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Necessity Specifications are Necessary for Robustness

ANONYMOUS AUTHOR(S)

Robust modules guarantee to do *only* what they are supposed to do – even in the presence of untrusted, malicious clients, and considering not just the direct behaviour of individual methods, but also the emergent behaviour from calls to more than one method. *Necessity* is a language for specifying robustness, based on novel necessity operators capturing temporal implication, and a proof logic that derives explicit robustness specifications from functional specifications. Soundness and an exemplar proof are mechanised in Coq.

1 INTRODUCTION: NECESSARY CONDITIONS AND ROBUSTNESS

Software needs to be both *correct* (programs do what they are supposed to) and *robust* (programs *only* do what they are supposed to). We use the term *robust* as a generalisation of *robust safety* [Bugliesi et al. 2011; Gordon and Jeffrey 2001; Swasey et al. 2017] whereby a module or process or ADT is *robustly safe* if its execution preserves some safety guarantees even when run together with unknown, unverified, potentially malicious client code. The particular safety guarantees vary across the literature. We are interested in *program-specific* safety guarantees which describe *necessary conditions* for some effect to take place. In this work we propose how to specify such necessary conditions, and how to prove that modules adhere to such specifications.

We motivate the need for necessary conditions for effects through an example: Correctness is traditionally specified through Hoare [1969] triples: a precondition, a code snippet, and a postcondition. For example, part of the functional specification of a `transfer` method for a bank module is that the source account's balance decreases:

$$S_{\text{correct}} \triangleq \{\text{pwd}=\text{src.pwd} \wedge \text{src.bal}=\text{b}\} \text{src.transfer}(\text{dst}, \text{pwd}) \{\text{src.bal}=\text{b}-100 \wedge \dots\}$$

Calling `transfer` on an account with the correct password will transfer the money.

Assuming termination, the precondition is a *sufficient* condition for the code snippet to behave correctly: the precondition (e.g. providing the right password) guarantees that the code (e.g. call the `transfer` function) will always achieve the postcondition (the money is transferred).

S_{correct} describes the *correct use* of the particular function, but is *not* concerned with the module's *robustness*. For example, can I pass an account to foreign untrusted code, in the expectation of receiving a payment, but without fear that a malicious client might use the account to steal my money [Miller et al. 2000]? A first attempt to specify robustness could be:

$$S_{\text{robust}_1} \triangleq \text{An account's balance does not decrease unless transfer was called with the correct password.}$$

Specification S_{robust_1} guarantees that it is not possible to take money out of the account without calling `transfer` and without providing the password. Calling `transfer` with the correct password is a *necessary condition* for (the effect of) reducing the account's balance.

S_{robust_1} is crucial, but not enough: it does not take account of the module's *emergent behaviour*, that is, does not cater for the potential interplay of several methods offered by the module. What if the module provided further methods which leaked the password? While no single procedure call is capable of breaking the intent of S_{robust_1} , a sequence of calls might. What we really need is

$$S_{\text{robust}_2} \triangleq \text{The balance of an account does not ever decrease in the future unless some external object now has access to the account's current password.}$$

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Necessity Specifications for Robustness

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Calling `transfer` on an account with the correct password will transfer the money.

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With $S_{\text{robust_2}}$, I can confidently pass my account to any, potentially untrusted context, where my password is not known; the payment I was expecting may or may not be made, but I know that my money will not be stolen [Miller 2011]. Note that $S_{\text{robust_2}}$ does not mention the names of any functions in the module, and thus can be expressed without reference to any particular API – indeed $S_{\text{robust_2}}$ can constrain *any* API with an account, an account balance, and a password.

Earlier work addressing robustness includes object capabilities [Birkedal et al. 2021; Devriese et al. 2016; Miller 2006], information control flow [Murray et al. 2013; Zdancewic and Myers 2001], correspondence assertions [Fournet et al. 2007], sandboxing [Patrignani and Garg 2021; Sammler et al. 2019], robust linear temporal logic [Anevlavis et al. 2022] – to name a few. Most of these propose *generic* guarantees (e.g. no dependencies from high values to low values), while we work with *problem-specific* guarantees (e.g. no decrease in balance without access to password). VERX [Permenev et al. 2020a] and Chainmail [Drossopoulou et al. 2020b] also work on problem-specific guarantees. Both these approaches are able to express necessary conditions like $S_{\text{robust_1}}$ using temporal logic operators and implication, and Chainmail is able to express $S_{\text{robust_2}}$, however neither have a proof logic *able* to prove adherence to such specifications.

1.1 Necessity

In this paper we introduce *Necessity*, the first approach that is able to both express and prove robustness specifications such as $S_{\text{robust_2}}$. Developing a specification language with a proof logic that is able to prove properties such as $S_{\text{robust_2}}$ and must tread a fine line: the language must be rich enough to express complex specifications; temporal operators are needed along with object capability style operators that describe *permission* and *provenance*, while also being simple enough that proof rules might be devised.

The first main contribution is three novel operators that merge temporal operators and implication and most importantly are both expressive enough to capture the examples we have found in the literature and provable. One such necessity operator is

$\text{from } A_{\text{curr}} \text{ to } A_{\text{fut}} \text{ onlyIf } A_{\text{nec}}$

This form says that a transition from a current state satisfying assertion A_{curr} to a future state satisfying A_{fut} is possible only if the necessary condition A_{nec} holds in the *current* state. Using this operator, we can formulate $S_{\text{robust_2}}$ as

```
1  $S_{\text{robust\_2}} \triangleq \text{from } a:\text{Account} \wedge a.\text{balance} == \text{bal} \text{ to } a.\text{balance} < \text{bal}$ 
2  $\text{onlyIf } \exists o. \{ \langle o \text{ external} \rangle \wedge \langle o \text{ access } a.\text{pwd} \rangle \}$ 
```

Namely, a transition from a current state where an account's balance is `bal`, to a future state where it has decreased, may *only* occur if in the current state some `external`, unknown client object has access to that account's password. More in §2.3.

Unlike Chainmail's temporal operators, the necessity operators are not first class, and may not appear in the assertions (e.g. A_{curr}). This simplification enabled us to develop our proof logic. Thus, we have reached a sweet spot between expressiveness and provability.

The second main contribution is a logic that enables us to prove that code obeys *Necessity* specifications. Our insight was that *Necessity* specifications are logically equivalent to the intersection of an *infinite* number of Hoare triples, i.e., $\text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_3$ is logically equivalent to $\forall \text{stmts}. \{A_1 \wedge \neg A_3\} \text{stmts} \{ \neg A_2 \}$. This leaves the challenge that there do not exist logics to reason about such infinite intersections.

We addressed that challenge through three further insights: (1) *Necessity* specifications of emergent behaviour can be built up from *Necessity* specifications of single-step executions, which (2) can be built from encapsulation and *finite* intersections of *Necessity* specifications of function calls,

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which (3) in turn can be obtained from *traditional* functional specifications. A strength of our work is that it is parametric with respect to assertion satisfaction, encapsulation, and functional specifications, all of which are well covered in the literature, and offer several off-the-shelf solutions.

1.2 Contributions and Paper Organization

The contributions of this work are:

- (1) A language to express *Necessity* specifications (§3), including three novel *Necessity* operators (§3.3) that combine implication and temporal operators.
- (2) A logic for proving a module's adherence to its *Necessity* specifications (§4), and a proof of soundness of the logic, (§4.5), both mechanised in Coq.
- (3) A proof in our logic that our bank module obeys $S_{\text{robust_2}}$ (§5), mechanised in Coq. And a proof that a richer bank module which uses ghostfields and confined classes obeys $S_{\text{robust_2}}$ (§F), also mechanised in Coq.
- (4) Examples taken from the literature (§3.4 and §C) specified in *Necessity*.

We place *Necessity* into the context of related work (§6) and consider our overall conclusions (§7). The Coq proofs of (2) and (3) above appear in the supplementary material, along with appendices containing expanded definitions and further examples. In the next section, (§2), we outline our approach using a bank as a motivating example.

2 OUTLINE OF OUR APPROACH

In this Section we outline our approach: we revisit our running example, the Bank Account (§2.1), introduce the three necessity operators (§2.2), give the *Necessity* specs (§2.3), outline how we model the open world (§2.4), give the main ideas of our proof system (§2.5) and outline how we use it to reason about adherence to *Necessity* specifications (§2.6).

2.1 Bank Account – three modules

Module `Modgood` consists of an empty `Password` class where each instance models a unique password, and an `Account` class with a password, and a balance, an `init` method to initialize the password, and a `transfer` method.

```

1 module Modgood
2   class Account
3     field balance:int
4     field pwd: Password
5     method transfer(dest:Account, pwd':Password) -> void
6       if this.pwd==pwd'
7         this.balance-=100
8         dest.balance+=100
9     method init(pwd':Password) -> void
10      if this.pwd==null
11        this.pwd=pwd'
12   class Password

```

We can capture the intended semantics of `transfer` through a functional specification with pre- and post- conditions and MODIFIES clauses as e.g., in Leavens et al.; Leino. The implementation of `transfer` in module `Modgood` meets this specification.

```

1 FuncSpec  $\triangleq$ 
2   method transfer(dest:Account, pwd':Password) -> void
3   ENSURES:
4     this.pwd=pwd'  $\wedge$  this $\neq$ dest  $\rightarrow$ 
5     this.balancepost=this.balancepre-100  $\wedge$  dest.balancepost=dest.balancepre+100

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11        this.pwd=pwd'
12   class Password

```

```

148 ENSURES:
149   this.pwd#pwd'  $\vee$  this=dest  $\rightarrow$ 
150   this.balancepost=this.balancepre  $\wedge$  dest.balancepost=dest.balancepre
151 MODIFIES: this.balance, dest.balance

```

Now consider the following alternative implementations: Mod_{bad} allows any client to reset an account's password at any time; $\text{Mod}_{\text{better}}$ requires the existing password in order to change it.

<pre> 154 module Mod_{bad} 155 class Account 156 field balance:int 157 field pwd: Password 158 method transfer(..) ... 159 ... as earlier ... 160 method init(...) ... 161 ... as earlier ... 162 method set(pwd': Password) 163 this.pwd=pwd' 164 class Password </pre>	<pre> 154 module Mod_{better} 155 class Account 156 field balance:int 157 field pwd: Password 158 method transfer(..) ... 159 ... as earlier ... 160 method set(pwd',pwd'': Password) 161 if (this.pwd==pwd') 162 this.pwd=pwd'' 163 class Password </pre>
--	--

Although the transfer method is the same in all three alternatives, and each one satisfies FuncSpec , code such as

```

an_account.set(42); an_account.transfer(rogue_account, 42)

```

is enough to drain an_account in Mod_{bad} without knowing the password.

2.2 The three necessity operators

We need a specification that rules out Mod_{bad} while permitting Mod_{good} and $\text{Mod}_{\text{better}}$. For this, we will be using one of the three necessity operators mentioned in §1.1. These operators are:

```

from Acurr to Afut onlyIf Anec
from Acurr next Afut onlyIf Anec
from Acurr to Afut onlyThrough Aintrm

```

The first operator was already introduced in §1.1: it says that a transition from a current state satisfying assertion A_{curr} to a future state satisfying A_{fut} is possible only if the necessary condition A_{nec} holds in the *current* state. The second operator says that a *one-step* transition from a current state satisfying assertion A_{curr} to a future state satisfying A_{fut} is possible only if A_{nec} holds in the *current* state. The third operator says that a change from A_{curr} to A_{fut} may happen only if A_{intrm} holds in some *intermediate* state.

Our assertions A , also allow for the use of capability operators, such as 1) having *access* to an object ($\langle \circ \text{ access } \circ' \rangle$) which means that \circ has a reference to \circ' , or 2) calling a method with on receiver with certain arguments, ($\langle \circ \text{ calls } \circ' .m(\text{args}) \rangle$), or 3) an object being *external*, where $\langle \circ \text{ external} \rangle$ means that \circ belongs to a class that is not defined in the current module, and thus its behaviour is unrestricted. These are the capability operators that we have adopted from Chainmail.

2.3 Bank Account – the right specification

We now return to our quest for a specification that rules out Mod_{bad} while permitting Mod_{good} and $\text{Mod}_{\text{better}}$. The catch is that the vulnerability present in Mod_{bad} is the result of *emergent* behaviour from the interactions of the *set* and *transfer* methods – even though $\text{Mod}_{\text{better}}$ also has a *set* method, it does not exhibit the unwanted interaction. This is exactly where a necessary condition can help: we want to avoid transferring money (or more generally, reducing an account's

We can capture the intended semantics of *transfer* through a functional specification with pre- and post- conditions and MODIFIES clauses as e.g., in [Leavens et al.](#); [Leino](#). The implementation of *transfer* in module Mod_{good} meets this specification.

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balance) *without* the existing account password. Phrasing the same condition the other way around rules out the theft: that money *can only* be transferred when the account's password is known.

In *Necessity* syntax, and recalling §1.1, and 2.2,

```

1  Srobust_1  $\triangleq$  from a:Account  $\wedge$  a.balance==bal next a.balance < bal
2                    onlyIf  $\exists o, a'. [(o \text{ external}) \wedge (o \text{ calls } a.\text{transfer}(a', a.\text{pwd}))]$ 
3
4  Srobust_2  $\triangleq$  from a:Account  $\wedge$  a.balance==bal to a.balance < bal
5                    onlyIf  $\exists o. [(o \text{ external}) \wedge (o \text{ access } a.\text{pwd})]$ 

```

S_{robust_1} does not fit the bill: all three modules satisfy it. But S_{robust_2} does fit the bill: Mod_{good} and $\text{Mod}_{\text{better}}$ satisfy S_{robust_2} , while Mod_{bad} does not.

A critical point of S_{robust_2} is that it is expressed in terms of observable effects (the account's balance is reduced: $a.\text{balance} < \text{bal}$) and the shape of the heap (external access to the password: $(o \text{ external}) \wedge (o \text{ access } a.\text{pwd})$) rather than in terms of individual methods such as `set` and `transfer`. This gives our specifications the vital advantage that they can be used to constrain *implementations* of a bank account with a balance and a password, irrespective of the API it offers, the services it exports, or the dependencies on other parts of the system.

This example also demonstrates that adherence to *Necessity* specifications is not monotonic: adding a method to a module does not necessarily preserve adherence to a specification, and while separate methods may adhere to a specification, their combination does not necessarily do so. For example, Mod_{good} satisfies S_{robust_2} , while Mod_{bad} does not. This is why we say that *Necessity* specifications capture a module's *emergent behaviour*.

2.3.1 How useful is S_{robust_2} ? One might think that S_{robust_2} was not useful: normally, there will exist somewhere in the heap at least one external object with access to the password – if no such object existed, then the money in the account *would be un-usable*. And if such an object did exist, then the guarantee given by S_{robust_2} might seem vacuous.

This is *not* so: there may exist scopes of execution from which such external objects with access to the password are not (transitively) reachable. In such scopes, S_{robust_2} guarantees that the balance of the account will not decrease. We illustrate this through the following code snippet:

```

1 module Mod1
2   ...
3   method cautious(untrusted:Object)
4     a = new Account
5     p = new Password
6     a.set(null, p)
7     ...
8     untrusted.make_payment(a)
9     ...

```

The method `cautious` has as argument an external object `untrusted`, of unknown provenance. It creates a new `Account` and initializes its password. In the scope of this method, external objects with access to the password are reachable: thus, during execution of line 7, or line 9 the balance may decrease.

Assume that class `Account` is from a module which satisfies S_{robust_2} . Assume also that the code in line 7 does not leak the password to `untrusted`. Then no external object reachable from the scope of execution of `make_payment` at line 8 has access to the password. Therefore, even though we are calling an untrusted object, S_{robust_2} guarantees that `untrusted` will not be able to take any money out of `a`.

and thus its behaviour is unrestricted. These are the capability operators that we have adopted from Chainmail.

2.3 Bank Account – the right specification

We now return to our quest for a specification that rules out Mod_{bad} while permitting Mod_{good} and $\text{Mod}_{\text{better}}$. The catch is that the vulnerability present in Mod_{bad} is the result of *emergent* behaviour from the interactions of the `set` and `transfer` methods – even though $\text{Mod}_{\text{better}}$ also has a `set` method, it does not exhibit the unwanted interaction. This is exactly where a necessary condition can help: we want to avoid transferring money (or more generally, reducing an account's balance) *without* the existing account password. Phrasing the same condition the other way around rules out the theft: that money *can only* be transferred when the account's password is known.

In *Necessity* syntax, and recalling §1.1, and 2.2,

```

1  Srobust_1  $\triangleq$  from a:Account  $\wedge$  a.balance==bal next a.balance < bal
2                    onlyIf  $\exists o, a'. [(o \text{ external}) \wedge (o \text{ calls } a.\text{transfer}(a', a.\text{pwd}))]$ 
3
4  Srobust_2  $\triangleq$  from a:Account  $\wedge$  a.balance==bal to a.balance < bal
5                    onlyIf  $\exists o. [(o \text{ external}) \wedge (o \text{ access } a.\text{pwd})]$ 

```

S_{robust_1} does not fit the bill: all three modules satisfy it. But S_{robust_2} does fit the bill: Mod_{good} and $\text{Mod}_{\text{better}}$ satisfy S_{robust_2} , while Mod_{bad} does not.

A critical point of S_{robust_2} is that it is expressed in terms of observable effects (the account's balance is reduced: $a.\text{balance} < \text{bal}$) and the shape of the heap (external access to the password: $(o \text{ external}) \wedge (o \text{ access } a.\text{pwd})$) rather than in terms of individual methods such as `set` and `transfer`. This gives our specifications the vital advantage that they can be used to constrain *implementations* of a bank account with a balance and a password, irrespective of the API it offers, the services it exports, or the dependencies on other parts of the system.

This example also demonstrates that adherence to *Necessity* specifications is not monotonic: adding a method to a module does not necessarily preserve adherence to a specification, and while separate methods may adhere to a specification, their combination does not necessarily do so. For example, Mod_{good} satisfies S_{robust_2} , while Mod_{bad} does not. This is why we say that *Necessity* specifications capture a module's *emergent behaviour*.

2.3.1 How useful is S_{robust_2} ? One might think that S_{robust_2} was not useful: normally, there will exist somewhere in the heap at least one external object with access to the password – if no such object existed, then nobody would be able to use the money of the account. And if such an object did exist, then the premise of S_{robust_2} would not hold, and thus the guarantee given by S_{robust_2} might seem vacuous.

This is *not* so: in scopes from which such external objects with access to the password are not (transitively) reachable, S_{robust_2} guarantees that the balance of the account will not decrease. We illustrate this through the following code snippet:

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9     ...

```

2.4 Internal and external modules, objects, and calls

Our work concentrates on guarantees made in an *open* setting; that is, a given module M must be programmed so that execution of M together with *any* external module M' will uphold these guarantees. In the tradition of visible states semantics, we are only interested in upholding the guarantees while M' , the *external* module, is executing. A module can temporarily break its own invariants, so long as the broken invariants are never visible externally.

We therefore distinguish between *internal* objects — instances of classes defined in M — and *external* objects defined in any other module. We also distinguish between *internal* calls (from either an internal or an external object) made to internal objects and *external* calls made to external objects. Looking at the code snippet from §2.3.1, the call to `set` on line 6 is an internal call, while the call to `make_payment` is an external call — from the external object `this` to the external object `untrusted`.

Because we only require guarantees while the external module is executing, we develop an *external states* semantics, where any internal calls are executed in one, large, step. With external steps semantics, the executing object (`this`) is always external. In line with other work in the literature [Albert et al. 2020; Grossman et al. 2017; Permenev et al. 2020b], we currently forbid calls from internal to external objects — further details on call-backs in §6.

For the purposes of the current work we are only interested in one internal, and one external module. But the interested reader might ask: what if there is more than one external module? The answer is that from the internal module's viewpoint, all external modules are considered as one; for this we provide a module linking operator with the expected semantics — more details in Def. 3.1 and §A. But from the external module's viewpoint, there may be more than one external module: for example, in §2.3.1, module `Mod1` is external to the module implementing class `Account`, and the module implementing the class of `untrusted` is external to `Mod1`.

2.5 Reasoning about Necessity

We will now outline the key ingredients of our logic with which we prove that modules obey *Necessity* specifications. We will use the auxiliary concept that an assertion A is *encapsulated* by a module M , if A can only be invalidated through a call to a method from M — more in §4.1.

The *Necessity* logic is based on the *crucial* insight that the specification

`from A_1 to A_2 onlyIf A_3`

is logically equivalent to

$\forall \text{stmts}. \{A_1 \wedge \neg A_3\} \text{stmts} \{ \neg A_2 \}$

— that is, with an *infinite* conjunction of Hoare triples. This leaves the challenge that *no* Hoare logics support such infinite conjunctions. Three *breakthroughs* helped us address that challenge:

From Hoare triples to per-call specs The Hoare triple $\{A_1 \wedge \neg A_3\} \times.m(y.s) \{ \neg A_2 \}$ is logically equivalent to the specification `from $(A_1 \wedge \neg \text{calls } \times.m(y.s))$ next A_2 onlyIf A_3` .

From per-call specs to per-step specs If an assertion A_2 is *encapsulated* by a module — and thus the only way from a state that satisfies A_2 to a state that does not, is through a call to a method in that module — then the *finite conjunction* that all methods of that module `from $(A_1 \wedge A_2 \wedge \neg \text{calls } \times.m(y.s))$ next $\neg A_2$ onlyIf A_3` is logically equivalent to `from $A_1 \wedge A_2$ next $\neg A_2$ onlyIf A_3` .

Proof logic for emergent behaviour combines several specifications to reason about the emergent behaviour, e.g., `from A_1 to A_2 onlyThrough A_3` and `from A_1 to A_3 onlyIf A_4` implies `from A_1 to A_2 onlyIf A_4` .

Thus, our system consists of four parts (five including functional specifications): (**Part 1**) assertion encapsulation, (**Part 2**) per-method specifications, (**Part 3**) per-step specifications, and (**Part 4**)

The method `cautious` has as argument an external object `untrusted`, of unknown provenance. It creates a new `Account` and initializes its password. In the scope of this method, external objects with access to the password are reachable: thus, during execution of line 7, or line 9 the balance may decrease.

Assume that class `Account` is from a module which satisfies $S_{\text{robust_2}}$. Assume also that the code in line 7 does not leak the password to `untrusted`. Then no external object reachable from the scope of execution of `make_payment` at line 8 has access to the password. Therefore, even though we are calling an untrusted object, $S_{\text{robust_2}}$ guarantees that `untrusted` will not be able to take any money out of `a`.

A proof sketch of the safety provided by $S_{\text{robust_2}}$ appears in Appendix H. Note that in this example, we have (at least) three modules: the internal module which defines class `Account` adhering to $S_{\text{robust_2}}$, the external module `Mod1`, and the external module which contains the class definition for `untrusted`. Our methodology allows the external module, `Mod1` to reason about its own code, and thus pass `a` to code from the second external module, without fear of losing money. In further work we want to make such arguments more generally applicable, and extend Hoare logics to encompass such proof steps.

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Our work concentrates on guarantees made in an *open* setting; that is, a given module M must be programmed so that execution of M together with *any* external module M' will uphold these guarantees. In the tradition of visible states semantics, we are only interested in upholding the guarantees while M' , the *external* module, is executing. A module can temporarily break its own invariants, so long as the broken invariants are never visible externally.

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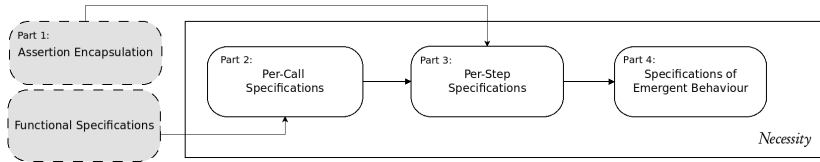


Fig. 1. Parts of *Necessity* Logic and their Dependencies. Note that gray parts with a dashed border indicate parts that are not part of *Necessity*, and on which *Necessity* is parametric.

specifications of emergent behaviour. The structure of the system, and the dependency of each part on preceding parts is given in Fig. 1. Functional specifications are used to prove per-method specifications, which coupled with assertion encapsulation is used to prove per-step specifications, which is used to prove specifications of emergent behaviour.

Our *Necessity* logic is parametric with respect to the way we ascertain whether an assertion is encapsulated and the way we obtain functional specifications. As a result we can leverage results from many different approaches. Further, our proofs of *Necessity* do not inspect method bodies: we rely on simple annotations to infer encapsulation, and on pre and post-conditions to infer per-method conditions.

2.6 Outline of the proof that $\text{Mod}_{\text{better}}$ obeys S_{robust_2}

For illustration, we outline a proof that $\text{Mod}_{\text{better}}$ adheres to S_{robust_2} . note that for illustration purposes, in this paper we show how assertion encapsulation can be proven based on simple annotations inspired by confinement types [Vitek and Bokowski 1999]; we could just as easily rely on other language mechanisms, e.g., ownership types, or even develop custom logics.

Part 1: Assertion Encapsulation.

We begin by proving that $\text{Mod}_{\text{better}}$ encapsulates:

- (A) The balance
- (B) The password
- (C) External accessibility to an account's password – that is, the property that no external object has access to the password may only be invalidated by calls to $\text{Mod}_{\text{better}}$.

Part 2: Per-Method Specifications

We prove that the call of any method from $\text{Mod}_{\text{better}}$ (`set` and `transfer`) satisfies:

- (D) If the balance decreases, then `transfer` was called with the correct password
- (E) If the password changes, then the method called was `set` with the correct password
- (F) It will not provide external accessibility to the password.

Part 3: Per-step Specifications

We then raise our results of Parts 1 and 2 to reason about arbitrary *single-step* executions:

- (F) By (A) and (D) only `transfer` and external access to the password may decrease the balance.
- (G) By (B) and (E) only `set` and external access to the password may change the password.
- (H) By (C) and (F) no step may grant external accessibility to an account's password.

Part 4: Specifications of Emergent Behaviour

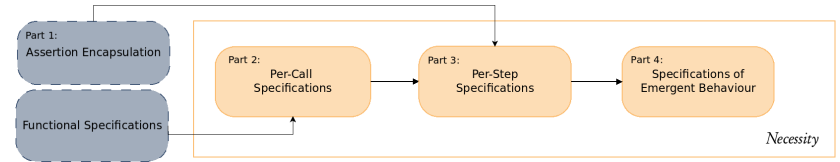


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$$\forall \text{stmts}. \{A_1 \wedge \neg A_3\} \text{stmts} \{ \neg A_2 \}$$

– that is, with an *infinite* conjunction of Hoare triples. This leaves the challenge that *usually*, Hoare logics *do not* support such infinite conjunctions. Three *ideas* helped us address that challenge:

From Hoare triples to per-call specs The Hoare triple $\{A_1 \wedge \neg A_3\} x.m(y.s) \{ \neg A_2 \}$ is logically equivalent to the specification `from (A1 ∧ calls x.m(y.s)) next A2 onlyIf A3`.

From per-call specs to per-step specs If an assertion A_2 is *encapsulated* by a module – and thus the only way from a state that satisfies A_2 to a state that does not, is through a call to a method in that module – then the *finite conjunction* that all methods of that module `from (A1 ∧ A2 ∧ calls x.m(y.s)) next $\neg A_2$ onlyIf A3` is logically equivalent to `from A1 ∧ A2 next $\neg A_2$ onlyIf A3`.

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Thus, our system consists of four parts (five including functional specifications): **(Part 1)** assertion encapsulation, **(Part 2)** per-method specifications, **(Part 3)** per-step specifications, and **(Part 4)** specifications of emergent behaviour. The structure of the system, and the dependency of each part on preceding parts is given in Fig. 1. Functional specifications are used to prove per-method specifications, which coupled with assertion encapsulation is used to prove per-step specifications, which is used to prove specifications of emergent behaviour.

Our *Necessity* logic is parametric with respect to the way we ascertain whether an assertion is encapsulated and the way we obtain functional specifications. As a result we can leverage results from many different approaches. Further, our proofs of *Necessity* do not inspect method bodies: we rely on simple annotations to infer encapsulation, and on pre and post-conditions to infer per-method conditions.

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For illustration, we outline a proof that $\text{Mod}_{\text{better}}$ adheres to S_{robust_2} . note that for illustration purposes, in this paper we show how assertion encapsulation can be proven based on simple annotations inspired by confinement types [Vitek and Bokowski 1999]; we could just as easily rely on other language mechanisms, e.g., ownership types, or even develop custom logics.

Part 1: Assertion Encapsulation.

We then raise our necessary conditions of Part 3 to reason about *arbitrary* executions:

- (I) A decrease in balance over any number of steps implies that some single intermediate step reduced the account's balance.
- (J) By (F) we know that step must be a call to `transfer` with the correct password.
- (K) When `transfer` was called, either
 - (K1) The password used was the current password, and thus by (H) we know that the current password must be externally known, satisfying $S_{\text{robust_2}}$, or
 - (K2) The password had been changed, and thus by (G) some intermediate step must have been a call to `set` with the current password. Thus, by (H) we know that the current password must be externally known, satisfying $S_{\text{robust_2}}$.

3 THE MEANING OF NECESSITY

In this section we define the *Necessity* specification language. We first define an underlying programming language, *Tool* (§3.1). We then define an assertion language, *Assert*, which can talk about the contents of the state, as well as about provenance, permission and control (§3.2). Finally, we define the syntax and semantics of our full language for writing *Necessity* specifications (§3.3).

3.1 Tool

Tool is a formal model of a small, imperative, sequential, class based, typed, object-oriented language. *Tool* is straightforward: Appendix A contains the full definitions. *Tool* is based on \mathcal{L}_{∞} [Drossopoulou et al. 2020b], with some small variations, as well as the addition of a simple type system – more in §4.1.3. A *Tool* state σ consists of a heap χ , and a stack ψ which is a sequence of frames. A frame ϕ consists of local variable map, and a continuation, *i.e.* a sequence of statements to be executed. A statement may assign to variables, create new objects and push them to the heap, perform field reads and writes on objects, or call methods on those objects.

Modules are mappings from class names to class definitions. Execution is in the context of a module M and a state σ , defined via unsurprising small-step semantics of the form $M, \sigma \rightsquigarrow \sigma'$. The top frame's continuation contains the statements to be executed next.

As discussed in §2.5, open world specifications need to be able to provide guarantees which hold during execution of an internal, known, trusted module M when linked together with any unknown, untrusted, module M' . These guarantees need only hold when the external module is executing; we are not concerned if they are temporarily broken by the internal module. Therefore, we are only interested in states where the executing object (`this`) is an external object. To express our focus on external states, we define the *external states semantics*, of the form $M'; M, \sigma \rightsquigarrow \sigma'$, where M' is the external module, and M is the internal module, and where we collapse all internal steps into one single step.

Definition 3.1 (External States Semantics). For modules M, M' , and states σ, σ' , we say that $M'; M, \sigma \rightsquigarrow \sigma'$ if and only if there exist $n \in \mathbb{N}$, and states $\sigma_0, \dots, \sigma_n$, such that

- $\sigma = \sigma_1$, and $\sigma' = \sigma_n$,
- $M' \circ M, \sigma_i \rightsquigarrow \sigma_{i+1}$ for all $i \in [0..n)$,
- $\text{classOf}(\sigma, \text{this}), \text{classOf}(\sigma', \text{this}) \in M'$,
- $\text{classOf}(\sigma_i, \text{this}) \in M$ for all $i \in (1..n)$.

The function $\text{classOf}(\sigma, _)$ is overloaded: applied to a variable, $\text{classOf}(\sigma, x)$ looks up the variable x in the top frame of σ , and returns the class of the corresponding object in the heap of σ ; applied to an address, $\text{classOf}(\sigma, \alpha)$ returns the class of the object referred by address α in the heap of σ .

We begin by proving that $\text{Mod}_{\text{better}}$ encapsulates:

- (A) The balance
- (B) The password
- (C) External accessibility to an account's password – that is, the property that no external object has access to the password may only be invalidated by calls to $\text{Mod}_{\text{better}}$.

Part 2: Per-Method Specifications

We prove that the call of any method from $\text{Mod}_{\text{better}}$ (`set` and `transfer`) satisfies:

- (D) If the balance decreases, then `transfer` was called with the correct password
- (E) If the password changes, then the method called was `set` with the correct password
- (F) It will not provide external accessibility to the password.

Part 3: Per-step Specifications

We then raise our results of Parts 1 and 2 to reason about arbitrary *single-step* executions:

- (F) By (A) and (D) only `transfer` and external access to the password may decrease the balance.
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- (H) By (C) and (F) no step may grant external accessibility to an account's password.

Part 4: Specifications of Emergent Behaviour

We then raise our necessary conditions of Part 3 to reason about *arbitrary* executions:

- (I) A decrease in balance over any number of steps implies that some single intermediate step reduced the account's balance.
- (J) By (F) we know that step must be a call to `transfer` with the correct password.
- (K) When `transfer` was called, either
 - (K1) The password used was the current password, and thus by (H) we know that the current password must be externally known, satisfying $S_{\text{robust_2}}$, or
 - (K2) The password had been changed, and thus by (G) some intermediate step must have been a call to `set` with the current password. Thus, by (H) we know that the current password must be externally known, satisfying $S_{\text{robust_2}}$.

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The module linking operator \circ , applied to two modules, $M' \circ M$, combines the two modules into one module in the obvious way, provided their domains are disjoint. Full details in Appendix A.

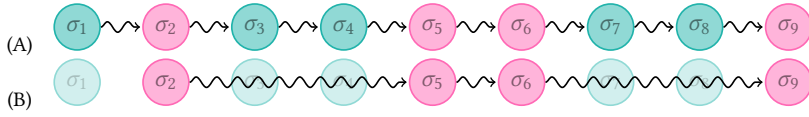


Fig. 2. External States Semantics (Def. 3.1), (A) $M' \circ M, \sigma_1 \rightsquigarrow \dots \rightsquigarrow \sigma_9$ (B) $M'; M, \sigma_2 \rightsquigarrow \dots \rightsquigarrow \sigma_9$, where $\text{classOf}(\sigma_1, \text{this}), \text{classOf}(\sigma_3, \text{this}), \text{classOf}(\sigma_5, \text{this}), \text{classOf}(\sigma_7, \text{this}), \text{classOf}(\sigma_9, \text{this}) \in M$, and where $\text{classOf}(\sigma_2, \text{this}), \text{classOf}(\sigma_4, \text{this}), \text{classOf}(\sigma_6, \text{this}), \text{classOf}(\sigma_8, \text{this}) \in M'$.

Fig. 2 inspired by Drossopoulou et al. [2020b] provides a simple graphical description of our external states semantics: (A) is the “normal” execution after linking two modules into one: $M' \circ M, \dots \rightsquigarrow \dots$ whereas (B) is the external states execution when M' is external, $M'; M, \dots \rightsquigarrow \dots$. Note that whether a module is external or internal depends on perspective – nothing in a module itself renders it internal or external. For example, in $M_1; M_2, \dots \rightsquigarrow \dots$ the external module is M_1 , while in $M_2; M_1, \dots \rightsquigarrow \dots$ the external module is M_2 .

We use the notation $M'; M, \sigma \rightsquigarrow^* \sigma'$ to denote zero or more steps starting at state σ and ending at state σ' , in the context of internal module M and external module M' . We are concerned **neither** with internal states **nor** states that can never arise. A state σ is *arising*, written $\text{Arising}(M', M, \sigma)$, if it may arise by external states execution starting at some initial configuration:

Definition 3.2 (Arising States). For modules M and M' , a state σ is called an *arising* state, formally $\text{Arising}(M', M, \sigma)$, if and only if there exists some σ_0 such that $\text{Initial}(\sigma_0)$ and $M'; M, \sigma_0 \rightsquigarrow^* \sigma$.

An *Initial* state's heap contains a single object of class `Object`, and its stack consists of a single frame, whose local variable map is a mapping from `this` to the single object, and whose continuation is any statement. (See Definitions A.5 and 3.2).

Applicability. While our work is based on a simple, imperative, typed, object oriented language with unforgeable addresses and private fields, we believe that it is applicable to several programming paradigms, and that unforgeability and privacy can be replaced by lower level mechanisms such as capability machines [Davis et al. 2019; Van Strydonck et al. 2022].

3.2 Assert

Assert is a subset of the *Chainmail* assertions language, i.e. a basic assertion language extended with object-capability assertions.

3.2.1 Syntax of Assert. The syntax of *Assert* is given in Definition 3.3. An assertion may be an expression, a query of the defining class of an object, the usual connectives and quantifiers, along with three non-standard assertion forms: (1) *Permission* and (2) *Provenance*, inspired by the capabilities literature, and (3) *Control* which allows tighter characterisation of the cause of effects – useful for the specification of large APIs.

- *Permission* ($\langle x \text{ access } y \rangle$): x has access to y .
- *Provenance* ($\langle x \text{ internal} \rangle$ and $\langle y \text{ external} \rangle$): x is internal, and y is external.
- *Control* ($\langle x \text{ calls } y.m(\bar{z}) \rangle$): x calls method m on object y with arguments \bar{z} .

Definition 3.3. Assertions (A) in *Assert* are defined as follows:

Modules are mappings from class names to class definitions. Execution is in the context of a module M and a state σ , defined via unsurprising small-step semantics of the form $M, \sigma \rightsquigarrow \sigma'$. The top frame's continuation contains the statement to be executed next.

As discussed in §2.5, open world specifications need to be able to provide guarantees which hold during execution of an internal, known, trusted module M when linked together with any unknown, untrusted, module M' . These guarantees need only hold when the external module is executing; we are not concerned if they are temporarily broken by the internal module. Therefore, we are only interested in states where the executing object (`this`) is an external object. To express our focus on external states, we define the *external states semantics*, of the form $M'; M, \sigma \rightsquigarrow \sigma'$, where M' is the external module, and M is the internal module, and where we collapse all internal steps into one single step.

Definition 3.1 (External States Semantics). For modules M, M' , and states σ, σ' , we say that $M'; M, \sigma \rightsquigarrow \sigma'$ if and only if there exist $n \in \mathbb{N}$, and states $\sigma_0, \dots, \sigma_n$, such that

- $\sigma = \sigma_0$, and $\sigma' = \sigma_n$,
- $M' \circ M, \sigma_i \rightsquigarrow \sigma_{i+1}$ for all $i \in [0..n)$,
- $\text{classOf}(\sigma, \text{this}), \text{classOf}(\sigma', \text{this}) \in M'$,
- $\text{classOf}(\sigma_i, \text{this}) \in M$ for all $i \in (1..n)$.

The function $\text{classOf}(\sigma, _)$ is overloaded: applied to a variable, $\text{classOf}(\sigma, x)$ looks up the variable x in the top frame of σ , and returns the class of the corresponding object in the heap of σ ; applied to an address, $\text{classOf}(\sigma, \alpha)$ returns the class of the object referred by address α in the heap of σ . The module linking operator \circ , applied to two modules, $M' \circ M$, combines the two modules into one module in the obvious way, provided their domains are disjoint. Full details in Appendix A.

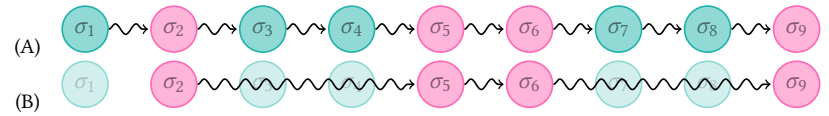


Fig. 2. External States Semantics (Def. 3.1), (A) $M' \circ M, \sigma_1 \rightsquigarrow \dots \rightsquigarrow \sigma_9$ and (B) $M'; M, \sigma_2 \rightsquigarrow \dots \rightsquigarrow \sigma_9$, where $\text{classOf}(\sigma_1, \text{this}), \text{classOf}(\sigma_3, \text{this}), \text{classOf}(\sigma_5, \text{this}), \text{classOf}(\sigma_7, \text{this}), \text{classOf}(\sigma_9, \text{this}) \in M$, and where $\text{classOf}(\sigma_2, \text{this}), \text{classOf}(\sigma_4, \text{this}), \text{classOf}(\sigma_6, \text{this}), \text{classOf}(\sigma_8, \text{this}) \in M'$.

Fig. 2 inspired by Drossopoulou et al. [2020b] provides a simple graphical description of our external states semantics: (A) is the “normal” execution after linking two modules into one: $M' \circ M, \dots \rightsquigarrow \dots$ whereas (B) is the external states execution when M' is external, $M'; M, \dots \rightsquigarrow \dots$. Note that whether a module is external or internal depends on perspective – nothing in a module itself renders it internal or external. For example, in $M_1; M_2, \dots \rightsquigarrow \dots$ the external module is M_1 , while in $M_2; M_1, \dots \rightsquigarrow \dots$ the external module is M_2 .

We use the notation $M'; M, \sigma \rightsquigarrow^* \sigma'$ to denote zero or more steps starting at state σ and ending at state σ' , in the context of internal module M and external module M' . We are **not** concerned with internal states or states that can never arise. A state σ is *arising*, written $\text{Arising}(M', M, \sigma)$, if it may arise by external states execution starting at some initial configuration:

Definition 3.2 (Arising States). For modules M and M' , a state σ is called an *arising* state, formally $\text{Arising}(M', M, \sigma)$, if and only if there exists some σ_0 such that $\text{Initial}(\sigma_0)$ and $M'; M, \sigma_0 \rightsquigarrow^* \sigma$.

An *Initial* state's heap contains a single object of class `Object`, and its stack consists of a single frame, whose local variable map is a mapping from `this` to the single object, and whose continuation is any statement. (See Definitions A.5 and 3.2).

$$A ::= e \mid e : C \mid \neg A \mid A \wedge A \mid A \vee A \mid \forall x.[A] \mid \exists x.[A] \mid \langle x \text{ access } y \rangle \mid \langle x \text{ internal} \rangle \mid \langle x \text{ external} \rangle \mid \langle x \text{ calls } y.m(\bar{z}) \rangle$$

3.2.2 Semantics of Assert. The semantics of *Assert* is given in Definition 3.4. We use the evaluation relation, $M, \sigma, e \hookrightarrow v$, which says that the expression e evaluates to value v in the context of state σ and module M . Note that expressions in *Tool* may be recursively defined, and thus evaluation need not **always** terminate. Nevertheless, the logic of A remains classical because recursion is restricted to expressions, and not generally to assertions. We have taken this approach from Drossopoulou et al. [2020b], which also contains a mechanized Coq proof that assertions are classical [Drossopoulou et al. 2020a]. The semantics of \hookrightarrow are unsurprising (see Fig. 10).

Shorthands: $[x]_\phi = v$ means that x maps to value v in the local variable map of frame ϕ , $[x]_\sigma = v$ means that x maps to v in the top most frame of σ 's stack, and $[x.f]_\sigma = v$ has the obvious meaning. The terms $\sigma.\text{stack}$, $\sigma.\text{contn}$, $\sigma.\text{heap}$ mean the stack, the continuation at the top frame of σ , and the heap of σ . The term $\alpha \in \sigma.\text{heap}$ means that α is in the domain of the heap of σ , and x *fresh* in σ means that x isn't in the variable map of the top frame of σ , while the substitution $\sigma[x \mapsto \alpha]$ is applied to the top frame of σ . $C \in M$ means that class C is in the domain of module M .

Definition 3.4 (Satisfaction of Assertions by a module and a state). We define satisfaction of an assertion A by a state σ with module M as:

- (1) $M, \sigma \models e$ iff $M, \sigma, e \hookrightarrow \text{true}$
- (2) $M, \sigma \models e : C$ iff $M, \sigma, e \hookrightarrow \alpha$ and $\text{classOf}(\sigma, \alpha) = C$
- (3) $M, \sigma \models \neg A$ iff $M, \sigma \not\models A$
- (4) $M, \sigma \models A_1 \wedge A_2$ iff $M, \sigma \models A_1$ and $M, \sigma \models A_2$
- (5) $M, \sigma \models A_1 \vee A_2$ iff $M, \sigma \models A_1$ or $M, \sigma \models A_2$
- (6) $M, \sigma \models \forall x.[A]$ iff $M, \sigma[x \mapsto \alpha] \models A$, for some x fresh in σ , and for all $\alpha \in \sigma.\text{heap}$.
- (7) $M, \sigma \models \exists x.[A]$ iff $M, \sigma[x \mapsto \alpha] \models A$, for some x fresh in σ , and for some $\alpha \in \sigma.\text{heap}$.
- (8) $M, \sigma \models \langle x \text{ access } y \rangle$ iff
 - (a) $[x.f]_\sigma = [y]_\sigma$ for some f ,
or
 - (b) $[x]_\sigma = [\text{this}]_\phi$, $[y]_\sigma = [z]_\phi$, and $z \in \phi.\text{contn}$ for some variable z , and some frame ϕ in $\sigma.\text{stack}$.
- (9) $M, \sigma \models \langle x \text{ internal} \rangle$ iff $\text{classOf}(\sigma, x) \in M$
- (10) $M, \sigma \models \langle x \text{ external} \rangle$ iff $\text{classOf}(\sigma, x) \notin M$
- (11) $M, \sigma \models \langle x \text{ calls } y.m(z_1, \dots, z_n) \rangle$ iff
 - (a) $\sigma.\text{contn} = (w := y'.m(z'_1, \dots, z'_n); s)$, for some variable w , and some statement s ,
 - (b) $M, \sigma \models x = \text{this}$ and $M, \sigma \models y = y'$,
 - (c) $M, \sigma \models z_i = z'_i$ for all $1 \leq i \leq n$

The assertion $\langle x \text{ access } y \rangle$ (defined in 8) requires that x has access to y either through a field of x (case 8a), or through some call in the stack, where x is the receiver and y is one of the arguments (case 8b). Note that access is not deep, and only refers to objects that an object has direct access to via a field or within the context of a current scope. The restricted form of access used in *Necessity* specifically captures a crucial property of robust programs in the open world: access to an object does not imply access to that object's internal data. For example, an object may have access to an account a , but a safe implementation of the account would never allow that object to leverage that access to gain direct access to $a.\text{pwd}$.

The assertion $\langle x \text{ calls } y.m(z_1, \dots, z_n) \rangle$ (defined in 11) requires that the current receiver (this) is x , and that it calls the method m on y with arguments z_1, \dots, z_n – It does *not* mean that somewhere

Applicability. While our work is based on a simple, imperative, typed, object oriented language with unforgeable addresses and private fields, we believe that it is applicable to several programming paradigms, and that unforgeability and privacy can be replaced by lower level mechanisms such as capability machines [Davis et al. 2019; Van Strydonck et al. 2022].

3.2 Assert

Assert is a basic assertion language extended with object-capability assertions.

3.2.1 Syntax of Assert. The syntax of *Assert* is given in Definition 3.3. An assertion may be an expression, a query of the defining class of an object, the usual connectives and quantifiers, along with three non-standard assertion forms: (1) *Permission* and (2) *Provenance*, inspired by the capabilities literature, and (3) *Control* which allows tighter characterisation of the cause of effects – useful for the specification of large APIs.

- *Permission* ($\langle x \text{ access } y \rangle$): x has access to y .
- *Provenance* ($\langle x \text{ internal} \rangle$ and $\langle y \text{ external} \rangle$): x is an internal (i.e. trusted) object, and y is an external (i.e. untrusted) object.
- *Control* ($\langle x \text{ calls } y.m(\bar{z}) \rangle$): x calls method m on object y with arguments \bar{z} .

Definition 3.3. Assertions (A) in *Assert* are defined as follows:

$$A ::= e \mid e : C \mid \neg A \mid A \wedge A \mid A \vee A \mid \forall x.[A] \mid \exists x.[A] \mid \langle x \text{ access } y \rangle \mid \langle x \text{ internal} \rangle \mid \langle x \text{ external} \rangle \mid \langle x \text{ calls } y.m(\bar{z}) \rangle$$

3.2.2 Semantics of Assert. The semantics of *Assert* is given in Definition 3.4. We use the evaluation relation, $M, \sigma, e \hookrightarrow v$, which says that the expression e evaluates to value v in the context of state σ and module M . Note that expressions in *Tool* may be recursively defined, and thus evaluation need not terminate. Nevertheless, the logic of A remains classical because recursion is restricted to expressions, and not generally to assertions. We have taken this approach from Drossopoulou et al. [2020b], which also contains a mechanized Coq proof that assertions are classical [Drossopoulou et al. 2020a]. The semantics of \hookrightarrow is unsurprising (see Fig. 11).

Shorthands: $[x]_\phi = v$ means that x maps to value v in the local variable map of frame ϕ , $[x]_\sigma = v$ means that x maps to v in the top most frame of σ 's stack, and $[x.f]_\sigma = v$ has the obvious meaning. The terms $\sigma.\text{stack}$, $\sigma.\text{contn}$, $\sigma.\text{heap}$ mean the stack, the continuation at the top frame of σ , and the heap of σ . The term $\alpha \in \sigma.\text{heap}$ means that α is in the domain of the heap of σ , and x *fresh* in σ means that x isn't in the variable map of the top frame of σ , while the substitution $\sigma[x \mapsto \alpha]$ is applied to the top frame of σ . $C \in M$ means that class C is in the domain of module M .

Definition 3.4 (Satisfaction of Assertions by a module and a state). We define satisfaction of an assertion A by a state σ with module M as:

- (1) $M, \sigma \models e$ iff $M, \sigma, e \hookrightarrow \text{true}$
- (2) $M, \sigma \models e : C$ iff $M, \sigma, e \hookrightarrow \alpha$ and $\text{classOf}(\sigma, \alpha) = C$
- (3) $M, \sigma \models \neg A$ iff $M, \sigma \not\models A$
- (4) $M, \sigma \models A_1 \wedge A_2$ iff $M, \sigma \models A_1$ and $M, \sigma \models A_2$
- (5) $M, \sigma \models A_1 \vee A_2$ iff $M, \sigma \models A_1$ or $M, \sigma \models A_2$
- (6) $M, \sigma \models \forall x.[A]$ iff $M, \sigma[x \mapsto \alpha] \models A$, for some x fresh in σ , and for all $\alpha \in \sigma.\text{heap}$.
- (7) $M, \sigma \models \exists x.[A]$ iff $M, \sigma[x \mapsto \alpha] \models A$, for some x fresh in σ , and for some $\alpha \in \sigma.\text{heap}$.
- (8) $M, \sigma \models \langle x \text{ access } y \rangle$ iff
 - (a) $[x.f]_\sigma = [y]_\sigma$ for some f ,
or

in the call stack there exists a call from x to $y.m(\dots)$. Note that in most cases, satisfaction of an assertion not only depends on the state σ , but also depends on the module in the case of expressions (1), class membership (2), and internal or external provenance (9 and 10).

We now define what it means for a module to satisfy an assertion: M satisfies A if any state arising from external steps execution of that module with any other external module satisfies A .

Definition 3.5 (Satisfaction of Assertions by a module). For a module M and assertion A , we say that $M \models A$ if and only if for all modules M' , and all σ , if $\text{Arising}(M', M, \sigma)$, then $M, \sigma \models A$.

In the current work we assume the existence of a proof system that judges $M \vdash A$, to prove satisfaction of assertions. We will not define such a judgement, but will rely on its existence later on for Theorem 4.4. We define soundness of such a judgement in the usual way:

Definition 3.6 (Soundness of Assert Provability). A judgement of the form $M \vdash A$ is *sound*, if for all modules M and assertions A , if $M \vdash A$ then $M \models A$.

3.2.3 Inside. We define a final shorthand predicate $\text{inside}(o)$ which states that only internal objects have access to o . The object o may be either internal or external.

Definition 3.7 (Inside). $\text{inside}(o) \triangleq \forall x. [\langle x \text{ access } o \rangle \Rightarrow \langle x \text{ internal} \rangle]$

inside is a very useful concept. For example, the balance of an account whose password is inside will not decrease in the next step. Often, API implementations contain objects whose capabilities, while crucial for the implementation, if exposed, would break the intended guarantees of the API. Such objects need to remain inside - see such an example in Section 5.

3.3 Necessity operators

The Necessity specification language extends *Assert* with four three novel necessity operators:

Only If $[\text{from } A_1 \text{ to } A_2 \text{ onlyIf } A]$: If an arising state satisfies A_1 , and after some execution, a state satisfying A_2 is reached, then the original state must have also satisfied A .

Single-Step Only If $[\text{from } A_1 \text{ next } A_2 \text{ onlyIf } A]$: If an arising state satisfies A_1 , and after a single step of execution, a state satisfying A_2 is reached, then the original state must have also satisfied A .

Only Through $[\text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A]$: If an arising state satisfies A_1 , and after some execution, a state satisfying A_2 is reached, then execution must have passed through some intermediate state satisfying A - the intermediate state satisfying A might be the starting state, the final state, or any state in between.

Necessity operators can explicitly constrain two or even three states, and implicitly constrain many states in between. The following specification $S_{\text{to_dcr_thr_call}}$ says that for an account's balance to go from 350 in one state down to 250 some subsequent state, transfer must have been called on that account in between:

```
1  $S_{\text{to\_dcr\_thr\_call}} \triangleq \text{from } a:\text{Account} \wedge a.\text{balance} == 350 \text{ to } a.\text{balance} == 250$ 
2  $\text{onlyThrough } \exists o, o', o''. (o \text{ calls } a.\text{transfer}(o', o''))$ 
```

$S_{\text{to_dcr_thr_call}}$ refers to two or even three different states: at the start, whenever the balance of a is 350, and after any number of steps whenever the balance of a is 250. The specification requires that such a change can only be caused by a call to transfer on a : that call could be in the current (starting) state, in which case presumably the balance will be 250 in the immediately following state, or within any number of intervening states.

(b) $[x]_\sigma = [\text{this}]_\phi$, $[y]_\sigma = [z]_\phi$, and $z \in \phi.\text{contn}$ for some variable z , and some frame ϕ in $\sigma.\text{stack}$.

(9) $M, \sigma \models \langle x \text{ internal} \rangle$ iff $\text{classOf}(\sigma, x) \in M$

(10) $M, \sigma \models \langle x \text{ external} \rangle$ iff $\text{classOf}(\sigma, x) \notin M$

(11) $M, \sigma \models \langle x \text{ calls } y.m(z_1, \dots, z_n) \rangle$ iff

(a) $\sigma.\text{contn} = (w := y'.m(z'_1, \dots, z'_n); s)$, for some variable w , and some statement s ,

(b) $M, \sigma \models x = \text{this}$ and $M, \sigma \models y = y'$,

(c) $M, \sigma \models z_i = z'_i$ for all $1 \leq i \leq n$

Quantification (defined in 6 and 7) is done over all objects on the heap. We do not include quantification over primitive types such as integers as Tool is too simple. The Coq mechanisation does include primitive types.

The assertion $\langle x \text{ access } y \rangle$ (defined in 8) requires that x has access to y either through a field of x (case 8a), or through some call in the stack, where x is the receiver and y is one of the arguments (case 8b). Note that access is not deep, and only refers to objects that an object has direct access to via a field or within the context of a current scope. The restricted form of access used in Necessity specifically captures a crucial property of robust programs in the open world: access to an object does not imply access to that object's internal data. For example, an object may have access to an account a , but a safe implementation of the account would never allow that object to leverage that access to gain direct access to $a.\text{pwd}$.

The assertion $\langle x \text{ calls } y.m(z_1, \dots, z_n) \rangle$ (defined in 11) describes the current innermost active call. It requires that the current receiver (this) is x , and that it calls the method m on y with arguments z_1, \dots, z_n - It does not mean that somewhere in the call stack there exists a call from x to $y.m(\dots)$. Note that in most cases, satisfaction of an assertion not only depends on the state σ , but also depends on the module in the case of expressions (1), class membership (2), and internal or external provenance (9 and 10).

We now define what it means for a module to satisfy an assertion: M satisfies A if any state arising from external steps execution of that module with any other external module satisfies A .

Definition 3.5 (Satisfaction of Assertions by a module). For a module M and assertion A , we say that $M \models A$ if and only if for all modules M' , and all σ , if $\text{Arising}(M', M, \sigma)$, then $M, \sigma \models A$.

In the current work we assume the existence of a proof system that judges $M \vdash A$, to prove satisfaction of assertions. We will not define such a judgement, but will rely on its existence later on for Theorem 4.4. We define soundness of such a judgement in the usual way:

Definition 3.6 (Soundness of Assert Provability). A judgement of the form $M \vdash A$ is *sound*, if for all modules M and assertions A , if $M \vdash A$ then $M \models A$.

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Definition 3.7 (Inside). $\text{inside}(o) \triangleq \forall x. [\langle x \text{ access } o \rangle \Rightarrow \langle x \text{ internal} \rangle]$

inside is a very useful concept. For example, the balance of an account whose password is inside will not decrease in the next step. Often, API implementations contain objects whose capabilities, while crucial for the implementation, if exposed, would break the intended guarantees of the API. Such objects need to remain inside - see such an example in Section 5.

3.3 Necessity operators

3.3.1 Syntax of Necessity Specifications. The Necessity specification language extends *Assert* with our three novel Necessity operators:

Relationship between Necessity Operators. The three *Necessity* operators are related by generality. *Only If* (*from* A_1 *to* A_2 *onlyIf* A) implies *Single-Step Only If* (*from* A_1 *next* A_2 *onlyIf* A), since if A is a necessary precondition for multiple steps, then it must be a necessary precondition for a single step. *Only If* also implies an *Only Through*, where the intermediate state is the starting state of the execution. There is no further relationship between *Single-Step Only If* and *Only Through*.

Relationship with Temporal Logic. Two of the three *Necessity* operators can be expressed in traditional temporal logic: *from* A_1 *to* A_2 *onlyIf* A can be expressed as $A_1 \wedge \Diamond A_2 \rightarrow A$, and *from* A_1 *next* A_2 *onlyIf* A can be expressed as $A_1 \wedge \bigcirc A_2 \rightarrow A$ (where \Diamond denotes any future state, and \bigcirc denotes the next state). Critically, *from* A_1 *to* A_2 *onlyThrough* A cannot be encoded in standard logics without “nominals” (explicit state references), because the state where A holds must be between the state where A_1 holds, and the state where A_2 holds; and this must be so on every execution path from A_1 to A_2 [Bräuner 2022; Brotherston et al. 2020]. TLA+, for example, cannot describe “only through” conditions [Lamport 2002], but we have found “only through” conditions critical to our proofs.

3.3.1 Semantics of Necessity Specifications. We define when a module M satisfies specifications S , written as $M \models S$, by cases over the four possible syntactic forms:

Definition 3.8 (Necessity Syntax).

$$S ::= A \mid \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_3 \mid \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \mid \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3$$

Definition 3.9 (Necessity Semantics). For any assertions A_1, A_2 , and A , we define

- $M \models A$ iff for all M', σ , if *Arising*(M', M, σ), then $M, \sigma \models A$. (see Def. 3.5)
- $M \models \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A$ iff for all M', σ, σ' , such that *Arising*(M', M, σ):

$$\left. \begin{array}{l} - M, \sigma \models A_1 \\ - M, \sigma' \triangleleft \sigma \models A_2 \\ - M'; M, \sigma \rightsquigarrow^* \sigma' \end{array} \right\} \Rightarrow M, \sigma \models A$$
- $M \models \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A$ iff for all M', σ, σ' , such that *Arising*(M', M, σ):

$$\left. \begin{array}{l} - M, \sigma \models A_1 \\ - M, \sigma' \triangleleft \sigma \models A_2 \\ - M'; M, \sigma \rightsquigarrow \sigma' \end{array} \right\} \Rightarrow M, \sigma \models A$$
- $M \models \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A$ iff for all M', σ_1, σ_n , such that *Arising*(M', M, σ_1):

$$\left. \begin{array}{l} - M, \sigma_1 \models A_1 \\ - M, \sigma_n \triangleleft \sigma_1 \models A_2 \\ - M'; M, \sigma_1 \rightsquigarrow^* \sigma_n \end{array} \right\} \Rightarrow \left(\forall \sigma_2, \dots, \sigma_{n-1}. \right. \\ \left. (\forall i \in [1..n). M'; M, \sigma_i \rightsquigarrow \sigma_{i+1} \Rightarrow \exists i \in [1..n]. M, \sigma_i \triangleleft \sigma_1 \models A) \right)$$

3.3.2 Adaptation. The definition of the necessity operators (Definition 3.9) is straightforward, apart from one quirk: we write $\sigma' \triangleleft \sigma$ (best read as “ σ' seen from σ ”, although one recalcitrant author prefers “ σ' adapted to σ ”) to deal with the fact that necessity operators can involve several states. To see the problem, consider a naive approach to giving semantics to $S_{\text{to_dcr_thr_call}}$: if $\sigma, \sigma \models a.\text{balance} == 350$, and $\sigma \rightsquigarrow^* \sigma'$ and $\sigma' \models a.\text{balance} == 250$, then $S_{\text{to_dcr_thr_call}}$ mandates that between σ and σ' there was a call to $a.\text{transfer}$. But if σ happened to have another

from A_1 *next* A_2 *onlyIf* A : If an arising state satisfies A_1 , and a single execution step reaches a state satisfying A_2 , then the original state must have also satisfied A .
from A_1 *to* A_2 *onlyIf* A : If an arising state satisfies A_1 and a number of execution steps reach a state satisfying A_2 , then the original state must have also satisfied A .
from A_1 *to* A_2 *onlyThrough* A : If an arising state satisfies A_1 , and a number of execution steps reach a state satisfying A_2 , then execution must have passed through some intermediate state satisfying A .

The syntax of *Necessity* specifications is given below

Definition 3.8. Syntax of Necessity Specifications

$$S ::= A \mid \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_3 \mid \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \mid \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3$$

As an example, we consider the following three specifications:

```

1 Snext_dcr_if_acc ≜ from a:Account ∧ a.balance==bal next a.balance < bal
2               onlyIf ∃ o. [(o external) ∧ (o access a.pwd)]
3 Sto_dcr_if_acc  ≜ from a:Account ∧ a.balance==bal to a.balance < bal
4               onlyIf ∃ o. [(o external) ∧ (o access a.pwd)]
5 Sto_dcr_thr_acc ≜ from a:Account ∧ a.balance==bal next a.balance < bal
6               onlyIf ∃ o. [(o external) ∧ (o access a.pwd)]

```

$S_{\text{next_dcr_if_acc}}$ requires that an account’s balance may decrease in *one step* (go from a state where the balance is `bal` to a state where it is less than `bal`) only if the password is accessible to an external object (in the original state an external object had access to the password). $S_{\text{to_dcr_if_acc}}$ requires that an account’s balance may decrease in *any number of steps* only if the password is accessible to an external object. $S_{\text{to_dcr_thr_acc}}$ requires that an account’s balance may decrease in *any number of steps* only if in *some intermediate state* the password was accessible to an external object – the *intermediate* state where the password is accessible to the external object might be the *starting* state, the *final* state, or any state in between.

3.3.2 Semantics of Necessity Specifications. We now define what it means for a module M to satisfy specification S , written as $M \models S$. The Definition 3.9 below is straightforward, apart from the use of the $\sigma' \triangleleft \sigma$ (best read as “ σ' seen from σ ”) to deal with the fact that execution might change the bindings in local variables. We explain this in detail in §3.3.3, but for now, the reader may ignore the applications of that operator and read $\sigma' \triangleleft \sigma$ as σ' , and also read $\sigma_k \triangleleft \sigma_1$ as σ_k . We illustrate the meaning of the three operators in Fig. 3.

Definition 3.9 (Semantics of Necessity Specifications). We define $M \models S$ by cases over the four possible syntactic forms. For any assertions A_1, A_2 , and A :

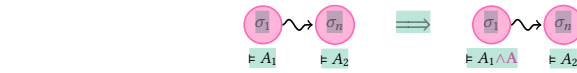
- $M \models A$ iff for all M', σ , if *Arising*(M', M, σ), then $M, \sigma \models A$. (see Def. 3.5)
- $M \models \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A$ iff for all M', σ, σ' , such that *Arising*(M', M, σ):

$$\left. \begin{array}{l} - M, \sigma \models A_1 \\ - M, \sigma' \triangleleft \sigma \models A_2 \\ - M'; M, \sigma \rightsquigarrow^* \sigma' \end{array} \right\} \Rightarrow M, \sigma \models A$$
- $M \models \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A$ iff for all M', σ, σ' , such that *Arising*(M', M, σ):

from A_1 to A_2 onlyIf A :



from A_1 next A_2 onlyIf A :



from A_1 to A_2 onlyThrough A :



Fig. 3. Illustrating the three Necessity operators

$$\left. \begin{array}{l} - M, \sigma \models A_1 \\ - M, \sigma' \triangleleft \sigma \models A_2 \\ - M'; M, \sigma \rightsquigarrow \sigma' \end{array} \right\} \Rightarrow M, \sigma \models A$$

- $M \models \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A$ iff for all $M', \sigma_1, \sigma_2, \dots, \sigma_n$, such that $\text{Arising}(M', M, \sigma_1)$:

$$\left. \begin{array}{l} - M, \sigma_1 \models A_1 \\ - M, \sigma_n \triangleleft \sigma_1 \models A_2 \\ - \forall i \in [1..n). M'; M, \sigma_i \rightsquigarrow \sigma_{i+1} \end{array} \right\} \Rightarrow \exists k. 1 \leq k \leq n \wedge M, \sigma_k \triangleleft \sigma_1 \models A$$

Revisiting the examples from the previous subsection, we obtain that all three modules satisfy $S_{\text{next_dcr_if_acc}}$. But Mod_{bad} does not satisfy $S_{\text{to_dcr_if_acc}}$: as already discussed in §2.1, with a of class `Account` implemented as in Mod_{bad} , starting in a state where no external object has access to a's password, and executing `a.set(42); a.transfer(rogue_account, 42)` leads to a state where the balance has decreased. All three modules satisfy $S_{\text{to_dcr_thr_acc}}$: namely, in all cases, the balance can only decrease if there was a call to `a.transfer(_, p)` where $p = a.\text{pwd}$, and since that call can only be made from an external object, p is externally known at the time of that call.

$\text{Mod}_{\text{good}} \models S_{\text{next_dcr_if_acc}}$	$\text{Mod}_{\text{bad}} \models S_{\text{next_dcr_if_acc}}$	$\text{Mod}_{\text{better}} \models S_{\text{next_dcr_if_acc}}$
$\text{Mod}_{\text{good}} \models S_{\text{to_dcr_if_acc}}$	$\text{Mod}_{\text{bad}} \not\models S_{\text{to_dcr_if_acc}}$	$\text{Mod}_{\text{better}} \models S_{\text{to_dcr_if_acc}}$
$\text{Mod}_{\text{good}} \models S_{\text{to_dcr_thr_acc}}$	$\text{Mod}_{\text{bad}} \models S_{\text{to_dcr_thr_acc}}$	$\text{Mod}_{\text{better}} \models S_{\text{to_dcr_thr_acc}}$

3.3.3 Adaptation. We now discuss the adaptation operator. To see the need, consider specification

```

1  Sto_dcr_thr_call ≜ from a:Account ∧ a.balance==350 next a.balance == 250
2  onlyIf ∃ o. [(o external) ∧ (o calls a.transfer(_, _, _))]

```

Without adaptation, the semantics of $S_{\text{to_dcr_thr_call}}$ would be: If $\dots, \sigma \models a.\text{balance}==350$, and $\dots, \sigma \rightsquigarrow^* \sigma'$ and $\sigma' \models a.\text{balance}==250$, then between σ and σ' there must be call to `a.transfer`. But if σ happened to have another account a_1 with balance 350, and if we reach σ' from σ by executing `a1.transfer(..., ...)`; $a=a_1$, then we would reach a σ' without `a.transfer` having been called: indeed, without the account a from σ having changed at all.

account $a1$ with balance 350, and if we reach σ' from σ by executing $a1.transfer(\dots)$; $a=a1$, then we would reach a σ' without $a.transfer$ having been called: indeed, without the account a from σ having changed at all! (Haskell programmers will probably feel at home here).

This is the remit of the adaptation operator: when we consider the future state, we must “see it from” the perspective of the current state; the binding for variables such as a must be from the current state, even though we may have assigned to them in the mean time. Thus, $\sigma' \triangleleft \sigma$ keeps the heap from σ' , and renames the variables in the top stack frame of σ' so that all variables defined in σ have the same bindings as in σ ; the continuation must be adapted similarly (see Fig. 3).

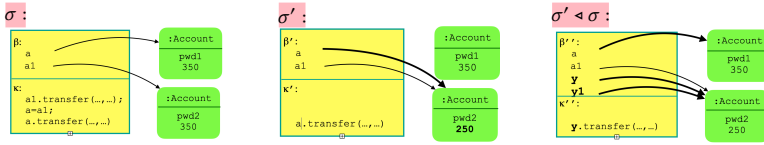


Fig. 3. Illustrating adaptation

Under adaptation, the semantics of $S_{to_dcr_thr_call}$ is: if $\dots, \sigma \models a.balance == 350$, and $\dots, \sigma \rightsquigarrow^* \sigma'$ and $\dots, \sigma' \triangleleft \sigma \models a.balance == 250$, some intermediate state's continuation contains a call to $a.transfer$; where every reference to a in any part of the necessity operator refers to the object bound to that variable in the initial state. Fig. 3 illustrates this semantics. In state σ the variable a points to an Account object with password $pwd1$, and balance 350; the variable $a1$ points to an Account object with password $pwd2$, and balance 350; and the continuation is $a1.transfer(\dots)$; $a=a1$; $a.transfer(\dots)$; \dots . We reach σ' by executing the first two statements from the continuation. Thus, $\sigma' \triangleleft \sigma \models a.balance == 250$. Moreover, in $\sigma' \triangleleft \sigma$ we introduce the fresh variables y and $y1$, and replace a and $a1$ by y and $y1$ in the continuation. This gives that $\sigma' \triangleleft \sigma \models _calls a1.transfer(\dots)$ and $\sigma' \triangleleft \sigma \models _calls a.transfer(\dots)$.

Definition 3.10 gives the full definition of the \triangleleft operator (equivalent to the adaptation operator from [Drossopoulou et al. 2020b]). We introduce fresh variables \bar{y} – as many as in the σ' top frame variable map – $dom(\beta') = \bar{x}$, and $|\bar{y}| = |\bar{x}|$. We extend σ' 's variable map (β'), so that it also maps \bar{y} in the way that σ'' 's variable map (β'') maps its local variables – $\beta'' = \beta[\bar{y} \mapsto \beta'(\bar{x})]$. We rename \bar{x} in σ' continuation to \bar{y} – $\kappa'' = [\bar{y}/\bar{x}]\kappa'$.

Definition 3.10. For any states σ, σ' , heaps χ, χ' , variable maps β, β' , and continuations κ, κ' , such that $\sigma = (\chi, (\beta, \kappa) : \psi)$, and $\sigma' = (\chi', (\beta', \kappa') : \psi')$, we define

- $\sigma' \triangleleft \sigma \triangleq (\chi', (\beta'', \kappa'') : \psi')$
where there exist variables \bar{y} such that
– $\beta'' = \beta[\bar{y} \mapsto \beta'(\bar{x})]$, and $\kappa'' = [\bar{y}/\bar{x}]\kappa'$
– $dom(\beta') = \bar{x}$, and $|\bar{y}| = |\bar{x}|$, and \bar{y} are fresh in β and β' .

Strictly speaking, \triangleleft does not define one unique state: Because variables \bar{y} are arbitrarily chosen, \triangleleft describes an infinite set of states. These states satisfy the same assertions and therefore are equivalent with each other. This is why it is sound to use \triangleleft as an operator, rather than as a set.

3.4 More Examples expressed in Necessity

In this section we introduce some further specification examples, and use them to elucidate finer points in the semantics of Necessity.

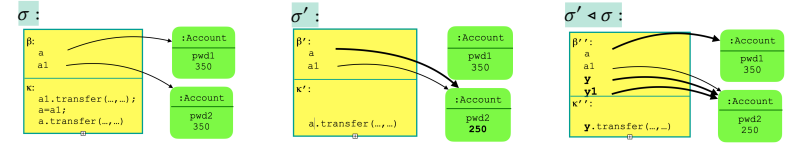


Fig. 4. Illustrating adaptation

In fact, with such a semantics, a module would satisfy $S_{to_dcr_thr_call}$ only if it did not support decrease of the balance by 100, or if states where an account's balance is 350 were unreachable!

This is the remit of the adaptation operator: when we consider the future state, we must “see it from” the perspective of the current state; the binding for variables such as a must be from the current state, even though we may have assigned to them in the mean time. Thus, $\sigma' \triangleleft \sigma$ keeps the heap from σ' , and renames the variables in the top stack frame of σ' so that all variables defined in σ have the same bindings as in σ ; the continuation must be adapted similarly (see Fig. 4).

Under adaptation, the semantics of $S_{to_dcr_thr_call}$ is: if $\dots, \sigma \models a.balance == 350$, and $\dots, \sigma \rightsquigarrow^* \sigma'$ and $\dots, \sigma' \triangleleft \sigma \models a.balance == 250$, then some intermediate state's continuation must contain a call to $a.transfer$; where, all variables bound in the initial state, σ , have the same bindings in $\sigma' \triangleleft \sigma$.

Fig. 4 illustrates the semantics of $\sigma' \triangleleft \sigma$. In σ the variable a points to an Account with password $pwd1$, and balance 350; the variable $a1$ points to an Account with password $pwd2$, and balance 350; and the continuation is $a1.transfer(\dots)$; $a=a1$; $a.transfer(\dots)$; \dots . We reach σ' by executing the first two statements from the continuation. Thus, $\sigma' \triangleleft \sigma \models a.balance == 250$. Moreover, in $\sigma' \triangleleft \sigma$ we introduce the fresh variables y and $y1$, and replace a and $a1$ by y and $y1$ in the continuation. This gives that $\sigma' \triangleleft \sigma \models _calls a1.transfer(\dots)$ and $\sigma' \triangleleft \sigma \models _calls a.transfer(\dots)$.

Definition 3.10 describes the \triangleleft operator in all detail (it is equivalent to, but not identical to the definition given in [Drossopoulou et al. 2020b]). We introduce fresh variables \bar{y} – as many as in the σ' top frame variable map – $dom(\beta') = \bar{x}$, and $|\bar{y}| = |\bar{x}|$. We extend σ' 's variable map (β'), so that it also maps \bar{y} in the way that σ'' 's variable map (β'') maps its local variables – $\beta'' = \beta[\bar{y} \mapsto \beta'(\bar{x})]$. We rename \bar{x} in σ' continuation to \bar{y} – $\kappa'' = [\bar{y}/\bar{x}]\kappa'$.

Definition 3.10. For any states σ, σ' , heaps χ, χ' , variable maps β, β' , and continuations κ, κ' , such that $\sigma = (\chi, (\beta, \kappa) : \psi)$, and $\sigma' = (\chi', (\beta', \kappa') : \psi')$, we define

- $\sigma' \triangleleft \sigma \triangleq (\chi', (\beta'', \kappa'') : \psi')$
where there exist variables \bar{y} such that $\beta'' = \beta[\bar{y} \mapsto \beta'(\bar{x})]$, and $\kappa'' = [\bar{y}/\bar{x}]\kappa'$, and $dom(\beta') = \bar{x}$, and $|\bar{y}| = |\bar{x}|$, and \bar{y} are fresh in β and β' .

Strictly speaking, \triangleleft does not define one unique state: Because variables \bar{y} are arbitrarily chosen, \triangleleft describes an infinite set of states. These states satisfy the same assertions and therefore are equivalent with each other. This is why it is sound to use \triangleleft as an operator, rather than as a set.

3.4 Expressiveness

We discuss expressiveness of Necessity operators, by comparing them with one another, with temporal operators, and with other examples from the literature.

Relationship between Necessity Operators. The three Necessity operators are related by generality. Only If (from A_1 to A_2 onlyIf A) implies Single-Step Only If (from A_1 next A_2 onlyIf A), since if A is a necessary precondition for multiple steps, then it must be a necessary precondition for

3.4.1 More examples of the Bank. Looking back at the examples from §2.3, it holds that

$\text{Mod}_{\text{good}} \models S_{\text{robust_1}}$ $\text{Mod}_{\text{bad}} \not\models S_{\text{robust_1}}$ $\text{Mod}_{\text{better}} \models S_{\text{robust_1}}$
 $\text{Mod}_{\text{good}} \models S_{\text{robust_2}}$ $\text{Mod}_{\text{bad}} \not\models S_{\text{robust_2}}$ $\text{Mod}_{\text{better}} \models S_{\text{robust_2}}$

Consider now another four *Necessity* specifications:

```

1  Snxt_dcr_if_acc  $\triangleq$  from a:Account  $\wedge$  a.balance==bal next a.balance < bal
2                    onlyIf  $\exists o. [(\circ \text{external}) \wedge (\circ \text{access a.pwd})]$ 
3
4  Snxt_dcr_if_call  $\triangleq$  from a:Account  $\wedge$  a.balance==bal next a.balance < bal
5                    onlyIf  $\exists o. [(\circ \text{external}) \wedge (\circ \text{calls a.transfer}(\_, \_, \_))]$ 
6
7  Sto_dcr_if_call  $\triangleq$  from a:Account  $\wedge$  a.balance==bal to a.balance < bal
8                    onlyIf  $\exists o. [(\circ \text{external}) \wedge (\circ \text{calls a.transfer}(\_, \_, \_))]$ 
9
10 Sto_dcr_thr_call  $\triangleq$  from a:Account  $\wedge$  a.balance==bal to a.balance < bal
11                   onlyThrough  $\exists o. [(\circ \text{external}) \wedge (\circ \text{calls a.transfer}(\_, \_, \_))]$ 

```

The specification $S_{\text{nxt_dcr_if_acc}}$ states that the balance of an account decreases in *one step*, only if an external object has access to the password. It is a weaker specification than $S_{\text{robust_2}}$, because it applies when the decrease takes place in *one step*, rather than in a *number* of steps. Even though Mod_{bad} does not satisfy $S_{\text{robust_2}}$, it does satisfy $S_{\text{nxt_dcr_if_acc}}$:

$\text{Mod}_{\text{good}} \models S_{\text{nxt_dcr_if_acc}}$ $\text{Mod}_{\text{bad}} \models S_{\text{nxt_dcr_if_acc}}$ $\text{Mod}_{\text{better}} \models S_{\text{nxt_dcr_if_acc}}$

The specifications $S_{\text{nxt_dcr_if_call}}$ and $S_{\text{to_dcr_if_call}}$ are similar: they both say that a decrease of the balance can only happen if the current statement is a call to transfer. The former considers a *single step*, while the latter allows for *any number* of steps. $S_{\text{robust_2}}$ is slightly different, because it says that such a decrease is only possible if some *intermediate* step called transfer. All three modules satisfy $S_{\text{nxt_dcr_if_call}}$. On the other hand, the code $\text{a1} = \text{new Account}; \text{a2.transfer}(\dots)$ decrements the balance of a2 and does call transfer but not as a first step; therefore, none of the modules satisfy $S_{\text{to_dcr_if_call}}$. That is:

$\text{Mod}_{\text{good}} \models S_{\text{nxt_dcr_if_call}}$ $\text{Mod}_{\text{bad}} \models S_{\text{nxt_dcr_if_call}}$ $\text{Mod}_{\text{better}} \models S_{\text{nxt_dcr_if_call}}$
 $\text{Mod}_{\text{good}} \not\models S_{\text{to_dcr_if_call}}$ $\text{Mod}_{\text{bad}} \not\models S_{\text{to_dcr_if_call}}$ $\text{Mod}_{\text{better}} \not\models S_{\text{to_dcr_if_call}}$

Finally, $S_{\text{to_dcr_thr_call}}$ is a weaker requirement than $S_{\text{to_dcr_if_call}}$, because it only asks that the transfer method is called in *some intermediate step*. All modules satisfy it:

$\text{Mod}_{\text{good}} \models S_{\text{to_dcr_thr_call}}$ $\text{Mod}_{\text{bad}} \models S_{\text{to_dcr_thr_call}}$ $\text{Mod}_{\text{better}} \models S_{\text{to_dcr_thr_call}}$

3.4.2 The DOM. This is the motivating example in [Devriese et al. 2016], dealing with a tree of DOM nodes: Access to a DOM node gives access to all its parent and children nodes, with the ability to modify the node's `property` – where `parent`, `children` and `property` are fields in class `Node`. Since the top nodes of the tree usually contain privileged information, while the lower nodes contain less crucial third-party information, we must be able to limit access given to third parties to only the lower part of the DOM tree. We do this through a `Proxy` class, which has a field `node` pointing to a `Node`, and a field `height`, which restricts the range of `Nodes` which may be modified through the use of the particular `Proxy`. Namely, when you hold a `Proxy` you can modify the `property` of all the descendants of the `height`-th ancestors of the node of that particular `Proxy`. We say that `pr` has *modification-capabilities* on `nd`, where `pr` is a `Proxy` and `nd` is a `Node`, if the `pr.height`-th parent of the node at `pr.node` is an ancestor of `nd`.

The specification DOMSpec states that the `property` of a node can only change if some external object presently has access to a node of the DOM tree, or to some `Proxy` with modification-capabilities to the node that was modified.

```

1  DOMSpec  $\triangleq$  from nd : Node  $\wedge$  nd.property = p to nd.property != p

```

a single step. *Only If* also implies an *Only Through*, where the intermediate state is the starting state of the execution. There is no further relationship between *Single-Step Only If* and *Only Through*.

Relationship with Temporal Logic. Two of the three *Necessity* operators can be expressed in traditional temporal logic: $\text{from } A_1 \text{ to } A_2 \text{ onlyIf } A$ can be expressed as $A_1 \wedge \Diamond A_2 \rightarrow A$, and $\text{from } A_1 \text{ next } A_2 \text{ onlyIf } A$ can be expressed as $A_1 \wedge \bigcirc A_2 \rightarrow A$ (where \Diamond denotes any future state, and \bigcirc denotes the next state). Critically, $\text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A$ cannot be encoded in temporal logics without “nominals” (explicit state references), because the state where A holds must be between the state where A_1 holds, and the state where A_2 holds; and this must be so on *every* execution path from A_1 to A_2 [Bräuner 2022; Brotherston et al. 2020]. TLA+, for example, cannot describe “only through” conditions [Lamport 2002], but we have found “only through” conditions critical to our proofs.

The DOM. This is the motivating example in [Devriese et al. 2016], dealing with a tree of DOM nodes: Access to a DOM node gives access to all its parent and children nodes, with the ability to modify the node's `property` – where `parent`, `children` and `property` are fields in class `Node`. Since the top nodes of the tree usually contain privileged information, while the lower nodes contain less crucial third-party information, we must be able to limit access given to third parties to only the lower part of the DOM tree. We do this through a `Proxy` class, which has a field `node` pointing to a `Node`, and a field `height`, which restricts the range of `Nodes` which may be modified through the use of the particular `Proxy`. Namely, when you hold a `Proxy` you can modify the `property` of all the descendants of the `height`-th ancestors of the node of that particular `Proxy`. We say that `pr` has *modification-capabilities* on `nd`, where `pr` is a `Proxy` and `nd` is a `Node`, if the `pr.height`-th parent of the node at `pr.node` is an ancestor of `nd`.

The specification DOMSpec states that the `property` of a node can only change if some external object presently has access to a node of the DOM tree, or to some `Proxy` with modification-capabilities to the node that was modified.

```

1  DOMSpec  $\triangleq$  from nd : Node  $\wedge$  nd.property = p to nd.property != p
2                    onlyIf  $\exists o. [(\circ \text{external}) \wedge$ 
3                           $(\exists \text{nd}':\text{Node}. [(\circ \text{access nd}')]) \vee$ 
4                           $\exists \text{pr}:\text{Proxy}, k:\mathbb{N}. [(\circ \text{access pr}) \wedge \text{nd.parent}^k = \text{pr.node.parent}^{\text{pr.height}}]]$ 

```

More examples. In order to investigate *Necessity*'s expressiveness, we used it for examples provided in the literature. In Appendix C, we compare with examples proposed by Drossopoulou et al. [2020b], and Permenev et al. [2020a].

4 PROVING NECESSITY

In this Section we provide a proof system for constructing proofs of the *Necessity* specifications defined in §3.3. As discussed in §2.5, such proofs consist of four parts:

- (Part 1) Proving Assertion Encapsulation (§4.1)
- (Part 2) Proving Per-Method *Necessity* specifications for a single internal method from the functional specification of that method (§4.2)
- (Part 3) Proving Per-Step *Necessity* specifications by combining per-method *Necessity* specifications (§4.3)
- (Part 4) Raising necessary conditions to construct proofs of properties of emergent behaviour (§4.4)

Part 1 is, to a certain extent, orthogonal to the main aims of our work; in this paper we propose a simple approach based on the type system, while also acknowledging that better solutions are


