## Q1. VaR during the 2008 Financial Crisis

All major banks use Value-at-Risk as a measure of market risk. As part of their disclosure to investors, banks report how they measure and manage market risk, including how they use Value-at-Risk. They also report on how their Value-at-Risk models have performed. Each study group is assigned to a bank as follows and reponsible for summarizing the VaR used by these banks during the 2008 financial crisis. Your group number can be found in the attached list. If your group number is greater than 10, please use the group corresponding to the last digit of your group number. For example, if your group number is 15, go with the same bank as group 5, i.e Barclays. Or if your group number is 20, please download data for Credit Suiss

- Download their 2008 annual reports (10-K for US firms and 20-F or 6-K for foreign firms) from SEC's website (https://www.sec.gov/edgar/searchedgar/companysearch.html). Find the section where market risk is discussed and write a short essay summarizing the bank's practices concerning the following:
- Technique used to compuate VaR
- What data is used to compuate VaR? Is more recent data weighted more heavily?
- Time horizon
- Confidence level
- Number of VaR exceptions in 2008 (days where loss exceeded VaR)
- Any changes to VaR methodology made as a result of the financial crisis?

## **Group 5 - Barclays Capital**

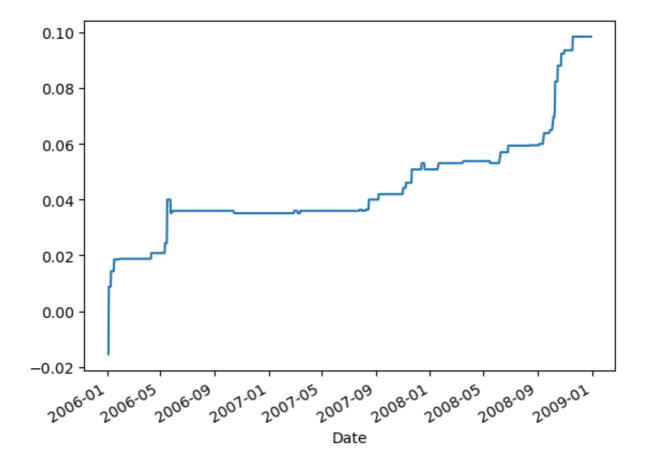
- Technique, Data, Time horizon
  - Barclays Capital uses the historical simulation method with a two year unweighted historical period.
- Confidence level
  - In 2008, the confidence level was changed to 95% from 98% as an increasing incidence of significant market movements made the existing measure more volatile and less effective for risk management purposes. Switching to 95% made DVaR more stable and consequently improved management, transparency and control of the market risk profile.
- Number of VaR exceptions in 2008 (days where loss exceeded VaR)
  - A green model is consistent with a good working DVaR model and is achieved formodels that have **four or less** back testing exceptions in a 12month period. For Barclays Capital's trading book, green model status was

maintained for 2008 and 2007.

- Any changes to VaR methodology made as a result of the financial crisis?
  - To further improve the control framework, formal daily monitoring of ES was started. This metric is the average of all the hypothetical losses beyond DVaR.Other controls include stress testing and scenario testing.
- 2. Download the daily stock price for the corresponding bank over 2006-2008 from Yahoo finance.
- On each day of 2008, compute the 99% 1-day VaR for the stock return using the historical method with all past data in the sample.

```
In [7]:
        import yfinance as yf
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        barclays_data = yf.download('BARC.L', start='2006-01-01', end='2008-12-31'
        barclays_data['Return'] = barclays_data['Adj Close'].pct_change()
        def historical_var(returns, confidence_level=0.99):
            # Sort returns from smallest to largest
            sorted_returns = returns.sort_values()
            # Calculate the index corresponding to the 1% quantile
            var_index = int((1 - confidence_level) * len(sorted_returns))
            # VaR is the return at that index
            var = - sorted_returns.iloc[var_index]
            return var
        return_2008 = barclays_data['Return']['2008':]
        historical_vars = barclays_data['Return'].expanding().apply(lambda x: his
        historical_vars_2008 = historical_vars['2008':]
        historical vars.plot()
       [********** 100%********** 1 of 1 completed
```

Out[7]: <Axes: xlabel='Date'>



• If you are at the end of 2008 and want to back-test this approach, what do you do and what do you conclude?

```
In [9]: # Count the number of days when the actual loss exceeded the VaR (VaR exc
var_exceptions = (return_2008 < -historical_vars_2008).sum()

# Show the VaR exceptions
print(var_exceptions)</pre>
```

• Comment on the relation with what you found in the annual report.

