Shift and Rotation Invariant Object Reconstruction Using the Bispectrum

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Abstract

Triple correlations and their Fourier transforms, called bispectra, have properties desirable for image sequence analysis. Specifically, the triple correlation of a 2-d sequence is shift-invariant, it vanishes for a zero-mean colored Gaussian random field, and can be used to uniquely recover the original sequence to within a linear phase shift. An FFT-based algorithm for reconstructing a 2-d sequence from its bispectrum is reviewed and tested. The bispectrum is also applied to estimate a randomly translating and rotating object from a sequence of noisy images. The technique does not require solution of the correspondence problem, and is insensitive to additive colored Gaussian noise of unknown spectral density. Some simulation results for the random translation case are presented.

1 Introduction

Consider the problem of reconstructing a randomly rotating and translating object from multiple noisy frames. Typically, the correspondence between frames would first be established so that averaging (or other filter function such as median filtering) can be applied to yield an improved estimate of the object, [1]. An alternative approach which does not require solution of the correspondence problem is to transform the image to a shift-invariant and high SNR domain, perform averaging or filtering in this domain, and then reconstruct an enhanced estimate of the object via an appropriate inverse transformation. Such a domain is that of higher than second-order correlations.

In this paper we employ *triple correlations* (or their corresponding Fourier transforms, called *bispectra*) for reconstructing an image of an object from noisy frames. Our motivation for using triple correlations comes from the following three facts.

- The triple correlation is shift-invariant.
- The triple correlation is invertible; i.e., we can uniquely reconstruct a two dimensional sequence from its triple correlation. The shift ambiguity is removed during reconstruction by specifying the centroid of the object to be at the origin.
- The triple correlation of a random zero-mean Gaussian field is identically zero; a fact which may be exploited to reduce additive noise effects.

It has been demonstrated in the 1-d case that significant processing gain may be achieved by utilizing the redundancy in the auto-triple correlation (or equivalently the bispectrum) when reconstructing the original signal [2]. The corresponding 2-d algorithm for reconstructing 2-d signals consists of calculating the auto-triple correlation of each frame (or equivalently the bispectrum of each frame), averaging the correlations (bispectra), and then reconstructing the object from the averaged correlation (bispectrum) [3,4].

Some investigators in optics, notably Lohmann [5,6], have used analysis of continuous deterministic signals without explicitly analyzing the noise effect. Here, discrete analysis is carried out for a deterministic signal in an additive random noise field. This approach yields discrete algorithms for implementation, and allows explicit treatment of the additive noise, [4]. A more general framework for ARMA modeling and phase reconstruction of multidimensional signals from higher-order correlations can be found in [7]. Use of higher-order statistics for signal detection and classification is treated in [8].

2 Background

The triple correlation of a zero-mean 2-d random field $f(\mathbf{m})$ is defined as

$$r_{3f}(\mathbf{n_1},\mathbf{n_2}) = E[f(\mathbf{m})f(\mathbf{m}+\mathbf{n_1})f(\mathbf{m}+\mathbf{n_2})], \qquad (1)$$

where E represents the expectation operator. A natural estimator for the triple correlation of a zero-mean image $f(\mathbf{m})$ defined by (1) is given by

$$f_3(\mathbf{n_1},\mathbf{n_2}) = \frac{1}{N^2}\sum_{\mathbf{m}} f(\mathbf{m})f(\mathbf{m}+\mathbf{n_1})f(\mathbf{m}+\mathbf{n_2}). \quad (2)$$

Let $o(\mathbf{m})$ denote a deterministic object in a noisy $N \times N$ image as

$$f_i(\mathbf{m}) = o(\mathbf{m}) + v_i(\mathbf{m}), \quad (3)$$

where $v(\mathbf{m})$ is a zero-mean additive noise process, and suppose there are L frames where i denotes the i^{th} frame. Averaging estimates of the triple correlation for each frame yields

