Midterm, Assignment 4, Quiz

① $\longrightarrow$ listed? NOT

$\boxed{\{\{\phi\}\}} \in \{\phi, \{\phi\}\}$?

membership

F, F F

E L F

Reasoning : NOT an element in the set

② $\phi \subset$ ~~$\{\phi, \{\phi\}\}$~~ ? True

F , F , T , T , T

E L F E-L D

empty set is ALWAYS a subset of any set

- Proof by contradiction.

2.1 x, y, z, w

a, b, c, d.

$m = a^2 + b^2$

$N^2 m = (Na)^2 + (Nb)^2$

$$m = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$$

even, odd, even+1, even/odd

$$= \left(p_1^{a_1} \cdots p_1^{a_k}\right) (p_2 \, p_4 \cdots)$$

even, even

$$= \left(\underbrace{p_1^{a_1/2} \cdots}_{N} p_k^{a_k/2}\right)^2 k.$$

$$3^3 \cdot 5^2 \cdot 7^5 \cdot 11^{27} = 3^2 3 \cdot 5^2 \cdot 7^4 \cdot 7 \cdot 11^{26} \cdot 11$$

$$= (3^2 5^2 7^4 11^{26})(3 \cdot 7 \cdot 11)$$

$$= (3 \cdot 5 \, 7^2 \, 11^{13})^2 \, (3 \cdot 7 \cdot 11)$$

$N^2 \quad . \quad m$

4.25

If $x^2 - 4x = y^2 - 4y$ and $x \neq y$, then $x+y = 4$.

Proof. Let's prove directly, since $x^2 - 4x = y^2 - 4y$,

therefore $x(x-4) = y(y-4)$

assuming $y \neq 0$ | $\boxed{\frac{x}{y} = \frac{y-4}{x-4}}$ may not work, could be undefined PAUSE

$y \to 0$ $x = 4?$ why can this be a problem?

can't divide by 0

Proof $x^2 - 4x = y^2 - 4y$

subtract by y^2 and add $4x$ on both sides

$x \neq y$ means $x-y \neq 0$ // so! you can divide by $x-y$

$x^2 - y^2 = 4x - 4y$ good?

$(x+y)\cancel{(x-y)} = 4\cancel{(x-y)}$ [good thing to remember].

i.e. $\frac{x-y}{x-y} = 1$ (ONLY true when $x-y \neq 0$)

so $x+y = 4$.

NO "proof by picture"

49 $A = (A-B) \cup (A \cap B)$.

$A \subseteq (A-B) \cup (A \cap B)$ & $(A-B) \cup (A \cap B) \subseteq A$

2-ways

set operations

$a \cdot (b+c) = a \cdot b + a \cdot c$

(1) $(A-B) \cup (A \cap B) = (A \cap \bar{B}) \cup (A \cap B)$

$= A \cap (\bar{B} \cup B) = A \cap U = A$

(2) $A \subseteq \underbrace{(A-B) \cup (A \cap B)}$

and $\boxed{x \in A}$, then to show $\underline{x \in A-B}$ or $\underline{x \in A \cap B}$

only 2 choices

n — ~~odd~~ / $\boxed{\text{even}}$

$x \in A$ and $x \notin A \cap B$ ~~$x \notin A$~~ or $x \notin B$

then $x \in A - B$ ✓

$A \subseteq (A-B) \cup (A \cap B)$

$x \in (A-B) \cup (A \cap B)$ $x \in A - B$ or $x \in A \cap B$

$x \in A$ (and $x \notin B$) $x \in A$ (and $x \in B$)

Therefore $x \in A$

\equiv

Think $\equiv$ is $=$ in mod world

* $a = a$ equality
* if $a = b$, then $b = a$
* if $a = b$ and $b = c$, then $a = c$

* $a \equiv a \bmod n$ (why?)

n

$3 \equiv 1 \bmod 2$ $3 = \frac{1}{2}$

($a \equiv b \bmod n$ → $a = \frac{b}{n}$ //

→ $a = kn + b$ //

→ $a-b$ is divisible by n //

$n \mid a-b$ for some $k \in \mathbb{Z}$

$a - b = kn$

$$\boxed{\begin{array}{c} a \equiv b \bmod n \\ \text{means } n \text{ divides } a-b \end{array}}$$

does n divide 0? Yes $0 = 0 \cdot n$

$0 = a - a$

$a \equiv a \bmod n$...

* If $a \equiv b \bmod n$, then $b \equiv a \bmod n$

How? $a = k \cdot n + b$ i.e. $a - b = k \cdot n$

$b - a = (-k)n$ $n \mid b - a$

if $a = b$ & $b = c$, then $a = c$

(*) if $a \equiv b \mod n$, $b \equiv c \mod n$

Then $a \equiv c \mod n$.

Proof: $a - b = k \cdot n$

$b - c = l \cdot n$

+

Direct proof

$$a - c = (k + l) \cdot n$$

$n \mid a - c$ i.e. $a \equiv c \mod n$

* Inequalities

AM & GM
arith mean, geom mean

$$\underbrace{AM \geq GM}\geq HM$$

→ harmonic mean

(2) $$\underbrace{\frac{a+b}{2} \geq \sqrt{ab}}$$

How to prove this? ✓

To show $\boxed{a + b - 2\sqrt{ab} \geq 0}$ MAIN

SCRATCH

$$a + b - 2\sqrt{a} \cdot \sqrt{b} \geq$$

$$(\sqrt{a} - \sqrt{b})^2 \geq 0$$ ✓

$(x-y)^2$

$x^2 + y^2 - 2xy$

Proof: Since $(\sqrt{a} - \sqrt{b})^2 \geq 0$, Thus $\frac{a+b}{2} \geq \sqrt{ab}$ //

Therefore $(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{a} \cdot \sqrt{b} \geq 0$; hence $a + b - 2\sqrt{ab} \geq 0$