

# Reasoning about External Calls

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In today's complex software, internal trusted code is tightly intertwined with external untrusted code. To reason about internal code, programmers must reason about the potential effects of calls to external code, even though that code is not trusted and may not even be available. The effects of external calls can be *limited* if internal code is programmed defensively, limiting potential effects by limiting access to the capabilities necessary to cause those effects.

This paper addresses the specification and verification of internal code that relies on encapsulation and object capabilities to limit the effects of external calls. We propose new assertions for access to capabilities, new specifications for limiting effects, and a Hoare logic to verify that a module satisfies its specification, even while making external calls. We illustrate the approach through a running example with mechanised proofs, and prove soundness of the Hoare logic.

CCS Concepts: • **Software and its engineering** → *Access protection*; **Formal software verification**; • **Theory of computation** → *Hoare logic*; • **Object oriented programming** → *Object capabilities*.

Additional Key Words and Phrases: Object Capabilities, Untrusted Code, Program Verification.

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## 1 Introduction

This paper addresses reasoning about *external calls* — when trusted internal code calls out to untrusted, unknown external code: In the code sketch to the right, an internal module,  $M_{intl}$ , has two methods. Method `m2` takes an untrusted parameter `untrst`, and calls an unknown external method `unkn` passing itself as an argument. The challenge is: What effects will that external call have? What if `untrst` calls back into  $M_{intl}$ ?

```
module  $M_{intl}$ 
  method m1 ..
    ... trusted code ...
  method m2(untrst:external)
    ... trusted code ...
    untrst.unkn(this)
    ... trusted code ...
```

*External calls need not have arbitrary effects.* If the programming language supports encapsulation (e.g. no address forging, privacy, etc.) then internal modules can be written so that effects are

- *Precluded*, i.e. guaranteed to *never happen*. E.g., a correct implementation of the DAO [24] can ensure that its balance never falls below the sum of the balances of its subsidiary accounts, *or*

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- *Limited*, i.e. they *may happen*, but only in well-defined circumstances. E.g., while the DAO does not preclude that a signatory's balance will decrease, it does ensure that the balance decreases only as a direct consequence of calls from the signatory.

Precluded effects are special case of limited effects, and have been studied extensively in the context of object invariants [8, 40, 66, 89, 108]. In this paper, we tackle the more general, and more subtle case of reasoning about limited effects for external calls.

*The Object Capability Model* combines the capability model of operating system security [68, 115] with pure object-oriented programming [1, 106, 110]. Capability-based operating systems reify resources as *capabilities* — unforgeable, distinct, duplicable, attenuable, communicable bitstrings which both denote a resource and grant rights over that resource. Effects can only be caused by invoking capabilities: controlling effects reduces to controlling capabilities.

Mark Miller's [83] *object-capability* model treats object references as capabilities. Building on early object-capability languages such as E [83, 86] and Joe-E [80], a range of recent programming languages and web systems [19, 53, 101] including Newspeak [16], AmbientTalk [32] Dart [15], Grace [11, 59], JavaScript (aka Secure EcmaScript [85]), and Wyvern [79] have adopted the object capability model. Security and encapsulation is encoded in the relationships between the objects, and the interactions between them. As argued in [42], object capabilities make it possible to write secure programs, but cannot by themselves guarantee that any particular program will be secure.

*Logics for Reasoning with Capabilities* have been proposed in [70, 98, 109, 112], primarily in object-based settings — our work operates in a class-based setting. More importantly, in these works, capabilities are statically embedded within objects, and authority is conferred through the object's interface. In contrast, our approach models capabilities in a manner that is, in some sense, the dual of theirs: capabilities are not intrinsic to objects but may be dynamically accessible to the callee—typically via ambient authority in the execution environment.

This distinction carries important implications for reasoning about effects. In [70, 98, 109, 112], one can argue that if an object is exported with an interface that omits certain capabilities, then the corresponding effects are ruled out. However, this argument hinges on the assumption that the exporting site has precise knowledge of the object's embedded capabilities. Our model requires no such assumption. Instead, we reason about potential effects based on the capabilities available in the receiving context. As long as the target context lacks access to specific capabilities, we can guarantee that the corresponding effects are unobservable —regardless of the object's interface.

Our earlier work [43, 75] follows a similar dynamic model of object capabilities, though without addressing external effects. Specifically, [43] introduces “holistic specifications” to capture a module's emergent behavior, while [75] proposes a logic for proving that modules devoid of external calls conform to their holistic specifications.

*This paper contributes* (1) *protection assertions* to limit access to object-capabilities, (2) a specification language to define how limited access to capabilities should limit effects, (3) a Hoare logic to reason about external calls and to prove that modules satisfy their specifications, (4) proof of soundness, (5) a worked illustrative example with a mechanised proof in Coq.

*Structure of this paper:* Sect. 2 outlines the main ingredients of our approach in terms of an example. Sect. 3 outlines a simple object-oriented language used for our work. Sect. 4 and 5 give syntax and semantics of assertions and specifications. Sect. 6 develops a Hoare logic to prove external calls, and modules' adherence to specifications, and summarises our proof of the running example (Coq at [74]). Sect. 7 outlines proof of soundness of the Hoare logic. Sect. 8 concludes. Full technical details can be found in the appendices in the extended version [41].

## 2 The problem and our approach

We introduce the problem through an example, and outline our approach. We work with a small, class-based object-oriented, sequential language similar to Joe-E [80] with modules, module-private fields (accessible only from methods from the same module), and unforgeable, un-enumerable addresses. We distinguish between *internal objects* — instances of our internal module  $M$ 's classes — and *external objects* defined in *any* number of external modules  $\bar{M}$ .<sup>1</sup> Private methods may only be called by objects of the same module, while public methods may be called by *any* object with a reference to the method receiver, and with actual arguments of dynamic types that match the declared formal parameter types.<sup>2</sup>

We are concerned with guarantees made in an *open* setting: Our internal module  $M$  must be programmed so that execution of  $M$  together with *any* unknown, arbitrary, external modules  $\bar{M}$  will satisfy these guarantees — without relying on any assumptions about  $\bar{M}$ 's code.<sup>3</sup> All we can rely on, is the guarantee that external code interacts with the internal code only through public methods; such a guarantee may be given by the programming language or by the underlying platform.<sup>4</sup>

### Shop — illustrating limited effects

Consider below the internal module  $M_{shop}$ , with classes `Item`, `Shop`, `Account`, and `Inventory`. Classes `Inventory` and `Item` are straightforward: we elide their details. Accounts hold a balance and have a key. Access to an `Account`, allows one to pay money into it, and access to an `Account` and its `Key`, allows one to withdraw money from it. A `Shop` has an `Account`, and a public method `buy` to allow a buyer — an external object — to buy and pay for an `Item`:

```

1 module Mshop
2   ...
3   class Shop
4     field acct:Account, invntry:Inventory, clients:external
5
6     public method buy(buyer:external, anItem:Item)
7       int price = anItem.price
8       int oldBlnc = this.acct.blnc
9       buyer.pay(this.acct, price)
10      if (this.acct.blnc == oldBlnc+price)
11        this.send(buyer, anItem)
12      else
13        buyer.tell("you have not paid me")
14
15      private method send(buyer:external, anItem:Item)    ....

```

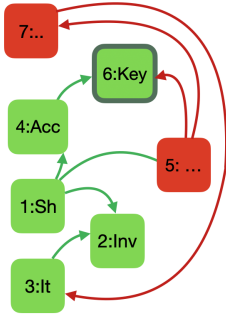
The sketch below shows a possible heap snippet.

<sup>1</sup>We use the notation  $\bar{z}$  for a sequence of  $z$ , i.e. for  $z_1, z_2, \dots, z_n$

<sup>2</sup>As in Joe-E, we leverage module-based privacy to restrict propagation of capabilities, and reduce the need for reference monitors etc, c.f. Sect 3 in [80].

<sup>3</sup>This is a critical distinction from e.g. cooperative approaches such as rely/guarantee [52, 111].

<sup>4</sup>Thus our approach is also applicable to inter-language safety.



External objects are red; internal objects are green. Each object has a number, and an abbreviated class name:  $o_1$ ,  $o_2$  and  $o_5$  are a Shop, an Inventory, and an external object. Curved arrows indicate field values:  $o_1$  has three fields, pointing to  $o_4$ ,  $o_5$  and  $o_2$ . Fields denote direct access. The transitive closure of direct access gives indirect (transitive) access:  $o_1$  has direct access to  $o_4$ , and indirect access to  $o_6$ . Object  $o_6$  — highlighted with a dark outline — is the key capability that allows withdrawal from  $o_4$ . The critical point in our code is the external call on line 9, where the Shop asks the buyer to pay the price of that item, by calling `pay` on `buyer` and passing the Shop’s account as an argument.

As `buyer` is an external object, the module  $M_{shop}$  has no method specification for `pay`, and no certainty about what its implementation might do. What are the possible effects of that external call? At line 9, the Shop hopes the buyer will have deposited the price into its account, but needs to be certain the buyer cannot have emptied that account instead. Can the Shop be certain?

Indeed, if

- (A) Prior to the call of `buy`, the `buyer` has no eventual access to the account’s key, — and
- (B)  $M_{shop}$  ensures that
  - (a) access to keys is not leaked to external objects, — and
  - (b) funds cannot be withdrawn unless the external entity responsible for the withdrawal (eventually) has access to the account’s key, — then
- (C) The external call on line 9 can never result in a decrease in the shop’s account balance.

The remit of this paper is to provide specification and verification tools that support arguments like the one above. This gives rise to the following two challenges: 1<sup>st</sup>: A specification language which describes access to capabilities and limited effects, 2<sup>nd</sup>: A Hoare Logic for adherence to such specifications.

## 2.1 1<sup>st</sup> Challenge: Specification Language

We want to give a formal meaning to the guarantee that for some effect,  $E$ , and an object  $o_c$  which is the capability for  $E$ :

$E$  (e.g. the account’s balance decreases) can be caused only by external objects calling

- (\*) methods on internal objects, and only if the causing object has access to  $o_c$  (e.g. the key).

The first task is to describe that effect  $E$  took place: if we find some assertion  $A$  (e.g. balance is  $\geq$  some value  $b$ ) which is invalidated by  $E$ , then, (\*) can be described by something like:

- (\*\*) If  $A$  holds, and if no external access to  $o_c$  then  $A$  holds in the future.

We next make more precise that “no external access to  $o_c$ ”, and that “ $A$  holds in the future”.

In a first attempt, we could say that “no external access to  $o_c$ ” means that no external object exists, nor will any external objects be created. This is too strong, however: it defines away the problem we are aiming to solve.

In a second attempt, we could say that “no external access to  $o_c$ ” means that no external object has access to  $o_c$ , nor will ever get access to  $o_c$ . This is also too strong, as it would preclude  $E$  from ever happening, while our remit is that  $E$  may happen but only under certain conditions.

This discussion indicates that the lack of external access to  $o_c$  is not a global property, and that the future in which  $A$  will hold is not permanent. Instead, they are both defined *from the perspective of the current point of execution*.

Thus:

If  $A$  holds, and if no external object reachable from the current point of execution has access to  $o_c$ , and if no internal objects pass  $o_c$  to external objects, then

$A$  holds in the future scoped by the current point of execution.

We will shortly formalize “reachable from the current point of execution” as *protection* in §2.1.1, and then “future scoped by the current point of execution” as *scoped invariants* in §2.1.2. Both of these definitions are in terms of the “current point of execution”:

The *Current Point of Execution* is characterized by the heap, and the activation frame of the currently executing method. Activation frames (frames for short) consist of a variable map and a continuation – the statements remaining to be executed in that method. Method calls push frames onto the stack of frames; method returns pop frames off. The frame on top of the stack (the most recently pushed frame) belongs to the currently executing method.

Fig. 1 illustrates the current point of execution. The left pane,  $\sigma_1$ , shows a state with the same heap as earlier, and where the top frame is  $\phi_1$  – it could be the state before a call to `buy`. The middle pane,  $\sigma_2$ , is a state where we have pushed  $\phi_2$  on top of the stack of  $\sigma_1$  – it could be a state during execution of `buy`. The right pane,  $\sigma_3$ , is a state where we have pushed  $\phi_3$  on top of the stack of  $\sigma_2$  – it could be a state during execution of `pay`.

States whose top frame has a receiver (`this`) which is an internal object are called *internal states*, the other states are called *external states*. In Fig 1, states  $\sigma_1$  and  $\sigma_2$  are internal, and  $\sigma_3$  is external.

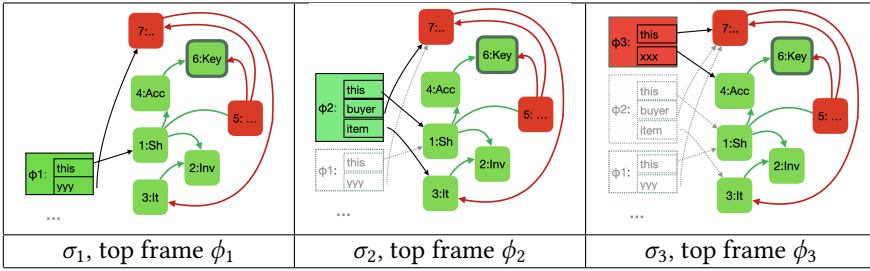


Fig. 1. The current point of execution before `buy`, during `buy`, and during `pay`. Frames  $\phi_1, \phi_2$  are green as their receiver (`this`) is internal;  $\phi_3$  is red as its receiver is external. Continuations are omitted.