$$\begin{aligned}\frac{1}{L}\sum_{i=1}^{L} f_{3,i}(\mathbf{n_1},\mathbf{n_2}) &= o_3(\mathbf{n_1},\mathbf{n_2}) + \frac{1}{L}\sum_{i=1}^{L} v_{3,i}(\mathbf{n_1},\mathbf{n_2})\\ &+[\sum_{\mathbf{m}} o(\mathbf{m})]\frac{1}{L}\sum_{i=1}^{L}[v_i(\mathbf{m})v_i(\mathbf{m}+\mathbf{n_1})\\ &+v_i(\mathbf{m})v_i(\mathbf{m}+\mathbf{n_2}) + v_i(\mathbf{m}+\mathbf{n_1})v_i(\mathbf{m}+\mathbf{n_2})]\\ &+\sum_{\mathbf{m}}[\frac{1}{L}\sum_{i=1}^{L} v_i(\mathbf{m})][o(\mathbf{m})o(\mathbf{m}+\mathbf{n_1})\\ &+o(\mathbf{m})o(\mathbf{m}+\mathbf{n_2}) + o(\mathbf{m}+\mathbf{n_1})o(\mathbf{m}+\mathbf{n_2})]. \quad (4)\end{aligned}$$

The fourth term in (4) vanishes due to the zero-mean of the noise. Similarly, the third term in (4) vanishes if the object is zero-mean. In practice the mean is subtracted before forming the estimate (2).

Thus, [2],

$$\frac{1}{L}\sum_{i=1}^{L} f_{3,i}(\mathbf{n_1},\mathbf{n_2}) \overset{N,L\to\infty}{\longrightarrow} o_3(\mathbf{n_1}\mathbf{n_2}) + r_{3v}(\mathbf{n_1},\mathbf{n_2}). \quad (5)$$

For $v(\mathbf{m})$ a zero-mean colored Gaussian random field, [9],

$$r_{3v}(\mathbf{n_1},\mathbf{n_2}) = 0. \quad (6)$$

Also, $v(\mathbf{m})$ can be non-Gaussian if it is i.i.d. and non-skewed (e.g., symmetrically distributed). Equation (5) states that the average triple correlation over multiple records of the deterministic zero-mean object plus random zero-mean noise is equal to the deterministic triple correlation of the object plus an estimate of the triple correlation of the noise. In the case of additive colored Gaussian noise the second term in (5) vanishes as the number of records L goes to infinity.

The discrete-time Fourier transform (DTFT) of an image $f(\mathbf{m})$ is defined as

$$F(e^{j\mathbf{u}}) = \sum_{\mathbf{m}} f(\mathbf{m})e^{-j\mathbf{u}\cdot\mathbf{m}}. \quad (7)$$

Denoting the four-dimensional DTFT of $f_3(\mathbf{n_1},\mathbf{n_2})$, as $F_3(\mathbf{u_1},\mathbf{u_2})$, it may be shown that

$$F_3(\mathbf{u_1},\mathbf{u_2}) = F(\mathbf{u_1})F(\mathbf{u_2})F(-\mathbf{u_1}-\mathbf{u_2}), \quad (8)$$

where $F(\mathbf{u})$ denotes the DTFT of the image $f(\mathbf{m})$.

Consider a translation of the object, $o(\mathbf{m}-\mathbf{m_o})$. The corresponding Fourier transform of the image is now, $e^{-j2\pi(\mathbf{u}\cdot\mathbf{m_o})}F(\mathbf{u})$, where $(\mathbf{u}\cdot\mathbf{m_o})$ represents the vector dot product. Substitution of this transform into (8) reveals that the bispectrum (and the triple correlation) are shift-invariant.

Therefore, we may form triple correlations of images of the object, where the object may be shifted but is otherwise unchanged, and average the correlations to reduce the additive noise effects. The resulting improved estimate of the triple correlation of the object may then be used to form an estimate of the object. This is possible since the triple correlation (or bispectrum) may be inverted to recover the original signal in amplitude and phase to within a shift ambiguity. Unlike the autocorrelation, higher-order correlations preserve phase information about the signal. A linear phase amibiguity arises in the triple correlation case due to the shift-invariance property. As will be seen in the following discussion of reconstruction from the bispectrum, the shift-invariance of the triple correlation allows us to place the centroid of the reconstructed object at the origin. Also, when dealing with a single frame, the original position of the object may be recovered by simply zero-padding the original image [2], so that after reconstruction any phase ambiguity will be apparent; i.e., the image will be circularly shifted within a field of zeros.