The discrepancy between the four or less VaR exceptions reported in Barclays' annual report and the 17 exceptions found through my analysis can be attributed to several factors:

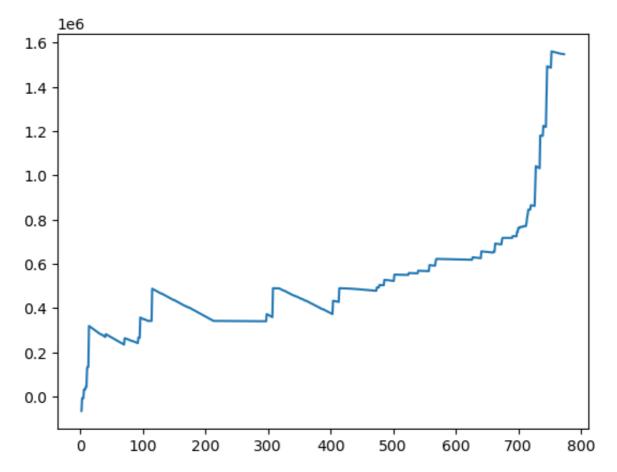
- **Data and Methodology:** The bank's reported VaR exceptions are based on its internal risk management systems and calculations, which may use different data or computational methods than the historical simulation you've performed.
- Risk Measures: The bank might measure risk differently, considering a variety of factors like liquidity adjustments, risk factor sensitivities, and tail risk measures, which could result in fewer exceptions.

- 3. Add to your dataset the daily stock price for all 10 banks over the same period.
- Use the historical method to compute the VaR for a portfolio with \$1m in the odd-numbered banks (1, 3, ...), \$2m in the even-numbered banks.

```
In [41]: import warnings
    warnings.simplefilter(action='ignore', category=FutureWarning)

In [38]: tickers = ['GS', 'UBS', 'JPM', 'C', 'BARC.L', 'MS', 'DB', 'BAC', 'BNP.PA'
    data = {}
    for ticker in tickers:
        data[ticker] = yf.download(ticker, start='2006-01-01', end='2008-12-3
    data = pd.concat([i for i in data.values()], keys=data.keys(), axis=1).so
    weights = np.ones(len(tickers)) * 1e6
    weights[1::2] = weights[1::2] * 2
    portfolio_returns = pd.Series(np.dot(data, weights))
    VaRs = - portfolio_returns.expanding().quantile(1-c)
    plt.plot(range(len(VaRs)), VaRs)
```

Out[38]: [<matplotlib.lines.Line2D at 0x13a49cc50>]

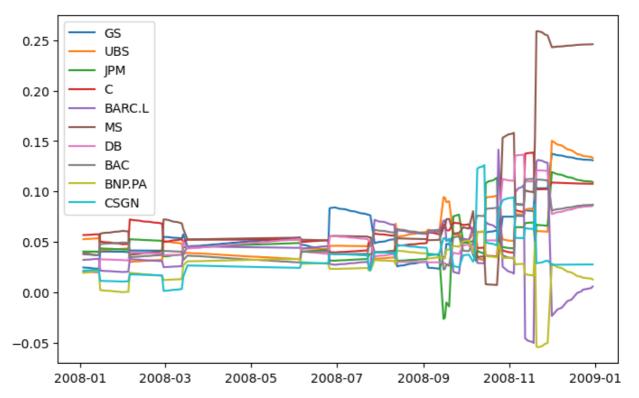


• Compute the DVaR and CVaR for each bank.

```
In [34]: c = 0.99
data = data.dropna()
```

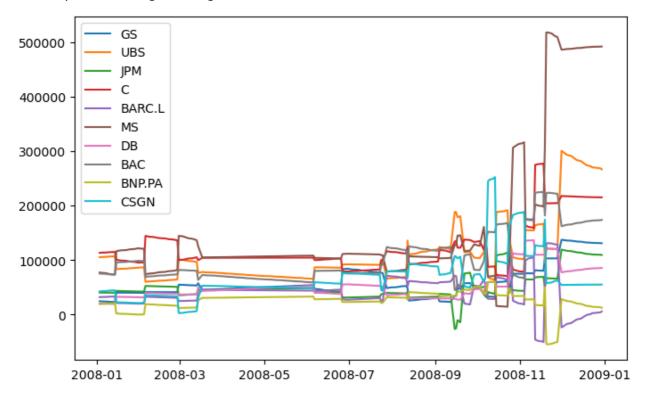
```
def VaRs(returns, weights, epsilon):
    portfolio_returns = pd.Series(np.dot(returns, weights))
    VaRs = - portfolio_returns.expanding().quantile(1-c)
   DVaRs = \{\}
    for i,ticker in zip(range(len(tickers)),tickers):
        new_weights = weights.copy()
        new_weights[i] += epsilon
        new_portfolio_returns = pd.Series(np.dot(returns, new_weights))
        DVaRs[tickers[i]] = (- new_portfolio_returns.expanding().quantile
        DVaRs[tickers[i]].index = returns.index
   DVaRs = pd.concat([i for i in DVaRs.values()], keys = DVaRs.keys(), a
   CVaRs = DVaRs.copy()
    for i,ticker in zip(range(10),tickers):
        CVaRs[ticker] = weights[i] * DVaRs[ticker]
    return VaR, DVaRs, CVaRs
epsilon = 1 # increase $1
VaR, DVaRs, CVaRs = VaRs(data, weights, epsilon)
plt.figure(figsize=(8,5))
for ticker in tickers:
    plt.plot(data['2008':].index, DVaRs['2008':][ticker], label=ticker)
plt.legend()
```

