The Identification of an Unknown Material Using Prism Spectrometry

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The refractive index (n) of a material can be expressed by the Cauchy formula, a function of wavelength (λ) and the material's characteristic coefficients i.e. A, B. With the aim of determining its material, a ray from a cadmium lamp is shone through a prism, and values of n are found with a spectrometer, leading to A = 1.617 $\pm$ 0.004 and B = (1.03 $\pm$ 0.01) $\times 10^{-2}$ μm^2. We discuss whether a specific identification of the material can be made, the potentially unrealistic size of the errors calculated, and the sources of these errors.

I Introduction

The refractive index (n) of an optical material is a measure of the bending of a ray of light as it passes from one medium to another, defined as the ratio of the speed of light in a vacuum to the speed of light in that material [1]. It varies for different wavelengths of light (λ).

n can be determined using prism spectrometry. The equation for the refractive index of a prism, for a particular λ, can be derived from Snell's law and is:

$$n = \frac{\sin(\frac{\omega+\phi}{2})}{\sin(\frac{\omega}{2})}, \tag{1}$$

where ϕ is the angle of minimum deviation, and ω is the angle of the prism. The angle of minimum deviation is the deviation of a ray passing through a prism when the exit and entrance angles of the ray are equal [2], as shown in figure 1.

Another way of expressing n is The Cauchy formula, which relates λ and n as follows:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ..., \tag{2}$$

where A, B and C are constants known as the Cauchy coefficients. Each material has its own characteristic Cauchy coefficients; hence a material can be identified from these values.

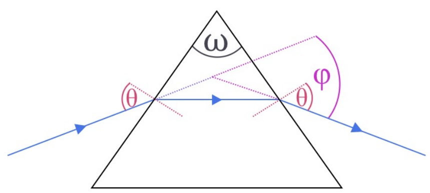

Figure 1: Diagram to aid definition of the angle of minimum deviation, ϕ. The entrance and exit angles are both θ.

In this work, a ray of light was refracted through a prism. The minimum deviation angles of the resultant spectral lines observed were measured in order to find n for each line, A, B, and hence the prism's material.

II Methods

A cadmium lamp was aligned with the stationary collimating telescope to produce a focused ray of light. The size of the telescope's aperture was adjusted so as to let through sufficient light for the ray to be clear, but not so much that it was too broad. Initially, the prism was removed from the platform and the direction, β, of an un-refracted light ray was found by measuring the position of the mobile telescope when opposite its stationary counterpart on each vernier scale.

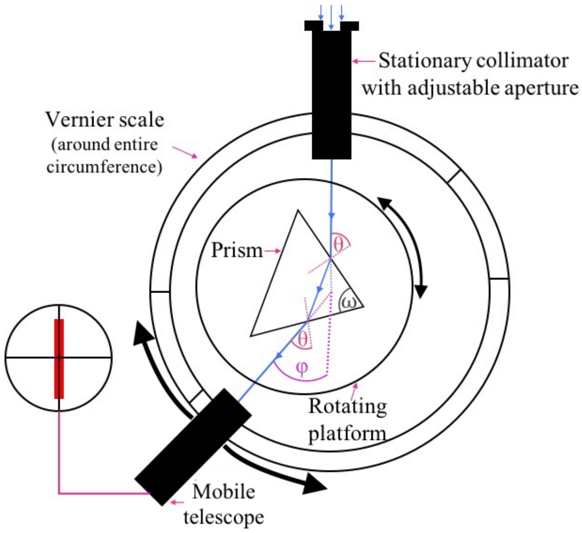

Figure 2: Schematic of the experimental setup. The mobile telescope rotates around the circumference of the apparatus, the whole of which is a vernier scale, which is read at two points. Also shown is view of a spectral line as seen through the mobile telescope.

The prism was returned to the spectrometer, and the mobile telescope roughly aligned with the direction of the refracted light. It was then rotated slowly via the platform to find the position of minimum deviation for each of the five spectral lines. At this position of inflection, the observed spectral line did not move, regardless of the direction of rotation. The telescope then was precisely aligned and focused so that the spectral line coincided with the telescope's crosshairs. The angular position of the telescope and hence of the spectral line, γ, was measured for each λ.

ϕ for each spectral line, the difference between γ and β, was then computed. Two repeats were taken for each λ and the mean calculated.

