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# Title

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## **Abstract**

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

# Contents

<b>1</b>	<b>Introduction</b>	<b>6</b>
1.1	Motivation . . . . .	6
1.2	What is a Milky Way globular cluster? . . . . .	6
1.3	Orbits & actions . . . . .	7
<b>2</b>	<b>Method &amp; Theory</b>	<b>9</b>
2.1	Density and kinematic profiles of globular clusters . . . . .	9
2.2	Orbits . . . . .	9
2.3	Actions . . . . .	10
<b>3</b>	<b>Analysis</b>	<b>12</b>
3.1	Description of the simulation . . . . .	12
3.2	Investigation in phase space . . . . .	12
3.2.1	Velocity dispersion . . . . .	15
3.2.2	Anisotropy . . . . .	16
3.2.3	Density profile . . . . .	16
3.2.4	Potential . . . . .	16
3.3	Investigations of orbits in action space . . . . .	17
3.3.1	Orbits . . . . .	17
3.3.2	Actions . . . . .	17
3.3.3	Integral of motions along orbits . . . . .	17
<b>4</b>	<b>Results &amp; Discussion</b>	<b>18</b>
4.1	Actions from different globular clusters . . . . .	18
4.2	Discussion & future perspectives . . . . .	18
<b>5</b>	<b>Conclusion</b>	<b>19</b>
<b>6</b>	<b>Acronyms</b>	<b>20</b>

# 1 Introduction

## 1.1 Motivation

In this thesis we will investigate stellar orbits of simulated globular clusters (GCs) and their integrals of motion to identify possible signatures of intermediate mass black holes (IMBHs).

## 1.2 What is a Milky Way globular cluster?

GCs are self-gravitating, gas-free systems of  $10^6$  to  $10^8$  stars which are spherically grouped. There are about 150 of them in the Milky Way (MW). Since they are some of the oldest stellar populations in the universe (approximately 13 Gyr), they contain much information about the assembly history and evolution of the MW. Formerly seen as very simple systems with only one stellar population and without rotation recent research revealed a much higher complexity of these systems. GCs are now known to host multiple stellar populations that challenge our understanding of their formation. Moreover, GCs now appear dynamically complex, presenting deviations from spherical symmetry, anisotropy in velocity space and significant internal rotation.

The typical stellar components of a globular cluster (GC) can be seen in a color magnitude diagram (CMD). In this CMD the visual magnitude is plotted against the B-V color. It's color coded by the mass of the star. A star's position can be interpreted as its evolution stage. Most of the stars are set in the main sequence. They fusion hydrogen in their cores. There are two main sequence lines one upon the other. These occur due to binary systems whose flux is given by the sum of the single fluxes of the single components, and therefore appear redder and more luminous. These binary systems represent about 5% of the stars in the GC. The position of the main sequence turn-off depends on the age of the system and therefore can be used as an indicator to determine the cluster's age. **explain isochrones** Bluewards of this turn off point following the trend of main sequence stars, there are so called "blue stragglers" which are remnants of stellar collisions or interacting binaries (B&T p.628). Continuing from the turn off point there is the red giant branch consisting of stars still fusing hydrogen but only in a shell surrounding a degenerate helium core. They are inflated with a radius much higher than the main sequence stars but have a much lower temperature. These are the brightest stars of a GC. On the upper part of the red giant branch lies the horizontal branch. Its stars burn helium in their core and hydrogen in a surrounding shell. In the

lower left corner white dwarfs are located. They are stellar remnants which have burnt all of their resources. In a typical GC dark stellar remnants like stellar black holes and neutron stars are present but not visualised in the CMD. [nochmal über CMD durchlesen](#)

Recent attention has been devoted to the search of IMBHs in the centre of GCs. In their centre GCs could host an intermediate mass black hole (IMBH). These could be the missing link between stellar mass black holes [mass](#) and super massive black holes (SMBHs) [mass](#) as origin of the super massive black hole (SMBH), expected to have a mass between  $10^3 - 10^4 M_{\odot}$ .

