9.48

$g \circ f : (0,1) \to \mathbb{R}$

$$(g\circ f)(x) = \frac{4x-1}{2\sqrt{x-x^2}}$$

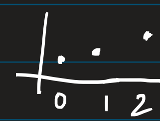
$f : (0,1) \to \mathbb{R}$, $\quad f(x) = 2x-1$

What's g?

$\{0,1\}$ 2 elements

$(0,1)$

$[0,1]$



domain

$[0,2]$ ×

$\{0,1,2\}$ ✓

$$g(f(x)) = \frac{4x-1}{2\sqrt{x-x^2}}$$

$$\parallel$$

$$g(2x-1) = \frac{4x-1}{2\sqrt{x-x^2}} \;//$$

$g(y)$ for any $y \in \operatorname{dom} g$

Replace x with something such that

$$2(\text{something}) - 1 = y \Leftarrow \text{something} = \frac{y+1}{2}$$

$$g(y) = g(2(\text{something})-1) = \frac{4\cdot\text{something} - 1}{2\sqrt{\text{somthing} - \text{somthing}^2}}$$

$$g(y) = g\left(2\left(\frac{y+1}{2}\right)-1\right) = \frac{4\left(\frac{y+1}{2}\right)-1}{2\sqrt{\frac{y+1}{2}\left(1-\frac{y+1}{2}\right)}}$$

$$= \text{simplify} \;//$$

9.42 (b) $A \xrightarrow{f} B \xrightarrow[\text{onto}]{g} C$

$g \circ f : A \to C$ is onto if g & f are onto

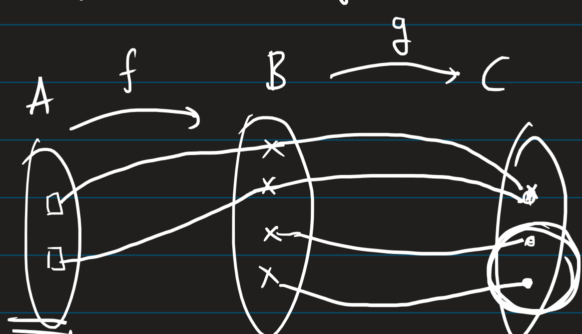
Let $c \in C$, to show $\exists a \in A$ st

$(g \circ f)(a) = c$ i.e. $g(f(a)) = c$

Let $c \in C$, since g is onto, $\exists\, b \in B$ st.

$g(b) = c$ we want f to be onto

False f is not onto g is onto and $g \circ f$ is not onto.



$A = \{1, 2\}$, $B = \{4, 5, 6, 7\}$ $C = \{8, 9, 10\}$

$g(4) = g(5) = 8$

$f(1) = 4$

$g(6) = 9$

$f(2) = 5$

$g(7) = 10$

① Injective, Surjective

$$f: A \longrightarrow B$$

* f is injective if whenever $f(x) = f(y)$, then $x = y$ ✓

  if whenever $x \neq y$, then $f(x) \neq f(y)$ ✓

  " distinct elements get mapped to distinct element "

* $$f: A \longrightarrow B$$

  range of f – $f(A) = \{ f(a) \mid a \in A \} \subseteq B$ ✓

f is <u>surjective</u> if $\underline{f(A) = B}$ i.e. whenever $\underline{B \subseteq f(A)}$

$$\boxed{\begin{array}{l} b \in B, \text{ then } b \in f(A) \text{ i.e. for some } a \in A \\ \qquad\qquad b = f(A) \end{array}}$$

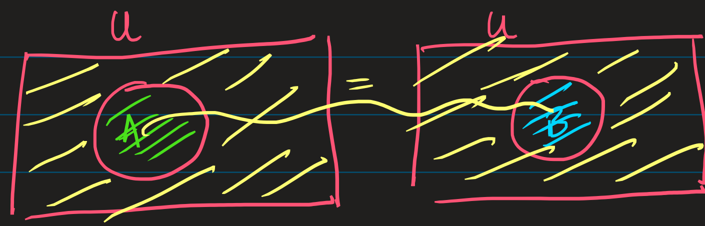
② U , $A, B \subseteq U$

Recall the notion of complement of a set

$$A^c = \overline{A} = U \setminus A = U - A = \{ x \in U \mid x \notin A \}$$

So, suppose $\overline{A} = \overline{B}$, is $\underline{A = B}$? //

$A \subseteq B$, $B \subseteq A$.



