In [1]:
```
import os
import yfinance as yf
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from math import *
from scipy.stats import norm, mvn
os.chdir('/Users/yiyujie/Desktop/program/Financial Risk Management')
```

In [2]:
```
import warnings
warnings.filterwarnings('ignore')
```

# Question 1. VaR for option on two underlying

1. We are interested in managing the risk of an option on two stocks with prices $S_{1,t}$ and $S_{2,t}$ . Assume a short rate r and:

$$\frac{dS_{1,t}}{S_{1,t}} = \mu_1 dt + \sigma_1 dW_{1,t}$$

$$\frac{dS_{2,t}}{S_{2,t}} = \mu_2 dt + \sigma_2 dW_{2,t}$$

$$corr(dW_{1,t}, dW_{2,t}) = \rho dt$$

where all the parameters are in daily units. Call $M(S_{1,t}, S_{2,t})$ the price of the option. Derive a formula for the 99% 1-day VaR for the option using the delta approach

$$\begin{aligned}
\triangle_1 &= \frac{\partial M}{\partial S_1}, \triangle_2 = \frac{\partial M}{\partial S_2} \\
dM(S_{1,t}, S_{2,t}) &= \frac{\partial M}{\partial S_1} dS_{1,t} + \frac{\partial M}{\partial S_2} dS_{2,t} = \triangle_1 dS_{1,t} + \triangle_2 dS_{2,t} \\
E[dM] &= [\triangle_1 S_{1,t}\mu_1 + \triangle_2 S_{2,t}\mu_2]dt \\
Std[dM] &= \sqrt{[\triangle_1^2 S_{1,t}^2 \sigma_1^2 + \triangle_2^2 S_{2,t}^2 \sigma_2^2 + 2\triangle_1 S_{1,t} \triangle_2 S_{2,t} \rho \sigma_1 \sigma_2]dt} \\
VaR &= -(E[dM_t] - 2.326 \times Std[dM_t])
\end{aligned}$$

2. Consider the case of a European option on the minimum of the two stock

price, with maturity T and final payoff: $M_T = max(min(S_{1,T}, S_{2,T}) - K, 0)$ From Stulz (1982) (attached paper), the price of the option when the time to maturity is $\tau$ is:

$$M(S_1, S_2) = S_1 N_2(\gamma_1 + \sigma_1\sqrt{\tau}, (ln(S_2/S_1) - \frac{1}{2}\sigma^2\sqrt{\tau})/(\sigma\sqrt{\tau}), (\rho\sigma_2 - \sigma_1)/\sigma)$$
$$+ S_2 N_2(\gamma_2 + \sigma_2\sqrt{\tau}, (ln(S_1/S_2) - \frac{1}{2}\sigma^2\sqrt{\tau})/(\sigma\sqrt{\tau}), (\rho\sigma_1 - \sigma_2)/\sigma)$$
$$- Ke^{-r\tau} N_2(\gamma_1, \gamma_2, \rho)$$

where

$$\gamma_1 = \frac{ln(S_1/K) + (r - \frac{1}{2}\sigma_1^2)\tau}{\sigma_1\sqrt{\tau}}$$
$$\gamma_1 = \frac{ln(S_2/K) + (r - \frac{1}{2}\sigma_2^2)\tau}{\sigma_2\sqrt{\tau}}$$
$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$

and $N_2(\alpha, \beta, \theta)$ is the bivariate cumulative standard normal distribution with upper limits $\alpha$ and $\beta$ and correlation $\theta$. That is, if $X_1$ and $X_2$ are standard normal with correlation $\theta$:

$$N_2(\alpha, \beta, \theta) = P(X_1 \leq \alpha, X_2 \leq \beta)$$

Assume: $r = 0.02\%, \sigma_1 = \sigma_2 = 1.5\%, \rho = 0.35, \mu_1 = \mu_2 = 0.025\%$, where again all parameters are in daily units. Further assume that at date 0, we have T = 6 months, and that $S_{1,0} = 99, S_{2,0} = 101$ and $K = 100$. Compute the price of the option at date 0.