The kinematic detection of IMBHs is usually based on the analysis of the velocity-dispersion profile in the inner few arcseconds around the crowded centre of a GC. This requires a combination of high angular resolution and high spectral resolving power. For this reason the detection of IMBHs remains highly controversial. Currently there are two different kinematic methods trying to detect IMBHs: [resolved kinematics & unresolved integrated light \[bianchini 2015\]](#) As an example there are the unresolved/integrated IFU kinematics which result in a signature of an IMBH for NGC 6388 and resolved/discrete kinematics which do not yield IMBHs. [include graph of Paolo's talk!!](#)

We propose to explore a new method to test if we can predict a signature of the IMBH going beyond the traditional phase-space analysis (i.e., analysis of velocity dispersion profiles), by exploiting the orbital information of the stars. This will be done by computing and comparing integrals of motions (e.g. actions) of the stars' orbits for simulations of GCs with and without IMBH.

### 1.3 Orbits & actions

An orbit is a path a unperturbed, collisionless tracer particle (e.g. a star) will move along in a gravitational potential. Orbits contain information about the gravitational potential generated by the mass distribution of a system in their position and velocity coordinates following Newtons 2nd law. Orbit distribution functions (DFs) describe which orbits are populated by how many tracers. From the orbit distribution function together with the overall potential we can draw inferences about the structure and evolution history of the system.

Historically observations of orbits enabled discoveries or confirmed them:

- Seen from the earth Mars' position moves over the sky as a loop called epicycle. That implies that the earth is not the centre of the universe! (C & O p.3)
- Neptune/Uranus [noch mal nachlesen](#)

- From rotation curves of galaxies we see that stars move faster than what expected by the presence of only the mass of luminous matter. There has to be more matter interacting via gravitational forces. This has led to the theory of dark matter. (Rubin 1980)
- Mercury's orbit differs hugely from calculated Kepler orbit. This is because of its migrating pericentre. Due to the proximity to the sun gravitational forces are so strong that we need to apply general relativity.
- The SMBH Sagittarius A\* was detected by observations of the orbits around the black hole and resulting mass calculations. (C & O p.923)

Some examples for orbit DFs are spiral galaxies where stars of different components (thin disc, thick disc, bulge, halo) are on different orbits (dynamical distinct) and have different metallicities (chemical distinct).

Actions are integrals of motion and are the distinct description of orbits. They are constant with time. Known for a long time they are extremely difficult to calculate. Actions of our solar system can be calculated easier since the potential is a Kepler potential. With nowadays supercomputers it is finally possible to compute actions of more complex and less explored systems. [how to describe orbits](#)

idea 1 ( $x(t), v(t)$ ) pretty complicated time evolution in 6 coordinates. idea 2 take quantities which are constant with time so called integrals of motions

beste set von integrals of motions are actions good to interpret



## 2 Method & Theory

### 2.1 Density and kinematic profiles of globular clusters

Our investigations of GCs in phase space consists in analysing the spatial distribution of stars (density profiles) and the kinematic profiles (such as velocity dispersion and anisotropy profiles). First we test the sphericity of the GC. Sphericity implies the usage of analytical methods that are very straight forward, especially for determining the potential of the globular cluster and then the actions in action space.

#### density profile

The velocity dispersion is the standard deviation of the mean velocity

$$\sigma_i = \sqrt{\langle (v_i - \langle v_i \rangle)^2 \rangle} = \sqrt{\langle v_i^2 - \langle v_i \rangle^2 \rangle} \quad i = r, \theta, \phi. \quad (1)$$

For a spherical system it is best to calculate them in spherical coordinates  $r, \theta, \phi$  respectively  $v_r, v_\theta, v_\phi$ . If the GC contains an IMBH the velocity dispersion towards the centre is increasing.

**what exactly describes anisotropy?** To quantify the anisotropy of the system we use the anisotropy parameter  $\beta$

$$\beta = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{\sigma_r^2}. \quad (2)$$

If  $\beta$  is positive the anisotropy is radial, if it is negative the anisotropy is tangential and if  $\beta \approx 0$  then the system is isotropic.

