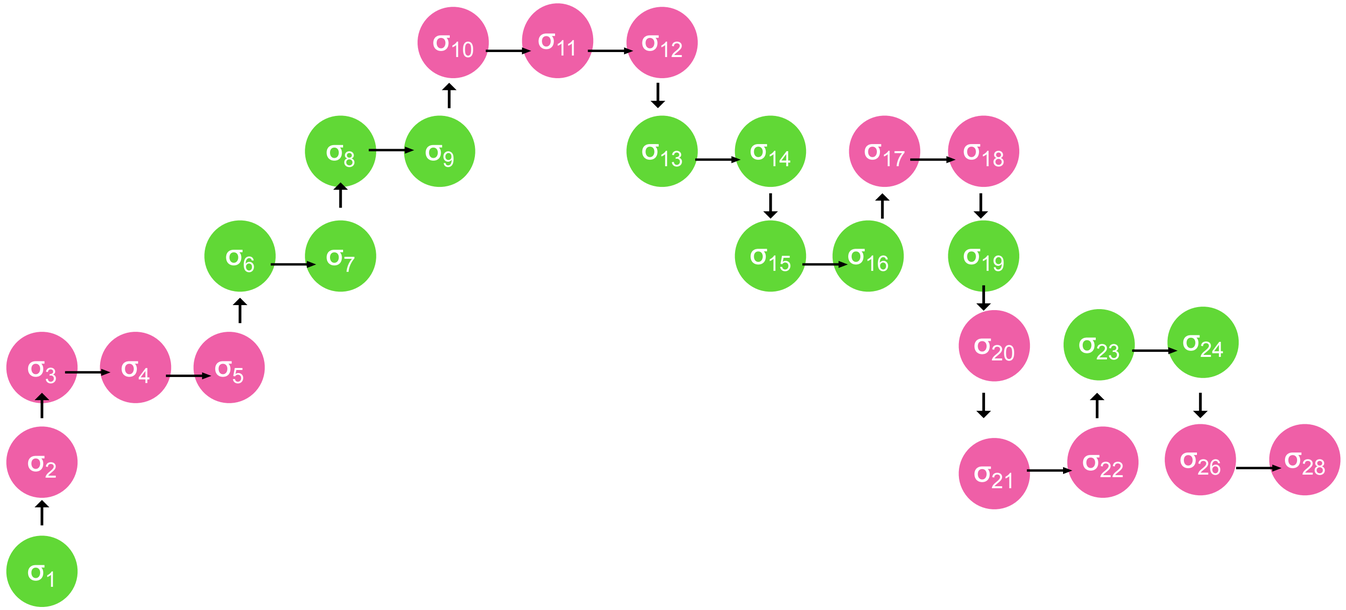

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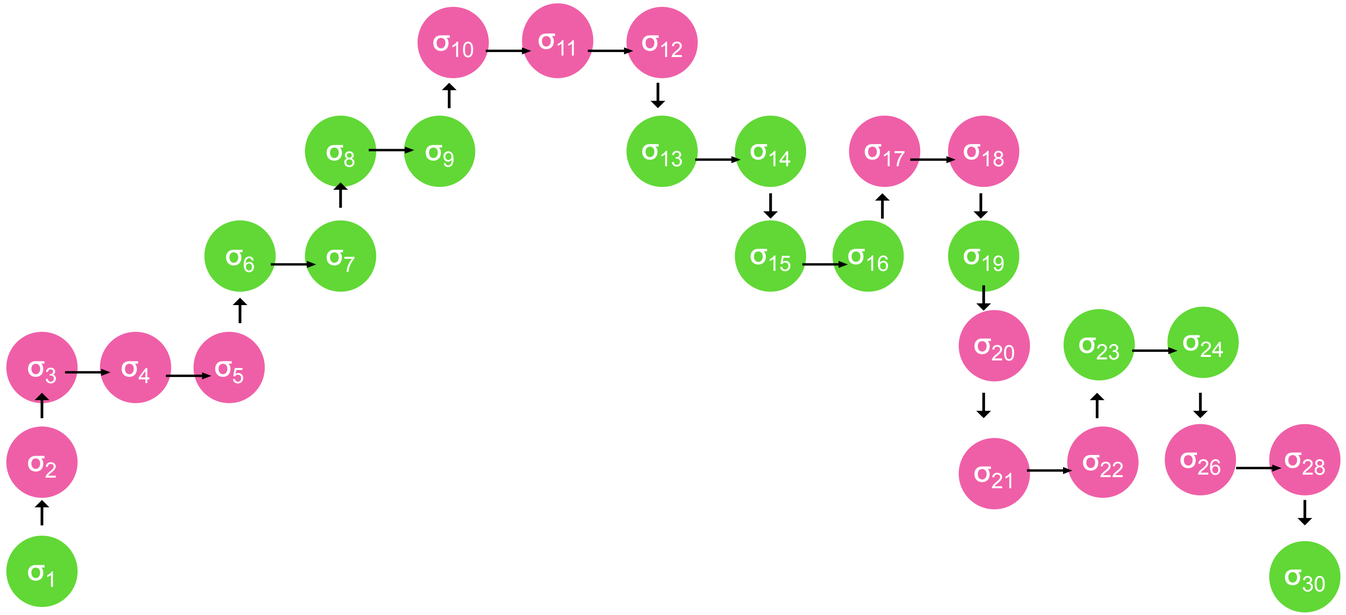

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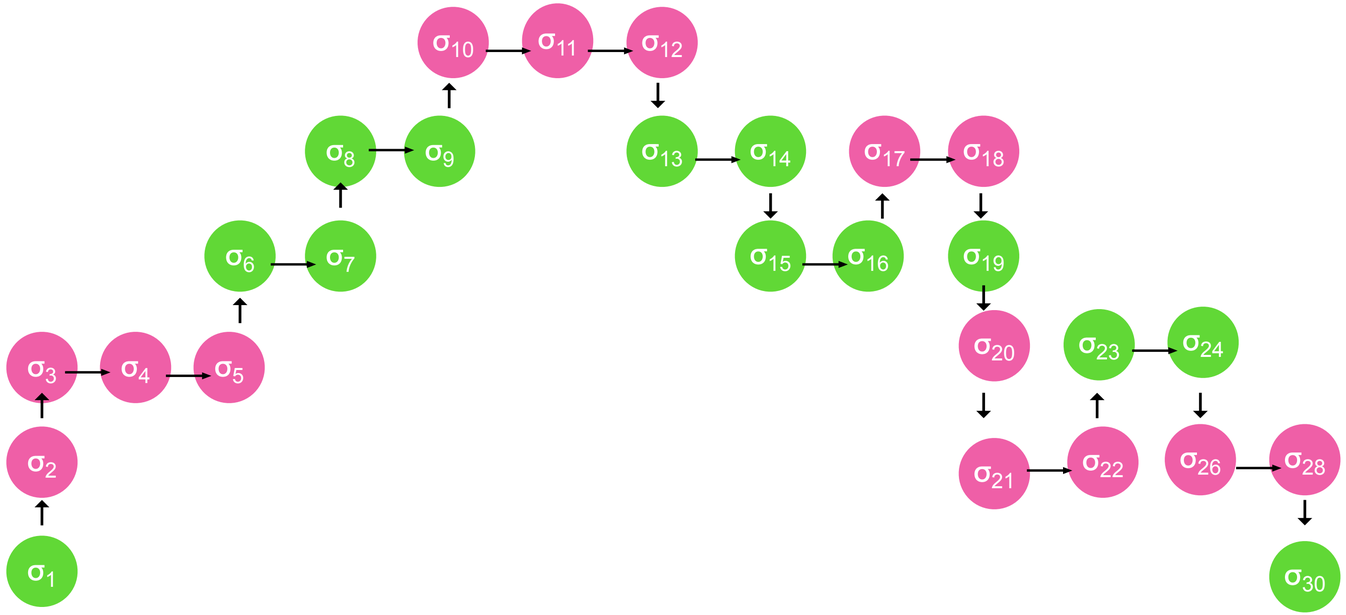
$M \models \forall x{:}C.\ \{ A \}$ means that



