For all nodes $\Delta x = \Delta y = \Delta = 0.01$

NODE 1

For
$$y = 0$$
, $\hat{n} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\frac{\partial T}{\partial y} \Big|_{T_1} = 0$$

For
$$\mathbf{x} = 0$$
, $\hat{n} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\nabla \mathbf{T} \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\frac{\partial T}{\partial x} \Big|_{T_1} = 0$$

$$\frac{\partial^2 T}{\partial x^2}\Big|_{T1} + \frac{\partial^2 T}{\partial y^2}\Big|_{T1} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{T2} - \frac{\partial T}{\partial x}\Big|_{T1}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T7} - \frac{\partial T}{\partial y}\Big|_{T1}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x}\Big|_{T2} + 0}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T7} + 0}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{T2} + \frac{\partial T}{\partial y}\Big|_{T7}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{T2} + \frac{\partial T}{\partial y}\Big|_{T7}$$

$$0 = \frac{T2 - T1}{\Delta x} + \frac{T7 - T1}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T2 - T1 + T7 - T1)$$

$$0 = T2 - 2T1 + T7$$

0 = 2T1 - T2 - T7

For
$$y = 0$$
, $\hat{n} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\frac{\partial T}{\partial y} \Big|_{T2} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T2} + \frac{\partial^2 T}{\partial y^2} \Big|_{T2} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T3} - \frac{\partial T}{\partial x} \Big|_{T2}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T8} - \frac{\partial T}{\partial y} \Big|_{T2}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T3} - \frac{\partial T}{\partial x} \Big|_{T2} + \frac{\partial T}{\partial y} \Big|_{T8} + 0}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T3} - \frac{\partial T}{\partial x} \Big|_{T2} + \frac{\partial T}{\partial y} \Big|_{T8} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T3} - \frac{\partial T}{\partial x} \Big|_{T2} + \frac{\partial T}{\partial y} \Big|_{T8}$$

$$0 = \frac{T3 - T2}{\Delta x} - \frac{T2 - T1}{\Delta x} + \frac{T8 - T2}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T3 - T2 - (T2 - T1) + T8 - T2)$$

$$0 = T3 - T2 - (T2 - T1) + T8 - T2$$

$$0 = T3 - T2 - T2 + T1 + T8$$

$$0 = -T1 + 3T2 - T3 - T8$$

For
$$y = 0$$
, $\hat{n} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\frac{\partial T}{\partial y} \Big|_{T3} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T3} + \frac{\partial^2 T}{\partial y^2} \Big|_{T3} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T4} - \frac{\partial T}{\partial x} \Big|_{T3} + \frac{\partial T}{\partial y} \Big|_{T9} - \frac{\partial T}{\partial y} \Big|_{T3}}{\Delta x}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T4} - \frac{\partial T}{\partial x} \Big|_{T3} + \frac{\partial T}{\partial y} \Big|_{T9} + 0}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T4} - \frac{\partial T}{\partial x} \Big|_{T3} + \frac{\partial T}{\partial y} \Big|_{T9} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T4} - \frac{\partial T}{\partial x} \Big|_{T3} + \frac{\partial T}{\partial y} \Big|_{T9}$$

$$0 = \frac{T4 - T3}{\Delta x} - \frac{T3 - T2}{\Delta x} + \frac{T9 - T3}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T4 - T3 - (T3 - T2) + T9 - T3)$$

$$0 = T4 - T3 - (T3 - T2) + T9 - T3$$

$$0 = T4 - T3 - T3 + T2 + T9 - T3$$

$$0 = T4 - T3 - T3 + T2 + T9$$

0 = -T2 + 3T3 - T4 - T9

For
$$y = 0$$
, $\hat{n} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\frac{\partial T}{\partial y} \Big|_{T_4} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T_4} + \frac{\partial^2 T}{\partial y^2} \Big|_{T_4} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T_5} - \frac{\partial T}{\partial x} \Big|_{T_4}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T_{10}} - \frac{\partial T}{\partial y} \Big|_{T_4}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T_5} - \frac{\partial T}{\partial x} \Big|_{T_4} + \frac{\partial T}{\partial y} \Big|_{T_{10}} + 0}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T_5} - \frac{\partial T}{\partial x} \Big|_{T_4} + \frac{\partial T}{\partial y} \Big|_{T_{10}} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T_5} - \frac{\partial T}{\partial x} \Big|_{T_4} + \frac{\partial T}{\partial y} \Big|_{T_{10}}$$

$$0 = \frac{T_5 - T_4}{\Delta x} - \frac{T_4 - T_3}{\Delta x} + \frac{T_{10} - T_4}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T_5 - T_4 - (T_4 - T_3) + T_{10} - T_4)$$

$$0 = T_5 - T_4 - (T_4 - T_3) + T_{10} - T_4$$

$$0 = T_5 - T_4 - T_4 + T_3 + T_{10} - T_4$$

$$0 = T_5 - 3T_4 + T_3 + T_{10}$$

$$0 = -T_3 + 3T_4 - T_5 - T_{10}$$

For
$$y = 0$$
, $\hat{n} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\frac{\partial T}{\partial y} \Big|_{T5} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T5} + \frac{\partial^2 T}{\partial y^2} \Big|_{T5} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T6} - \frac{\partial T}{\partial x} \Big|_{T5}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T11} - \frac{\partial T}{\partial y} \Big|_{T5}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T6} - \frac{\partial T}{\partial x} \Big|_{T5}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T11} + 0}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T6} - \frac{\partial T}{\partial x} \Big|_{T5} + \frac{\partial T}{\partial y} \Big|_{T11} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T6} - \frac{\partial T}{\partial x} \Big|_{T5} + \frac{\partial T}{\partial y} \Big|_{T11}$$

$$0 = \frac{T6 - T5}{\Delta x} - \frac{T5 - T4}{\Delta x} + \frac{T11 - T5}{\Delta y}$$

$$0 = \frac{1}{\Delta} (TT6 - T5 - (T5 - T4) + T11 - T5)$$

$$0 = T6 - T5 - (T5 - T4) + T11 - T5$$

$$0 = T6 - T5 - T5 + T4 + T11$$

$$0 = T6 - T5 - T5 + T4 + T11$$

For
$$y = 0$$
, $\hat{n} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\frac{\partial T}{\partial y} \Big|_{T6} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T6} + \frac{\partial^2 T}{\partial y^2} \Big|_{T6} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{TCD} - \frac{\partial T}{\partial x} \Big|_{T6}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T12} - \frac{\partial T}{\partial y} \Big|_{T6}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{TCD} - \frac{\partial T}{\partial x} \Big|_{T6}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T12} - 0}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{TCD} - \frac{\partial T}{\partial x} \Big|_{T6} + \frac{\partial T}{\partial y} \Big|_{T12} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{TCD} - \frac{\partial T}{\partial x} \Big|_{T6} + \frac{\partial T}{\partial y} \Big|_{T12}$$