3 DFT Reconstruction

Several algorithms have been reported in [2] for reconstructing a 1-d signal from its higher-order correlations, or from the Fourier transforms of the higher-order correlations, called *polyspectra*. A 2-d FFT-based recursive algorithm for reconstructing the discrete Fourier transform (DFT) of an image from its bispectrum has been developed in [4], and is briefly reviewed here.

Consider a Cartesian sampled image given by $f(\mathbf{m})$. Denoting the 2-d DFT of the image as $F(\mathbf{k})$, the *discrete bispectrum* of the image is given by

$$F_3(\mathbf{k_1},\mathbf{k_2}) = F(\mathbf{k_1})F(\mathbf{k_2})F^*(\mathbf{k_1}+\mathbf{k_2}). \quad (9)$$

where * denotes complex conjugation, and it is assumed that $f(\mathbf{m})$ is real valued.

Using the substitutions $\mathbf{k_2} = \mathbf{0}$ and $\mathbf{k_1} \mapsto \mathbf{k_1} - \mathbf{k_2}$ in (9) yields

$$F_3(\mathbf{k_1},\mathbf{0}) = F(\mathbf{k_1})F(\mathbf{0})F^*(\mathbf{k_1}) \quad (10)$$

and

$$F_3(\mathbf{k_1}-\mathbf{k_2},\mathbf{k_2}) = F(\mathbf{k_1}-\mathbf{k_2})F(\mathbf{k_2})F^*(\mathbf{k_1}). \quad (11)$$

Eliminating $F^*(\mathbf{k_1})$ and solving for $F(\mathbf{k_1})$ in (10) and (11) yields

$$F(\mathbf{k_1}) = \frac{F_3(\mathbf{k_1},\mathbf{0})}{F_3(\mathbf{k_1}-\mathbf{k_2},\mathbf{k_2})F(\mathbf{0})}F(\mathbf{k_1}-\mathbf{k_2})F(\mathbf{k_2}). \quad (12)$$

Equation (12) may be used recursively to compute the 2-d DFT $F(\mathbf{k})$ from its bispectrum $F_3(\mathbf{k_1},\mathbf{k_2})$.

An alternative to (12) is to rearrange (11) into

$$F^*(\mathbf{k_1}) = \frac{F_3(\mathbf{k_1}-\mathbf{k_2},\mathbf{k_2})}{F(\mathbf{k_1}-\mathbf{k_2})F(\mathbf{k_2})}. \quad (13)$$

Note that this form does not rely on the mean value of the image being non-zero, i.e., does not rely on $F(\mathbf{0})$ being non-zero. This is important since noise reduction relies on the assumption of zero-mean. In practice the mean is subtracted before estimating the bispectrum.

The redundancy in the bispectrum implies that one or more values for $\mathbf{k_2}$ may be allowed for each choice of $\mathbf{k_1}$ in (12) or (13). Let $\mathbf{k_2} \in \mathbf{R_{k_1}}$ represent the set of allowable choices for $\mathbf{k_2}$ given $\mathbf{k_1}$. The allowable choices are $\mathbf{0} < \mathbf{k_2} < \mathbf{k_1}$. For $\mathbf{k_2} = (k_{21}, k_{22})$, there are $(k_{21}+1)(k_{22}+1)-2$ elements in $\mathbf{R_{k_1}}$, since $\mathbf{k_2} = \mathbf{0}$ and $\mathbf{k_2} = \mathbf{k_1}$ are excluded. We can exploit this redundancy in (12) by averaging over $\mathbf{R_{k_1}}$ as

$$\hat{F}(\mathbf{k_1}) = \frac{1}{(k_{21}+1)(k_{22}+1)-2} \times \sum_{\mathbf{k_2} \in \mathbf{R_{k_1}}} \frac{F_3(\mathbf{k_1}, \mathbf{0})}{F_3(\mathbf{k_1}-\mathbf{k_2}, \mathbf{k_2})\hat{F}(\mathbf{0})} \hat{F}(\mathbf{k_1}-\mathbf{k_2})\hat{F}(\mathbf{k_2}) \qquad (14)$$