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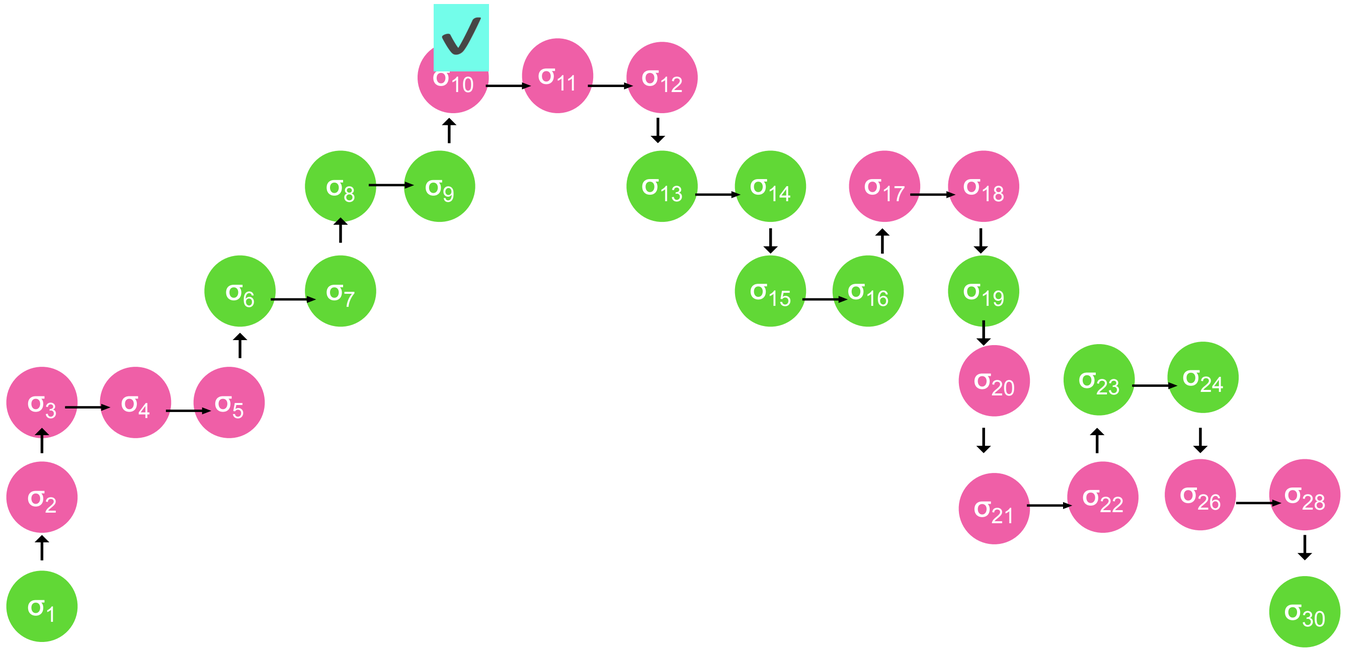

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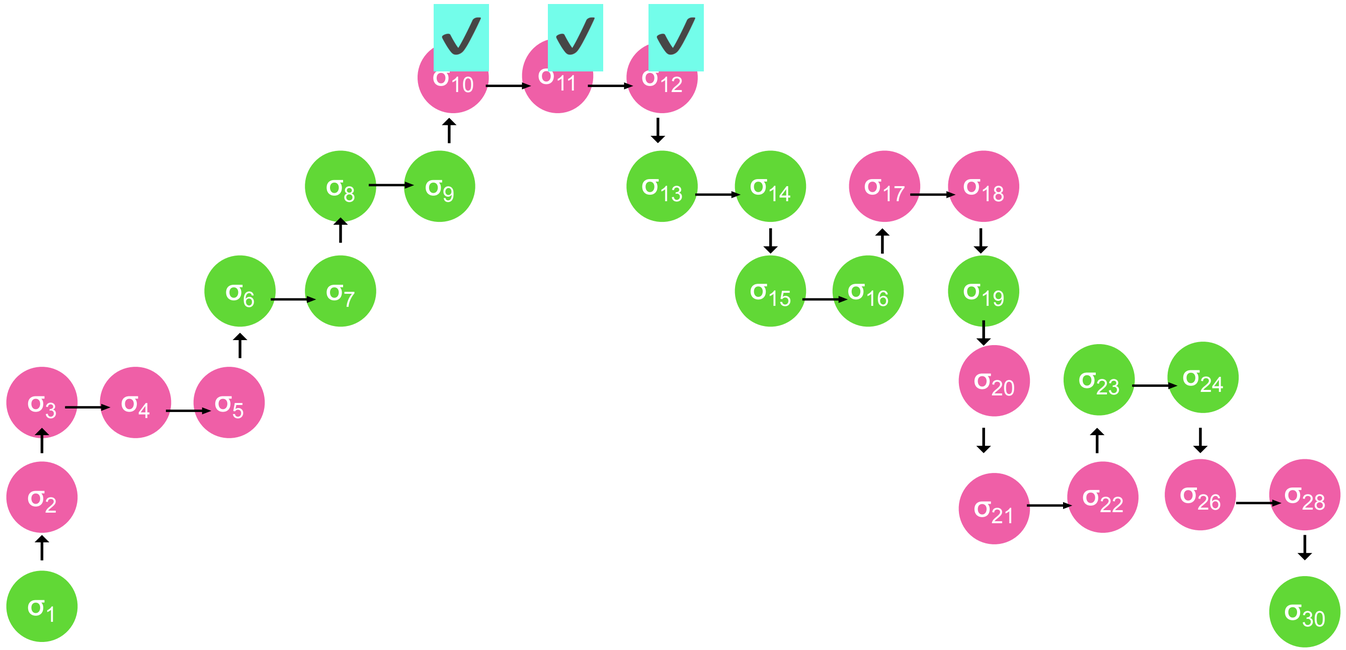
$M \models \forall x{:}C.\ \{\ A\ \}$ means that $M, \sigma_{10} \models \alpha{:}C \wedge A[\alpha/x]$ implies $M, \sigma_{11} \models A[\alpha/x]$
$M, \sigma_{12} \models A[\alpha/x]$



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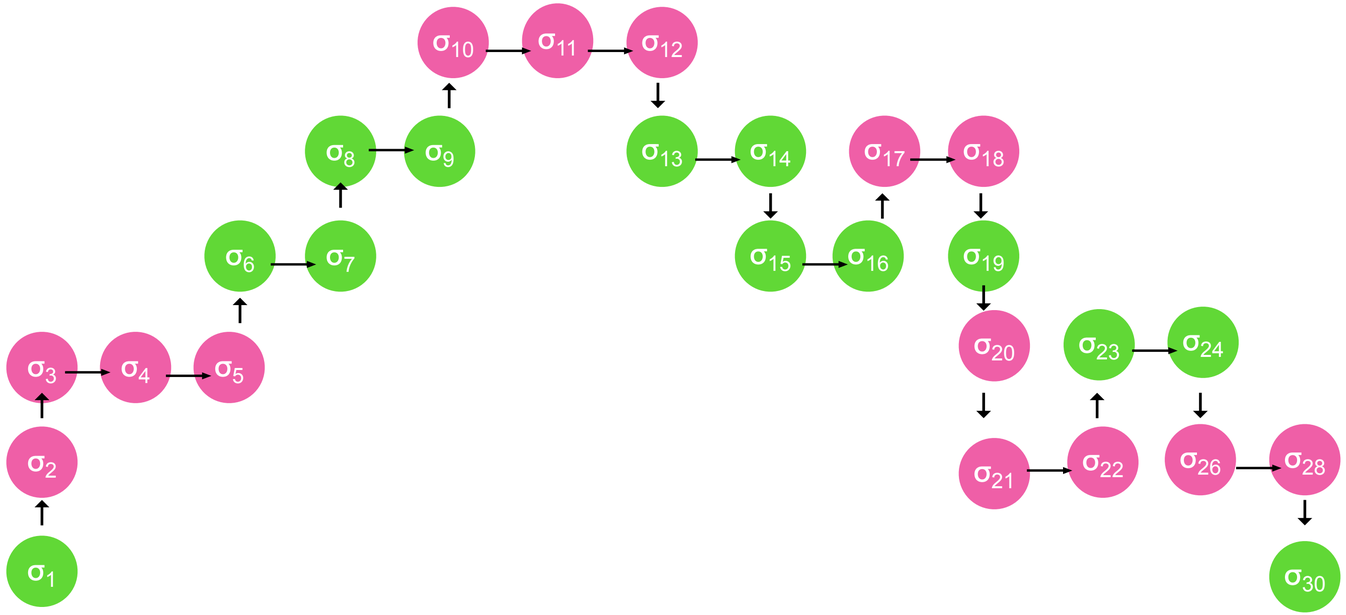