$$0 = \frac{TCD - T6}{\Delta x} - \frac{T6 - T5}{\Delta x} + \frac{T12 - T6}{\Delta y}$$

$$0 = \frac{1}{\Delta} (TCD - T6 - (T6 - T5) + T12 - T6)$$

$$0 = TCD - T6 - (T6 - T5) + T12 - T6$$

$$0 = TCD - T6 - T6 + T5 + T12$$

$$40 = -T5 + 3T6 - T12$$

For
$$x = 0$$
, $\hat{n} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\frac{\partial T}{\partial x} \Big|_{T7} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T7} + \frac{\partial^2 T}{\partial y^2} \Big|_{T7} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T8} - \frac{\partial T}{\partial x} \Big|_{T7}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T13} - \frac{\partial T}{\partial y} \Big|_{T7}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T8} + 0}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T13} - \frac{\partial T}{\partial y} \Big|_{T7}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T8} + \frac{\partial T}{\partial y} \Big|_{T13} - \frac{\partial T}{\partial y} \Big|_{T7} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T8} + \frac{\partial T}{\partial y} \Big|_{T13} - \frac{\partial T}{\partial y} \Big|_{T7}$$

$$0 = \frac{T8 - T7}{\Delta x} + \frac{T13 - T7}{\Delta y} - \frac{T7 - T1}{\Delta y}$$

$$0 = \frac{1}{\Delta} (TT8 - T7 + T13 - T7 - (T7 - T1))$$

$$0 = T8 - T7 + T13 - T7 - (T7 - T1)$$

$$0 = T8 - T7 + T13 - T7 - T7 + T1$$

$$0 = T8 - 3T7 + T13 + T1$$

$$0 = -T1 + 3T7 - T8 - T13$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T8} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T8} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T9} - \frac{\partial T}{\partial x} \bigg|_{T8}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \bigg|_{T14} - \frac{\partial T}{\partial y} \bigg|_{T8}}{\Delta y} \\ &0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T9} - \frac{\partial T}{\partial x} \bigg|_{T8} + \frac{\partial T}{\partial y} \bigg|_{T14} - \frac{\partial T}{\partial y} \bigg|_{T8} \right) \\ &0 = \frac{\partial T}{\partial x} \bigg|_{T9} - \frac{\partial T}{\partial x} \bigg|_{T8} + \frac{\partial T}{\partial y} \bigg|_{T14} - \frac{\partial T}{\partial y} \bigg|_{T8} \\ &0 = \frac{T9 - T8}{\Delta x} - \frac{T8 - T7}{\Delta x} + \frac{T14 - T8}{\Delta y} - \frac{T8 - T2}{\Delta y} \\ &0 = \frac{1}{\Delta} (T9 - T8 - (T8 - T7) + T14 - T8 - (T8 - T2)) \\ &0 = T9 - T8 - (T8 - T7) + T14 - T8 - (T8 - T2) \\ &0 = T9 - T8 - T8 + T7 + T14 - T8 - T8 + T2 \\ &0 = T9 - 4T8 + T7 + T14 + T2 \\ &0 = -T2 - T7 + 4T8 - T9 - T14 \end{aligned}$$

$$\frac{\partial^{2}T}{\partial x^{2}}\Big|_{T9} + \frac{\partial^{2}T}{\partial y^{2}}\Big|_{T9} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{T10} - \frac{\partial T}{\partial x}\Big|_{T9}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T15} - \frac{\partial T}{\partial y}\Big|_{T9}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{T10} - \frac{\partial T}{\partial x}\Big|_{T9} + \frac{\partial T}{\partial y}\Big|_{T15} - \frac{\partial T}{\partial y}\Big|_{T9}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{T10} - \frac{\partial T}{\partial x}\Big|_{T9} + \frac{\partial T}{\partial y}\Big|_{T15} - \frac{\partial T}{\partial y}\Big|_{T9}$$

$$0 = \frac{T10 - T9}{\Delta x} - \frac{T9 - T8}{\Delta x} + \frac{T15 - T9}{\Delta y} - \frac{T9 - T3}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T10 - T9 - (T9 - T8) + T15 - T9 - (T9 - T3))$$

$$0 = T10 - T9 - (T9 - T8) + T15 - T9 - (T9 - T3)$$

$$0 = T10 - T9 - T9 + T8 + T15 - T9 - T9 + T3$$

$$0 = T10 - 4T9 + T8 + T15 + T3$$

$$0 = -T3 - T8 + 4T9 - T10 - T15$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T10} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T10} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T11} - \frac{\partial T}{\partial x} \bigg|_{T10}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \bigg|_{T16} - \frac{\partial T}{\partial y} \bigg|_{T10}}{\Delta y} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T11} - \frac{\partial T}{\partial x} \bigg|_{T10} + \frac{\partial T}{\partial y} \bigg|_{T16} - \frac{\partial T}{\partial y} \bigg|_{T10} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T11} - \frac{\partial T}{\partial x} \bigg|_{T10} + \frac{\partial T}{\partial y} \bigg|_{T16} - \frac{\partial T}{\partial y} \bigg|_{T10} \\ 0 &= \frac{T11 - T10}{\Delta x} - \frac{T10 - T9}{\Delta x} + \frac{T16 - T10}{\Delta y} - \frac{T10 - T4}{\Delta y} \end{aligned}$$

$$0 &= \frac{1}{\Delta} (T11 - T10 - (T10 - T9) + T16 - T10 - (T10 - T4))$$

$$0 &= T11 - T10 - (T10 - T9) + T16 - T10 - (T10 - T4)$$

$$0 &= T11 - T10 - T10 + T9 + T16 - T10 - T10 + T4$$

$$0 &= T11 - 4T10 + T9 + T16 + T4$$

$$0 &= T11 - 4T10 - T10 - T11 - T16$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T11} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T11} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T12} - \frac{\partial T}{\partial x} \bigg|_{T11}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \bigg|_{T17} - \frac{\partial T}{\partial y} \bigg|_{T11}}{\Delta y} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T12} - \frac{\partial T}{\partial x} \bigg|_{T11} + \frac{\partial T}{\partial y} \bigg|_{T17} - \frac{\partial T}{\partial y} \bigg|_{T11} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T12} - \frac{\partial T}{\partial x} \bigg|_{T11} + \frac{\partial T}{\partial y} \bigg|_{T17} - \frac{\partial T}{\partial y} \bigg|_{T11} \\ 0 &= \frac{T12 - T11}{\Delta x} - \frac{T11 - T10}{\Delta x} + \frac{T17 - T11}{\Delta y} - \frac{T11 - T5}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T12 - T11 - (T11 - T10) + T17 - T11 - (T11 - T5)) \\ 0 &= T12 - T11 - (T11 - T10) + T17 - T11 - (T11 - T5) \\ 0 &= T12 - 4T11 + T10 + T17 + T5 \\ 0 &= T12 - 4T11 + T10 + T17 + T5 \end{aligned}$$

