The Verification of Coulomb's Law and Determination of the Permittivity of Free Space, ϵ_0

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Coulomb's law gives the relationship between the force and distance between two charges. We verify Coulomb's law by taking measurements of the force exerted by a conducting sphere, charged with 25 kV, on a conducting metal plate. The sphere's charge is found using the method of Faraday's ice pail connected to a 10 nF capacitor. By the method of image charges, we adapted Coulomb's law to describe this arrangement, hence finding a value of ϵ_0, (6.6$\pm$0.8)$\times 10^{-14}$ F m^{-1}. We discuss this value's disagreement with the accepted value, identify sources of error in the results, and justify the decision to confirm that Coulomb's law holds.

I Introduction

Coulomb's law states that the magnitude of electromagnetic force, F, between two charges of magnitudes q_1 and q_2 is inversely proportional to r, the distance between them [1]. This can be written in equation form as

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \tag{1}$$

where ϵ_0 is the permittivity of free space. The accepted value of ϵ_0 is 8.854 187 817... $\times$ 10^{-12} F m^{-1} [2].

Coulomb's law can be adapted to describe the electromagnetic force when, instead of two charges, there is one object (with charge of magnitude q) and a conducting plate. The plate mirrors the object's charge and it can hence be imagined that there is an object equally and oppositely charged on the other side of the plate, equidistant. This is called the method of image charges. Hence equation (1) becomes

$$F = \frac{q^2}{4\pi\epsilon_0 r^2} \tag{2}$$

where r is the separation of the charged object q and the imaginary object. This is twice the distance between q and the conducting plate. It is assumed that the plate is infinitely large and that the electric field is uniform.

In this experiment, an attempt is made to verify Coulomb's law, and the value of ϵ_0 is determined through measurements of the force exerted by a conducting sphere of known charge on a metal plate, for various distances between the sphere and plate.

II Methods

A metal sphere of radius r = 19.8$\pm$0.1 mm was charged using a 25 kV power supply. By Faraday's Ice Pail method, the sphere induced a charge in a circuit connected to a 10 nF capacitor. From this and the voltage across the capacitor, q was found to be

$$q = C \times V \tag{3}$$

where C is the capacitance of the capacitor, and V is voltage. To find V, five measurements of voltage were taken; the mean of these was 2.86 V. Using this value, q was calculated as 2.86×10^{-8} C.

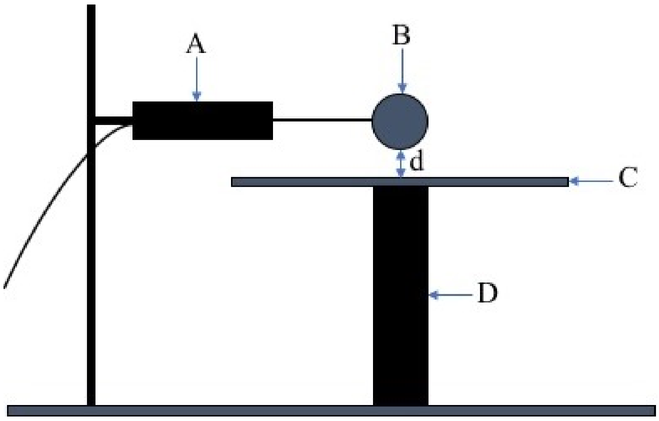

Figure 1: Diagram of the apparatus; force-meter with "mobile-cassy" readout (A), conductive sphere with insulating handle (B) and circular metal plate (C) on movable mount with built-in vernier scale (D) to vary its height.

The diameter of the sphere was measured using vernier callipers and halved to obtain the radius. The height of the sphere above the plate, d, was measured using a vernier scale built into the plate's movable mount. The vernier scale was set to zero with the sphere as close to the plate as possible. The force-meter was zeroed with the uncharged sphere attached i.e. when electromagnetic force due to the sphere was zero.

For various values of d, the sphere was charged, its handle inserted into the force-meter, and the centre of the metal plate placed under the sphere, as shown in in figure 1. The force exerted was then measured by the force-meter. Four repeat measurements were taken for each value of d to reduce error.