### 2.1.1 Protection.

**Protection** Object  $o$  is *protected from* object  $o'$ , formally  $\langle o \rangle \not\Leftarrow o'$ , if no external object indirectly accessible from  $o'$  has direct access to  $o$ . Object  $o$  is *protected*, formally  $\langle o \rangle$ , if no external object indirectly accessible from the current frame has direct access to  $o$ , and if the receiver is external then  $o$  is not an argument.<sup>5</sup> More in Def. 4.4.

Fig. 2 illustrates *protected* and *protected from*. Object  $o_6$  is not protected in states  $\sigma_1$  and  $\sigma_2$ , but is protected in state  $\sigma_3$ . This is so, because the external object  $o_5$  is indirectly accessible from the top frame in  $\sigma_1$  and in  $\sigma_2$ , but not from the top frame in  $\sigma_3$  – in general, calling a method (pushing a frame) can only ever *decrease* the set of indirectly accessible objects. Object  $o_4$  is protected in states  $\sigma_1$  and  $\sigma_2$ , and not protected in state  $\sigma_3$  because though neither object  $o_5$  nor  $o_7$  have direct access to  $o_4$ , in state  $\sigma_3$  the receiver is external and  $o_4$  is one of the arguments.

<sup>5</sup>An object has direct access to another object if it has a field pointing to the latter; it has indirect access to another object if there exists a sequence of field accesses (direct references) leading to the other object; an object is indirectly accessible from the frame if one of the frame’s variables has indirect access to it.

heap	$\sigma_1$	$\sigma_2$	$\sigma_3$
$\dots \models \langle o_6 \rangle \leftarrow * o_4$	$\sigma_1 \not\models \langle o_6 \rangle$	$\sigma_2 \not\models \langle o_6 \rangle$	$\sigma_3 \models \langle o_6 \rangle$
$\dots \not\models \langle o_6 \rangle \leftarrow * o_5$	$\sigma_1 \models \langle o_4 \rangle$	$\sigma_2 \models \langle o_4 \rangle$	$\sigma_3 \not\models \langle o_4 \rangle$

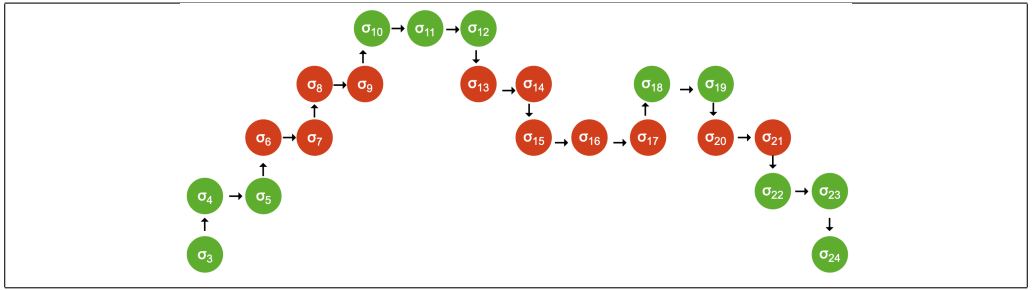
Fig. 2. *Protected from and Protected.* – continuing from Fig. 1.

If a protected object  $o$  is never passed to external objects (*i.e.* never leaked) then  $o$  will remain protected during the whole execution of the current method, including during any nested calls. This is the case even if  $o$  was not protected before the call to the current method. We define *scoped invariants* to describe property preservation within the current and all its nested calls.

**2.1.2 Scoped Invariants.** We build on the concept of history invariants [27, 67, 69] and define:

**Scoped invariants**  $\forall \bar{x} : \bar{C}. \{A\}$  expresses that if an external state  $\sigma$  has objects  $\bar{x}$  of class  $\bar{C}$ , and satisfies  $A$ , then all external states which are part of the *scoped future* of  $\sigma$  will also satisfy  $A$ . The scoped future contains all states which can be reached through any program execution steps, including further method calls and returns, but stopping just before returning from the call active in  $\sigma$ <sup>6</sup> – *c.f.* Def 3.2.

Fig. 3 shows the states of an unspecified execution starting at internal state  $\sigma_3$  and terminating at internal state  $\sigma_{24}$ . It distinguishes between steps within the same method ( $\rightarrow$ ), method calls ( $\uparrow$ ), and method returns ( $\downarrow$ ). The scoped future of  $\sigma_6$  consists of  $\sigma_6$ - $\sigma_{21}$ , but does not contain  $\sigma_{22}$  onwards, since scoped future stops before returning. Similarly, the scoped future of  $\sigma_9$  consists of  $\sigma_9$ ,  $\sigma_{10}$ ,  $\sigma_{11}$ ,  $\sigma_{12}$ ,  $\sigma_{13}$ , and  $\sigma_{14}$ , and does not include, *e.g.*,  $\sigma_{15}$ , or  $\sigma_{18}$ .

Fig. 3. *Execution.* Green disks represent internal states; red disks external states.

The scoped invariant  $\forall \bar{x} : \bar{C}. \{A_0\}$  guarantees that if  $A_0$  holds in  $\sigma_8$ , then it will also hold in  $\sigma_9$ ,  $\sigma_{13}$ , and  $\sigma_{14}$ ; it doesn't have to hold in  $\sigma_{10}$ ,  $\sigma_{11}$ , and  $\sigma_{12}$  as these are internal states. Nor does it have to hold at  $\sigma_{15}$  as it is not part of  $\sigma_9$ 's scoped future.

**Example 2.1.** The following scoped invariants

$$S_1 \triangleq \forall a : \text{Account}. \{\langle a \rangle\} \quad S_2 \triangleq \forall a : \text{Account}. \{\langle a.\text{key} \rangle\}$$

$$S_3 \triangleq \forall a : \text{Account}, b : \text{int}. \{\langle a.\text{key} \rangle \wedge a.\text{blnce} \geq b\}$$

guarantee that accounts are not leaked ( $S_1$ ), keys are not leaked ( $S_2$ ), and that the balance does not decrease unless there is unprotected access to the key ( $S_3$ ).

This example illustrates three crucial properties of our invariants:

<sup>6</sup>Here lies the difference to history invariants, which consider *all* future states, including returning from the call active in  $\sigma$ .

**Conditional:** They are *preserved*, but unlike object invariants, they do not always hold. *E.g.*, `buy` cannot assume  $\langle a.\text{key} \rangle$  holds on entry, but guarantees that if it holds on entry, then it will still hold on exit.

**Scoped:** They are preserved during execution of a specific method but not beyond its return. It is, in fact, expected that the invariant will eventually cease to hold after its completion. For instance, while  $\langle a.\text{key} \rangle$  may currently hold, it is possible that an earlier (thus quiescent) method invocation frame has direct access to `a.key` – without such access, `a` would not be usable for payments. Once control flow returns to the quiescent method (*i.e.* enough frames are popped from the stack)  $\langle a.\text{key} \rangle$  will no longer hold.

**Modular:** They describe externally observable effects (*e.g.* `key` stays protected) across whole modules, rather than the individual methods (*e.g.* `set`) making up a module's interface. Our example specifications will characterize *any* module defining accounts with a `blnce` and a `key` – even as ghost fields – irrespective of their APIs.

**Example 2.2.** We now use the features from the previous section to specify methods.

```
S4 ≜ { ⟨this.accnt.key⟩↔× buyer ∧ this.accnt.blnc = b }
      public Shop :: buy(buyer : external, anItem : Item)
      { this.accnt.blnc ≥ b } || { ... }
```

$S_4$  guarantees that if the `key` was protected from `buyer` before the call, then the balance will not decrease<sup>7</sup>. It does *not* guarantee `buy` will only be called when  $\langle \text{this.accnt.key} \rangle \leftrightarrow \times \text{buyer}$  holds: as a public method, `buy` can be invoked by external code that ignores all specifications.

**Example 2.3.** We illustrate the meaning of our specifications using three versions ( $M_{\text{good}}$ ,  $M_{\text{bad}}$ , and  $M_{\text{fine}}$ ) of the  $M_{\text{shop}}$  module [75]; these all share the same `transfer` method to withdraw money:

```
module Mgood
  class Shop ... as earlier ...
  class Account
    field blnce:int
    field key:Key
    public method transfer(dest:Account, key':Key, amt:nat)
      if (this.key==key') this.blnc-=amt; dest.blnc+=amt
    public method set(key':Key)
      if (this.key==null) this.key=key'
```

Now consider modules  $M_{\text{bad}}$  and  $M_{\text{fine}}$ , which differ from  $M_{\text{good}}$  only in their `set` methods. Whereas  $M_{\text{good}}$ 's `key` is fixed once it is `set`,  $M_{\text{bad}}$  allows any client to `set` an account's `key` at any time, while  $M_{\text{fine}}$  requires the existing `key` in order to replace it.

```
module Mbad                                module Mfine
  ... as earlier ...                        ... as earlier ...
  public method set(key':Key)                public method set(key',key'':Key)
    this.key=key'                            if (this.key==key') this.key=key''
```

Thus, in all three modules, the `key` is a capability which *enables* the withdrawal of the money. Moreover, in  $M_{\text{good}}$  and  $M_{\text{fine}}$ , the `key` capability is a necessary precondition for withdrawal of money, while in  $M_{\text{bad}}$  it is not. Using  $M_{\text{bad}}$ , it is possible to start in a state where the account's `key` is unknown, modify the `key`, and then withdraw the money. Code such as

```
k=new Key; acc.set(k); acc.transfer(rogue_accnt,k,1000)
```

is enough to drain `acc` in  $M_{\text{bad}}$  without knowing the `key`. Even though `transfer` in  $M_{\text{bad}}$  is “safe”

<sup>7</sup>We ignore the ... for the time being.



when considered in isolation, it is not safe when considered in conjunction with other methods from the same module.

$M_{good}$  and  $M_{fine}$  satisfy  $S_2$  and  $S_3$ , while  $M_{bad}$  satisfies neither. So if  $M_{bad}$  was required to satisfy either  $S_2$  or  $S_3$  then it would be rejected by our inference system as not safe. None of the three versions satisfy  $S_1$  because `pay` could leak an `Account`.

## 2.2 2<sup>nd</sup> Challenge: A Hoare logic for adherence to specifications

*Hoare Quadruples.* Scoped invariants require quadruples, rather than classical triples. Specifically,

$$\forall x : \overline{C}. \{A\}$$

asserts that if an external state  $\sigma$  satisfies  $x : \overline{C} \wedge A$ , then all its *scoped* external future states will also satisfy  $A$ . For example, if  $\sigma$  was an external state executing a call to `Shop : :buy`, then a *scoped* external future state could be reachable during execution of the call `pay`. This implies that we consider not only states at termination but also external states reachable *during* execution of statements. To capture this, we extend traditional Hoare triples to quadruples of form

$$\{A\} \text{ stmt } \{A'\} \parallel \{A''\}$$

promising that if a state satisfies  $A$  and executes *stmt*, any terminating state will satisfy  $A'$ , and and any intermediate external states reachable during execution of *stmt* satisfy  $A''$  – c.f. Def. 5.2.

We assume an underlying Hoare logic of triples  $M \vdash_{ul} \{A\} \text{ stmt } \{A'\}$ , which does not have the concept of protection – that is, the assertions  $A$  in the  $\vdash_{ul}$ -triples do not mention protection. We then embed the  $\vdash_{ul}$ -logic into the quadruples logic ( $\vdash_{ul}$ -triples whose statements do not contain method calls give rise to quadruples in our logic – see rule below). We extend assertions  $A$  so they may mention protection and add rules about protection (e.g. newly created objects are protected – see rule below), and add usual conditional and substructural rules. More in Fig. 6 and Fig. 15 in [41].

$$\frac{M \vdash_{ul} \{A\} \text{ stmt } \{A'\} \quad \text{stmt calls no methods}}{M \vdash \{A\} \text{ stmt } \{A'\} \parallel \{A''\}} \quad \frac{}{M \vdash \{true\} u = \text{new } C \{ \langle u \rangle \} \parallel \{A\}}$$

*Well-formed modules and External Calls.* A module is well-formed, if its specification is well-formed, its public methods preserve the module’s scoped invariants, and all methods satisfy their specifications – c.f. Fig. 8. E.g., to prove that `Shop : :buy` satisfies  $S_4$ , taking  $stmts_b$  for the body of `buy`, we have to prove:

$$(0) \quad \frac{\{ \text{buyer} : \text{extl} \wedge \langle \text{this}.\text{acct}.\text{key} \rangle \ltimes \text{buyer} \wedge \text{this}.\text{acct}.\text{blnce} = b \} \text{ stmts}_b}{\{ \text{this}.\text{acct}.\text{blnce} \geq b \} \parallel \{ \dots \}}$$

The proof proceeds using our Hoare quadruples logic. The treatment of external calls is of special interest. Here, the challenge is the external call on line 9. We need to establish the Hoare quadruple:

$$(1) \quad \frac{\{ \text{buyer} : \text{extl} \wedge \langle \text{this}.\text{acct}.\text{key} \rangle \ltimes \text{buyer} \wedge \text{this}.\text{acct}.\text{blnce} = b \} \text{ buyer.pay}(\text{this}.\text{acct}, \text{price})}{\{ \text{this}.\text{acct}.\text{blnce} \geq b \} \parallel \{ \dots \}}$$

which says that if the shop’s account’s key is protected from `buyer`, then the account’s balance will not decrease after the call of `pay`.

To prove (1), we aim to use  $S_3$ , but this is not straightforward:  $S_3$  requires  $\langle \text{this}.\text{acct}.\text{key} \rangle$ , which is not provided by the precondition of (1). More alarmingly,  $\langle \text{this}.\text{acct}.\text{key} \rangle$  may *not hold* at the time of the call. For example, in state  $\sigma_2$  (Fig. 2), which could initiate the call to `pay`, we have  $\sigma_2 \models \langle o_4.\text{key} \rangle \ltimes o_7$ , but  $\sigma_2 \not\models \langle o_4.\text{key} \rangle$ .

