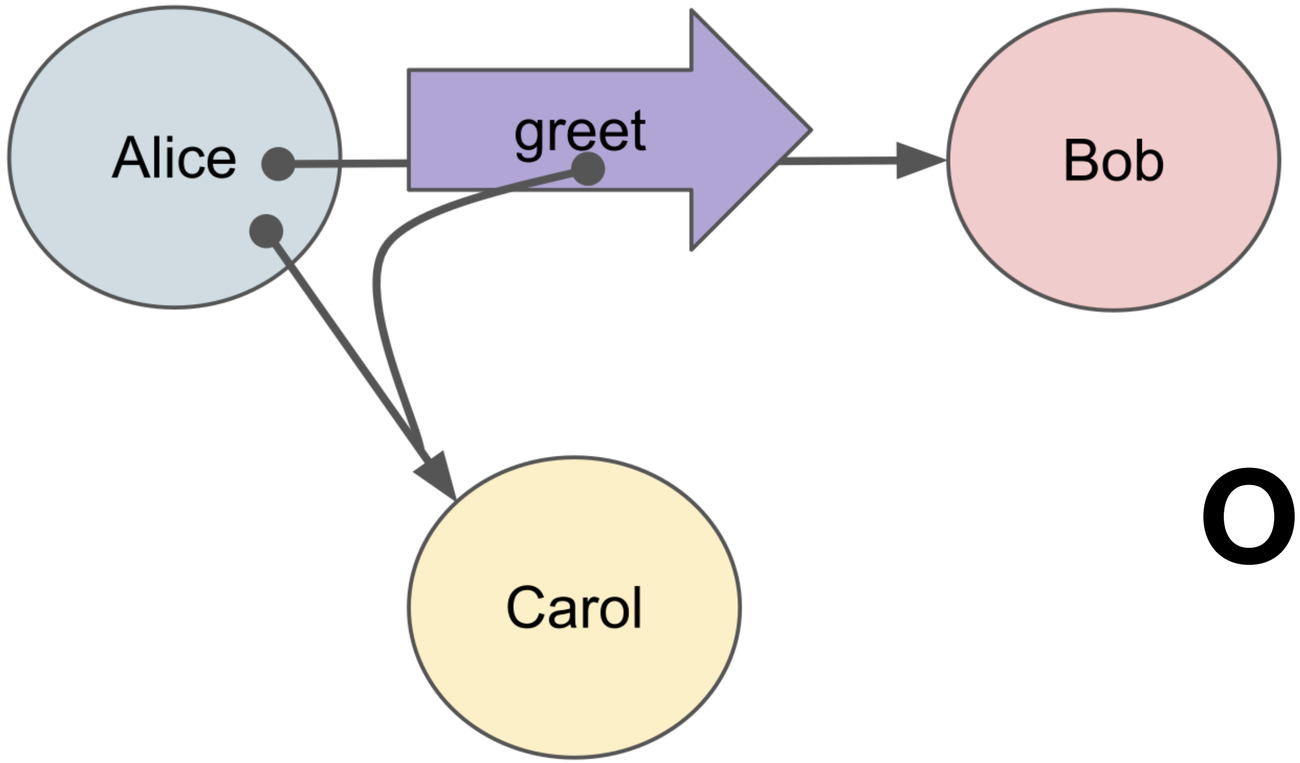


Reasoning with Object Capabilities

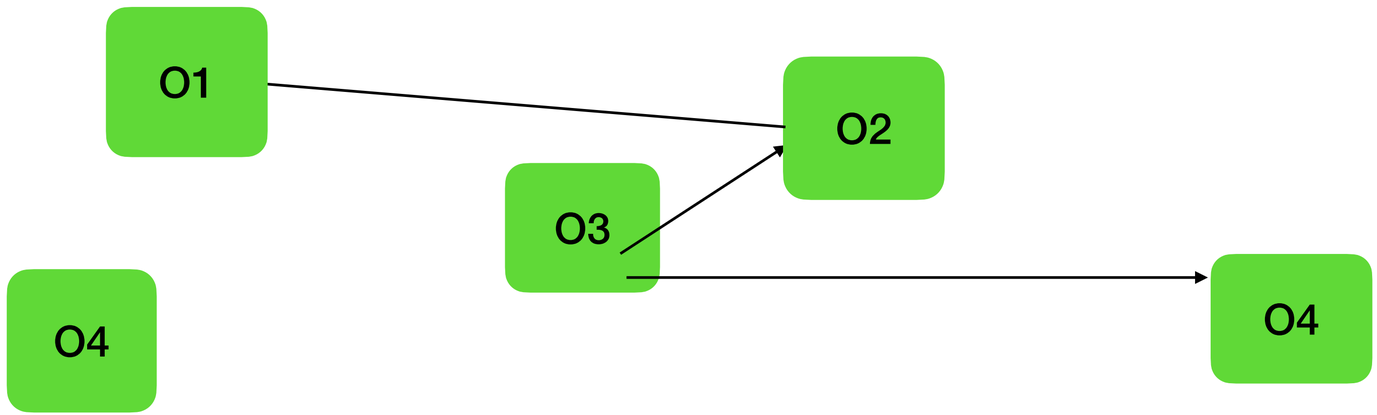
POCL 2024

James Noble **Susan Eisenbach** **Julian Mackay**

and in earlier works

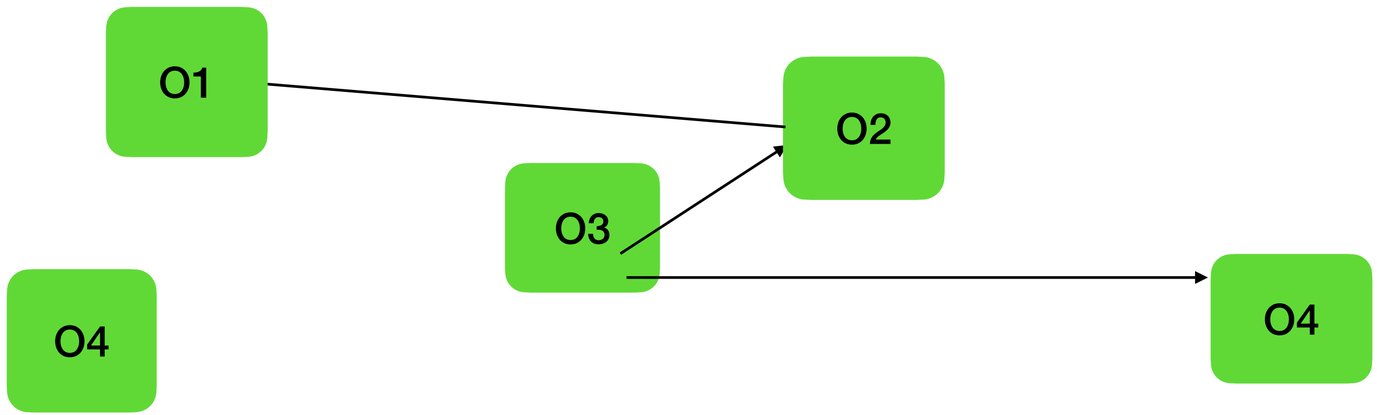
Mark Miller **Toby Murray**

Our research Question



Our research Question

Reason about how our, internal, trusted objects



Our research Question

Reason about how our, internal, trusted objects can interact securely with the
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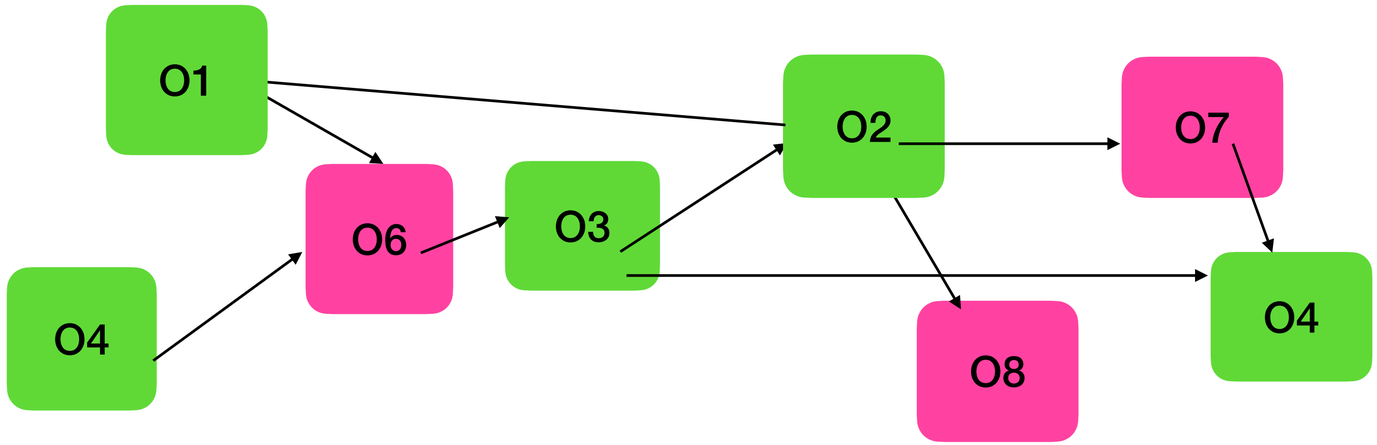

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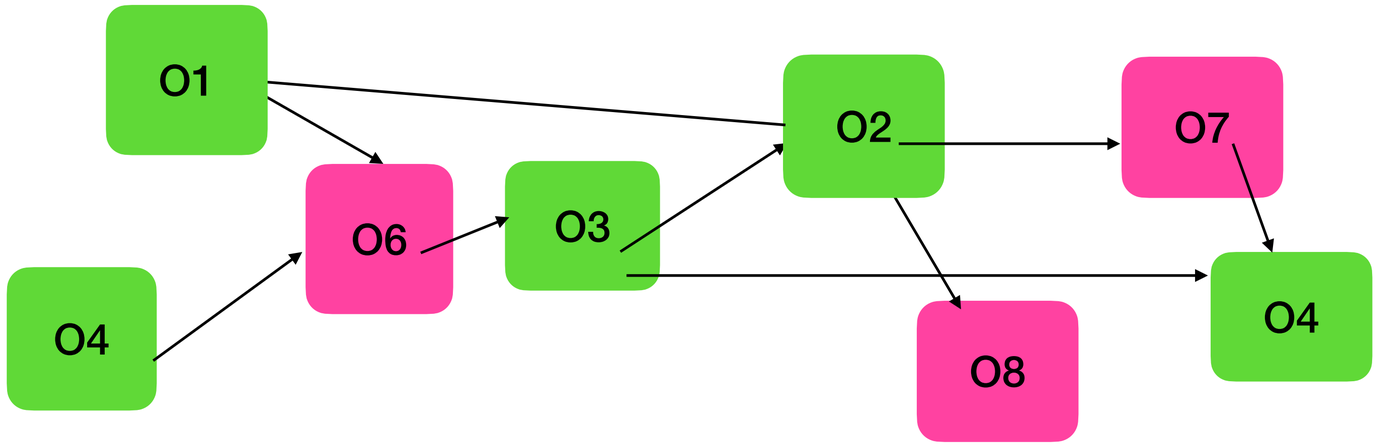

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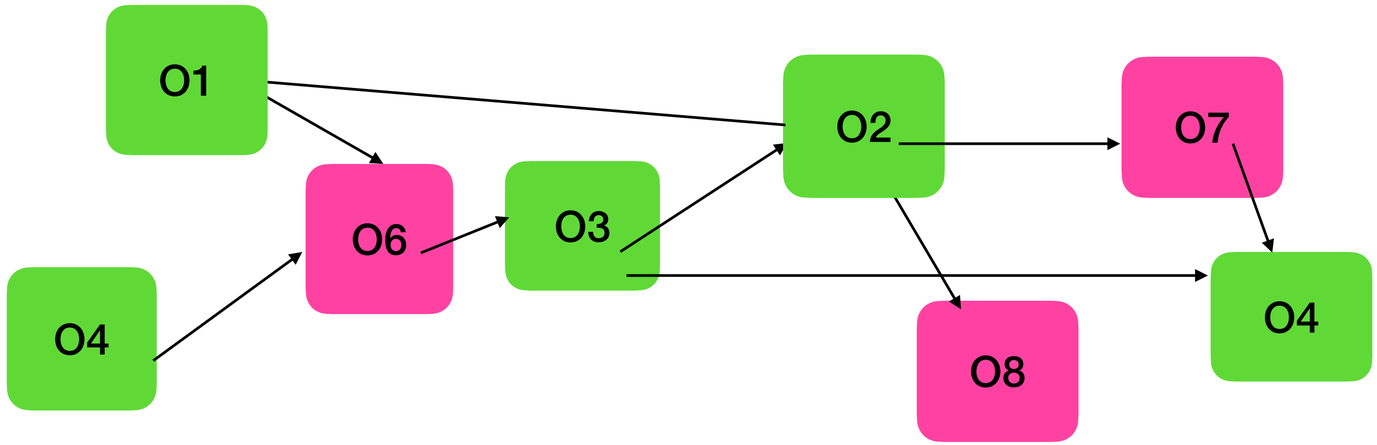
Literature:

*"A **capability** describes a transferable right to perform one (or more) operations."*

"References cannot be forged.
Capability transferred only through messages or through creation."