```

onlyIf  $\exists o. [ \langle o \text{ external} \rangle \wedge$ 
      (  $\exists nd': \text{Node}. [ \langle o \text{ access } nd' \rangle ] \vee$ 
       $\exists pr: \text{Proxy}, k: \mathbb{N}. [ \langle o \text{ access } pr \rangle \wedge nd.\text{parent}^k = pr.\text{node}.\text{parent}^{pr.\text{height}} ]$  )

```

3.4.3 Expressiveness. In order to investigate *Necessity*'s expressiveness, we used it for examples provided in the literature. In this section we considered the DOM, proposed by Devriese et al. [2016]. In Appendix C, we compare with examples proposed by Drossopoulou et al. [2020b], and Permenev et al. [2020a].

4 PROVING NECESSITY

In this Section we provide a proof system for constructing proofs of the *Necessity* specifications defined in §3.3. As discussed in §2.5, four concerns are involved in the proof of *Necessity* specifications:

- (Part 1) Proving Assertion Encapsulation (§4.1)
- (Part 2) Proving Per-Method *Necessity* specifications from functional specifications for a single internal method (§4.2)
- (Part 3) Proving Per-Step *Necessity* specifications by combining per-method *Necessity* specifications (§4.3)
- (Part 4) Raising necessary conditions to construct proofs of emergent behaviour (§4.4)

4.1 Assertion Encapsulation

A key component of constructing *Necessity* proofs is identifying properties that require internal (and thus known) computation to be invalidated. We refer to this property as *Assertion Encapsulation*. *Necessity* is parametric over the details of the encapsulation model [Noble et al. 2003]; Appendix B and Figure 12 present a rudimentary system that is sufficient to support our example proof. The key judgement we rely upon is *assertion encapsulation* that describes whether an assertion is encapsulated within a module.

4.1.1 Assertion Encapsulation Semantics. Assertion encapsulation models the notion that if an assertion A' is encapsulated by module M , then the validity of that assertion can only be changed via that module. In Tool, that means by calls to objects defined in M but that are accessible from the outside. We provide an definition: A' is encapsulated if whenever we go from state σ to σ' , and when the value of A' changes (i.e. to $\neg A'$) then we must have called a method on one of M 's internal objects. In fact we rely on a slightly more subtle underlying definition, "conditional" encapsulation where $M \models A \Rightarrow \text{Enc}(A')$ expresses that in states which satisfy A , the assertion A' cannot be invalidated, unless a method from M was called.

Definition 4.1 (Assertion Encapsulation). An assertion A' is *encapsulated* by module M and assertion A , written as $M \models A \Rightarrow \text{Enc}(A')$, if and only if for all external modules M' , and all states σ, σ' such that $\text{Arising}(M', M, \sigma)$:

$$\left. \begin{array}{l} - M'; M, \sigma \rightsquigarrow \sigma' \\ - M, \sigma' \triangleleft \sigma \models \neg A' \\ - M, \sigma \models A \wedge A' \end{array} \right\} \Rightarrow \exists x, m, \bar{z}. (M, \sigma \models \langle \text{calls } x.m(\bar{z}) \rangle \wedge \langle x \text{ internal} \rangle)$$

This definition uses adaptation $\sigma' \triangleleft \sigma$ because we have to interpret one assertion in two different states. Revisiting the examples from §2, both Mod_{bad} and $\text{Mod}_{\text{better}}$ encapsulate the balance of an account, because any change to it requires calling a method on an internal object.

$$\text{Mod}_{\text{bad}} \models a: \text{Account} \Rightarrow \text{Enc}(a.\text{balance} = \text{bal})$$

$$\text{Mod}_{\text{better}} \models a: \text{Account} \Rightarrow \text{Enc}(a.\text{balance} = \text{bal})$$

possible. For Parts 2-4, we came up with the key ideas outlined in §2.5, which we develop in more detail in §4.2-§4.4.

4.1 Assertion Encapsulation

Necessity proofs often leverage the fact that some assertions cannot be invalidated unless some internal (and thus known) computation took place. We refer to this property as *Assertion Encapsulation*. In this work, we define the property $M \models A' \Rightarrow \text{Enc}(A)$, which states that under the conditions described by assertion A' , the assertion A is encapsulated by module M . We do not mandate how this property should be derived – instead, we rely on a judgment $M \vdash A' \Rightarrow \text{Enc}(A)$ provided by some external system. Thus, *Necessity* is parametric over the derivation of the encapsulation judgment; in fact, several ways to do that are possible [Clarke and Drossopoulou 2002; Leino and Müller 2004; Noble et al. 2003]. In Appendix B and Figure 13 we present a rudimentary system that is sufficient to support our example proof.

4.1.1 Assertion Encapsulation Semantics. As we said earlier, an assertion A is encapsulated by a module M under condition A' , if in all possible states which arise from execution of module M with any other external module M' , and which satisfy A' , the validity of A can only be changed via computations internal to that module – i.e., via a call to a method from M . In Tool, that means by calls to objects defined in M but accessible from the outside.

Definition 4.1 (Assertion Encapsulation). An assertion A is *encapsulated* by module M and assertion A' , written as $M \models A' \Rightarrow \text{Enc}(A)$, if and only if for all external modules M' , and all states σ, σ' such that $\text{Arising}(M', M, \sigma)$:

$$\left. \begin{array}{l} - M'; M, \sigma \rightsquigarrow \sigma' \\ - M, \sigma \models A \wedge A' \\ - M, \sigma' \triangleleft \sigma \models \neg A \end{array} \right\} \Rightarrow \exists x, m, \bar{z}. (M, \sigma \models \langle \text{calls } x.m(\bar{z}) \rangle \wedge \langle x \text{ internal} \rangle)$$

Note that this definition uses adaptation, $\sigma' \triangleleft \sigma$. The application of the adaptation operator is necessary because we interpret the assertion A in the current state, σ , while we interpret the assertion $\neg A$ in the future state, $\sigma' \triangleleft \sigma$.

Revisiting the examples from §2, both Mod_{bad} and $\text{Mod}_{\text{better}}$ encapsulate the equality of the balance of an account to some value bal : This equality can only be invalidated through calling methods on internal objects.

$$\text{Mod}_{\text{bad}} \models a: \text{Account} \Rightarrow \text{Enc}(a.\text{balance} = \text{bal})$$

$$\text{Mod}_{\text{better}} \models a: \text{Account} \Rightarrow \text{Enc}(a.\text{balance} = \text{bal})$$

Moreover, the property that an object is only accessible from module-internal objects is encapsulated, that is, for all o , and all modules M :

$$M \models o: \text{Object} \Rightarrow \text{Enc}(\text{inside}(o))$$

This is so because any object which is only internally accessible can become externally accessible only via an internal call.

In general, code that does not contain calls to a given module is guaranteed not to invalidate any assertions encapsulated by that module. Assertion encapsulation has been used in proof systems to address the frame problem [Banerjee and Naumann 2005b; Leino and Müller 2004].

4.1.2 Deriving Assertion Encapsulation. Our logic does not deal with, nor rely on, the specifics of how encapsulation is derived. Instead, it relies on an encapsulation judgment and expects it to be sound:

Definition 4.2 (Encapsulation Soundness). A judgement of the form $M \vdash A' \Rightarrow \text{Enc}(A)$ is *sound*, if for all modules M , and assertions A and A' , if

Therefore, code that does not contain calls to a `certain` module is guaranteed not to invalidate any assertions encapsulated by that module. Assertion encapsulation has been used in proof systems to address the frame problem [Banerjee and Naumann 2005b; Leino and Müller 2004].

4.1.2 Proving Assertion Encapsulation. Our logic does not rely on the specifics of the encapsulation model, but only its soundness:

Definition 4.2 (Encapsulation Soundness). A judgement of the form $M \vdash A \Rightarrow \text{Enc}(A)$ is *sound*, if for all modules M , and assertions A_1 and A_2 , if $M \vdash A_1 \Rightarrow \text{Enc}(A_2)$ then $M \models A_1 \Rightarrow \text{Enc}(A_2)$.

The key consequence of soundness is that an object inside a module (`inside(o)`) will always be encapsulated, in the sense that it can only leak out of the module via an internal call.

4.1.3 Types. To allow for an easy way to judge encapsulation of assertions, we assume a very simple type system, where field, method arguments and method results are annotated with classes, and the type system checks that field assignments, method calls, and method returns adhere to these expectations. Because the type system is so simple, we do not include its specification in the paper. Note however, that the type system has one further implication: modules are typed in isolation, thereby implicitly prohibiting method calls from internal objects to external objects.

Based on this type system, we define a predicate $\text{Enc}_e(e)$, in Appendix B, which asserts that any object read during the evaluation of e are internal. Thus, any assertion that only involves $\text{Enc}_e(_)$ expressions is encapsulated, more in Appendix B.

Finally, a further small addition to the type system assists the knowledge that an object is `inside`: Classes may be annotated as `confined`. A `confined` object cannot be accessed by external objects; that is, it is always `inside`. The type system needs to ensure that objects of `confined` type are never returned from method bodies, this is even simpler than in [Vitek and Bokowski 1999]. Again, we omit the detailed description of this simple type system.

4.2 Per-Method Necessity Specifications

In this section we detail how we use functional specifications to prove per-method *Necessity* specifications. That is, for some method m in class C , we prove a specification of the form:

$$\text{from } A_1 \wedge x : C \wedge \langle _ \text{ calls } x.m(_) \rangle \text{ next } A_2 \text{ onlyIf } A$$

Thus, A is a necessary precondition for reaching A_2 from A_1 via a method call m to an object of class C . Our first breakthrough observation (§1.1) is that if a precondition and a certain statement is *sufficient* to achieve a particular result, then the negation of that precondition is *necessary* to achieve the negation of the result after executing that statement.

For the use in functional specifications, we define *Classical assertions*, a subset of *Assert*, comprising only those assertions that are commonly present in other specification languages. They are restricted to expressions, class assertions, the usual connectives, negation, implication, and the usual quantifiers.

Definition 4.3. Classical assertions, P, Q , are defined as follows

$$P, Q ::= e \mid e : C \mid P \wedge P \mid P \vee P \mid P \longrightarrow P \mid \neg P \mid \forall x.[P] \mid \exists x.[P]$$

We assume that there exists some functional specification proof system that allows us to prove specifications of the form $M \vdash \{P\} \leq \{Q\}$. This implies that we can also have guarantees of

$$M \vdash \{P\} \text{ res} = x.m(\bar{z}) \{Q\}$$

That is, the execution of $x.m(\bar{z})$ with the precondition P results in a program state that satisfies postcondition Q , where the returned value is represented by `res` in Q . As we have previously

$$M \vdash A' \Rightarrow \text{Enc}(A) \text{ implies } M \models A' \Rightarrow \text{Enc}(A).$$

Types for Assertion Encapsulation. Even though the derivation of assertion encapsulation is not the focus of this paper, for illustrative purposes, we will outline now a very simple type system which supports such derivations: We assume that field declarations, method arguments and method results are annotated with class names, and that classes may be annotated as `confined`. A `confined` object is not accessed by external objects; that is, it is always `inside`.

The type system then checks that field assignments, method calls, and method returns adhere to these expectations, and in particular, that objects of `confined` type are never returned from method bodies – this is a simplified version of the type system described in [Vitek and Bokowski 1999]. Because the type system is so simple, we do not include its formalization in the paper. Note however, that the type system has one further implication: modules are typed in isolation, thereby implicitly prohibiting method calls from internal objects to external objects.

Based on this type system, we define a predicate $\text{Enc}_e(e)$, in Appendix B, which asserts that any objects read during the evaluation of e are internal. Thus, any assertion that only involves $\text{Enc}_e(_)$ expressions is encapsulated – more in Appendix B.

4.2 Per-Method Necessity Specifications

In this section we detail how we use functional specifications to prove per-method *Necessity* specifications of the form

$$\text{from } A_1 \wedge x : C \wedge \langle _ \text{ calls } x.m(\bar{z}) \rangle \text{ next } A_2 \text{ onlyIf } A$$

where C is a class, and m a method in C .

The first key idea in §2.5 is that if a precondition and a certain statement is *sufficient* to achieve a particular result, then the negation of that precondition is *necessary* to achieve the negation of the result after executing that statement. Specifically, $\{P\} \leq \{Q\}$ implies that $\neg P$ is a *necessary precondition* for $\neg Q$ to hold following the execution of s .

For the use in functional specifications, we define *Classical assertions*, a subset of *Assert*, comprising only those assertions that are commonly present in other specification languages. They are restricted to expressions, class assertions, the usual connectives, negation, implication, and the usual quantifiers.

Definition 4.3. Classical assertions, P, Q , are defined as follows

$$P, Q ::= e \mid e : C \mid P \wedge P \mid P \vee P \mid P \longrightarrow P \mid \neg P \mid \forall x.[P] \mid \exists x.[P]$$

We assume that there exists some proof system that derives functional specifications of the form $M \vdash \{P\} \leq \{Q\}$. This implies that we can also have guarantees of

$$M \vdash \{P\} \text{ res} = x.m(\bar{z}) \{Q\}$$

That is, the execution of $x.m(\bar{z})$ with the precondition P results in a program state that satisfies postcondition Q , where the returned value is represented by `res` in Q . We further assume that such a proof system is sound, i.e. that if $M \vdash \{P\} \leq \{Q\}$, then for every program state σ that satisfies P , the execution of the method call $x.m(\bar{z})$ results in a program state satisfying Q . As we have previously discussed (see §2.5), we build *Necessity* specifications on top of functional specifications using the fact that validity of $\{P\} \leq \{Q\}$ implies that $\neg P$ is a necessary precondition to $\neg Q$ being true after execution of $\text{res} = x.m(\bar{z})$.

Proof rules for per-method specifications are given in Figure 5. Note that the receiver x in the rules in 5 is implicitly an internal object. This is because we only have access to internal code, and thus are only able to prove the validity of the associated Hoare triple.

$$\begin{array}{c}
\frac{M \vdash \{x : C \wedge P_1 \wedge \neg P\} \text{res} = x.m(\bar{z}) \{\neg P_2\}}{M \vdash \text{from } P_1 \wedge x : C \wedge \underline{\text{calls}} x.m(\bar{z}) \text{ next } P_2 \text{ onlyIf } P} \quad (\text{IF1-CLASSICAL}) \\
\\
\frac{M \vdash \{x : C \wedge \neg P\} \text{res} = x.m(\bar{z}) \{\text{res} \neq y\}}{M \vdash \text{from inside}(y) \wedge x : C \wedge \underline{\text{calls}} x.m(\bar{z}) \text{ next } \neg \text{inside}(y) \text{ onlyIf } P} \quad (\text{IF1-INSIDE})
\end{array}$$

Fig. 4. Per-Method *Necessity* specifications

discussed (see §2.5), we build *Necessity* specifications on top of functional specifications using the fact that $\neg P$ is a necessary pre-condition to $\neg Q$ being true after the method execution.

Proof rules for per-method specifications are given in Figure 4. IF1-CLASSICAL and IF1-INSIDE raise functional specifications to *Necessity* specifications. These are rules whose conclusion have the form Single-Step Only If.

IF1-CLASSICAL states that if by some classical logic, the execution of $x.m(\bar{z})$, with precondition $P \wedge \neg P_1$, has a resulting state satisfying postcondition $\neg P_2$, then P_1 is a *necessary* precondition to the resulting state satisfying P_2 .

IF1-INSIDE states that a method which does not return an object y preserves the “insideness” of y . In more detail, IF1-INSIDE states that if P is a necessary precondition for returning an object y , then it follows that P is a necessary precondition for a call $x.m(\bar{z})$ to result in an external object gaining access to y . At first glance this rule might seem unsound, however the restriction on external calls ensures soundness of this rule. There are only four ways an object x might gain access to another object y : (1) y is created by x as the result of a new expression, (2) y is written to some field of x , (3) y is passed to x as an argument to a method call on x , or (4) y is returned to x as the result of a method call from an object z that has access to y . The rules in Fig. 4 are only concerned with effects on program state resulting from a method call to some internal object, and thus (1) and (2) need not be considered as neither object creation or field writes may result in an external object gaining access from an internal object. Since we are only concerned with describing how internal objects grant access to external objects, our restriction on external method calls within internal code prohibits (3) from occurring. Finally, (4) is described by IF1-INSIDE. In further work we plan to weaken the restriction on external method calls, and will strengthen this rule. Note that IF1-INSIDE is essentially a specialized version of IF1-CLASSICAL for the $\text{inside}(_)$ predicate. Since $\text{inside}(_)$ is not a classical assertion, we cannot use functional specifications to reason about necessary conditions for invalidating $\text{inside}(_)$.

4.3 Per-Step *Necessity* Specifications

We now use our second breakthrough observation (§1.1): if an assertion is encapsulated, and all methods within the internal module have the same necessary pre-condition to invalidating that assertion, then we are able to generalize per-method *Necessity* specifications to per-step *Necessity* specifications. We present a selection of the rules whose conclusion is of the form Single Step Only If in Fig. 5. The full rule set can be found in Fig. 16.

IF1-INTERNAL lifts a per-method *Necessity* specifications to a per-step *Necessity* specification. Any *Necessity* specification which is satisfied for any method calls sent to any object in a module, is satisfied for *any step*, even an external step, provided that the effect involved, i.e. going from A_1 states to A_2 states, is encapsulated.

The remaining rules are more standard, and are reminiscent of the Hoare logic rule of consequence. We present a few of the more interesting rules here:

$$\begin{array}{c}
\frac{M \vdash \{x : C \wedge P_1 \wedge \neg P\} \text{res} = x.m(\bar{z}) \{\neg P_2\}}{M \vdash \text{from } P_1 \wedge x : C \wedge \underline{\text{calls}} x.m(\bar{z}) \text{ next } P_2 \text{ onlyIf } P} \quad (\text{IF1-CLASSICAL}) \\
\\
\frac{M \vdash \{x : C \wedge \neg P\} \text{res} = x.m(\bar{z}) \{\text{res} \neq y\}}{M \vdash \text{from inside}(y) \wedge x : C \wedge \underline{\text{calls}} x.m(\bar{z}) \text{ next } \neg \text{inside}(y) \text{ onlyIf } P} \quad (\text{IF1-INSIDE})
\end{array}$$

Fig. 5. Per-Method *Necessity* specifications

IF1-CLASSICAL states that if the execution of $x.m(\bar{z})$, with precondition $P \wedge \neg P_1$, leads to a state satisfying postcondition $\neg P_2$, then P_1 is a *necessary* precondition to the resulting state satisfying P_2 .

IF1-INSIDE states that if the precondition $\neg P$ guarantees that the result of the call $x.m(\bar{z})$ is not y , then P is a necessary pre-condition to invalidate $\text{inside}(y)$ by calling $x.m(\bar{z})$. This is sound, because the premise of IF1-INSIDE implies that P is a necessary precondition for the call $x.m(\bar{z})$ to return an object y ; this, in turn, implies that P is a necessary precondition for the call $x.m(\bar{z})$ to result in an external object gaining access to y . The latter implication is valid because the rule is applicable only to external states semantics, which means that the call $x.m(\bar{z})$ is a call from an external object to some internal object x . Namely, there are only four ways an object o might gain access to another object o' : (1) o' is created by o as the result of a new expression, (2) o' is written to some field of o , (3) o' is passed to o as an argument to a method call on o , or (4) o' is returned to o as the result of a method call from an object o'' that has access to o' . The rule IF1-INSIDE is only concerned with effects on program state resulting from a method call to some internal object, and thus (1) and (2) need not be considered as neither object creation or field writes may result in an external object gaining access to an object that is only internally accessible. Since we are only concerned with describing how internal objects grant access to external objects, our restriction on external method calls within internal code prohibits (3) from occurring. Finally, (4) is described by IF1-INSIDE. In further work we plan to weaken the restriction on external method calls, and will strengthen this rule. Note that IF1-INSIDE is essentially a specialized version of IF1-CLASSICAL for the $\text{inside}(_)$ predicate. Since $\text{inside}(_)$ is not a classical assertion, we cannot use functional specifications to reason about necessary conditions for invalidating $\text{inside}(_)$.

4.3 Per-Step *Necessity* Specifications

$$\begin{array}{c}
\left[\frac{\text{for all } C \in \text{dom}(M) \text{ and } m \in M(C).\text{mths},}{M \vdash \text{from } A_1 \wedge x : C \wedge \underline{\text{calls}} x.m(\bar{z}) \text{ next } A_2 \text{ onlyIf } A_3} \right] \\
\frac{M \vdash A_1 \longrightarrow \neg A_2 \quad M \vdash A_1 \Rightarrow \text{Enc}(A_2)}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3} \quad (\text{IF1-INTERNAL}) \\
\\
\frac{M \vdash A_1 \longrightarrow A'_1 \quad M \vdash A_2 \longrightarrow A'_2 \quad M \vdash A'_3 \longrightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ next } A'_2 \text{ onlyIf } A'_3}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3} \quad (\text{IF1} \longrightarrow) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \vee A' \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A} \quad (\text{IF1-VE}) \\
\\
\frac{\forall y. M \vdash \text{from } ([y/x]A_1) \text{ next } A_2 \text{ onlyIf } A}{M \vdash \text{from } \exists x.[A_1] \text{ next } A_2 \text{ onlyIf } A} \quad (\text{IF1-}\exists_1)
\end{array}$$

Fig. 6. Selected rules for Single-Step Only If

$$\begin{array}{c}
\frac{\left[\begin{array}{l} \text{for all } C \in \text{dom}(M) \text{ and } m \in M(C).\text{mths}, \\ M \vdash \text{from } A_1 \wedge x : C \wedge (_ \text{calls } x.m(\bar{z})) \text{ next } A_2 \text{ onlyIf } A_3 \\ M \vdash A_1 \rightarrow \neg A_2 \quad M \vdash A_1 \Rightarrow \text{Enc}(A_2) \end{array} \right]}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3} \quad (\text{IF1-INTERNAL}) \\
\\
\frac{M \vdash A_1 \rightarrow A'_1 \quad M \vdash A_2 \rightarrow A'_2 \quad M \vdash A'_3 \rightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ next } A'_2 \text{ onlyIf } A'_3}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3} \quad (\text{IF1} \rightarrow) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \vee A' \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A} \quad (\text{IF1-}\vee\text{E}) \\
\\
\frac{\forall y, M \vdash \text{from } ([y/x]A_1) \text{ next } A_2 \text{ onlyIf } A}{M \vdash \text{from } \exists x. [A_1] \text{ next } A_2 \text{ onlyIf } A} \quad (\text{IF1-}\exists_1)
\end{array}$$

Fig. 5. Selected rules for Single-Step *Only If*

$$\begin{array}{c}
\frac{M \vdash \text{from } A \text{ next } \neg A \text{ onlyIf } A'}{M \vdash \text{from } A \text{ to } \neg A \text{ onlyThrough } A'} \quad (\text{CHANGES}) \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_1) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_3 \text{ to } A_2 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_2) \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{IF}) \\
\\
M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_2 \quad (\text{END})
\end{array}$$

Fig. 6. Selected rules for *Only Through* – the rest can be found in Figure 17

$$\begin{array}{c}
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \quad (\text{IF-TRANS}) \\
\\
M \vdash \text{from } x : C \text{ to } \neg x : C \text{ onlyIf false} \quad (\text{IF-CLASS}) \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_1 \quad (\text{IF-START})
\end{array}$$

Fig. 7. Selected rules for *Only If* – the rest can be found in Figure 18

The rule for implication (IF1 \rightarrow) may strengthen properties of either the starting or ending state, or weaken the necessary precondition. The disjunction elimination rule (IF1 \vee E) mirrors typical disjunction elimination rules, with a variation stating that if it is not possible to reach the end state from one branch of the disjunction, then we can eliminate that branch.

Two rules support existential elimination on the left hand side. We present one here. IF1 \exists_1 states that if any single step of execution starting from a state satisfying $[y/x]A_1$ for all possible y , reaching some state satisfying A_2 has A as a necessary precondition, it follows that any single step execution starting in a state where such a y exists, and ending in a state satisfying A_2 , must have A as a necessary precondition.

4.4 Emergent *Necessity* Specifications

We present our third breakthrough (§2.5): per-step *Necessity* specifications are raised to multiple step *Necessity* specifications, allowing the specification of emergent behaviour. Figure 6 presents

$$\begin{array}{c}
\frac{M \vdash \text{from } A \text{ next } \neg A \text{ onlyIf } A'}{M \vdash \text{from } A \text{ to } \neg A \text{ onlyThrough } A'} \quad (\text{CHANGES}) \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_1) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_3 \text{ to } A_2 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_2) \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{IF}) \\
\\
M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_2 \quad (\text{END})
\end{array}$$

Fig. 7. Selected rules for *Only Through* – the rest can be found in Figure 18

$$\begin{array}{c}
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \quad (\text{IF-TRANS}) \\
\\
M \vdash \text{from } x : C \text{ to } \neg x : C \text{ onlyIf false} \quad (\text{IF-CLASS}) \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_1 \quad (\text{IF-START})
\end{array}$$

Fig. 8. Selected rules for *Only If* – the rest can be found in Figure 19

The second key idea in §2.5 allows us to leverage several per-method *Necessity* specifications to obtain one per-step *Necessity* specification: Namely, if an assertion is encapsulated, and all methods within the internal module require the same condition to the invalidation of that assertion, then this condition is a necessary, program-wide, single-step condition to the invalidation of that assertion.

In this section we present a selection of the rules whose conclusion is of the form Single Step *Only If* in Fig. 6. The full rule set can be found in Fig. 17.

IF1-INTERNAL lifts a set of per-method *Necessity* specifications to a per-step *Necessity* specification. Any *Necessity* specification which is satisfied for all method calls sent to any object in a module, is satisfied for *any step*, even an external step, provided that the effect involved, *i.e.* going from A_1 states to A_2 states, is encapsulated.

The remaining rules are more standard, and are reminiscent of the Hoare logic rule of consequence. We present a few of the more interesting rules here:

The rule for implication (IF1 \rightarrow) may strengthen properties of either the starting or ending state, or weaken the necessary precondition. The disjunction elimination rule (IF1 \vee E) mirrors typical disjunction elimination rules, with a variation stating that if it is not possible to reach the end state from one branch of the disjunction, then we can eliminate that branch.

Two rules support existential elimination on the left hand side. IF1 \exists_1 states that if any single step of execution starting from a state satisfying $[y/x]A_1$ for all possible y , reaching some state satisfying A_2 has A as a necessary precondition, it follows that any single step execution starting in a state where such a y exists, and ending in a state satisfying A_2 , must have A as a necessary precondition. The other rule can be found in Fig. 17.

4.4 Emergent *Necessity* Specifications

The third key idea in §2.5 allows us to leverage several per-step *Necessity* specifications to obtain multiple-step *Necessity* specifications, and thus enables the description of the module's emergent behaviour. We combine per-step *Necessity* specifications into multiple-step *Necessity* specifications, as well as several multiple step *Necessity* specifications into further multiple step *Necessity* specifications.

some of the rules with conclusion *Only Through*, while Figure 7 provides some of the rules with conclusion *Only If*. The full rules can be found in Appendix D.

CHANGES, in Figure 6, states that if A' is a necessary condition for the satisfaction of A to change in *one* step, then it is also a necessary condition for the satisfaction of A to change in *any number* of steps. This is sound, because if the satisfaction of some assertion changes over time, then *then* there must be some specific intermediate state where that change occurred. CHANGES is an important enabler for proofs of emergent properties. It is this rule that ultimately connects program execution to encapsulated properties.

It may seem natural that CHANGES should take the more general form:

$$\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3}$$

This would not be sound because a transition from a state satisfying A_1 to one satisfying A_2 is not required to occur in a single step; however a change in satisfaction for a specific assertion (i.e. A to $\neg A$) can only take place in a single step.

TRANS₁ and TRANS₂ are transitivity rules that state that necessary conditions to reach intermediate states or proceed from intermediate states are themselves necessary intermediate states. Finally, *Only Through* includes END, stating that the ending condition is a necessary intermediate condition. Moreover, any *Only If* specification entails the corresponding *Only Through* specification (If). *Only If* also includes a transitivity rule (If-TRANS), but since the necessary condition must be true in the beginning state, there is only a single rule. If-CLASS expresses that an object's class never changes. Finally, any starting condition is itself a necessary precondition (If-START).

4.5 Soundness of the Necessity Logic

THEOREM 4.4 (SOUNDNESS). Assuming a sound Assert proof system, $M \vdash A$, and a sound encapsulation inference system, $M \vdash A \Rightarrow \text{Enc}(A')$, and *and* that on top of these systems we built the Necessity logic according to the rules in Figures 4, and 5, and 7, and 6, then, for all modules M , and all Necessity specifications S :

$$M \vdash S \quad \text{implies} \quad M \models S$$

PROOF. by induction on the derivation of $M \vdash S$. \square

Theorem 4.4 demonstrates that the Necessity logic is sound with respect to the semantics of Necessity specifications. The Necessity logic parametric wrt to the algorithms for proving validity of assertions $M \vdash A$, and assertion encapsulation ($M \vdash A \Rightarrow \text{Enc}(A')$), and is sound provided that these two proof systems are sound.

The mechanized proof of Theorem 4.4 in Coq can be found in the associated artifact. The Coq formalism deviates slightly from the system as presented here, mostly in the formalization of the Assert language. The Coq version of Assert restricts variable usage to expressions, and allows only addresses to be used as part of non-expression syntax. For example, in the Coq formalism we can write assertions like $x.f == \text{this}$ and $x == \alpha_y$ and $\langle \alpha_x \text{ access } \alpha_y \rangle$, but we cannot write assertions like $\langle x \text{ access } y \rangle$, where x and y are variables, and α_x and α_y are addresses. The reason for this restriction in the Coq formalism is to avoid spending significant effort encoding variable renaming and substitution, a well-known difficulty for languages such as Coq. This restriction does not affect the expressiveness of our Coq formalism: we are able to express assertions such as $\langle x \text{ access } y \rangle$, by using addresses and introducing equality expressions to connect variables to address, i.e. $\langle \alpha_x \text{ access } \alpha_y \rangle \wedge \alpha_x == x \wedge \alpha_y == y$. The Coq formalism makes use of the CpdTactics [Chlipala 2019] library of tactics to discharge some proofs.

Figure 7 presents some of the rules with conclusion *Only Through*, while Figure 8 provides some of the rules with conclusion *Only If*. The full rules can be found in Appendix D.

CHANGES, in Figure 7, states that if A' is a necessary condition for the satisfaction of A to change in *one* step, then it is also a necessary condition for the satisfaction of A to change in *any number* of steps. This is sound, because if the satisfaction of some assertion changes over time, then there must be some specific intermediate state where that change occurred. CHANGES is an important enabler for proofs of emergent properties: Since Necessity specifications are concerned with necessary conditions for change, their proofs typically hinge around such necessary conditions for certain properties to change. For example, under what conditions may our account's balance decrease?

It might seem natural that CHANGES had the more general form:

$$\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3} \quad ((\text{CHANGESUNBOUND}))$$

(CHANGESUNBOUND) is not sound because the conclusion of the rule describes transitions from a state satisfying A_1 to one satisfying A_2 which may occur over several steps, while the premise describes a transition that takes place over one single step. Such a concern does not apply to (CHANGES), because a change in satisfaction for a specific assertion (i.e. A to $\neg A$) can only take place in a single step.

TRANS₁ and TRANS₂ are rules about transitivity. They state that necessary conditions to reach intermediate states or proceed from intermediate states are themselves necessary intermediate states. Any *Only If* specification entails the corresponding *Only Through* specification (If). Finally, END states that the ending condition is a necessary intermediate condition.

Only If also includes a rule for transitivity (If-TRANS), but since the necessary condition must be true in the beginning state, there is only a single rule. If-CLASS expresses that an object's class never changes. Finally, any starting condition is itself a necessary precondition (If-START).

4.5 Soundness of the Necessity Logic

THEOREM 4.4 (SOUNDNESS). Assuming a sound Assert proof system, $M \vdash A$, and a sound encapsulation inference system, $M \vdash A \Rightarrow \text{Enc}(A')$, and that on top of these systems we built the Necessity logic according to the rules in Figures 5, 6, 7, and 8, then, for all modules M , and all Necessity specifications S :

$$M \vdash S \quad \text{implies} \quad M \models S$$

PROOF. by induction on the derivation of $M \vdash S$. \square

Theorem 4.4 demonstrates that the Necessity logic is sound with respect to the semantics of Necessity specifications. The Necessity logic parametric wrt to the algorithms for proving validity of assertions $M \vdash A$, and assertion encapsulation ($M \vdash A \Rightarrow \text{Enc}(A')$), and is sound provided that these two proof systems are sound.

The mechanized proof of Theorem 4.4 in Coq can be found in the associated artifact. The Coq formalism deviates slightly from the system as presented here, mostly in the formalization of the Assert language. The Coq version of Assert restricts variable usage to expressions, and allows only addresses to be used as part of non-expression syntax. For example, in the Coq formalism we can write assertions like $x.f == \text{this}$ and $x == \alpha_y$ and $\langle \alpha_x \text{ access } \alpha_y \rangle$, but we cannot write assertions like $\langle x \text{ access } y \rangle$, where x and y are variables, and α_x and α_y are addresses. The reason for this restriction in the Coq formalism is to avoid spending significant effort encoding variable renaming and substitution, a well-known difficulty for languages such as Coq. This restriction does not affect the expressiveness of our Coq formalism: we are able to express assertions such

5 PROVING THAT $\text{MOD}_{\text{better}}$ SATISFIES $S_{\text{robust_2}}$

We now revisit our example from §1 and 2, and outline a proof that $\text{MOD}_{\text{better}}$ satisfies $S_{\text{robust_2}}$. A summary of this proof has already been discussed in §2.5. A more complex variant of this example that employs **ghost fields and a ledger of accounts** can be found in Appendix G.

Recall that an Account includes at least a field or ghost field called `balance`, and a method called `transfer`.

We first rephrase $S_{\text{robust_2}}$ to use the `inside(_)` predicate.

```
1  $S_{\text{robust\_2}} \triangleq$  from a:Account  $\wedge$  a.balance=bal
2   to a.balance < bal onlyIf  $\neg$ inside(a.pwd)
```

We next revisit the functional specification from §2.1 and derive the following PRE- and POST-conditions. The first two pairs of PRE-, POST-conditions correspond to the first two ENSURES clauses from §2.1, while the next two pairs correspond to the MODIFIES-clause. The current expression in terms of PRE- and POST-conditions is weaker than the one in §2.1, and is not modular, but is **appropriate** for proving adherence to $S_{\text{robust_2}}$.

```
1 FuncSpec'  $\triangleq$ 
2   method transfer(dest:Account, pwd':Password) -> void
3     (PRE: this.balance=bal1  $\wedge$  dest.balance=bal2  $\wedge$  this.pwd=pwd'  $\wedge$  this#dest
4       POST: this.balance=bal1-100  $\wedge$  dest.balance=bal2+100)
5     (PRE: this.balance=bal1  $\wedge$  dest.balance=bal2  $\wedge$  (this.pwd#pwd'  $\vee$  this=dest)
6       POST: this.balance=bal1  $\wedge$  dest.balance=bal2)
7     (PRE: a:Account  $\wedge$  a.balance=bal  $\wedge$  a#this  $\wedge$  a#dest
8       POST: a.balance=bal)
9     (PRE: a:Account  $\wedge$  a.pwd=pwd1
10      POST: a.pwd=pwd1)
```

5.1 Part 1: Assertion Encapsulation

The first part of the proof demonstrates that the `balance`, `pwd`, and external accessibility to the password are encapsulated properties. That is, for the `balance` to change (i.e. for `a.balance = bal` to be invalidated), internal computation is required. We use a **conservative approach to an** encapsulation system, detailed in App. B, and provide the proof steps below.

aEnc and **balanceEnc** state that `a` and `a.balance` are encapsulated. Moreover, **balEnc**

BalEncaps:	
aEnc: $\text{Mod}_{\text{better}} \vdash a:\text{Account} \wedge a.\text{balance}=\text{bal} \Rightarrow \text{Enc}_e(a)$	by $\text{Enc}_e\text{-Obj}$
balanceEnc: $\text{Mod}_{\text{better}} \vdash a:\text{Account} \wedge a.\text{balance}=\text{bal} \Rightarrow \text{Enc}_e(a.\text{balance})$	by aEnc and $\text{Enc}\text{-FIELD}$
balEnc: $\text{Mod}_{\text{better}} \vdash a:\text{Account} \wedge a.\text{balance}=\text{bal} \Rightarrow \text{Enc}_e(\text{bal})$	by $\text{Enc}_e\text{-INT}$
$\text{Mod}_{\text{better}} \vdash a:\text{Account} \wedge a.\text{balance}=\text{bal} \Rightarrow \text{Enc}(a.\text{balance}=\text{bal})$	
by balanceEnc , balEnc , $\text{Enc}\text{-Eq}$, and $\text{Enc}\text{-=}$	

states that since `bal` is an integer, its value is constant and may not change, and thus is encapsulated. These combine to prove that the assertion `a.balance = bal` is encapsulated. Note: it may seem odd to say that the integer `bal` or the address `a` is encapsulated; but remember that their value cannot change, and thus they vacuously satisfy the definition of encapsulation.

as $\langle x \text{ access } y \rangle$, by using addresses and introducing equality expressions to connect variables to address, i.e. $\langle \alpha_x \text{ access } \alpha_y \rangle \wedge \alpha_x == x \wedge \alpha_y == y$. The Coq formalism makes use of the `CpdtTactics` [Chlipala 2019] library of tactics to discharge some proofs.

5 PROVING THAT $\text{MOD}_{\text{better}}$ SATISFIES $S_{\text{robust_2}}$

We now revisit our example from §1 and §2, and outline a proof that $\text{MOD}_{\text{better}}$ satisfies $S_{\text{robust_2}}$. A summary of this proof has already been discussed in §2.5. A more complex variant of this example that employs can be found in Appendix G. It demonstrates dealing with modules consisting of several classes some of which are confined, and which use ghost fields defined through functions; it also demonstrates proofs of assertion encapsulation of assertions which involve reading the values of several fields. Mechanised versions of the proofs in both this Section, and Appendix G can be found in the associated Coq artifact in `simple_bank_account.v` and `bank_account.v` respectively.

Recall that an Account includes at least a field (or ghost field) called `balance`, and a method called `transfer`.

We first rephrase $S_{\text{robust_2}}$ to use the `inside(_)` predicate.

```
1  $S_{\text{robust\_2}} \triangleq$  from a:Account  $\wedge$  a.balance=bal
2   to a.balance < bal onlyIf  $\neg$ inside(a.pwd)
```

We next revisit the functional specification from §2.1 and derive the following PRE- and POST-conditions. The first two pairs of PRE-, POST-conditions correspond to the first two ENSURES clauses from §2.1, while the next two pairs correspond to the MODIFIES-clause. The current expression in terms of PRE- and POST-conditions is weaker than the one in §2.1, and is not modular, but is **sufficient** for proving adherence to $S_{\text{robust_2}}$.