$$\frac{\partial^{2}T}{\partial x^{2}}\Big|_{T12} + \frac{\partial^{2}T}{\partial y^{2}}\Big|_{T12} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{TCD} - \frac{\partial T}{\partial x}\Big|_{T12}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T18} - \frac{\partial T}{\partial y}\Big|_{T12}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{TCD} - \frac{\partial T}{\partial x}\Big|_{T12} + \frac{\partial T}{\partial y}\Big|_{T18} - \frac{\partial T}{\partial y}\Big|_{T12}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{TCD} - \frac{\partial T}{\partial x}\Big|_{T12} + \frac{\partial T}{\partial y}\Big|_{T18} - \frac{\partial T}{\partial y}\Big|_{T12}$$

$$0 = \frac{TCD - T12}{\Delta x} - \frac{T12 - T11}{\Delta x} + \frac{T18 - T12}{\Delta y} - \frac{T12 - T6}{\Delta y}$$

$$0 = \frac{1}{\Delta}(TCD - T12 - (T12 - T11) + T18 - T12 - (T12 - T6))$$

$$0 = TCD - T12 - (T12 - T11) + T18 - T12 - (T12 - T6)$$

$$0 = TCD - T12 - T12 + T11 + T18 - T12 - T12 + T6$$

$$0 = 40 - 4T12 + T11 + T18 + T6$$

$$40 = -T6 - T11 + 4T12 - T18$$

For
$$x = 0$$
, $\hat{n} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\frac{\partial T}{\partial x} \Big|_{T13} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T13} + \frac{\partial^2 T}{\partial y^2} \Big|_{T13} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T14} - \frac{\partial T}{\partial x} \Big|_{T13} + \frac{\frac{\partial T}{\partial y} \Big|_{T19} - \frac{\partial T}{\partial y} \Big|_{T13}}{\Delta x}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T14} + 0}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T19} - \frac{\partial T}{\partial y} \Big|_{T13}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T14} + \frac{\partial T}{\partial y} \Big|_{T19} - \frac{\partial T}{\partial y} \Big|_{T13} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T14} + \frac{\partial T}{\partial y} \Big|_{T19} - \frac{\partial T}{\partial y} \Big|_{T13}$$

$$0 = \frac{T14 - T13}{\Delta x} + \frac{T19 - T13}{\Delta y} - \frac{T13 - T7}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T14 - T13 + T19 - T13 - (T13 - T7))$$

$$0 = T14 - T13 + T19 - T13 - (T13 - T7)$$

$$0 = T14 - T13 + T19 - T13 - T13 + T7$$

$$0 = T14 - T13 + T19 - T13 - T13 + T7$$

0 = -T7 + 3T13 - T14 - T19

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T14} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T14} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T15} - \frac{\partial T}{\partial x} \bigg|_{T14} + \frac{\partial T}{\partial y} \bigg|_{T20} - \frac{\partial T}{\partial y} \bigg|_{T14}}{\Delta x} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T15} - \frac{\partial T}{\partial x} \bigg|_{T14} + \frac{\partial T}{\partial y} \bigg|_{T20} - \frac{\partial T}{\partial y} \bigg|_{T14} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T15} - \frac{\partial T}{\partial x} \bigg|_{T14} + \frac{\partial T}{\partial y} \bigg|_{T20} - \frac{\partial T}{\partial y} \bigg|_{T14} \\ 0 &= \frac{T15 - T14}{\Delta x} - \frac{T14 - T13}{\Delta x} + \frac{T20 - T14}{\Delta y} - \frac{T14 - T8}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T15 - T14 - (T14 - T13) + T20 - T14 - (T14 - T8)) \\ 0 &= T15 - T14 - (T14 - T13) + T20 - T14 - (T14 - T8) \\ 0 &= T15 - 4T14 + T13 + T20 - T14 - T14 + T8 \\ 0 &= T15 - 4T14 + T13 + T20 + T8 \\ 0 &= -T8 - T13 + 4T14 - T15 - T20 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T15} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T15} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T16} - \frac{\partial T}{\partial x} \bigg|_{T15} + \frac{\partial T}{\partial y} \bigg|_{T21} - \frac{\partial T}{\partial y} \bigg|_{T15}}{\Delta x} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T16} - \frac{\partial T}{\partial x} \bigg|_{T15} + \frac{\partial T}{\partial y} \bigg|_{T21} - \frac{\partial T}{\partial y} \bigg|_{T15} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T16} - \frac{\partial T}{\partial x} \bigg|_{T15} + \frac{\partial T}{\partial y} \bigg|_{T21} - \frac{\partial T}{\partial y} \bigg|_{T15} \\ 0 &= \frac{T16 - T15}{\Delta x} - \frac{T15 - T14}{\Delta x} + \frac{T21 - T15}{\Delta y} - \frac{T15 - T9}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T16 - T15 - (T15 - T14) + T21 - T15 - (T15 - T9)) \\ 0 &= T16 - T15 - (T15 - T14) + T21 - T15 - (T15 - T9) \\ 0 &= T16 - 4T15 + T14 + T21 + T9 \\ 0 &= T16 - 4T15 + T14 + T21 + T9 \\ 0 &= -T9 - T14 + 4T15 - T16 - T21 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T16} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T16} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T17} - \frac{\partial T}{\partial x} \bigg|_{T16} + \frac{\frac{\partial T}{\partial y} \bigg|_{T22} - \frac{\partial T}{\partial y} \bigg|_{T16}}{\Delta x} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T17} - \frac{\partial T}{\partial x} \bigg|_{T16} + \frac{\partial T}{\partial y} \bigg|_{T22} - \frac{\partial T}{\partial y} \bigg|_{T16} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T17} - \frac{\partial T}{\partial x} \bigg|_{T16} + \frac{\partial T}{\partial y} \bigg|_{T22} - \frac{\partial T}{\partial y} \bigg|_{T16} \\ 0 &= \frac{T17 - T16}{\Delta x} - \frac{T16 - T15}{\Delta x} + \frac{T22 - T16}{\Delta y} - \frac{T16 - T10}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T17 - T16 - (T16 - T15) + T22 - T16 - (T16 - T10)) \\ 0 &= T17 - T16 - (T16 - T15) + T22 - T16 - (T16 - T10) \\ 0 &= T17 - T16 - T16 + T15 + T22 - T16 - T16 + T10 \\ 0 &= T17 - 4T16 + T15 + T22 + T10 \\ 0 &= -T10 - T15 + 4T16 - T17 - T22 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T17} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T17} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T18} - \frac{\partial T}{\partial x} \bigg|_{T17} + \frac{\frac{\partial T}{\partial y} \bigg|_{T23} - \frac{\partial T}{\partial y} \bigg|_{T17}}{\Delta y} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T18} - \frac{\partial T}{\partial x} \bigg|_{T17} + \frac{\partial T}{\partial y} \bigg|_{T23} - \frac{\partial T}{\partial y} \bigg|_{T17} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T18} - \frac{\partial T}{\partial x} \bigg|_{T17} + \frac{\partial T}{\partial y} \bigg|_{T23} - \frac{\partial T}{\partial y} \bigg|_{T17} \\ 0 &= \frac{T18 - T17}{\Delta x} - \frac{T17 - T16}{\Delta x} + \frac{T23 - T17}{\Delta y} - \frac{T17 - T11}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T18 - T17 - (T17 - T16) + T23 - T17 - (T17 - T11)) \\ 0 &= T18 - T17 - (T17 - T16) + T23 - T17 - (T17 - T11) \\ 0 &= T18 - 4T17 + T16 + T23 + T11 \\ 0 &= T18 - 4T17 + T16 + T23 + T11 \\ 0 &= -T11 - T16 + 4T17 - T18 - T23 \end{aligned}$$