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Our Insights:

Capabilities are necessary conditions for *effects* (not operations)

***Tracking access* to capabilities is paramount.**

Example
and
Four Challenges

Three Modules

.. assuming all methods are public, and fields are private

```
module Mod_good
  class Account
    field balance:int
    field pwd: Password
    method transfer(dest:Account, pwd':Password) -> void
      if this.pwd==pwd'
        this.balance-=100
        dest.balance+=100
     method init(pwd':Password) -> void
      if this.pwd==null
        this.pwd=pwd'
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module Mod_better
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    method set(pwd',pwd'': Password)
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```

Challenge_2: An inference system, such that

$M_{good} \vdash S$

$M_{bad} \nvdash S$

$M_{better} \vdash S$

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module Mod_bad
  class Account
    field balance:int
    field pwd: Password
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Challenge_3: Inference system should be algorithmic

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      if (this.pwd==pwd')
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  class Password
```

```
class Shop

  fld myAccount : Account
  fld inventory : Inventory

  void buy(buyer: Object, anItem: Item)
    int price = anItem.price
    int oldBalance = this.myAccount.balance
    buyer.payMe(myAccount,price)
    if (this.myAccount.balance == oldBalance+price)
       this.send(buyer,anItem)
    else
       buyer.tell("you have not paid me")
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class Shop

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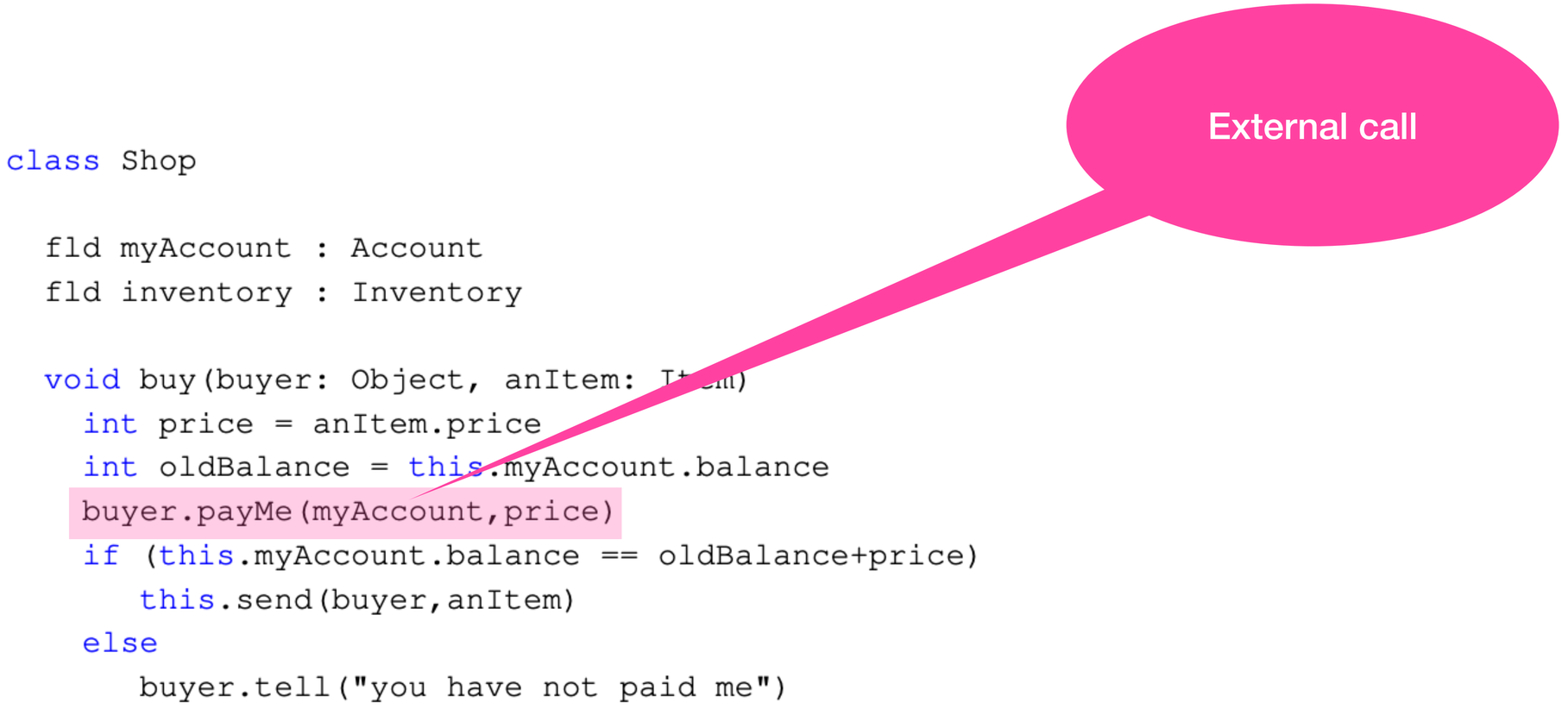
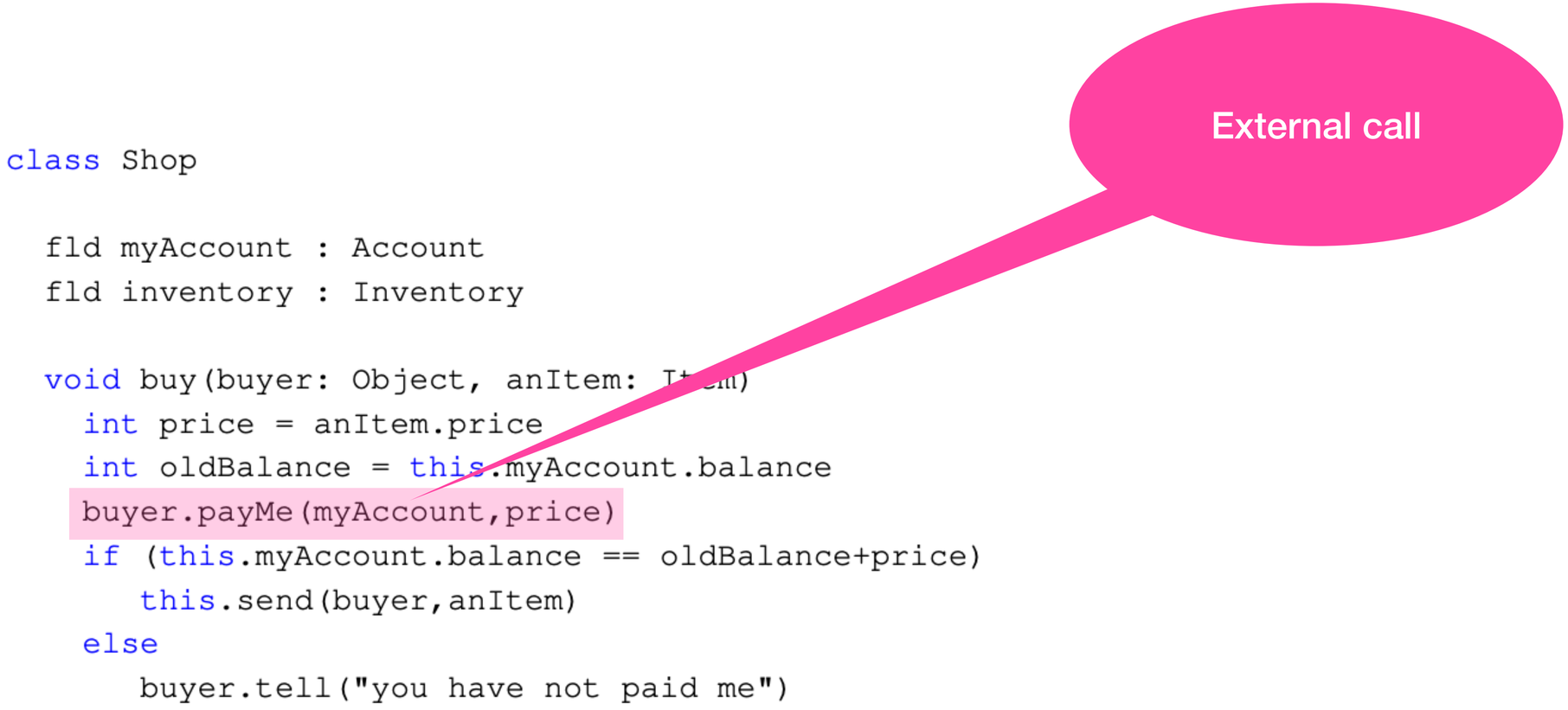


If `Account` comes from a “good” module,
and upon call, `buyer` has no eventual access to `myAccount.password`,



If `Account` comes from a “good” module,
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then
the balance of `myAccount` does not decrease by the call `payMe`.

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class Shop

  fld myAccount : Account
  fld inventory : Inventory

  void buy(buyer: Object, anItem: Item)
    int price = anItem.price
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Challenge_4: An inference system, such that we can prove **external** calls.

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So: “The password enables withdrawal from the account”?

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~~***So***: "The password enables withdrawal from the account"?~~

Or: "Without the password call of withdraw will fail"?

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So: $\forall$ s:Statement. {without a.password $\wedge$ a.balance=b} s { a.balance >= b }

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Motto:
Capability is a *necessary* condition for some effect

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In general: ∀ x1:C1,x2:C2… (| A |) (| A' |)

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Challenge_1_a : Meaning of x **prt**

