Midterm (4)

$P(x) \Rightarrow Q(x)$ to be true for all $x \in T$

① $P(x)$ is false for all $x \in T$

② $P(x)$ and $Q(x)$ is true for all $x \in T$

(1) $P \wedge Q$ are false for all $x \in T$

- atleast one of them is false //
- $P(x)$ or $Q(x)$
  $\hookrightarrow F$ $\hookrightarrow F$

P – F T
– T Q is T T

(2) $Q(x)$ is true for all $x \in T$.

(3) $P(x)$ is false

T T

(4) $P(x)$ is true $\propto$ $Q(x)$ is true T
$Q(x)$ is false F

F

(☆) $(\Rightarrow)$ T F for P
= T & T for P & Q
$\wedge$ T both are T
=
$\vee$ T at least is T

Quiz Topics

① Logical Equivalence

② Proof — Direct / Contrapositive / Cases. (☆)

When a particular method should be used?

Judgement Call. Do problems to develop of judgement.

Prove if $n > 0$, then $n^2 > 0$

Proof: Consider an arbitrary $n > 0$

$$n^2 = n \cdot n > n \cdot 0 = 0$$

(Fact about inequality – multiplying by a positive number doesn't change the sign)

smaller $<$ $\leq$

Contrapositive: if $n^2 \leq 0$, then $n \leq 0$

(can $n^2 < 0$ happen?)

$n^2 = 0$, then $n = 0$ (Tautology)

* $\mathbb{N}$ first natural number?

1

0

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

$= \cancel{\{0, 1, 2, \ldots\}}$

$\mathbb{N} := \mathbb{Z}_{>0}$ positive integers

$\mathbb{Z}_{\geq 0}$ non-negative integers.

* P: x is odd

Q: $3x - 1$ is even

R: $5x - 2$ is odd

$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

$P \Rightarrow R \equiv \neg R \Rightarrow \neg P$

$\neg R \longrightarrow \neg P$

$\neg Q \longrightarrow \neg P$

$P \Rightarrow Q$

$\neg R \Rightarrow \neg P$

$P \Rightarrow Q$

"Transitivity" ends of one thing to begin

Q: $3x-1$ is odd

P: about x

R: $5x-2$ is even

$Q \quad \boxed{*} \quad R$: x is even

$R \quad \boxed{*} \quad Q$

$3x-1$ is odd

$3x$ is even

$\boxed{x \text{ is even}}$

$\neg Q \rightarrow * \rightarrow \neg R$

$\neg R \rightarrow * \rightarrow \neg Q$

$\mathbb{N} = \mathbb{Z}_{>0}$

* suppose $n \le 5$

what can n be?

$n \in \{1, 2, 3, 4, 5\}$ ✓

* $\boxed{ab = 6}$ // $\boxed{c + d = 6}$ //

what can a, b, c, d be?

a	b
1	6
2	3
6	1
3	2

c	d
1	5
3	3
5	1
4	2
2	4
3	3

a	b
1	6
2	3

c	d
1	5
2	4
3	3

$a = 1, \quad b = -1$

-1

1, 1

* $a, b \in \mathbb{Z}$, $ab = 1$

what can a, b be?

a	b
1	1
-1	-1

HW

$ab = 1 \Rightarrow a, b \in \{\pm 1\}$

Let's assume that $a, b \notin \{\pm 1\}$ i.e. $a, b \in \mathbb{Z}$ st $|a| > 1$

if $|a| > 1$ and $ab = 1$
then $b = 1/|a| \notin \mathbb{Z}$

(Proof by contradiction)

$a = 2$, $b = 1/2$

* $a \equiv b \mod n$ iff $n \mid a-b$ ~ $a = b + nl$, $l \in \mathbb{Z}$

a special kind of equality

with one rule $\boxed{kn \equiv 0 \mod n}$

$4 \mod 3 \equiv 3 + 1 \mod 3$

$\equiv 0 + 1 \mod 3$

$\equiv 1 \mod 3$

$\mod n \rightarrow 0, \ldots, n-1$ ✓

* Writing Proofs

↳ Indicate method of proof (how you're "directly" ~ ~ inducti...)

$P \Rightarrow Q$.
Assume $\neg Q$ ...

↳ Written proof is not for you or about you
it's written for the reader

The audience are your peers

✓ Solving the problem || and $\boxed{\text{writing proof}}$

↳ Don't abbreviate words

↳ You can use "symbols" — $\mathbb{Z}$ // — $=$ — $a \in \mathbb{R}$ fine

↳ NOT write symbolically are logical symbols

$\wedge$ and, $\vee$ or, $\Rightarrow$ implies, $\exists$ there exists, $\forall$ for all.

$A = \{$

$A \cap B = \{x \in A \text{ and } x \in B\}$

logical symbols — develop math thinking

— NOT in math writing