Fig. 2 provides insights into addressing this issue. Upon entering the call, in state  $\sigma_3$ , we find that  $\sigma_3 \models \langle o_4.\text{key} \rangle$ . More generally, if  $\langle \text{this}.\text{acct}.\text{key} \rangle \ltimes \text{buyer}$  holds before the call to `pay`, then  $\langle \text{this}.\text{acct}.\text{key} \rangle$  holds upon entering the call. This is because any objects indirectly



accessible during `pay` must have been indirectly accessible from the call's receiver (`buyer`) or arguments (`this.accnt` and `price`) when `pay` was called.

In general, if  $\langle x \rangle \leftarrow^* y_i$  holds for all  $y_i$ , before a call  $y_0.m(y_1, \dots, y_n)$ , then  $\langle x \rangle$  holds upon entering the call. Here we have  $\langle \text{this.accnt.key} \rangle \leftarrow^* \text{buyer}$  by precondition. We also have that `price` is a scalar and therefore  $\langle \text{this.accnt.key} \rangle \leftarrow^* \text{price}$ . And the type information gives that all fields transitively accessible from an `Account` are scalar or internal; this gives that  $\langle \text{this.accnt.key} \rangle \leftarrow^* \text{this.accnt}$ . This enables the application of  $S_3$  in (1). The corresponding Hoare logic rule is shown in Fig. 7.

## Summary

In an open world, external objects can execute arbitrary code, invoke any public internal methods, access any other external objects, and even collude with each another. The external code may be written in the same or a different programming language than the internal code – all we need is that the platform protects direct external read/write of the internal private fields, while allowing indirect manipulation through calls of public methods.

The conditional and scoped nature of our invariants is critical to their applicability. Protection is a local condition, constraining accessible objects rather than imposing a structure across the whole heap. Scoped invariants are likewise local: they do not preclude some effects from the whole execution of a program, rather the effects are precluded only in some local contexts. While `a.blnc` may decrease in the future, this can only happen in contexts where an external object has direct access to `a.key`. Enforcing these local conditions is the responsibility of the internal module: precisely because these conditions are local, they can be enforced locally within a module, irrespective of all the other modules in the open world.

## 3 The underlying programming language $\mathcal{L}_{ul}$

### 3.1 $\mathcal{L}_{ul}$ syntax and runtime configurations

This work is based on  $\mathcal{L}_{ul}$ , a minimal, imperative, sequential, class based, typed, object-oriented language. We believe, however, that the work can easily be adapted to any capability safe language with some form of encapsulation, and that it can also support inter-language security, provided that the platform offers means to protect a module's private state; cf capability-safe hardware as in Cheri [31]. Wrt to encapsulation and capability safety,  $\mathcal{L}_{ul}$  supports private fields, private and public methods, unforgeable addresses, and no ambient authority (no static methods, no address manipulation). To reduce the complexity of our formal models, as is usually done, e.g. [34, 57, 95],  $\mathcal{L}_{ul}$  lacks many common languages features, omitting static fields and methods, interfaces, inheritance, subsumption, exceptions, and control flow. In our examples, we use numbers and booleans – these can be encoded.

Fig. 4 shows the  $\mathcal{L}_{ul}$  syntax. Statements, *stmt*, are three-address instructions, method calls, or empty,  $\epsilon$ . Expressions, *e*, are ghost code; as such, they may appear in assertions but not in statements, and have no side-effects [22, 45]. Expressions may contain fields, *e.f*, or ghost-fields,  $e_0.gf(\bar{e})$ . The meaning of *e* is module-dependent; e.g. `a.blnc` is a field lookup in  $M_{good}$ , but in a module which stores balances in a table it would be a recursive lookup through that table – c.f. example in §A.3 in [41].<sup>8</sup> In line with most tools, we support ghost-fields, but they are not central to our work.

$\mathcal{L}_{ul}$  states,  $\sigma$ , consist of a heap  $\chi$  and a stack. A stack is a sequence of frames,  $\phi_1 \dots \phi_n$ . A frame,  $\phi$ , consists of a local variable map and a continuation, i.e. the statements to be executed. The top frame, i.e. the frame most recently pushed onto the stack, in a state  $(\phi_1 \dots \phi_n, \chi)$  is  $\phi_n$ .

*Notation.* We adopt the following unsurprising notation:

<sup>8</sup>For convenience, *e.gf* is short for *e.gf()*. Thus, *e.gf* may be simple field lookup in some modules, or ghost-field in others.

$Mdl$	$::= \overline{C} \mapsto CDef$	<b>Module Def.</b>	$fld$	$::= \text{field } f : T$	<b>Field Def.</b>
$CDef$	$::= \text{class } C \{ \overline{fld}; \overline{meth}; \overline{gfld}; \}$	<b>Class Def.</b>	$T$	$::= C$	<b>Type</b>
$meth$	$::= p \text{ method } m (\overline{x:T}): T \{ s \}$	<b>Method Def.</b>	$p$	$::= \text{private} \mid \text{public}$	<b>Privacy</b>
$stmt$	$::= x := y \mid x := v \mid x := y.f \mid x.f := y \mid x := y_0.m(\overline{y}) \mid x := \text{new } C \mid stmt; stmt \mid \epsilon$				
$gfld$	$::= \text{ghost } gf(\overline{x:T}) \{ e \} : T$				<b>Ghosts</b>
$e$	$::= x \mid v \mid e.f \mid e.gf(\overline{e})$				
$\sigma$	$::= (\overline{\phi}, \chi)$	<b>Program State</b>	$C, f, m, gf, x, y$	$::= \text{Identifier}$	
$\phi$	$::= (\overline{x \mapsto v}; s)$	<b>Frame</b>	$o$	$::= (C; \overline{f \mapsto v})$	<b>Object</b>
$\chi$	$::= (\overline{\alpha \mapsto o})$	<b>Heap</b>	$v$	$::= \alpha \mid \text{null}$	<b>Value</b>

Fig. 4.  $\mathcal{L}_{ul}$  Syntax. We use  $x, y, z$  for variables,  $C, D$  for class identifiers,  $f$  for field identifier,  $gf$  for ghost field identifiers,  $m$  for method identifiers,  $\alpha$  for addresses.

- An object is uniquely identified by the address that points to it. We shall be talking of objects  $o, o'$  when talking less formally, and of addresses,  $\alpha, \alpha', \alpha_1, \dots$  when more formal.
- $x, x', y, z, u, v, w$  are variables;  $dom(\phi)$  and  $Rng(\phi)$  indicate the variable map in  $\phi$ ;  $dom(\sigma)$  and  $Rng(\sigma)$  indicate the variable map in the top frame in  $\sigma$
- $\alpha \in \sigma$  means that  $\alpha$  is defined in the heap of  $\sigma$ , and  $x \in \sigma$  means that  $x \in dom(\sigma)$ . Conversely,  $\alpha \notin \sigma$  and  $x \notin \sigma$  have the obvious meanings.  $[\alpha]_\sigma$  is  $\alpha$ ; and  $[x]_\sigma$  is the value to which  $x$  is mapped in the top-most frame of  $\sigma$ 's stack, and  $[e.f]_\sigma$  looks up in  $\sigma$ 's heap the value of  $f$  for the object  $[e]_\sigma$ .
- $\phi[x \mapsto \alpha]$  updates the variable map of  $\phi$ , and  $\sigma[x \mapsto \alpha]$  updates the top frame of  $\sigma$ .  $A[e/x]$  is textual substitution where we replace all occurrences of  $x$  in  $A$  by  $e$ .
- As usual,  $\overline{q}$  stands for sequence  $q_1, \dots, q_n$ , where  $q$  can be an address, a variable, a frame, an update or a substitution. Thus,  $\sigma[\overline{x \mapsto \alpha}]$  and  $A[\overline{e/y}]$  have the expected meaning.
- $\phi.\text{cont}$  is the continuation of frame  $\phi$ , and  $\sigma.\text{cont}$  is the continuation in the top frame.
- $text_1 \stackrel{\text{txt}}{=} text_2$  expresses that  $text_1$  and  $text_2$  are the same text.
- We define the depth of a stack as  $|\phi_1 \dots \phi_n| \triangleq n$ . For states,  $|(\overline{\phi}, \chi)| \triangleq |\overline{\phi}|$ . The operator  $\sigma[k]$  truncates the stack up to the  $k$ -th frame:  $(\phi_1 \dots \phi_k \dots \phi_n, \chi)[k] \triangleq (\phi_1 \dots \phi_k, \chi)$
- $Vs(stmt)$  returns the variables which appear in  $stmt$ . For example,  $Vs(u := y.f) = \{u, y\}$ .

### 3.2 $\mathcal{L}_{ul}$ Execution

Fig. 5 describes  $\mathcal{L}_{ul}$  execution by a small steps operational semantics with shape  $\overline{M}; \sigma \rightarrow \sigma'$ .  $\overline{M}$  stands for one or more modules, where a module,  $M$ , maps class names to class definitions. The functions  $classOf(\sigma, x)$ ,  $Meth(\overline{M}, C, m)$ ,  $fields(\overline{M}, C)$ ,  $SameModule(x, y, \sigma, \overline{M})$ , and  $Prms(\sigma, \overline{M})$ , return the class of  $x$ , the method  $m$  for class  $C$ , the fields for class  $C$ , whether  $x$  and  $y$  belong to the same module, and the formal parameters of the method currently executing in  $\sigma$  – c.f. Defs A.2 – A.7 in [41]. Initial states,  $Initial(\sigma)$ , contain a single frame with single variable `this` pointing to a single object in the heap and a continuation, c.f. A.8 in [41].

The semantics is unsurprising: The top frame's continuation ( $\sigma.\text{cont}$ ) contains the statement to be executed next. We dynamically enforce a simple form of module-wide privacy: Fields may be read or written only if they belong to an object (here  $y$ ) whose class comes from the same module as the class of the object reading or writing the fields (`this`).<sup>9</sup> Wlog, to simplify some proofs we require, as in Kotlin, that method bodies do not assign to formal parameters.

<sup>9</sup>More fine-grained privacy, e.g. C++ private fields or ownership types, would provide all the guarantees needed in our work.

$$\begin{array}{c}
\frac{\sigma.\text{cont} \stackrel{\text{txt}}{=} x := y.f; \text{stmt} \quad x \notin \text{Prms}(\sigma, \bar{M}) \quad \text{SameModule}(\text{this}, y, \sigma, \bar{M})}{\bar{M}, \sigma \twoheadrightarrow \sigma[x \mapsto \lfloor y.f \rfloor_\sigma][\text{cont} \mapsto \text{stmt}]} \quad (\text{READ}) \\
\\
\frac{\sigma.\text{cont} \stackrel{\text{txt}}{=} x.f := y; \text{stmt} \quad \text{SameModule}(\text{this}, x, \sigma, \bar{M})}{\bar{M}, \sigma \twoheadrightarrow \sigma[\lfloor x \rfloor_\sigma.f \mapsto \lfloor y \rfloor_\sigma][\text{cont} \mapsto \text{stmt}]} \quad (\text{WRITE}) \\
\\
\frac{\sigma.\text{cont} \stackrel{\text{txt}}{=} x := \text{new } C; \text{stmt} \quad x \notin \text{Prms}(\sigma, \bar{M}) \quad \text{fields}(\bar{M}, C) = \bar{f} \quad \alpha \text{ fresh in } \sigma}{\bar{M}, \sigma \twoheadrightarrow \sigma[x \mapsto \alpha][\alpha \mapsto (C; \bar{f} \mapsto \text{null})][\text{cont} \mapsto \text{stmt}]} \quad (\text{NEW}) \\
\\
\frac{\begin{array}{l} \phi_n.\text{cont} \stackrel{\text{txt}}{=} u := y_0.m(\bar{y}); \_ \quad u \notin \text{Prms}((\bar{\phi} \cdot \phi_n, \chi), \bar{M}) \\ \text{Meth}(\bar{M}, \text{classOf}((\phi_n, \chi), y_0), m) = p \ C :: m(\bar{x} : T) : T \{ \text{stmt} \} \\ p = \text{public} \vee \text{SameModule}(\text{this}, y_0, (\phi_n, \chi), \bar{M}) \end{array}}{\bar{M}, (\bar{\phi} \cdot \phi_n, \chi) \twoheadrightarrow (\bar{\phi} \cdot \phi_n \cdot (\text{this} \mapsto \lfloor y_0 \rfloor_{\phi_n}, x \mapsto \lfloor y \rfloor_{\phi_n}; \text{stmt}), \chi)} \quad (\text{CALL}) \\
\\
\frac{\phi_{n+1}.\text{cont} \stackrel{\text{txt}}{=} \epsilon \quad \phi_n.\text{cont} \stackrel{\text{txt}}{=} x := y_0.m(\bar{y}); \text{stmt}}{\bar{M}, (\bar{\phi} \cdot \phi_n \cdot \phi_{n+1}, \chi) \twoheadrightarrow (\bar{\phi} \cdot \phi_n[x \mapsto \lfloor \text{res} \rfloor_{\phi_{n+1}}][\text{cont} \mapsto \text{stmt}], \chi)} \quad (\text{RETURN})
\end{array}$$

Fig. 5.  $\mathcal{L}_{ul}$  operational Semantics

Private methods may be called only if the class of the callee (the object whose method is being called – here  $y_0$ ) comes from the same module as the class of the caller (here  $\text{this}$ ). Public methods may always be called. When a method is called, a new frame is pushed onto the stack; this frame maps  $\text{this}$  and the formal parameters to the values for the receiver and other arguments, and the continuation to the body of the method. Method bodies are expected to store their return values in the implicitly defined variable  $\text{res}$ <sup>10</sup>. When the continuation is empty ( $\epsilon$ ), the frame is popped and the value of  $\text{res}$  from the popped frame is stored in the variable map of the top frame.