The distance from the plate to the centre of the charged sphere is the sum of r and d. The distance between the sphere and its image charge, R, is given by the equation

$$R = 2(r + d). \tag{4}$$

Substituting into Coulomb's law in the form of equation (2) gives

$$F = \frac{q^2}{4\pi\epsilon_0 R^2} \tag{5}$$

Assuming that Coulomb's law holds, i.e. the data follows this relationship, ϵ_0 can be obtained by putting equation (5) in the form $y = mx + c$

$$F = \frac{q^2}{4\pi\epsilon_0}\frac{1}{R^2} \tag{6}$$

and rearranging the gradient term to obtain

$$\epsilon_0 = \frac{q^2}{4\pi \times gradient}. \tag{7}$$

III Results

The raw data gathered shows force "decaying" as R increased, suggesting an inverse square relationship. This was tested by plotting force against $\frac{1}{R^2}$ (figure 2). The last four data points we took were not plotted, as at distances from the plate below 4 mm, the sphere discharged too rapidly for an accurate value of force to be measured.

Although there are more points below the line of best fit (obtained using the method of least squares) than above, two-thirds of the data points lie with the line within their error bars. It can therefore be stated that there is an almost linear relationship between the two quantities, confirming that F is proportional to $\frac{1}{R^2}$.

For the sake of obtaining a value of ϵ_0, it has been assumed that the relationship is totally linear. Using the method described in section II, ϵ_0 was found to be $(6.6\pm0.8)\times10^{-14}$ F m^{-1}, to 1 significant figure. This not consistent with the accepted value stated in section I.

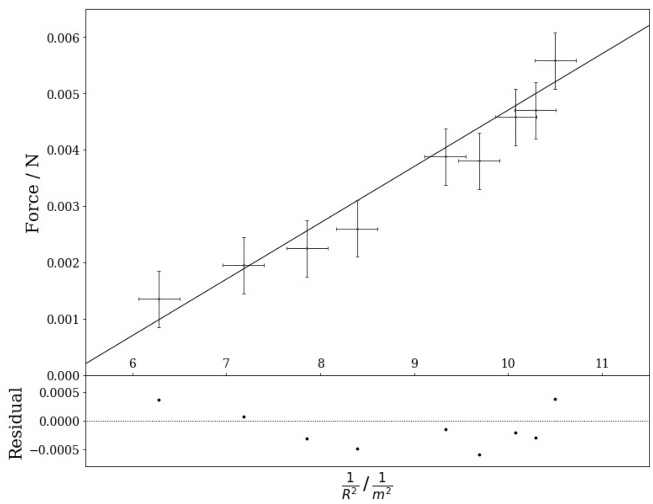

Figure 2: Graph to show the relationship between force and $\frac{1}{R^2}$ and plot of residuals.

IV Discussion

For a set of data perfectly obeying Coulomb's law, the expected result would be a linear plot with the line of best fit intersecting the origin. This was not the case with the data gathered in this experiment. Although there is a linear relationship between force and $\frac{1}{R^2}$, the line of best fit intersects the x-axis rather than the origin. This implies systematic error. It is unclear where this is from.

The ϵ_0 obtained in this experiment was a factor of 134 smaller than the accepted value, meaning the two values are not even of the same order of magnitude. The discrepancy between the value of ϵ_0 obtained and the accepted value could be due to poor experimental technique. Fitting a straight line to the data points despite the obvious curve could also have affected the value of ϵ_0. However, without this line, no value could have been calculated, and as is discussed in section III, the data have a sufficiently linear relationship that such a line is still appropriate.

Care was taken to zero equipment before use, but despite this, when the vernier scale was "zeroed", there was a small gap between the ball and parts of the plate. This is due to the plate being slightly sloped. As values of d were only in the order of a few mm, even a very slight slope could cause error; e.g. for d = 5 mm, a gap of 0.5 mm due to the slope would cause 10% error.

In order to calculate ϵ_0, F and $\frac{1}{R^2}$ were interpreted as having a completely linear relationship. However, it is clear from the graph and residual plot that the data points follow a slight curve. This is possibly due to the discharge of the sphere between being charged at the 25 kV power supply, being mounted in the force-meter, and the value of F being measured, which follows the exponential relationship[3]

$$q = Q_0 e^{\frac{-t}{RC}}. \tag{8}$$

The amount by which the sphere discharged varied, as the time between charging and measuring force varied by a couple of seconds between measurements. Consequently, the charge of the sphere was inconsistent. There is therefore greater variation between the values of F for each value of R than there would be if the sphere had had the exact same charge for each measurement. If the experiment was redone, keeping the time interval between charging the sphere and measuring force constant would reduce random error. Additionally, if the sphere had less time to discharge in transit (i.e. if the distance between power supply and experimental set up was shorter), the value of q would be larger at the plate, reducing percentage error, leading to a more precise value of ϵ_0.

