Induction hypothesis

$$\frac{a(1-r^{k+1})}{1-r}$$

$$a + ar + \cdots + ar^{k-1} + ar^k = \frac{a(1-r^k)}{1-r} + ar^k$$

$$= \frac{a(1-r^k) + ar^k(1-r)}{1-r}$$

$$= \left(\frac{a - \cancel{ar^k} + \cancel{ar^k} - ar^{k+1}}{1-r}\right)$$

Ind. $7 \mid 3^{4(k+1)+1} - 5^{2(k+1)-1} = 3^{4k+5} - 5^{2k+1}$ //

$$3^{4k+1} - 5^{2k-1} = 7l$$

here

$$3^4 \cdot 3^{\boxed{4k+1}} - 5^2 \cdot 5^{\boxed{2k-1}}$$

$$\underbrace{3^{4k+1}} = \underbrace{7l + 5^{2k-1}}$$

$$3^4(7l + 5^{2k-1}) - 5^2 \, 5^{2k-1}$$

81
-25
$56 = 7 \times 8$

$$= 7(3^4 l) + (\overset{81-25}{3^4 - 5^2})\, 5^{2k-1}$$

divisibly by 7

$2^n > n^3$ for $n \geq 10$

$2^k > k^3$ //

To show $2^{k+1} > (k+1)^3$

$2^{k+1} = 2 \cdot 2^k > 2k^3$

Now To show -

$2k^3 > (k+1)^3$

$= k^3 + 3k + 3k^2 + 1$

To show $k^3 > 3k + 3k^2 + 1$ ||

To show $\boxed{k^3 - 3k - 3k^2 - 1 > 0}$ $+ 6k^2$

$k^3 - 3k + 3k^2 - 1 > 6k^2$

$(k-1)^3 > 6k^2$

* $k \Rightarrow k+1$ ✓ $\qquad \frac{a_1}{a_1} \geq 1$

How does k imply $k+1$?

Let's look at an example $(1 \Rightarrow 2)$

$(2 \Rightarrow 3)$

Assume $P(2)$ to be true

$$\boxed{(a_1+a_2)\left(\frac{1}{a_1}+\frac{1}{a_2}\right)} \geq 2^2 = 4 \qquad T$$

use this to prove

$$P(3): \quad \underbrace{(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)}_{x} \geq 3^2 = 9$$

$(x+a_3)\left(y+\frac{1}{a_3}\right)$, xy

$$(x+a_3)\left(y+\frac{1}{a_3}\right) = xy + \frac{x}{a_3} + a_3 y + \frac{a_3}{a_3}$$

$$= xy + \frac{x}{a_3} + a_3 y + 1$$

$$\geq 4 + 1 + \underbrace{\frac{x}{a_3} + y a_3}_{\geq 4}$$

To show $\frac{x}{a_3} + y a_3 = \frac{a_1}{a_3} + \frac{a_2}{a_3} + \frac{a_3}{a_1} + \frac{a_2}{a_1} \geq 4$.

$$\boxed{\frac{a_1}{a_3} + \frac{a_3}{a_1} \geq 2} \quad \& \quad \boxed{\frac{a_2}{a_3} + \frac{a_3}{a_2} \geq 2}$$

We're done if we show this.

Section : 2-variable AM-GM

$$\boxed{\frac{x+y}{2} \geq \sqrt{xy}} \quad \text{for +ve } x, y$$

to $x = \frac{a_3}{a_1}$, $y = \frac{a_1}{a_3}$ & to $x = \frac{a_3}{a_2}$ & $y = \frac{a_2}{a_3}$

$k \Rightarrow k+1$

$$(\underbrace{a_1 + a_2 + \cdots + a_k}_{X} + a_{k+1}) \left(\underbrace{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_k}}_{Y} + \frac{1}{a_{k+1}}\right)$$

$$\frac{X}{a_{k+1}} + Y a_{k+1} = \underbrace{\frac{a_1}{a_{k+1}} + \frac{a_{k+1}}{a_1}}_{2} + \underbrace{\frac{a_2}{a_{k+1}} + \frac{a_{k+1}}{a_2}}_{2} + \cdots + \underbrace{\frac{a_k}{a_{k+1}} + \frac{a_{k+1}}{a_k}}_{2}$$

$$2 + 2 + \quad 2$$

$$= 2k$$

$$\geq \underbrace{k^2}_{P(k)} + \underbrace{1}_{a_{k+1}/a_{k+1}} + 2k = (k+1)^2 \quad \blacksquare$$

$n | 3$ then $2 | F_n$ assume strong induction.

$$F_n = F_{n-2} + F_{n-1}$$

$3 | n$

$$= F_{n-2} + F_{n-2} + F_{n-3},$$

$F_k = F_{k-1} + F_{k-2}$

$k = n-1$

$$= 2F_{n-2} + F_{n-3}$$

if $3 | n$, then $3 | n-3$, by ind hyp

$\rightarrow$ F_n is even $\qquad 2 | F_{n-3}$

F_n is even then $3 | n$

So, let's prove $3 \nmid n$ then F_n is odd

On the numbers $\{1, 2, 4, 5, 7, 8, 10, 11, \ldots\}$

assume $3 \nmid k$, then F_k is odd for $k < n$.

$$F_n = F_{n-1} + F_{n-2}$$

$$= 2F_{n-2} + F_{n-3}$$

since $3 \nmid n$, therefore $3 \nmid n-3 < n$ so by ind hyp

F_{n-3} is odd

F_n is odd as a sum of even + odd number.

$a_1 = \underset{}{1}^{=2^0}, \quad a_2 = 2 = 2^1$

$$a_n = a_{n-1} + 2a_{n-2}$$

$n=3, \quad a_3 = a_2 + 2a_1 = 2+2(1) = 4 = 2^2$

$n=4, \quad a_4 = a_3 + 2a_2 = 4+2(2) = 8 = 2^3$

$n=5, \quad a_5 = a_4 + 2a_3 = 8+2(4) = 16 = 2^4$

$n=6, \quad a_6 = a_5 + 2a_4 = 16+2(16) = 32 = 2^5$

Conjecture: $\boxed{a_n = 2^{n-1}}$ $\quad a_n = a_{n-1} + 2a_{n-2}$

$P(1)$, $\quad P(k) \Rightarrow P(k+1)$