```
1 FuncSpec'  $\triangleq$ 
2   method transfer(dest:Account, pwd':Password) -> void
3     (PRE: this.balance=bal1  $\wedge$  dest.balance=bal2  $\wedge$  this.pwd=pwd'  $\wedge$  this#dest
4       POST: this.balance=bal1-100  $\wedge$  dest.balance=bal2+100)
5     (PRE: this.balance=bal1  $\wedge$  dest.balance=bal2  $\wedge$  (this.pwd#pwd'  $\vee$  this=dest)
6       POST: this.balance=bal1  $\wedge$  dest.balance=bal2)
7     (PRE: a:Account  $\wedge$  a.balance=bal  $\wedge$  a#this  $\wedge$  a#dest
8       POST: a.balance=bal)
9     (PRE: a:Account  $\wedge$  a.pwd=pwd1
10      POST: a.pwd=pwd1)
```

5.1 Part 1: Assertion Encapsulation

The first part of the proof demonstrates that the `balance`, `pwd`, and external accessibility to the password are encapsulated properties. That is, for the `balance` to change (i.e. for `a.balance = bal` to be invalidated), or for or for the encapsulation of `a.pwd` to be broken (ie for a transition from `inside(a.pwd)` to `\neg inside(a.pwd)`), internal computation is required.

We use a simple encapsulation system, detailed in Appendix B, and provide the proof steps below. **aEnc** and **balanceEnc** state that `a` and `a.balance` satisfy the Enc_e predicate. That is, if any objects' contents are to be looked up during execution of these expressions, then these objects are internal. $\text{Enc}_e(a)$ holds because no object's contents is looked up, while $\text{Enc}_e(a.\text{balance})$ holds because `balance` is a field of `a`, and `a` is internal.

Moreover, **balEnc** states that `bal` satisfies the Enc_e predicate – it is an integer, and no object look-up is involved in its calculation. **balanceEnc** and **balEnc** combine to prove that the

We similarly prove that $a.pwd$ is encapsulated (**PwdEncaps**), and that $inside(a.pwd)$ is encapsulated (**PwdInsideEncaps**). That is, if only internal objects have access to an account's pwd , then only internal computation may grant to an external object access to pwd .

PwdEncaps:	
$Mod_{better} \vdash a:Account \Rightarrow Enc(a.pwd=p)$	by $ENC_e\text{-Obj}$, $ENC\text{-FIELD}$, and $ENC\text{-Eq}$
PwdInsideEncaps:	
$Mod_{better} \vdash a:Account \Rightarrow Enc(inside(a.balance))$	by $ENC\text{-INSIDE}$

5.2 Part 2: Per-Method Necessity Specifications

Part 2 proves necessary preconditions for each method in the module interface. We employ the crucial observation that we can build necessary preconditions on top of classical Hoare logic (§4.2). **SetBalChange** uses classical Hoare logic to prove that the `set` method in `Account` never modifies the balance. We then use $IF1\text{-CLASSICAL}$ and our *Necessity* logic to prove that if it ever did change (a logical absurdity), then `transfer` must have been called. **SetPwdLeak**

SetBalChange:	
$\{a, a':Account \wedge a'.balance=bal\}$ $a.set(_, _)$ $\{a'.balance = bal\}$	by functional specification
$\{a, a':Account \wedge a'.balance = bal \wedge \neg false\}$ $a.set(_, _)$ $\{\neg a'.balance = bal < bal\}$	by Hoare logic
from $a, a':Account \wedge a'.balance=bal \wedge (_ calls a.set(_, _))$ next $a'.balance < bal$ onlyIf false	by $IF1\text{-CLASSICAL}$
from $a, a':Account \wedge a'.balance=bal \wedge (_ calls a.set(_, _))$ next $a'.balance < bal$ onlyIf $(_ calls a.transfer(_, a'.pwd))$	by ABSURD and $IF1 \rightarrow$

demonstrates how we employ classical Hoare logic to prove that a method does not leak access to some data (in this case the pwd). Using $IF1\text{-INSIDE}$, we reason that since the return value of `set` is `void`, and `set` is prohibited from making external method calls, no call to `set` can result in an object (external or otherwise) gaining access to the pwd .

SetPwdLeak:	
$\{a:Account \wedge a':Account \wedge a.pwd == p\}$ $res=a'.set(_, _)$ $\{res != pwd\}$	by functional specification
$\{a:Account \wedge a':Account \wedge a.pwd == p \wedge \neg false\}$ $res=a'.set(_, _)$ $\{res != p\}$	by Hoare logic
from $inside(pwd) \wedge a, a':Account \wedge a.pwd=p \wedge (_ calls a'.set(_, _))$ next $\neg inside(_)$ onlyIf false	by $IF1\text{-INSIDE}$

In the same manner as **SetBalChange** and **SetPwdLeak**, we also prove **SetPwdChange**, **TransferBalChange**, **TransferPwdLeak**, and **TransferPwdChange**. We provide their statements, but omit their proofs.

5.3 Part 3: Per-Step Necessity Specifications

Part 3 builds upon the proofs of Parts 1 and 2 to construct proofs of necessary preconditions, not for single method execution, but any single execution step. That is, a proof that for *any* single step in program execution, certain changes in program state require specific preconditions.

BalEncaps:	
aEnc: $Mod_{better} \vdash a:Account \wedge a.balance=bal \Rightarrow Enc_e(a)$	by $ENC_e\text{-Obj}$
balanceEnc: $Mod_{better} \vdash a:Account \wedge a.balance=bal \Rightarrow Enc_e(a.balance)$	by $aEnc$ and $ENC\text{-FIELD}$
balEnc: $Mod_{better} \vdash a:Account \wedge a.balance=bal \Rightarrow Enc_e(bal)$	by $ENC_e\text{-INT}$
$Mod_{better} \vdash a:Account \wedge a.balance=bal \Rightarrow Enc(a.balance=bal)$	by $balanceEnc$, $balEnc$, $ENC\text{-Eq}$, and $ENC\text{-=}$

assertion $a.balance = bal$ is encapsulated – only internal object lookups are involved in the validity of that assertion, and therefore only internal computation may cause it to be invalidated.

Using similar reasoning, we prove that $a.pwd$ is encapsulated (**PwdEncaps**), and that $inside(a.pwd)$ is encapsulated (**PwdInsideEncaps**).

PwdEncaps:	
$Mod_{better} \vdash a:Account \Rightarrow Enc(a.pwd=p)$	by $ENC_e\text{-Obj}$, $ENC\text{-FIELD}$, and $ENC\text{-Eq}$
PwdInsideEncaps:	
$Mod_{better} \vdash a:Account \Rightarrow Enc(inside(a.balance))$	by $ENC\text{-INSIDE}$

5.2 Part 2: Per-Method Necessity Specifications

Part 2 proves necessary preconditions for each method in the module interface. We employ the rules from §4.2 which describe how to derive necessary preconditions from functional specifications.

SetBalChange uses a functional specification and a rule of consequence to prove that the `set` method in `Account` never modifies the balance. We then use $IF1\text{-CLASSICAL}$ and our *Necessity* logic to prove that if it ever did change (a logical absurdity), then `transfer` must have been called.

SetBalChange:	
$\{a, a':Account \wedge a'.balance=bal\}$ $a.set(_, _)$ $\{a'.balance = bal\}$	by functional specification
$\{a, a':Account \wedge a'.balance = bal \wedge \neg false\}$ $a.set(_, _)$ $\{\neg a'.balance < bal\}$	by rule of consequence
from $a, a':Account \wedge a'.balance=bal \wedge (_ calls a.set(_, _))$ next $a'.balance < bal$ onlyIf false	by $IF1\text{-CLASSICAL}$
from $a, a':Account \wedge a'.balance=bal \wedge (_ calls a.set(_, _))$ next $a'.balance < bal$ onlyIf $(_ calls a.transfer(_, a'.pwd))$	by ABSURD and $IF1 \rightarrow$

Similarly, in **SetPwdLeak** we employ functional specifications to prove that a method does not leak access to some data (in this case the pwd). Using $IF1\text{-INSIDE}$, we reason that since the return value of `set` is `void`, and `set` is prohibited from making external method calls, no call to `set` can result in an object (external or otherwise) gaining access to the pwd .

In the same manner as **SetBalChange** and **SetPwdLeak**, we also prove **SetPwdChange**, **TransferBalChange**, **TransferPwdLeak**, and **TransferPwdChange**. We provide their statements, but omit their proofs.

SetPwdChange:
 from $a, a': \text{Account} \wedge a'.\text{pwd} = p \wedge \neg \text{calls } a.\text{set}(_, _)$
 next $\neg a.\text{pwd} = p \quad \text{onlyif } \neg \text{calls } a'.\text{set}(a'.\text{pwd}, _)$ by If1-CLASSICAL

TransferBalChange:
 from $a, a': \text{Account} \wedge a'.\text{balance} = \text{bal} \wedge \neg \text{calls } a.\text{transfer}(_, _)$
 next $a'.\text{balance} < \text{bal} \quad \text{onlyif } \neg \text{calls } a'.\text{transfer}(_, a'.\text{pwd})$ by If1-CLASSICAL

TransferPwdLeak:
 from $\text{inside}(\text{pwd}) \wedge a, a': \text{Account} \wedge a.\text{pwd} = p \wedge \neg \text{calls } a'.\text{transfer}(_, _)$
 next $\neg \text{inside}(_) \quad \text{onlyif false}$ by If1-INSIDE

TransferPwdChange:
 from $a, a': \text{Account} \wedge a'.\text{pwd} = p \wedge \neg \text{calls } a.\text{transfer}(_, _)$
 next $\neg a.\text{pwd} = p \quad \text{onlyif } \neg \text{calls } a'.\text{set}(a'.\text{pwd}, _)$ by If1-CLASSICAL

BalanceChange: by BalEncaps, SetBalChange, TransferBalChange, and If1-INTERNAL
 from $a: \text{Account} \wedge a.\text{balance} = \text{bal}$
 next $a.\text{balance} < \text{bal} \quad \text{onlyif } \neg \text{calls } a.\text{transfer}(_, a.\text{pwd})$

PasswordChange: by PwdEncaps, SetPwdChange, TransferPwdChange, and If1-INTERNAL
 from $a: \text{Account} \wedge a.\text{pwd} = p$
 next $\neg a.\text{pwd} = \text{bal} \quad \text{onlyif } \neg \text{calls } a.\text{set}(a.\text{pwd}, _)$

PasswordLeak: by PwdInsideEncaps, SetPwdLeak, TransferPwdLeak, and If1-INTERNAL
 from $a: \text{Account} \wedge a.\text{pwd} = p \wedge \text{inside}(p)$
 next $\neg \text{inside}(p) \quad \text{onlyif false}$

5.4 Part 4: Emergent Necessity Specifications

Part 4 raises necessary preconditions for single execution steps proven in Part 3 to the level of an arbitrary number of execution steps in order to prove specifications of emergent behaviour. The proof of S_{robust_2} takes the following form:

- (1) If the balance of an account decreases, then by BalanceChange there must have been a call to transfer in Account with the correct password.
- (2) If there was a call where the Account's password was used, then there must have been an intermediate program state when some external object had access to the password.
- (3) Either that password was the same password as in the starting program state, or it was different:
 - (Case A) If it is the same as the initial password, then since by PasswordLeak it is impossible to leak the password, it follows that some external object must have had access to the password initially.
 - (Case B) If the password is different from the initial password, then there must have been an intermediate program state when it changed. By PasswordChange we know that this must have occurred by a call to set with the correct password. Thus, there must be a some intermediate program state where the initial password is known. From here we proceed by the same reasoning as (Case A).

6 RELATED WORK

Program specification and verification has a long and proud history [Hatcliff et al. 2012; Hoare 1969; Leavens et al. 2007; Leino 2010; Leino and Schulte 2007; Pearce and Groves 2015; Summers and

SetPwdLeak: by functional specification
 $\{a: \text{Account} \wedge a': \text{Account} \wedge a.\text{pwd} == p\}$
 $\text{res} = a'.\text{set}(_, _)$
 $\{ \text{res} != \text{pwd} \}$
 { $a: \text{Account} \wedge a': \text{Account} \wedge a.\text{pwd} == p \wedge \neg \text{false}$ } by rule of consequence
 $\text{res} = a'.\text{set}(_, _)$
 $\{ \text{res} != p \}$
 from $\text{inside}(\text{pwd}) \wedge a, a': \text{Account} \wedge a.\text{pwd} = p \wedge \neg \text{calls } a'.\text{set}(_, _)$
 next $\neg \text{inside}(_) \quad \text{onlyif false}$ by If1-INSIDE

SetPwdChange: by If1-CLASSICAL
 from $a, a': \text{Account} \wedge a'.\text{pwd} = p \wedge \neg \text{calls } a.\text{set}(_, _)$
 next $\neg a.\text{pwd} = p \quad \text{onlyif } \neg \text{calls } a'.\text{set}(a'.\text{pwd}, _)$

TransferBalChange: by If1-CLASSICAL
 from $a, a': \text{Account} \wedge a'.\text{balance} = \text{bal} \wedge \neg \text{calls } a.\text{transfer}(_, _)$
 next $a'.\text{balance} < \text{bal} \quad \text{onlyif } \neg \text{calls } a'.\text{transfer}(_, a'.\text{pwd})$

TransferPwdLeak: by If1-INSIDE
 from $\text{inside}(\text{pwd}) \wedge a, a': \text{Account} \wedge a.\text{pwd} = p \wedge \neg \text{calls } a'.\text{transfer}(_, _)$
 next $\neg \text{inside}(_) \quad \text{onlyif false}$

TransferPwdChange: by If1-CLASSICAL
 from $a, a': \text{Account} \wedge a'.\text{pwd} = p \wedge \neg \text{calls } a.\text{transfer}(_, _)$
 next $\neg a.\text{pwd} = p \quad \text{onlyif } \neg \text{calls } a'.\text{set}(a'.\text{pwd}, _)$

5.3 Part 3: Per-Step Necessity Specifications

Part 3 builds upon the proofs of Parts 1 and 2 to construct proofs of necessary preconditions, not for single method execution, but for any single execution step. That is, a proof that for any single step in program execution, changes in program state require specific preconditions.

BalanceChange: by BalEncaps, SetBalChange, TransferBalChange, and If1-INTERNAL
 from $a: \text{Account} \wedge a.\text{balance} = \text{bal}$
 next $a.\text{balance} < \text{bal} \quad \text{onlyif } \neg \text{calls } a.\text{transfer}(_, a.\text{pwd})$

PasswordChange: by PwdEncaps, SetPwdChange, TransferPwdChange, and If1-INTERNAL
 from $a: \text{Account} \wedge a.\text{pwd} = p$
 next $\neg a.\text{pwd} = \text{bal} \quad \text{onlyif } \neg \text{calls } a.\text{set}(a.\text{pwd}, _)$

PasswordLeak: by PwdInsideEncaps, SetPwdLeak, TransferPwdLeak, and If1-INTERNAL
 from $a: \text{Account} \wedge a.\text{pwd} = p \wedge \text{inside}(p)$
 next $\neg \text{inside}(p) \quad \text{onlyif false}$

5.4 Part 4: Emergent Necessity Specifications

Part 4 raises necessary preconditions for single execution steps proven in Part 3 to the level of an arbitrary number of execution steps in order to prove specifications of emergent behaviour. The proof of S_{robust_2} takes the following form:

- (1) If the balance of an account decreases, then by BalanceChange there must have been a call to transfer in Account with the correct password.
- (2) If there was a call where the Account's password was used, then there must have been an intermediate program state when some external object had access to the password.

S_{robust_2}	
from a:Account \wedge a.balance=bal to a.balance < bal onlyThrough ($_$ calls a.transfer($_,$ a.pwd))	by CHANGES and BalanceChange
from a:Account \wedge a.balance=bal to b.balance(a) < bal onlyThrough \neg inside(a.pwd)	by \rightarrow , CALLER-EXT, and CALLS-ARGS
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.balance < bal onlyThrough \neg inside(a.pwd) \wedge (a.pwd=p \vee a.pwd \neq p)	by \rightarrow and EXCLUDED MID- DLE
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.balance < bal onlyThrough (\neg inside(a.pwd) \wedge a.pwd=p) \vee (\neg inside(a.pwd) \wedge a.pwd \neq p)	by \rightarrow
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.balance < bal onlyThrough \neg inside(p) \vee a.pwd \neq p	by \rightarrow
Case A (\neginside(p)):	
from a:Account \wedge a.balance=bal \wedge a.pwd=p to \neg inside(p) onlyIf inside(p) \vee \neg inside(p)	by If- \rightarrow and EXCLUDED MIDDLE
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to \neg inside(p) onlyIf \neg inside(p)	by VE and PasswordLeak
Case B (a.pwd \neq p):	
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to a.pwd \neq p onlyThrough ($_$ calls a.set(p, $_$))	by CHANGES and PASS- WORDCHANGE
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.pwd \neq p onlyThrough \neg inside(p)	by VE and PasswordLeak
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.pwd \neq p onlyIf \neg inside(p)	by Case A and TRANS
from a:Account \wedge a.balance=bal \wedge a.pwd=p to b.balance(a) < bal onlyIf \neg inside(p)	by Case A, Case B, If- \vee I ₂ , and If- \rightarrow

[Drossopoulou 2010]. These verification techniques assume a closed system, where modules can be trusted to coöperate — Design by Contract [Meyer 1992] explicitly rejects “defensive programming” with an “absolute rule” that calling a method in violation of its precondition is always a bug.

Open systems, by definition, must interact with untrusted code: they cannot rely on callers’ obeying method preconditions. [Miller 2006; Miller et al. 2013] define the necessary approach as *defensive consistency*: “An object is *defensively consistent* when it can defend its own invariants and provide correct service to its well behaved clients, despite arbitrary or malicious misbehaviour by its other clients.” [Murray 2010] made the first attempt to formalise defensive consistency and correctness in a programming language context. Murray’s model was rooted in counterfactual causation [Lewis 1973]: an object is *defensively consistent* when the addition of untrustworthy clients cannot cause well-behaved clients to be given incorrect service. Murray formalised defensive consistency abstractly, without a specification language for describing effects.

The security community has developed a similar notion of “robust safety” that originated in type systems for process calculi, ensuring protocols behave correctly in the presence of “an arbitrary hostile opponent” [Bugliesi et al. 2011; Gordon and Jeffrey 2001]. More recent work has applied robust safety in the context of programming languages. For example, [Swasey et al. 2017] present a logic for object capability patterns, drawing on verification techniques for security and information flow. They prove a robust safety property that ensures interface objects (“low values”) will never leak internal implementations (“high values”) to arbitrary attackers. Similarly, [Schaefer et al. 2018] have added support for information-flow security using refinement to ensure correctness (in this case confidentiality) by construction.

(3) Either that password was the same password as in the starting program state, or it was different:

(Case A) If it is the same as the initial password, then since by PasswordLeak it is impossible to leak the password, it follows that some external object must have had access to the password initially.

(Case B) If the password is different from the initial password, then there must have been an intermediate program state when it changed. By PasswordChange we know that this must have occurred by a call to set with the correct password. Thus, there must be a some intermediate program state where the initial password is known. From here we proceed by the same reasoning as (Case A).

S_{robust_2}	
from a:Account \wedge a.balance=bal to a.balance < bal onlyThrough ($_$ calls a.transfer($_,$ a.pwd))	by CHANGES and BalanceChange
from a:Account \wedge a.balance=bal to b.balance(a) < bal onlyThrough \neg inside(a.pwd)	by \rightarrow , CALLER-EXT, and CALLS-ARGS
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.balance < bal onlyThrough \neg inside(a.pwd) \wedge (a.pwd=p \vee a.pwd \neq p)	by \rightarrow and EXCLUDED MID- DLE
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.balance < bal onlyThrough (\neg inside(a.pwd) \wedge a.pwd=p) \vee (\neg inside(a.pwd) \wedge a.pwd \neq p)	by \rightarrow
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.balance < bal onlyThrough \neg inside(p) \vee a.pwd \neq p	by \rightarrow
Case A (\neginside(p)):	
from a:Account \wedge a.balance=bal \wedge a.pwd=p to \neg inside(p) onlyIf inside(p) \vee \neg inside(p)	by If- \rightarrow and EXCLUDED MIDDLE
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to \neg inside(p) onlyIf \neg inside(p)	by VE and PasswordLeak
Case B (a.pwd \neq p):	
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to a.pwd \neq p onlyThrough ($_$ calls a.set(p, $_$))	by CHANGES and PASS- WORDCHANGE
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.pwd \neq p onlyThrough \neg inside(p)	by VE and PasswordLeak
from a:Account \wedge a.balance=bal \wedge a.pwd=p to a.pwd \neq p onlyIf \neg inside(p)	by Case A and TRANS
from a:Account \wedge a.balance=bal \wedge a.pwd=p to b.balance(a) < bal onlyIf \neg inside(p)	by Case A, Case B, If- \vee I ₂ , and If- \rightarrow

6 RELATED WORK

Program specification and verification has a long and proud history [Hatcliff et al. 2012; Hoare 1969; Leavens et al. 2007; Leino 2010; Leino and Schulte 2007; Pearce and Groves 2015; Summers and Drossopoulou 2010]. These verification techniques assume a closed system, where modules can be trusted to coöperate — Design by Contract [Meyer 1992] explicitly rejects “defensive programming” with an “absolute rule” that calling a method in violation of its precondition is always a bug.

Open systems, by definition, must interact with untrusted code: they cannot rely on callers’ obeying method preconditions. [Miller 2006; Miller et al. 2013] define the necessary approach as *defensive consistency*: “An object is *defensively consistent* when it can defend its own invariants and provide correct service to its well behaved clients, despite arbitrary or malicious misbehaviour by its other clients.” [Murray 2010] made the first attempt to formalise defensive consistency and

[Devriese et al. 2016] have deployed powerful theoretical techniques to address similar problems to *Necessity*. They show how step-indexing, Kripke worlds, and representing objects as state machines with public and private transitions can be used to reason about object capabilities. They have demonstrated solutions to a range of exemplar problems, including the DOM wrapper (replicated in §3.4.1) and a mashup application. Their distinction between public and private transitions is similar to our distinction between internal and external objects.

Necessity differs from Swasey, Schaefer’s, and Devriese’s work in a number of ways: They are primarily concerned with mechanisms that ensure encapsulation (aka confinement) while we abstract away from any mechanism. They use powerful mathematical techniques which the users need to understand in order to write their specifications, while *Necessity* users only need to understand first order logic. Finally, none of these systems offer the kinds of necessity assertions addressing control flow, provenance, and permission that are at the core of *Necessity*’s approach.

By enforcing encapsulation, all these approaches are reminiscent of techniques such as ownership types [Clarke et al. 1998; Noble et al. 1998], which also can protect internal implementation objects behind encapsulation boundaries. [Banerjee and Naumann 2005a,b] demonstrated that by ensuring confinement, ownership systems can enforce representation independence. *Necessity* relies on an implicit form of ownership types [Vitek and Bokowski 1999], where inside objects are encapsulated behind a boundary consisting of all the internal objects that are accessible outside their defining module [Noble et al. 2003]. Compare *Necessity*’s definition of inside — all references to o are from objects x that are within M (here internal to M): $\forall x. [\langle x \text{ access } o \rangle \Rightarrow \langle x \text{ internal} \rangle]$ with the containment invariant from Clarke et al. [2001] — all references to o are from objects x whose representation is within ($<$): o ’s owner: $(\forall x. [\langle x \text{ access } o \rangle \Rightarrow \text{rep}(x) <: \text{owner}(o)])$.

In early work, [Drossopoulou and Noble 2014] sketched a specification language to specify six correctness policies from [Miller 2006]. They also sketched how a trust-sensitive example (escrow) could be verified in an open world [Drossopoulou et al. 2015]. More recently, [Drossopoulou et al. 2020b] presents the *Chainmail* language for “holistic specifications” in open world systems. Like *Necessity*, *Chainmail* is able to express specifications of *permission*, *provenance*, and *control*; *Chainmail* also includes *spatial* assertions and a richer set of temporal operators, but no proof system. *Necessity*’s restrictions mean we can provide the proof system that *Chainmail* lacks.

The recent VerX tool is able to verify a range of specifications for Solidity contracts automatically [Permenev et al. 2020a]. VerX includes temporal operators, predicates that model the current invocation on a contract (similar to *Necessity*’s “calls”), access to variables, but has no analogues to *Necessity*’s permission or provenance assertions. Unlike *Necessity*, VerX includes a practical tool that has been used to verify a hundred properties across case studies of twelve Solidity contracts. Also unlike *Necessity*, VerX’s own correctness has not been formalised or mechanistically proved.

Like *Necessity*, VerX [Permenev et al. 2020a] and *Chainmail* [Drossopoulou et al. 2020b] also work on problem-specific guarantees. Both approaches can express necessary conditions like S_{robust_1} using temporal logic operators and implication. For example, S_{robust_1} could be written:

$$a:\text{Account} \wedge a.\text{balance} == \text{bal} \wedge \langle \text{next } a.\text{balance} < \text{bal} \rangle \\ \rightarrow \exists o, a'. \langle o \text{ calls } a.\text{transfer}(a', a.\text{password}) \rangle$$

However, to express S_{robust_2} , one also needs capability operators which talk about provenance and permission. VerX does not support capability operators, and thus cannot express S_{robust_2} , while *Chainmail* does support capability operators, and can express S_{robust_2} .

Moreover, temporal operators in VerX and *Chainmail* are first class, i.e. may appear in any assertions and form new assertions. This makes VerX and *Chainmail* very expressive, and allows specifications which talk about any number of points in time. However, this expressivity comes at the cost of making it very difficult to develop a logic to prove adherence to such specifications.

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O'Hearn and Raad et al. developed Incorrectness logics to reason about the presence of bugs, based on a Reverse Hoare Logic [de Vries and Koutavas 2011]. Classical Hoare triples $\{P\} C \{Q\}$ express that starting at states satisfying P and executing C is sufficient to reach only states that satisfy Q (soundness), while incorrectness triples $[P_i] C_i [Q_i]$ express that starting at states satisfying P_i and executing C_i is sufficient to reach all states that satisfy Q_i and possibly some more (completeness). From our perspective, classical Hoare logics and Incorrectness logics are both about sufficiency, whereas here we are concerned with *Necessity*.

In practical open systems, especially web browsers, defensive consistency / robust safety is typically supported by sandboxing: dynamically separating trusted and untrusted code, rather than relying on static verification and proof. Google's Caja [Miller et al. 2008], for example, uses proxies and wrappers to sandbox web pages. Sandboxing has been validated formally: [Maffeis et al. 2010] develop a model of JavaScript and show it prevents trusted dependencies on untrusted code. [Dimoulas et al. 2014] use dynamic monitoring from function contracts to control objects flowing around programs; [Moore et al. 2016] extends this to use fluid environments to bind callers to contracts. [Sammler et al. 2019] develop λ_{sandbox} , a low-level language with built in sandboxing, separating trusted and untrusted memory. λ_{sandbox} features a type system, and Sammler et al. show that sandboxing achieves robust safety. Sammler et al. address a somewhat different problem domain than *Necessity* does, low-level systems programming where there is a possibility of forging references to locations in memory. Such a domain would subvert *Necessity*, in particular a reference to x could always be guessed thus the assertion `inside(x)` would no longer be encapsulated.

Callbacks. *Necessity* does not –yet– support calls of external methods from within internal modules. While this is a limitation, it is common in the related literature. For example, VerX [Permenev et al. 2020b] work on effectively call-back free contracts, while [Grossman et al. 2017] and [Albert et al. 2020] drastically restrict the effect of a callback on a contract. In further work we are planning to incorporate callbacks by splitting internal methods at the point where a call to an external method appears. This would be an adaptation of Bräm et al.'s approach, who split methods into the call-free subparts, and use the transitive closure of the effects of all functions from a module to overapproximate the effect of an external call. One useful simplification was proposed by Permenev et al. [2020b]: in “effectively callback free” methods, meaning that we could include callbacks while also only requiring at most one functional specification per-method.

7 CONCLUSION

This paper presents *Necessity*, a specification language for a program's emergent behaviour. *Necessity* specifications constrain when effects can happen in some future state (“onlyIf”), in the immediately following state (“next”), or on an execution path (“onlyThrough”).

We have developed a proof system to prove that modules meet their specifications. Our proof system exploits the pre and postconditions of functional specifications to infer per method *Necessity* specifications, generalises those to cover any single execution step, and then combines them to capture a program's emergent behaviour.

We have proved our system sound, and used it to prove a bank account example correct: the Coq mechanisation is detailed in the appendices and available as an artifact.

In future work we want to consider more than one external module – c.f. §2.4, and expand a Hoare logic so as to make use of *Necessity* specifications, and reason about calls into unknown code – c.f. §2.3.1. We want to work on supporting callbacks. We want to develop a logic for encapsulation rather than rely on a type system. Finally we want to develop logics about reasoning about risk and trust [Drossopoulou et al. 2015].

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However, to express $S_{\text{robust_2}}$, one also needs capability operators which talk about provenance and permission. VerX does not support capability operators, and thus cannot express $S_{\text{robust_2}}$, while *Chainmail* does support capability operators, and can express $S_{\text{robust_2}}$.

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A Tool

We introduce Tool, a simple, typed, class-based, object-oriented language that underlies the *Necessity* specifications introduced in this paper. Tool includes ghost fields, recursive definitions that may only be used in the specification language. To reduce the complexity of our formal models, Tool lacks many common languages features, omitting static fields and methods, interfaces, inheritance, subsumption, exceptions, and control flow. These features are well-understood: their presence (or absence) would not change the results we claim nor the structures of the proofs of those results. Similarly, while Loo is typed, we don't present or mechanise its type system. Our results and proofs rely only upon type soundness — in fact, we only need that an expression of type T (where T is a class C declared in module M) will evaluate to an instance of some class from M , with the same confinement status as C . Featherweight Java extended with modules and assignment will more than suffice [Igarashi et al. 2001].

A.1 Syntax

The syntax of Tool is given in Fig. 8. Tool modules (M) map class names (C) to class definitions (*ClassDef*). A class definition consists of an optional annotation `confined`, a list of field definitions, ghost field definitions, and method definitions. Fields, ghost fields, and methods all have types: types are classes. Ghost fields may be optionally annotated as `internal`, requiring the argument to have an internal type, and the body of the ghost field to only contain references to internal objects. This is enforced by the limited type system of Tool. A program state (σ) is represented as a heap (χ), stack (ψ) pair, where a heap is a map from addresses (α) to objects (o), and a stack is a non-empty list of frames (ϕ). A frame consists of a local variable map and a continuation (c) that represents the statements that are yet to be executed (s), or a hole waiting to be filled by a method return in the frame above ($x := \bullet; s$). A statement is either a field read ($x := y.f$), a field write ($x.f := y$), a method call ($x := y.m(\bar{z})$), a constructor call (`new C(\bar{x})`), a method return statement (`return x`), or a sequence of statements ($s; s$).

Tool also includes syntax for expressions e that may be used in writing specifications or the definition of ghost fields.

A.2 Semantics

Tool is a simple object oriented language, and the operational semantics (given in Fig. 9 and discussed later) do not introduce any novel or surprising features. The operational semantics make use of several helper definitions that we define here.

We provide a definition of reference interpretation in Definition A.1

Definition A.1. For a program state $\sigma = (\chi, \phi : \psi)$, we provide the following function definitions:

- $\lfloor x \rfloor_\sigma \triangleq \phi.(\text{local})(x)$
- $\lfloor \alpha.f \rfloor_\sigma \triangleq \chi(\alpha).(\text{flds})(f)$
- $\lfloor x.f \rfloor_\sigma \triangleq \lfloor \alpha.f \rfloor_\sigma$ where $\lfloor x \rfloor_\sigma = \alpha$

That is, a variable x , or a field access on a variable $x.f$ has an interpretation within a program state of value v if x maps to v in the local variable map, or the field f of the object identified by x points to v .

Definition A.2 defines the class lookup function an object identified by variable x .

Definition A.2 (Class Lookup). For program state $\sigma = (\chi, \phi : \psi)$, class lookup is defined as

$$\text{classOf}(\sigma, x) \triangleq \chi(\lfloor x \rfloor_\sigma).(\text{class})$$

Definition A.3 defines the method lookup function for a method call m on an object of class C .

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Definition A.3 defines the method lookup function for a method call m on an object of class C .

1422	x, y, z	Variable
1423	C, D	Class Id.
1424	$T ::= C$	Type
1425	f	Field Id.
1426	g	Ghost Field Id.
1427	m	Method Id.
1428	α	Address Id.
1429	$i \in \mathbb{Z}$	Integer
1430	$v ::= \alpha \mid i \mid \text{true} \mid \text{false} \mid \text{null}$	Value
1431	$e ::= x \mid v \mid e + e \mid e = e \mid e < e$ $\mid \text{if } e \text{ then } e \text{ else } e \mid e.f \mid e.g(e)$	Expression
1432	$o ::= \{\text{class} := C; \text{flds} := \bar{f} \mapsto v\}$	Object
1433	$s ::= x := y.f \mid x.f := y \mid x := y.m(\bar{z})$ $\mid \text{new } C(\bar{x}) \mid \text{return } x \mid s; s$	Statement
1434	$c ::= s \mid x := \bullet; s$	Continuation
1435	$\chi ::= \bar{\alpha} \mapsto \bar{o}$	Heap
1436	$\phi ::= \{\text{local} := \bar{x} \mapsto \bar{v}; \text{contn} := c\}$	Frame
1437	$\psi ::= \phi \mid \phi : \psi$	Stack
1438	$\sigma ::= (\text{heap} := \chi, \text{stack} := \psi)$	Program Config.
1439	$\text{meth} ::= \text{method } m(\bar{x} : \bar{T})\{s\}$	Method Def.
1440	$\text{fld} ::= \text{field } f : T$	Field Def.
1441	$\text{gfld} ::= \text{ghost } g(x : T)\{e\} : T \mid \text{ghost intrnl } g(x : T)\{e\} : T$	Ghost Field Def.
1442	$\text{An} ::= \text{confined}$	Class Annotation
1443	$\text{CDef} ::= [\text{An}] \text{ class } C \{ \text{constr} := (\bar{x} : \bar{T})\{s\}; \text{flds} := \bar{f}ld; \text{gflds} := \bar{g}fld; \text{meths} := \bar{m}th \}$	Class Def.
1444	$\text{Mdl} ::= \bar{C} \mapsto \text{ClassDef}$	Module Def.

Fig. 8. Tool Syntax

Definition A.3 (Method Lookup). For module M , class C , and method name m , method lookup is defined as

$$\text{Meth}(M, C, m) \triangleq M(C).\text{meths}(m)$$

Fig. 9 gives the operational semantics of Tool. Program state σ_1 reduces to σ_2 in the context of module M if $M, \sigma_1 \leadsto \sigma_2$. The semantics in Fig. 9 are unsurprising, but it is notable that reads (READ) and writes (WRITE) are restricted to the class that the field belongs to.

While the small-step operational semantics of Tool is given in Fig. 9, specification satisfaction is defined over an abstracted notion of the operational semantics that models the open world, called *external states semantics*. That is, execution occurs in the context of not just an internal, trusted module, but an external, untrusted module. We borrow the definition of external states semantics from Drossopoulou et al., along with the related definition of module linking, given in Definition A.4.

Definition A.4. For all modules M and M' , if the domains of M and M' are disjoint, we define the module linking function as $M \circ M' \triangleq M \cup M'$.

That is, given an internal, module M , and an external module M' , we take their linking as the union of the two if their domains are disjoint.

An *Initial* program state contains a single frame with a single local variable `this` pointing to a single object in the heap of class `Object`, and a continuation.

1471	x, y, z	Variable
1472	C, D	Class Id.
1473	$T ::= C$	Type
1474	f	Field Id.
1475	g	Ghost Field Id.
1476	m	Method Id.
1477	α	Address Id.
1478	$i \in \mathbb{Z}$	Integer
1479	$v ::= \alpha \mid i \mid \text{true} \mid \text{false} \mid \text{null}$	Value
1480	$e ::= x \mid v \mid e + e \mid e = e \mid e < e$ $\mid \text{if } e \text{ then } e \text{ else } e \mid e.f \mid e.g(e)$	Expression
1481	$o ::= \{\text{class} := C; \text{flds} := \bar{f} \mapsto v\}$	Object
1482	$s ::= x := y.f \mid x.f := y \mid x := y.m(\bar{z})$ $\mid \text{new } C(\bar{x}) \mid \text{return } x \mid s; s$	Statement
1483	$c ::= s \mid x := \bullet; s$	Continuation
1484	$\chi ::= \bar{\alpha} \mapsto \bar{o}$	Heap
1485	$\phi ::= \{\text{local} := \bar{x} \mapsto \bar{v}; \text{contn} := c\}$	Frame
1486	$\psi ::= \phi \mid \phi : \psi$	Stack
1487	$\sigma ::= (\text{heap} := \chi, \text{stack} := \psi)$	Program Config.
1488	$\text{meth} ::= \text{method } m(\bar{x} : \bar{T})\{s\}$	Method Def.
1489	$\text{fld} ::= \text{field } f : T$	Field Def.
1490	$\text{gfld} ::= \text{ghost } g(x : T)\{e\} : T \mid \text{ghost intrnl } g(x : T)\{e\} : T$	Ghost Field Def.
1491	$\text{An} ::= \text{confined}$	Class Annotation
1492	$\text{CDef} ::= [\text{An}] \text{ class } C \{ \text{constr} := (\bar{x} : \bar{T})\{s\}; \text{flds} := \bar{f}ld; \text{gflds} := \bar{g}fld; \text{meths} := \bar{m}th \}$	Class Def.
1493	$\text{Mdl} ::= \bar{C} \mapsto \text{ClassDef}$	Module Def.

Fig. 9. Tool Syntax

Definition A.3 (Method Lookup). For module M , class C , and method name m , method lookup is defined as

$$\text{Meth}(M, C, m) \triangleq M(C).\text{meths}(m)$$

Fig. 10 gives the operational semantics of Tool. Program state σ_1 reduces to σ_2 in the context of module M if $M, \sigma_1 \leadsto \sigma_2$. The semantics in Fig. 10 are unsurprising, but it is notable that reads (READ) and writes (WRITE) are restricted to the class that the field belongs to.

While the small-step operational semantics of Tool is given in Fig. 10, specification satisfaction is defined over an abstracted notion of the operational semantics that models the open world, called *external states semantics*. That is, execution occurs in the context of not just an internal, trusted module, but an external, untrusted module. We borrow the definition of external states semantics from Drossopoulou et al., along with the related definition of module linking, given in Definition A.4.

Definition A.4. For all modules M and M' , if the domains of M and M' are disjoint, we define the module linking function as $M \circ M' \triangleq M \cup M'$.

That is, given an internal, module M , and an external module M' , we take their linking as the union of the two if their domains are disjoint.

An *Initial* program state contains a single frame with a single local variable `this` pointing to a single object in the heap of class `Object`, and a continuation.

$$\begin{array}{c}
\frac{\sigma_1 = (\chi, \phi_1 : \psi) \quad \sigma_2 = (\chi, \phi_2 : \phi'_1 : \psi) \quad \phi_1.(\text{contn}) = (x := y.m(\bar{z}); s) \quad \phi'_1 = \phi_1[\text{contn} := (x := \bullet; s)] \quad \text{Meth}(M, \text{classOf}(\sigma_1, x), m) = m(p : T)\{\text{body}\} \quad \phi_2 = \{\text{local} := ([\text{this} \mapsto [x]_{\sigma_1}][p_i \mapsto [z_i]_{\sigma_1}], \text{contn} := \text{body})\}}{M, \sigma_1 \leadsto \sigma_2} \text{ (CALL)} \\
\\
\frac{\sigma_1 = (\chi, \phi_1 : \psi) \quad \sigma_2 = (\chi, \phi_2 : \psi) \quad \phi_1.(\text{contn}) = (x := y.f; s) \quad [x.f]_{\sigma_1} = v \quad \phi_2 = \{\text{local} := \phi_1.(\text{local}), \text{contn} := s\} \quad \text{classOf}(\sigma_1, \text{this}) = \text{classOf}(\sigma_1, y)}{M, \sigma_1 \leadsto \sigma_2} \text{ (READ)} \\
\\
\frac{\sigma_1 = (\chi_1, \phi_1 : \psi) \quad \sigma_2 = (\chi_2, \phi_2 : \psi) \quad \phi_1.(\text{contn}) = (x.f := y; s) \quad [y]_{\sigma_1} = v \quad \phi_2 = \{\text{local} := \phi_1.(\text{local}), \text{contn} := s\} \quad \chi_2 = \chi_1[[x]_{\sigma_1}.f \mapsto v] \quad \text{classOf}(\sigma_1, \text{this}) = \text{classOf}(\sigma_2, x)}{M, \sigma_1 \leadsto \sigma_2} \text{ (WRITE)} \\
\\
\frac{\sigma_1 = (\chi, \phi : \psi) \quad \phi.(\text{contn}) = (x := \text{new } C(\bar{z}); s) \quad M(C).(\text{constr}) = (p : T)\{s'\} \quad \phi' = \{\text{local} := [\text{this} \mapsto \alpha], [p_i \mapsto [z_i]_{\sigma_1}], \text{contn} := s'\} \quad \sigma_2 = (\chi[\alpha \mapsto \{\text{class} := C, \text{flds} := \bar{f} \mapsto \text{null}], \phi' : \phi[\text{contn} := (x := \bullet; s)] : \psi)}{M, \sigma_1 \leadsto \sigma_2} \text{ (NEW)} \\
\\
\frac{\sigma_1 = (\chi, \phi_1 : \phi_2 : \psi) \quad \phi_1.(\text{contn}) = (\text{return } x; s) \text{ or } \phi_1.(\text{contn}) = (\text{return } x) \quad \phi_2.(\text{contn}) = (y := \bullet; s) \quad \sigma_2 = (\chi, \phi_2[y \mapsto [x]_{\sigma_1}] : \psi)}{M, \sigma_1 \leadsto \sigma_2} \text{ (RETURN)}
\end{array}$$

Fig. 9. Tool operational Semantics

$$\begin{array}{c}
M, \sigma, v \hookrightarrow v \quad \text{(E-VAL)} \quad M, \sigma, x \hookrightarrow [x]_{\sigma} \quad \text{(E-VAR)} \\
\\
\frac{M, \sigma, e_1 \hookrightarrow i_1 \quad M, \sigma, e_2 \hookrightarrow i_2 \quad i_1 + i_2 = i}{M, \sigma, e_1 + e_2 \hookrightarrow i} \text{ (E-ADD)} \quad \frac{M, \sigma, e_1 \hookrightarrow v \quad M, \sigma, e_2 \hookrightarrow v}{M, \sigma, e_1 = e_2 \hookrightarrow \text{true}} \text{ (E-EQ}_1\text{)} \\
\\
\frac{M, \sigma, e_1 \hookrightarrow v_1 \quad M, \sigma, e_2 \hookrightarrow v_2 \quad v_1 \neq v_2}{M, \sigma, e_1 = e_2 \hookrightarrow \text{false}} \text{ (E-EQ}_2\text{)} \quad \frac{M, \sigma, e \hookrightarrow \text{true} \quad M, \sigma, e_1 \hookrightarrow v}{M, \sigma, e \hookrightarrow v} \text{ (E-IF}_1\text{)} \\
\\
\frac{M, \sigma, e \hookrightarrow \text{false} \quad M, \sigma, e_2 \hookrightarrow v}{M, \sigma, e \hookrightarrow v} \text{ (E-IF}_2\text{)} \quad \frac{M, \sigma, e \hookrightarrow \alpha}{M, \sigma, e.f \hookrightarrow [\alpha.f]_{\sigma}} \text{ (E-FIELD)} \\
\\
\frac{M, \sigma, e_1 \hookrightarrow \alpha \quad M, \sigma, e_2 \hookrightarrow v' \quad \text{ghost } g(x : T)\{e\} : T' \in M(\text{classOf}(\sigma, \alpha)).(\text{gflds}) \quad M, \sigma, [v'/x]e \hookrightarrow v}{M, \sigma, e_1.g(e_2) \hookrightarrow v} \text{ (E-GHOST)}
\end{array}$$

Fig. 10. Tool expression evaluation

Definition A.5 (Initial Program State). A program state σ is said to be an initial state ($\text{Initial}(\sigma)$) if and only if

- $\sigma.\text{heap} = [\alpha \mapsto \{\text{class} := \text{Object}; \text{flds} := \emptyset\}]$ and
- $\sigma.\text{stack} = \{\text{local} := [\text{this} \mapsto \alpha]; \text{contn} := s\}$

for some address α and some statement s .

Finally, we provide a semantics for expression evaluation is given in Fig. 10. That is, given a module M and a program state σ , expression e evaluates to v if $M, \sigma, e \hookrightarrow v$. Note, the evaluation of expressions is separate from the operational semantics of Tool, and thus there is no restriction on field access.

$$\begin{array}{c}
\sigma_1 = (\chi, \phi_1 : \psi) \quad \sigma_2 = (\chi, \phi_2 : \phi'_1 : \psi) \quad \phi_1.(\text{contn}) = (x := y.m(\bar{z}); s) \quad \phi'_1 = \phi_1[\text{contn} := (x := \bullet; s)] \quad \text{Meth}(M, \text{classOf}(\sigma_1, x), m) = m(p : T)\{\text{body}\} \quad \phi_2 = \{\text{local} := ([\text{this} \mapsto [x]_{\sigma_1}][p_i \mapsto [z_i]_{\sigma_1}], \text{contn} := \text{body})\}}{M, \sigma_1 \leadsto \sigma_2} \text{ (CALL)} \\
\\
\frac{\sigma_1 = (\chi, \phi_1 : \psi) \quad \sigma_2 = (\chi, \phi_2 : \psi) \quad \phi_1.(\text{contn}) = (x := y.f; s) \quad [x.f]_{\sigma_1} = v \quad \phi_2 = \{\text{local} := \phi_1.(\text{local}), \text{contn} := s\} \quad \text{classOf}(\sigma_1, \text{this}) = \text{classOf}(\sigma_1, y)}{M, \sigma_1 \leadsto \sigma_2} \text{ (READ)} \\
\\
\frac{\sigma_1 = (\chi_1, \phi_1 : \psi) \quad \sigma_2 = (\chi_2, \phi_2 : \psi) \quad \phi_1.(\text{contn}) = (x.f := y; s) \quad [y]_{\sigma_1} = v \quad \phi_2 = \{\text{local} := \phi_1.(\text{local}), \text{contn} := s\} \quad \chi_2 = \chi_1[[x]_{\sigma_1}.f \mapsto v] \quad \text{classOf}(\sigma_1, \text{this}) = \text{classOf}(\sigma_2, x)}{M, \sigma_1 \leadsto \sigma_2} \text{ (WRITE)} \\
\\
\frac{\sigma_1 = (\chi, \phi : \psi) \quad \phi.(\text{contn}) = (x := \text{new } C(\bar{z}); s) \quad M(C).(\text{constr}) = (p : T)\{s'\} \quad \phi' = \{\text{local} := [\text{this} \mapsto \alpha], [p_i \mapsto [z_i]_{\sigma_1}], \text{contn} := s'\} \quad \sigma_2 = (\chi[\alpha \mapsto \{\text{class} := C, \text{flds} := \bar{f} \mapsto \text{null}], \phi' : \phi[\text{contn} := (x := \bullet; s)] : \psi)}{M, \sigma_1 \leadsto \sigma_2} \text{ (NEW)} \\
\\
\frac{\sigma_1 = (\chi, \phi_1 : \phi_2 : \psi) \quad \phi_1.(\text{contn}) = (\text{return } x; s) \text{ or } \phi_1.(\text{contn}) = (\text{return } x) \quad \phi_2.(\text{contn}) = (y := \bullet; s) \quad \sigma_2 = (\chi, \phi_2[y \mapsto [x]_{\sigma_1}] : \psi)}{M, \sigma_1 \leadsto \sigma_2} \text{ (RETURN)}
\end{array}$$

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$$\begin{array}{c}
M, \sigma, v \hookrightarrow v \quad \text{(E-VAL)} \quad M, \sigma, x \hookrightarrow [x]_{\sigma} \quad \text{(E-VAR)} \\
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\\
\frac{M, \sigma, e_1 \hookrightarrow v_1 \quad M, \sigma, e_2 \hookrightarrow v_2 \quad v_1 \neq v_2}{M, \sigma, e_1 = e_2 \hookrightarrow \text{false}} \text{ (E-EQ}_2\text{)} \quad \frac{M, \sigma, e \hookrightarrow \text{true} \quad M, \sigma, e_1 \hookrightarrow v}{M, \sigma, e \hookrightarrow v} \text{ (E-IF}_1\text{)} \\
\\
\frac{M, \sigma, e \hookrightarrow \text{false} \quad M, \sigma, e_2 \hookrightarrow v}{M, \sigma, e \hookrightarrow v} \text{ (E-IF}_2\text{)} \quad \frac{M, \sigma, e \hookrightarrow \alpha}{M, \sigma, e.f \hookrightarrow [\alpha.f]_{\sigma}} \text{ (E-FIELD)} \\
\\
\frac{M, \sigma, e_1 \hookrightarrow \alpha \quad M, \sigma, e_2 \hookrightarrow v' \quad \text{ghost } g(x : T)\{e\} : T' \in M(\text{classOf}(\sigma, \alpha)).(\text{gflds}) \quad M, \sigma, [v'/x]e \hookrightarrow v}{M, \sigma, e_1.g(e_2) \hookrightarrow v} \text{ (E-GHOST)}
\end{array}$$

Fig. 11. Tool expression evaluation

Definition A.5 (Initial Program State). A program state σ is said to be an initial state ($\text{Initial}(\sigma)$) if and only if

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- $\sigma.\text{stack} = \{\text{local} := [\text{this} \mapsto \alpha]; \text{contn} := s\}$

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Finally, we provide a semantics for expression evaluation is given in Fig. 11. That is, given a module M and a program state σ , expression e evaluates to v if $M, \sigma, e \hookrightarrow v$. Note, the evaluation of expressions is separate from the operational semantics of Tool, and thus there is no restriction on field access.

B ENCAPSULATION

Assertion encapsulation (Definition 4.1) is critical to our approach. Assertion encapsulation ensures that a change in satisfaction of an assertion can only depend on computation *internal* to the module in which the assertion is encapsulated — this is related to the footprint of an assertion [Banerjee and Naumann 2005b; Leino and Müller 2004]. If the footprint of an assertion is contained within a module, then that assertion is encapsulated, however there are assertions that are encapsulated by a module whose footprint is not contained within the module. Specifically, the assertion `insidex` is not contained within an module M since its due to the universal quantification contained withing `insidex`, the footprint consists of portions of the heap that are external to M . `insidex` is encapsulated by M since if only objects that derive from M have access to x , it follows that a method call on M is required to gain access to x . Necessity Logic itself does not depend on the details of the encapsulation scheme — only that we can determine whether an assertion is encapsulated within a particular part of the program. For reasons of simplicity, we have adopted an encapsulation model for Tool based on Vitek and Bokowski's *Confined Types* [1999] (and we rely on their proof). Confined types partition the objects accessible to code within a module, based on those objects' defining classes and modules:

- instances of non-confined classes constitute their defining module's encapsulation boundary [Noble et al. 2003], and may be accessed anywhere.
- instances of confined classes are encapsulated inside their defining module.
- instances of confined classes defined in *other* modules are not accessible elsewhere
- instances of non-confined classes defined in *other* modules are visible, however methods may only be invoked on such objects when the confinement system guarantees the particular instance is only accessible inside *this* module.

Tool's Confined Types rely on three syntactic restrictions to enforce this encapsulation model:

- confined class declarations must be annotated.
- confined objects may not be returned by methods of non-confined classes.
- Ghost fields may be annotated as `intrnl`; if so, they must only refer to objects inside their defining module — i.e. either defined directly in that module, or instances of non-confined classes defined in *other* modules where those particular instances are only ever accessed within the defining module.

We define internally evaluated expressions ($Enc_e(_)$) whose evaluation only inspects internal objects or primitives (i.e. integers or booleans).

Definition B.1 (Internally Evaluated Expressions). For all modules M , assertions A , and expressions e , $M \models A \Rightarrow Enc_e(e)$ if and only if for all heaps χ , stacks ψ , and frames ϕ such that $M, (\chi, \phi : \psi) \models A$, we have for all values v , such that $M, (\chi, \phi : \psi), e \hookrightarrow v$ then $M, (\chi', \phi' : \psi), e \hookrightarrow v$, where

- χ' is the internal portion of χ , i.e.
 $\chi' = \{\alpha \mapsto o \mid \alpha \mapsto o \in \chi \wedge o.(cname) \in M\}$ and
- $\phi'.(local)$ is the internal portion of the $\phi.(local)$ i.e.
 $\phi' = \{x \mapsto v \mid x \mapsto v \in \chi \wedge (v \in \mathbb{Z} \vee v = \text{true} \vee v = \text{false}) \vee (\exists \alpha, v = \alpha \wedge classOff(\chi, \phi : \psi), \alpha) \in M\}$

The encapsulation proof system consists of two relations

- Purely internal expressions: $M \vdash A \Rightarrow Enc_e(e)$ and
- Assertion encapsulation: $M \vdash A \Rightarrow Enc(A')$

Fig. 11 gives proof rules for an expression comprising purely internal objects. Primitives are Enc_e ($ENC_e\text{-INT}$, $ENC_e\text{-NULL}$, $ENC_e\text{-TRUE}$, and $ENC_e\text{-FALSE}$). Addresses of internal objects are Enc_e

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Tool's Confined Types rely on three syntactic restrictions to enforce this encapsulation model:

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- Ghost fields may be annotated as `intrnl`; if so, they must only refer to objects inside their defining module — i.e. either defined directly in that module, or instances of non-confined classes defined in *other* modules where those particular instances are only ever accessed within the defining module.

We define internally evaluated expressions ($Enc_e(_)$) whose evaluation only inspects internal objects or primitives (i.e. integers or booleans).

Definition B.1 (Internally Evaluated Expressions). For all modules M , assertions A , and expressions e , $M \models A \Rightarrow Enc_e(e)$ if and only if for all heaps χ , stacks ψ , and frames ϕ such that $M, (\chi, \phi : \psi) \models A$, we have for all values v , such that $M, (\chi, \phi : \psi), e \hookrightarrow v$ then $M, (\chi', \phi' : \psi), e \hookrightarrow v$, where

- χ' is the internal portion of χ , i.e.
 $\chi' = \{\alpha \mapsto o \mid \alpha \mapsto o \in \chi \wedge o.(cname) \in M\}$ and
- $\phi'.(local)$ is the internal portion of the $\phi.(local)$ i.e.
 $\phi' = \{x \mapsto v \mid x \mapsto v \in \chi \wedge (v \in \mathbb{Z} \vee v = \text{true} \vee v = \text{false}) \vee (\exists \alpha, v = \alpha \wedge classOff(\chi, \phi : \psi), \alpha) \in M\}$

The encapsulation proof system consists of two relations

- Purely internal expressions: $M \vdash A \Rightarrow Enc_e(e)$ and
- Assertion encapsulation: $M \vdash A \Rightarrow Enc(A')$

Fig. 12 gives proof rules for an expression comprising purely internal objects. Primitives are Enc_e ($ENC_e\text{-INT}$, $ENC_e\text{-NULL}$, $ENC_e\text{-TRUE}$, and $ENC_e\text{-FALSE}$). Addresses of internal objects are Enc_e

(ENC_e-OBJ). Field accesses with internal types of Enc_e expressions are themselves Enc_e (ENC_e-FIELD). Ghost field accesses annotated as Enc_e on Enc_e expressions are themselves Enc_e (ENC_e-GHOST).

$$\begin{array}{c}
\frac{}{M \vdash A \Rightarrow Enc_e(i)} \text{ (ENC}_e\text{-INT)} \quad \frac{}{M \vdash A \Rightarrow Enc_e(\text{null})} \text{ (ENC}_e\text{-NULL)} \\
\frac{}{M \vdash A \Rightarrow Enc_e(\text{true})} \text{ (ENC}_e\text{-TRUE)} \quad \frac{}{M \vdash A \Rightarrow Enc_e(\text{false})} \text{ (ENC}_e\text{-FALSE)} \\
\frac{M \vdash A \longrightarrow \alpha : C \quad C \in M}{M \vdash A \Rightarrow Enc_e(\alpha)} \text{ (ENC}_e\text{-OBJ)} \\
\frac{M \vdash A \Rightarrow Enc_e(e) \quad M \vdash A \longrightarrow e : C \quad [\text{field_f} : D] \in M(C).(\text{flds}) \quad D \in M}{M \vdash A \Rightarrow Enc_e(e.f)} \text{ (ENC}_e\text{-FIELD)} \\
\frac{M \vdash A \Rightarrow Enc_e(e_1) \quad M \vdash A \longrightarrow e_1 : C \quad \text{ghost intrnl } g(x : _) \{e\} \in M(C).(\text{gflds})}{M \vdash A \Rightarrow Enc_e(e_1.g(e_2))} \text{ (ENC}_e\text{-GHOST)}
\end{array}$$

Fig. 11. Internal Proof Rules

Fig. 12 gives proof rules for whether an assertion is encapsulated, that is whether a change in satisfaction of an assertion requires interaction with the internal module. An `Intrl` expression is also an encapsulated assertion (ENC-EXP). A field access on an encapsulated expression is an encapsulated expression. Binary and ternary operators applied to encapsulated expressions are themselves encapsulated assertions (ENC=_e, ENC+_e, ENC-<_e, ENC-If). An internal object may only lose access to another object via internal computation (ENC-INTACCESS). Only internal computation may grant external access to an `inside(_)` object (ENC-INSIDE₁). If an object is `inside(_)`, then nothing (not even internal objects) may gain access to that object except by internal computation (ENC-INSIDE₂). If an assertion A_1 implies assertion A_2 , then A_1 implies the encapsulation of any assertion that A_2 does. Further, if an assertion is encapsulated, then any assertion that is implied by it is also encapsulated. These two rules combine into an encapsulation rule for consequence (ENC-CONSEQ).

C MORE ABOUT THE EXPRESSIVENESS OF NECESSITY SPECIFICATIONS

C.1 ERC20

The ERC20 [The Ethereum Wiki 2018] is a widely used token standard describing the basic functionality of any Ethereum-based token contract. This functionality includes issuing tokens, keeping track of tokens belonging to participants, and the transfer of tokens between participants. Tokens may only be transferred if there are sufficient tokens in the participant's account, and if either they (using the `transfer` method) or someone authorized by the participant (using the `transferFrom` method) initiated the transfer.

We specify these necessary conditions here using *Necessity*. Firstly, `ERC20Spec1` says that if the balance of a participant's account is ever reduced by some amount m , then that must have occurred as a result of a call to the `transfer` method with amount m by the participant, or the `transferFrom` method with the amount m by some other participant.