$$\frac{\partial^{2}T}{\partial x^{2}}\Big|_{T18} + \frac{\partial^{2}T}{\partial y^{2}}\Big|_{T18} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{TCD} - \frac{\partial T}{\partial x}\Big|_{T18}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T24} - \frac{\partial T}{\partial y}\Big|_{T18}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{TCD} - \frac{\partial T}{\partial x}\Big|_{T18} + \frac{\partial T}{\partial y}\Big|_{T24} - \frac{\partial T}{\partial y}\Big|_{T18}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{TCD} - \frac{\partial T}{\partial x}\Big|_{T18} + \frac{\partial T}{\partial y}\Big|_{T24} - \frac{\partial T}{\partial y}\Big|_{T18}$$

$$0 = \frac{TCD - T18}{\Delta x} - \frac{T18 - T17}{\Delta x} + \frac{T24 - T18}{\Delta y} - \frac{T18 - T12}{\Delta y}$$

$$0 = \frac{1}{\Delta}(TCD - T18 - (T18 - T17) + T24 - T18 - (T18 - T12))$$

$$0 = TCD - T18 - (T18 - T17) + T24 - T18 - (T18 - T12)$$

$$0 = TCD - T18 - T18 + T17 + T24 - T18 - T18 + T12$$

$$0 = 40 - 4T18 + T17 + T24 + T12$$

$$40 = -T12 - T17 + 4T18 - T24$$

For
$$x = 0$$
, $\hat{n} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\frac{\partial T}{\partial x} \Big|_{T19} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T19} + \frac{\partial^2 T}{\partial y^2} \Big|_{T19} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T20} - \frac{\partial T}{\partial x} \Big|_{T19} + \frac{\partial T}{\partial y} \Big|_{T25} - \frac{\partial T}{\partial y} \Big|_{T19}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T20} + 0}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T25} - \frac{\partial T}{\partial y} \Big|_{T19}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T20} + \frac{\partial T}{\partial y} \Big|_{T25} - \frac{\partial T}{\partial y} \Big|_{T19} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T20} + \frac{\partial T}{\partial y} \Big|_{T25} - \frac{\partial T}{\partial y} \Big|_{T19}$$

$$0 = \frac{T20 - T19}{\Delta x} + \frac{T25 - T19}{\Delta y} - \frac{T19 - T13}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T20 - T19 + T25 - T19 - (T19 - T13))$$

$$0 = T20 - T19 + T25 - T19 - T19 + T13$$

$$0 = T20 - T19 + T25 - T19 - T19 + T13$$

$$0 = T20 - T19 + T25 + T13$$

$$0 = T20 - T13 + 3T19 - T20 - T25$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T20} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T20} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T21} - \frac{\partial T}{\partial x} \bigg|_{T20}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \bigg|_{T26} - \frac{\partial T}{\partial y} \bigg|_{T20}}{\Delta y} \\ &0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T21} - \frac{\partial T}{\partial x} \bigg|_{T20} + \frac{\partial T}{\partial y} \bigg|_{T26} - \frac{\partial T}{\partial y} \bigg|_{T20} \right) \\ &0 = \frac{\partial T}{\partial x} \bigg|_{T21} - \frac{\partial T}{\partial x} \bigg|_{T20} + \frac{\partial T}{\partial y} \bigg|_{T26} - \frac{\partial T}{\partial y} \bigg|_{T20} \\ &0 = \frac{T21 - T20}{\Delta x} - \frac{T20 - T19}{\Delta x} + \frac{T26 - T20}{\Delta y} - \frac{T20 - T14}{\Delta y} \end{aligned}$$

$$0 = \frac{1}{\Delta} (T21 - T20 - (T20 - T19) + T26 - T20 - (T20 - T14))$$

$$0 = T21 - T20 - (T20 - T19) + T26 - T20 - (T20 - T14)$$

$$0 = T21 - T20 - T20 + T19 + T26 - T20 - T20 + T14$$

$$0 = T21 - T20 - T20 + T19 + T26 - T20 - T20 + T14$$

$$0 = T21 - T20 - T20 + T19 + T26 + T14$$

$$0 = T21 - T20 - T20 + T19 + T26 + T14$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T21} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T21} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T22} - \frac{\partial T}{\partial x} \bigg|_{T21} + \frac{\partial T}{\partial y} \bigg|_{T27} - \frac{\partial T}{\partial y} \bigg|_{T21}}{\Delta x} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T22} - \frac{\partial T}{\partial x} \bigg|_{T21} + \frac{\partial T}{\partial y} \bigg|_{T27} - \frac{\partial T}{\partial y} \bigg|_{T21} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T22} - \frac{\partial T}{\partial x} \bigg|_{T21} + \frac{\partial T}{\partial y} \bigg|_{T27} - \frac{\partial T}{\partial y} \bigg|_{T21} \\ 0 &= \frac{T22 - T21}{\Delta x} - \frac{T21 - T20}{\Delta x} + \frac{T27 - T21}{\Delta y} - \frac{T21 - T15}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T22 - T21 - (T21 - T20) + T27 - T21 - (T21 - T15)) \\ 0 &= T22 - T21 - (T21 - T20) + T27 - T21 - (T21 - T15) \\ 0 &= T22 - T21 - T21 + T20 + T27 - T21 - T21 + T15 \\ 0 &= T22 - 4T21 + T20 + T27 + T15 \\ 0 &= -T15 - T20 + 4T21 - T22 - T27 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T22} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T22} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T23} - \frac{\partial T}{\partial x} \bigg|_{T22} + \frac{\partial T}{\partial y} \bigg|_{T28} - \frac{\partial T}{\partial y} \bigg|_{T22}}{\Delta x} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T23} - \frac{\partial T}{\partial x} \bigg|_{T22} + \frac{\partial T}{\partial y} \bigg|_{T28} - \frac{\partial T}{\partial y} \bigg|_{T22} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T23} - \frac{\partial T}{\partial x} \bigg|_{T22} + \frac{\partial T}{\partial y} \bigg|_{T28} - \frac{\partial T}{\partial y} \bigg|_{T22} \\ 0 &= \frac{T23 - T22}{\Delta x} - \frac{T22 - T21}{\Delta x} + \frac{T28 - T22}{\Delta y} - \frac{T22 - T16}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T23 - T22 - (T22 - T21) + T28 - T22 - (T22 - T16)) \\ 0 &= c23 - T22 - (T22 - T21) + T28 - T22 - (T22 - T16) \\ 0 &= T23 - T22 - T22 + T21 + T28 - T22 - T22 + T16 \\ 0 &= T23 - 4T22 + T21 + T28 + T16 \\ 0 &= -T16 - T21 + 4T22 - T23 - T28 \end{aligned}$$