In [3]:
```
# Parameters
r = 0.0002 # Convert daily parameters to annual scale
mu_1 = mu_2 = 0.00025
sigma1 = sigma2 = 0.015 # Convert daily parameters to annual scale
rho = 0.35  # correlation
T = 0.5 * 252   # 6 months
S10 = 99
S20 = 101
K = 100

def option_price(S10, S20, K, r, sigma1, sigma2, rho, T, mu_1, mu_2):
    sigma = np.sqrt(sigma1**2 + sigma2**2 - 2 * rho * sigma1 * sigma2)

    # γ1, γ2 calculations
    gamma1 = (log(S10/K) + (r - 0.5 * sigma1**2) * T) / (sigma1 * sqrt(T)
    gamma2 = (log(S20/K) + (r - 0.5 * sigma2**2) * T) / (sigma2 * sqrt(T)

    # Arguments for bivariate normal distributions
    lower = np.array([-np.inf, -np.inf])
    upper1 = np.array([gamma1 + sigma1 * sqrt(T), (log(S20/S10) - 0.5 * s
    upper2 = np.array([gamma2 + sigma2 * sqrt(T), (log(S10/S20) - 0.5 * s
    upper3 = np.array([gamma1, gamma2])
    cov1 = np.array([[1, (rho * sigma2 - sigma1) / sigma], [(rho * sigma2
```

```
    cov2 = np.array([[1, (rho * sigma1 - sigma2) / sigma], [(rho * sigma1
    cov3 = np.array([[1, rho], [rho, 1]])

    # Bivariate normal calculations
    p1, _ = mvn.mvnun(lower, upper1, np.array([0, 0]), cov1) # means
    p2, _ = mvn.mvnun(lower, upper2, np.array([0, 0]), cov2)
    p3, _ = mvn.mvnun(lower, upper3, np.array([0, 0]), cov3)

    return S10 * p1 + S20 * p2 - K * exp(-r * T) * p3, p1, p2 # Option pr

price, p1, p2 = option_price(S10, S20, K, r, sigma1, sigma2, rho, T, mu_1
price
```

Out[3]: 7.558996338280906

3. Compute the 99% 1-day VaR for the option using the formula you have derived in question 1.

$$\begin{aligned}
\triangle_1 &= N_2(\gamma_1 + \sigma_1\sqrt{\tau}, (ln(S_2/S_1) - \frac{1}{2}\sigma^2\sqrt{\tau})/(\sigma\sqrt{\tau}), (\rho\sigma_2 - \sigma_1)/\sigma) \\
\triangle_2 &= N_2(\gamma_2 + \sigma_2\sqrt{\tau}, (ln(S_1/S_2) - \frac{1}{2}\sigma^2\sqrt{\tau})/(\sigma\sqrt{\tau}), (\rho\sigma_1 - \sigma_2)/\sigma) \\
E[dM_t] &= [\triangle_1 S_{1,t}\mu_1 + \triangle_2 S_{2,t}\mu_2]dt \\
Var[dM_t] &= [\triangle_1^2 S_{1,t}^2\sigma_1^2 + \triangle_2^2 S_{2,t}^2\sigma_2^2 + 2\triangle_1 S_{1,t}\triangle_2 S_{2,t}\rho\sigma_1\sigma_2]dt \\
VaR &= -(E[dM_t] - 2.326 \times Var[dM_t])
\end{aligned}$$

In [4]:
```
delta_1 = (option_price(S10+0.5, S20, K, r, sigma1, sigma2, rho, T, mu_1,
delta_2 = (option_price(S10, S20+0.5, K, r, sigma1, sigma2, rho, T, mu_1,
dt = 1
EM = (delta_1 * S10 * mu_1 + delta_2 * S20 * mu_2 ) * dt
VarM = np.sqrt(((delta_1 * S10 * sigma1)**2 + (delta_2 * S20 * sigma2)**2
print("The 99% 1-day VaR is ", -(EM - 2.326 * VarM))
```

```
The 99% 1-day VaR is  1.6911693414597475
```

4. Compute the 99% 1-day VaR for the option using simulations. Compare to the results of the previous question and explain the intuition behind this result.

In [5]:
```
mu_1 = mu_2 = 0.00025
sigma1 = sigma2 = 0.015
rho = 0.35
S10 = 99
S20 = 101
K = 100

dt = 1  # Daily time step
n_steps = int(T)  # Number of time steps
np.random.seed(42)
```

```
mu = [0, 0]
cov = [[1, rho],[rho, 1]]

N = 1000000
S1 = np.ones(N) * S10
S2 = np.ones(N) * S20

Z = np.random.multivariate_normal(mu, cov, N)
S1 = S1 * np.exp((mu_1 - 0.5 * sigma1**2) * dt + sigma1 * np.sqrt(dt) * Z
S2 = S2 * np.exp((mu_1 - 0.5 * sigma2**2) * dt + sigma2 * np.sqrt(dt) * Z

VaR = price - option_price(pd.Series(S1).quantile(0.01), pd.Series(S2).qu
print("The 99% 1-day VaR using simulations is ", VaR)
```

```
The 99% 1-day VaR using simulations is  1.8901210954040621
```

5. Optional: Derive a formula for the 99%-VaR for the option using the delta-gamma approach. Implement the formula and compare your results to the other two approaches. Explain the intuition for this result.