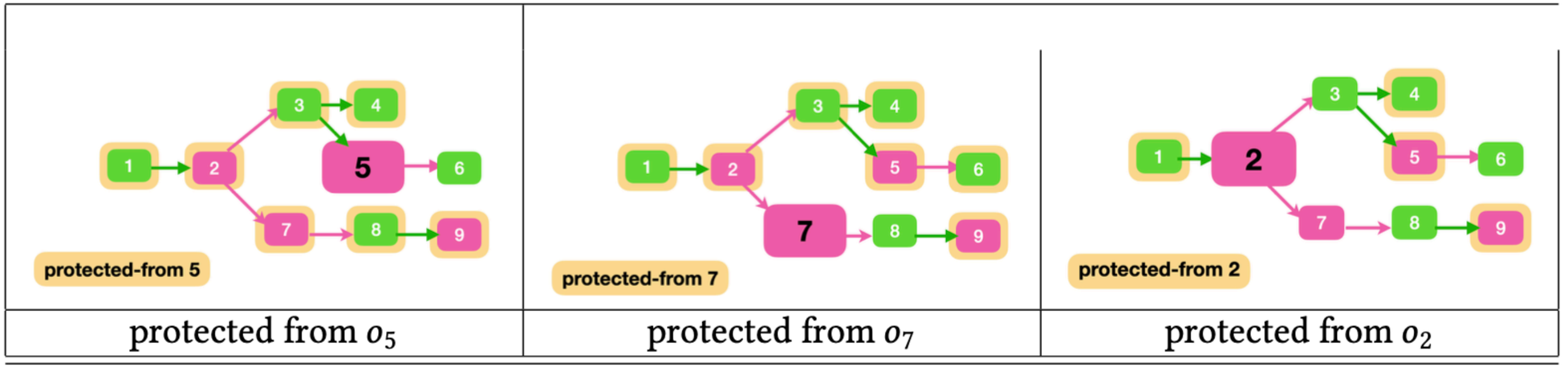
Challenge_1_b : Meaning of ∀ x1:C1,x2:C2… (| A |) (| A' |)

Challenge_1_a : Meaning of x **prt**

Remember:

…how our, internal, trusted objects will interact securely with the external, untrusted, potentially malicious, objects.

Def: o **prt-frm** o’ Iff o=/=o’ and the penultimate object on any path from o’ to o is internal.



protected from o_5	protected from o_7	protected from o_2

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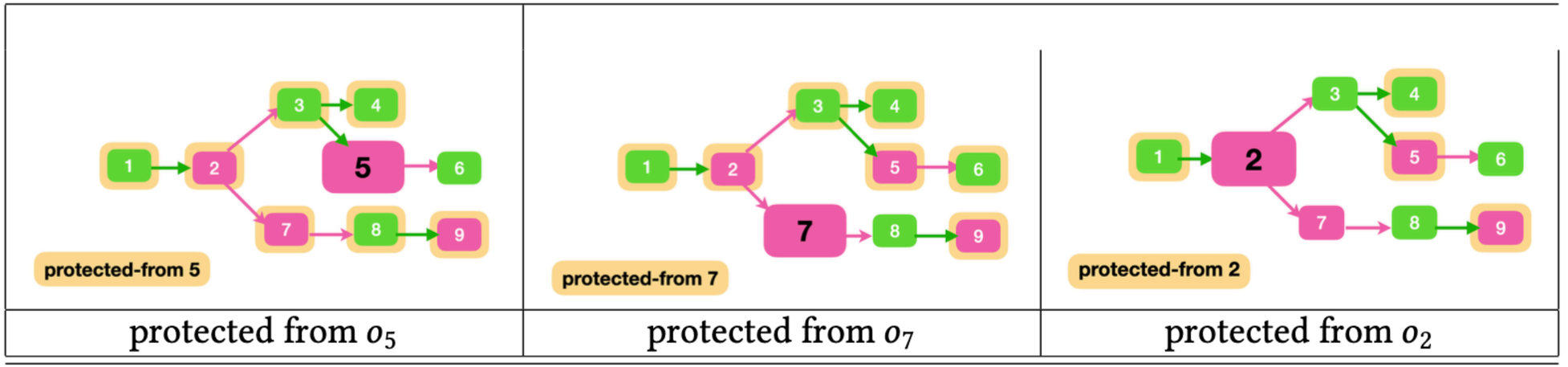

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Def: o **prt-frm o'**, if **extr** o' and any path from o' to o goes through an internal object.

Def: o **prt** , if o **prt-from** any external object accessible from top frame

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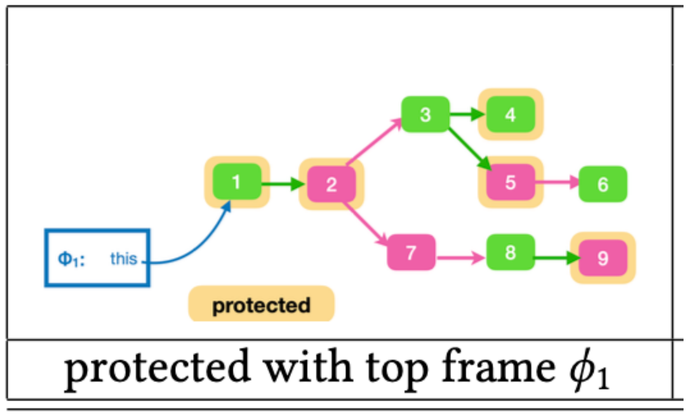

protected with top frame ϕ_1

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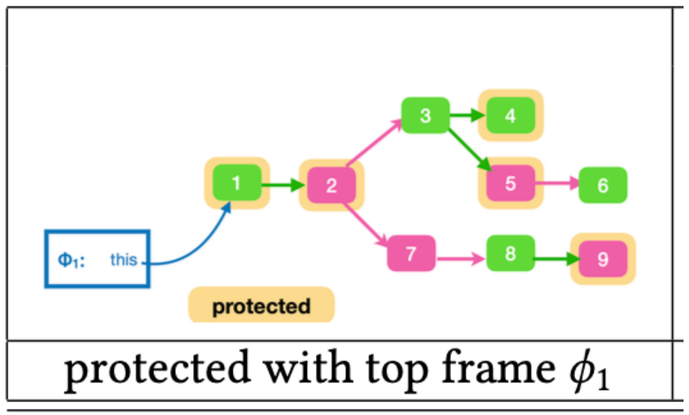

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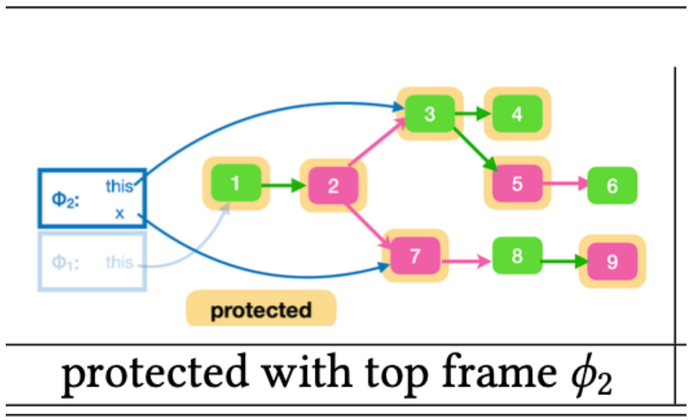

protected with top frame ϕ_2

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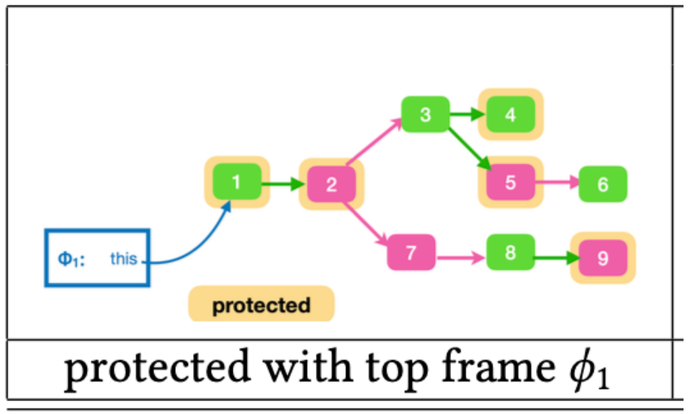

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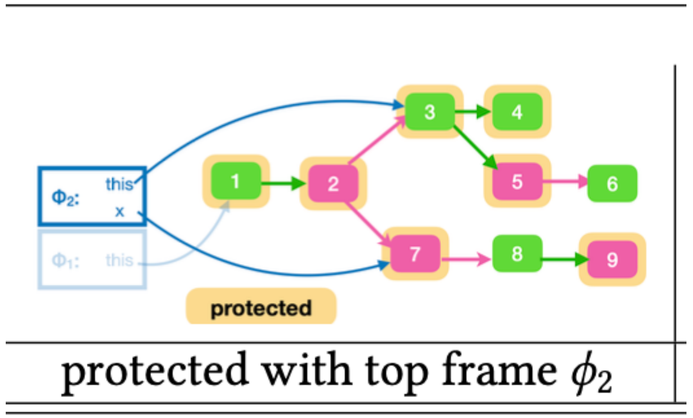

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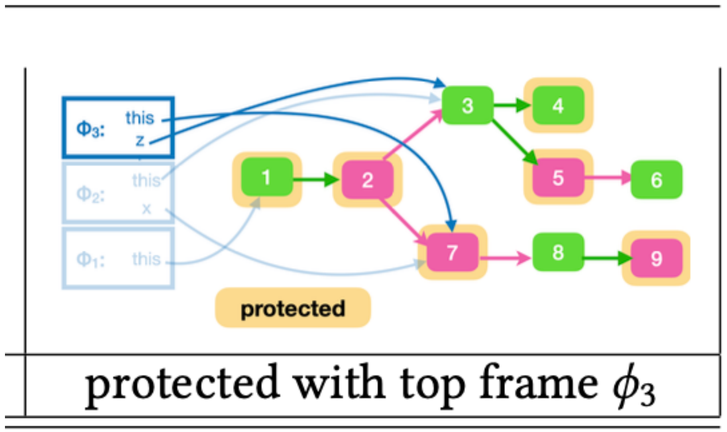

protected with top frame ϕ_3

Challenge_1_a : Meaning of x **prt**

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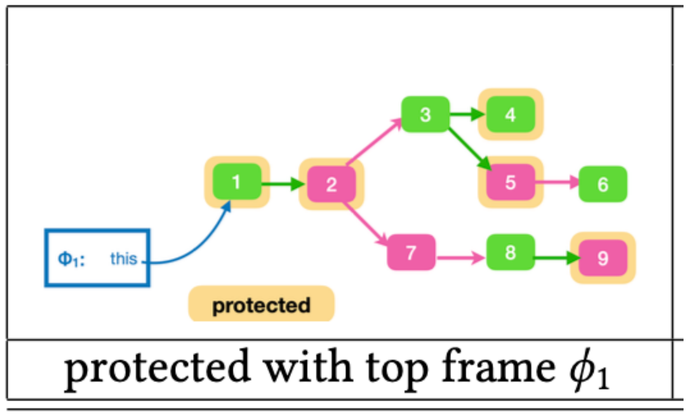
Motto:
Tracking access to Capabilities



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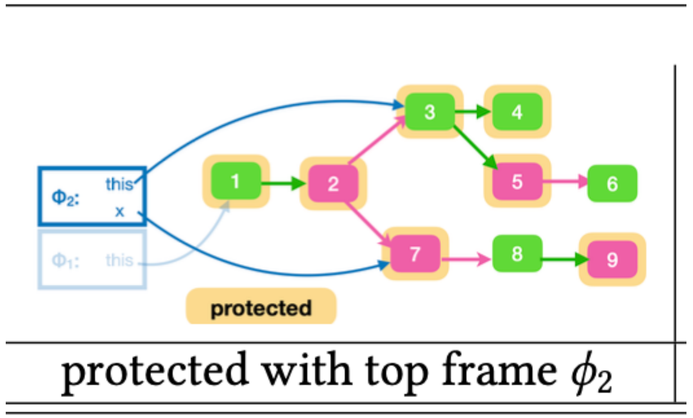

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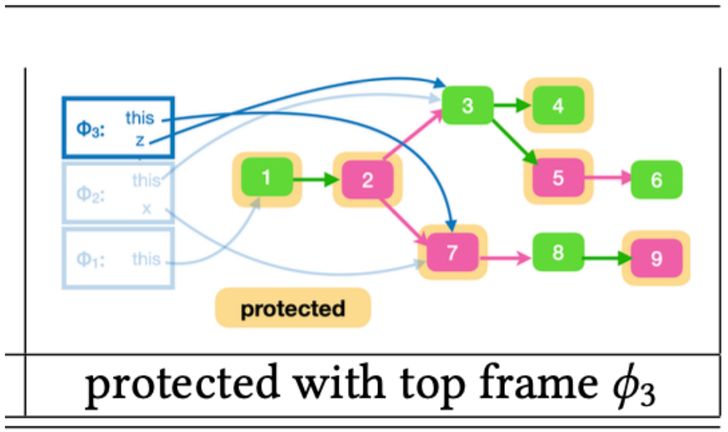

protected with top frame ϕ_3

Challenge_1_b : Meaning of ∀ x1:C1,x2:C2… (| A |) (| A’ |)

Definition

M ⊨ ∀ x1:C1,x2:C2… (| A |) (| A’ |)

iff

∧

Challenge_1_b : Meaning of ∀ x1:C1,x2:C2… (| A |) (| A' |)

Definition

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iff

For all modules M',
For all states σ arising from execution of (M, M') ∧
For all objects o1, .. on globally accessible at σ of class C1, … Cn,
For all states σ' in the future from σ whithout returning from σ 's top frame,

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For all states σ' in the future from σ whithout returning from σ 's top frame,

If

M, σ ⊨ o1:C1 ∧ … ∧ on:Cn ∧ A[o1/x1,… on/xn]

then

M, σ' ⊨ A'[o1/x1,… on/xn]

Putting it together

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Therefore,

$\forall$ x: Object (| **prt** x $\wedge$ A(x) |) (| A(x) |)

guarantees that

Putting it together

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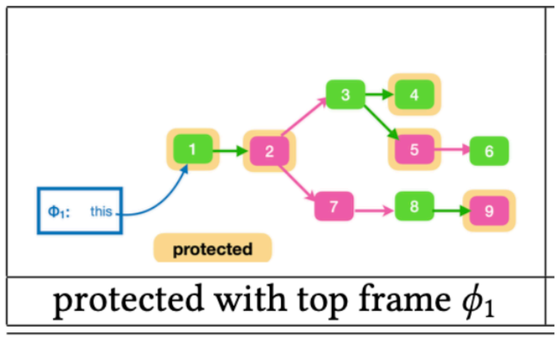
guarantees that

if we start below,



then

A(o1), A(o2), A(o4),
A(o5), A(o9)

will be preserved

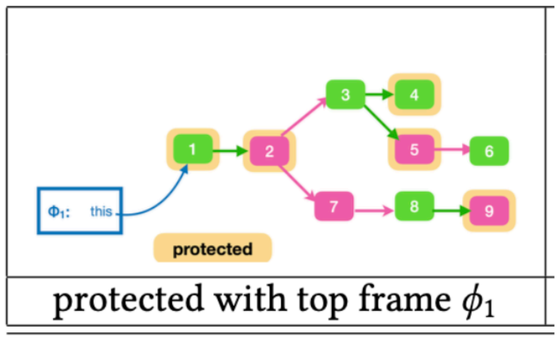
Putting it together

Therefore,

∀ x: Object (| **prt** x ∧ A(x) |) (| A(x) |)

guarantees that

if we start below,



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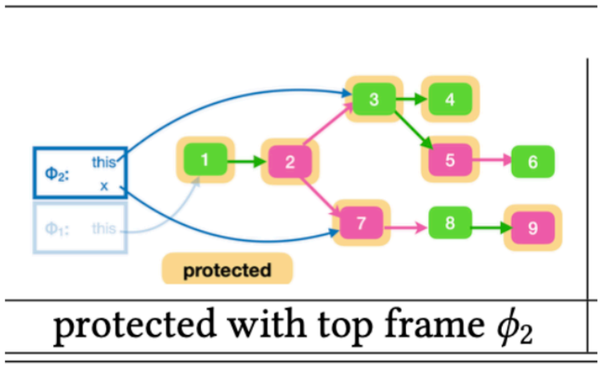

then

A(o1), A(o2), A(o3), A(o4),
A(o5), A(o7), A(o9)

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Putting it together