Thus, when  $\bar{M}; \sigma \twoheadrightarrow \sigma'$  is within the same method we have  $|\sigma'| = |\sigma|$ ; when it is a call we have  $|\sigma'| = |\sigma| + 1$ ; and when it is a return we have  $|\sigma'| = |\sigma| - 1$ . Fig. 3 from §2 distinguishes  $\twoheadrightarrow$  execution steps into: steps within the same call ( $\rightarrow$ ), entering a method ( $\uparrow$ ), returning from a method ( $\downarrow$ ). Therefore  $\bar{M}; \sigma_8 \twoheadrightarrow \sigma_9$  is a step within the same call,  $\bar{M}; \sigma_9 \twoheadrightarrow \sigma_{10}$  is a method entry with  $\bar{M}$ ;  $\sigma_{12} \twoheadrightarrow \sigma_{13}$  the corresponding return. In general,  $\bar{M}; \sigma \twoheadrightarrow^* \sigma'$  may involve any number of calls or returns: e.g.  $\bar{M}; \sigma_{10} \twoheadrightarrow^* \sigma_{15}$ , involves no calls and two returns.

### 3.3 Fundamental Concepts

The novel features of our assertions – protection and scoped invariants – are both defined in terms of the current point of execution. Therefore, for the semantics of our assertions we need to represent calls and returns, scoped execution, and (in)directly accessible objects.

**3.3.1 Method Calls and Returns.** These are characterized through pushing/popping frames :  $\sigma \nabla \phi$  pushes frame  $\phi$  onto the stack of  $\sigma$ , while  $\sigma \Delta$  pops the top frame and updates the continuation and variable map.

**Definition 3.1.** Given a state  $\sigma$ , and a frame  $\phi$ , we define

- $\sigma \nabla \phi \triangleq (\bar{\phi} \cdot \phi, \chi)$  if  $\sigma = (\bar{\phi}, \chi)$ .
- $\sigma \Delta \triangleq (\bar{\phi} \cdot (\phi_n[\text{cont} \mapsto \text{stmt}][x \mapsto \lfloor \text{res} \rfloor_{\phi_n}], \chi)$  if  $\sigma = (\bar{\phi} \cdot \phi_n \cdot \phi_{n+1}, \chi)$ , and  $\phi_n(\text{cont}) \stackrel{\text{txt}}{=} x := y_0.m(\bar{y}); \text{stmt}$

<sup>10</sup>For ease of presentation, we omit assignment to  $\text{res}$  in methods returning  $\text{void}$ .

Consider Fig. 3 again:  $\sigma_8 = \sigma_7 \nabla \phi$  for some  $\phi$ , and  $\sigma_{15} = \sigma_{14} \Delta$ .

**3.3.2 Scoped Execution.** In order to give semantics to scoped invariants (introduced in §2.1.2 and to be fully defined in Def. 5.4), we need a new definition of execution, called *scoped execution*.

**Definition 3.2** (Scoped Execution). Given modules  $\bar{M}$ , and states  $\sigma, \sigma_1, \sigma_n$ , and  $\sigma'$ , we define:

- $\bar{M}; \sigma \rightsquigarrow \sigma' \triangleq \bar{M}; \sigma \twoheadrightarrow \sigma' \wedge |\sigma| \leq |\sigma'|$
- $\bar{M}; \sigma_1 \rightsquigarrow^* \sigma_n \triangleq \sigma_1 = \sigma_n \vee \exists \sigma_2, \dots, \sigma_{n-1}. \forall i \in [1..n] [ \bar{M}; \sigma_i \twoheadrightarrow \sigma_{i+1} \wedge |\sigma_i| \leq |\sigma_{i+1}| ]$
- $\bar{M}; \sigma \rightsquigarrow_{fin}^* \sigma' \triangleq \bar{M}; \sigma \rightsquigarrow^* \sigma' \wedge |\sigma| = |\sigma'| \wedge \sigma'.cont = \epsilon$

Consider Fig. 3 : Here  $|\sigma_8| \leq |\sigma_9|$  and thus  $\bar{M}; \sigma_8 \rightsquigarrow \sigma_9$ . Also,  $\bar{M}; \sigma_{14} \twoheadrightarrow \sigma_{15}$  but  $|\sigma_{14}| \not\leq |\sigma_{15}|$  (this step returns from the active call in  $\sigma_{14}$ ), and hence  $\bar{M}; \sigma_{14} \not\rightsquigarrow \sigma_{15}$ . Finally, even though  $|\sigma_8| = |\sigma_{18}|$  and  $\bar{M}; \sigma_8 \twoheadrightarrow^* \sigma_{18}$ , we have  $\bar{M}; \sigma_8 \not\rightsquigarrow^* \sigma_{18}$ : This is so, because the execution  $\bar{M}; \sigma_8 \twoheadrightarrow^* \sigma_{18}$  goes through the step  $\bar{M}; \sigma_{14} \twoheadrightarrow \sigma_{15}$  and  $|\sigma_8| \not\leq |\sigma_{15}|$  (this step returns from the active call in  $\sigma_8$ ).

The relation  $\rightsquigarrow^*$  contains more than the transitive closure of  $\rightsquigarrow$ . E.g.,  $\bar{M}; \sigma_9 \rightsquigarrow^* \sigma_{13}$ , even though  $\bar{M}; \sigma_9 \rightsquigarrow \sigma_{12}$  and  $\bar{M}; \sigma_{12} \not\rightsquigarrow^* \sigma_{13}$ . Lemma 3.3 says that the value of the parameters does not change during execution of the same method. Appendix B in [41] discusses proofs, and further properties.

**Lemma 3.3.** For all  $\bar{M}, \sigma, \sigma'$ :  $\bar{M}; \sigma \rightsquigarrow^* \sigma' \wedge |\sigma| = |\sigma'| \implies \forall x \in Prms(\bar{M}, \sigma). [ \lfloor x \rfloor_\sigma = \lfloor x \rfloor_{\sigma'} ]$

**3.3.3 Reachable Objects, Locally Reachable Objects, and Well-formed States.** To define protection (no external object indirectly accessible from the top frame has access to the protected object, c.f. § 2.1.1) we first define reachability. An object  $\alpha$  is *locally reachable*, i.e.  $\alpha \in LocRchbl(\sigma)$ , if it is reachable from the top frame on the stack of  $\sigma$ .

**Definition 3.4.** We define

- $Rchbl(\alpha, \sigma) \triangleq \{ \alpha' \mid \exists n \in \mathbb{N}. \exists f_1, \dots, f_n. [ \lfloor \alpha.f_1 \dots f_n \rfloor_\sigma = \alpha' ] \}$ .
- $LocRchbl(\sigma) \triangleq \{ \alpha \mid \exists x \in dom(\sigma) \wedge \alpha \in Rchbl(\lfloor x \rfloor_\sigma, \sigma) \}$ .

In well-formed states,  $\bar{M} \models \sigma$ , the value of a parameter in any callee ( $\sigma[k]$ ) is also the value of some variable in the caller ( $\sigma[k-1]$ ), and any address reachable from any frame ( $LocRchbl(\sigma[k])$ ) is reachable from some formal parameter of that frame.

**Definition 3.5** (Well-formed states). For modules  $\bar{M}$ , and states  $\sigma, \sigma'$ :

$$\begin{aligned} \bar{M} \models \sigma &\triangleq \forall k \in \mathbb{N}. [ 1 < k \leq |\sigma| \implies \\ &[ \forall x \in Prms(\sigma[k], \bar{M}). [ \exists y. \lfloor x \rfloor_{\sigma[k]} = \lfloor y \rfloor_{\sigma[k-1]} ] \quad \wedge \\ &LocRchbl(\sigma[k]) = \bigcup_{z \in Prms(\sigma[k], \bar{M})} Rchbl(\lfloor z \rfloor_{\sigma[k]}, \sigma) ] \end{aligned}$$

Lemma 3.6 says that (1) execution preserves well-formedness, and (2) any object which is locally reachable after pushing a frame was locally reachable before pushing that frame.

**Lemma 3.6.** For all modules  $\bar{M}$ , states  $\sigma, \sigma'$ , and frame  $\phi$ :

- (1)  $\bar{M} \models \sigma \wedge \bar{M}, \sigma \twoheadrightarrow \sigma' \implies \bar{M} \models \sigma'$
- (2)  $\sigma' = \sigma \nabla \phi \wedge \bar{M} \models \sigma' \implies LocRchbl(\sigma') \subseteq LocRchbl(\sigma)$

## 4 Assertions

Our assertions are standard (e.g. properties of the values of expressions, connectives, quantification etc.) or about protection (i.e.  $\langle e \rangle \leftarrow \times e$  and  $\langle e \rangle$ ).

**Definition 4.1.** Assertions,  $A$ , are defined as follows:

$$A ::= e \mid e : C \mid \neg A \mid A \wedge A \mid \forall x : C. A \mid e : \text{extl} \mid \langle e \rangle \leftarrow e \mid \langle e \rangle \quad 11$$

$Fv(A)$  returns the free variables in  $A$ ; for example,  $Fv(a : \text{Account} \wedge \forall b : \text{int}. [a.\text{blnce} = b]) = \{a\}$ .

**Definition 4.2** (Shorthands). We write  $e : \text{intl}$  for  $\neg(e : \text{extl})$ , and  $\text{extl}$ . resp.  $\text{intl}$  for  $\text{this} : \text{extl}$  resp.  $\text{this} : \text{intl}$ . Forms such as  $A \rightarrow A'$ ,  $A \vee A'$ , and  $\exists x : C. A$  can be encoded.

Satisfaction of Assertions by a module and a state is expressed through  $M, \sigma \models A$  and defined by cases on the shape of  $A$ , in definitions 4.3 and 4.4.  $M$  is used to look up the definitions of ghost fields, and to find class definitions to determine whether an object is external.

#### 4.1 Semantics of assertions – first part

To determine satisfaction of an expression, we use the evaluation relation,  $M, \sigma, e \hookrightarrow v$ , which says that the expression  $e$  evaluates to value  $v$  in the context of state  $\sigma$  and module  $M$ . Ghost fields may be recursively defined, thus evaluation of  $e$  might not terminate. Nevertheless, the logic of assertions remains classical because recursion is restricted to expressions.

**Definition 4.3** (Satisfaction of Assertions – first part). We define satisfaction of an assertion  $A$  by a state  $\sigma$  with module  $M$  as:

- (1)  $M, \sigma \models e \triangleq M, \sigma, e \hookrightarrow \text{true}$
- (2)  $M, \sigma \models e : C \triangleq M, \sigma, e \hookrightarrow \alpha \wedge \text{classOf}(\alpha, \sigma) = C$
- (3)  $M, \sigma \models \neg A \triangleq M, \sigma \not\models A$
- (4)  $M, \sigma \models A_1 \wedge A_2 \triangleq M, \sigma \models A_1 \wedge M, \sigma \models A_2$
- (5)  $M, \sigma \models \forall x : C. A \triangleq \forall \alpha. [ M, \sigma \models \alpha : C \implies M, \sigma \models A[\alpha/x] ]$
- (6)  $M, \sigma \models e : \text{extl} \triangleq \exists C. [ M, \sigma \models e : C \wedge C \notin M ]$

Note that while execution takes place in the context of one or more modules,  $\overline{M}$ , satisfaction of assertions considers *exactly one* module  $M$  – the internal module.  $M$  is used to look up the definitions of ghost fields, and to determine whether objects are external.

#### 4.2 Semantics of Assertions - second part

In §2.1.1 we introduced protection – we will now formalize this concept.

An object is protected from another object,  $\langle \alpha \rangle \leftarrow \alpha_o$ , if the two objects are not equal, and no external object reachable from  $\alpha_o$  has a field pointing to  $\alpha$ . This ensures that the last element on any path leading from  $\alpha_o$  to  $\alpha$  in an internal object.

An object is protected,  $\langle \alpha \rangle$ , if no external object reachable from any of the current frame's arguments has a field pointing to  $\alpha$ ; and furthermore, if the receiver is external, then no parameter to the current method call directly refers to  $\alpha$ . This ensures that no external object reachable from the current receiver or arguments can “obtain”  $\alpha$ , where obtain  $\alpha$  is either direct access through a field, or by virtue of the method's receiver being able to see all the arguments.

**Definition 4.4** (Satisfaction of Assertions – Protection). – continuing definitions in 4.3:

- (1)  $M, \sigma \models \langle e \rangle \leftarrow e_o \triangleq \exists \alpha, \alpha_o. [$ 
  - (a)  $M, \sigma, e \hookrightarrow \alpha \wedge M, \sigma, e_o \hookrightarrow \alpha_o \wedge \alpha \neq \alpha_o,$
  - (b)  $\forall \alpha' \in \text{Rchbl}(\alpha_o, \sigma). \forall f. [ M, \sigma \models \alpha' : \text{extl} \implies [\alpha'.f]_\sigma \neq \alpha ] ]$ .
- (2)  $M, \langle e \rangle \models \triangleq \exists \alpha. [$ 
  - (a)  $M, \sigma, e \hookrightarrow \alpha,$
  - (b)  $M, \sigma \models \text{extl} \implies \forall x \in \sigma. M, \sigma \models x \neq \alpha,$

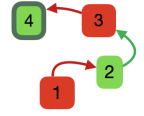
<sup>11</sup>Addresses in assertions as e.g. in  $\alpha.\text{blnce} > 700$ , are useful when giving semantics to universal quantifiers c.f. Def. 4.3.(5), when the local map changes e.g. upon call and return, and in general, for scoped invariants, c.f. Def. 5.4.

$$(c) \forall \alpha' \in \text{LocRchbl}(\sigma). \forall f. [M, \sigma \models \alpha' : \text{extl} \implies [\alpha'.f]_\sigma \neq \alpha] \quad ].$$

We illustrate “protected” and “protected from” in Fig. 2 in §2. and Fig. 12 in App. C in [41]. In general,  $\langle \alpha \rangle \leftarrow \alpha_o$  ensures that  $\alpha_o$  will get access to  $\alpha$  only if another object grants that access. Similarly,  $\langle \alpha \rangle$  ensures that during execution of the current method, no external object will get direct access to  $\alpha$  unless some internal object grants that access<sup>12</sup>. Thus, protection together with protection preservation (*i.e.* no internal object gives access) guarantee lack of eventual external access.