$$\triangle_1 = \frac{\partial M}{\partial S_1}, \triangle_2 = \frac{\partial M}{\partial S_2}, \Gamma_1 = \frac{\partial^2 M}{\partial S_1^2}, \Gamma_2 = \frac{\partial^2 M}{\partial S_2^2}$$

$$dM(S_{1,t}, S_{2,t}) = \triangle_1 dS_{1,t} + \triangle_2 dS_{2,t} + \frac{1}{2}\Gamma_1 dS_{1,t}^2 + \frac{1}{2}\Gamma_2 dS_{2,t}^2$$

$$E[dS_{i,t}] = \mu_i S_{i,t} dt, E[dS_{i,t}^2] = (\sigma_i S_{i,t})^2 dt,$$

$$E[dM_t] = dt \sum_{i=1}^{2} \left[\triangle_i \mu_i S_{i,t} + \frac{1}{2}\Gamma_i (\sigma_i S_{i,t})^2\right]$$

$$dM_t - E[dM_t] = \triangle_1 \sigma_1 S_{1,t} dW_t + \triangle_2 \sigma_2 S_{2,t} dW_t$$

$$Std[dM_t] = \sqrt{[\triangle_1^2 S_{1,t}^2 \sigma_1^2 + \triangle_2^2 S_{2,t}^2 \sigma_2^2 + 2\triangle_1 S_{1,t} \triangle_2 S_{2,t} \rho \sigma_1 \sigma_2] dt}$$

$$VaR = -(E[dM_t] - 2.326 \times Std[dM_t])$$

In [7]:
```
gamma_1 = (option_price(S10+1, S20, K, r, sigma1, sigma2, rho, T, mu_1, m
gamma_2 = (option_price(S10, S20+1, K, r, sigma1, sigma2, rho, T, mu_1, m
EM = (delta_1 * S10 * mu_1 + delta_2 * S20 * mu_2 + (gamma_1*(sigma1*S10)
VarM = np.sqrt(((delta_1 * S10 * sigma1)**2 + (delta_2 * S20 * sigma2)**2
print("The 99% 1-day VaR is ", -(EM - 2.326 * VarM))
```

```
The 99% 1-day VaR is  1.7185754175226657
```

6. Now assume you are a trader in the real world and you do not know for sure that the model for the underlying is correct. What other types of risks would you worry about? If you had to worry about just one more risk, what woumld it be? Explain (quantitatively) how you get to this conclusion.

- The model relies on several assumptions, such as constant volatility and correlation, which may not hold in reality. If these assumptions are incorrect, the option price could be significantly misestimated.
- The parameters used in the model (e.g., volatilities, correlation, drift rates) are estimated from historical data. If the future behavior of these parameters deviates from past behavior, the model's predictions will be inaccurate.

# Q2 Case Study: Implementing Quantitative Risk Management and VaR in a Chinese Investment Bank

You should buy the case study material using this link: https://hbsp.harvard.edu/import/1169954.

1. Explain the objectives and priorities of each player: Jasper Wang, Jianguo Lu, and Charles Pan. What is motivating the different players? What tensions existed among their different objectives?

**Jasper Wang**

- **Objectives**: He is primarily focused on **rationalizing the trading function and enhancing risk management practice**s within the firm. He aims to introduce more quantitative and systematic approaches to trading.
- **Motivations**: He is motivated by the need to bring more discipline and predictability to the trading operations, potentially increasing efficiency and reducing risk.
- **Tensions**: His push for more structured and quantitative methods may clash with traders who are accustomed to more discretionary and less systematic approaches. This could create resistance among those who prefer traditional methods of trading.

**Jianguo Lu**

- **Objectives**: His primary objective is to maintain and potentially enhance the profitability of the trading operations. He is likely focused on strategies that have **proven successful in the past** and may be wary of significant changes that could disrupt current performance.
- **Motivations**: He is motivated by short-term profitability and the success of existing trading strategies. He may prioritize maintaining current trading methods that have historically yielded positive results.
- **Tensions**: His preference for maintaining the status quo may conflict with Jasper's desire for modernization and systematic approaches. This can create

tension as the two perspectives struggle to align.

**Charles Pan**

- **Objectives**: He is likely focused on overseeing the **broader strategic direction** of the firm's trading operations and ensuring that any changes align with the firm's overall goals.
- **Motivations**: He is motivated by the long-term success and sustainability of the firm's trading practices. He needs to balance innovation with proven methods to ensure overall stability and growth.
- **Tensions**: He may find himself in the middle of the tension between Jasper and Jianguo, needing to mediate and find a compromise that allows for both innovation and stability. His role would be to ensure that changes are implemented smoothly without causing significant disruptions to profitability.

2. Why does Jasper choose to make the VaR model the first step towards rationalizing the trading function? What is the appeal of the VaR model generally?

- Jasper chose the Value at Risk (VaR) model as the initial step towards rationalizing the trading function because it provides a quantifiable measure of risk. VaR offers a standardized way to assess and compare the risk levels of different trading strategies.
- **Appeal of the VaR Model**: VaR is widely accepted in the financial industry as a reliable measure of market risk. It quantifies the potential loss in value of a portfolio over a defined period for a given confidence interval. And VaR is often used for regulatory reporting and compliance, making it a practical choice for aligning the firm's risk management practices with industry standards, since it focusing on tail risks, which are critical in trading environments.

3. Why do Jianguo and the traders resist the VaR model? Do you think their pattern of resistance to risk management is unique to China, or might it be found elsewhere too?

- They think VaR Model cannot accurately capture the complexities of financial markets and it overrely on the historical data.
- The resistance to the VaR model is not unique to China, since it failed during the 2008 financial crisis. The institutions that heavily relied on VaR suffered major unexpected losses during the 2008 financial crisis, leading to widespread skepticism.

4. Using the spreadsheet provided, run backtests of the VaR predictions against actual daily gains or losses for both the S&P 500 index and the Shanghai Composite index.

- (a) Starting with a lookback period of three months, observe the number of exceptions in all years for both the Shanghai and S&P indexes. How do they compare?

- The S&P 500 index generally shows a slightly higher average number of exceptions (15.33) compared to the Shanghai Composite index (14.13).
- The percent of exceptions for the S&P 500 is also higher on average (6.1%) compared to the Shanghai Composite index (5.6%).
- Both indices experienced their highest number of exceptions during periods of high market volatility, such as the financial crisis in 2007-2008.

- (b) Trym different lookback periods (say, 3, 6 and 9 months) to see if the length of the period changes your conclusions.

| exceptions | 3m | 6m | 9m |
|---|---|---|---|
| S&P | 15.33,2007 (24), 2008 (25) | 14.13, 2007 (26), 2008 (26) | 14.2, 2007 (31), 2008 (29) |
| Shanghai Composite index | 14.13, 2001 (21), 2008 (19) | 12.67, 2001 (23), 2008 (19) | 12.4, 2001 (26), 2008 (23) |

- Stability: For both indices, the percent of exceptions does not drastically change with different lookback periods, indicating some level of stability in the VaR model's predictive power.
- Sensitivity: The S&P 500 index shows some sensitivity to the length of the lookback period, particularly in years with high volatility (e.g., 2007-2008).
- Model Validation: The VaR model with a 95% confidence level generally performs consistently across different lookback periods. The percent exceptions hover around the expected 5%, with some deviations during periods of high market volatility.

- (c) Given that Jasper's VaR model assumes a 95% confidence level, how well does the backtest validate the model?

```
A 95% confidence level implies that we expect exceptions
(actual losses exceeding VaR) to occur about 5% of the
```

`time.`

- The backtest results suggest that Jasper's VaR model generally performs well but shows some limitations during periods of high market volatility, particularly for the S&P 500 index.
- The Shanghai Composite index results indicate a better fit, with exceptions closely aligning with the expected 5%.
- Which suggests us that during periods of high market volatility, additional risk measures such as stress testing and scenario analysis should complement the VaR model.

5. How might Jasper use the backtest results to bolster his case for introducing the VaR model?

- The backtests show that the VaR model, with a 95% confidence level, generally captures the expected number of exceptions (around 5%). This consistency can build confidence among stakeholders that the model is reliable for daily risk assessment.
- By highlighting the periods (e.g., 2007-2008) where exceptions exceeded expectations, Jasper can argue that the VaR model provides critical insights into market volatility and can help in preparing for potential financial stress.

6. How successful do you think Jasper will be in his attempt to implement Western risk management practices? What advice would you give to someone in a role similar to his?

- I think it will be hard at first since the financial environment seems so different between China and Western. But there are some regulate support since the evolving regulatory landscape in China is becoming more stringent post-crisis. So I think if Jasper can show how this method bring clear benefits then he will successfully implement the VaR model.
- **Suggestions:** Engage with regulators to ensure the model aligns with regulatory expectations and can be used to demonstrate compliance.

7. What is the current regulation environment of risk followed by Chinese banks and how has it evolved since the crisis? (Find information beyond the case study material)

- Since the financial crisis, Chinese regulators have significantly strengthened the

regulatory framework for banks, for example, increased capital adequacy requirements, enhanced liquidity coverage ratios, and more stringent risk management standards.
- There has been a focus on regulating shadow banking activities to prevent excessive credit growth and ensure financial stability.
- Regular stress testing has become a crucial tool for assessing the resilience of Chinese banks to economic shocks.

In [ ]:

```

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