### 2.2 Orbits

[] In a dynamical system the mass distribution is described by the form of theoretically existent orbits  $(\vec{x}(t), \vec{v}(t))$ . Position and velocity are linked with six coordinates and contain all information about the potential. With Newton's 2nd law we get the connection between potential  $\Phi(\vec{r})$  and acceleration  $\vec{a}$  which is

$$\vec{F}(\vec{r}) = -\nabla\Phi(\vec{r}) = m \cdot \vec{a}.$$

Since the system is spherically potential and force only depend on the distance from the centre  $r$ . The potential can be derived from the Poisson's equation

$$\Delta\Phi(r) = 4\pi G\rho(r) \quad (3)$$

with the density  $\rho$  depending only on the distance as well. Due to the spherical symmetry the potential can be calculated by

$$\Phi(r) = -\frac{G}{r} \int_0^r dM(r') - G \int_r^\infty \frac{dM(r')}{r'} = -4\pi G \left[ \frac{1}{r} \int_0^r dr' r'^2 \rho(r') + \int_r^\infty dr' r' \rho(r') \right] \quad (4)$$

(Binney & Tremaine eq. 2.28). We calculate the density by binning the masses on logarithmic equally distributed shells **write density formula?** and solve the integrals of the Poisson's equation numerically by using the Gauss-Legendre quadrature

$$\int_a^b f(x) dx = \frac{b-a}{2} \sum_{i=1}^n w_i f\left(\frac{b-a}{2} x_i + \frac{a+b}{2}\right)$$

where the points  $x_i$  and the weights  $w_i$  are derived from the Legendre polynomials. Since the orbit is described by position and velocity at each time step we use the numerical leapfrog method which is a second-order time reversible integrator.  $X_i$  and  $v_i$  are calculated by

$$\begin{aligned} x_{i+1} &= x_i + v_i \Delta t + \frac{a_i(x_i)}{2} \Delta t^2 \\ v_{i+1} &= v_i + \frac{a(x_{i+1}) + a(x_i)}{2} \Delta t. \end{aligned}$$

## 2.3 Actions

Stars in spherical symmetric potential are fully described by their actions

$$J_i = \frac{1}{2\pi} \oint_{\gamma_i} \vec{p} \cdot d\vec{q} \quad i = r, \theta, \phi \quad (5)$$

which are used as coordinates in action space. These actions are integrals of motion. For most potentials actions can't be described analytically. The actions of a spherical system are derived from angular momentum, energy and potential. Only the potential is depending in  $r$ . Energy and angular momentum as well as resulting actions are constant over time and orbit. The azimuthal action  $J_\phi$  and the latitudinal action  $J_\theta$  can be evaluated simply. To calculate the radial action  $J_r$  we have to solve an integral

numerically. Actions of a spherical potential are found to be

$$J_\phi = L_z, \quad (6)$$

$$J_\theta = L - |L_z|, \quad (7)$$

$$J_r = \frac{1}{\pi} \int_{r_{min}}^{r_{max}} dr \sqrt{2E - 2\Phi(r) - \frac{L^2}{r^2}}. \quad (8)$$

Should I write down the derivation of the actions (B&T p.220) or just the results (B&T p.221, 3.221,3.223,3.224)?

The pericenter  $r_{min}$  and the apocenter  $r_{max}$  as well as the guiding star radius  $r_g$  can be found in the effective potential

$$V_L(r) = V(r) + \frac{L^2}{2mr^2}. \text{ m should be left out right?} \quad (9)$$

Bartelman Theo 1 Skript p59 formula 6.27.

In the peri- and apocenter the effective potential equals the total energy since the stars do not have any kinetic energy there. That results in following function which is to solve:

$$\left(\frac{1}{r}\right)^2 + \frac{2 \cdot (\Phi - E)}{L^2} = 0.$$

The guiding star radius is the distance at which a star with given total angular momentum would have a circular orbit. This is at the minimum of the effective potential. To get  $r_g$  we have to solve

$$r \sqrt{r \frac{\partial \Phi}{\partial r}} - |L| = 0$$

where  $\sqrt{r \frac{\partial \Phi}{\partial r}} = v_{circ}$  is the circular velocity. This distance is used to have a better comparison of the actions since in the snapshot the stars are at a random position on their orbit.

plot v\_eff plot from barthelman skript?

