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Bachelor Thesis in Physics
submitted by

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Title

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Abstract

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Contents

1 Introduction

1.1 Motivation

In this thesis we will apply integrals of motion as actions to globular cluster (GC)s to investigate possible signatures of intermediate mass black hole (IMBH)s.

1.2 What is a globular cluster in the Milky Way?

GCs are self-gravitating, gas-free systems of 10^6 to 10^8 stars which are spherically grouped. There are about 150 of them in the Milky Way (MW). As some of the oldest stellar populations in the universe we can obtain much information about the evolution of the MW. Formerly seen as very simple system with only one stellar population and without rotation recent research revealed a much higher complexity of these systems. nachlesen, wie das genau mit stellar populations und rotation und models ist
In this color magnitude diagram (CMD) the visual magnitude is plotted against the B-V color. It's color coded by the mass of the star. A star's position can be interpreted as its evolution stage. Most of the stars are set in the main sequence. They fusion hydrogen in their cores. There are two main sequence lines one upon the other. These occur due to binary systems. These binary systems represent about percentage % of the stars in the GC. The main sequence turn-off is depending on the age of the system and is used as indicator for such. Beyond this turn off point there are so called blue stragglers which are remnants of stellar collisions (B&T p.628). Continuing from the turn off point there is the red giant branch consisting of stars still fusing hydrogen but only in a shell surrounding a degenerate helium core. They are inflated with a radius much higher than the main sequence stars but have only a very low temperature. At the end of the red giant branch lies the horizontal branch. Its stars have sun-like masses and burn helium in their core and hydrogen in a surrounding shell. In the lower left corner white dwarfs are located. They are stellar remnants which have burnt all of their resources. nochmal über CMD durchlesen und alle branches erwähnen In 3.1. we will compare the CMD to isochrones which describe the actual distribution of stars in the different stages for a given age and metallicity.
In their centre GCs could contain an IMBH. These could be the missing link between stellar mass black holes and super massive black hole (SMBH) as origin of the SMBH. Currently there are two different kinematic methods trying to detect IMBHs. As an example there are the unresolved/integrated IFU kinematics which result in a signature

of an IMBH for NGC 6388 and resolved/discrete kinematics which don't yield IMBHs. include graph of paolo's talk? To get more certain methods we test if we can derive a signature of the IMBH by computing and comparing actions of the stars.

1.3 Actions & orbits

Orbits contain all information about the potential of a system in their position and velocity coordinates following Newtons 2nd law. From the orbit distribution function we can draw inferences about the history of the system. Since there are many possible orbits but the stars are only on some of them questions are raised. How do they come on these special orbits? What do these orbits tell us about the evolution of the system? There are some examples when orbits enabled discoveries or confirmed them:

- Seen from the earth Mars' position moves over the sky as a loop. That implies that the earth is not the centre of the universe!
- Neptune/Uranus noch mal nachlesen
- From rotation curves of galaxies we see that stars move faster than expected by mass of luminous matter. There has to be more matter interacting via gravitational forces. This has led to the theory of dark matter.
- Mercury's orbit differs hugely from calculated Kepler orbit. This is because of its migrating pericentre. Due to the proximity to the sun gravitational forces are so strong that we need to apply general relativity.
- The SMBH Sagittarius A* was detected by observations of the orbits around the black hole and resulting mass calculations.

Some examples for orbit distribution functions are the asteroid belt, the distribution function of moons around planets which allows feedback connections to formation and bonding of different planet types and spiral galaxies where stars of different parts(thin disc, thick disc, bulge, halo) have different orbits (dynamical distinct) and different metallicity (chemical distinct).

Actions are integrals of motion and are the distinct description of orbits. They are constant with time. Known for a long time they are extremly difficult to calculate. Actions of our solar system can be calculated easier since we know the potential. With nowadays supercomputers it's finally possible to compute actions of more complex and less explored systems.

2 Method & Theory

2.1 Observed kinematics in globular clusters

Our investigations of GCs in phase space consists star distribution plots, the velocity dispersion and the anisotropy parameter. First we test the sphericity of the GC. Sphericity implies the usage of analytical methods especially for determing the potential of the globular cluster and the actions in action space.
The velocity dispersion is the standard deviation of the mean velocity

$$\sigma_i = \sqrt{\langle(v_i - \langle v_i \rangle)^2\rangle} = \sqrt{\langle v_i^2 - \langle v_i \rangle^2 \rangle} \qquad i = r, \theta, \phi. \tag{1}$$

For a spherical system it's best to calculate them in spherical coordinates r, θ, ϕ respectively v_r, v_θ, v_ϕ. If the GC contains an IMBH the velocity dispersion towards the centre is increasing.
what exactly describes anisotropy? To quantify the anisotropy of the system we use the anisotropy parameter β

$$\beta = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{\sigma_r^2}. \tag{2}$$

If β is positive the anisotropy is radial and if it's negative the anisotropy is tangential.

2.2 Orbits

In a dynamical system the mass distribution is described by the form of theoretically existent orbits $(\vec{x}(t), \vec{v}(t))$. Position and velocity are linked with six coordinates and contain all information about the potential. With Newton's 2nd law we get the connection between potential $\Phi(\vec{r})$and acceleration $\vec{a}$ which is

$$\vec{F}(\vec{r}) = -\nabla\Phi(\vec{r}) = m \cdot \vec{a}.$$

Since the system is spherically potential and force only depend on the distance from the centre r. The potential can be derived from the Poisson's equation

$$\Delta\Phi(r) = 4\pi G\rho(r) \tag{3}$$

with the density ρ depending only on the distance as well. Due to the spherical symmetry the potential can be calculated by

$$\Phi(r) = -\frac{G}{r}\int_0^r \mathrm{d}M(r') - G\int_r^\infty \frac{\mathrm{d}M(r')}{r'} = -4\pi G\left[\frac{1}{r}\int_0^r \mathrm{d}r' r'^2\rho(r') + \int_r^\infty \mathrm{d}r' r'\rho(r')\right] \tag{4}$$

(Binney & Tremaine eq. 2.28). We calculate the density by binning the masses on logarithmic equally distributed shells write density formula? and solve the integrals of the Poisson's equation numerically by using the Gauss-Legendre quadrature

$$\int_a^b f(x)dx = \frac{b-a}{2}\sum_{i=1}^{n} w_i f\left(\frac{b-a}{2}x_i + \frac{a+b}{2}\right)$$

where the points x_i and the weights w_i are derived from the Legendre polynomials. Since the orbit is described by position and velocity at each time step we use the numerical leapfrog method which is a second-order time reversible integrator. X_i and v_i are calculated by

$$x_{i+1} = x_i + v_i\Delta t + \frac{a_i(x_i)}{2}\Delta t^2$$
$$v_{i+1} = v_i + \frac{a(x_{i+1}) + a(x_i)}{2}\Delta t.$$

2.3 Actions

Stars in spherical symmetric potential are fully described by their actions

$$J_i = \frac{1}{2\pi}\oint_{\gamma_i} \vec{p}\cdot d\vec{q} \qquad\qquad i = r, \theta, \phi \tag{5}$$

which are used as coordinates in action space These actions are integrals of motion. For most potentials actions can't be described analytically. The actions of a spherical system are derived from angular momentum, energy and potential. Only the potential is depending in r. Energy and angular momentum as well as resulting actions are constant over time and orbit. The azimuthal action J$_\phi$ and the latitudinal action J$_\theta$ can be evaluated simply. To calculate the radial action J$_r$ we have to solve an integral numerically. Actions of a spherical potential are found to be

$$J_\phi = L_z, \tag{6}$$

$$J_\theta = L - |L_z|, \tag{7}$$

$$J_r = \frac{1}{\pi}\int_{r_{min}}^{r_{max}} \mathrm{d}r\sqrt{2E - 2\Phi(r) - \frac{L^2}{r^2}}. \tag{8}$$

Should I write down the derivation of the actions (B&T p.220) or just the results (B&T p.221, 3.221,3.223,3.224)?

The pericenter r_{min} and the apocenter r_{max} as well as the guiding star radisu r_g can be found in the effective potential

$$V_L(r) = V(r) + \frac{L^2}{2mr^2}. \text{ m should be left out right?} \tag{9}$$

Bartelman Theo 1 Skript p59 formula 6.27.

In the peri- and apocenter the effective potential equals the total energy since the stars don't have any kinetic energy there. That results in following function which is to solve:

$$\left(\frac{1}{r}\right)^2 + \frac{2\cdot(\Phi - E)}{L^2} = 0.$$

The guiding star radius is the distance at which a star with given total angular momentum would have a circular orbit. This is at the minimum of the effective potential. To get r_g we have to solve

$$r\sqrt{r\frac{\partial\Phi}{\partial r}} - |L| = 0$$

where $\sqrt{r\frac{\partial\Phi}{\partial r}} = v_{circ}$ is the circular velocity. This distance is used to have a better comparison of the actions since in the snapshot the stars are at a random position on their orbit.

plot v_eff plot from barthelman skript?

3 Analysis

3.1 Description of the simulation

Should I do this for every simulations or only for the first with IMBH?
where simulation comes from and what it is & description of output

To get a familiar with the simulation we first have a look at the scatter plot of the star positions.

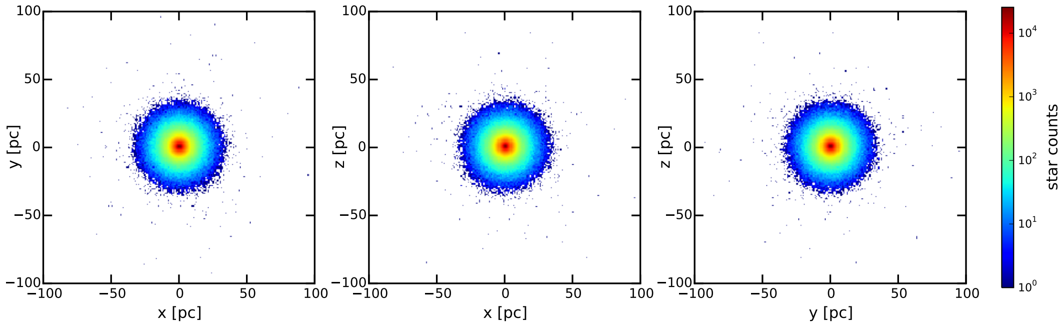


Figure 1: Position scatter plot. The stars are distributed spherically with most of the stars in the inner part. It is spread until 100 pc.

include same plot with velocities?
As we said in section 2.1 it is important to test the sphericity of a system. We will do this by splitting the GC into octants and compare their mass or number since we will introduce mass density plots only later density mass density sphericity plots. As you see they're acceptable overlaying within their errors.
As mentioned in 1.2 the CMD is showing a star's evolution stage dependend on its position. If you don't know age or metallicity of the system you can plot isochrones on the CMD. Isochrones are curves of evolutionary stages of stars having the same age and metallicity but different masses. You can plot several to your CMD to determine which one fits best. This will give you age and metallicity of the system. cmd with isocrones plots

3.2 Investigation in phase space

First we will investigate the GC in phase space. We will start with the velocity dispersion and the anisotropy parameter then we will have a density profile and from that get the potential. for all 4 simulations or only for the first with IMBH? or with on plot containing two or all of them?

3.2.1 Velocity dispersion

With (1) from section 2.1 we can calculate the velocity dispersion for each coordinate r, θ, ϕ. For every bin we take the same amount of stars and calculate the dispersion. To compare both simulations we plot the dispersion over the effective radius. The half mass radius of the simulation with IMBH is 4.1pc and of the simulation without IMBH it's 7.9pc. As expected there is a rise in the centre for the simulation with IMBH. This is due the high gravitational potential of the IMBH which disturbs the dynamics of close stars.

3.2.2 Anisotropy

Anisotropy can be calculated from (2) in 2.1. It is binned the same way as for the velocity dispersion and again dependent on the effective radii. In the center of both GCs there is nearly the same anisotropy. Both are positive and rising. That means the systems are radial anisotropic. The GC with IMBH is most radial anisotropic in it's center at about 4 effective radii. The other GC is becoming more radial anisotropic the more far from the centre it is.