V Conclusions

The plot of the data shows a linear relationship between F and $\frac{1}{R^2}$. It can therefore be concluded that electromagnetic force is inversely proportional to the square of the separation between charges, and Coulomb's law holds. However, the value of ϵ_0, $(6.6\pm0.8)\times10^{-14}$ F m^{-1}, was inaccurate and there was significant systematic error in the experiment.

References

[1] R. Feynman, R. Leighton and M. Sands, "The Feynman Lectures on Physics, Volume II", 2nd edition, Pearson: Addison Wesley, USA (2006), p4-2

[2] NIST, "CODATA Value: electric constant", https://physics.nist.gov/cgi-bin/cuu/Value?ep0, (Accessed 10-02-2018)

[3] H. Young and R. Freedman, "University Physics with Modern Physics", 13th edition, Pearson, UK (2015)

[4] I. Hughes and T. Hase, "Measurements and their Uncertainties", Oxford University Press, UK (2010), p. 16

[5] I. Hughes and T. Hase, "Measurements and their Uncertainties", Oxford University Press, UK (2010), back cover

VI Error Appendix

VI.1 Errors in single measurements

For readings from analogue equipment, error was taken to be half of the smallest interval on the equipment's scale, leading to errors in the radius of the sphere, r, and distance of of the plate from the sphere, d, being

$$\alpha_r = \pm 0.1mm \tag{9}$$

and

$$\alpha_d = \pm 0.1mm. \tag{10}$$

The reading from the force-meter was constantly fluctuating. This is because it was a very sensitive instrument, and so was affected by external forces in the room, e.g. the bench being bumped, turbulence in the air. Therefore, error in force α_F was not found from the last digit on the display, as would usually be convention for digital equipment, or from the standard deviation, and instead is

$$\alpha_F = \pm 0.5mN. \tag{11}$$

This was determined from the difference between the highest and lowest values displayed in quick succession by the force-meter, which was generally around 1 mN. It was up to me and my partner to decide which of the displayed values was the most appropriate to record. The value approximately midway between the highest and lowest displayed in the space of a couple of seconds was chosen. Using data logging software to pinpoint the initial value of force before the sphere discharged too greatly would have reduced error in this measurement. The rapid discharging of the sphere also added error to the values of F obtained.

Multiple values of voltage, V, were taken in order to determine q, and from these, standard error was calculated using the formula[4]

$$\alpha_V = \frac{\sigma_{N-1}}{\sqrt{N}}. \tag{12}$$

The capacitor stated an error of 5%, so

$$\alpha_C = 5\% \times C. \tag{13}$$

The error in the gradient of the graph, which was obtained using Excel's LINEST function, is

$$\alpha_{gradient} = \pm(8 \times 10^{-5}). \tag{14}$$

VI.2 Combining errors

All combined errors were calculated using the calculus method[5]. In retrospect, the functional approach may have been more efficient.

Error in R, and consequently in $\frac{1}{R^2}$, were found using

$$\alpha_R = \sqrt{(\alpha_r)^2 + (\alpha_d)^2} \tag{15}$$

and F

$$\alpha_{R^{-2}} = |-2R^{-3}|\alpha_R. \tag{16}$$

Similarly, error in q and in q^2 were calculated with

$$\alpha_q = q\sqrt{(\frac{\alpha_C}{C})^2 + (\frac{\alpha_V}{V})^2} \tag{17}$$

and

$$\alpha_{q^2} = |2q|\alpha_q. \tag{18}$$

α_{q^2} was then used along with $\alpha_{gradient}$ to calculate the error in ϵ_0:

$$\alpha_{\epsilon_0} = \epsilon_0\sqrt{(\frac{\alpha_{q^2}}{q^2})^2 + (\frac{\alpha_{gradient}}{gradient})^2}. \tag{19}$$