$$\frac{\partial^{2}T}{\partial x^{2}}\Big|_{T23} + \frac{\partial^{2}T}{\partial y^{2}}\Big|_{T23} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{T24} - \frac{\partial T}{\partial x}\Big|_{T23}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T29} - \frac{\partial T}{\partial y}\Big|_{T23}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{T24} - \frac{\partial T}{\partial x}\Big|_{T23} + \frac{\partial T}{\partial y}\Big|_{T29} - \frac{\partial T}{\partial y}\Big|_{T23}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{T24} - \frac{\partial T}{\partial x}\Big|_{T23} + \frac{\partial T}{\partial y}\Big|_{T29} - \frac{\partial T}{\partial y}\Big|_{T23}$$

$$0 = \frac{T24 - T23}{\Delta x} - \frac{T23 - T22}{\Delta x} + \frac{T29 - T23}{\Delta y} - \frac{T23 - T17}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T24 - T23 - (T23 - T22) + T29 - T23 - (T23 - T17))$$

$$0 = T24 - T23 - (T23 - T22) + T29 - T23 - (T23 - T17)$$

$$0 = T24 - T23 - T23 + T22 + T29 - T23 - T23 + T17$$

$$0 = T24 - 4T23 + T22 + T29 + T17$$

$$0 = -T17 - T22 + 4T23 - T24 - T29$$

Let α denote the ambient temperature.

For
$$y = 0$$
, $\hat{n} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$3 \times \nabla T \cdot \hat{n} = 20a - 20T_{24}$$

$$\nabla T \cdot \hat{n} = \frac{20a - 20T_{24}}{3} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{\partial T}{\partial y} \Big|_{T24}$$

$$\frac{\partial T}{\partial y} \Big|_{T24} = \frac{20a - 20T_{24}}{3}$$

For diagonal component, $n = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$||n|| = \sqrt{1^2 + 1^2} \\ = \sqrt{2}$$

$$\hat{n} = \frac{n}{\|n\|} = \frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\nabla T \cdot \hat{n} = \frac{20a - 20T_{24}}{3} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \left(\frac{\partial T}{\partial x} \Big|_{T24} + \frac{\partial T}{\partial y} \Big|_{T24} \right)$$

$$\begin{split} \frac{\partial T}{\partial x}\Big|_{T24} &+ \frac{\partial T}{\partial y}\Big|_{T24} = \frac{\sqrt{2}(20a - 20T_{24})}{3} \\ \frac{\partial T}{\partial x}\Big|_{T24} &= \frac{\sqrt{2}(20a - 20T_{24})}{3} - \frac{\partial T}{\partial y}\Big|_{T24} \\ \frac{\partial T}{\partial x}\Big|_{T24} &= \frac{\sqrt{2}(20a - 20T_{24})}{3} - \frac{20a - 20T_{24}}{3} \\ \frac{\partial T}{\partial x}\Big|_{T24} &= \frac{20a - 20T_{24}}{3} (\sqrt{2} - 1) \end{split}$$

$$\frac{\partial^2 T}{\partial x^2}\Big|_{T24} + \frac{\partial^2 T}{\partial y^2}\Big|_{T24} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{T24} - \frac{\partial T}{\partial x}\Big|_{T23}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T24} - \frac{\partial T}{\partial y}\Big|_{T18}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{T24} - \frac{\partial T}{\partial x}\Big|_{T23} + \frac{\partial T}{\partial y}\Big|_{T24} - \frac{\partial T}{\partial y}\Big|_{T18}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{T24} - \frac{\partial T}{\partial x}\Big|_{T23} + \frac{\partial T}{\partial y}\Big|_{T24} - \frac{\partial T}{\partial y}\Big|_{T18}$$

$$0 = \frac{20a - 20T_{24}}{3}(\sqrt{2} - 1) - \frac{(T24 - T23)}{\Delta x} + \frac{20a - 20T_{24}}{3} - \frac{(T24 - T18)}{\Delta y}$$

$$0 = \frac{\sqrt{2}(20a - 20T_{24})}{3} - \frac{20a - 20T_{24}}{3} - \frac{(T24 - T23)}{\Delta x} + \frac{20a - 20T_{24}}{3} - \frac{(T24 - T18)}{\Delta y}$$

$$0 = \frac{\sqrt{2}(20a - 20T_{24})}{3} - \frac{(T24 - T23)}{\Delta x} - \frac{(T24 - T18)}{\Delta y}$$

$$0 = \frac{20a\sqrt{2}}{3} - \frac{20T_{24}\sqrt{2}}{3} + \frac{-T24 + T23}{\Delta x} + \frac{-T24 + T18}{\Delta y}$$

$$0 = \frac{20a\sqrt{2}}{3} - \frac{20T_{24}\sqrt{2}}{3} + \frac{-T24 + T23 - T24 + T18}{0.01}$$

$$0 = \frac{20a\sqrt{2}}{300} - \frac{20T_{24}\sqrt{2}}{300} - T24 + T23 - T24 + T18$$

$$0 = \frac{a\sqrt{2}}{15} - \frac{T_{24}\sqrt{2}}{15} - T24 + T23 - T24 + T18$$

$$\frac{a\sqrt{2}}{15} = \left(\frac{30 + \sqrt{2}}{15}\right)T24 - T23 - T18$$

For a = 20

$$\frac{20\sqrt{2}}{15} = \left(\frac{30 + \sqrt{2}}{15}\right)T24 - T23 - T18$$

$$\frac{4\sqrt{2}}{3} = \left(\frac{30 + \sqrt{2}}{15}\right)T24 - T23 - T18$$

For
$$\mathbf{x} = 0$$
, $\hat{n} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\frac{\partial T}{\partial x} \Big|_{T25} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T25} + \frac{\partial^2 T}{\partial y^2} \Big|_{T25} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T26} - \frac{\partial T}{\partial x} \Big|_{T25}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T30} - \frac{\partial T}{\partial y} \Big|_{T25}}{\Delta y}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T26} + 0}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{T30} - \frac{\partial T}{\partial y} \Big|_{T25}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T26} + \frac{\partial T}{\partial y} \Big|_{T30} - \frac{\partial T}{\partial y} \Big|_{T25} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T26} + \frac{\partial T}{\partial y} \Big|_{T30} - \frac{\partial T}{\partial y} \Big|_{T25}$$