By measuring ϕ for each of the spectral lines, n of the prism for each of the five λ were calculated using equation (1). The prism was assumed to be equilateral, i.e. ω = 60°.

For visible λ, it is generally sufficient to only consider the first two terms of the Cauchy formula [3]. Therefore, it is suitable to plot a graph of n against $\frac{1}{\lambda^2}$, from which a linear dependence should be observed. The equation of the line of best fit is:

$$n(\lambda) = B\frac{1}{\lambda^2} + A. \tag{3}$$

Subsequently the Cauchy coefficients for the prism can be found (B = gradient and A = y intercept) and compared, along with n for each spectral line's λ, with existing data to determine the material of the prism.

III Results

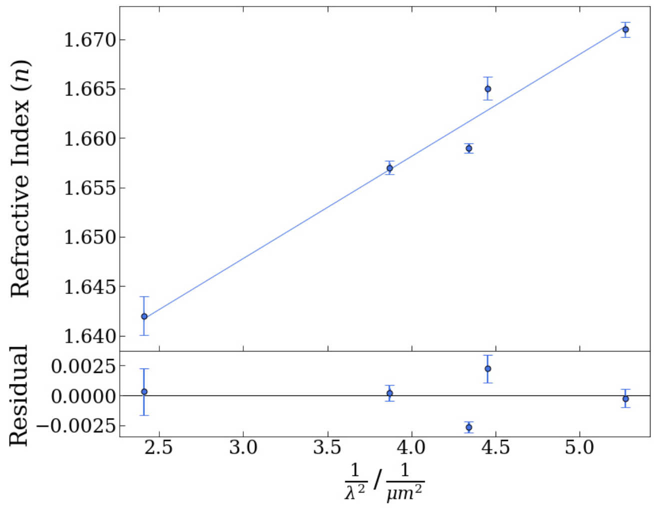

Figure 3: Graph of refractive index against $\frac{1}{\lambda^2}$ for a cadmium lamp. Vertical error bars represent standard error; the horizontal error bars, which are of the order of 10^{-9} m^2, are too small to be seen.

We observe a linear relationship between n and $\frac{1}{\lambda^2}$. Despite two anomalous data points, two-thirds of the data points lie on the line of best fit [6]. The average of the residuals is -2.09 $\times 10^{-5}$, relatively close to 0, and the residual plot has no particular shape; this shows that the model chosen is appropriate to the data.

The method described in section II is hence used to obtain A = 1.617 $\pm$ 0.004 and B = (1.03 $\pm$ 0.01) $\times 10^{-2}$ μm^2.

IV Discussion

The obtained A and B were compared to the corresponding literature values for various materials. Barium flint glass was found to be the closest match, with A = 1.670 and B = 0.00743 μm^2 [4].

The range of n values found in this work (approximately 1.64 - 1.67) was also compared to accepted n values for various materials. Again, flint glasses (with n in the range 1.57 - 1.75) were found to correspond best [5]. Specific n values were then compared to aid in more precise identification. Most literature stated n for λ = 589 nm:

λ/nm	n (3 sf)	Material	Source
589	1.66	Dense flint glass	[1]
589	1.62	Medium flint glass	[1]
589	1.59	Barium flint glass	[3]

The cadmium lamp produces no spectral line at this λ, so in order to make a comparison, n was calculated for λ = 589 nm using the Cauchy formula with our A and B:

$$n_{589nm} = 1.647 \pm 0.005. \tag{4}$$

Usually, as the accepted values of n, A and B for these materials lie outside of the range of the error (each differ from the experimental value by more than three standard errors), n_{589nm} would not be stated to agree with any of the values above. However, it should be noted that the errors stated in this work are smaller than errors in reality. There was assumed to be no error in ω, resulting in no contribution from this to α_n. Additionally, the error in λ is only 0.1 nm, obtained from the precision of the λ values given in the lab. It is unknown how accurately these λ were determined; α_λ is likely larger than the given value. Therefore, comparisons were made by considering percentage differences.

n_{589nm} is 0.79% different from the accepted n value for dense flint glass, 1.6% different for medium flint glass, and 3.5% different for barium flint glass. A and B for barium flint glass are 3.3% and 28% different to our values respectively. As dense flint glass' n is within 1% of n_{589nm}, it is reasonable to suggest that this is the material of the prism. However, the accepted A and B for dense flint glass, 1.7280 and 0.01342 μm^2, are 3.5% and 30% different to our experimentally determined values respectively. It can therefore not be confirmed that the material is dense flint glass.