```

ERC20Spec1  $\triangleq$  from e : ERC20  $\wedge$  e.balance(p) = m + m'  $\wedge$  m > 0
  next e.balance(p) = m'
  onlyIf  $\exists p' p''. [\langle p' \text{ calls } e.\text{transfer}(p, m) \rangle \vee$ 
    e.allowed(p, p'')  $\geq m \wedge \langle p'' \text{ calls } e.\text{transferFrom}(p', m) \rangle]$ 

```

(ENC_e-OBJ). Field accesses with internal types of Enc_e expressions are themselves Enc_e (ENC_e-FIELD). Ghost field accesses annotated as Enc_e on Enc_e expressions are themselves Enc_e (ENC_e-GHOST).

$$\begin{array}{c}
\frac{}{M \vdash A \Rightarrow Enc_e(i)} \text{ (ENC}_e\text{-INT)} \quad \frac{}{M \vdash A \Rightarrow Enc_e(\text{null})} \text{ (ENC}_e\text{-NULL)} \\
\frac{}{M \vdash A \Rightarrow Enc_e(\text{true})} \text{ (ENC}_e\text{-TRUE)} \quad \frac{}{M \vdash A \Rightarrow Enc_e(\text{false})} \text{ (ENC}_e\text{-FALSE)} \\
\frac{M \vdash A \longrightarrow \alpha : C \quad C \in M}{M \vdash A \Rightarrow Enc_e(\alpha)} \text{ (ENC}_e\text{-OBJ)} \\
\frac{M \vdash A \Rightarrow Enc_e(e) \quad M \vdash A \longrightarrow e : C \quad [\text{field_f} : D] \in M(C).(\text{flds}) \quad D \in M}{M \vdash A \Rightarrow Enc_e(e.f)} \text{ (ENC}_e\text{-FIELD)} \\
\frac{M \vdash A \Rightarrow Enc_e(e_1) \quad M \vdash A \longrightarrow e_1 : C \quad \text{ghost intrnl } g(x : _) \{e\} \in M(C).(\text{gflds})}{M \vdash A \Rightarrow Enc_e(e_1.g(e_2))} \text{ (ENC}_e\text{-GHOST)}
\end{array}$$

Fig. 12. Internal Proof Rules

Fig. 13 gives proof rules for whether an assertion is encapsulated, that is whether a change in satisfaction of an assertion requires interaction with the internal module. An `Intrl` expression is also an encapsulated assertion (ENC-EXP). A field access on an encapsulated expression is an encapsulated expression. Binary and ternary operators applied to encapsulated expressions are themselves encapsulated assertions (ENC=_e, ENC+_e, ENC-<_e, ENC-If). An internal object may only lose access to another object via internal computation (ENC-INTACCESS). Only internal computation may grant external access to an `inside(_)` object (ENC-INSIDE₁). If an object is `inside(_)`, then nothing (not even internal objects) may gain access to that object except by internal computation (ENC-INSIDE₂). If an assertion A_1 implies assertion A_2 , then A_1 implies the encapsulation of any assertion that A_2 does. Further, if an assertion is encapsulated, then any assertion that is implied by it is also encapsulated. These two rules combine into an encapsulation rule for consequence (ENC-CONSEQ).

$$\begin{array}{c}
\frac{M \vdash A \Rightarrow \text{Enc}_e(e)}{M \vdash A \Rightarrow \text{Enc}(e)} \text{ (ENC-EXP)} \quad \frac{M \vdash A \Rightarrow \text{Enc}_e(e)}{M \vdash A \Rightarrow \text{Enc}(e.f)} \text{ (ENC-FIELD)} \\
\frac{M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(e_1 = e_2)} \text{ (ENC-=)} \quad \frac{M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(e_1 + e_2)} \text{ (ENC-+)} \\
\frac{M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(e_1 < e_2)} \text{ (ENC-<)} \\
\frac{M \vdash A \Rightarrow \text{Enc}(e) \quad M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(\text{if } e \text{ then } e_1 \text{ else } e_2)} \text{ (ENC-IF)} \\
\frac{M \vdash A \rightarrow \langle x \text{ internal} \rangle}{M \vdash A \Rightarrow \text{Enc}(\langle x \text{ access } y \rangle)} \text{ (ENC-INTACCESS)} \quad \frac{M \vdash A \Rightarrow \text{Enc}(\text{inside}(x))}{M \vdash A \Rightarrow \text{Enc}(\langle x \text{ access } y \rangle)} \text{ (ENC-INSIDE}_1\text{)} \\
\frac{M \vdash A \rightarrow \text{inside}(x)}{M \vdash A \Rightarrow \text{Enc}(\neg \langle x \text{ access } y \rangle)} \text{ (ENC-INSIDE}_2\text{)} \\
\frac{M \vdash A_1 \rightarrow A_2 \quad M \vdash A \rightarrow A' \quad M \vdash A_2 \Rightarrow \text{Enc}(A)}{M \vdash A_1 \Rightarrow \text{Enc}(A')} \text{ (ENC-CONSEQ)}
\end{array}$$

Fig. 12. Assertion Encapsulation Proof Rules

Secondly, ERC20Spec2 specifies under what circumstances some participant p' is authorized to spend m tokens on behalf of p : either p approved p' , p' was previously authorized, or p' was authorized for some amount $m + m'$, and spent m' .

```

ERC20Spec2  $\triangleq$  from  $e : \text{ERC20} \wedge p : \text{Object} \wedge p' : \text{Object} \wedge m : \text{Nat}$ 
  next  $e.\text{allowed}(p, p') = m$ 
  onlyIf  $\langle p \text{ calls } e.\text{approve}(p', m) \rangle \vee$ 
     $(e.\text{allowed}(p, p') = m \wedge$ 
       $\neg (\langle p' \text{ calls } e.\text{transferFrom}(p, \_) \rangle \vee$ 
         $\langle p \text{ calls } e.\text{allowed}(p, \_) \rangle)) \vee$ 
       $\exists p''. [e.\text{allowed}(p, p') = m + m' \wedge \langle p' \text{ calls } e.\text{transferFrom}(p'', m') \rangle]$ 

```

C.2 DAO

The Decentralized Autonomous Organization (DAO) [Christoph Jentsch 2016] is a well-known Ethereum contract allowing participants to invest funds. The DAO famously was exploited with a re-entrancy bug in 2016, and lost \$50M. Here we provide specifications that would have secured the DAO against such a bug. DAOspec1 says that no participant's balance may ever exceed the ether remaining in DAO.

```

DAOspec1  $\triangleq$  from  $d : \text{DAO} \wedge p : \text{Object}$ 
  to  $d.\text{balance}(p) > d.\text{ether}$ 
  onlyIf false

```

Note that DAOspec1 enforces a class invariant of DAO, something that could be enforced by traditional specifications using class invariants. The second specification DAOspec2 states that if after some single step of execution, a participant's balance is m , then either

- (a) this occurred as a result of joining the DAO with an initial investment of m ,
- (b) the balance is 0 and they've just withdrawn their funds, or

$$\begin{array}{c}
\frac{M \vdash A \Rightarrow \text{Enc}_e(e)}{M \vdash A \Rightarrow \text{Enc}(e)} \text{ (ENC-EXP)} \quad \frac{M \vdash A \Rightarrow \text{Enc}_e(e)}{M \vdash A \Rightarrow \text{Enc}(e.f)} \text{ (ENC-FIELD)} \\
\frac{M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(e_1 = e_2)} \text{ (ENC-=)} \quad \frac{M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(e_1 + e_2)} \text{ (ENC-+)} \\
\frac{M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(e_1 < e_2)} \text{ (ENC-<)} \\
\frac{M \vdash A \Rightarrow \text{Enc}(e) \quad M \vdash A \Rightarrow \text{Enc}(e_1) \quad M \vdash A \Rightarrow \text{Enc}(e_2)}{M \vdash A \Rightarrow \text{Enc}(\text{if } e \text{ then } e_1 \text{ else } e_2)} \text{ (ENC-IF)} \\
\frac{M \vdash A \rightarrow \langle x \text{ internal} \rangle}{M \vdash A \Rightarrow \text{Enc}(\langle x \text{ access } y \rangle)} \text{ (ENC-INTACCESS)} \quad \frac{M \vdash x : C \Rightarrow \text{Enc}(\text{inside}(x))}{M \vdash A \Rightarrow \text{Enc}(\langle x \text{ access } y \rangle)} \text{ (ENC-INSIDE}_1\text{)} \\
\frac{M \vdash A \rightarrow \text{inside}(x)}{M \vdash A \Rightarrow \text{Enc}(\neg \langle x \text{ access } y \rangle)} \text{ (ENC-INSIDE}_2\text{)} \\
\frac{M \vdash A_1 \rightarrow A_2 \quad M \vdash A \rightarrow A' \quad M \vdash A_2 \Rightarrow \text{Enc}(A)}{M \vdash A_1 \Rightarrow \text{Enc}(A')} \text{ (ENC-CONSEQ)}
\end{array}$$

Fig. 13. Assertion Encapsulation Proof Rules

(c) the balance was m to begin with

```
DAOSpec2  $\triangleq$  from d : DAO  $\wedge$  p : Object
  next d.balance(p) = m
  onlyIf {p calls d.repay(_)}  $\wedge$  m = 0  $\vee$  {p calls d.join(m)}  $\vee$  d.balance(p) = m
```

C.3 Safe

[Drossopoulou et al. 2020b] used as a running example a Safe, where a treasure was secured within a Safe object, and access to the treasure was only granted by providing the correct password. Using *Necessity*, we express SafeSpec, that requires that the treasure cannot be removed from the safe without knowledge of the secret.