*Discussion.* Lack of eventual direct access is a central concept in the verification of code with calls to and callbacks from untrusted code. It has already been over-approximated in several different ways, *e.g.* 2nd-class [94, 116] or borrowed (“2nd-hand”) references [14, 23], textual modules [75], information flow [109], runtime checks [4], abstract data type exports [70], separation-based invariants Iris [48, 99], – more in § 8. In general, protection is applicable in more situations (*i.e.* is less restrictive) than most of these approaches, although more restrictive than the ideal “lack of eventual access”.

An alternative definition might consider  $\alpha$  as protected from  $\alpha_o$ , if any path from  $\alpha_o$  to  $\alpha$  goes through at least one internal object. With this definition,  $o_4$  would be protected from  $o_1$  in the heap shown here. However,  $o_1$  can make a call to  $o_2$ , and this call could return  $o_3$ . Once  $o_1$  has direct access to  $o_3$ , it can also get direct access to  $o_4$ . The example justifies our current definition.



### 4.3 Preservation of Assertions

Program logics require some form of framing, *i.e.* conditions under which satisfaction of assertions is preserved across program execution. This is the subject of the current subsection.

Def. 4.5 turns an assertion  $A$  to the equivalent variable-free form by replacing all free variables from  $A$  by their values in  $\sigma$ . Then, Lemma 4.5 says that satisfaction of an assertion is not affected by replacing free variables by their values, nor by changing the state's continuation.

**Definition and Lemma 4.5.** For all  $M, \sigma, stmt, A$ , and  $\bar{x}$  where  $\bar{x} = Fv(A)$ :

- $\sigma[A] \triangleq A[\bar{x}]_{\sigma/\bar{x}}$
- $M, \sigma \models A \iff M, \sigma \models \sigma[A] \iff M, \sigma[\text{cont} \mapsto stmt] \models A$

We now move to assertion preservation across method call and return.

**4.3.1 Stability.** In most program logics, satisfaction of variable-free assertions is preserved when pushing/popping frames – *i.e.* immediately after entering a method or returning from it. But this is not so for our assertions, where protection depends on the heap but also on the range of the top frame. *E.g.*, Fig. 2:  $\sigma_2 \not\models \langle o_6 \rangle$ , but after pushing a frame, we have  $\sigma_3 \models \langle o_6 \rangle$ .

Assertions which do not contain  $\langle \_ \rangle$  are called  $Stbl(\_)$ , while assertions which do not contain  $\langle \_ \rangle$  in *negative* positions are called  $Stb^+(\_)$ . Sect 4.3.2 shows some examples. Lemma 4.6 says that  $Stbl(\_)$  assertions are preserved when pushing/popping frames, and  $Stb^+(\_)$  assertions are preserved when pushing internal frames. *C.f.* Appendix D in [41] for definitions and proofs.

**Lemma 4.6.** For all states  $\sigma$ , frames  $\phi$ , all assertions  $A$  with  $Fv(A) = \emptyset$

- $Stbl(A) \implies [M, \sigma \models A \iff M, \sigma \nabla \phi \models A]$
- $Stb^+(A) \wedge M \cdot \bar{M} \models \sigma \nabla \phi \wedge M, \sigma \nabla \phi \models \text{intl} \wedge M, \sigma \models A \implies M, \sigma \nabla \phi \models A$

<sup>12</sup>This is in line with the motto “only connectivity begets connectivity” from [83].

While  $Stb^+$  assertions are preserved when pushing internal frames, they are *not* necessarily preserved when pushing external frames nor when popping frames (c.f. Ex. 4.7).

**Example 4.7.** Fig. 2 illustrates that

- $Stb^+$  not necessarily preserved by External Push: Namely,  $\sigma_2 \models \langle o_4 \rangle$ , pushing frame  $\phi_3$  with an external receiver and  $o_4$  as argument gives  $\sigma_3$ , we have  $\sigma_3 \not\models \langle o_4 \rangle$ .
- $Stb^+$  not necessarily preserved by Pop: Namely,  $\sigma_3 \models \langle o_6 \rangle$ , returning from  $\sigma_3$  would give  $\sigma_2$ , and we have  $\sigma_2 \not\models \langle o_6 \rangle$ .

We work with  $Stb^+$  assertions (the  $Stbl$  requirement is too strong). But we need to address the lack of preservation of  $Stb^+$  assertions for external method calls and returns. We do the former through *adaptation* ( $\neg\forall$  in Sect 6.2.2), and the latter through *deep satisfaction* (§7).

**4.3.2 Encapsulation.** As external code is unknown, it could, in principle, have unlimited effect and invalidate any assertion, and thus make reasoning about external calls impossible. However, because fields are private, assertions which read internal fields only, cannot be invalidated by external execution steps. Reasoning about external calls relies on such *encapsulated* assertions.

Judgment  $M \vdash \text{Enc}(A)$ , defined in D.4 in [41], requires that  $A$  looks up the contents of internal objects only, that  $A$  does not contain  $\langle \_ \rangle$  in negative positions, nor does it contain  $\langle \_ \rangle \leftarrow x$  in any position. Lemma 4.8 says that  $M \vdash \text{Enc}(A)$  implies that any external scoped execution step which involves  $M$  and any set of other modules  $\bar{M}$  preserves satisfaction of  $A$ .

**Lemma 4.8** (Encapsulation). For all modules  $M$ , and assertions  $A$ :

- $M \vdash \text{Enc}(A) \implies \forall \bar{M}, \sigma, \sigma'. [M, \sigma \models (A \wedge \text{extl}) \wedge M \cdot \bar{M}; \sigma \rightsquigarrow \sigma' \implies M, \sigma' \models \sigma[A]]$

The example below compares the judgments  $Stbl(\_)$ ,  $Stb^+(\_)$ , and  $\text{Enc}(\_)$ .

	$z.f \geq 3$	$\langle x \rangle$	$\neg(\langle x \rangle)$	$\langle y \rangle \leftarrow x$	$\neg(\langle y \rangle \leftarrow x)$
$Stbl(\_)$	✓	✗	✗	✓	✓
$Stb^+(\_)$	✓	✓	✗	✓	✓
$\text{Enc}(\_)$	✓	✓	✗	✗	✗

## 5 Specifications

### 5.1 Syntax, Semantics, and Examples of Specifications

**Definition 5.1** (Specifications Syntax). We define the syntax of specifications,  $S$ :

$$\begin{aligned}
 S &::= \forall \bar{x} : \bar{C}. \{A\} \mid \{A\} p C :: m(\bar{y} : \bar{C}) \{A\} \parallel \{A\} \mid S \wedge S \\
 p &::= \text{private} \mid \text{public}
 \end{aligned}$$

In Def. 5.6 later on we describe well-formedness of  $S$ , but we first discuss semantics and some examples. We use quadruples involving states:  $\bar{M}; M \models \{A\} \sigma \{A'\} \parallel \{A''\}$  says that if  $\sigma$  satisfies  $A$ , then any terminating scoped execution of its continuation  $(\bar{M} \cdot M; \sigma \rightsquigarrow_{fin}^* \sigma')$  will satisfy  $A'$ , and any intermediate reachable external state  $(\bar{M} \cdot M; \sigma \rightsquigarrow^* \sigma'')$  will satisfy the “mid-condition”,  $A''$ .

**Definition 5.2.** For modules  $\bar{M}, M$ , state  $\sigma$ , and assertions  $A, A'$  and  $A''$ , we define:

- $\bar{M}; M \models \{A\} \sigma \{A'\} \parallel \{A''\} \triangleq \forall \sigma', \sigma''. [$   
 $M, \sigma \models A \implies [ \bar{M} \cdot M; \sigma \rightsquigarrow_{fin}^* \sigma' \implies M, \sigma' \models A' ] \wedge$   
 $[ \bar{M} \cdot M; \sigma \rightsquigarrow^* \sigma'' \implies M, \sigma'' \models (\text{extl} \rightarrow \sigma[A'']) ] ]$

**Example 5.3.** Consider  $\dots; \dots \models \{A_1\} \sigma_4 \{A_2\} \parallel \{A_3\}$  for Fig. 3. It means that if  $\sigma_4$  satisfies  $A_1$ , then  $\sigma_{23}$  will satisfy  $A_2$ , while  $\sigma_6$ – $\sigma_9$ ,  $\sigma_{13}$ – $\sigma_{17}$ , and  $\sigma_{20}$ – $\sigma_{21}$  will satisfy  $A_3$ . It does not imply anything about  $\sigma_{24}$  because  $\dots; \sigma_4 \not\rightsquigarrow^* \sigma_{24}$ . Similarly, if  $\sigma_8$  satisfies  $A_1$  then  $\sigma_{14}$  will satisfy  $A_2$ , and  $\sigma_8, \sigma_9, \sigma_{13}, \sigma_{14}$  will satisfy  $A_3$ , while making no claims about  $\sigma_{10}, \sigma_{11}, \sigma_{12}$ , nor about  $\sigma_{15}$  onwards.



Now we define  $M \models \overline{\forall x : C}. \{A\}$  to mean that if an external state  $\sigma$  satisfies  $A$ , then all future external states reachable from  $\sigma$ —including nested calls and returns but *stopping* before returning from the active call in  $\sigma$ —also satisfy  $A$ . And  $M \models \{A_1\} p D :: m(\overline{y : D}) \{A_2\} \parallel \{A_3\}$  means that scoped execution of a call to  $m$  from  $D$  in states satisfying  $A_1$  leads to final states satisfying  $A_2$  (if it terminates), and to intermediate external states satisfying  $A_3$ .

**Definition 5.4** (Semantics of Specifications). We define  $M \models S$  by cases over  $S$ :

- (1)  $M \models \overline{\forall x : C}. \{A\} \triangleq \forall \overline{M}, \sigma. [ \overline{M}; M \models \{ \text{ext} \perp \wedge \overline{x : C} \wedge A \} \sigma \{A\} \parallel \{A\} ]$ .
- (2)  $M \models \{A_1\} p D :: m(\overline{y : D}) \{A_2\} \parallel \{A_3\} \triangleq$   
 $\forall \overline{M}, \sigma, y_0, \overline{y}. [ \sigma.\text{cont} \stackrel{\text{txt}}{=} u := y_0.m(y_1..y_n) \implies$   
 $\overline{M}; M \models \{ y_0 : D, \overline{y : D} \wedge A[y_0/\text{this}] \} \sigma \{A_2[u/\text{res}, y_0/\text{this}] \} \parallel \{A_3\} ]$
- (3)  $M \models S \wedge S' \triangleq M \models S \wedge M \models S'$

Fig. 3 in §2.1.2 illustrated the meaning of  $\overline{\forall x : C}. \{A_0\}$ . Moreover,  $M_{\text{good}} \models S_2 \wedge S_3 \wedge S_4$ , and  $M_{\text{fine}} \models S_2 \wedge S_3 \wedge S_4$ , while  $M_{\text{bad}} \not\models S_2$ . More examples below and in Appendix E.1.1 in [41].

**Example 5.5** (Scoped Invariants and Method Specs).  $S_5$  says that non-null keys are immutable:

$$S_5 \triangleq \forall a : \text{Account}, k : \text{Key}. \{ \text{null} \neq k = a.\text{key} \}$$

$S_9$  guarantees that `set` preserves the protectedness of any account, and any key.

$$S_9 \triangleq \{ a : \text{Account}, a' : \text{Account} \wedge \langle a \rangle \wedge \langle a'.\text{key} \rangle \}$$

$$\text{public Account} :: \text{set}(\text{key}' : \text{Key})$$

$$\{ \langle a \rangle \wedge \langle a'.\text{key} \rangle \} \parallel \{ \langle a \rangle \wedge \langle a'.\text{key} \rangle \}$$

Note that  $a, a'$  are disjoint from `this` and the formal parameters of `set`. In that sense,  $a$  and  $a'$  are universally quantified; a call of `set` will preserve protectedness for *all* accounts and their keys.

## 5.2 Well-formedness of Specifications

We now define what it means for a specification to be well-formed:

**Definition 5.6.** *Well-formedness* of specifications,  $\vdash S$ , is defined by cases on  $S$ :

- $\vdash \overline{\forall x : C}. \{A\} \triangleq Fv(A) \subseteq \{\overline{x}\} \wedge M \vdash \text{Enc}(\overline{x : C} \wedge A)$ .
- $\vdash \{x : \overline{C'} \wedge A\} p C :: m(\overline{y : C}) \{A'\} \parallel \{A''\} \triangleq$   
 $[ \text{res}, \text{this} \notin \overline{x}, \overline{y} \wedge Fv(A) \subseteq \overline{x}, \overline{y}, \text{this} \wedge Fv(A') \subseteq \overline{x}, \overline{y}, \text{this}, \text{res} \wedge Fv(A'') \subseteq \overline{x}$   
 $\wedge \text{Stb}^+(A) \wedge \text{Stb}^+(A') \wedge M \vdash \text{Enc}(\overline{x : C'} \wedge A'') ]$
- $\vdash S \wedge S' \triangleq \vdash S \wedge \vdash S'$ .