Therefore,

∀ x: Object (| **prt** x ∧ A(x) |) (| A(x) |)

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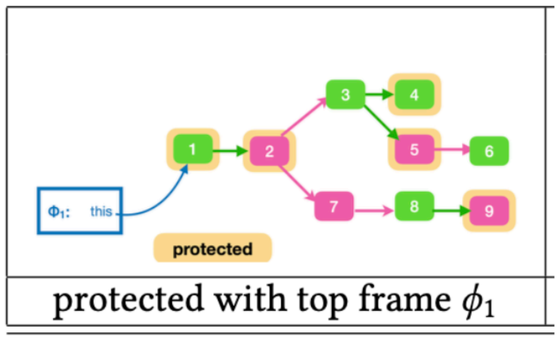

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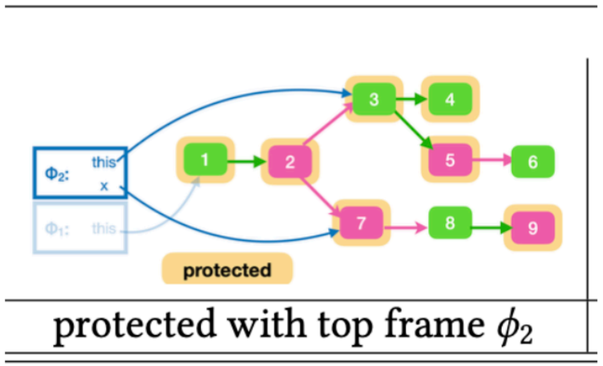

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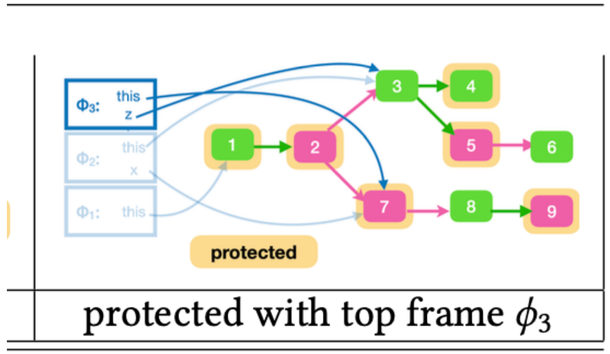

then

A(o1), A(o2), A(o4),
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will be preserved

Challenge_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{better} \models S$

$S =$

$\forall$ a:Account (| **prt** a |) (| **prt** a |)

$\wedge$

$\forall$ a:Account (| **prt** a.password |) (| **prt** a.password |)

$\wedge$

$\forall$ a:Account. $\forall$ b:Num (| **prt** a $\wedge$ a.balance = b |) (| a.balance = b |)

$\wedge$

$\forall$ a:Account. $\forall$ b:Num (| **prt** a.password $\wedge$ a.balance = b |) (| a.balance $\geq$ b |)

Challenge_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{better} \models S$

API - agnostic:
a.balance, a.password can be ghost

Talks about effects

$S =$

$\forall$ a:Account (| **prt** a |) (| **prt** a |)

$\wedge$

$\forall$ a:Account (| **prt** a.password |) (| **prt** a.password |)

$\wedge$

$\forall$ a:Account. $\forall$ b:Num (| **prt** a $\wedge$ a.balance = b |) (| a.balance = b |)

$\wedge$

$\forall$ a:Account. $\forall$ b:Num (| **prt** a.password $\wedge$ a.balance = b |) (| a.balance $\geq$ b |)

Talks about emergent behaviour

Challenge_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{better} \models S$

$M_{better} \models \forall$ a:Account (| **prt** a |) (| **prt** a |)

$M_{better} \models \forall$ a:Account (| **prt** a.password |) (| **prt** a.password |)

$M_{better} \models \forall$ a:Account. $\forall$ b:Num (| **prt** a $\wedge$ a.balance = b |) (| a.balance = b |)

$M_{better} \models \forall$ a:Account. $\forall$ b:Num (| **prt** a.password $\wedge$ a.balance = b |) (| a.balance $\geq$ b |)

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Challenge_2: An inference system, such that

$M_{good} \vdash S$
$M_{bad} \nvdash S$
$M_{better} \vdash S$

Challenge_4: An inference system, such that we can prove external calls

In the context of arbitrary, unlimited calls from internal to external,
and arbitrary, unlimited calls from external to internal,

Challenge_2: An inference system, such that

$M_{good} \vdash S$
$M_{bad} \nvdash S$
$M_{better} \vdash S$

In the context of arbitrary, unlimited calls from internal to external,
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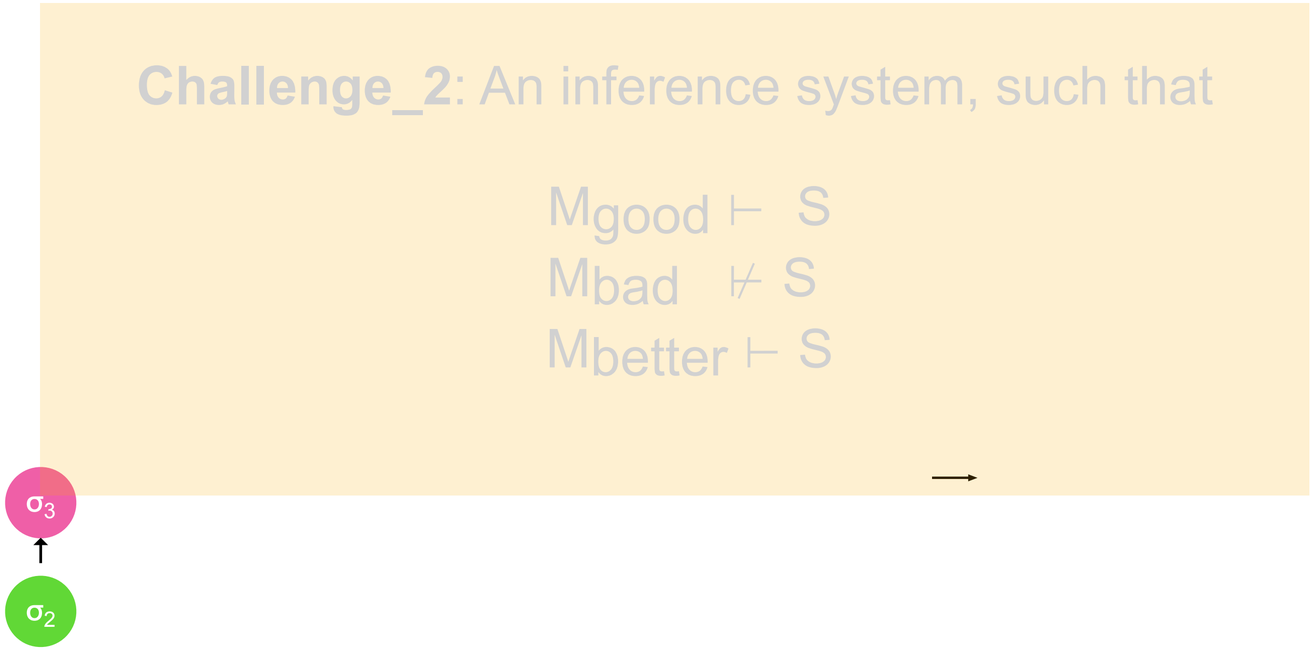

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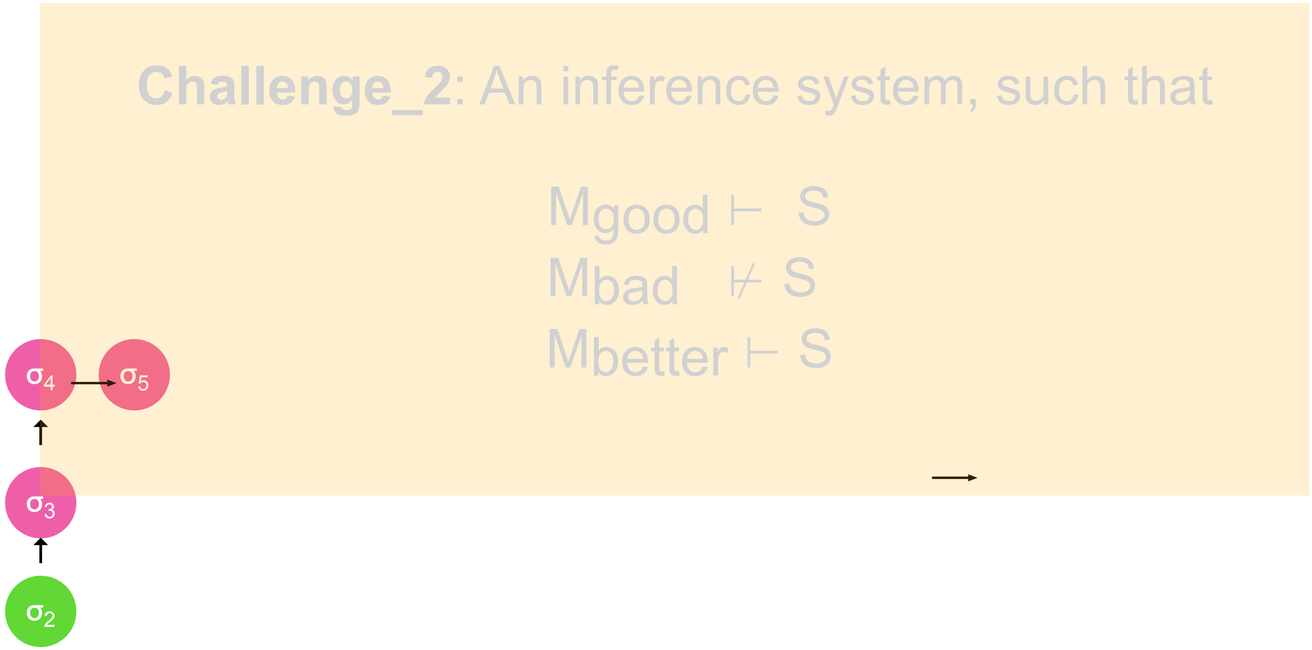

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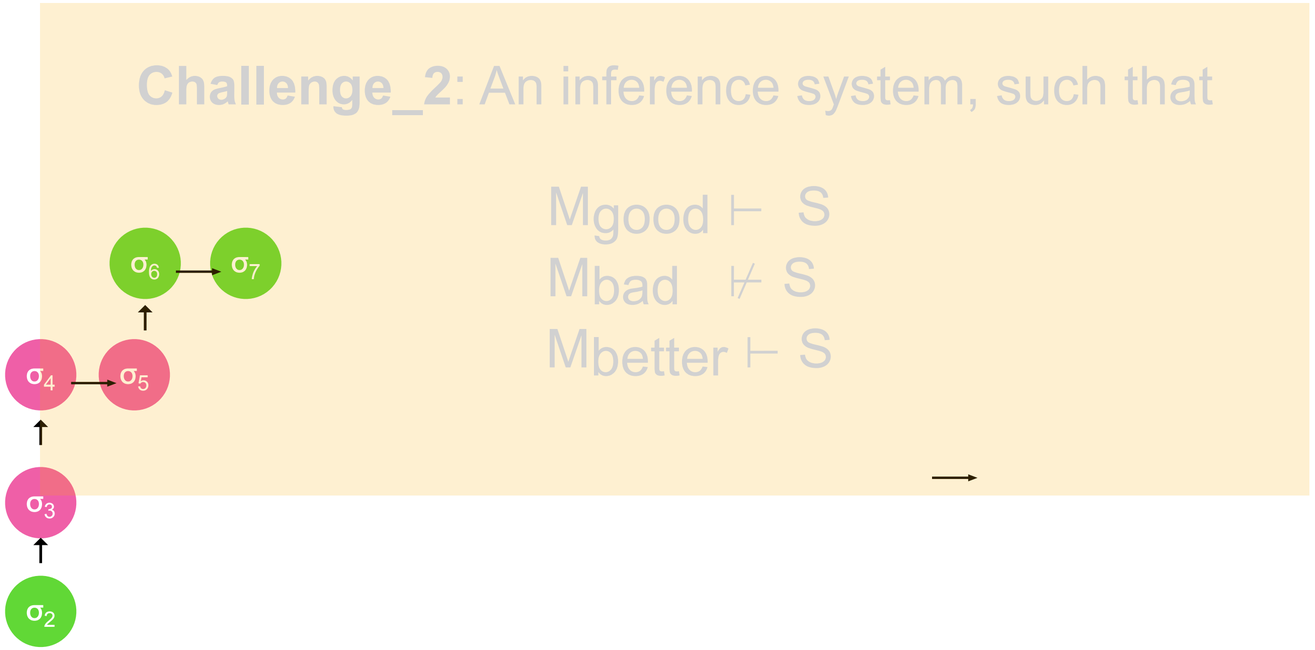

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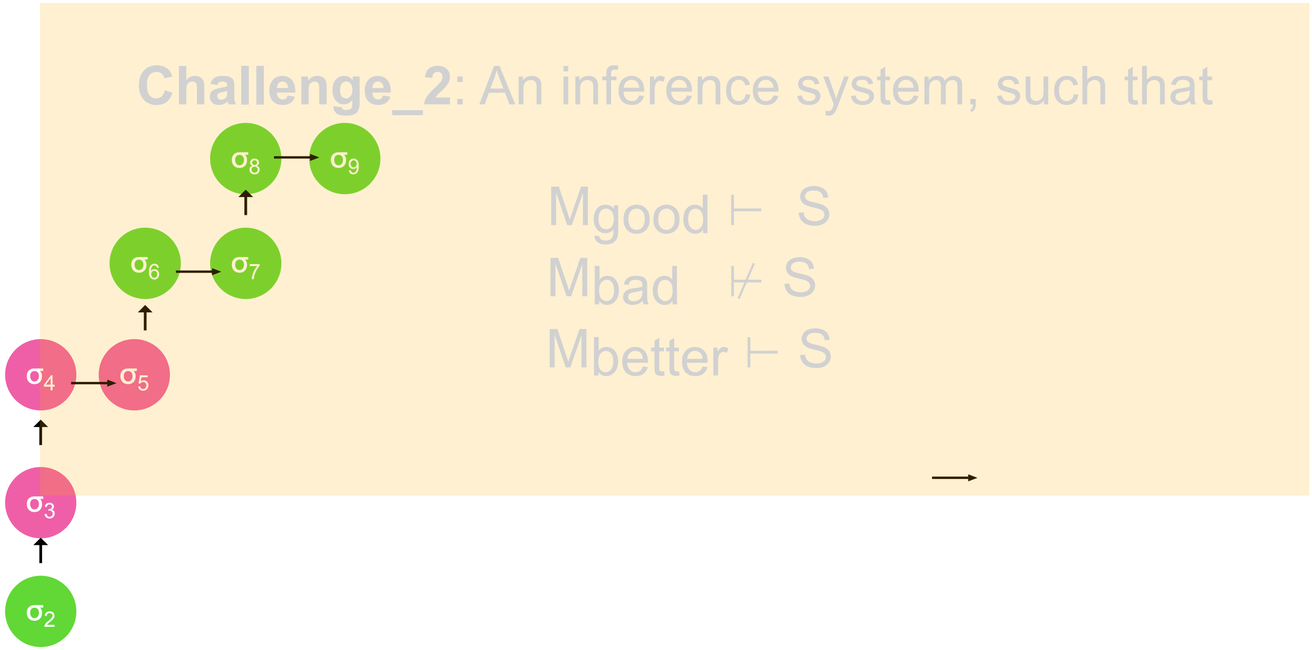

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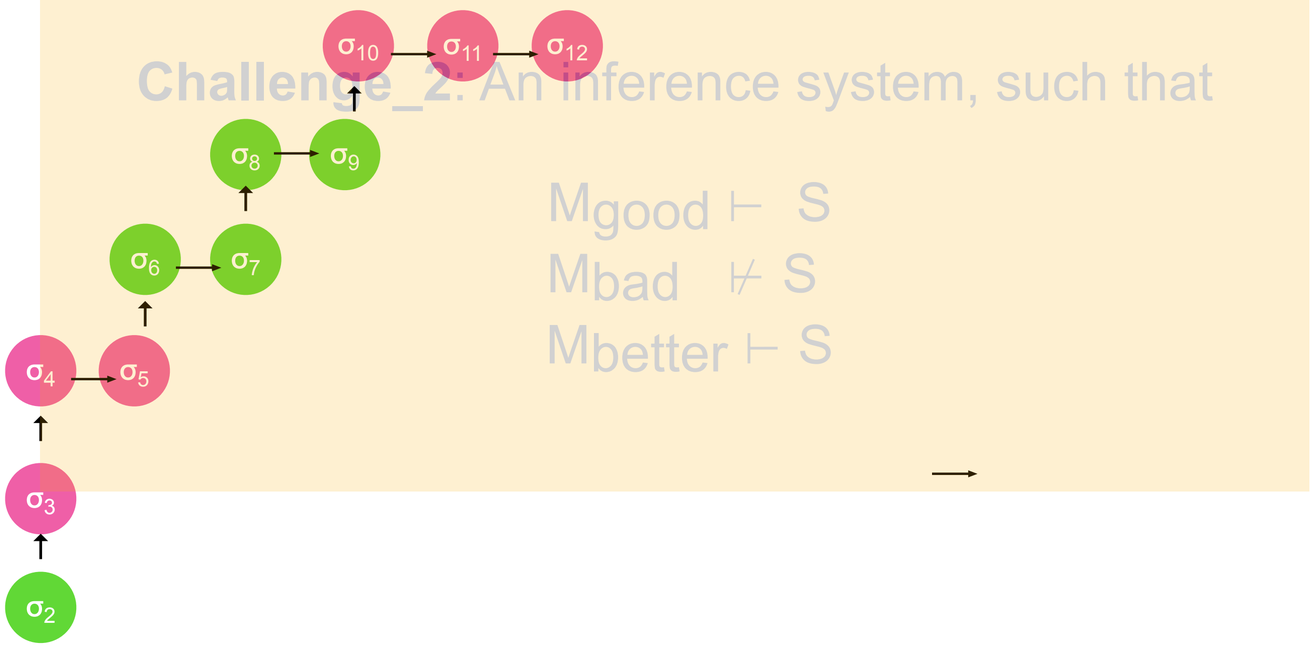

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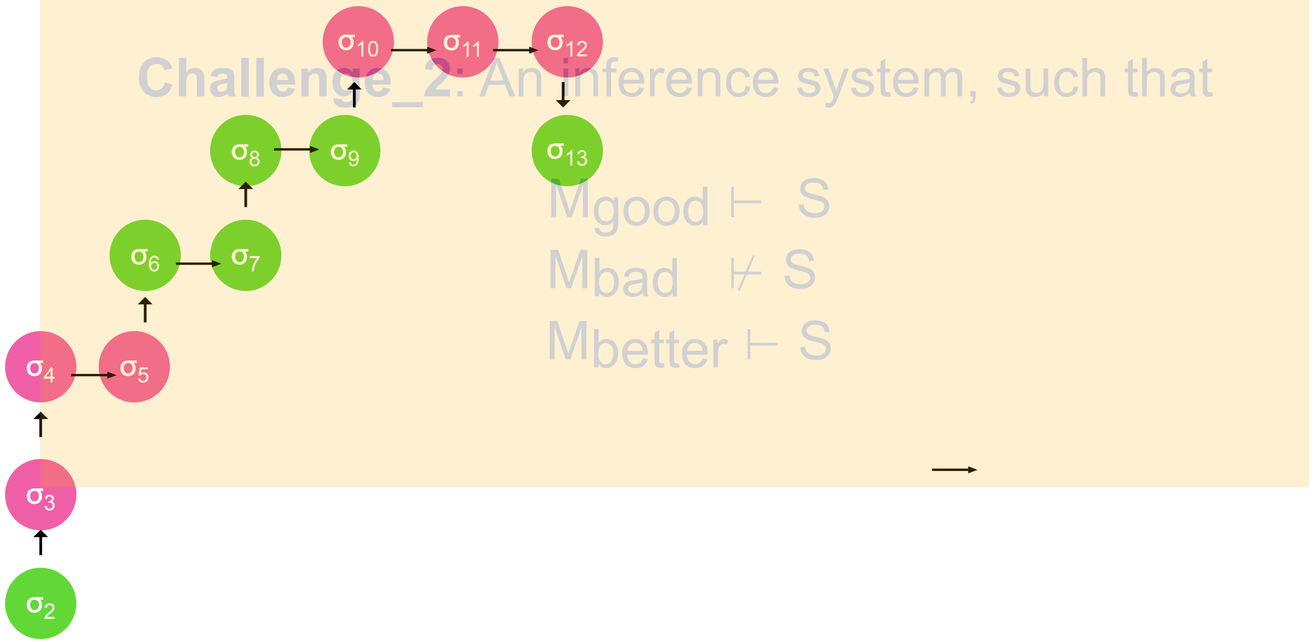

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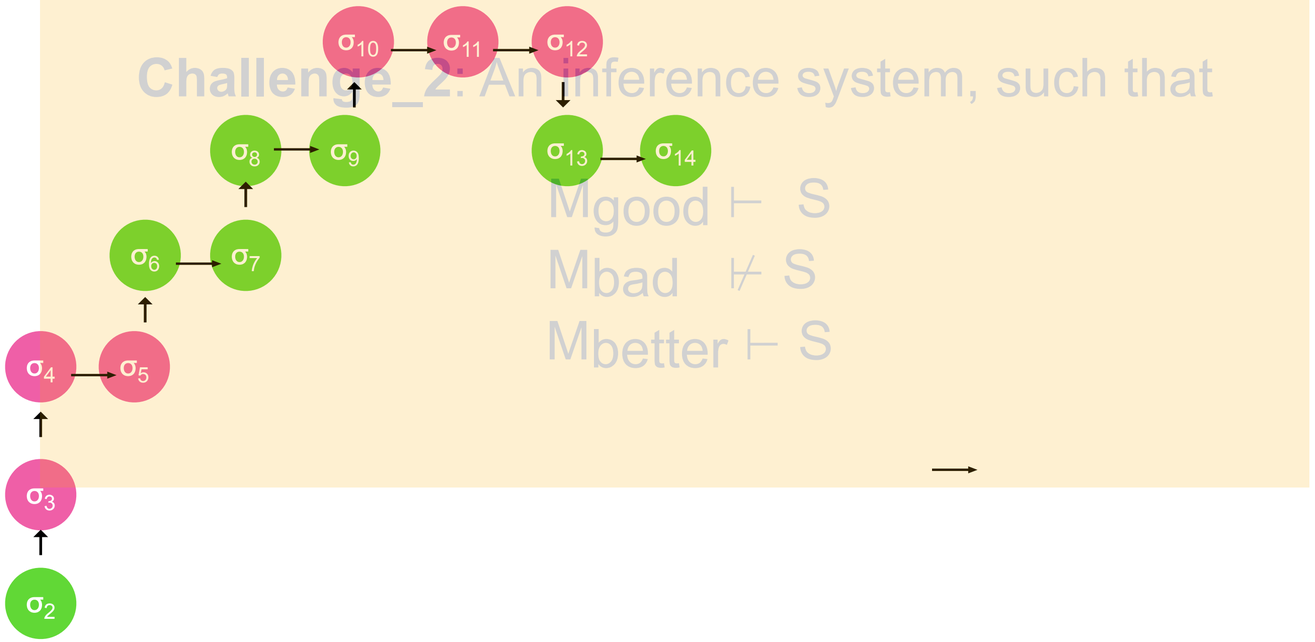