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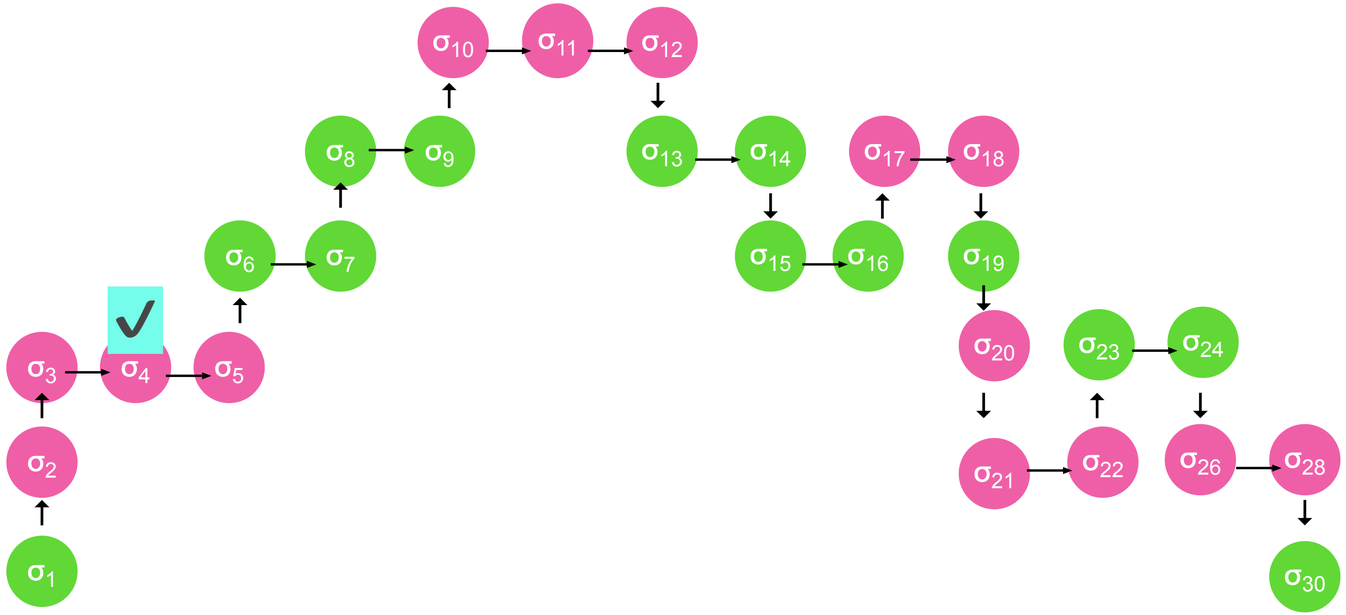

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$M \models \forall x{:}C.\ \{\ A\ \}$ means that $M, \sigma_4 \models \alpha{:}C \wedge A[\alpha/x]$ implies $M, \sigma_5 \models A[\alpha/x]$

$M, \sigma_6 \models A[\alpha/x]$

$M, \sigma_{10} \models A[\alpha/x]$

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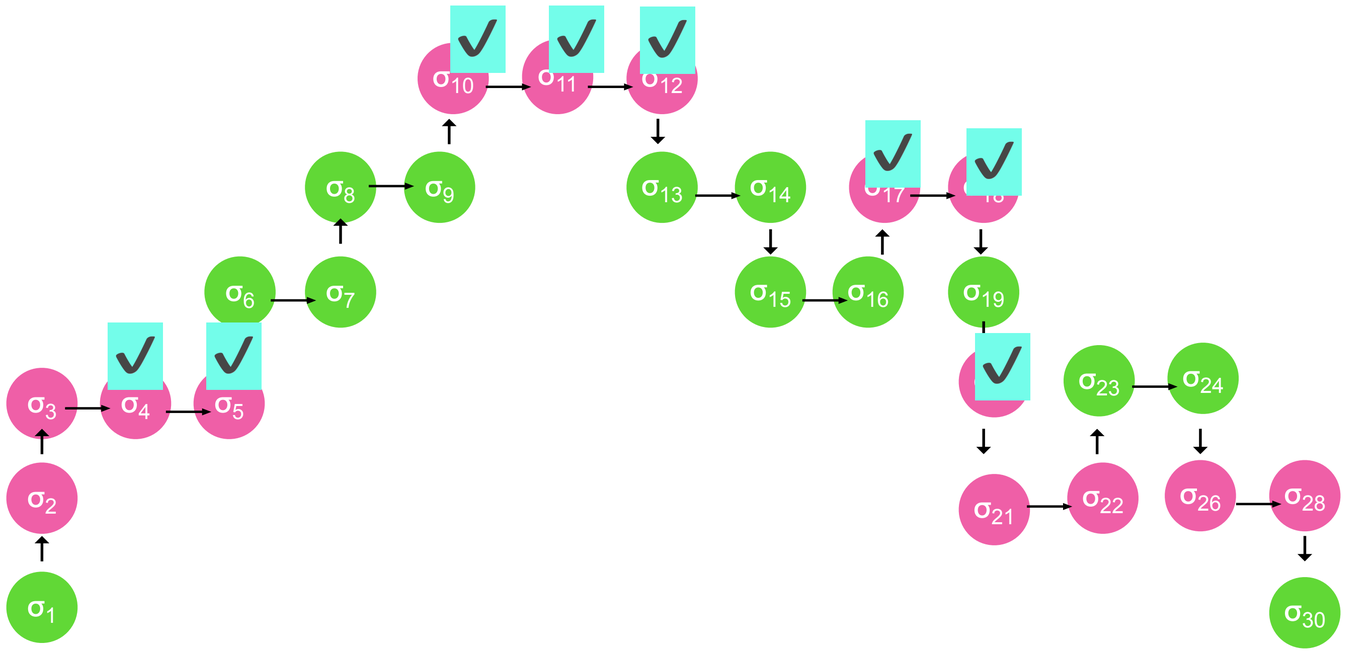
$M, \sigma_{18} \models A[\alpha/x]$

$M, \sigma_{20} \models A[\alpha/x]$



Challenge_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{fine} \models S$

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S1 $\triangleq$ $\forall$ a:Account. { ⟪a⟫ }

S2 $\triangleq$ $\forall$ a:Account. { ⟪a.key ⟫ }

S4 $\triangleq$ $\forall$ a:Account, b:Num. { ⟪a.key ⟫ $\wedge$ a.blnce $\geq$ b }

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S5 $\triangleq$ { ⟪this.accnt.key ⟫ $\not\hookleftarrow$ buyer $\wedge$ this.accnt.blnce =b }

Shop::buy(buyer:external, item:Item)

{ this.accnt.blnce $\geq$ b }

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$M_{bad} \not\models S2$ $M_{bad} \not\models S4$ $M_{good} \models S2 \wedge S4 \wedge S5$ $M_{fine} \models S2 \wedge S4 \wedge S5$

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$M_{bad} \not\models S4$

$M_{bad} \not\models S1$

$M_{good} \models S2 \wedge S4 \wedge S5$

$M_{good} \not\models S1$

$M_{fine} \models S2 \wedge S4 \wedge S5$

$M_{fine} \not\models S1$

Challenge_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{fine} \models S$

API - agnostic:
a.blnce, a.key can be ghost

Talk about effects

Talk about emergent behaviour

$S1 \triangleq \forall$ a:Account. { ⟪a⟫ }

$S2 \triangleq \forall$ a:Account. { ⟪a.key ⟫ }

$S4 \triangleq \forall$ a:Account, b:Num. { ⟪a.key ⟫ ∧ a.blnce ≥ b }

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Shop::buy(buyer:external, item:Item)
{ this.accnt.blnce ≥ b }

$M_{bad} \not\models S2$

$M_{bad} \not\models S4$

$M_{bad} \not\models S1$

$M_{good} \models S2 \wedge S4 \wedge S5$

$M_{good} \not\models S1$

$M_{fine} \models S2 \wedge S4 \wedge S5$

$M_{fine} \not\models S1$

Challenge_2: An inference system, such that

$M_{good} \vdash S$
$M_{bad} \nvdash S$
$M_{fine} \vdash S$

Challenge_2: An inference system, such that

$$M_{good} \vdash S$$
$$M_{bad} \nvdash S$$
$$M_{fine} \vdash S$$

In the context of arbitrary, unlimited calls from internal to external,
and arbitrary, unlimited calls from external to internal.

Challenge_2: An inference system, such that …

Three Stages

Assume an underlying Hoare logic of triples with usual meaning $M \vdash_{ul} \{ A \}\, s \,\{ A' \}$

1st stage Expand it to Hoare logic of triples with usual meaning $M \vdash \{A\}\, s \,\{A'\}$

2nd Stage Expand triples to quadruples $M \vdash \{A\}\, s \,\{A'\} \parallel \{A''\};$

Which promises that

- termination of s leads to a state satisfying A'
- intermediate external states satisfy A"

3rd Stage Rules for module satisfying a specification $M \vdash S$

Challenge_2: An inference system, such that …

1st stage

We extend some underlying Hoare logic to a Hoare logic of triples with usual meaning

$$\textsc{extend}\ \frac{M \vdash_{ul} \{ A \}\ s\ \{ A' \} \quad s \text{ contains no method call}}{M \vdash \{A\}\ s\ \{A'\}}$$

$$\textsc{types-1}\ \frac{s \text{ contains no method call}}{M \vdash \{x : C\}\ s\ \{x : C\}}$$

Challenge_2: An inference system, such that …

2nd stage

We expand triples to quadruples

INV

$$\frac{M \vdash \{A\}\, s\, \{A'\}}{M \vdash \{A\}\, s\, \{A'\} \parallel \{A''\}}$$

TYPES-2

$$\frac{M \vdash \{A\}\, s\, \{A'\} \parallel \{A''\}}{M \vdash \{x : C \wedge A\}\, s\, \{x : C \wedge A'\} \parallel \{A''\}}$$

COMBINE

$$\frac{M \vdash \{A_1\}\, s\, \{A_2\} \parallel \{A\} \qquad M \vdash \{A_3\}\, s\, \{A_4\} \parallel \{A\}}{M \vdash \{A_1 \wedge A_3\}\, s\, \{A_2 \wedge A_4\} \parallel \{A\}}$$

SEQU

$$\frac{M \vdash \{A_1\}\, s_1\, \{A_2\} \parallel \{A\} \qquad M \vdash \{A_2\}\, s_2\, \{A_3\} A}{M \vdash \{A_1\}\, s_1; s_2\, \{A_3\} \parallel \{A\}}$$

CONSEQU

$$\frac{M \vdash \{A_2\}\, s\, \{A_3\} \parallel \{A_4\} \qquad M \vdash A_1 \rightarrow A_2 \qquad M \vdash A_3 \rightarrow A_5 \qquad M \vdash A_4 \rightarrow A_6}{M \vdash \{A_2\}\, s\, \{A_5\} \parallel \{A_6\}}$$

Challenge_2: An inference system, such that …

2nd stage

We introduce triples for protection

$$\frac{u \overset{\text{txt}}{\neq} x}{M \vdash \{\, true \,\}\ u = \texttt{new}\ C\ \{\, \langle u \rangle \ \wedge \langle u \rangle \leftarrow\!\!\!\!\times\ x \,\}} \quad [\textsc{prot-new}]$$

$$\frac{\begin{array}{c} stmt \text{ is free of method cals, or assignment to } z \\ M \vdash \{\, e = z \,\}\ stmt\ \{\, e = z \,\} \end{array}}{M \vdash \{\ \langle e \rangle\ \}\ stmt\ \{\ \langle e \rangle\ \}} \quad [\textsc{prot-1}]$$

$$\frac{\begin{array}{c} stmt \text{ is either } x := y \text{ or } x := y.f, \text{ and } z, z' \overset{\text{txt}}{\neq} x \\ M \vdash \{\ z = e \wedge z' = e'\ \}\ stmt \{\ z = e \wedge z' = e'\ \} \end{array}}{M \vdash \{\ \langle e \rangle \leftarrow\!\!\!\!\times\ e'\ \}\ stmt\ \{\ \langle e \rangle \leftarrow\!\!\!\!\times\ e'\ \}} \quad [\textsc{prot-2}]$$

$$\frac{x \overset{\text{txt}}{\neq} z}{M \vdash \{\ \langle y.f \rangle \leftarrow\!\!\!\!\times\ z\ \}\ x = y.f\ \{\ \langle x \rangle \leftarrow\!\!\!\!\times\ z\ \}} \quad [\textsc{prot-3}]$$

$$\frac{}{M \vdash \{\ \langle x \rangle \leftarrow\!\!\!\!\times\ z\ \wedge\ \langle x \rangle \leftarrow\!\!\!\!\times\ y'\ \}\ y.f = y'\ \{\ \langle x \rangle \leftarrow\!\!\!\!\times\ z\ \}} \quad [\textsc{prot-4}]$$

Challenge_2: An inference system, such that …

3rd stage

$$\frac{M \vdash \mathscr{S}pec(M)}{\vdash M}\ \textsc{WellFrm_Mod} \qquad \frac{M \vdash S_1 \qquad M \vdash S_2}{M \vdash S_1 \wedge S_2}\ \textsc{Comb_Spec}$$

Challenge_2: An inference system, such that …

3rd stage

$$\frac{???}{M \vdash \forall \overline{x : C}\{A\}}\ \textsc{invariant}$$

Challenge_2: An inference system, such that …

$$\frac{M \vdash Encps(\overline{x : C} \wedge A)}{M \vdash \forall \overline{x : C}\{A\}}\ \textbf{INVARIANT}$$

Challenge_2: An inference system, such that …

$$\frac{M \vdash Encps(\overline{x : C} \wedge A) \qquad \forall D, m : \quad \texttt{mBody}(m, D, M) = \texttt{public}\ (\overline{y : D})\{\ stmt\ \} \implies}{M \vdash \forall \overline{x : C}.\{A\}}\ \textsc{invariant}$$

Challenge_2: An inference system, such that …

$$\textbf{INVARIANT}$$

$$M \vdash Encps(\overline{x : C} \wedge A)$$

$$- \quad \forall D, m : \quad \texttt{mBody}(m, D, M) = \texttt{public}\ (\overline{y : D})\{\ stmt\ \} \implies$$

$$\frac{M \vdash \{\ \texttt{this} : \texttt{D}, \overline{y : D},\ \overline{x : C} \wedge A \wedge A \text{-}\!\!\triangledown(\texttt{this}, \overline{y})\ \}\ stmt\ \{\ A \wedge A \text{-}\!\!\triangledown \texttt{res}\ \} \parallel \{\ A\ \}}{M \vdash \forall \overline{x : C}.\{A\}}$$

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Protection is “relative” to a frame;
Our $-\!\!\triangledown$ operator

helps us switch to callee’s view

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Remit_3: An inference system, such that we can prove external calls

Challenge_4: An inference system, such we can prove external calls

$$\frac{}{M \vdash \{\ y_0 : \texttt{ext} \quad , \quad ???\ \}\ u := y_0.m(y_1, ..y_n)\ \{\ ???\ \}\ \|\ \{\ ??\ \}}\ [\textsc{Call\_Ext}]$$

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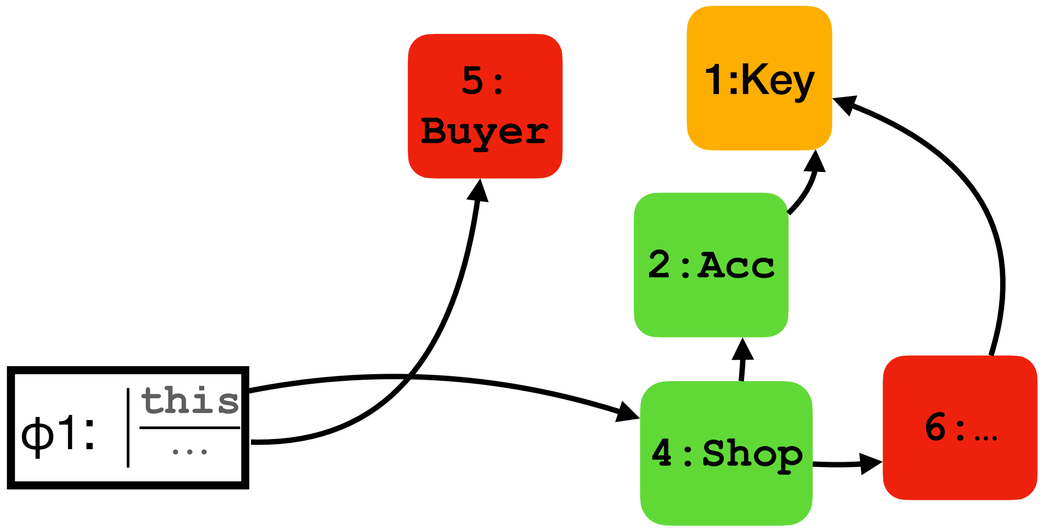
Challenge_4a: From Caller to Callee

We consider `Shop`'s method pay,



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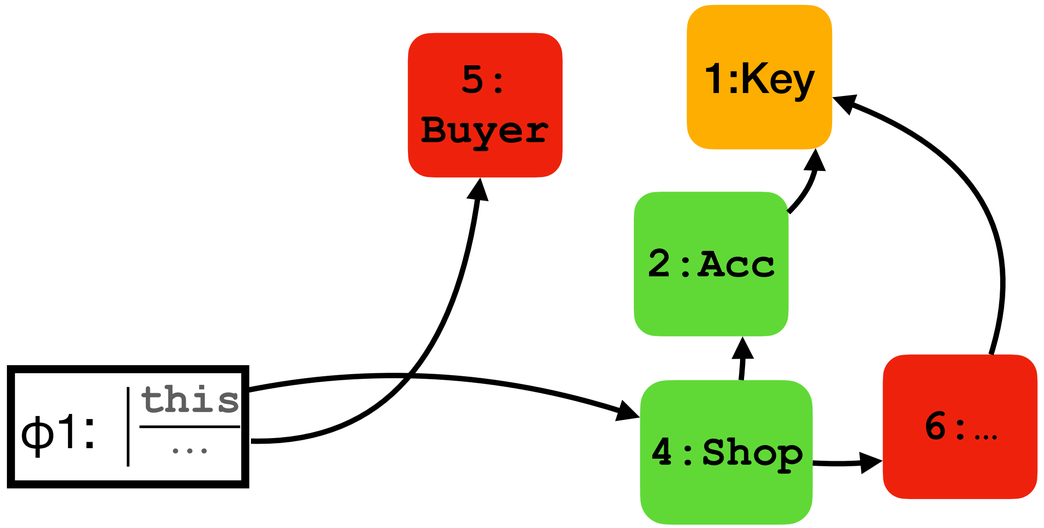
$$\{\ \texttt{this.accnt.blnce} \geq b\ \} \ldots$$



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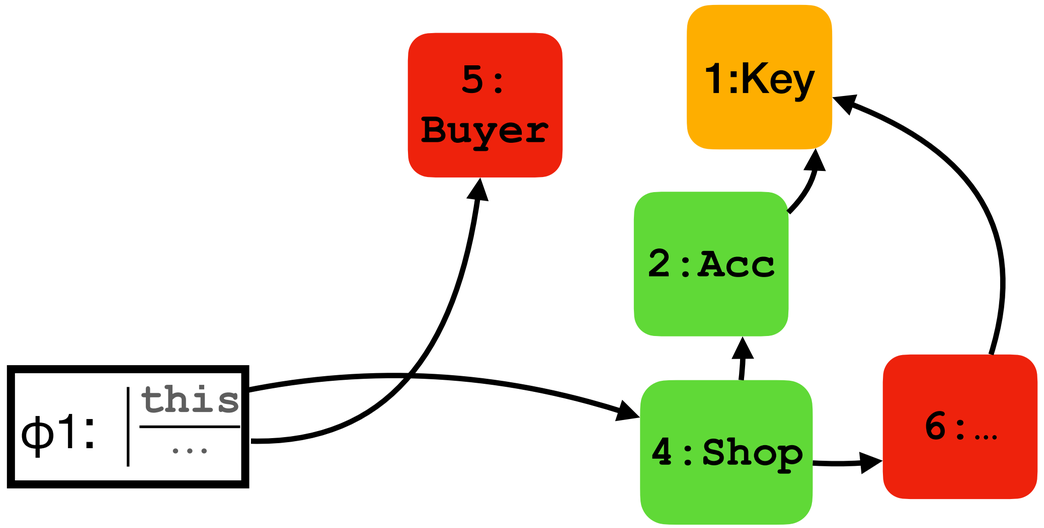

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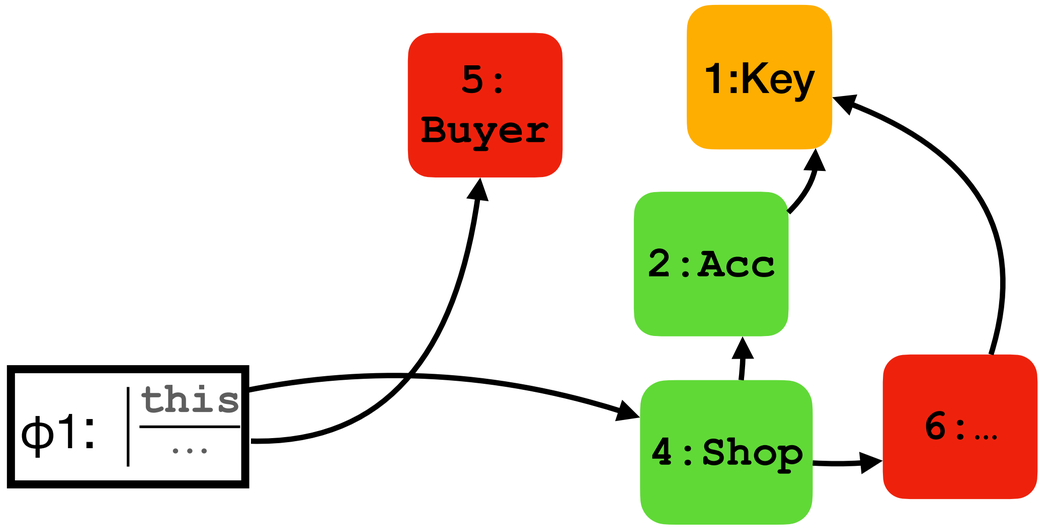

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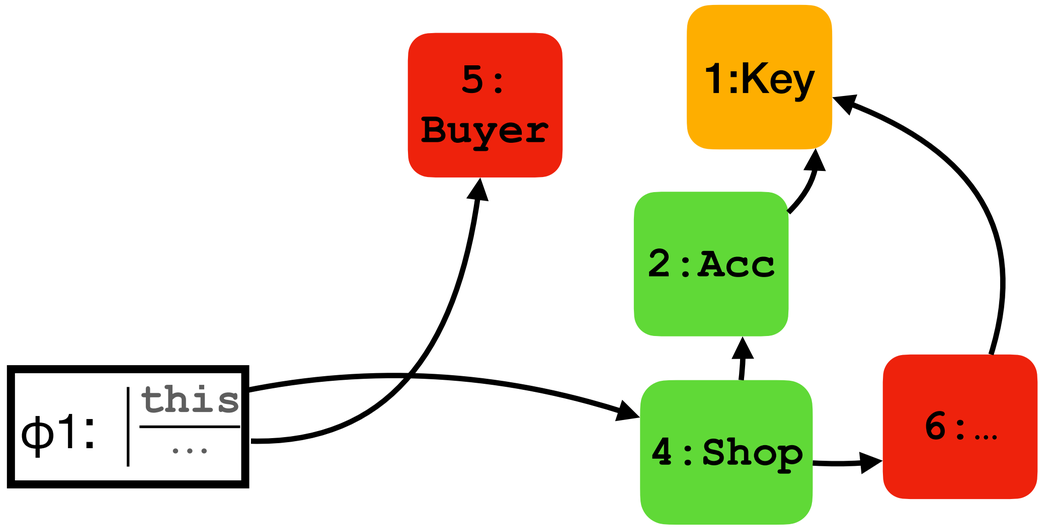

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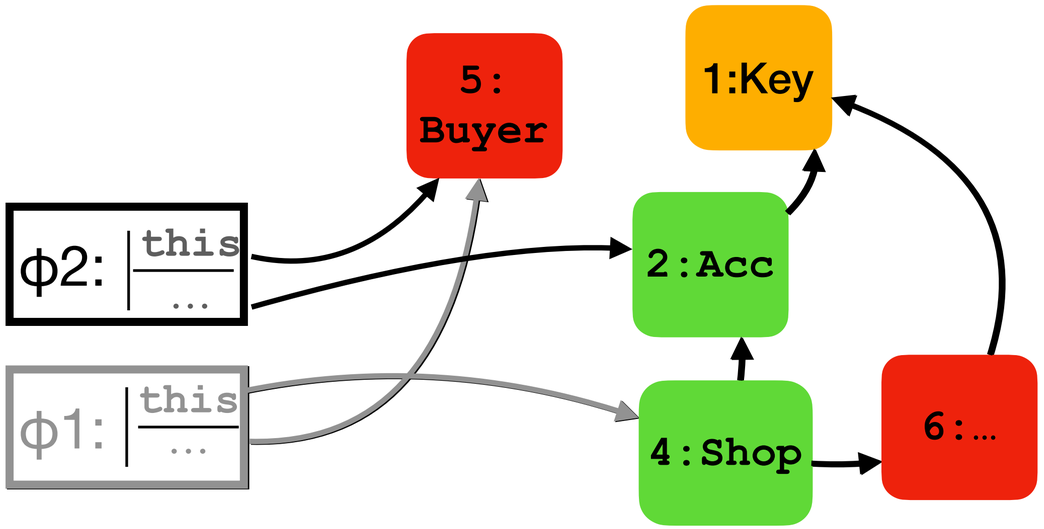

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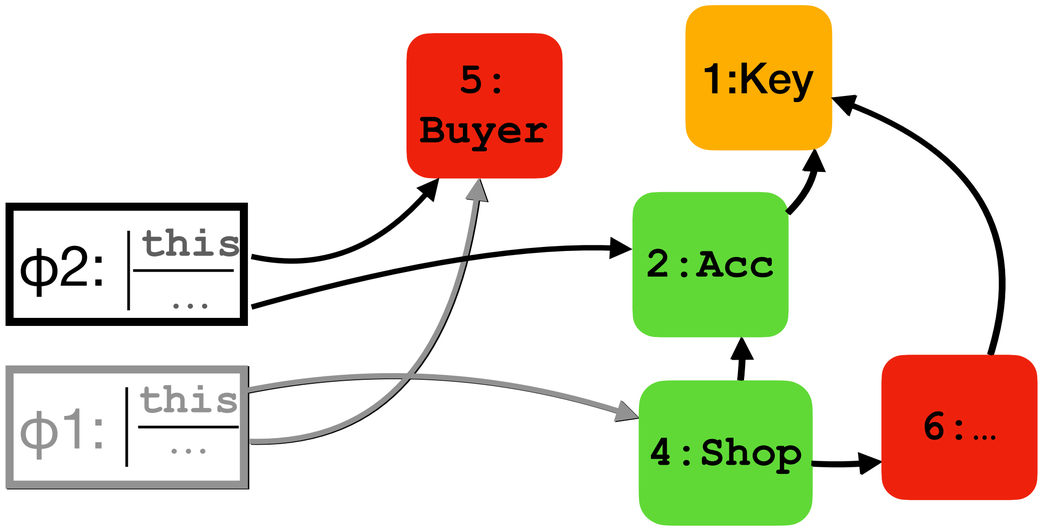

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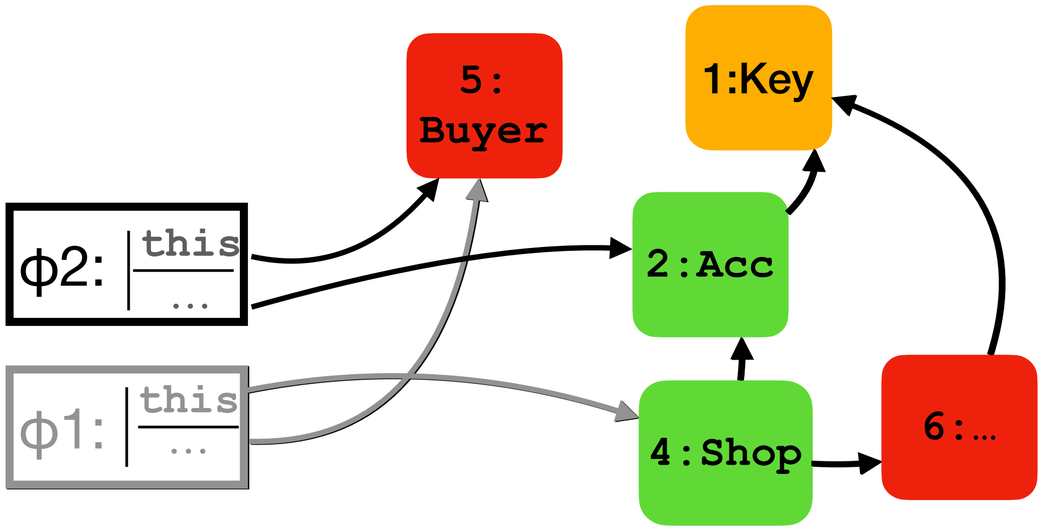

Therefore, we need an operator which mediates asserions between viewpoint of the callee and viewpoint of caller.

Challenge_4a: From Caller to Callee — The $-\!\!\triangledown$ operator

Definition 6.1. [The $-\!\!\triangledown$ operator]

$$\begin{array}{rcl@{\qquad}rcl}
\langle e\rangle -\!\!\triangledown \overline{y} & \triangleq & \langle e\rangle \leftrightarrow\!\!\!\!\times\ \overline{y} & (A_1 \wedge A_2) -\!\!\triangledown \overline{y} & \triangleq & (A_1 -\!\!\triangledown \overline{y}) \wedge (A_2 -\!\!\triangledown \overline{y}) \\
(\langle e\rangle \leftrightarrow\!\!\!\!\times\ \overline{u}) -\!\!\triangledown \overline{y} & \triangleq & \langle e\rangle \leftrightarrow\!\!\!\!\times\ \overline{u} & (\forall x : C.A) -\!\!\triangledown \overline{y} & \triangleq & \forall x : C.(A -\!\!\triangledown \overline{y}) \\
(e : \texttt{extl}) -\!\!\triangledown \overline{y} & \triangleq & e : \texttt{extl} & (\neg A) -\!\!\triangledown \overline{y} & \triangleq & \neg(A -\!\!\triangledown \overline{y}) \\
e -\!\!\triangledown \overline{y} & \triangleq & e & (e : C) -\!\!\triangledown \overline{y} & \triangleq & e : C
\end{array}$$

Lemma 6.2. For states σ, assertions A, so that $Stb^{+}(A)$ and $Fv(A) = \emptyset$, frame ϕ, variables $y_0, \overline{y}$:

$$\begin{array}{lcl}
(2)\ M, \sigma \models A -\!\!\triangledown Rng(\phi) & \Longrightarrow & M, \sigma \triangledown \phi \models A \\
(3)\ M, \sigma \triangledown \phi \models A \wedge \texttt{extl} & \Longrightarrow & M, \sigma \models A -\!\!\triangledown Rng(\phi)
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Challenge_4a: From Caller to Callee — The $-\triangledown$ operator

$\triangledown$ translates an assertion from the view of the callee to that of the caller.

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\end{array}
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Example: ⟪this.accnt.key⟫ $-\triangledown$ buyer = ⟪this.accnt.key⟫ $+$ buyer

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▽ translates

Protection is "relative" to a frame;

▽ operator switches assertion to callee's viewpoint

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Challenge_4: An inference system, such we can prove external calls

$$\frac{}{M \vdash \{\ y_0 : \texttt{ext} \quad , \quad \} \ u := y_0.m(y_1, ..y_n)\ \{\ ???\ \} \parallel \{\ ??\ \}}\ [\textsc{Call\_Ext}]$$

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Using [Call_Ext] we can, indeed, prove

$$\{\ \mathtt{buyer}:\mathtt{extl} \wedge \langle\!\langle \mathtt{this.accnt.key} \rangle\!\rangle \leftrightarrow \mathtt{buyer} \wedge \mathtt{this.accnt.blnce} = b\ \}$$

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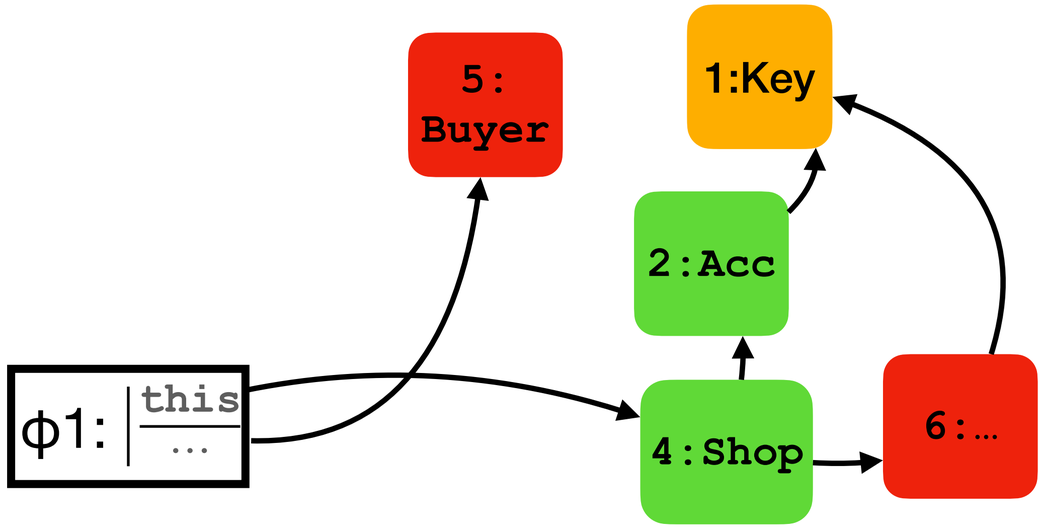


Using our quadruples, we have proven

$$M_{good} \vdash S2 \wedge S4 \wedge S5$$

$$M_{fine} \vdash S2 \wedge S4 \wedge S5$$

Moreover, we have proven

$$\vdash M \wedge M \vdash \{A\}\ stmt\ \{A'\} \,||\, \{A''\} \Rightarrow M \models \{A\}\ stmt\ \{A'\} \,||\, \{A''\}$$

$$\vdash M \Rightarrow M \models S$$

Summary

- Distinction between external/internal objects
- 《e 》: expresses that e is protected from reachable external objects
- Specifications talk about necessary conditions for effect:

  ∀ x: … { 《e 》 ∧ A }

  means that A is preserved as long as **capability** e is ptotected
- API-agrnositc spec,
- “Algorithmic” inference system system,
- Reason with open calls
- Protaction, 《e 》 relative to frame. Use —▽ to switch view

Surprises

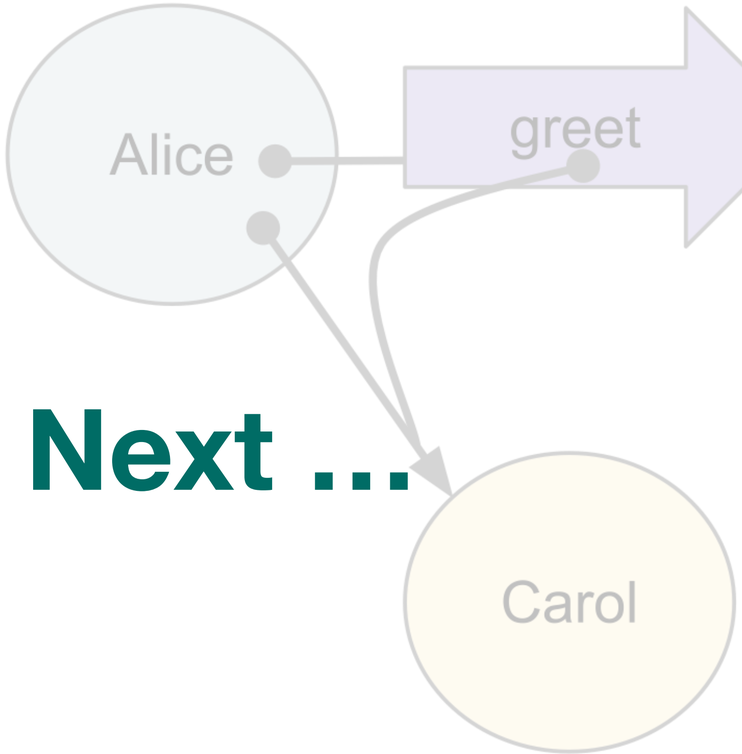
- Frame-related concepts
  - Protected object
  - Scoped Invarinats
  - ▽ to switch view to callee frame
- Started with *necessary conditions*,
  but ended up using *sufficient conditions* to reason about them
- Started with temporal logics,
  but ended up using invariants
- Hoare logic extensions
- Invariants — twenty years later …

A Unified Framework for Verification Techniques for Object Invariants

S. Drossopoulou[1], A. Francalanza[2], P. Müller[3], and A. J. Summers[1]

[1] Imperial College London,
[2] University of Southampton,
[3] Microsoft Research, Redmond

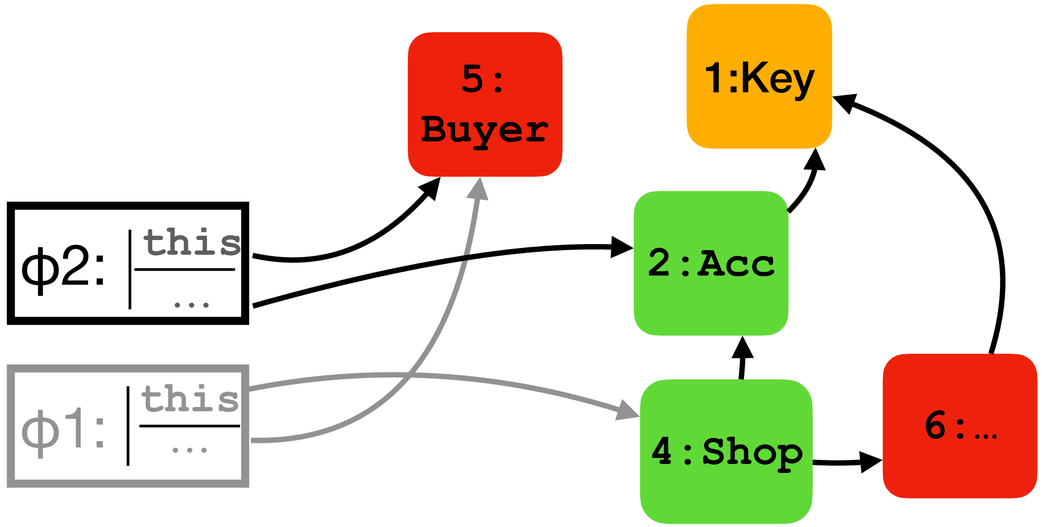
Abstract. Object invariants define the consistency of objects. They have subtle semantics because of call-backs, multi-object invariants and subclassing. Several visible-state verification techniques for object in-



Next …

- Modules
- Mechanize proofs
- Completeness?
- Revisit protection:
  - What if more than one capability for an effect?
  - Ownership types, membranes etc?
  - Instance-level protection?
  - assertions rather than objects to protect
- Other programming Paradigms
- Better interaction with underlying Hoare logics
- Tool

Thank You!


5 : Buyer
1:Key
ϕ2: this …
2 : Acc
ϕ1: this …
4 : Shop
6 : …


ϕ1: this …


ϕ1:
this
...