3.2.3 Density profile

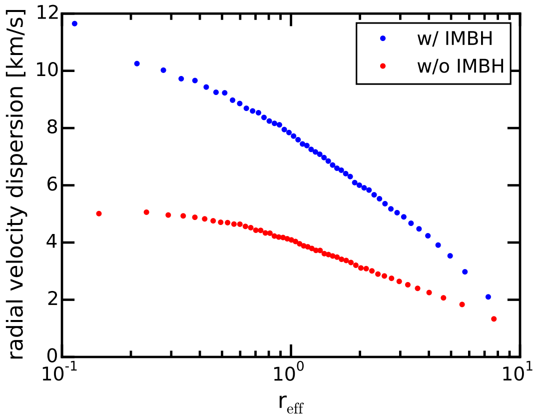
The density profile shows the density of the system over its radius. The bins are chosen so that the radii are equidistant on a logarithmic scale and that they are at least 100 stars per bin to have a reliable stochastic. Outside of the cluster the density is set to 0. In the innermost part the density is set to be let's see what Glenn is saying... plots

3.2.4 Potential

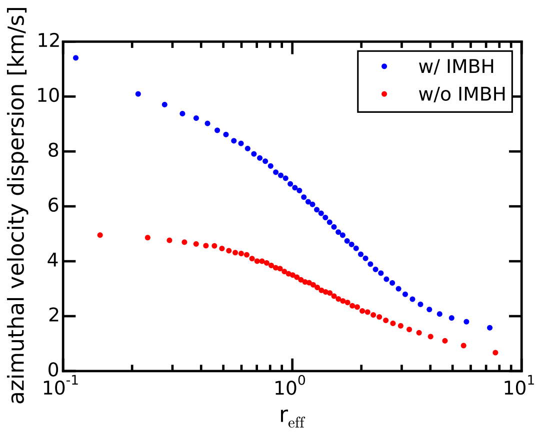
potential daraus

3.3 Investigations of orbits in action space

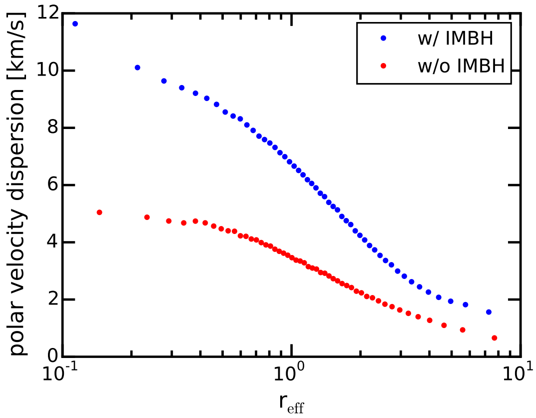
wilma class



(a) Radial velocity dispersions



(b) Azimuthal velocity dispersions



(c) Polar velocity dispersions

Figure 2: Velocity dispersions. They are binned in a way that each bin contains the same amount of stars. We can see that the velocity dispersion of the simulation with IMBH rises towards the centre whereas the simulation without IMBH stays nearly on one level in the inner part.

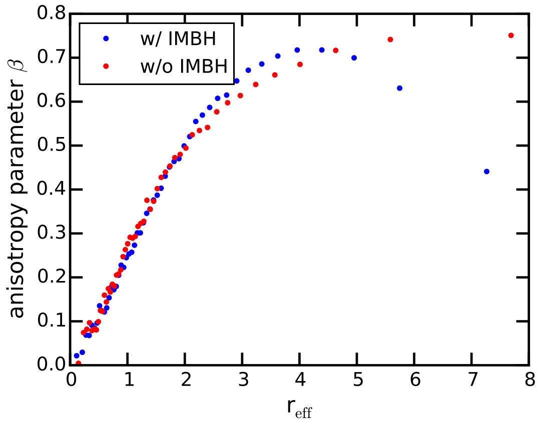


Figure 3: Anisotropy parameter β. Both simulations are radial anisotropic. The simulation with IMBH has a peak at 4 effective radii where it is most radial anisotropic. The other simulation is rising until the end. The far more outside a star is the higher its radial anisotropy is.

3.3.1 Orbits

3.3.2 Actions

3.3.3 Integral of motions along orbits

4 Results & Discussion

only triangle plots

4.1 Actions from different globular clusters

4.2 Discussion & future perspectives

do the same distinguishing the mass of the stars
redo the work with only observational light data

5 Conclusion

6 Acronyms

CMD color magnitude diagram

GC globular cluster

IMBH intermediate mass black hole

MW Milky Way

SMBH super massive black hole