Def 5.6's requirements about free variables are relatively straightforward – c.f. §E.1.1 in [41].

Def 5.6's requirements about encapsulation are motivated by Def. 5.4. If  $\overline{x : C} \wedge A$  in the scoped invariant were not encapsulated, then it could be invalidated by some external code, and it would be impossible to ever satisfy Def. 5.4(1). Similarly, if a method specification's mid-condition,  $A''$ , could be invalidated by some external code, then it would be impossible to ever satisfy Def. 5.4(2).

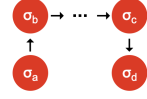
Def 5.6's requirements about stability are motivated by our Hoare logic rule for internal calls, [CALL\_INT], Fig 7. The requirement  $\text{Stb}^+(A)$  for the method's precondition gives that  $A$  is preserved when an internal frame is pushed, c.f. Lemma 4.6. The requirement  $\text{Stb}^+(A')$  for the method's postcondition gives, in the context of deep satisfaction, that  $A'$  is preserved when an internal frame is popped, c.f. Lemma G.42 in [41]. This is crucial for soundness of [CALL\_INT].

In §Appendix E.1.2 in [41] we discuss the encapsulation requirements, and why these do not restrict expressiveness.

### 5.3 Discussion

*Difference with Object and History Invariants.* Our scoped invariants are similar to, but different from, history invariants and object invariants. We compare through an example:

State  $\sigma_a$  is making a call transitioning to  $\sigma_b$ , execution of  $\sigma_b$ 's continuation eventually results in  $\sigma_c$ , and  $\sigma_c$  returns to  $\sigma_d$ . Suppose all four states are external, and the module guarantees  $\forall x : \text{Object}. \{A\}$ , and  $\sigma_a \not\models A$ , but  $\sigma_b \models A$ . Scoped invariants ensure  $\sigma_c \models A$ , but allow  $\sigma_d \not\models A$ .



*History invariants* [27, 67, 69], instead, consider all future states including any method returns, and therefore would require that  $\sigma_d \models A$ . Thus, they are, for our purposes, both *unenforceable* and overly *restrictive*. *Unenforceable*: Take  $A \stackrel{\text{txt}}{=} \langle \text{acc}. \text{key} \rangle$ , assume in  $\sigma_a$  a path to an external object which has access to  $\text{acc}. \text{key}$ , assume that path is unknown in  $\sigma_b$ : then, the transition from  $\sigma_b$  to  $\sigma_c$  cannot eliminate that path—hence,  $\sigma_d \not\models \langle \text{acc}. \text{key} \rangle$ . *Restrictive*: Take  $A \stackrel{\text{txt}}{=} \langle \text{acc}. \text{key} \rangle \wedge a.\text{blncc} \geq b$ ; then, requiring  $A$  to hold in all states from  $\sigma_a$  until termination would prevent all future withdrawals from  $a$ , rendering the account useless.

*Object invariants* [8, 66, 81, 82, 89], on the other hand, expect invariants to hold in all (visible) states, here would require, e.g. that  $\sigma_a \models A$ . Thus, they are *inapplicable* for us: They would require, e.g., that for all  $\text{acc}$ , in all (visible) states,  $\langle \text{acc}. \text{key} \rangle$ , and thus prevent *any* withdrawals from *any* account in *any* state.

*Difference between Postconditions and Invariants.* In all method specification examples so far, the post-condition and mid-condition were identical. However, this need not be so. Assume a method `tempLeak` defined in `Account`, with an external argument `extArg`, and method body:

```
extArg.m(this.key); this.key:=new Key
```

Then, the assertion  $\langle \text{this}. \text{key} \rangle$  is invalidated by the external call `extArg.m(this.key)`, but is established by `this.key:=new Key`. Therefore,  $\langle \text{this}. \text{key} \rangle$  is a valid post-condition but not a valid mid-condition. The specification of `tempLeak` could be

```
StempLeak  ≜  { true }
              public Account :: tempLeak(extArg: external)
              { ⟨this.key⟩ } || { true }
```

*Expressiveness* In §E.2 in [41] we argue the expressiveness of our approach through a sequence of capability patterns studied in related approaches from the literature [33, 75, 96, 98, 109] and written in our specification language. These approaches are based on temporal logics [75, 96], or on extensions of Coq/Iris [33, 98, 109], and do not offer Hoare logic rules for external calls.

## 6 Hoare Logic

We develop an inference system for adherence to our specifications. We distinguish three phases:

**First Phase:** We assume an underlying Hoare logic,  $M \vdash_{ul} \{A\} \text{ stmt } \{A'\}$ , and extend it to a logic  $M \vdash \{A\} \text{ stmt } \{A'\}$  with the expected meaning, i.e. (\*) execution of statement *stmt* in a state satisfying  $A$  will lead to a state satisfying  $A'$ . These triples only apply to *stmt*'s that do not contain method calls (even internal) – this is so, because method calls may make further calls to external methods. In our extension we introduce judgements which talk about protection.

**Second Phase:** We develop a logic of quadruples  $M \vdash \{A\} \text{ stmt } \{A'\} \parallel \{A''\}$ . These promise (\*) and in addition, that (\*\*) any intermediate external states reachable during execution of that *stmt* satisfy the *mid-condition*  $A''$ . We incorporate the triples from the first phase, introduce mid-conditions, give the usual substructural rules, and deal with method calls. For internal calls we use the methods' specs. For external calls, we use the module's invariants.

**Third Phase:** We prove adherence to our specifications. For method specifications we require that the body maps the precondition to the postcondition and preserves the method's mid-condition. For module invariants we require that they are preserved by all public methods of the module.

*Preliminaries:* The judgement  $\vdash M : S$  expresses that  $S$  is part of  $M$ 's specification. In particular, it allows *safe renamings*. These renamings are a convenience, akin to the Barendregt convention, and allow simpler Hoare logic rules – *c.f.* Sect. 6.3, Def. F.1 in [41], and Ex. F.2 in [41]. We also require an underlying Hoare logic with judgements  $M \vdash_{ul} \{A\} stmt \{A'\}$  – *c.f.* Ax. F.3 in [41].

## 6.1 First Phase: Triples

In Fig. 6 we introduce our triples, of the form  $M \vdash \{A\} stmt \{A'\}$ . These promise, as expected, that any execution of  $stmt$  in a state satisfying  $A$  leads to a state satisfying  $A'$ .

$$\begin{array}{c}
 \text{[EMBED\_UL]} \\
 \frac{\text{\textit{stmt} contains no method call} \quad Stbl(A) \quad Stbl(A') \quad M \vdash_{ul} \{A\} stmt \{A'\}}{M \vdash \{A\} stmt \{A'\}} \\
 \\
 \text{[PROT-NEW]} \\
 \frac{u \stackrel{\text{txt}}{\neq} x}{M \vdash \{true\} u = \text{new } C \{ \langle u \rangle \wedge \langle u \rangle \Leftarrow x \}} \\
 \\
 \text{[PROT-1]} \\
 \frac{\text{\textit{stmt} is free of meth. calls, or assignment to } z} \quad \frac{M \vdash \{A \wedge e = z\} stmt \{e = z\}}{M \vdash \{A \wedge \langle e \rangle\} stmt \{ \langle e \rangle \}} \\
 \\
 \text{[PROT-2]} \\
 \frac{\text{\textit{stmt} is either } x := y \text{ or } x := y.f \quad z, z' \stackrel{\text{txt}}{\neq} x}{M \vdash \{A \wedge z = e \wedge z' = e'\} stmt \{z = e \wedge z' = e'\}} \\
 \frac{M \vdash \{A \wedge \langle e \rangle \Leftarrow e'\} stmt \{ \langle e \rangle \Leftarrow e'\}}{M \vdash \{A \wedge \langle e \rangle \Leftarrow e'\} stmt \{ \langle e \rangle \Leftarrow e'\}} \\
 \\
 \text{[PROT-3]} \\
 \frac{x \stackrel{\text{txt}}{\neq} z}{M \vdash \{ \langle y.f \rangle \Leftarrow z \} x = y.f \{ \langle x \rangle \Leftarrow z \}} \\
 \\
 \text{[PROT-4]} \\
 \frac{A \stackrel{\text{txt}}{=} \langle x \rangle \Leftarrow z \wedge \langle x \rangle \Leftarrow y' \quad A' \stackrel{\text{txt}}{=} \langle x \rangle \Leftarrow z}{M \vdash \{A\} y.f = y' \{A'\}}
 \end{array}$$

Fig. 6. Embedding the Underlying Hoare Logic, and Protection

With rule EMBED\_UL in Fig. 6, any triple  $\{A\} stmt \{A'\}$  whose statement does not contain a method call, and which can be proven in the underlying Hoare logic, can also be proven in our logic. In PROT-1, we see that protection of an object  $o$  is preserved by internal code which does not call any methods: namely any heap modifications will only affect internal objects, and this will not expose new, unmitigated external access to  $o$ . PROT-2, PROT-3 and PROT-4 describe the preservation of relative protection. Proofs of soundness for these rules can be found in App.G.5 in [41].

## 6.2 Second Phase: Quadruples

**6.2.1 Introducing mid-conditions, and substructural rules.** We now introduce quadruple rules. Rule MID embeds triples  $M \vdash \{A\} s \{A'\}$  into quadruples  $M \vdash \{A\} s \{A'\} \parallel \{A''\}$ . This is sound, because  $stmt$  is guaranteed not to contain method calls (by lemma F.5 in [41]).

$$\text{[MID]} \\
 \frac{M \vdash \{A\} stmt \{A'\}}{M \vdash \{A\} stmt \{A'\} \parallel \{A''\}}$$

Substructural quadruple rules appear in Fig.15 in [41], and are as expected: Rules SEQU and CONSEQU are the usual rules for statement sequences and consequence, adapted to quadruples. Rule COMBINE combines two quadruples for the same statement into one. Rule ABSURD allows us to deduce anything out of `false` precondition, and CASES allows for case analysis. These rules apply to *any* statements – even those containing method calls.

**6.2.2 Adaptation.** In the outline of the Hoare proof of the external call in §2.2, we saw that an assertion of the form  $\langle x \rangle \leftarrow \bar{y}$  at the call site may imply  $\langle x \rangle$  at entry to the call. More generally, the  $\neg\forall$  operator adapts an assertion from the view of the callee to that of the caller, and is used in the Hoare logic for method calls. It is defined below.

**Definition 6.1.** [The  $\neg\forall$  operator]

$$\begin{array}{ll} \langle e \rangle \neg\forall \bar{y} \triangleq \langle e \rangle \leftarrow \bar{y} & (A_1 \wedge A_2) \neg\forall \bar{y} \triangleq (A_1 \neg\forall \bar{y}) \wedge (A_2 \neg\forall \bar{y}) \\ (\langle e \rangle \leftarrow \bar{u}) \neg\forall \bar{y} \triangleq \langle e \rangle \leftarrow \bar{u} & (\forall x : C.A) \neg\forall \bar{y} \triangleq \forall x : C.(A \neg\forall \bar{y}) \\ (e : \text{extl}) \neg\forall \bar{y} \triangleq e : \text{extl} & (\neg A) \neg\forall \bar{y} \triangleq \neg(A \neg\forall \bar{y}) \\ e \neg\forall \bar{y} \triangleq e & (e : C) \neg\forall \bar{y} \triangleq e : C \end{array}$$

Only the first equation in Def. 6.1 is interesting: for  $e$  to be protected at a callee with arguments  $\bar{y}$ , it should be protected from these arguments – thus  $\langle e \rangle \neg\forall \bar{y} = \langle e \rangle \leftarrow \bar{y}$ . The notation  $\langle e \rangle \leftarrow \bar{y}$  stands for  $\langle e \rangle \leftarrow y_0 \wedge \dots \wedge \langle e \rangle \leftarrow y_n$ , assuming that  $\bar{y} = y_0, \dots, y_n$ .

Lemma 6.2 states that indeed,  $\neg\forall$  adapts assertions from the callee to the caller, and is the counterpart to the  $\nabla$ . In particular: (1):  $\neg\forall$  turns an assertion into a stable assertion. (2): If the caller,  $\sigma$ , satisfies  $A \nabla \text{Rng}(\phi)$ , then the callee,  $\sigma \nabla \phi$ , satisfies  $A$ . (3): When returning from external states, an assertion implies its adapted version. (4): When calling from external states, an assertion implies its adapted version.

**Lemma 6.2.** For states  $\sigma$ , assertions  $A$ , so that  $\text{Stb}^+(A)$  and  $\text{Fv}(A) = \emptyset$ , frame  $\phi$ , variables  $y_0, \bar{y}$ :

- (1)  $\text{Stbl}(A \neg\forall (y_0, \bar{y}))$
- (2)  $M, \sigma \models A \neg\forall \text{Rng}(\phi) \implies M, \sigma \nabla \phi \models A$
- (3)  $M, \sigma \nabla \phi \models A \wedge \text{extl} \implies M, \sigma \models A \neg\forall \text{Rng}(\phi)$
- (4)  $M, \sigma \models A \wedge \text{extl} \wedge M \cdot \bar{M} \models \sigma \nabla \phi \implies M, \sigma \nabla \phi \models A \neg\forall \text{Rng}(\phi)$

Proofs in Appendix F.5 in [41]. Example 6.3 demonstrates the need for  $\text{extl}$  requirement in (3).