The method of least squares was used to fit a line to the data, but a weighted fit would have been more appropriate for this heteroscedastic data set, reducing the effect of random error on the final values of A and B.

The two anomalous data points for the 480.0 nm and 474.0 nm lines may be a result of mispositioning the mobile telescope's crosshairs; an effort was made to align these with the same edge of the spectral line each measurement (rather than the centre of the line, which could cause random error), but for the fainter, bluer lines, this became more difficult. The small error bar for the 480.0 nm anomaly suggests a systematic error; this could be a repeated parallax error from reading the scale at a different angle from the other measurements.

V Conclusions

The Cauchy coefficients, A = 1.62 $\pm$ 0.0040 and B = (1.03 $\pm$ 0.096) $\times 10^{-2}$ μm^2, were found from n for each of the cadmium lamp's spectral line. On the grounds that n for dense flint glass is within 1% of n_{589nm}, it is probable that this is the material of the prism. However, this cannot be confirmed as B for dense flint glass is in definite disagreement with the value found in this work. The material of the prism was therefore concluded to be flint glass, but no more specific identification can be made.

References

[1] H. Young and R. Freedman, 'University Physics with Modern Physics', 13th edition, Pearson, UK (2015)

[2] 'Minimum Deviation by a Prism' http://www.mtholyoke.edu/~mpeterso/classes/phys103/geomopti/MinDev.html (accessed 04-03-18)

[3] F. Jenkins and H. White, 'Fundamentals of Optics', 3rd edition, McGraw-Hill Book Company, Inc., USA (1957)

[4] 'Cauchy's equation' https://en.wikipedia.org/wiki/Cauchy%27s_equation (accessed 08-03-18)

[5] 'Index of Refraction', http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/indrf.html (accessed 02-03-18)

[6] I. Hughes and T. Hase, "Measurements and their Uncertainties", Oxford University Press, UK (2010)

VI Error Appendix

VI.1 Errors in Single Measurements

Error in λ was obtained from the precision of the values of λ given in the lab:

$$\alpha_\lambda = 0.1nm. \tag{5}$$

Errors in ϕ were calculated using standard error rather than using the precision of the vernier scale:

$$\alpha_\phi = \frac{\sigma}{\sqrt{N}}, \tag{6}$$

where σ is the standard deviation in the mean and N is the number of repeat measurements, which in this case was 2 [6]. The choice to use this method is in part due to the difference between repeat values being larger than the smallest division on the scale, 30 arcseconds. There are also factors other than instrumental precision effecting ϕ to consider, such as parallax when reading the vernier scale, and dirt and chips on the prism's surface causing extra refraction.

Using gloves to handle the prism could have reduced dirt on the prism and hence error in ϕ.

Errors in A and B were calculated using Excel's LINEST function:

$$\alpha_A = 0.004 \tag{7}$$

$$\alpha_B = 9.6 \times 10^{-16} m^2. \tag{8}$$

Using a weighted fit instead might have reduced these errors.

VI.2 Combining Errors

Combined errors were found using the functional approach [6]:

$$\alpha_{\frac{1}{\lambda^2}} = |\frac{1}{(\lambda + \alpha_\lambda)^2} - \frac{1}{\lambda^2}| \tag{9}$$

$$\alpha_n = |\frac{\sin(\frac{\omega + (\phi + \alpha_\phi)}{2}) - \sin(\frac{\omega + \phi}{2})}{sin(\frac{\omega}{2})}|. \tag{10}$$

$\alpha_{\frac{1}{\lambda^2}}$ and α_n are plotted as error bars on figure 3.

The error in n_{589nm} involved the combination of errors from various sources, and so was calculated by the functional approach:

$$\alpha_n^A = |((A + \alpha_A) + \frac{B}{\lambda^2}) - (A + \frac{B}{\lambda^2})|, \tag{11}$$

$$\alpha_n^B = |(A + \frac{(B + \alpha_B)}{\lambda^2}) - (A + \frac{B}{\lambda^2})|, \tag{12}$$

leading to

$$\alpha_{n_{589nm}} = \sqrt{(\alpha_n^A)^2 + (\alpha_n^B)^2}. \tag{13}$$