Out[34]: <matplotlib.legend.Legend at 0x13a2e4590>



```
In [35]: plt.figure(figsize=(8,5))
    for ticker in tickers:
        plt.plot(data['2008':].index, CVaRs['2008':][ticker], label=ticker)
    plt.legend()
```

Out[35]: <matplotlib.legend.Legend at 0x13a3e4210>



· Comment on the results.

For most of banks, the daily CVaR increased for the year 2008. And Morgan Stanley, UBS have the high increase in risk over the period. It represents that in the fourth quarter of 2008, the volatility of financial market rocketed up.

• If you had to make a recommendation on how to tilt this portfolio, what would it be based on the data you have

Reducing Exposure to High-Risk Entities: Banks with the highest DVaR and CVaR, such as MS and UBS, may be candidates for reduced exposure to limit potential losses in turbulent market conditions.

## Q2. Short questions

1. Prove that if 8 people are born in a three-year period, at least 3 of them are born within the same one-year period. What does it have to do with the class?

We can use the Pigeonhole Principle.

- Pigeons: the 8 people.
- Pigeonholes: the 3 one-year periods within the three-year span.

By the Pigeonhole Principle, at least one of the one-year periods during the threeyear span must contain at least three people.

This calculation involves understanding the worst losses that could be incurred in an investment in a given time period given a specific confidence interval. The connection between this problem and VaR lies in the conceptualization of distributions and the worst-case scenarios:

- Distribution: Just as we distributed people among years, VaR involves distributing potential losses across various scenarios and finding the worst expected loss under normal market conditions.
- Worst-case Scenario: The Pigeonhole Principle can analogously be applied to understand the concentration of risk (where "years" are different market conditions and "people" could represent losses or risks associated), ensuring that planners prepare for scenarios where risk might be concentrated anomalously.
- 2. What is the ten-day 99% VaR of a portfolio with a five-day 98% VaR of \$10 million?

```
In [161... from scipy.stats import norm
    day_5_98 = 1e7
    day_1_98 = day_5_98 / np.sqrt(5)
    day_1_99 = day_1_98 * norm.ppf(0.01) / norm.ppf(0.02)
    day_10_99 = day_1_99 * np.sqrt(10)
    day_10_99
```

Out [161... 16019254.823386569

3. What is the probability of having more than one exception in the same month?

Use the answer this question to come up with a test of a VaR measure based on bunching.

```
P(exception > 1) = 1 - P(noexception) - P(exception = 1) = 1 - c^n - nc^{n-1}(
```

- c is the confidence level of the VaR
- if c = 0.99, n = 21, then P(exception > 1) = 0.01851.
- If the exception of VaR is larger than 0.01851, this could indicate that the VaR
  model is not accurate, and the actual risk could be higher than the VaR measure
  suggests. This kind of statistical testing is often used to back-test VaR models to
  ensure they are performing within expected parameters.

> 4. The next regular FOMC meeting is scheduled for the end of this month. How would you estimate the 2-day 99% VaR of investing \$1m in the S&P500 a day before the announcement? Bonus question: Provide a number.

```
In [6]: SP500 = yf.download('^GSPC', start='2022-4-21', end='2024-4-21',progress=
        SP500['return'] = SP500['Close'].pct_change()
        SP500['VaR_1day'] = SP500['return'].quantile(0.01)
        SP500['VaR_2day'] = SP500['VaR_1day'] * np.sqrt(2)
        VaR = -SP500['VaR_2day'].iloc[-1] * 1e6
        VaR
```

Out[6]: 47642.10953116461

```
In [ ]:
```