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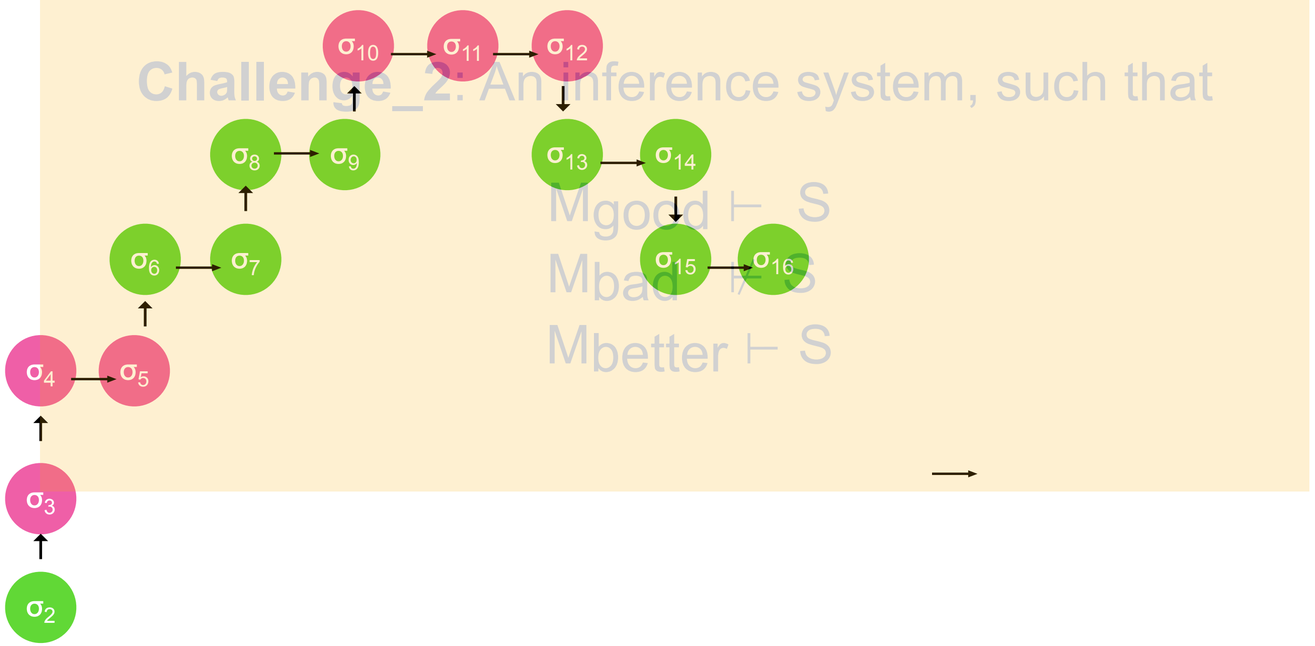

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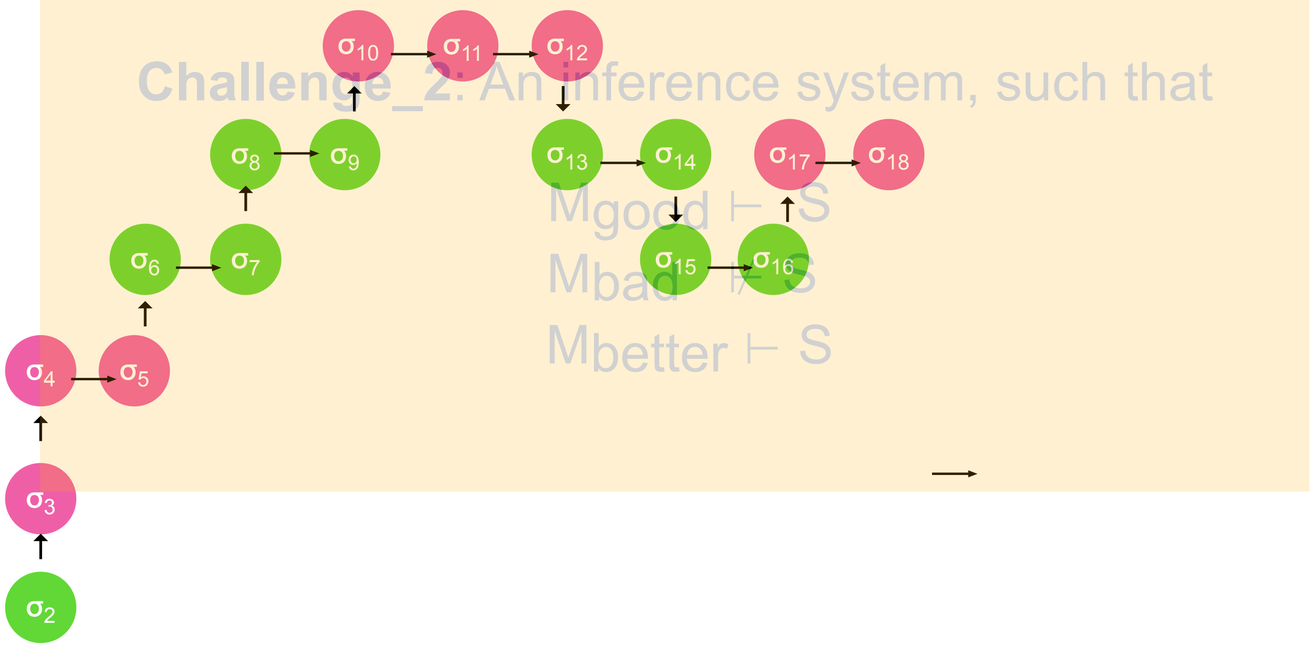

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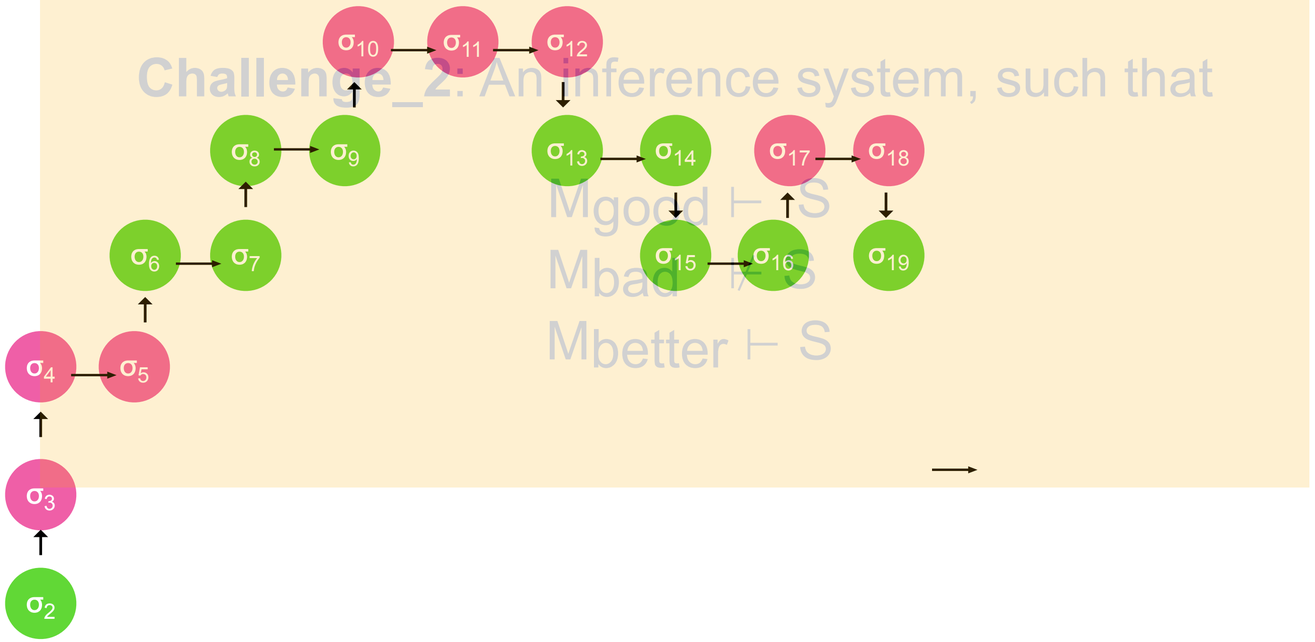

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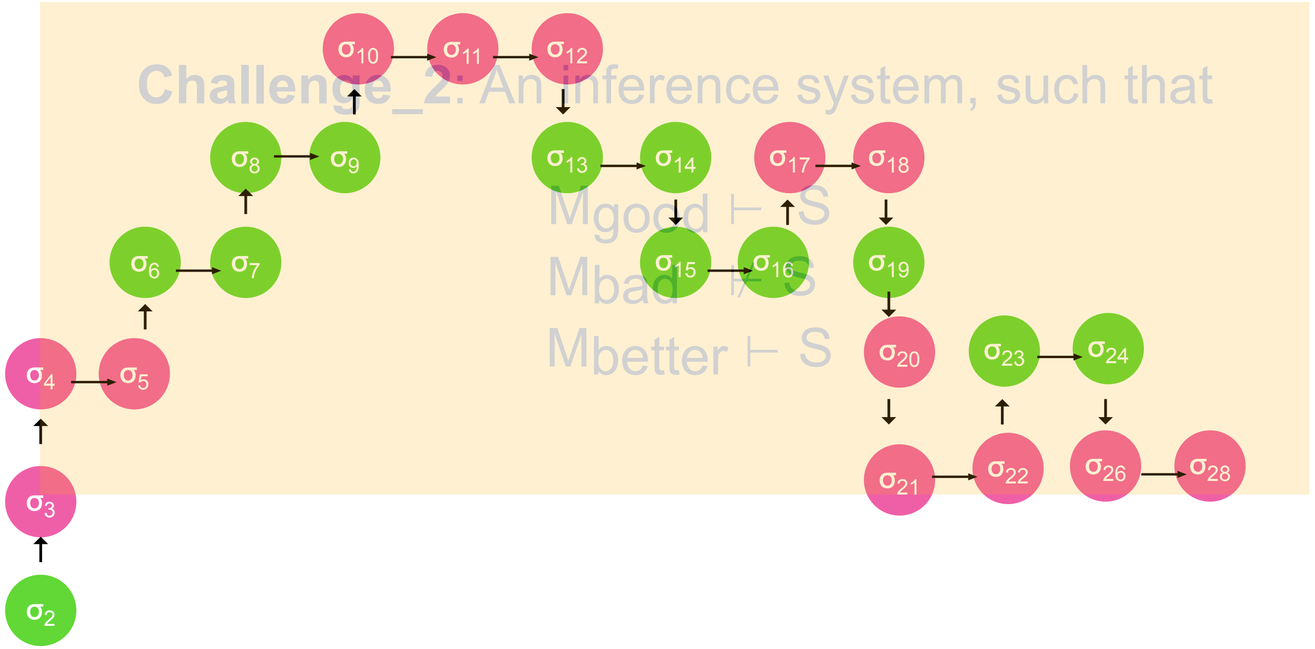

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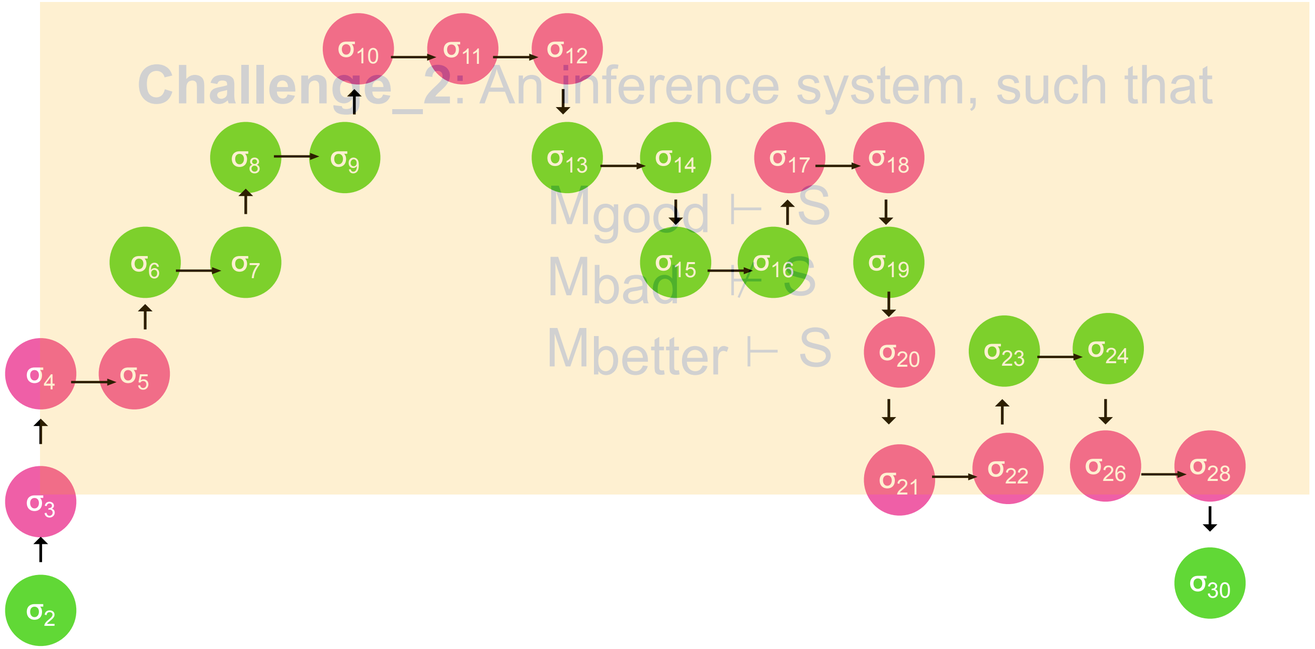

Challenge_2/4: An inference system, such that …

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it can only be invalidated through calls to methods from M.

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For example:

$$\texttt{Mod}_{\texttt{bad}} \vDash \textit{Encaps}(\texttt{a}:\texttt{Account} \wedge \texttt{a.balance} = \texttt{bal})$$
$$\texttt{Mod}_{\texttt{better}} \vDash \textit{Encaps}(\texttt{a}:\texttt{Account} \wedge \texttt{a.balance} = \texttt{bal})$$

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Assume two further modules, Mod_{ul} and Mod_{pl}, which use ledgers to keep a map between accounts and their balances, which export functions that allow the update of this map. In Mod_{ul} the ledger is *not* protected, while in Mod_{pl} the ledger *is* protected.

$$Mod_{ul} \nvDash \mathit{Encaps}(\mathtt{a:Account} \wedge \mathtt{a.balance} = \mathtt{bal})$$
$$Mod_{pl} \vDash \mathit{Encaps}(\mathtt{a:Account} \wedge \mathtt{a.balance} = \mathtt{bal})$$

Challenge_2/4: An inference system, such that …

$$\frac{\begin{array}{l}\forall D \in M, m \textit{ with } \texttt{mBody}(m, D, M) = \overline{y : D}\{\ s\ \} \\ \quad m \textit{ private} \\ \quad\quad \textit{for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \} \\ \quad m \textit{ public} \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \quad ??? \quad \}\ s\ \{\ \quad ??? \quad \} \\ \quad\quad \textit{and for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \quad ??? \quad \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \}\end{array}}{\vdash M}\ \textsc{Module\_Well\_Formed}$$

Challenge_2/4: An inference system, such that …

MODULE_WELL_FORMED

$$\frac{\begin{array}{l}\forall D \in M, m \text{ with } \texttt{mBody}(m, D, M) = \overline{y : D}\{\ s\ \} \\ \quad m \text{ private} \\ \quad\quad \text{for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \} \\ \quad m \text{ public} \\ \quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathcal{I}nvs(M) \text{ ??}\ \}\ s\ \{\ \mathcal{I}nvs(M) \text{ ??}\ \} \\ \quad \text{and for all its specs } A_{pre}, A_{post} : \\ \quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathcal{I}nvs(M) \text{ ?? } \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \}\end{array}}{\vdash M}$$

$$\begin{array}{lcl}\mathcal{I}nvs(\forall \overline{x : C}.(\!| A |\!)) & \triangleq & \forall \overline{x}.[\ \overline{x : C} \rightarrow A\ ] \\ \mathcal{I}nvs(\forall \overline{x : C}.(\!| A |\!)(\!| A |\!)) & \triangleq & \overline{x : C} \wedge A \\ \mathcal{I}nvs(S_1 \wedge S_2) & \triangleq & \mathcal{I}nvs(S_1) \wedge_{S_2} \mathcal{I}nvs(S_2) \\ \mathcal{I}nvs(M) & \triangleq & \mathcal{I}nvs(HS(M))\end{array}$$

Challenge_2/4: An inference system, such that …

MODULE_WELL_FORMED