$$0 = \frac{T26 - T25}{\Delta x} + \frac{T30 - T25}{\Delta y} - \frac{T25 - T19}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T26 - T25 + T30 - T25 - (T25 - T19))$$

$$0 = T26 - T25 + T30 - T25 - (T25 - T19)$$

$$0 = T26 - T25 + T30 - T25 - T25 + T19$$

$$0 = T26 - T25 + T30 - T25 - T25 + T19$$

$$0 = T26 - T25 + T30 + T19$$

$$0 = T26 - T25 + T30 + T19$$

$$\frac{\partial^{2}T}{\partial x^{2}}\Big|_{T26} + \frac{\partial^{2}T}{\partial y^{2}}\Big|_{T26} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{T27} - \frac{\partial T}{\partial x}\Big|_{T26} + \frac{\frac{\partial T}{\partial y}\Big|_{T31} - \frac{\partial T}{\partial y}\Big|_{T26}}{\Delta x}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{T27} - \frac{\partial T}{\partial x}\Big|_{T26} + \frac{\partial T}{\partial y}\Big|_{T31} - \frac{\partial T}{\partial y}\Big|_{T26}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{T27} - \frac{\partial T}{\partial x}\Big|_{T26} + \frac{\partial T}{\partial y}\Big|_{T31} - \frac{\partial T}{\partial y}\Big|_{T26}$$

$$0 = \frac{T27 - T26}{\Delta x} - \frac{T26 - T25}{\Delta x} + \frac{T31 - T26}{\Delta y} - \frac{T26 - T20}{\Delta y}$$

$$0 = \frac{1}{\Delta}(T27 - T26 - (T26 - T25) + T31 - T26 - (T26 - T20))$$

$$0 = T27 - T26 - (T26 - T25) + T31 - T26 - (T26 - T20)$$

$$0 = T27 - T26 - T26 + T25 + T31 - T26 - T26 + T20$$

$$0 = T27 - T26 - T26 + T25 + T31 + T20$$

$$0 = T27 - T25 + T26 - T27 - T31$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T27} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T27} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T28} - \frac{\partial T}{\partial x} \bigg|_{T27} + \frac{\partial T}{\partial y} \bigg|_{T32} - \frac{\partial T}{\partial y} \bigg|_{T27}}{\Delta y} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T28} - \frac{\partial T}{\partial x} \bigg|_{T27} + \frac{\partial T}{\partial y} \bigg|_{T32} - \frac{\partial T}{\partial y} \bigg|_{T27} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T28} - \frac{\partial T}{\partial x} \bigg|_{T27} + \frac{\partial T}{\partial y} \bigg|_{T32} - \frac{\partial T}{\partial y} \bigg|_{T27} \\ 0 &= \frac{T28 - T27}{\Delta x} - \frac{T27 - T26}{\Delta x} + \frac{T32 - T27}{\Delta y} - \frac{T27 - T21}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T28 - T27 - (T27 - T26) + T32 - T27 - (T27 - T21)) \\ 0 &= T28 - T27 - (T27 - T26) + T32 - T27 - (T27 - T21) \\ 0 &= T28 - T27 - T27 + T26 + T32 - T27 - T27 + T21 \\ 0 &= T28 - 4T27 + T26 + T32 + T21 \\ 0 &= -T21 - T26 + 4T27 - T28 - T32 \end{aligned}$$

$$\frac{\partial^{2}T}{\partial x^{2}}\Big|_{T28} + \frac{\partial^{2}T}{\partial y^{2}}\Big|_{T28} = 0 = \frac{\frac{\partial T}{\partial x}\Big|_{T29} - \frac{\partial T}{\partial x}\Big|_{T28}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T33} - \frac{\partial T}{\partial y}\Big|_{T28}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{T29} - \frac{\partial T}{\partial x}\Big|_{T28} + \frac{\partial T}{\partial y}\Big|_{T33} - \frac{\partial T}{\partial y}\Big|_{T28}\right)$$

$$0 = \frac{\partial T}{\partial x}\Big|_{T29} - \frac{\partial T}{\partial x}\Big|_{T28} + \frac{\partial T}{\partial y}\Big|_{T33} - \frac{\partial T}{\partial y}\Big|_{T28}$$

$$0 = \frac{T29 - T28}{\Delta x} - \frac{T28 - T27}{\Delta x} + \frac{T33 - T28}{\Delta y} - \frac{T28 - T22}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T29 - T28 - (T28 - T27) + T33 - T28 - (T28 - T22))$$

$$0 = T29 - T28 - (T28 - T27) + T33 - T28 - (T28 - T22)$$

$$0 = T29 - T28 - T28 + T27 + T33 - T28 - T28 + T22$$

$$0 = T29 - 4T28 + T27 + T33 + T22$$

$$0 = T29 - 4T28 + T27 + T33 + T22$$

$$0 = -T22 - T27 + 4T28 - T29 - T33$$

Let a denote the ambient temperature.

For diagonal component, $n = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$||n|| = \sqrt{1^2 + 1^2} \\ = \sqrt{2}$$

$$\hat{n} = \frac{n}{\|n\|} = \frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{split} \nabla T \cdot \hat{n} &= \frac{20a - 20T_{29}}{3} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \left(\frac{\partial T}{\partial x} \Big|_{T29} + \frac{\partial T}{\partial y} \Big|_{T29} \right) \\ \frac{\partial T}{\partial x} \Big|_{T29} &+ \frac{\partial T}{\partial y} \Big|_{T29} = \frac{\sqrt{2}(20a - 20T_{29})}{3} \end{split}$$