## 3 Analysis

### 3.1 Description of the simulation

name of the simulation	# of patches	total mass [ $M_{\odot}$ ]	mass of the IMBH [ $M_{\odot}$ ]	$r_m$ [pc]
SIM 1 - IMBH	1026735	308533.2	10102	4.13
SIM 2 - IMBH	271946	115149.6	12254	2.50
SIM 3 - NOIMBH	468627	172671.3	0	7.89
SIM 4 - NOIMBH	1851556	669844.3	0	5.41

Table 1: Overview of the data of the simulations. **Question @ Paolo: Should # of patches include IMBH?**

**where simulation comes from and what it is & description of output**

To get a familiar with the simulation we first have a look at the scatter plot of the star positions.

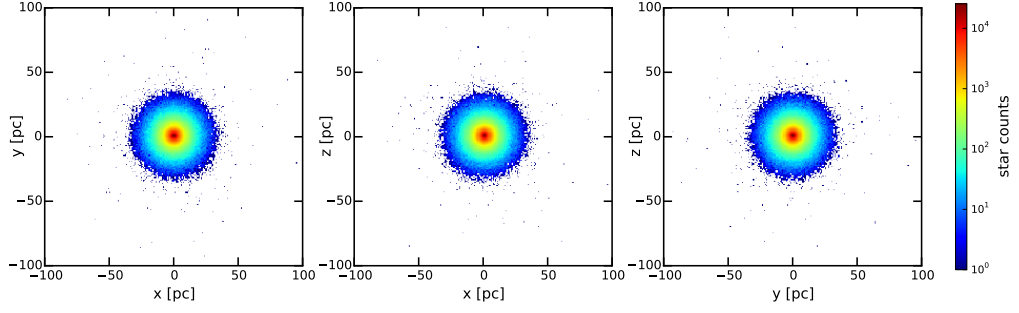
**include same plot with velocities!!**

As we said in section 2.1 it is important to test the sphericity of a system. We will do this by splitting the GC into octants and compare their mass **or number since we will introduce mass density plots only later density mass density sphericity plots**. As you see they're acceptable overlaying **within their errors**.

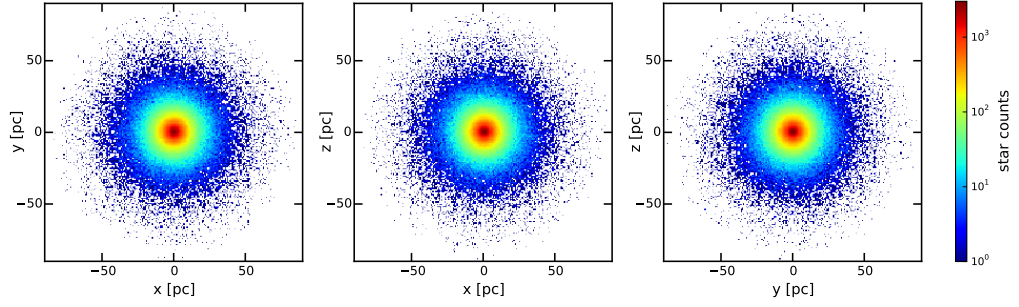
As mentioned in 1.2 the CMD is showing a star's evolution stage dependend on its position. If you do not know age or metallicity of the system you can fit isochrones on the CMD. Isochrones are curves of evolutionary stages of stars having the same age and metallicity but different masses. We plot several to our CMD to determine which one fits best. This will give us the age and the metallicity of the system. **cmd with isocrones plots**