$$\frac{\begin{array}{l}\forall D \in M, m \text{ with } \texttt{mBody}(m, D, M) = \overline{y : D}\{\ s\ \} \\ \quad m \text{ private} \\ \quad\quad \text{for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \} \\ \quad m \text{ public} \\ \quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathscr{I}nvs(M) \multimap \overline{y}\ \}\ s\ \{\ \mathscr{I}nvs(M) \multimap \overline{y}\ \} \\ \quad\quad \text{and for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathscr{I}nvs(M) \multimap \overline{y} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \}\end{array}}{\vdash M}$$

$$\begin{array}{rcl@{\qquad}rcl}
(\,\mathbf{prt}\ e\,) \multimap \overline{y} & \triangleq & e\ \mathbf{prt\text{–}frm}\ \overline{y} & (A_1 \wedge A_2) \multimap \overline{y} & \triangleq & (A_1 \multimap \overline{y}) \wedge (A_2 \multimap \overline{y}) \\
(\,e\ \mathbf{prt\text{–}frm}\ \overline{u}\,) \multimap \overline{y} & \triangleq & e\ \mathbf{prt\text{–}frm}\ \overline{u} & (\forall x : C.A) \multimap \overline{y} & \triangleq & \forall x : C.(A \multimap \overline{y}) \\
(\,\mathbf{intl}\ e\,) \multimap \overline{y} & \triangleq & \mathbf{intl}\ e & (\neg A) \multimap \overline{y} & \triangleq & \neg(A \multimap \overline{y}) \\
e \multimap \overline{y} & \triangleq & e & (e : C) \multimap \overline{y} & \triangleq & e : C
\end{array}$$

Challenge_2/4: An inference system, such that …

$$\frac{\begin{array}{l}\forall D \in M, m \text{ with } \texttt{mBody}(m, D, M) = \overline{y : D}\{\ s\ \} \\ \quad m \text{ private} \\ \quad\quad \text{for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \} \\ \quad m \text{ public} \\ \quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathscr{I}nvs(M)\ \neg\triangledown\overline{y}\ \}\ s\ \{\ \mathscr{I}nvs(M)\ \neg\triangledown\overline{y}\ \} \\ \quad\quad \text{and for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathscr{I}nvs(M)\ \neg\triangledown\overline{y} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \}\end{array}}{\vdash M}\ \textsc{Module\_Well\_Formed}$$

Lemma 3.6. *For any states σ, σ', assertion A, addresses $\overline{\alpha}$, variables $\overline{x}$, $\overline{y}$, $\overline{z}$ disjoint with one another, and such that $fv(A) \subseteq \overline{x}$:*

(1) $M, \sigma[\overline{x \mapsto \alpha}] \models A\ \neg\triangledown\overline{y} \implies M,\ (\sigma[\overline{x \mapsto \alpha}])\ \triangledown\ \overline{y} \models A$

(2) $M, ((\sigma[\overline{x \mapsto \alpha}])\ \triangledown\ (\overline{y}, \overline{z})) \models A \implies M, \sigma[\overline{x \mapsto \alpha}] \models A\ \neg\triangledown\overline{y}$

Challenge_2/4: An inference system, such that …

$$\textsc{Module\_Well\_Formed}$$

$$\frac{\begin{array}{l}\forall D \in M, m \textit{ with } \texttt{mBody}(m, D, M) = \overline{y : D}\{\ s\ \} \\ \quad m \textit{ private} \\ \quad\quad \textit{for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \} \\ \quad m \textit{ public} \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathscr{I}nvs(M)\ \neg\nabla\overline{y}\ \}\ s\ \{\ \mathscr{I}nvs(M)\ \neg\nabla\overline{y}\ \} \\ \quad\quad \textit{and for all its specs } A_{pre}, A_{post} : \\ \quad\quad\quad M \vdash \{\ \texttt{this} : \texttt{D} \wedge \overline{y : D} \wedge \mathscr{I}nvs(M)\ \neg\nabla\overline{y} \wedge A_{pre}\ \}\ s\ \{\ A_{post}\ \}\end{array}}{\vdash M}$$