$$\begin{split} \frac{\partial^2 T}{\partial x^2} \bigg|_{T29} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T29} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T29} - \frac{\partial T}{\partial x} \bigg|_{T28}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \bigg|_{T29} - \frac{\partial T}{\partial y} \bigg|_{T23}}{\Delta y} \\ &0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T29} - \frac{\partial T}{\partial x} \bigg|_{T28} + \frac{\partial T}{\partial y} \bigg|_{T29} - \frac{\partial T}{\partial y} \bigg|_{T23} \right) \\ &0 = \frac{\partial T}{\partial x} \bigg|_{T29} + \frac{\partial T}{\partial y} \bigg|_{T29} - \frac{\partial T}{\partial x} \bigg|_{T29} - \frac{\partial T}{\partial y} \bigg|_{T23} \\ &0 = \frac{\sqrt{2}(20a - 20T_{29})}{3} - \frac{(T29 - T28)}{\Delta x} - \frac{(T29 - T23)}{\Delta y} \\ &0 = \frac{20a\sqrt{2}}{3} - \frac{20T_{29}\sqrt{2}}{3} + \frac{-T29 + T28}{\Delta x} + \frac{-T29 + T23}{\Delta y} \\ &0 = \frac{20a\sqrt{2}}{300} - \frac{20T_{29}\sqrt{2}}{300} - T29 + T28 - T29 + T23 \\ &0 = \frac{a\sqrt{2}}{15} - \frac{T_{29}\sqrt{2}}{15} - T29 + T28 - T29 + T23 \\ &\frac{a\sqrt{2}}{15} = \left(\frac{30 + \sqrt{2}}{15} \right) T29 - T28 - T23 \end{split}$$

For a = 20

$$\frac{20\sqrt{2}}{15} = \left(\frac{30 + \sqrt{2}}{15}\right)T29 - T28 - T23$$
$$\frac{4\sqrt{2}}{3} = \left(\frac{30 + \sqrt{2}}{15}\right)T29 - T28 - T23$$

For
$$x = 0$$
, $\hat{n} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\nabla T \cdot \hat{n} = 0 = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\frac{\partial T}{\partial x} \Big|_{T30} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{T30} + \frac{\partial^2 T}{\partial y^2} \Big|_{T30} = 0 = \frac{\frac{\partial T}{\partial x} \Big|_{T31} - \frac{\partial T}{\partial x} \Big|_{T30} + \frac{\partial T}{\partial y} \Big|_{TAB} - \frac{\partial T}{\partial y} \Big|_{T30}}{\Delta x}$$

$$0 = \frac{\frac{\partial T}{\partial x} \Big|_{T31} + 0}{\Delta x} + \frac{\frac{\partial T}{\partial y} \Big|_{TAB} - \frac{\partial T}{\partial y} \Big|_{T30}}{\Delta y}$$

$$0 = \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \Big|_{T31} + \frac{\partial T}{\partial y} \Big|_{TAB} - \frac{\partial T}{\partial y} \Big|_{T30} \right)$$

$$0 = \frac{\partial T}{\partial x} \Big|_{T31} + \frac{\partial T}{\partial y} \Big|_{TAB} - \frac{\partial T}{\partial y} \Big|_{T30}$$

$$0 = \frac{T31 - T30}{\Delta x} + \frac{TAB - T30}{\Delta y} - \frac{T30 - T25}{\Delta y}$$

$$0 = \frac{1}{\Delta} (T31 - T30 + TAB - T30 - (T30 - T25))$$

$$0 = T31 - T30 + TAB - T30 - (T30 - T25)$$

$$0 = T31 - T30 + TAB - T30 - T30 + T25$$

$$0 = T31 - T30 + TAB - T30 - T30 + T25$$

$$0 = T31 - T30 + TAB - T30 - T30 + T25$$

$$0 = T31 - T30 + T4B - T30 - T30 + T25$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T31} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T31} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T32} - \frac{\partial T}{\partial x} \bigg|_{T31}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \bigg|_{TAB} - \frac{\partial T}{\partial y} \bigg|_{T31}}{\Delta y} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T32} - \frac{\partial T}{\partial x} \bigg|_{T31} + \frac{\partial T}{\partial y} \bigg|_{TAB} - \frac{\partial T}{\partial y} \bigg|_{T31} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T32} - \frac{\partial T}{\partial x} \bigg|_{T31} + \frac{\partial T}{\partial y} \bigg|_{TAB} - \frac{\partial T}{\partial y} \bigg|_{T31} \\ 0 &= \frac{T32 - T31}{\Delta x} - \frac{T31 - T30}{\Delta x} + \frac{TAB - T31}{\Delta y} - \frac{T31 - T26}{\Delta y} \end{aligned}$$

$$0 &= \frac{1}{\Delta} (T32 - T31 - (T31 - T30) + TAB - T31 - (T31 - T26))$$

$$0 &= T32 - T31 - (T31 - T30) + TAB - T31 - (T31 - T26)$$

$$0 &= T32 - T31 - T31 + T30 + TAB - T31 - T31 + T26$$

$$0 &= T32 - 4T31 + T30 + TAB + T26$$

$$70 &= -T26 - T30 + 4T31 - T32$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} \bigg|_{T32} + \frac{\partial^2 T}{\partial y^2} \bigg|_{T32} &= 0 = \frac{\frac{\partial T}{\partial x} \bigg|_{T33} - \frac{\partial T}{\partial x} \bigg|_{T32}}{\Delta x} + \frac{\frac{\partial T}{\partial y} \bigg|_{TAB} - \frac{\partial T}{\partial y} \bigg|_{T32}}{\Delta y} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x} \bigg|_{T33} - \frac{\partial T}{\partial x} \bigg|_{T32} + \frac{\partial T}{\partial y} \bigg|_{TAB} - \frac{\partial T}{\partial y} \bigg|_{T32} \right) \\ 0 &= \frac{\partial T}{\partial x} \bigg|_{T33} - \frac{\partial T}{\partial x} \bigg|_{T32} + \frac{\partial T}{\partial y} \bigg|_{TAB} - \frac{\partial T}{\partial y} \bigg|_{T32} \\ 0 &= \frac{T33 - T32}{\Delta x} - \frac{T32 - T31}{\Delta x} + \frac{TAB - T32}{\Delta y} - \frac{T32 - T27}{\Delta y} \\ 0 &= \frac{1}{\Delta} (T33 - T32 - (T32 - T31) + TAB - T32 - (T32 - T27)) \\ 0 &= T33 - T32 - (T32 - T31) + TAB - T32 - (T32 - T27) \\ 0 &= T33 - T32 - T32 + T31 + TAB - T32 - T32 + T27 \\ 0 &= T33 - 4T32 + T31 + TAB + T27 \end{aligned}$$

Let α denote the ambient temperature.