③ A & B be finite sets

How many functions are their from $A \longrightarrow B$

$B^A := \{\text{all functions from } A \longrightarrow B\}$

What is $\# B^A = |B^A| = ?$

Take any function $f: A \to B$,

$|A| = n$ & $A = \{a_1, \dots, a_n\}$

What is $f(a_1), \dots\dots, f(a_n)$?

if $|B| = m$

m choices, m, ..., m

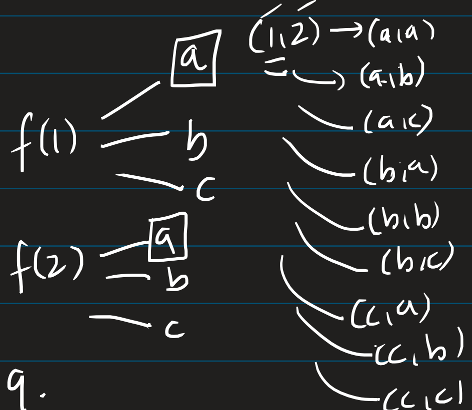
in total m^n choices

$= |B|^{|A|}$

$|B^A| = |B|^{|A|}$

$A = 2$, $B = 3$.

$A = \{1, 2\}$ $B = \{a, b, c\}$



$|B^A| = |B|^{|A|} = 3^2 = 9.$

Remarks

$\rightarrow$ If $|B| = 2$, $B = \{0,1\}$

$$B^A = 2^A$$

$$|B^A| = 2^{|A|} = |P(A)|$$

Is there a bijection from the set of functions $\underline{A} \longrightarrow \{0,1\}$ and the power set of A? Yes!

$$\underline{F} : P(A) \longrightarrow 2^A$$

$$S \subseteq A \longmapsto F(S) := f_S$$

where
$$f_S(a) = \begin{cases} 1 & a \in S \\ 0 & a \notin S \end{cases}$$

"indicator function of S"

$\rightarrow$

$f : \underline{X} \longrightarrow \underline{Y}$ that is may or may not be surjective or injective

Question : Can we make f surjective by "editing" its target?

Answer : $f : X \longrightarrow f(X)$ is surjective!

Question : Can we make f injective by "editing" its source ?

Answer : $f : X \longrightarrow Y$ a function

define a relation R on X as follows

$$x R y \quad \text{iff} \quad f(x) = f(y)$$

Fact: R is an equivalence relation.

Take the set of equivalence classes wrt R

$= X/R$ ("quotient set")

$$\tilde{f} : X/R \longrightarrow Y$$

$$\tilde{f}([x]) := f(x)$$

Claim: $\tilde{f}$ is injective

Consider $[x], [y] \in X/R$ st

$$\tilde{f}([x]) = \tilde{f}([y])$$

$$\Rightarrow \quad f(x) = f(y)$$

$$\Rightarrow \quad x R y$$

$$\Rightarrow \quad [x] = [y]$$

Hence $\tilde{f}$ is injective.

stitching our previous remarks.

$f: X \to Y$ and R as above

X/R and $f(X)$ are bijective.

$\tilde{f}: X/R \longrightarrow Y$ injective

$\tilde{f}: X/R \longrightarrow \tilde{f}(X)$ " & surjective

$= f(X)$

so $\tilde{f}$ is a bijection b/w X/R & $f(X)$.

$g = h$ $\quad \underbrace{g(x) = h(x) \qquad g(f(y))}= h(f(y))$

f is one-one ✓

$g \circ f = h \circ f$ but $g \neq h$ (equality happens if f was $\boxed{\text{surjective}}$)

$f: \mathbb{Z} \longrightarrow \mathbb{Z}$, $x \longmapsto 2x$ is one-one.

$\underline{g(2x)} = g \circ f(x) = h \circ f(x) = \underline{h(2x)}$

$$g(n) = \begin{cases} 1 & n \text{ is even} \\ 0 & \text{o/w} \end{cases} \qquad h(n) = \begin{cases} 1 & n \text{ is even} \\ -1 & n \text{ is odd} \end{cases}$$

$g \neq h$ since, e.g. $g(3) = 0$ & $h(3) = -1$.