$$\textsc{Weaken}\quad \frac{M \vdash S \qquad M \vdash S \sqsubseteq S'}{M \vdash S'}$$

$$\textsc{Multi}\quad \frac{M \vdash S \qquad M \vdash S'}{M \vdash S \wedge S'}$$

Challenge_2/4: … prove method bodies ….

EXTEND

$$\frac{M \vdash_{ul} \{ A \}\, s\, \{ A' \}}{M \vdash \{ A \}\, s\, \{ A' \}}$$

COMBINE

$$\frac{M \vdash \{ A_1 \}\, s\, \{ A_1' \} \quad M \vdash \{ A_2 \}\, s\, \{ A_2' \}}{M \vdash \{ A_1 \wedge A_2 \}\, s\, \{ A_1' \wedge A_2' \}}$$

SEQU

$$\frac{M \vdash \{ A \}\, s_1\, \{ A'' \} \quad M \vdash \{ A'' \}\, s_2\, \{ A' \}}{M \vdash \{ A \}\, s_1;\, s_2\, \{ A' \}}$$

CONSEQU

$$\frac{M \vdash A \rightarrow A'' \quad M \vdash \{ A'' \}\, s\, \{ A''' \} \quad M \vdash A''' \rightarrow A'}{M \vdash \{ A \}\, s\, \{ A' \}}$$

[IntCall_WithSpec]

$$\frac{M, C, m : (A_1,\ A_2) \qquad fv(A_1) = \overline{x}}{M \vdash \{\ \mathbf{intl}\, y_0 \wedge y : C \wedge A_1[\overline{y}/\mathtt{this}, \overline{x}]\ \}\ u := y_0.m(y_1, ..y_n)\ \{\ A_2[u/result, \overline{y}/\mathtt{this}, \overline{x}]\ \}}$$

Challenge_2/4: …. prove external calls

[CallAndAlias]

$$\frac{x \overset{\text{txt}}{\neq} u \overset{\text{txt}}{\neq} x'}{M \vdash \{\ x = x' \ \wedge\ (\ \mathbf{extl}\, y_0 \ \rightarrow\ \mathscr{I}nvs(M)\ \neg\triangledown\overline{y})\ \}\ u := y_0.m(y_1,..y_n)\ \{\ x = x'\ \}}$$

[CallNonAlias]

$$\frac{x \overset{\text{txt}}{\neq} u \overset{\text{txt}}{\neq} x'}{M \vdash \{\ x \neq x' \ \wedge\ (\ \mathbf{extl}\, y_0 \ \rightarrow\ \mathscr{I}nvs(M)\ \neg\triangledown\overline{y})\ \}\ u := y_0.m(y_1,..y_n))\ \{\ x \neq x'\ \}}$$

Challenge_2/4: …. prove external calls

$$\frac{}{M \vdash \{\ \mathbf{extl}\, y_0 \ \wedge\ \mathscr{I}nvs(M) \multimap \forall \overline{y}\ \}\ u := y_0.m(y_1, ..y_n)\ \{\ \mathscr{I}nvs(M) \multimap \forall \overline{y}\ \}}\ [\textsc{ExtCal}]$$

Challenge_2/4: …. prove external calls

$$\frac{\vdash M : \quad ???}{M \vdash \{\ \mathbf{extl}\, y_0 \ \wedge\ \overline{x : C} \ \wedge\ \ ???\ \ \wedge\ \mathit{Invs}(M) \multimap \overline{\forall y}\ \}\ u := y_0.m(y_1, ..y_n)\ \{\ \ ???\ \ \}}\ [\textsc{ExtCall\_WithSpec\_Weak}]$$

Challenge_2/4: …. prove external calls

[ExtCall_WithSpec_Weak]

$$\frac{\vdash M : \quad ???}{M \vdash \{\ \mathbf{extl}\, y_0 \ \wedge\ \overline{x : C} \ \wedge\ \ ???\ \ \wedge\ \mathit{Invs}(M) \multimap \forall \overline{y}\ \}\ u := y_0.m(y_1, ..y_n)\ \{\ \ ???\ \ \}}$$

Motto:
Capability is a *necessary* condition for some effect

Challenge_2/4: …. prove external calls

$$\frac{\vdash\ M\ :\ \forall\overline{x:C}.(\!|A_1|\!)(\!|A_2|\!)}{M\ \vdash\ \{\ \mathbf{extl}\,y_0\ \wedge\ \overline{x:C}\ \wedge\ \ ???\ \ \wedge\ \mathit{Invs}(M)\ \neg\forall\overline{y}\ \}\ u := y_0.m(y_1,..y_n)\ \{\ \ ???\ \ \}}\ [\textsc{ExtCall\_WithSpec\_Weak}]$$

Motto:
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Challenge_2/4: …. prove external calls

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Motto:
Capability is a *necessary* condition for some effect

Challenge_2/4: …. Revisiting the Shop example