yielding an improved estimate for $F(\mathbf{k_1})$. If a particular choice of $\mathbf{k_2}$ produces a zero denominator then that term is excluded from the average. A further reduction in computation of (14) is possible by considering only those values of the bispectrum in a single non-redundant region, e.g., using the bispectrum symmetry $F_3(\mathbf{k_1}, \mathbf{k_2}) = F_3(\mathbf{k_2}, \mathbf{k_1})$. As a result, half of the allowable values of $\mathbf{k_2} \in \mathbf{R_{k_1}}$ yield redundant computations in (14). Similar statements are true regarding use of (13). Note that the complex conjugate symmetry of the DFT may be exploited when estimating $\hat{F}(\mathbf{k})$, so that only $\frac{N^2}{2}+2$ points of $\hat{F}(\mathbf{k})$ need be computed directly.

The shift-invariance of the bispectrum implies that reconstruction is unique to within a phase ambiguity. This ambiguity can be exploited during reconstruction to centroid the object at the origin. The k^{th} derivative of the *continuous* Fourier transform, evaluated at the origin, is proportional to the k^{th} moment of the image, [10]. A similar property is true for the discrete case. As a result specifying the phase of the points $\hat{F}(0,1)$ and $\hat{F}(1,0)$ to be zero will yield a reconstruction such that the object is centroided about the origin. The magnitude of $\hat{F}(0,1)$ and $\hat{F}(1,0)$ are found from (12) or (13) using an estimate of the mean for the value of $\hat{F}(\mathbf{0})$. For computation of all other points of $\hat{F}(\mathbf{k})$ it is important that $F(\mathbf{0}) = 0$ to suppress the additive noise component. When multiple frames are available an estimate of the mean may be obtained by averaging the means of the frames.

Let $\hat{\phi}(\mathbf{k})$ denote the phase of $\hat{F}(\mathbf{k})$. As a consequence of setting $\hat{\phi}(1,0) = \hat{\phi}(0,1) = 0$, it is necessary to apply a phase correction factor. A linear phase factor in the DFT corresponds to multiplying by $e^{-j\frac{2\pi}{N}\mathbf{m_o}}$ in the frequency domain, or equivalently shifting in the time domain by $\mathbf{m_o}$, *where* $\mathbf{m_o}$ *has purely integer components.* In specifying $\hat{\phi}(0,1) = \hat{\phi}(1,0) = 0$, it is not clear that the implied phase shift corresponds to a linear shift of the DFT. Alternatively, it is not true in general that the resulting estimates $\hat{F}(0,\frac{N}{2})$, $\hat{F}(\frac{N}{2},0)$ and $\hat{F}(\frac{N}{2},\frac{N}{2})$ will be real valued, as required if the resulting reconstructed image is to be real valued. The phase correction will ensure that the phase of these points will be zero. After (12) or (13) are used to compute $\hat{F}(k,l)$ the phase corrected DFT is given by

$$\hat{F}_\phi(k,l) = \hat{F}(k,l)\exp[-j\frac{2k}{N}\hat{\phi}(\frac{N}{2},0) - j\frac{2l}{N}\hat{\phi}(0,\frac{N}{2})]. \qquad (15)$$

Finally, $\hat{F}_\phi(\mathbf{0})$ may be set to the mean previously estimated.

Note also that just as high variance periodogram estimates require smoothing, estimates of the bispectrum also require smoothing [11]. The latter guarantees consistency of the sample estimates, and may be accomplished by window weighting the image, or by using a smoothing operation on the bispectrum such as local averaging.

4 Random Translation

The shift-invariance and invertibility of the bispectrum can be exploited when multiple noisy views of a randomly translated object are available. Let

$$f(\mathbf{m}) = o(\mathbf{m}) + v(\mathbf{m}) \qquad (16)$$

be an image, where $o(\mathbf{m})$ is the object and $v(\mathbf{m})$ is an additive random Gaussian field independent of $o(\mathbf{m})$ and not necessarily white. Let

$$f_i(\mathbf{m}) = o(\mathbf{m}-\mathbf{m_i}) + v_i(\mathbf{m}) \qquad (17)$$

be the i^{th} frame, where the object is randomly translated by $\mathbf{m_i}$, and $v_i(\mathbf{m})$ represents an independent sample set of the noise field $v(\mathbf{m})$. Let $F_i(\mathbf{k})$ be the DFT of the i^{th} frame, so that

