Negate

[If m is even and n is odd, then $m+n$ is odd.

1. Identify statements and the logical operator involved

① $\Rightarrow$ R

② $\wedge$ ⑤ $m+n$ is odd

③ P: m is even

④ Q: n is odd

$$P \wedge Q \Rightarrow R$$

② Negate this statement //

$$\neg(P \wedge Q \Rightarrow R) \equiv \left[\underbrace{(P \wedge Q) \wedge (\neg R)}\right]$$

$$\left[\neg(P \Rightarrow Q) \equiv P \wedge \neg Q\right]$$

③ Replace w/ english sentences

m is odd and n is even, and $m+n$ is $\underbrace{\text{(not odd)}}$
even.

$$\underbrace{\neg(A \Rightarrow B) \equiv (A) \wedge \neg B)}$$

$A \sim P \wedge Q$ $\quad$ $(\underbrace{P \Rightarrow Q})$ $\equiv$ $\underbrace{\neg Q \Rightarrow \neg P}$

$B \sim R$

$(P \wedge Q) \wedge (\sim R)$ $\quad$ contrapositive

$\neg(P \Rightarrow Q) \equiv$

n^2

$n \equiv 1 \bmod 3$

$n^2 \equiv 1^2 \bmod 3$

$\underbrace{n^2 \equiv 1 \equiv n \bmod 3}$

$\begin{pmatrix} a = b \\ c = b \end{pmatrix}$ transitive relation of equality

$\boxed{c = b = a}$

n $n = 1$ in mod 3 world

$n^2 = 1$ in mod 3

$n^2 = 1 = n$ in mod 3 world.

1. Logical equivalence (___) [2 statements are l.e. if they have the same truth table]
2. Proofs

↳ Direct proofs
↳ Proof by contrapositive
↳ Proof by cases

PAUSE

1. • $\Rightarrow$ (false ; true)
   T, T
 • $\wedge$ (true iff only when both are true)
 • $\vee$ (true only when atleast on is true)

Show

$((P \wedge Q) \Rightarrow R)$

$\equiv \quad (P \wedge \sim R) \Rightarrow (\sim Q)$

$\left[A \Rightarrow B \equiv \underline{\neg A} \vee \underline{B} \right]$

④ Definitions...

$(P \wedge Q) \Rightarrow R \quad \equiv \quad \neg(P \wedge Q) \vee R$

De-Morgan's $\quad \equiv \quad (\neg P \vee \neg Q) \vee R$

associativity $((A \vee B) \vee C \equiv A \vee (B \vee C))$ $\quad \equiv \quad \neg P \vee (\neg Q \vee R)$

commutativity $(A \vee B = B \vee A)$ $\quad \equiv \quad \neg P \vee (R \vee \neg Q)$

associativity $\quad \equiv \quad \neg (P \wedge \neg R) \vee \neg Q$

$\equiv (P \wedge \neg R) \Rightarrow (\neg Q)$ ☐

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1 Direct Proof

2. Proof by Contrapositive

3. Proof by cases

$n \in \mathbb{Z}_{\geq 0} = \{x \in \mathbb{Z} \mid x \geq 0\}$

[If $n > 0$, then $n^2 > 0$]

* Direct proof $\left[\begin{array}{l} n \cdot n > 0 \\ n^2 > 0 \end{array} \right]$ [inequalities don't change when positive numbers are involved]

tautology.

* $\underset{n>0}{P} \Rightarrow \underset{n^2>0}{Q} \quad \equiv \quad \underset{n^2 \leq 0}{\neg Q} \Rightarrow \underset{n \leq 0}{\neg P} \quad \equiv \quad \boxed{n^2 = 0 \Rightarrow n = 0}$

$n \in \mathbb{N}$ Suppose $n \le 5$

"what can n be?"

$n \in \{1, 2, 3, 4, 5\}$

$\mathbb{N}$ in this class, don't include 0.

Suppose $ab = 6$

[$\underline{\underline{\mathbb{N}}}$, $\mathbb{Z}_{>0}$ positive integers
$\mathbb{Z}_{\geq 0}$ non-negative integers]

$ab = 6$ $\qquad$ $\underline{a, b \in \mathbb{Z}_{>0}}$

"What can a, b be?"

→ 2, 3 and 1, 6
3, 2 and 6, 1

$a + b = 6$

"what can a, b be"

→ 1, 5 $\quad$ 5, 1
2, 4 $\quad$ 4, 2
3, 3

$6 = 2 \cdot 3 = 1 \cdot 2 \cdot 3$ ...

Given a statement like this, this is your "scratchwork" to figure out the cases.

Case I: $\boxed{a = 3, b = 2}$ $\qquad$ Case II: $\boxed{a = 1, b = 6}$.

$xy = 6$, $x, y \in \cancel{\mathbb{R}}\ \mathbb{Z}_{>0}$ 2 options

1,6 2,3

"what can x, y be?"

$\{x \in \mathbb{R},\ x \neq 0\}$ $\{y = 6/x\}$

Prove $n \in \mathbb{Z}$, then one element of the set

$$\{n-3, n-2, n-1, n, n+1, n+2, n+3\}$$

is divisible by 7.

$\boxed{n}$

$\boxed{n+3}$

$\{(n+3)-3, (n+3)-2$

Proof: (mod 7)

Consider n mod 7

Case I. $n \equiv 1 \bmod 7$

$n-1 \equiv 0 \bmod 7$

Therefore $7 \mid n-1$

($a \mid b$ a divides b)

Case II $n \equiv 2 \bmod 7$

Case III $n \equiv 3 \bmod 7$

Case IV $n \equiv 4 \bmod 7$

$\boxed{n-4 \equiv 0 \bmod 7}$ agreed?

$\underbrace{n-4+7}$ $\equiv 7 \bmod 7$ $\equiv 0 \bmod 7$

$n+3 \equiv 0 \bmod 7.$

$n-5 \equiv n+2$

$-5 \equiv 2 \bmod 7$

$-6 \equiv 1 \bmod 7.$

Direct Proof Statement proof

"Strategy / Idea of a proof" Scratch

vs

written proof

$P \Rightarrow Q$

- Indicate method of proof $\neg Q$ ⟶
- ~~Yours~~ A proof is not for you, a proof written to be read
- → use words, not symbols. $\vee$ $\wedge$ "or" "and" ($\Rightarrow$) implies logical operator Thing
-
-

$\forall$ for all

$\exists$ there exists

$\neg(\exists) \equiv \forall$

for all

$\boxed{\text{for all } x \text{ if } P\text{, then } Q}$

contrapositive

$\forall$ ~~there exist~~ x, if $\neg Q$, then $\neg P$

$\forall x,\ P \Rightarrow Q \equiv$

$\forall x,\ \neg Q \Rightarrow \neg P$

Contrapositive

$$P \implies Q \equiv \underbrace{\neg Q \implies \neg P}$$

both contrapositive

$\underbrace{\forall x \in \mathbb{R}}$, $\underline{\underline{\text{negation}}}$

[illegible]

$$\underline{\underline{\forall x}},\ P \implies Q \qquad \equiv$$

$$\underline{\underline{\forall x}},\ \neg Q \implies \neg P$$