```
class Shop

  fld myAccount : Account
  fld inventory : Inventory

  void buy(buyer: Object, anItem: Item)
    int price = anItem.price
    int oldBalance = this.myAccount.balance
    buyer.payMe(myAccount,price)
    if (this.myAccount.balance == oldBalance+price)
       this.send(buyer,anItem)
    else
       buyer.tell("you have not paid me")
```

Challenge_2/4: …. Revisiting the Shop exam

External call

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`Shop` part of `Banking` module;
`buy` a public method of `Shop`.

Challenge_2/4: …. Revisiting the Shop exam


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`buy` satisfies

PRE: `this.myAccount.passdw` **`prt-frm`** `buyer && this.myAccount.balance==b`
POST: `this.myAccount.balance >= b`

Challenge_2/4: …. Revisiting the Shop exam


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This spec *derivable* from the `Banking` **spec**

Challenge_2/4: …. Revisiting the Shop exam

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`buy` possible spec

PRE: `myAccount.balance=b && this.inventory = list`

POST: `myAccount.balance=b+anItem.price -> this.inventory = list\anItem`

`buy` satisfies **implicitly**

PRE: `this.myAccount.passdw` **`prt-frm`** `buyer && this.myAccount.balance==b`

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🤔🤔 this spec is implied 🤔🤔

Of course, we have to prove that `buy` preserves `Banking`'s spec
(as per rule Module_Well_Fomed)

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This spec *derivable* from the `Banking` spec

Challenge_3: The inference system should be algorithmic

Happy!

Convinced!

Surprised!

Summary

- Distinction between external/internal objects
- Specifications talk about necessary conditions for effect:
  ∀ x: Object (| **denial of capability** ∧ A |) (| A |)
  means that **capability** is needed in order to invalidate A
- **prt** x: expresses that capability x is protected from external objects
- Design “Fineties”
  - **prt** x only protects from *locally-relevant* objects;
  - ∀ x: ..(|..|)(|…|) tallks about *globally-relevant* objects
  - “future is shallow”
- API-agrnositc spec, “Algorithmic” inference system system, open calls
- Hand-written soundness and adherence proof

Happy!

Convinced:

Object capabilities are about necessary conditions for effects caused by external objects.

Surprised!

Happy!

Convinced!

Surprised:

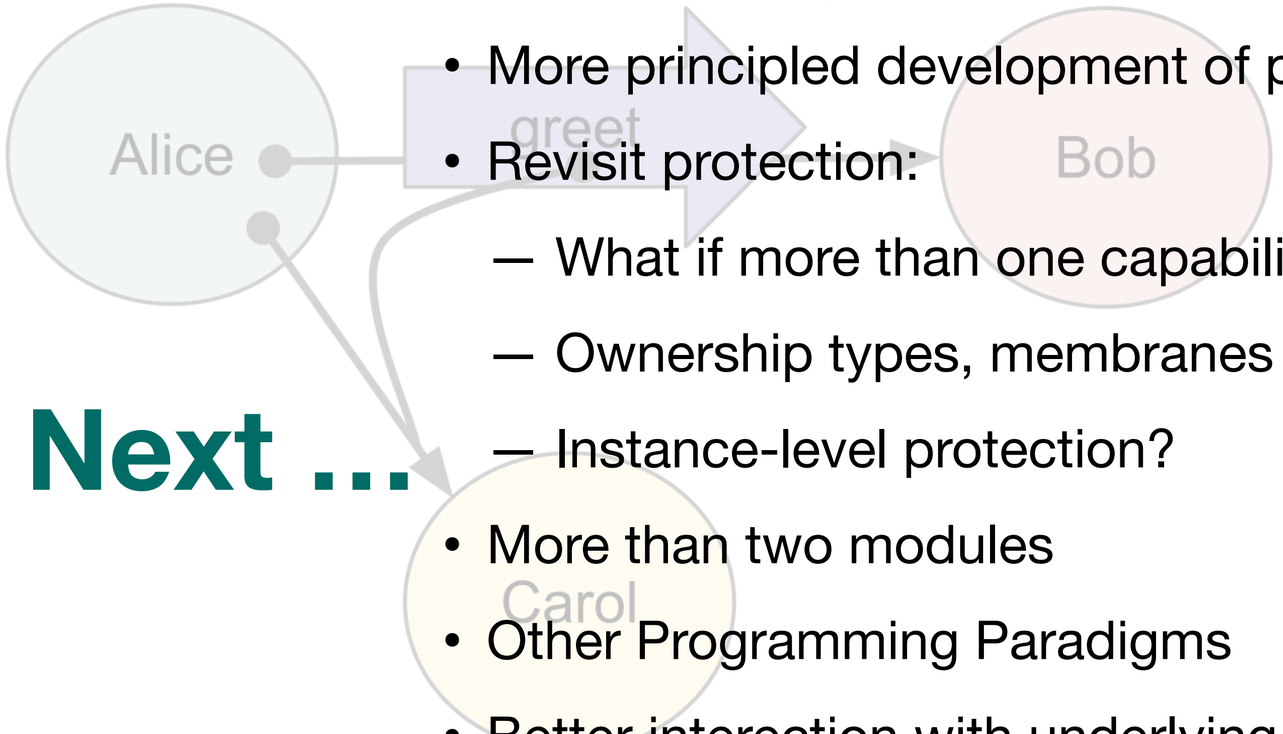
* talk about *necessary* conditions, but reason with *sufficient* conditions.

* No need for temporal logic specifications.

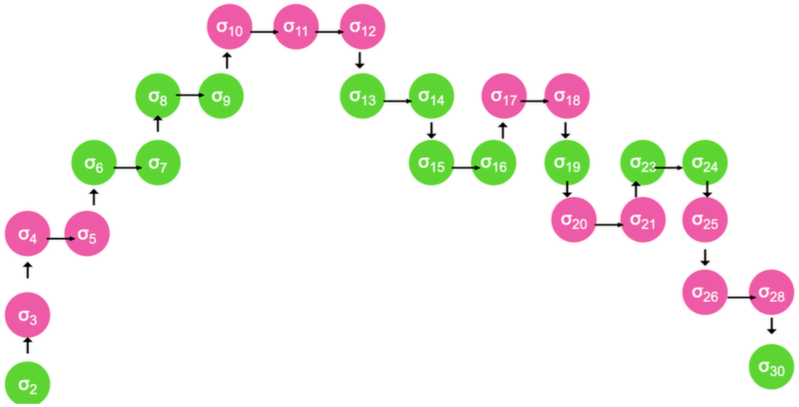
* Hoare-logic extension

Next …

- Smooth the edges
- Mechanize proofs
- Completeness?
- Adversarial logic
- More principled development of ptr-Hoare rules
- Revisit protection:
  - What if more than one capability for an effect?
  - Ownership types, membranes etc?
  - Instance-level protection?
- More than two modules
- Other Programming Paradigms
- Better interection with underlying Hoare logics, esp. “modifies” clauses, *
- Parametric with language and Hoare Logic
- Inference of Classic Specs given Invariants
- Tool



the original execution:



the summarised execution: