1) Prove:

P(A,B|K) = P(A|B,K)P(B|K)

left = P(A,B|K) = P(A,B,K)/P(K)

right = P(A|B,K)*P(B|K) = P(A,B,K)/P(B,K)*P(B,K)/P(K) = P(A,B,K)/P(K) = left

P(A|B,K) = P(B|A,K)P(A|K)/P(B|K)

left = P(A,B,K)/P(B,K)

right = P(B,A,K)/P(A,K)*P(A,K)/P(K)/(P(B,K)/P(K)) = P(A,B,K)/P(B,K) = left

2.

P(oil) = .5

P(gas,~oil) = .2

P(~oil,~gas) = .3

P(test|oil) = .9

P(test|gas,~oil) = .3

P(test|~oil,~gas) = .1

Find out P(oil|test)

= P(test|oil)*P(oil)/P(test)

P(test) = P(test,oil) + P(test,~oil,gas) + P(test,~oil,~gas)

= P(test|oil)*P(oil) + P(test|~oil,gas)*P(~oil,gas) + P(test|~oil,~gas)*P(~oil,~gas)

= .9*.5 + .3*.2 + .1*.3 = .54

P(oil|test) = .9*.5 / .54 = .83

3.

The world:

World	Black	Square	One	Pr(w)
1	t	t	t	2/13
2	t	t	f	4/13
3	t	f	t	1/13
4	t	f	f	2/13
5	f	t	t	1/13
6	f	t	f	1/13
7	f	f	t	1/13
8	f	f	f	1/13

$P(\alpha_1)$ = 2/13+ 4/13+1/13+2/13 = 9/13

$P(\alpha_2)$ = 2/13+ 4/13+ 1/13*2 = 8/13

$P(\alpha_3)$ = P(~One & ~Black | Square) = 2/13 + 4/13 + 1/13*2 + 1/13 = 9/13

Two sentences:

- Given the object is not Black(γ), the object is Square(α) and the object is One(β) is independent;
  Proof: In world 5-8:
  P(One| Square, ~Black)=P(One | ~Black)= 1/2
- Given the object is Black (γ), the object is not Square(α) and the object is One(β) is independent.
  Proof: In world 1-4:
  P(One| Square, Black)=P(One | Black)=1/3

4.

a)

I(A,{},B)
I(B,{},A)
I(C, A, DBE)
I(D, AB, CE)
I(E,B,ACDFG)
I(F, CD, ABE)
I(G, F, ACDBEH)
I(H, EF, ACDBG)

b)

- False, because ACFHE is not block, H(converge) is open since H in {BH}.
- True, because all paths either have to go through D or H. D(sequential) is blocked since D in {D}; H(converge) is blocked since H not in {D}.
- False, because BEH is not blocked, since E(sequential) is open.

c)

P(a, b, c, d, e, f, g, h) = P(a)P(b)P(c | a)P(d | a, b)P(e | b)P(f | c, d)P(h | f, e)P(g | f)

d)

P(A = 0, B = 0) = P(A=0) * P(B = 0) = 0.24
Since A and B are independent.

P(E = 1| A = 1) = P(E=1) since E and A are independent.
P(E =1) = P(E = 1, B = 0) + P(E = 1 , B = 1)
= P(E = 1|B = 0)*P(B = 0) + P(E = 1|B = 1)*P(B = 1)
= .9*.3 + .1*.7 = 0.34
Therefore P(E = 1| A = 1) = 0.34