$$F_{3,i}(\mathbf{k_1}, \mathbf{k_2}) = F_i(\mathbf{k_1})F_i(\mathbf{k_2})F_i^*(\mathbf{k_1}+\mathbf{k_2}) \qquad (18)$$

is the discrete bispectrum if the i^{th} frame. Due to the shift-invariance of the bispectrum

$$F_{3,i}(\mathbf{k_1}, \mathbf{k_2}) \simeq O_3(\mathbf{k_1}, \mathbf{k_2}) + V_{3,i}(\mathbf{k_1}, \mathbf{k_2}) \qquad (19)$$

where $O_3(\mathbf{k_1}, \mathbf{k_2})$ is the bispectrum of the object and $V_{3,i}(\mathbf{k_1}, \mathbf{k_2})$ is the bispectrum of the i^{th} sample set of the noise. Thus, given L available frames the bispectra are averaged to form the estimate

$$\hat{F}_3(\mathbf{k_1}, \mathbf{k_2}) = \frac{1}{L}\sum_{i=1}^{L} F_{3,i}(\mathbf{k_1}, \mathbf{k_2}). \qquad (20)$$

$\hat{F}_3(\mathbf{k_1}, \mathbf{k_2})$ represents an improved estimate of the bispectrum of the object, due both to the averaging of the bispectra of the views of the object and the averaging of the bispectra of the noise sample sets. Under the Gaussian noise assumption

$$\frac{1}{L}\sum_{i=1}^{L} V_{3,i}(\mathbf{k_1},\mathbf{k_2}) \stackrel{L\to\infty}{\longrightarrow} 0. \tag{21}$$

Reconstruction of the object can now be performed by inverting the bispectrum.

The continuous version of this algorithm has been implemented optically on astronomical data to remove atmospheric turbulence effects [12,13]. It does not require knowledge of the object translation vector $\mathbf{m_i}$. Thus, the bispectrum technique offers the advantages of eliminating the correspondence problem, use of the FFT, and allowing averaging in the bispectrum-domain. Use of the Radon transform has also been suggested for estimating the bispectrum of a randomly translated object [14]. This approach reduces the dimensionality of the problem but requires reconstruction from projections.

5 Random Rotation

In this section it is again assumed that multiple noisy views of an object in an additive random Gaussian field are available. However, it is now assumed that the centroid of the object is always at the same fixed location (at the center of the frame, say), but that the object is randomly rotated about the centroid from frame to frame. It is desired to reconstruct the object given L noisy frames.

An algorithm for rotation-invariance has been suggested by Lohmann [15]. This algorithm relies on 1-d Fourier series formed by treating constant radius rings in the image as periodic sequences. In practice the Fourier series coefficients are found using the DFT, taken around each ring. These DFT's can be used to form discrete bispectra for each ring, and the bispectra can then be averaged over the image set. The dimensionality of the problem has been reduced, but the method leads to a phase correspondence problem between the rings after they are reconstructed which requires a solution from the DFT-domain.

A new algorithm for rotation-invariance is now developed which exploits the shift-invariance of the bispectrum. This algorithm relies on the following property of the discrete bispectrum given by (9), that *the discrete bispectrum is invariant under a circular shift of the image-domain sequence.*This is shown as follows. Let

$$y(n_1,n_2) = x(((n_1-m_1))_N,((n_2-m_2))_N) \tag{22}$$

be a circularly shifted version of the $N\times N$ image $x(n_1,n_2)$, where $(\cdot)_N$ denotes a modulo-N operation. The respective DFT's of $y(n_1,n_2)$ and $x(n_1,n_2)$ are related by, [16],

$$Y(k_1,k_2) = e^{j\frac{2\pi}{N}(k_1m_1+k_2m_2)}X(k_1,k_2). \tag{23}$$

Inserting $Y(k_1,k_2)$ into (9) reveals that the discrete bispectrum of $y(n_1,n_2)$ and $x(n_1,n_2)$ are identical.

Let

$$f_i(r,\phi) = o(r,\phi-\phi_i)+v_i(r,\phi) \tag{24}$$

represent regular polar samples of the i^{th} image of an object, where $o(r,\phi-\phi_i)$ is the centroided object randomly rotated by ϕ_i, and $v_i(r,\phi)$ represents samples of an additive random Gaussian field. Let $F_i(\omega,\theta)$ represent regular polar samples of the DTFT of $f_i(r,\phi)$. It is known that $F_i(\omega,\theta)$ can be obtained with the use of Fourier and Hankel transforms; e.g., see [17]. However, it can readily be shown that the bispectrum formed by substituting $F_i(\omega,\theta)$ into (8) is *not* rotation-invariant. An alternative is to form for each frame

$$\tilde{F}(k,l) \triangleq \sum_r\sum_\phi f(r,\phi)e^{-jkr}e^{-jl\phi}. \tag{25}$$

Substitution of $\tilde{F}(k,l)$ into (8) reveals that the bispectrum formed with $\tilde{F}(k,l)$ *is* rotation-invariant. The definition of $\tilde{F}(k,l)$ may be thought of as mapping the polar coordinate system to a Cartesian coordinate system and then taking the DTFT in the new Cartesian system. Translation in ϕ in the new Cartesian system is equivalent to rotation in the original polar system. In practice (25) is computed using the DFT. Now, rotation corresponds to a circular shift in the new Cartesian system. However, as shown above, the discrete bispectrum is invariant to a circular shift. In order to avoid any circular shifting in r which may occur when inverting the bispectrum, it is only necessary to zero-pad $f(r,\phi)$ in the r variable. It is important to note that $\tilde{F}(k,l)$ does *not* represent samples of the DTFT of the original image, but merely exists to transform the rotation problem into an equivalent translation problem.

After forming $\tilde{F}_i(k,l)$ for each image, a bispectrum-based translation-invariant algorithm may be applied, as presented in section 4. The bispectra are computed for each frame from $\tilde{F}_i(k,l)$, an averaged bispectrum is formed, and then inverted, yielding an estimate of $\tilde{F}(k,l)$. Finally, (25) is inverted to yield an estimate $\hat{f}(r,\phi)$.

6 Translation and Rotation

In the preceding, algorithms have been described for reconstructing a translating object and reconstructing a rotating object with known centroid. An outcome of reconstructing a translated object is that the centroid of the reconstructed object is known, and in practice is easily placed at the origin. This suggests an algorithm [15] whereby the translation is first removed by forming the bispectrum and then reconstructing to remove translation; followed by a rotation-invariant algorithm to arrive at an estimate of the object. Thus, an algorithm for reconstructing a randomly rotated and translated object is as follows:

1. Form the bispectrum and reconstruct *for each frame.* This results in a series of frames in which the object has a common centroid but remains randomly rotated about the centroid.

2. A rotation-invariant bispectrum-based algorithm, as described in section 5, is applied to obtain an estimate of the object.

Note that the purpose of step one is only to remove the translation ambiguity, and averaging of bispectra occurs only in step two. This algorithm is computationally expensive in that the bispectrum is formed twice for each frame, once during step one, and once during step two. Also, a reconstruction algorithm must be applied to each of the bispectra found in step one.

7 Simulations

Simulations were performed to verify the reconstruction algorithm of section 3 and reconstruction of a randomly translating object presented in section 4. Figure 1 depicts the 64×64 pixel binary object "Calvin." Figure 2 depicts a reconstruction from the bispectrum of object "Calvin." The 2-d FFT of the object was found and the mean set to zero. Equation (9) was used to compute the bispectrum, so that the recursion of (13) could be applied. Averaging was performed when using (13). Finally, the mean was restored and the inverse FFT applied. No noise was added so that smoothing of the bispectrum estimates was not necessary for exact reconstruction. The reconstructed object is centroided about the origin which is at the upper left of the frame.

Next, Gaussian white noise was added to 10 frames of "Calvin" with random translations at a signal-to-noise ratio (SNR) of $-10dB$. SNR is defined as the sum of the squares of the signal over the sum of the squares of the noise. A single frame with SNR of $-10dB$ is shown in figure 3. The object is at the upper left in the frame. The reconstructed object is shown in figure 4. The algorithm of section 4 was used, so that the 10 bispectra were averaged before reconstruction. For ease of simulation, no local averaging of the bispectra was performed.