```
SafeSpec  $\triangleq$  from s : Safe  $\wedge$  s.treasure != null
  to s.treasure == null
  onlyIf  $\neg$  inside(s.secret)
```

The module SafeModule described below satisfies SafeSpec.

```
module SafeModule
  class Secret{}
  class Treasure{}
  class Safe{
    field treasure : Treasure
    field secret : Secret
    method take(scr : Secret){
      if (this.secret==scr) then {
        t=treasure
        this.treasure = null
        return t
      }
    }
  }
```

C.4 Crowdsale

Necessity is able to encode the motivating example of Permenev et al. [2020a]: an escrow smart contract that ensures that the contract may not be coerced to pay out or refund more money than has been raised. The motivating Crowdsale example consists of a Crowdsale contract for crowd sourcing funding. A Crowdsale object consists of an Escrow object, an amount raised, a funding goal, and a closing time in which the goal must be met for the fund to be successful. An Escrow consists of a ledger of investors and how much they have invested. There are several properties that Permenev et al. [2020a] sought to encode, and we have provided the encoding of those specifications in Fig. 15. R0 states that if an investor claims a refund from an escrow, then the balance of the escrow decreases by the amount the investor had deposited in the escrow. R1 states that if at anytime the escrow has not yet succeeded, then the deposits must be less than the balance of the escrow. R2_1 and R2_2 combine to express a single property: no one may ever withdraw and then subsequently claim a refund or visa versa. R3 states that if the funding goal is ever met, then no one may subsequently claim a refund.

D MORE NECESSITY LOGIC RULES

Here we give the complete version of the rules in Fig. 5, Fig. 6, and Fig. 7.

C MORE ABOUT THE EXPRESSIVENESS OF NECESSITY SPECIFICATIONS

C.1 ERC20

The ERC20 [The Ethereum Wiki 2018] is a widely used token standard describing the basic functionality of any Ethereum-based token contract. This functionality includes issuing tokens, keeping track of tokens belonging to participants, and the transfer of tokens between participants. Tokens may only be transferred if there are sufficient tokens in the participant's account, and if either they (using the transfer method) or someone authorized by the participant (using the transferFrom method) initiated the transfer.

We specify these necessary conditions here using *Necessity*. Firstly, ERC20Spec1 says that if the balance of a participant's account is ever reduced by some amount m , then that must have occurred as a result of a call to the transfer method with amount m by the participant, or the transferFrom method with the amount m by some other participant.

```
ERC20Spec1  $\triangleq$  from e : ERC20  $\wedge$  e.balance(p) = m + m'  $\wedge$  m > 0
  next e.balance(p) = m'
  onlyIf  $\exists$  p' p''. [{p' calls e.transfer(p, m)}  $\vee$ 
    e.allowed(p, p'')  $\geq$  m  $\wedge$  {p'' calls e.transferFrom(p', m)}]
```

Secondly, ERC20Spec2 specifies under what circumstances some participant p' is authorized to spend m tokens on behalf of p : either p approved p' , p' was previously authorized, or p' was authorized for some amount $m + m'$, and spent m' .

```
ERC20Spec2  $\triangleq$  from e : ERC20  $\wedge$  p : Object  $\wedge$  p' : Object  $\wedge$  m : Nat
  next e.allowed(p, p') = m
  onlyIf {p calls e.approve(p', m)}  $\vee$ 
    (e.allowed(p, p') = m  $\wedge$ 
       $\neg$  ({p' calls e.transferFrom(p, _)}  $\vee$ 
        {p calls e.allowed(p, _)}))  $\vee$ 
     $\exists$  p''. [e.allowed(p, p') = m + m'  $\wedge$  {p' calls e.transferFrom(p'', m')}]
```

C.2 DAO

The Decentralized Autonomous Organization (DAO) [Christoph Jentsch 2016] is a well-known Ethereum contract allowing participants to invest funds. The DAO famously was exploited with a re-entrancy bug in 2016, and lost \$50M. Here we provide specifications that would have secured the DAO against such a bug. DAOSpec1 says that no participant's balance may ever exceed the ether remaining in DAO.

```
DAOSpec1  $\triangleq$  from d : DAO  $\wedge$  p : Object
  to d.balance(p) > d.ether
  onlyIf false
```

Note that DAOSpec1 enforces a class invariant of DAO, something that could be enforced by traditional specifications using class invariants. The second specification DAOSpec2 states that if after some single step of execution, a participant's balance is m , then either

- (a) this occurred as a result of joining the DAO with an initial investment of m ,
- (b) the balance is 0 and they've just withdrawn their funds, or
- (c) the balance was m to begin with

```
DAOSpec2  $\triangleq$  from d : DAO  $\wedge$  p : Object
  next d.balance(p) = m
  onlyIf {p calls d.repay(_)}  $\wedge$  m = 0  $\vee$  {p calls d.join(m)}  $\vee$  d.balance(p) = m
```

small changes over Julian's Using *Necessity*, we express *SafeSpec*, that requires that the treasure cannot be removed from the safe without knowledge of the secret.

```

1765 SafeSpec  $\triangleq$  from s : Safe  $\wedge$  s.treasure != null
1766 to s.treasure == null
1767 onlyIf  $\neg$  inside(s.secret)

```

The module *SafeModule* described below satisfies *SafeSpec*.

```

1771 module SafeModule
1772   class Secret{}
1773   class Treasure{}
1774   class Safe{
1775     field treasure : Treasure
1776     field secret : Secret
1777     method take(scr : Secret){
1778       if (this.secret==scr) then {
1779         t=treasure
1780         this.treasure = null
1781         return t }
1782     }
1783   }

```

C.3 Crowdsale

Necessity is able to encode the motivating example of [Permenev et al. \[2020a\]](#): an escrow smart contract that ensures that the contract may not be coerced to pay out or refund more money than has been raised. The motivating Crowdsale example consists of a Crowdsale contract for crowd sourcing funding. A Crowdsale object consists of an Escrow object, an amount raised, a funding goal, and a closing time in which the goal must be met for the fund to be successful. An Escrow consists of a ledger of investors and how much they have invested. There are several properties that [Permenev et al. \[2020a\]](#) sought to encode, and we have provided the encoding of those specifications in Fig. 16. R0 states that if an investor claims a refund from an escrow, then the balance of the escrow decreases by the amount the investor had deposited in the escrow. R1 states that if at anytime the escrow has not yet succeeded, then the deposits must be less than the balance of the escrow. R2_1 and R2_2 combine to express a single property: no one may ever withdraw and then subsequently claim a refund or visa versa. R3 states that if the funding goal is ever met, then no one may subsequently claim a refund.

```

1716 1 class Crowdsale {
1717 2   Escrow escrow;
1718 3   closeTime, raised, goal : int;
1719 4   method init() {
1720 5     if escrow == null
1721 6       escrow := new Escrow(new Object);
1722 7       closeTime := now + 30 days;
1723 8       raised := 0;
1724 9       goal := 10000 * 10**18;
1725 10  }
1726 11  method invest(investor : Object, value : int) {
1727 12    if raised < goal
1728 13      escrow.deposit(investor, value);
1729 14      raised += value;
1730 15  }
1731 16  method close() {
1732 17    if now > closeTime || raised >= goal
1733 18      if raised >= goal
1734 19        escrow.close();
1735 20      else
1736 21        escrow.refund();
1737 22  }
1738 23  }

```

Fig. 13. Crowdsale Contract

E ASSERT LOGIC

Fig. 19 presents some rules the *Assert* proof system relies upon, of the form $M \vdash A$. These rules are relatively simple, with none presenting any surprising results, and would be straightforward, but rather time-consuming, to prove sound in the Coq mechanisation. CALLER-EXT, CALLER-RECV, CALLER-ARGS, and CLASS-INT are simple properties that arise from the semantics of *Assert*. FLD-CLASS and INSIDE-INT are directly drawn from the simple type system of Tool. ABSURD and EXCLUDED MIDDLE are common logical properties.

F Mod_{best} – A MORE INTERESTING BANK ACCOUNT MODULE

We now revisit the bank account example, and present Mod_{best} in Figure 20. Mod_{best} is more interesting than Mod_{better}, as it allows us to demonstrate how *Necessity* logic deals with challenges that come with more complex data structures and specifications. These challenges are

- (1) Specifications defined using ghost fields – in this case `b.balance(a)` returns the balance of account `a` in Bank `b`.
- (2) Modules with several classes and methods; they all must be considered when constructing proofs about emergent behaviour.
- (3) The construction of a proof of assertion encapsulation. Such a proof is necessary here because the ghost field `balance` reads several fields. We use our simple confinement system, captured by confined classes in Tool.

In Mod_{best}, the balance of an account is kept in a ledger rather than in the account itself. Mod_{best} consists of three classes: (1) *Account* that maintains a password, (2) *Bank*, a public interface for

```

1814 1 class Crowdsale {
1815 2   Escrow escrow;
1816 3   closeTime, raised, goal : int;
1817 4   method init() {
1818 5     if escrow == null
1819 6       escrow := new Escrow(new Object);
1820 7       closeTime := now + 30 days;
1821 8       raised := 0;
1822 9       goal := 10000 * 10**18;
1823 10  }
1824 11  method invest(investor : Object, value : int) {
1825 12    if raised < goal
1826 13      escrow.deposit(investor, value);
1827 14      raised += value;
1828 15  }
1829 16  method close() {
1830 17    if now > closeTime || raised >= goal
1831 18      if raised >= goal
1832 19        escrow.close();
1833 20      else
1834 21        escrow.refund();
1835 22  }
1836 23  }

```

Fig. 14. Crowdsale Contract

```

1765 1 confined class Escrow {
1766 2   owner, beneficiary : Object;
1767 3   mapping(Object => uint256) deposits;
1768 4   OPEN, SUCCESS, REFUND : Object;
1769 5   state : Object;
1770 6   method init(o : Object, b : Object) {
1771 7     if owner == null || beneficiary == null
1772 8       owner := o;
1773 9       beneficiary := b;
1774 10      OPEN := new Object; SUCCESS := new Object; REFUND := new Object;
1775 11      state := OPEN;
1776 12
1777 13   method close() {state = SUCCESS;}
1778 14   method refund() {state = REFUND;}
1779 15   method deposit(value : int, p : Object) {
1780 16     deposits[p] := deposits[p] + value;
1781 17   }
1782 18   method withdraw() {
1783 19     if state == SUCCESS
1784 20       return this.balance;
1785 21   }
1786 22   method claimRefund(p : Object) {
1787 23     if state == REFUND
1788 24       int amount := deposits[p];
1789 25       deposits[p] := 0;
1790 26       return amount;
1791 27   }
1792 28 }

```

Fig. 14. Escrow Contract

```

1793 1 (R0)  $\triangleq e : \text{Escrow} \wedge \langle \_ \text{calls } e.\text{claimRefund}(p) \rangle$ 
1794 2   next e.balance = nextBal onlyIf nextBal = e.balance - e.deposits(p)
1795 3 (R1)  $\triangleq e : \text{Escrow} \wedge e.\text{state} \neq e.\text{SUCCESS} \rightarrow \text{sum}(\text{deposits}) \leq e.\text{balance}$ 
1796 4 (R2_1)  $\triangleq e : \text{Escrow} \wedge \langle \_ \text{calls } e.\text{withdraw}(\_) \rangle$ 
1797 5   to  $\langle \_ \text{calls } e.\text{claimRefund}(\_) \rangle$  onlyIf false
1798 6 (R2_2)  $\triangleq e : \text{Escrow} \wedge \langle \_ \text{calls } e.\text{claimRefund}(\_) \rangle$ 
1799 7   to  $\langle \_ \text{calls } e.\text{withdraw}(\_) \rangle$  onlyIf false
1800 8 (R3)  $\triangleq c : \text{Crowdsale} \wedge \text{sum}(\text{deposits}) \geq c.\text{escrow.goal}$ 
1801 9   to  $\langle \_ \text{calls } c.\text{escrow.claimRefund}(\_) \rangle$  onlyIf false
1802
1803
1804
1805
1806

```

Fig. 15. Encoding VerX Crowdsale Example in Necessity

transferring money from one account to another, and (3) Ledger, a private class, annotated as confined, used to map Account objects to their balances.

A Bank has a Ledger field, a method for transferring funds between accounts (transfer), and a ghost field, for looking up the balance of an account at a bank (balance). A Ledger is a mapping from Accounts to their balances. For brevity our implementation only includes two accounts (acc1 and acc2), but it is easy to see how this could extend to a Ledger of arbitrary

```

1863 1 confined class Escrow {
1864 2   owner, beneficiary : Object;
1865 3   mapping(Object => uint256) deposits;
1866 4   OPEN, SUCCESS, REFUND : Object;
1867 5   state : Object;
1868 6   method init(o : Object, b : Object) {
1869 7     if owner == null || beneficiary == null
1870 8       owner := o;
1871 9       beneficiary := b;
1872 10      OPEN := new Object; SUCCESS := new Object; REFUND := new Object;
1873 11      state := OPEN;
1874 12
1875 13   method close() {state = SUCCESS;}
1876 14   method refund() {state = REFUND;}
1877 15   method deposit(value : int, p : Object) {
1878 16     deposits[p] := deposits[p] + value;
1879 17   }
1880 18   method withdraw() {
1881 19     if state == SUCCESS
1882 20       return this.balance;
1883 21   }
1884 22   method claimRefund(p : Object) {
1885 23     if state == REFUND
1886 24       int amount := deposits[p];
1887 25       deposits[p] := 0;
1888 26       return amount;
1889 27   }
1890 28 }

```

Fig. 15. Escrow Contract

```

1891 1 (R0)  $\triangleq e : \text{Escrow} \wedge \langle \_ \text{calls } e.\text{claimRefund}(p) \rangle$ 
1892 2   next e.balance = nextBal onlyIf nextBal = e.balance - e.deposits(p)
1893 3 (R1)  $\triangleq e : \text{Escrow} \wedge e.\text{state} \neq e.\text{SUCCESS} \rightarrow \text{sum}(\text{deposits}) \leq e.\text{balance}$ 
1894 4 (R2_1)  $\triangleq e : \text{Escrow} \wedge \langle \_ \text{calls } e.\text{withdraw}(\_) \rangle$ 
1895 5   to  $\langle \_ \text{calls } e.\text{claimRefund}(\_) \rangle$  onlyIf false
1896 6 (R2_2)  $\triangleq e : \text{Escrow} \wedge \langle \_ \text{calls } e.\text{claimRefund}(\_) \rangle$ 
1897 7   to  $\langle \_ \text{calls } e.\text{withdraw}(\_) \rangle$  onlyIf false
1898 8 (R3)  $\triangleq c : \text{Crowdsale} \wedge \text{sum}(\text{deposits}) \geq c.\text{escrow.goal}$ 
1899 9   to  $\langle \_ \text{calls } c.\text{escrow.claimRefund}(\_) \rangle$  onlyIf false
1900
1901
1902
1903
1904

```

Fig. 16. Encoding VerX Crowdsale Example in Necessity

$$\begin{array}{c}
\text{for all } C \in \text{dom}(M) \text{ and } m \in M(C).\text{mths}, \quad M \vdash \text{from } A_1 \wedge x : C \wedge \langle _ \text{calls } x.m(\bar{z}) \rangle \text{next } A_2 \text{ onlyIf } A_3 \\
\hline
M \vdash A_1 \longrightarrow \neg A_2 \quad M \vdash A_1 \Rightarrow \text{Enc}(A_2) \quad (\text{If1-INTERNAL}) \\
M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3 \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A} \quad (\text{If1-If}) \\
\\
\frac{M \vdash A_1 \longrightarrow A'_1 \quad M \vdash A_2 \longrightarrow A'_2 \quad M \vdash A'_3 \longrightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ next } A'_2 \text{ onlyIf } A'_3}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3} \quad (\text{If1} \longrightarrow) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A'_1 \text{ next } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \vee A'_1 \text{ next } A_2 \text{ onlyIf } A \vee A'} \quad (\text{If1-}\vee I_1) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ next } A'_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ next } A_2 \vee A'_2 \text{ onlyIf } A \vee A'} \quad (\text{If1-}\vee I_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \vee A' \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A} \quad (\text{If1-}\vee E) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \wedge A'} \quad (\text{If1-}\wedge I) \quad \frac{\forall y, M \vdash \text{from } ([y/x]A_1) \text{ next } A_2 \text{ onlyIf } A}{M \vdash \text{from } \exists x.[A_1] \text{ next } A_2 \text{ onlyIf } A} \quad (\text{If1-}\exists_1) \\
\\
\frac{\forall y, M \vdash \text{from } A_1 \text{ next } ([y/x]A_2) \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ next } \exists x.[A_2] \text{ onlyIf } A} \quad (\text{If1-}\exists_2)
\end{array}$$

Fig. 16. Single-Step Necessity Specifications

size. Ledger is annotated as `confined`, so Tool's Confined Types will ensure the necessary encapsulation. Finally, an `Account` has some password object, and methods to authenticate a provided password (`authenticate`), and to change the password (`changePass`).

Figures 21, 22, and 23 give pre- and postcondition specifications for `Modbest`. Informally, these functional specifications state that

- (1) no method returns the password,
- (2) the `transfer` method in `Ledger` results in a decreased balance to the `from Account`,
- (3) and the `transfer` method in `Bank` results in a decreased balance to the `from Account` *only if* the correct password is supplied, and
- (4) every other method in `Modbest` never modifies any balance in any `Bank`.

D MORE NECESSITY LOGIC RULES

Here we give the complete version of the rules in Fig. 6, Fig. 7, and Fig. 8.

$$\begin{array}{c}
\text{for all } C \in \text{dom}(M) \text{ and } m \in M(C).\text{mths}, \quad M \vdash \text{from } A_1 \wedge x : C \wedge \langle _ \text{calls } x.m(\bar{z}) \rangle \text{next } A_2 \text{ onlyIf } A_3 \\
\hline
M \vdash A_1 \longrightarrow \neg A_2 \quad M \vdash A_1 \Rightarrow \text{Enc}(A_2) \quad (\text{If1-INTERNAL}) \\
M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3 \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A} \quad (\text{If1-If}) \\
\\
\frac{M \vdash A_1 \longrightarrow A'_1 \quad M \vdash A_2 \longrightarrow A'_2 \quad M \vdash A'_3 \longrightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ next } A'_2 \text{ onlyIf } A'_3}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A_3} \quad (\text{If1} \longrightarrow) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A'_1 \text{ next } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \vee A'_1 \text{ next } A_2 \text{ onlyIf } A \vee A'} \quad (\text{If1-}\vee I_1) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ next } A'_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ next } A_2 \vee A'_2 \text{ onlyIf } A \vee A'} \quad (\text{If1-}\vee I_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \vee A' \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A} \quad (\text{If1-}\vee E) \\
\\
\frac{M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ next } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \wedge A'} \quad (\text{If1-}\wedge I) \quad \frac{\forall y, M \vdash \text{from } ([y/x]A_1) \text{ next } A_2 \text{ onlyIf } A}{M \vdash \text{from } \exists x.[A_1] \text{ next } A_2 \text{ onlyIf } A} \quad (\text{If1-}\exists_1) \\
\\
\frac{\forall y, M \vdash \text{from } A_1 \text{ next } ([y/x]A_2) \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ next } \exists x.[A_2] \text{ onlyIf } A} \quad (\text{If1-}\exists_2)
\end{array}$$

Fig. 17. Single-Step Necessity Specifications

$$\begin{array}{c}
\frac{M \vdash \text{from } A \text{ next } \neg A \text{ onlyIf } A'}{M \vdash \text{from } A \text{ to } \neg A \text{ onlyThrough } A'} \quad (\text{CHANGES}) \\
\\
\frac{M \vdash A_1 \longrightarrow A'_1 \quad M \vdash A_2 \longrightarrow A'_2 \quad M \vdash A'_3 \longrightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ to } A'_2 \text{ onlyThrough } A'_3}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3} \quad (\longrightarrow) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \quad M \vdash \text{from } A'_1 \text{ to } A_2 \text{ onlyThrough } A'}{M \vdash \text{from } A_1 \vee A'_1 \text{ to } A_2 \text{ onlyThrough } A \vee A'} \quad (\vee I_1) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \quad M \vdash \text{from } A_1 \text{ to } A'_2 \text{ onlyThrough } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \vee A'_2 \text{ onlyThrough } A \vee A'} \quad (\vee I_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A' \text{ onlyThrough false} \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \vee A'} \quad (\vee E_1) \quad \frac{M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false} \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \vee A'}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\vee E_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_1) \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_3 \text{ to } A_2 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{If}) \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_2 \quad (\text{END}) \\
\\
\frac{\forall y. M \vdash \text{from } ([y/x]A_1) \text{ to } A_2 \text{ onlyThrough } A}{M \vdash \text{from } \exists x. [A_1] \text{ to } A_2 \text{ onlyThrough } A} \quad (\exists_1) \\
\\
\frac{\forall y. M \vdash \text{from } A_1 \text{ to } ([y/x]A_2) \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\exists_2)
\end{array}$$

Fig. 17. Only Through

$$\begin{array}{c}
\frac{M \vdash \text{from } A \text{ next } \neg A \text{ onlyIf } A'}{M \vdash \text{from } A \text{ to } \neg A \text{ onlyThrough } A'} \quad (\text{CHANGES}) \\
\\
\frac{M \vdash A_1 \longrightarrow A'_1 \quad M \vdash A_2 \longrightarrow A'_2 \quad M \vdash A'_3 \longrightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ to } A'_2 \text{ onlyThrough } A'_3}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3} \quad (\longrightarrow) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \quad M \vdash \text{from } A'_1 \text{ to } A_2 \text{ onlyThrough } A'}{M \vdash \text{from } A_1 \vee A'_1 \text{ to } A_2 \text{ onlyThrough } A \vee A'} \quad (\vee I_1) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \quad M \vdash \text{from } A_1 \text{ to } A'_2 \text{ onlyThrough } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \vee A'_2 \text{ onlyThrough } A \vee A'} \quad (\vee I_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A' \text{ onlyThrough false} \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \vee A'} \quad (\vee E_1) \quad \frac{M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false} \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A \vee A'}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\vee E_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_1) \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_3 \text{ to } A_2 \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{TRANS}_2) \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\text{If}) \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_2 \quad (\text{END}) \\
\\
\frac{\forall y. M \vdash \text{from } ([y/x]A_1) \text{ to } A_2 \text{ onlyThrough } A}{M \vdash \text{from } \exists x. [A_1] \text{ to } A_2 \text{ onlyThrough } A} \quad (\exists_1) \\
\\
\frac{\forall y. M \vdash \text{from } A_1 \text{ to } ([y/x]A_2) \text{ onlyThrough } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A} \quad (\exists_2)
\end{array}$$

Fig. 18. Only Through

$$\begin{array}{c}
\frac{M \vdash A_1 \longrightarrow A'_1 \quad M \vdash A_2 \longrightarrow A'_2 \quad M \vdash A'_3 \longrightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ to } A'_2 \text{ onlyIf } A'_3}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_3} \text{ (IF-}\longrightarrow\text{)} \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A'_1 \text{ to } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \vee A'_1 \text{ to } A_2 \text{ onlyIf } A \vee A'} \text{ (IF-}\vee\text{I}_1\text{)} \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ to } A'_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \vee A'_2 \text{ onlyIf } A \vee A'} \text{ (IF-}\vee\text{I}_2\text{)} \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \vee A' \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-}\vee\text{E)} \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \wedge A'} \text{ (IF-}\wedge\text{I)} \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-TRANS)} \\
\\
M \vdash \text{from } x : C \text{ to } \neg x : C \text{ onlyIf false} \text{ (IF-CLASS)} \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_1 \text{ (IF-START)} \\
\\
\frac{\forall y, M \vdash \text{from } ([y/x]A_1) \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } \exists x.[A_1] \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-}\exists_1\text{)} \quad \frac{\forall y, M \vdash \text{from } A_1 \text{ to } ([y/x]A_2) \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-}\exists_2\text{)}
\end{array}$$

Fig. 18. Only If

$$\begin{array}{c}
M \vdash \langle x \text{ calls } y.m(\bar{z}) \rangle \longrightarrow \langle x \text{ external} \rangle \text{ (CALLER-EXT)} \\
\\
M \vdash \langle x \text{ calls } y.m(\bar{z}) \rangle \longrightarrow \langle x \text{ access } y \rangle \text{ (CALLER-RECV)} \\
\\
M \vdash \langle x \text{ calls } y.m(\dots, z_i, \dots) \rangle \longrightarrow \langle x \text{ access } z_i \rangle \text{ (CALLER-ARGS)} \\
\\
\frac{C \in M}{M \vdash x : C \longrightarrow \langle x \text{ internal} \rangle} \text{ (CLASS-INT)} \quad \frac{(\text{field}_f : D) \in M(C).(\text{flds})}{M \vdash e : C \longrightarrow e.f : D} \text{ (FLD-CLASS)} \\
\\
\frac{(\text{class confined } C\{.;.\}) \in M}{M \vdash x : C \longrightarrow \text{inside}(x)} \text{ (INSIDE-INT)} \quad M \vdash \text{false} \longrightarrow A \text{ (ABSURD)} \\
\\
M \vdash A \vee \neg A \text{ (EXCLUDED MIDDLE)}
\end{array}$$

Fig. 19. Properties of the Assert proof system.

$$\begin{array}{c}
\frac{M \vdash A_1 \longrightarrow A'_1 \quad M \vdash A_2 \longrightarrow A'_2 \quad M \vdash A'_3 \longrightarrow A_3 \quad M \vdash \text{from } A'_1 \text{ to } A'_2 \text{ onlyIf } A'_3}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_3} \text{ (IF-}\longrightarrow\text{)} \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A'_1 \text{ to } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \vee A'_1 \text{ to } A_2 \text{ onlyIf } A \vee A'} \text{ (IF-}\vee\text{I}_1\text{)} \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ to } A'_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \vee A'_2 \text{ onlyIf } A \vee A'} \text{ (IF-}\vee\text{I}_2\text{)} \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \vee A' \quad M \vdash \text{from } A' \text{ to } A_2 \text{ onlyThrough false}}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-}\vee\text{E)} \\
\\
\frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A'}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A \wedge A'} \text{ (IF-}\wedge\text{I)} \quad \frac{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyThrough } A_3 \quad M \vdash \text{from } A_1 \text{ to } A_3 \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-TRANS)} \\
\\
M \vdash \text{from } x : C \text{ to } \neg x : C \text{ onlyIf false} \text{ (IF-CLASS)} \quad M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A_1 \text{ (IF-START)} \\
\\
\frac{\forall y, M \vdash \text{from } ([y/x]A_1) \text{ to } A_2 \text{ onlyIf } A}{M \vdash \text{from } \exists x.[A_1] \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-}\exists_1\text{)} \quad \frac{\forall y, M \vdash \text{from } A_1 \text{ to } ([y/x]A_2) \text{ onlyIf } A}{M \vdash \text{from } A_1 \text{ to } A_2 \text{ onlyIf } A} \text{ (IF-}\exists_2\text{)}
\end{array}$$

Fig. 19. Only If

```

19611 module Modbest
19612   class Account
19613     field password:Object
19614     method authenticate(pwd:Object):bool
19615       {return pwd == this.password}
19616     method changePass(pwd:Object, newPwd:Object):void
19617       {if pwd == this.password
19618         this.password := newPwd}
19619   confined class Ledger
19620     field acc1:Account
19621     field bal1:int
19622     field acc2:Account
19623     field bal2:int
19624     ghost intrnl balance(acc):int=
19625       if acc == acc1
19626         bal1
19627       else if acc == acc2
19628         bal2
19629       else -1
19630     method transfer(amt:int, from:Account, to:Account):void
19631       {if from == acc1 && to == acc2
19632         bal1 := bal1 - amt
19633         bal2 := bal2 + amt
19634       else if from == acc2 && to == acc1
19635         bal1 := bal1 + amt
19636         bal2 := bal2 - amt}
19637   class Bank
19638     field book:Ledger
19639     ghost intrnl balance(acc):int=book.balance(acc)
19640     method transfer(pwd:Object, amt:int, from:Account, to:Account):void
19641       {if (from.authenticate(pwd))
19642         book.transfer(amt, from, to)}

```

Fig. 20. Mod_{best} – a more interesting bank account implementation

E ASSERT LOGIC

Fig. 20 presents some rules the *Assert* proof system relies upon, of the form $M \vdash A$. These rules are relatively simple, with none presenting any surprising results, and would be straightforward, but rather time-consuming, to prove sound in the Coq mechanisation. CALLER-EXT, CALLER-RECV, CALLER-ARGS, and CLASS-INT are simple properties that arise from the semantics of *Assert*. FLD-CLASS and INSIDE-INT are directly drawn from the simple type system of Tool. ABSURD and EXCLUDED MIDDLE are common logical properties.

$$\begin{array}{c}
M \vdash \langle x \text{ calls } y.m(\bar{z}) \rangle \longrightarrow \langle x \text{ external} \rangle \quad (\text{CALLER-EXT}) \\
M \vdash \langle x \text{ calls } y.m(\bar{z}) \rangle \longrightarrow \langle x \text{ access } y \rangle \quad (\text{CALLER-RECV}) \\
M \vdash \langle x \text{ calls } y.m(\dots, z_i, \dots) \rangle \longrightarrow \langle x \text{ access } z_i \rangle \quad (\text{CALLER-ARGS}) \\
\frac{C \in M}{M \vdash x : C \longrightarrow \langle x \text{ internal} \rangle} \quad (\text{CLASS-INT}) \quad \frac{(\text{field_f} : D) \in M(C).(\text{flds})}{M \vdash e : C \longrightarrow e.f : D} \quad (\text{FLD-CLASS}) \\
\frac{(\text{class confined } C\{; _ \}) \in M}{M \vdash x : C \longrightarrow \text{inside}(x)} \quad (\text{INSIDE-INT}) \quad M \vdash \text{false} \longrightarrow A \quad (\text{ABSURD}) \\
M \vdash A \vee \neg A \quad (\text{EXCLUDED MIDDLE})
\end{array}$$

Fig. 20. Properties of the *Assert* proof system.

F Mod_{best} – A MORE INTERESTING BANK ACCOUNT MODULE

We now revisit the bank account example, and present Mod_{best} in Figure 21. Mod_{best} is more interesting than $\text{Mod}_{\text{better}}$, as it allows us to demonstrate how *Necessity* logic deals with challenges that come with more complex data structures and specifications. These challenges are

- (1) Specifications defined using ghost fields – in this case $b.\text{balance}(a)$ returns the balance of account a in Bank b .
- (2) Modules with several classes and methods; they all must be considered when constructing proofs about emergent behaviour.
- (3) The construction of a proof of assertion encapsulation. Such a proof is necessary here because the ghost field `balance` reads several fields. We use our simple confinement system, captured by confined classes in Tool.