**Example 6.3** (When returning from internal states,  $A$  does not imply  $A \neg\forall \text{Rng}(\phi)$ ). In Fig. 2 we have  $\sigma_2 = \sigma_1 \nabla \phi_2$ , and  $\sigma_2 \models \langle o_1 \rangle$ , and  $o_1 \in \text{Rng}(\phi_2)$ , but  $\sigma_1 \not\models \langle o_1 \rangle \leftarrow o_1$ .

**6.2.3 Reasoning about calls.** is described in Fig. 7.  $\text{CALL\_INT}$  for internal methods, whether public or private; and  $\text{CALL\_EXT\_ADAPT}$  and  $\text{CALL\_EXT\_ADAPT\_STRONG}$  for external methods.

$$\begin{array}{c} \text{[CALL\_INT]} \\ \frac{\vdash M : \{A_1\} p C :: m(\overline{x:C}) \{A_2\} \parallel \{A_3\} \quad A'_1 = A_1[y_0, \bar{y}/\text{this}, \bar{x}] \quad A'_2 = A_2[y_0, \bar{y}, u/\text{this}, \bar{x}, \text{res}]}{M \vdash \{y_0 : C, \bar{y} : \overline{C \wedge A'_1} \} u := y_0.m(y_1, \dots, y_n) \{A'_2\} \parallel \{A_3\}} \\ \text{[CALL\_EXT\_ADAPT]} \\ \frac{\vdash M : \overline{\forall x : C. \{A\}}}{M \vdash \{y_0 : \text{extl} \wedge \overline{x : C} \wedge A \neg\forall (y_0, \bar{y}) \} u := y_0.m(y_1, \dots, y_n) \{A \neg\forall (y_0, \bar{y})\} \parallel \{A\}} \\ \text{[CALL\_EXT\_ADAPT\_STRONG]} \\ \frac{\vdash M : \overline{\forall x : C. \{A\}}}{M \vdash \{y_0 : \text{extl} \wedge \overline{x : C} \wedge A \wedge A \neg\forall (y_0, \bar{y}) \} u := y_0.m(y_1, \dots, y_n) \{A \wedge A \neg\forall (y_0, \bar{y})\} \parallel \{A\}} \end{array}$$

Fig. 7. Hoare Quadruples for Internal and External Calls – here  $\bar{y}$  stands for  $y_1, \dots, y_n$

For internal calls, we start, as usual, by looking up the method's specification, and substituting the formal by the actual parameters  $(\text{this}, \bar{x} \text{ by } y_0, \bar{y})$ .  $\text{CALL\_INT}$  is as expected: we require

the precondition, and guarantee the postcondition and mid-condition. `CALL_INT` is applicable whether the method is public or private.

For external calls, we consider the module's invariants. If the module promises to preserve  $A$ , *i.e.* if  $\vdash M : \forall \bar{x} : \bar{D}. \{A\}$ , and if its adapted version,  $A \rightarrow (y_0, \bar{y})$ , holds before the call, then it also holds after the call (`CALL_EXT_ADAPT`). If, in addition, the un-adapted version also holds before the call, then it also holds after the call (`CALL_EXT_ADAPT_STRONG`).

Notice that internal calls, `CALL_INT`, require the *un-adapted* method precondition (*i.e.*  $A'_1$ ), while external calls, both `CALL_EXT_ADAPT` and `CALL_EXT_ADAPT_STRONG`, require the *adapted* invariant (*i.e.*  $A \rightarrow (y_0, \bar{y})$ ). This is sound, because internal callees preserve  $Stb^+(\_)$ -assertions – *c.f.* Lemma 4.6. On the other hand, external callees do not necessarily preserve  $Stb^+(\_)$ -assertions – *c.f.* Ex. 4.7. Therefore, in order to guarantee that  $A$  holds upon entry to the callee, we need to know that  $A \rightarrow (y_0, \bar{y})$  held at the caller site – *c.f.* Lemma 6.2.

Remember that popping frames does not necessarily preserve  $Stb^+(\_)$  assertions – *c.f.* Ex. 4.7. Nevertheless, `CALL_INT` guarantees the unadapted version,  $A$ , upon return from the call. This is sound, because of our *deep satisfaction* of assertions – more in Sect. 7.

*Polymorphic Calls.* Our rules do not *directly* address a scenario where the receiver may be internal or external, and where the choice about this is made at runtime. However, such scenarios are *indirectly* supported, through our rules of consequence and case-split. More in Appendix H.6 in [41].

**Example 6.4** (Proving external calls). We continue our discussion from §2.2 on how to establish the Hoare triple (1) :

$$(1?) \quad \begin{array}{l} \{ \text{buyer} : \text{ext1} \wedge \langle \text{this.accnt.key} \rangle \leftarrow \text{buyer} \wedge \text{this.accnt.blnc} = b \} \\ \text{buyer.pay}(\text{this.accnt}, \text{price}) \\ \{ \text{this.accnt.blnc} \geq b \} \parallel \{ \langle a.\text{key} \rangle \wedge a.\text{blnc} \geq b \} \end{array}$$

We use  $S_3$ , which says that  $\forall a : \text{Account}, b : \text{int}. \{ \langle a.\text{key} \rangle \wedge a.\text{blnc} \geq b \}$ . We can apply rule `CALL_EXT_ADAPT`, by taking  $y_0 \triangleq \text{buyer}$ , and  $\bar{x} : \bar{D} \triangleq a : \text{Account}, b : \text{int}$ , and  $A \triangleq \langle a.\text{key} \rangle \wedge a.\text{blnc} \geq b$ , and  $m \triangleq \text{pay}$ , and  $\bar{y} \triangleq \text{this.accnt}, \text{price}$ , and provided that we can establish that

$$(2?) \quad \langle \text{this.accnt.key} \rangle \leftarrow (\text{buyer}, \text{this.accnt}, \text{price})$$

holds. Using type information, we get that all fields transitively accessible from `this.accnt.key`, or `price` are internal or scalar. This implies

$$(3) \quad \langle \text{this.accnt.key} \rangle \leftarrow \text{this.accnt} \wedge \langle \text{this.accnt.key} \rangle \leftarrow \text{price}$$

Using then Def. 6.1, we can indeed establish that

$$(4) \quad \langle \text{this.accnt.key} \rangle \leftarrow (\text{buyer}, \text{this.accnt}, \text{price}) = \langle \text{this.accnt.key} \rangle \leftarrow \text{buyer}$$

Then, by application of the rule of consequence, (4), and the rule `CALL_EXT_ADAPT`, we can establish (1). More details in §H.3 in [41].

### 6.3 Third phase: Proving adherence to Module Specifications

In Fig. 8 we define the judgment  $\vdash M$ , which says that  $M$  has been proven to be well formed.

`WELLFRM_MOD` and `COMB_SPEC` say that  $M$  is well-formed if its specification is well-formed (according to Def. 5.6), and if  $M$  satisfies all conjuncts of the specification. `METHOD` says that a module satisfies a method specification if the body satisfies the corresponding pre-, post- and midcondition. In the postcondition we also ask that  $A \rightarrow \text{res}$ , so that `res` does not leak any of the values that  $A$  promises will be protected. `INVARIANT` says that a module satisfies a specification  $\forall \bar{x} : \bar{C}. \{A\}$ , if the method body of each public method has  $A$  as its pre-, post- and midcondition. Moreover, the precondition is strengthened by  $A \rightarrow (\text{this}, \bar{y})$  – this is sound because the caller is external, and by Lemma 6.2, part (4).

$$\begin{array}{c}
\frac{\vdash \mathcal{S}pec(M) \quad \text{WELLFRM\_MOD} \quad M \vdash \mathcal{S}pec(M)}{\vdash M} \quad \frac{M \vdash S_1 \quad \text{COMB\_SPEC} \quad M \vdash S_2}{M \vdash S_1 \wedge S_2} \\
\\
\text{METHOD} \\
\frac{\text{mBody}(m, D, M) = p(\overline{y : D})\{stmt\} \quad M \vdash \{ \text{this} : D, \overline{y : D} \wedge A_1 \} stmt \{ A_2 \wedge A_2 \neg res \} \parallel \{ A_3 \}}{M \vdash \{ A_1 \} p D :: m(\overline{y : D}) \{ A_2 \} \parallel \{ A_3 \}} \\
\\
\text{INVARIANT} \\
\frac{\forall D, m. \text{mBody}(m, D, M) = \text{public}(\overline{y : D})\{stmt\} \implies \quad M \vdash \{ \text{this} : D, \overline{y : D}, \overline{x : C} \wedge A \wedge A \neg (\text{this}, \overline{y}) \} stmt \{ A \wedge A \neg res \} \parallel \{ A \}}{M \vdash \forall x : C. \{ A \}}
\end{array}$$

Fig. 8. Methods' and Modules' Adherence to Specification

*Barendregt* In METHOD we implicitly require the free variables in a method's precondition not to overlap with variables in its body, unless they are the receiver or one of the parameters ( $Vs(stmt) \cap Fv(A_1) \subseteq \{\text{this}, y_1, \dots, y_n\}$ ). And in INVARIANT we require the free variables in  $A$  (which are a subset of  $\overline{x}$ ) not to overlap with the variable in  $stmt$  ( $Vs(stmt) \cap \overline{x} = \emptyset$ ). This can easily be achieved through renamings, c.f. Def. H.3 in [41].

**Example 6.5** (Proving a public method). Consider the proof that `Account :: set` from  $M_{fine}$  satisfies  $S_2$ . Applying rule INVARIANT, we need to establish:

$$\begin{array}{l}
\{ \dots a : \text{Account} \wedge \langle a.\text{key} \rangle \wedge \langle a.\text{key} \rangle \not\Leftarrow (\text{key}', \text{key}'') \} \\
(5?) \quad \text{body\_of\_set\_in\_Account\_in\_}M_{fine} \\
\{ \langle a.\text{key} \rangle \wedge \langle a.\text{key} \rangle \neg res \} \parallel \{ \langle a.\text{key} \rangle \}
\end{array}$$

Given the conditional statement in `set`, and with the obvious treatment of conditionals (c.f. Fig. 15 in [41]), among other things, we need to prove for the `true`-branch that:

$$\begin{array}{l}
\{ \dots \langle a.\text{key} \rangle \wedge \langle a.\text{key} \rangle \not\Leftarrow (\text{key}', \text{key}'') \wedge \text{this}.\text{key} = \text{key}' \} \\
(6?) \quad \text{this}.\text{key} := \text{key}'' \\
\{ \langle a.\text{key} \rangle \} \parallel \{ \langle a.\text{key} \rangle \}
\end{array}$$

We can apply case-split (c.f. Fig. 15 in [41]) on whether `this=a`, and thus a proof of (7?) and (8?), would give us a proof of (6?):

$$\begin{array}{l}
\{ \dots \langle a.\text{key} \rangle \wedge \langle a.\text{key} \rangle \not\Leftarrow (\text{key}', \text{key}'') \wedge \text{this}.\text{key} = \text{key}' \wedge \text{this} = a \} \\
(7?) \quad \text{this}.\text{key} := \text{key}'' \\
\{ \langle a.\text{key} \rangle \} \parallel \{ \langle a.\text{key} \rangle \}
\end{array}$$

and also

$$\begin{array}{l}
\{ \dots \langle a.\text{key} \rangle \wedge \langle a.\text{key} \rangle \not\Leftarrow (\text{key}', \text{key}'') \wedge \text{this}.\text{key} = \text{key}' \wedge \text{this} \neq a \} \\
(8?) \quad \text{this}.\text{key} := \text{key}'' \\
\{ \langle a.\text{key} \rangle \} \parallel \{ \langle a.\text{key} \rangle \}
\end{array}$$

If `this.key=key' ∧ this=a`, then `a.key=key'`. But  $\langle a.\text{key} \rangle \not\Leftarrow \text{key}'$  and PROT-NEQ from Fig. 16 in [41] give `a.key ≠ key'`. So, by contradiction (c.f. Fig. 15 in [41]), we can prove (7?). If `this ≠ a`, then we obtain from the underlying Hoare logic that the value of `a.key` did not change. Thus, by rule PROT\_1, we obtain (8?). More details in §H.5 in [41].

On the other hand, `set` from  $M_{bad}$  cannot be proven to satisfy  $S_2$ , because it requires proving

$$\begin{array}{l}
\{ \dots \langle a.\text{key} \rangle \wedge \langle a.\text{key} \rangle \not\Leftarrow (\text{key}', \text{key}'') \} \\
(???) \quad \text{this}.\text{key} := \text{key}'' \\
\{ \{ \langle a.\text{key} \rangle \} \parallel \{ \langle a.\text{key} \rangle \} \}
\end{array}$$

and without the condition `this.key=key'` there is no way we can prove (???)

## 6.4 Our Example Proven

Using our Hoare logic, we have developed a mechanised proof in Coq, that, indeed,  $M_{good} \vdash S_2 \wedge S_3$ . This proof is part of the accompanying artifact.

Our proof models  $\mathcal{L}_{ul}$ , the assertion language, the specification language, and the Hoare logic from §6.1, §6.2, §6.3, §F in [41] and Def. 6.1. In keeping with the start of §6, our proof assumes the existence of an underlying Hoare logic, and several, standard, properties of that underlying logic, the assertions logic (e.g. equality of objects implies equality of field accesses) and of type systems (e.g. fields of objects of different types cannot be aliases of one another). All assumptions are clearly indicated in the associated artifact. Appendix H in [41] outlines that proof.

## 7 Soundness

We now give a synopsis of the proof of soundness of the logic from §6, and outline the two most interesting aspects: deep satisfaction, and summarized execution.