8 Conclusions

A recursive FFT-based algorithm for image reconstruction from the bispectrum has been reviewed and tested. The shift-invariance and noise reduction properties of the triple correlation have been exploited to obtain estimates of a randomly translating and rotating object from multiple noisy frames. The bispectrum-based approach does not require solution of the correspondence problem, and is independent of object shape or connectedness. Simulation results are shown for a randomly translating object in additive Gaussian white noise. Successful reconstruction from 10 frames at $-10dB$ SNR per frame has been demonstrated.

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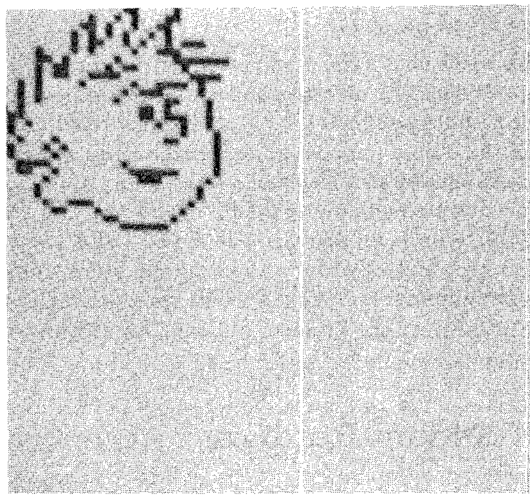

Figure 1
Object "Calvin."

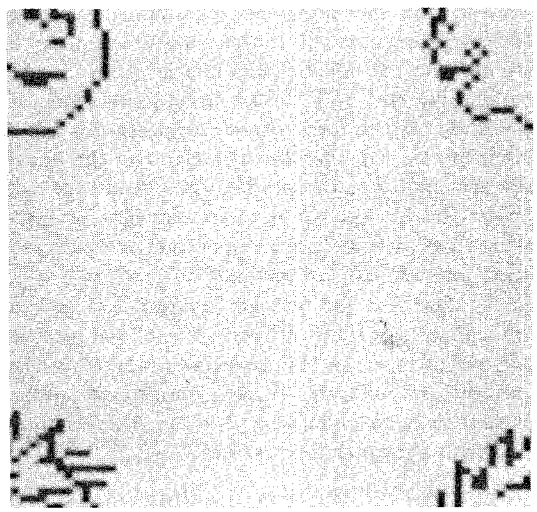

Figure 2
Reconstruction from bispectrum of figure 1.

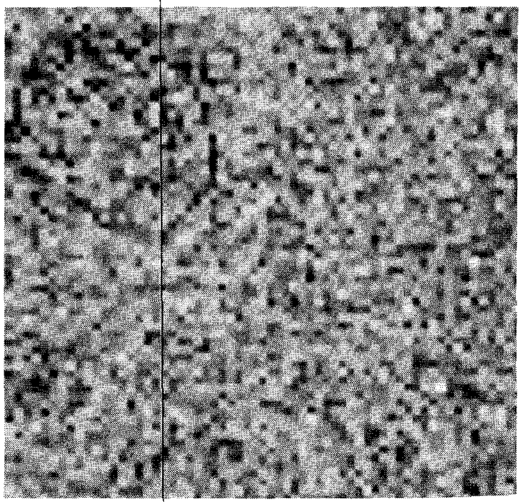

Figure 3
Object "Calvin" in white Gaussian noise. SNR $= -10dB$.

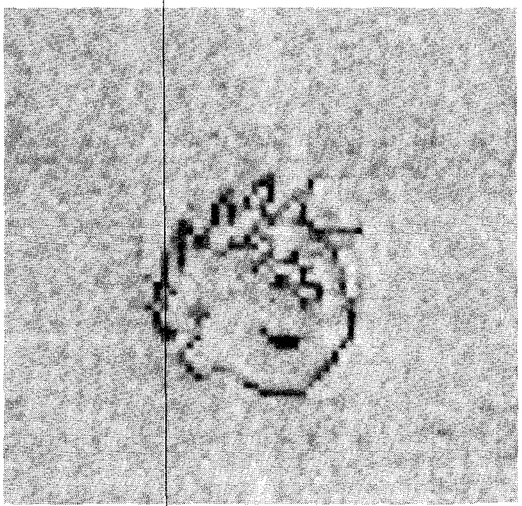

Figure 4
Reconstruction from 10 frames with random translation.

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