### 3.2 Investigation in phase space

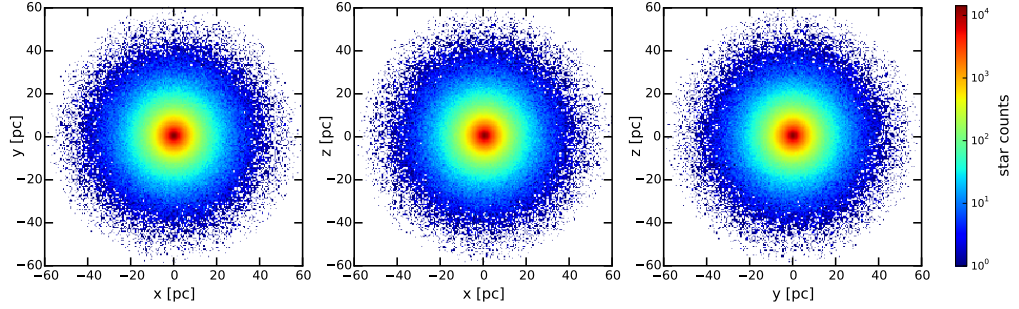
First we will investigate the GC in phase space for the set of simulations that w will use throughout this work. We will start with the velocity dispersion and the anisotropy parameter then we will have a density profile and from that get the potential. **for all 4 simulations or only for the first with IMBH? or with on plot containing two or all of them?**



(a) SIM 1. The GC is spread until 100 pc with most of the stars located in the inner 40 pc.

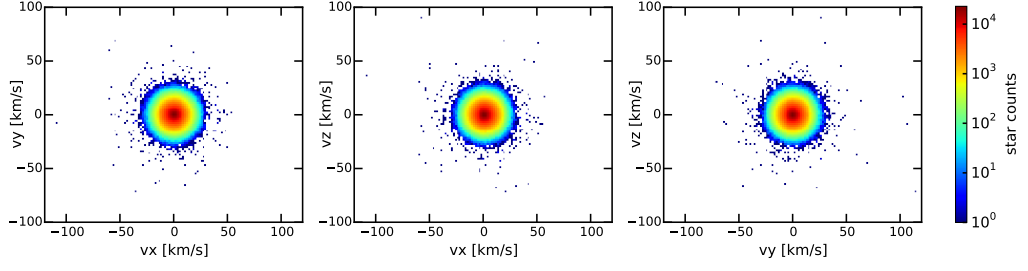


(b) SIM 3. The GC is spread until 90 pc.

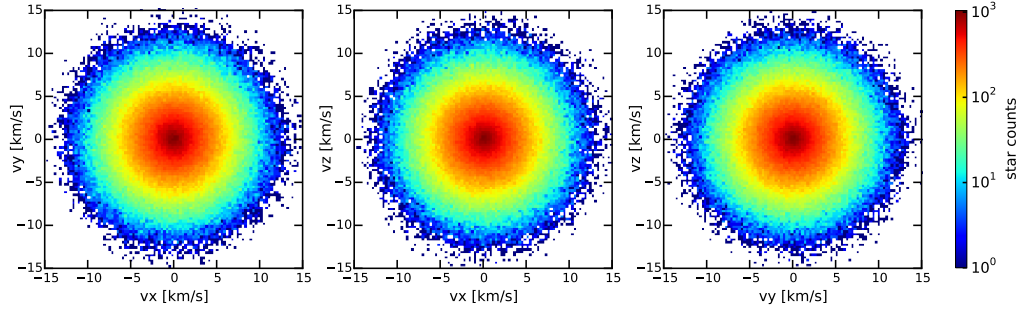


(c) SIM 4. The GC is spread until 60 pc.

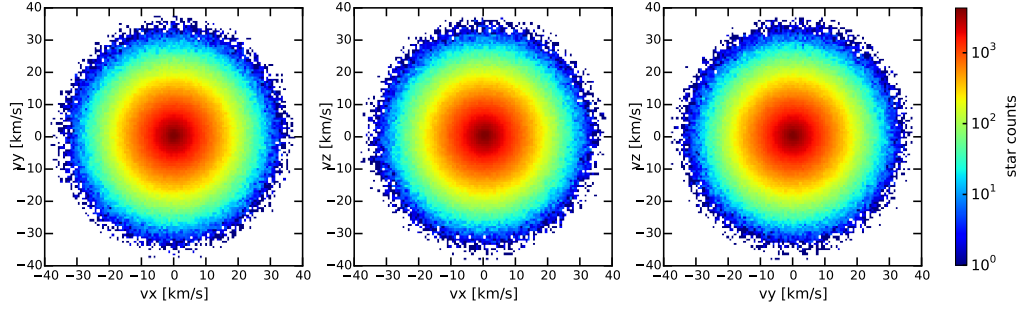
Figure 1: Position scatter plots. The stars are distributed spherically with most of the stars in the inner part. The stars of the GC with IMBH are less spread in the outer parts despite very few which are far outside. In the GCs without IMBH the stars in the outer part are less accumulated but the furthestmost stars still in the main sphere.