*Deep Satisfaction* We are faced with the problem that assertions are not always preserved when popping the top frame (c.f. Ex. 4.7), while we need to be able to argue that method return preserves post-conditions. For this, we introduce a “deeper” notion of assertion satisfaction, which requires that an assertion not only is satisfied from the viewpoint of the top frame, but also from the viewpoint of all frames from  $k$ -th frame onwards:  $M, \sigma, k \models A$  says that  $\forall j. [k \leq j \leq |\sigma| \Rightarrow M, \sigma[j] \models A]$ . Accordingly, we introduce *deep specification satisfaction*,  $\overline{M}; M \models_{deep} \{A\} \sigma \{A'\} \parallel \{A''\}$ , which promises for all  $k \leq |\sigma|$ , if  $M, \sigma, k \models A$ , and if scoped execution of  $\sigma$ 's continuation leads to final state  $\sigma'$  and intermediate external state  $\sigma''$ , then  $M, \sigma', k \models A'$ , and  $M, \sigma'', k \models A''$  - c.f. App.G.3 in [41].

Here how deep satisfaction addresses this problem: Assume state  $\sigma_1$  right before entering a call,  $\sigma_2$  and  $\sigma_3$  at start and end of the call's body, and  $\sigma_4$  upon return. If a pre-condition holds at  $\sigma_1$ , then it holds for a  $k \leq |\sigma_1|$ ; hence, if the postcondition holds for  $k$  at  $\sigma_3$ , and because  $|\sigma_3| = |\sigma_1| + 1$ , it also holds for  $\sigma_4$ . Deep satisfaction is stronger than shallow (i.e. specification satisfaction as in Def. 5.2).

**Lemma 7.1.** For all  $\overline{M}, M, A, A', A'', \sigma$ :

$$\bullet \overline{M}; M \models_{deep} \{A\} \sigma \{A'\} \parallel \{A''\} \implies \overline{M}; M \models \{A\} \sigma \{A'\} \parallel \{A''\}$$

*Soundness of the Triples Logic* We require the assertion logic,  $M \vdash A$ , and the underlying Hoare logic,  $M \vdash_{ul} \{A\} stmt \{A'\}$ , to be sound. Such sound logics do exist. Namely, one can build an assertion logic,  $M \vdash A$ , by extending a logic which does not talk about protection, through the addition of structural rules which talk about protection; this extension preserves soundness - c.f. App. G.1 in [41]. Moreover, since the assertions  $A$  and  $A'$  in  $M \vdash_{ul} \{A\} stmt \{A'\}$  may, but need not, talk about protection, one can take a Hoare logic from the literature as the  $\vdash_{ul}$ -logic.

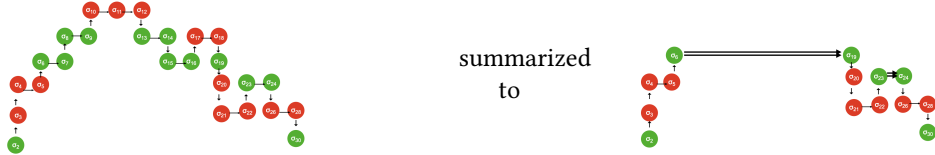
We then prove soundness of the rules about protection from Fig. 6, and, based on this, we prove soundness of the inference system for triples - c.f. Appendix G.5 in [41].

**Theorem 7.2.** For module  $M$  such that  $\vdash M$ , and for any assertions  $A, A', A''$  and statement  $stmt$ :

$$M \vdash \{A\} stmt \{A'\} \implies M \models_{deep} \{A\} stmt \{A'\} \parallel \{A''\}$$

*Summarised Execution.* Execution of an external call may consist of any number of external transitions, interleaved with calls to public internal methods, which in turn may make any number of further internal calls (public or private), and these, again may call external methods. For the proof of soundness, internal and external transitions use different arguments. For external transitions we consider small steps and argue in terms of preservation of encapsulated properties, while for internal calls, we use large steps, and appeal to the method's specification. Therefore, we define *sumarized* executions, where internal calls are collapsed into one, large step, e.g. below:





Lemma G.28 in [41] says that any terminating execution starting in an external state consists of a sequence of external states interleaved with terminating executions of public methods. Lemma G.29 in [41] says that such an execution preserves an encapsulated assertion  $A$  provided that all these finalising internal executions also preserve  $A$ .

*Soundness of the Quadruples Logic* Proving soundness of our quadruples requires induction on the execution in some cases, and induction on the derivation of the quadruples in others. We address this through a well-founded ordering that combines both, *c.f.* Def. G.22 in [41] and Lemma G.23 in [41]. Finally, in §G.16 in [41], we prove soundness:

**THEOREM 7.3.** *For module  $M$ , assertions  $A, A', A''$ , state  $\sigma$ , and specification  $S$ :*

$$\begin{aligned} (A) : & \vdash M \wedge M \vdash \{A\} \text{stmt}\{A'\} \parallel \{A''\} \implies M \models_{\text{deep}} \{A\} \text{stmt}\{A'\} \parallel \{A''\} \\ (B) : & M \vdash S \implies M \models_{\text{deep}} S \end{aligned}$$

## 8 Conclusion: Summary, Related Work and Further Work

*Our motivation* comes from the OCAP approach to security, whereby object capabilities guard against unsanctioned effects. Miller [83, 85] advocates *defensive consistency*: whereby “An object is defensively consistent when it can defend its own invariants and provide correct service to its well behaved clients, despite arbitrary or malicious misbehaviour by its other clients.” Defensively consistent modules are hard to design and verify, but make it much easier to make guarantees about systems composed of multiple components [90].

*Our work* aims to elucidate such guarantees. We want to formalize and prove that [44]:

*Lack of eventual access implies that certain properties will be preserved, even in the presence of external calls.*

For this, we had to model the concept of lack of eventual access, determine the temporal scope of the preservation, and develop a Hoare logic framework to formally prove such guarantees.

For lack of eventual access, we introduced protection, a property of all the paths of all external objects accessible from the current stack frame. For the temporal scope of preservation, we developed scoped invariants, which ensure that a given property holds as long as we have not returned from the current method. (top of current stack has not been popped yet). For our Hoare logic, we introduced an adaptation operator, which translates assertions between the caller’s and callee’s frames. Finally, to prove the soundness of our approach, we developed the notion of deep satisfaction, which mandates that an assertion must be satisfied from a particular stack frame onward. Thus, most concepts in this work are *scope-aware*, as they depend on the current stack frame.

With these concepts, we developed a specification language for modules limiting effects, a Hoare Logic for proving external calls, protection, and adherence to specifications, and have proven it sound.

*Lack of Eventual Access* Efforts to restrict “eventual access” have been extensively explored, with Ownership Types being a prominent example [20, 26]. These types enforce encapsulation boundaries to safeguard internal implementations, thereby ensuring representation independence and defensive consistency [6, 25, 92]. Ownership is fundamental to key systems like Rust’s memory safety [60, 62], Scala’s Concurrency [50, 51], Java’s heap analyses [54, 87, 97], and it plays a critical role in program

verification [13, 65] including Spec# [8, 9], universes [35, 36, 72], Borrowable Fractional Ownership [91], and recently has been integrated into languages like OCAML [71, 76].

Ownership types are closely related to the notion of protection: both are scoped relative to a frame. However, ownership requires an object to control some part of the path, while protection demands that module objects control the endpoints of paths.

In future work we want to explore how to express protection within Ownership Types, with the primary challenge being how to accommodate capabilities accessible to some external objects while still inaccessible to others. Moreover, tightening some rules in our current Hoare logic (e.g. Def. 4.4) may lead to a native Hoare logic of ownership. Also, recent approaches like the Alias Calculus [63, 102], Reachability Types [7, 113] and Capturing Types [12, 17, 117] abstract fine-grained method-level descriptions of references and aliases flowing into and out of methods and fields, and likely accumulate enough information to express protection. Effect exclusion [73] directly prohibits nominated effects, but within a closed, fully-typed world.

*Temporal scope of the guarantee* Starting with loop invariants[47, 55], property preservation at various granularities and durations has been widely and successfully adapted and adopted [8, 27, 40, 56, 66, 67, 69, 81, 82, 89]. In our work, the temporal scope of the preservation guarantee includes all nested calls, until termination of the currently executing method, but not beyond. We compare with object and history invariants in §3.3.2.

Such guarantees are maintained by the module as a whole. Drossopoulou et al. [43] proposed “holistic specifications” which take an external perspective across the interface of a module. Mackay et al. [75] builds upon this work, offering a specification language based on *necessary* conditions and temporal operators. Neither of these systems support any kind of external calls. Like [43, 75] we propose “holistic specifications”, albeit without temporal logics, and with sufficient conditions. In addition, we introduce protection, and develop a Hoare logic for protection and external calls.

*Hoare Logics* were first developed in Hoare’s seminal 1969 paper [55], and have inspired a plethora of influential further developments and tools. We shall discuss a few only.

Separation logics [58, 100] reason about disjoint memory regions. Incorporating Separation Logic’s powerful framing mechanisms will pose several challenges: We have no specifications and no footprint for external calls. Because protection is “scope-aware”, expressing it as a predicate would require quantification over all possible paths and variables within the current stack frame. We may also require a new separating conjunction operator. Hyper-Hoare Logics [30, 39] reason about the execution of several programs, and could thus be applied to our problem, if extended to model all possible sequences of calls of internal public methods.

Incorrectness Logic [93] under-approximates postconditions, and thus reasons about the presence of bugs, rather than their absence. Our work, like classical Hoare Logic, over-approximates postconditions, and differs from Hoare and Incorrectness Logics by tolerating interactions between verified code and unverified components. Interestingly, even though in earlier works[43, 75] we employed *necessary* conditions for effects (*i.e.* under-approximate pre-conditions), we can, instead, employ *sufficient* conditions for the lack of effects (over-approximate postconditions). Incorporating our work into Incorrectness Logic might require under-approximating eventual access, while protection over-approximates it.

Rely-Guarantee [52, 111] and Deny-Guarantee [38] distinguish between assertions guaranteed by a thread, and those a thread can reply upon. Our Hoare quadruples are (roughly) Hoare triples plus the “guarantee” portion of rely-guarantee. When a specification includes a guarantee, that guarantee must be maintained by every “atomic step” in an execution [52], rather than just at method boundaries as in visible states semantics [40, 89, 107]. In concurrent reasoning, this is

because shared state may be accessed by another cooperating thread at any time: while in our case, it is because unprotected state may be accessed by an untrusted component within the same thread.

*Models and Hoare Logics for the interaction with the the external world* Murray [90] made the first attempt to formalise defensive consistency, to tolerate interacting with any untrustworthy object, although without a specification language for describing effects (i.e. when an object is correct).

Cassez et al. [21] propose one approach to reason about external calls. Given that external callbacks are necessarily restricted to the module’s public interface, external callsites are replaced with a generated `externalcall()` method that nondeterministically invokes any method in that interface. Rao et al. [99]’s Iris-Wasm is similar. WASM’s modules are very loosely coupled: a module has its own byte memory and object table. Iris-Wasm ensures models can only be modified via their explicitly exported interfaces.

Swasey et al. [109] designed OCPL, a logic that separates internal implementations (“high values”) from interface objects (“low values”). OCPL supports defensive consistency (called “robust safety” after the security literature [10]) by ensuring low values can never leak high values, and prove object-capability patterns, such as sealer/unsealer, caretaker, and membrane. RustBelt [60] developed this approach to prove Rust memory safety using Iris [61], and combined with RustHorn [78] for the safe subset, produced RustHornBelt [77] that verifies both safe and unsafe Rust programs. Similar techniques were extended to C [103].

Devriese et al. [33] deploy step-indexing, Kripke worlds, and public/private state machines to model problems including the DOM wrapper and a mashup application. Their distinction between public and private transitions is similar to our distinction between internal and external objects. VMSL is an Iris-based separation logic for virtual machines to assure defensive consistency [70]. Cerise uses Iris invariants to support proofs of programs with outgoing calls and callbacks, on capability-safe CPUs [48]. [?] supports several different wrappers on the same object. Besides the distinction between the object-based and class-based approach, our work differs from [33, 48, 70, 77, 99, 109?] in that they follow a “static” capability model whereby authority is conferred though an object’s interface, whereas we follow a “dynamic” model whereby authority may be stored in the callee’s context.

Anderson et al. [3] extend memory safety arguments to “stack safety”, ensuring that method calls and returns are well bracketed, and preserving the integrity of caller and callee. Schaefer et al. [104] adopt an information-flow approach to ensure confidentiality by construction.

*Smart Contracts* also pose the problem of external calls. Rich-Ethereum [18] assumes instance-private and unaliased contract fields, and employs a “finished” flag to manage callbacks. Scilla [105], a minimalist functional alternative to Ethereum, helps prevent common contract errors.

The VerX tool can verify specifications for Solidity contracts automatically [96]. VerX’s specification language is based on temporal logic. It is restricted to “effectively call-back free” programs [2, 49], delaying any callbacks until the incoming call to the internal object has finished.

CONSol [114] provides a specification language for smart contracts, checked at runtime [46]. SCIO\* [4], implemented in F\*, supports both verified and unverified code. Both CONSol and SCIO\* are similar to gradual verification techniques [28, 118] that insert dynamic checks between verified and unverified code, and contracts for general access control [29, 37, 88].

*Future work* includes the application of our techniques to languages that rely on lexical nesting for access control, e.g. Javascript [84], to languages that support ownership types, e.g. Rust, leveraged for verification [5, 64, 77], and to features such as uniqueness [71, 76]. These different language paradigms may lead to a refined view of the meaning of protection. We also want to investigate tighter integrations with the underlying Hoare logic and its framing operators.

## Data Availability Statement

Accompanying this paper, we include a Rocq model for Chainmail, along with a proof for the Shop case study from Sect. 2 [74].

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