HW5

1. 1)

P	Q	P ⇒ ¬Q	Q ⇒ ¬P
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Model(P ⇒ ¬Q) is the same with Models (Q ⇒ ¬P)

2)

P	Q	P ⇔ ¬Q	(P ∧ ¬Q) ∨ (¬P ∧ Q)
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

Models (P ⇔ ¬Q) is the same with Models ((P ∧ ¬Q) ∨ (¬P ∧ Q)).

2. 1)

Smoke	Fire	A: smoke ⇒ Fire	B: ¬smoke ⇒ ¬Fire	A ⇒ B
T	T	T		T
T	F	F		T
F	T	T		F
F	F	T		T

models is not all ¬¬ world, therefore neither.

2)

Smoke	Fire	Heat	A: Smoke ⇒ Fire	B: (Smoke ∨ Heat) ⇒ Fire	A ⇒ B
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

models is not all world, ⇒ neither.

3)

Smoke	Fire	Heat	A: (Smoke ∧ Heat) ⇒ Fire	B: (Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)	A ⇔ B
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T

T	F	F	(T)FT	T	(T)FT	
F	T	T	(T)TF	T	T	
F	T	F	T	FT	T	
F	F	T	T	TT	
F	F	F	T	TT	

models are the whole world, therefore <u>Valid</u>

3. a) A: unicore is mythical
B: unicore is mortal
C: unicore is mammal
D: unicore is horned
E: unicore is magic

$\Delta: \begin{cases} A \Rightarrow \neg B \\ \neg A \Rightarrow C \wedge B \\ (\neg B \vee C) \Rightarrow D \\ D \equiv E \end{cases}$

$\Delta: \begin{cases} A \Rightarrow \neg B \\ \neg A \Rightarrow C \\ \neg A \Rightarrow B \\ (\neg B \vee C) \Rightarrow D \\ D \equiv E \end{cases}$

b)

CNF: $\neg A \lor \neg B$

    $A \lor C$

    $A \lor B$

    $B \lor D$

    $\neg C \lor D$

    $\neg D \lor E$

(c) 1) prove $\Delta \land \neg A$ is UNSAT using Resolution.

$\neg A$	$\neg A$
$A \lor C$	C
$A \lor B$ $\Rightarrow$	B $\Rightarrow$ no conflict
$B \lor D$	D
$\neg C \lor D$	E
$\neg D \lor E$	

prove $\Delta \land A$ is UNSAT.

A	A
$\neg B$	
$A \lor C$	
$A \lor B$ $\Rightarrow$	$\Rightarrow$ no conflict
$B \lor D$	D
$\neg C \lor D$	$\neg C \lor D$
$\neg D \lor E$	E

Therefore, cannot conclude A = i.e, unicorn is mythical

2) prove unicorn is magic $\Leftrightarrow$ $\Delta \land \neg E$ is UNSAT
using Resolution

$\neg A \lor \neg B$	$\neg B$	$\neg B$
$A \lor C$	A	A
$A \lor B$ $\Rightarrow$		$\Rightarrow$
$B \lor D$	D	D
$\neg C \lor D$	$\neg C$	$\neg C$
$\neg D \lor E$	$V D$	$\{\}$
$\neg E$		

3) Prove unicorn is horned. ⟺ $\Delta \wedge \neg D$ is unSAT using Resolution.

$\neg A \vee \neg B$	$\neg B$	$\neg B$
$A \vee C$	A	A
$A \vee B$		

$B \vee D$	$\Rightarrow D$	$\Rightarrow D$
$\neg C \vee D$	$\neg C$	$\neg C$
$\neg D \vee E$	E	E
$\neg D$	$\neg D$	$\{\ \}$ conflict

Therefore $\Delta \wedge \neg D$ is unSAT.