```

2121 module Modbest
2122   class Account
2123     field password:Object
2124     method authenticate(pwd:Object):bool
2125       {return pwd == this.password}
2126     method changePass(pwd:Object, newPwd:Object):void
2127       {if pwd == this.password
2128        this.password := newPwd}
2129   confined class Ledger
2130     field acc1:Account
2131     field bal1:int
2132     field acc2:Account
2133     field bal2:int
2134     ghost intrnl balance(acc):int=
2135       if acc == acc1
2136         bal1
2137       else if acc == acc2
2138         bal2
2139       else -1
2140     method transfer(amt:int, from:Account, to:Account):void
2141       {if from == acc1 && to == acc2
2142        bal1 := bal1 - amt
2143        bal2 := bal2 + amt
2144        else if from == acc2 && to == acc1
2145        bal1 := bal1 + amt
2146        bal2 := bal2 - amt}
2147   class Bank
2148     field book:Ledger
2149     ghost intrnl balance(acc):int=book.balance(acc)
2150     method transfer(pwd:Object, amt:int, from:Account, to:Account):void
2151       {if (from.authenticate(pwd))
2152        book.transfer(amt, from, to)}

```

Fig. 21. Mod_{best} – a more interesting bank account implementation

In Mod_{best} , the balance of an account is kept in a ledger rather than in the account itself. Mod_{best} consists of three classes: (1) `Account` that maintains a password, (2) `Bank`, a public interface for transferring money from one account to another, and (3) `Ledger`, a private class, annotated as confined, used to map `Account` objects to their balances.

```

20101 module Modbest
20112   class Account
20113     field password : Object
20114     method authenticate(pwd : Object) : bool
20115       (PRE: a : Account ∧ b : Bank
20116        POST: b.balance(a)old == b.balance(a)new)
20117     (PRE: a : Account
20118      POST: res != a.password)
20119     (PRE: a : Account
20120      POST: a.passwordold == a.passwordnew)
20121     {return pwd == this.password}
20122   method changePassword(pwd : Object, newPwd : Object) : void
20123     (PRE: a : Account
20124      POST: res != a.password)
20125     (PRE: a : Account ∧ b : Bank
20126      POST: b.balance(a)old == b.balance(a)new)
20127     (PRE: a : Account ∧ pwd != this.password
20128      POST: a.passwordold = a.passwordnew)
20129     {if pwd == this.password
20130      this.password := newPwd}
20131
20132   confined class Ledger
20133     continued in Fig.22
20134
20135   class Bank
20136     continued in Fig.23
20137   ...

```

Fig. 21. Mod_{best} functional specifications, 1st part

2157 A Bank has a Ledger field, a method for transferring funds between accounts (transfer),
 2158 and a ghost field, for looking up the balance of an account at a bank (balance). A Ledger is
 2159 a mapping from Accounts to their balances. For brevity our implementation only includes two
 2160 accounts (acc1 and acc2), but it is easy to see how this could extend to a Ledger of arbitrary
 2161 size. Ledger is annotated as confined, so Tool's Confined Types will ensure the necessary
 2162 encapsulation. Finally, an Account has some password object, and methods to authenticate a
 2163 provided password (authenticate), and to change the password (changePass).

Figures 22, 23, and 24 give pre- and postcondition specifications for Mod_{best}. Informally, these functional specifications state that

- (1) no method returns the password,
- (2) the transfer method in Ledger results in a decreased balance to the from Account,
- (3) and the transfer method in Bank results in a decreased balance to the from Account only if the correct password is supplied, and
- (4) every other method in Mod_{best} never modifies any balance in any Bank.

```

2172 module Modbest
2173   class Account
2174     field password : Object
2175     method authenticate(pwd : Object) : bool
2176       (PRE: a : Account ∧ b : Bank
2177        POST: b.balance(a)old == b.balance(a)new)
2178     (PRE: a : Account
2179      POST: res != a.password)
2180     (PRE: a : Account
2181      POST: a.passwordold == a.passwordnew)
2182     {return pwd == this.password}
2183   method changePassword(pwd : Object, newPwd : Object) : void
2184     (PRE: a : Account
2185      POST: res != a.password)
2186     (PRE: a : Account ∧ b : Bank
2187      POST: b.balance(a)old == b.balance(a)new)
2188     (PRE: a : Account ∧ pwd != this.password
2189      POST: a.passwordold = a.passwordnew)
2190     {if pwd == this.password
2191      this.password := newPwd}
2192
2193   confined class Ledger
2194     continued in Fig.23
2195
2196   class Bank
2197     continued in Fig.24
2198   ...

```

Fig. 22. Mod_{best} functional specifications, 1st part

```

20591   confined class Ledger
20602       field acc1 : Account
20613       field bal1 : int
20624       field acc2 : Account
20635       field bal2 : int
20646       ghost intrnl balance(acc) : int =
20657         if acc == acc1
20668           bal1
20679         else if acc == acc2
20690           bal2
20701         else -1
20712   method transfer(amt : int, from : Account, to : Account) : void
20723     (PRE: a : Account  $\wedge$  b : Bank  $\wedge$  (a != acc1  $\wedge$  a != acc2)
20734     POST: b.balance(a)old == b.balance(a)new)
20745     (PRE: a : Account
20756     POST: res != a.password)
20767     (PRE: a : Account
20778     POST: a.passwordold == a.passwordnew)
20789     {if from == acc1 && to == acc2
20800       bal1 := bal1 - amt
20811       bal2 := bal2 + amt
20822     else if from == acc2 && to == acc1
20833       bal1 := bal1 + amt
20844       bal2 := bal2 - amt}

```

Fig. 22. Mod_{best} functional specifications, 2nd part

```

20821   class Bank
20832       field book : Ledger
20843       ghost intrnl balance(acc) : int = book.balance(acc)
20854       method transfer(pwd : Object, amt : int, from : Account, to : Account) : void
20865         (PRE: a : Account  $\wedge$  b : Bank  $\wedge$   $\neg$  (a == acc1  $\wedge$  a == acc2)
20876         POST: b.balance(a)old a = b.balance(a)new)
20887         (PRE: a : Account
20898         POST: res != a.password)
20909         (PRE: a : Account
20920         POST: a.passwordold == a.passwordnew)
20931         {if (from.authenticate(pwd))
20942           book.transfer(amt, from, to)}

```

Fig. 23. Mod_{best} functional specifications, 3rd part

```

22061   confined class Ledger
22072       field acc1 : Account
22083       field bal1 : int
22094       field acc2 : Account
22105       field bal2 : int
22116       ghost intrnl balance(acc) : int =
22127         if acc == acc1
22138           bal1
22149         else if acc == acc2
22160           bal2
22171         else -1
22182   method transfer(amt : int, from : Account, to : Account) : void
22193     (PRE: a : Account  $\wedge$  b : Bank  $\wedge$  (a != acc1  $\wedge$  a != acc2)
22204     POST: b.balance(a)old == b.balance(a)new)
22215     (PRE: a : Account
22226     POST: res != a.password)
22237     (PRE: a : Account
22248     POST: a.passwordold == a.passwordnew)
22259     {if from == acc1 && to == acc2
22270       bal1 := bal1 - amt
22281       bal2 := bal2 + amt
22292     else if from == acc2 && to == acc1
22303       bal1 := bal1 + amt
22314       bal2 := bal2 - amt}

```

Fig. 23. Mod_{best} functional specifications, 2nd part

```

22291   class Bank
22302       field book : Ledger
22313       ghost intrnl balance(acc) : int = book.balance(acc)
22324       method transfer(pwd : Object, amt : int, from : Account, to : Account) : void
22335         (PRE: a : Account  $\wedge$  b : Bank  $\wedge$   $\neg$  (a == acc1  $\wedge$  a == acc2)
22346         POST: b.balance(a)old a = b.balance(a)new)
22357         (PRE: a : Account
22368         POST: res != a.password)
22379         (PRE: a : Account
22390         POST: a.passwordold == a.passwordnew)
22401         {if (from.authenticate(pwd))
22412           book.transfer(amt, from, to)}

```

Fig. 24. Mod_{best} functional specifications, 3rd part

G PROOF OF MOD_{best} 'S ADHERENCE TO $S_{\text{robust_2}}$

We now describe the poof that MOD_{best} 's adheres to $S_{\text{robust_2}}$: the accompanying Coq formalism includes a mechanized version.

Even though both the implementation and the specification being proven differ from those in §2, the structure of the proofs do retain broad similarities. In particular the proof in this section follows the outline of our reasoning given in Sec. 2.5. Namely, we prove:

- (1) encapsulation of the account's balance and password;
- (2) *per-method Necessity* specifications on all MOD_{best} methods,
- (3) *per-step Necessity* specifications for changing the balance and password, and finally
- (4) the *emergent Necessity* specification $S_{\text{robust_2}}$.

We now discuss each of these four parts of the proof.

G.1 Part 1: Assertion Encapsulation

Using the rules for proving $\text{Enc}_e()$ and $\text{Enc}()$ from Appendix B we prove encapsulation of $b.\text{balance}(a)$ as below

BalanceEncaps:	
aEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(a)$	by $\text{ENC}_e\text{-Obj}$
bEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(b)$	by $\text{ENC}_e\text{-Obj}$
getBalEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(b.\text{balance}(a))$	by aEnc, bEnc, and $\text{ENC}_e\text{-GHOST}$
balEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(\text{bal})$	by $\text{ENC}_e\text{-INT}$
$\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}(b.\text{balance}(a)=\text{bal})$	

We omit the proof of $\text{Enc}(a.\text{password}=\text{pwd})$, as its construction is very similar to that of $\text{Enc}(b.\text{balance}(a)=\text{bal})$.

G.2 Part 2: Per-Method Necessity Specifications

We now provide proofs for per-method specifications, working from method pre- and postconditions. functional specifications. Here we focus on proofs of *authenticate* from the *Account* class.

There are two *per-method Necessity* specifications that we need to prove of *authenticate*:

AuthBalChange: any change to the balance of an account may only occur if call to *transfer* on the *Bank* with the correct password is made. This may seem counter-intuitive as it is not possible to make two method calls (*authenticate* and *transfer*) at the same time, however we are able to prove this by first proving the absurdity that *authenticate* is able to modify any balance.

AuthPwdLeak: any call to *authenticate* may only invalidate *inside(a.password)* (for any account *a*) if *false* is first satisfied – clearly an absurdity.

AuthBalChange. First we use the functional specification of the *authenticate* method in *Account* to prove that a call to *authenticate* can only result in a decrease in balance in a single step if there were in fact a call to *transfer* to the *Bank*. This may seem odd at first, and

G PROOF OF MOD_{best} 'S ADHERENCE TO $S_{\text{robust_2}}$

We now describe the poof that MOD_{best} 's adheres to $S_{\text{robust_2}}$: the accompanying Coq formalism includes a mechanized version.

Even though both the implementation and the specification being proven differ from those in §2, the structure of the proofs do retain broad similarities. In particular the proof in this section follows the outline of our reasoning given in Sec. 2.5. Namely, we prove:

- (1) encapsulation of the account's balance and password;
- (2) *per-method Necessity* specifications on all MOD_{best} methods,
- (3) *per-step Necessity* specifications for changing the balance and password, and finally
- (4) the *emergent Necessity* specification $S_{\text{robust_2}}$.

Mechanised versions of the proofs found in this Appendix can be found in the associated Coq artifact in *bank_account.v*.

We now discuss each of these four parts of the proof.

G.1 Part 1: Assertion Encapsulation

Using the rules for proving $\text{Enc}_e()$ and $\text{Enc}()$ from Appendix B we prove encapsulation of $b.\text{balance}(a)$ as below

BalanceEncaps:	
aEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(a)$	by $\text{ENC}_e\text{-Obj}$
bEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(b)$	by $\text{ENC}_e\text{-Obj}$
getBalEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(b.\text{balance}(a))$	by aEnc, bEnc, and $\text{ENC}_e\text{-GHOST}$
balEnc: $\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}_e(\text{bal})$	by $\text{ENC}_e\text{-INT}$
$\text{Mod}_{\text{best}} \vdash b, b':\text{Bank} \wedge a:\text{Account} \wedge b.\text{balance}(a)=\text{bal} \Rightarrow \text{Enc}(b.\text{balance}(a)=\text{bal})$	

We omit the proof of $\text{Enc}(a.\text{password}=\text{pwd})$, as its construction is very similar to that of $\text{Enc}(b.\text{balance}(a)=\text{bal})$.

G.2 Part 2: Per-Method Necessity Specifications

We now provide proofs for per-method specifications, working from method pre- and postconditions. functional specifications. It said "These proof steps are quite verbose" ... – please do not say that, put it in a positive way Here we focus on proofs of *authenticate* from the *Account* class.

There are two *per-method Necessity* specifications that we need to prove of *authenticate*:

AuthBalChange: any change to the balance of an account may only occur if call to *transfer* on the *Bank* with the correct password is made. This may seem counter-intuitive as it is not possible to make two method calls (*authenticate* and *transfer*) at the same time, however we are able to prove this by first proving the absurdity that *authenticate* is able to modify any balance.

AuthPwdLeak: any call to *authenticate* may only invalidate *inside(a.password)* (for any account *a*) if *false* is first satisfied – clearly an absurdity.

impossible to prove, however we leverage the fact that we are first able to prove that `false` is a necessary condition to decreasing the balance, or in other words, it is not possible to decrease the balance by a call to `authenticate`. We then use the proof rule `ABSD` to prove our desired necessary condition. This proof is presented as `AuthBalChange` below.

AuthBalChange:	
$\{a, a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal}\}$ $a.\text{authenticate}(\text{pwd})$ $\{b.\text{balance}(a') == \text{bal}\}$	by functional specifications
$\{a, a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal} \wedge \neg \text{false}\}$ $a.\text{authenticate}(\text{pwd})$ $\{\neg b.\text{balance}(a') < \text{bal}\}$	by Hoare logic
from $a, a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal} \wedge (_ \text{calls } a.\text{authenticate}(\text{pwd}))$ next $b.\text{balance}(a') < \text{bal}$ onlyIf <code>false</code>	by If1-CLASSICAL
from $a: \text{Account} \wedge a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal} \wedge (_ \text{calls } a.\text{authenticate}(\text{pwd}))$ next $b.\text{balance}(a') < \text{bal}$ onlyIf $(_ \text{calls } b.\text{transfer}(a'.\text{password}, \text{amt}, a', \text{to}))$	by <code>ABSD</code> and If1 \rightarrow

AuthPwdLeak. The proof of `AuthPwdLeak` is given below, and is proven by application of Hoare logic rules and If1-INSIDE.

AuthPwdLeak:	
$\{a: \text{Account} \wedge a': \text{Account} \wedge a.\text{password} == \text{pwd}\}$ $\text{res} = a'.\text{authenticate}(_)$ $\{\text{res} != \text{pwd}\}$	by functional specification
$\{a: \text{Account} \wedge a': \text{Account} \wedge a.\text{password} == \text{pwd} \wedge \neg \text{false}\}$ $\text{res} = a'.\text{authenticate}(_)$ $\{\text{res} != \text{pwd}\}$	by Hoare logic
from <code>inside(pwd) \wedge a, a': Account \wedge a.password=pwd \wedge ($_ \text{calls } a'.\text{authenticate}(_)$)</code> next <code>$\neg \text{inside}(_)$ onlyIf false</code>	by If1-INSIDE

G.3 Part 3: Per-Step Necessity Specifications

The next step is to construct proofs of necessary conditions for *any* possible step in our external state semantics. In order to prove the final result in the next section, we need to prove three per-step Necessity specifications: `BalanceChange`, `PasswordChange`, and `PasswordLeak`.

```

1 BalanceChange  $\triangleq$  from  $a: \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a) = \text{bal}$ 
2 next  $b.\text{balance}(a) < \text{bal}$  onlyIf  $(\_ \text{calls } b.\text{transfer}(a.\text{password}, \_, a, \_))$ 
3
4 PasswordChange  $\triangleq$  from  $a: \text{Account} \wedge a.\text{password} = p$ 
5 next  $\neg a.\text{password} != p$  onlyIf  $(\_ \text{calls } a.\text{changePass}(a.\text{password}, \_))$ 
6
7 PasswordLeak  $\triangleq$  from  $a: \text{Account} \wedge a.\text{password} = p \wedge \text{inside} < p >$ 
8 next  $\neg \text{inside} < p >$  onlyIf false

```

We provide the proofs of these in Appendix F, but describe the construction of the proof of `BalanceChange` here: by application of the rules/results `AuthBalChange`, `changePassBalChange`, `Ledger :: TransferBalChange`, `Bank :: TransferBalChange`, `BalanceEncaps`, and If1-INTERNAL.

G.4 Part 4: Emergent Necessity Specifications

Finally, we combine our module-wide single-step Necessity specifications to prove emergent behaviour of the entire system. Informally the reasoning used in the construction of the proof of `Srobust_2` can be stated as

- (1) If the balance of an account decreases, then by `BalanceChange` there must have been a call to `transfer` in `Bank` with the correct password.

AuthBalChange. First we use the functional specification of the `authenticate` method in `Account` to prove that a call to `authenticate` can only result in a decrease in balance in a single step if there were in fact a call to `transfer` to the `Bank`. This may seem odd at first, and impossible to prove, however we leverage the fact that we are first able to prove that `false` is a necessary condition to decreasing the balance, or in other words, it is not possible to decrease the balance by a call to `authenticate`. We then use the proof rule `ABSD` to prove our desired necessary condition. This proof is presented as `AuthBalChange` below.

AuthBalChange:	
$\{a, a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal}\}$ $a.\text{authenticate}(\text{pwd})$ $\{b.\text{balance}(a') == \text{bal}\}$	by functional specifications
$\{a, a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal} \wedge \neg \text{false}\}$ $a.\text{authenticate}(\text{pwd})$ $\{\neg b.\text{balance}(a') < \text{bal}\}$	by Hoare logic
from $a, a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal} \wedge (_ \text{calls } a.\text{authenticate}(\text{pwd}))$ next $b.\text{balance}(a') < \text{bal}$ onlyIf <code>false</code>	by If1-CLASSICAL
from $a: \text{Account} \wedge a': \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a') = \text{bal} \wedge (_ \text{calls } a.\text{authenticate}(\text{pwd}))$ next $b.\text{balance}(a') < \text{bal}$ onlyIf $(_ \text{calls } b.\text{transfer}(a'.\text{password}, \text{amt}, a', \text{to}))$	by <code>ABSD</code> and If1 \rightarrow

AuthPwdLeak. The proof of `AuthPwdLeak` is given below, and is proven by application of Hoare logic rules and If1-INSIDE.

Do we want to show the other proofs? Or at least list what else is proven?

AuthPwdLeak:	
$\{a: \text{Account} \wedge a': \text{Account} \wedge a.\text{password} == \text{pwd}\}$ $\text{res} = a'.\text{authenticate}(_)$ $\{\text{res} != \text{pwd}\}$	by functional specification
$\{a: \text{Account} \wedge a': \text{Account} \wedge a.\text{password} == \text{pwd} \wedge \neg \text{false}\}$ $\text{res} = a'.\text{authenticate}(_)$ $\{\text{res} != \text{pwd}\}$	by Hoare logic
from <code>inside(pwd) \wedge a, a': Account \wedge a.password=pwd \wedge ($_ \text{calls } a'.\text{authenticate}(_)$)</code> next <code>$\neg \text{inside}(_)$ onlyIf false</code>	by If1-INSIDE

Per-method Specifications on Methods confined Classes. It is notable that proofs of per-method specifications are trivial since the type system prevents external access and thus external method calls objects of `confined` classes. While this does not arise in the example detailed in §5, we use it in this example to prove necessary pre-conditions on methods in `Ledger`. We don't detail these here, however proofs of these Lemmas can be found in `bank_account.v` in the associated Coq artifact.

G.3 Part 3: Per-Step Necessity Specifications

The next step is to construct proofs of necessary conditions for *any* possible step in our external state semantics. In order to prove the final result in the next section, we need to prove three per-step Necessity specifications: `BalanceChange`, `PasswordChange`, and `PasswordLeak`.

```

2344 BalanceChange  $\triangleq$  from  $a: \text{Account} \wedge b: \text{Bank} \wedge b.\text{balance}(a) = \text{bal}$ 
2345 next  $b.\text{balance}(a) < \text{bal}$  onlyIf  $(\_ \text{calls } b.\text{transfer}(a.\text{password}, \_, a, \_))$ 
2346
2347 PasswordChange  $\triangleq$  from  $a: \text{Account} \wedge a.\text{password} = p$ 
2348 next  $\neg a.\text{password} != p$  onlyIf  $(\_ \text{calls } a.\text{changePass}(a.\text{password}, \_))$ 
2349
2350 PasswordLeak  $\triangleq$  from  $a: \text{Account} \wedge a.\text{password} = p \wedge \text{inside} < p >$ 
2351 next  $\neg \text{inside} < p >$  onlyIf false

```


- (2) If there was a call where the `Account`'s password was used, then there must have been an intermediate program state when some external object had access to the password.
- (3) Either that password was the same password as in the starting program state, or it was different:
- (Case A) If it is the same as the initial password, then since by `PasswordLeak` it is impossible to leak the password, it follows that some external object must have had access to the password initially.
- (Case B) If the password is different from the initial password, then there must have been an intermediate program state when it changed. By `PasswordChange` we know that this must have occurred by a call to `changePassword` with the correct password. Thus, there must be a some intermediate program state where the initial password is known. From here we proceed by the same reasoning as (Case A).

S_{robust_2}:	
from a:Account \wedge b:Bank \wedge b.balance(a)=bal to b.balance(a) < bal onlyThrough (calls b.transfer(a.password,_,a_))	by CHANGES and BalanceChange
from a:Account \wedge b:Bank \wedge b.balance(a)=bal to b.balance(a) < bal onlyThrough $\exists o. ((o \text{ external}) \wedge (o \text{ access a.password}))$	by \rightarrow , CALLER-EXT, and CALLS-ARGS
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to b.balance(a) < bal onlyThrough \neg inside(a.password)	by \rightarrow
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to b.balance(a) < bal onlyThrough \neg inside(a.password) \wedge (a.password=pwd \vee a.password != pwd)	by \rightarrow and EXCLUDED MID- DLE
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to b.balance(a) < bal onlyThrough (\neg inside(a.password) \wedge a.password=pwd) \vee (\neg inside(a.password) \wedge a.password != pwd)	by \rightarrow
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to b.balance(a) < bal onlyThrough \neg inside(pwd) \vee a.password != pwd	by \rightarrow
Case A (\neginside(pwd)):	
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to \neg inside(pwd) onlyIf inside(pwd) \vee \neg inside(pwd)	by If- \rightarrow and EXCLUDED MIDDLE
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to \neg inside(pwd) onlyIf \neg inside(pwd)	by \vee E and PasswordLeak
Case B (a.password != pwd):	
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to a.password != pwd onlyThrough (calls a.changePass(pwd,_))	by CHANGES and PASS- WORDCHANGE
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to a.password != pwd onlyThrough \neg inside(pwd)	by \vee E and PasswordLeak
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to a.password != pwd onlyIf \neg inside(pwd)	by Case A and TRANS
from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd to b.balance(a) < bal onlyIf \neg inside(pwd)	by Case A, Case B, If- \vee I ₂ , and If- \rightarrow

We provide the proofs of these in Appendix F, but describe the construction of the proof of `BalanceChange` here: by application of the rules/results `AuthBalChange`, `changePassBalChange`, `Ledger::TransferBalChange`, `Bank::TransferBalChange`, `BalanceEncaps`, and `If1-INTERNAL`. **somehif missing. Where is Appendix F? MUST BE FIXED NOW**

G.4 Part 4: Emergent Necessity Specifications

Finally, we combine our module-wide single-step *Necessity* specifications to prove emergent behaviour of the entire system. Informally the reasoning used in the construction of the proof of `Srobust_2` can be stated as

- (1) If the balance of an account decreases, then by `BalanceChange` there must have been a call to `transfer` in `Bank` with the correct password.
- (2) If there was a call where the `Account`'s password was used, then there must have been an intermediate program state when some external object had access to the password.
- (3) Either that password was the same password as in the starting program state, or it was different:
 - (Case A) If it is the same as the initial password, then since by `PasswordLeak` it is impossible to leak the password, it follows that some external object must have had access to the password initially.
 - (Case B) If the password is different from the initial password, then there must have been an intermediate program state when it changed. By `PasswordChange` we know that this must have occurred by a call to `changePassword` with the correct password. Thus, there must be a some intermediate program state where the initial password is known. From here we proceed by the same reasoning as (Case A).

2402	<i>S</i>robust_2:	
2403	from a:Account \wedge b:Bank \wedge b.balance(a)=bal	by CHANGES and
2404	to b.balance(a) < bal onlyThrough ($_$ calls b.transfer(a.password, $_$,a, $_$))	BalanceChange
2405	from a:Account \wedge b:Bank \wedge b.balance(a)=bal	by \longrightarrow , CALLER-EXT, and
2406	to b.balance(a) < bal onlyThrough \exists c. { (c external) \wedge (c access a.password) }	CALLS-ARGS
2407	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by \longrightarrow
2408	to b.balance(a) < bal onlyThrough \neg inside(a.password)	
2409	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by \longrightarrow and EXCLUDED MID-
2410	to b.balance(a) < bal onlyThrough \neg inside(a.password) \wedge (a.password=pwd \vee a.password != pwd)	DLE
2411	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by \longrightarrow
2412	to b.balance(a) < bal onlyThrough (\neg inside(a.password) \wedge a.password=pwd) \vee	
2413	(\neg inside(a.password) \wedge a.password != pwd)	
2414	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by \longrightarrow
2415	to b.balance(a) < bal onlyThrough \neg inside(pwd) \vee a.password != pwd	
2416	Case A (\neginside(pwd)):	
2417	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by If- \longrightarrow and EXCLUDED
2418	to \neg inside(pwd) onlyIf inside(pwd) \vee \neg inside(pwd)	MIDDLE
2419	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by \vee E and
2420	to \neg inside(pwd) onlyIf \neg inside(pwd)	PasswordLeak
2421	Case B (a.password != pwd):	
2422	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by CHANGES and PASS-
2423	to a.password != pwd onlyThrough ($_$ calls a.changePass(pwd, $_$))	WORDCHANGE
2424	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by \vee E and
2425	to a.password != pwd onlyThrough \neg inside(pwd)	PasswordLeak
2426	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by Case A and TRANS
2427	to a.password != pwd onlyIf \neg inside(pwd)	
2428	from a:Account \wedge b:Bank \wedge b.balance(a)=bal \wedge a.password=pwd	by Case A, Case B, If- \vee I ₂ ,
2429	to b.balance(a) < bal onlyIf \neg inside(pwd)	and If- \longrightarrow
2430		
2431		
2432		
2433		
2434		
2435		
2436		
2437		
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H PROOF OF GUARANTEE OF SAFETY IN §2.3.1

In this section we provide a proof sketch that $S_{\text{robust_2}}$ ensures our balance does not decrease in contexts with no access to our password. This property is expressed in §2.3.1, and the example is repeated below.

```

1 module Mod1
2   ...
3   method cautious(untrusted:Object)
4     a = new Account
4595     p = new Password
4606     a.set(null,p)
4617     ...
4628     untrusted.make_payment(a)
4639     ...

```

The guarantee for the above code snippet is that as long as `untrusted` does not have external access (whether transitive or direct) to `a.pwd` before the call on line 7, then `a.balance` will not decrease during the execution of line 8. This property is expressed and proven in Theorem H.1.

THEOREM H.1 ($S_{\text{robust_2}}$ GUARANTEES ACCOUNT SAFETY). *Let BankMdl be some module that satisfies $S_{\text{robust_2}}$, let M be any external module, and $\sigma_1 = (\chi_1, \phi_1 : \psi_1)$ be some Arising program state, Arising($M, \text{BankMdl}, \sigma_1$).*

If

- *the continuation of ϕ_1 is*

```

a = new Account;
p = new Password;
a.set(null,p);
s;
untrusted.make_payment(a, z1, ..., zn); ...

```

- $\sigma_2 = (\chi_2, \phi_2 : \psi_2)$ is the program state immediately preceding the execution of s
- $\sigma_3 = (\chi_3, \phi_3 : \psi_3)$ is the program state immediately following the execution of s
- $\sigma_4 = (\chi_4, \phi_4 : \psi_4)$ is the program state immediately following the execution of `untrusted.make_payment(a, z1, ..., zn)`
- *for all objects $o \in \chi_3$ which are transitively accessible (i.e. the transitive closure of `_access_`) from `untrusted` or from `z1,...,zn`:*
 $\text{BankMdl}; \sigma_3 \models \langle o \text{ access } a.\text{pwd} \rangle, \text{ implies } \text{BankMdl}; \sigma_3 \models \langle o \text{ internal} \rangle,$
- $\text{BankMdl}; \sigma_3 \models a.\text{balance} = b$

then

- $\text{BankMdl}; \sigma_4 \models a.\text{balance} \geq b.$

Proof Idea

The challenge in constructing a proof is that $S_{\text{robust_2}}$ is not directly applicable to σ_3 since there may exist external objects that have access to `a.password`, depending on the contents of the code in `s`. For example, if `s` is the empty code, then $\sigma_1(\text{this})$ has access to `a`.

To address this challenge, we will create a program state, say σ'_3 . In the new program state σ'_3 there will be no external access to `a.password`. Also, σ'_3 must be similar enough to σ_3 so that the execution of `untrusted.make_payment(a, z1, ..., zn)` in σ_3 is effectively equivalent to that σ'_3 are effectively equivalent. Moreover, σ'_3 must also be *Arising* for us to apply the *Necessity* specification $S_{\text{robust_2}}$ to it. This throws up a new challenge: σ'_3 is not necessarily

Arising in BankMdl and M . We address the latter challenge by creating a new module, M' , such that $\text{Arising}(M', \text{BankMdl}, \sigma'_3)$.

Proof Sketch

We construct M' from M by 1) modifying all methods in all classes in M so that all methods are duplicated: a) the original version, and b) a version almost identical to that in M with the addition that it keeps track of all the objects which contain fields pointing to any objects of the `Password` class, 2) We add to all classes in M a method called `nullify` that compares the contents of each of its fields with the method's argument, and if they are equal overwrites the field with `null`, 3) all method calls are replaced by those in part 1a, except of the body of `make_payment`, 4) we modify the code in `s` (and any methods called from it) so that it also keeps track of the current value of `a.pwd`, 5) after `s` and before the call `untrusted.make_payment(a, z1, ..., zn)` we insert code which runs through the list created in part 1, and calls `nullify` with the current value of `a.pwd` by `null` as its argument.

By starting with the same initial configuration which reached σ_3 , but now using M' as the external module, we reach σ'_3 , that is, $\text{Arising}(M', \text{BankMdl}, \sigma'_3)$. Moreover, σ'_3 satisfies the premise of $S_{\text{robust_2}}$. We execute `untrusted.make_payment(a, z1, ..., zn)` in the context of σ'_3 and reach σ'_4 . We apply $S_{\text{robust_2}}$, and obtain that $\text{BankMdl}; \sigma'_4 \models a.\text{balance} \geq b$.

We use the latter fact, to conclude that $\text{BankMdl}; \sigma_4 \models a.\text{balance} \geq b$. Namely, σ_3 and σ'_3 are equivalent – up to renaming of addresses – for all all the objects which are reachable from `o, z1, ..., zn`, and for all objects from BankMdl . Therefore, the execution of `make_payment` in $M; \text{BankMdl}$ and σ_3 will be "equivalent" to that in $M'; \text{BankMdl}$ and σ'_3 . Therefore, σ_4 and σ'_4 are equivalent – up to renaming of addresses – for all all the objects which are reachable from `o, z1, ..., zn` and for all objects from BankMdl . This gives us that $\text{BankMdl}; \sigma_4 \models a.\text{balance} \geq b$.