For
$$\mathbf{x} = 0$$
, $\hat{n} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$3 \times \nabla \mathbf{T} \cdot \hat{n} = 20a - 20T_{33}$$

$$\nabla T \cdot \hat{n} = \frac{20a - 20T_{33}}{3} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{\partial T}{\partial x} \Big|_{T33}$$

$$\frac{\partial T}{\partial x} \Big|_{T33} = \frac{20a - 20T_{33}}{3}$$

For diagonal component, $n = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$||n|| = \sqrt{1^2 + 1^2} \\ = \sqrt{2}$$

$$\hat{n} = \frac{n}{\|n\|} = \frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\nabla T \cdot \hat{n} = \frac{20a - 20T_{33}}{3} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \left(\frac{\partial T}{\partial x} \Big|_{T33} + \frac{\partial T}{\partial y} \Big|_{T33} \right)$$

$$\begin{split} \frac{\partial T}{\partial x}\Big|_{T33} &+ \frac{\partial T}{\partial y}\Big|_{T33} = \frac{\sqrt{2}(20a - 20T_{33})}{3} \\ \frac{\partial T}{\partial y}\Big|_{T33} &= \frac{\sqrt{2}(20a - 20T_{33})}{3} - \frac{\partial T}{\partial x}\Big|_{T33} \\ \frac{\partial T}{\partial y}\Big|_{T33} &= \frac{\sqrt{2}(20a - 20T_{33})}{3} - \frac{20a - 20T_{33}}{3} \\ \frac{\partial T}{\partial y}\Big|_{T33} &= \frac{20a - 20T_{33}}{3} (\sqrt{2} - 1) \end{split}$$

$$\begin{split} \frac{\partial^2 T}{\partial x^2}\Big|_{T33} + \frac{\partial^2 T}{\partial y^2}\Big|_{T33} &= 0 = \frac{\frac{\partial T}{\partial x}\Big|_{T33} - \frac{\partial T}{\partial x}\Big|_{T32}}{\Delta x} + \frac{\frac{\partial T}{\partial y}\Big|_{T33} - \frac{\partial T}{\partial y}\Big|_{T28}}{\Delta y} \\ 0 &= \frac{1}{\Delta} \left(\frac{\partial T}{\partial x}\Big|_{T33} - \frac{\partial T}{\partial x}\Big|_{T32} + \frac{\partial T}{\partial y}\Big|_{T33} - \frac{\partial T}{\partial y}\Big|_{T28}\right) \\ 0 &= \frac{\partial T}{\partial x}\Big|_{T33} - \frac{\partial T}{\partial x}\Big|_{T32} + \frac{\partial T}{\partial y}\Big|_{T33} - \frac{\partial T}{\partial y}\Big|_{T28} \end{split}$$

$$0 = \frac{20a - 20T_{33}}{3} - \frac{(T33 - T32)}{\Delta x} + \frac{20a - 20T_{33}}{3}(\sqrt{2} - 1) - \frac{(T33 - T28)}{\Delta y}$$

$$0 = \frac{20a - 20T_{33}}{3} + \frac{\sqrt{2}(20a - 20T_{33})}{3} - \frac{20a - 20T_{33}}{3} - \frac{(T33 - T32)}{\Delta x} - \frac{(T33 - T28)}{\Delta y}$$

$$0 = \frac{\sqrt{2}(20a - 20T_{33})}{3} - \frac{(T33 - T32)}{\Delta x} - \frac{(T33 - T28)}{\Delta y}$$

$$0 = \frac{20a\sqrt{2}}{3} - \frac{20T_{33}\sqrt{2}}{3} + \frac{-T33 + T32}{\Delta x} + \frac{-T33 + T28}{\Delta y}$$

$$0 = \frac{20a\sqrt{2}}{3} - \frac{20T_{33}\sqrt{2}}{3} + \frac{-T33 + T32 - T33 + T28}{0.01}$$

$$0 = \frac{20a\sqrt{2}}{300} - \frac{20T_{33}\sqrt{2}}{300} - T33 + T32 - T33 + T28$$

$$0 = \frac{a\sqrt{2}}{15} - \frac{T_{33}\sqrt{2}}{15} - T33 + T32 - T33 + T28$$

$$\frac{a\sqrt{2}}{15} = \left(\frac{30 + \sqrt{2}}{15}\right)T33 - T32 - T28$$

For a = 20

$$\frac{20\sqrt{2}}{15} = \left(\frac{30 + \sqrt{2}}{15}\right)T33 - T32 - T28$$
$$\frac{4\sqrt{2}}{3} = \left(\frac{30 + \sqrt{2}}{15}\right)T33 - T32 - T28$$

Summary of each node

NODE 1
$$0 = 2T1 - T2 - T7$$

NODE 2
$$0 = -T1 + 3T2 - T3 - T8$$

NODE 3
$$0 = -T2 + 3T3 - T4 - T9$$

NODE 4
$$0 = -T3 + 3T4 - T5 - T10$$

NODE 5
$$0 = -T4 + 3T5 - T6 - T11$$

NODE 6
$$40 = -T5 + 3T6 - T12$$

$$0 = -T1 + 3T7 - T8 - T13$$

$$0 = -T2 - T7 + 4T8 - T9 - T14$$

NODE 9

$$0 = -T3 - T8 + 4T9 - T10 - T15$$

NODE 10

$$0 = -T4 - T9 + 4T10 - T11 - T16$$

NODE 11

$$0 = -T5 - T10 + 4T11 - T12 - T17$$

NODE 12

$$40 = -T6 - T11 + 4T12 - T18$$

NODE 13

$$0 = -T7 + 3T13 - T14 - T19$$

NODE 14

$$0 = -T8 - T13 + 4T14 - T15 - T20$$

NODE 15

$$0 = -T9 - T14 + 4T15 - T16 - T21$$

NODE 16

$$0 = -T10 - T15 + 4T16 - T17 - T22$$

NODE 17

$$0 = -T11 - T16 + 4T17 - T18 - T23$$

NODE 18

$$40 = -T12 - T17 + 4T18 - T24$$

NODE 19

$$0 = -T13 + 3T19 - T20 - T25$$

NODE 20

$$0 = -T14 - T19 + 4T20 - T21 - T26$$

NODE 21

$$0 = -T15 - T20 + 4T21 - T22 - T27$$

$$0 = -T16 - T21 + 4T22 - T23 - T28$$

$$0 = -T17 - T22 + 4T23 - T24 - T29$$

$$\frac{4\sqrt{2}}{3} = -T18 - T23 + \left(\frac{30 + \sqrt{2}}{15}\right)T24$$

NODE 25

$$0 = -T19 + 3T25 - T26 - T30$$

NODE 26

$$0 = -T20 - T25 + 4T26 - T27 - T31$$

NODE 27

$$0 = -T21 - T26 + 4T27 - T28 - T32$$

NODE 28

$$0 = -T22 - T27 + 4T28 - T29 - T33$$

NODE 29

$$\frac{4\sqrt{2}}{3} = -T23 - T28 + \left(\frac{30 + \sqrt{2}}{15}\right)T29$$

NODE 30

$$70 = -T25 + 3T30 - T31$$

NODE 31

$$70 = -T26 - T30 + 4T31 - T32$$

NODE 32

$$70 = -T27 - T31 + 4T32 - T33$$

$$\frac{4\sqrt{2}}{3} = -T28 - T32 + \left(\frac{30 + \sqrt{2}}{15}\right)T33$$