(a) SIM 1. The stars' velocities are spread until 120 km/s with most of them reaching 30 km/s.



(b) SIM 3. The stars' velocities are spread until 15 km/s.

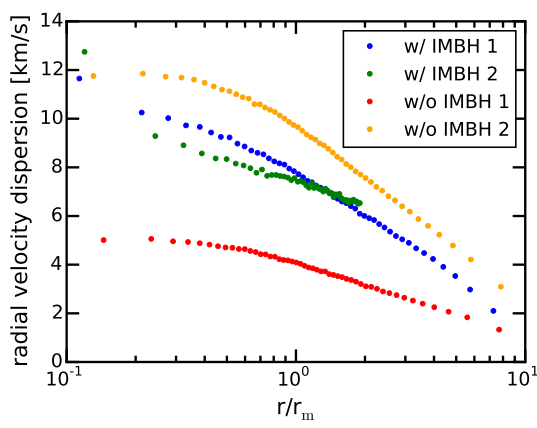


(c) SIM 4. The stars' velocities are spread until 40 km/s.

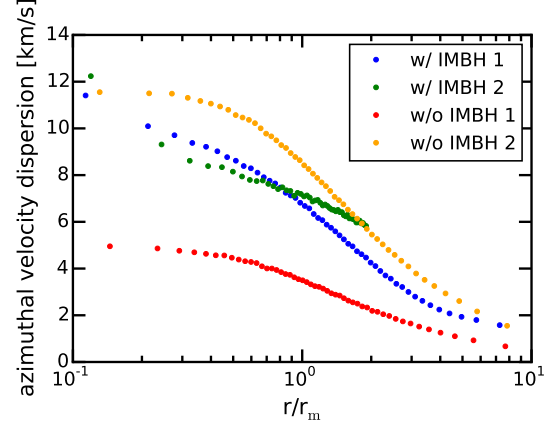
Figure 2: Velocity scatter plots. The velocities are spherically distributed. Most of the stars have low or no velocity while a few have high velocities in different directions. Like the star distribution the velocity distribution of the GC with IMBH has some velocities outside the main sphere whereas the GCs without IMBH contains all velocities inside the main shell.

### 3.2.1 Velocity dispersion

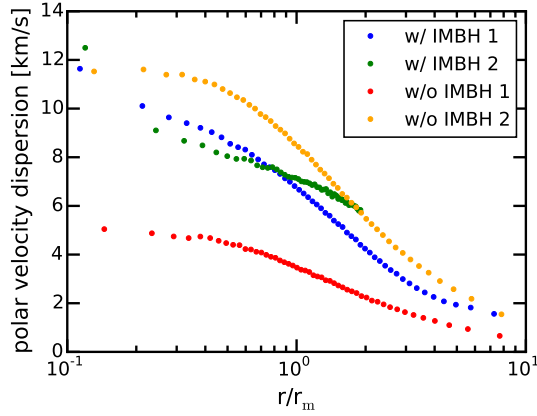
With (1) from section 2.1 we can calculate the velocity dispersion for each coordinate  $r, \theta, \phi$ . For every bin we take the same amount of stars and calculate the dispersion. To compare both simulations we plot the dispersion over the effective radius. The half mass radius of the simulation with IMBH is 4.1pc and of the simulation without IMBH it is 7.9pc. As expected there is a rise in the centre for the simulation with IMBH. This is