5 :
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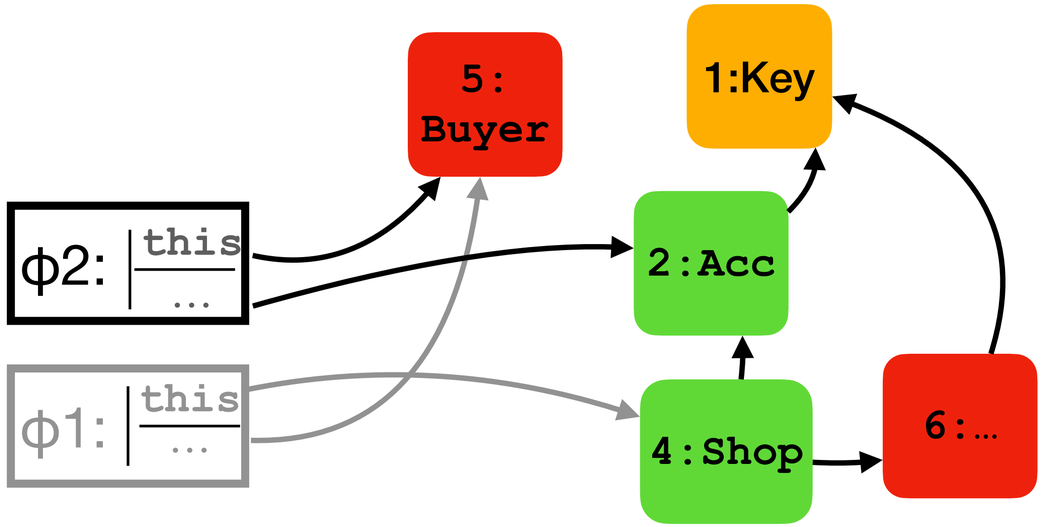
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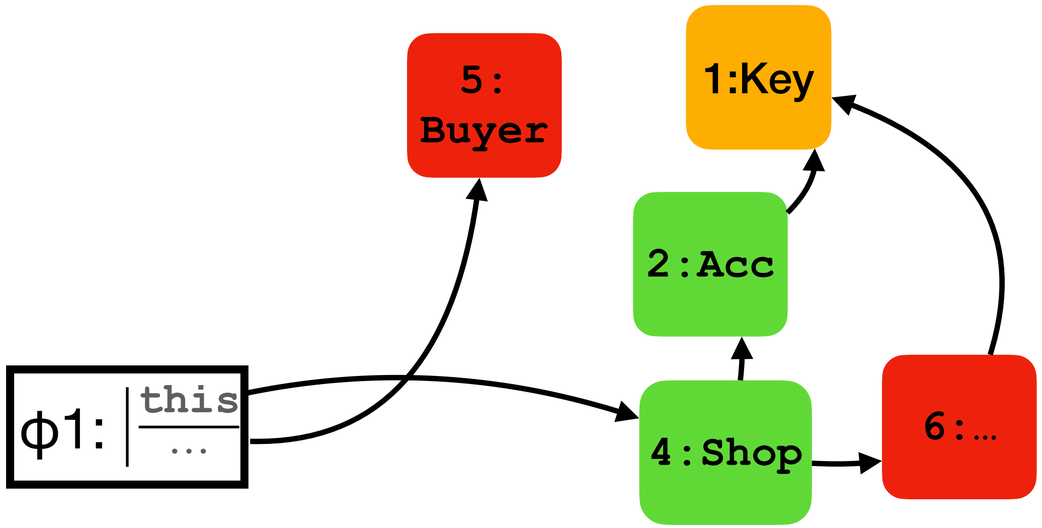
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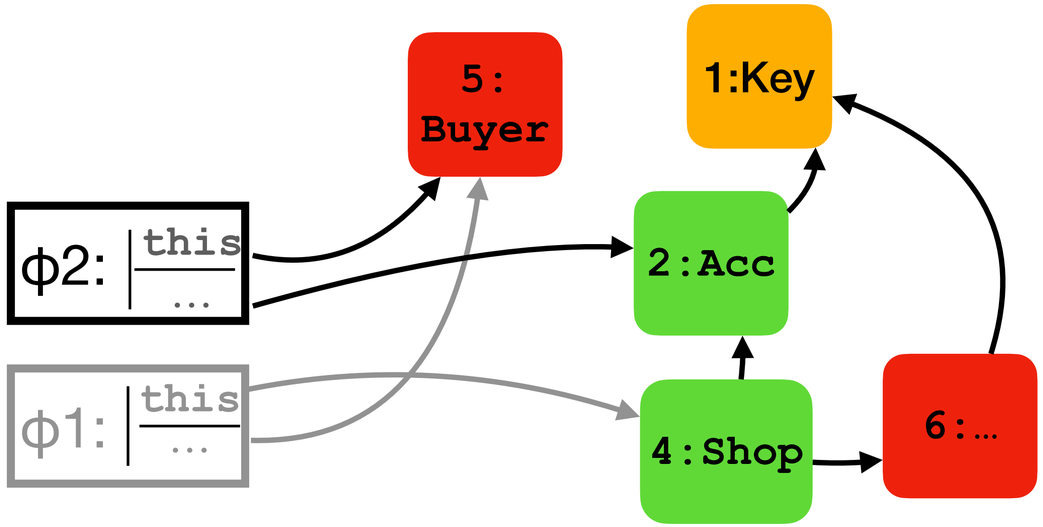
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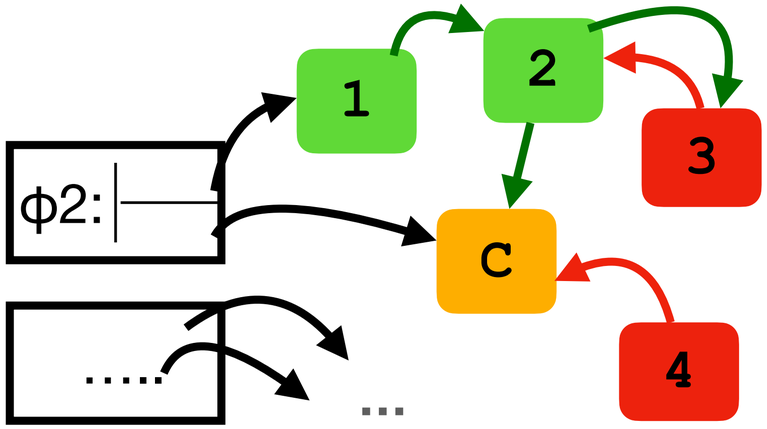
ϕ1: this …
4 : Shop
6 : …


1
2
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ϕ2:
C
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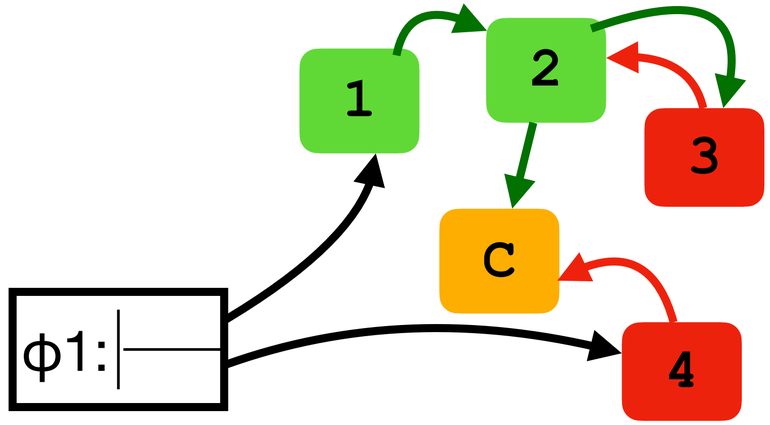
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ϕ1:
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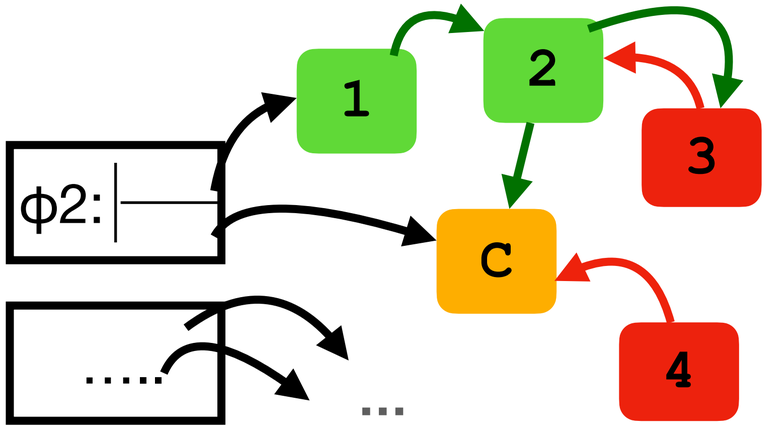
1
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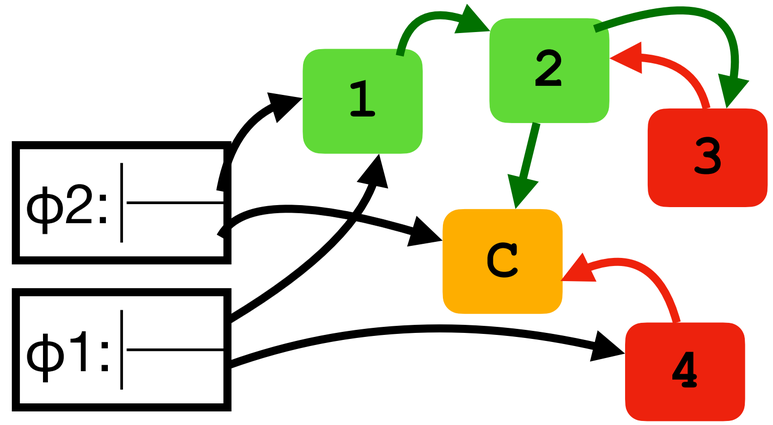
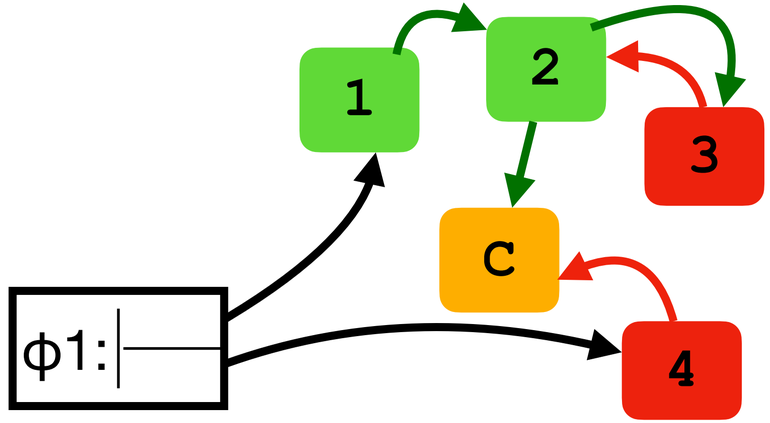

1
2
3
ϕ2:
C
ϕ1:
4


1
2
3
C
ϕ1:
4