(a) Radial velocity dispersions



(b) Azimuthal velocity dispersions



(c) Polar velocity dispersions

Figure 3: Velocity dispersion profiles as a function of the radius in units of the effective radius  $r_{\text{eff}}$ . They are binned in a way that each bin contains the same amount of stars. We can see that the velocity dispersion of the simulation with IMBH rises towards the centre whereas the simulation without IMBH exhibits a cored profile.

due the high gravitational potential of the IMBH which disturbs the dynamics of close

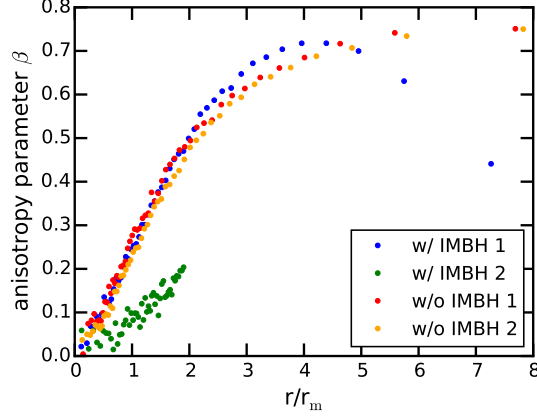


Figure 4: Anisotropy parameter  $\beta$ . Both simulations are radial anisotropic. The simulation with IMBH has a peak at 4 effective radii where it is most radial anisotropic. The other simulation is rising until the end. The far more outside a star is the higher its radial anisotropy is.

stars.

### 3.2.2 Anisotropy

Anisotropy can be calculated from (2) in 2.1. It is binned the same way as for the velocity dispersion and again dependent on the effective radii. In the center of both GCs there is nearly the same anisotropy. Both are positive and rising. That means the systems are radial anisotropic. The GC with IMBH is most radial anisotropic in its center at about 4 effective radii. The other GC is becoming more radial anisotropic the more far from the centre it is.

### 3.2.3 Density profile

The density profile shows the density of the system over its radius. The bins are chosen so that the radii are equidistant on a logarithmic scale and that they are at least 100 stars per bin to have a reliable stochastic. Outside of the cluster the density is set to 0. In the innermost part the density is set to be as the innermost point. plots

### 3.2.4 Potential

From the density profile we can compute the potential as described in 2.2. It is composed by the potential given from the stars and if there is one the potential of the IMBH



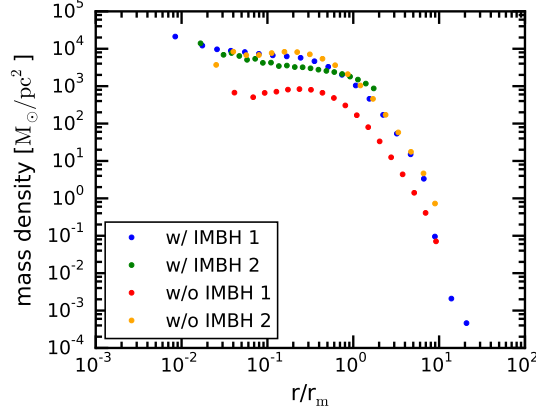


Figure 5: Mass density profiles. The density in  $\frac{M_{\odot}}{pc^2}$  is plotted against the effective radius. The density of the GC with IMBH is everywhere larger than the density of the GC without IMBH. In the centre there is a raise in the density of the GC with IMBH whereas the other GC stays approximately on the same level. Both start decreasing at about  $0.5 r_{eff}$ .

expressed as Kepler potential.

### 3.3 Investigations of orbits in action space

wilma class

#### 3.3.1 Orbits

#### 3.3.2 Actions

#### 3.3.3 Integral of motions along orbits

## 4 Results & Discussion

only triangle plots

### 4.1 Actions from different globular clusters

### 4.2 Discussion & future perspectives

do the same distinguishing the mass of the stars

redo the work with only observational-like data

## 5 Conclusion

## 6 Acronyms

**CMD** color magnitude diagram

**DFs** distribution functions

**GC** globular cluster

**GCs** globular clusters

**IMBH** intermediate mass black hole

**IMBHs** intermediate mass black holes

**MW** Milky Way

**SMBH** super massive black hole

**SMBHs** super massive black holes