Section 01C (01/11)

25 November 2020 14:06

$\underbrace{\text{power set}}$, cardinality, cartesian products, union/intersection

↳ indexed

A, set

↳ finite

$P(A) = \{ B \mid B \subseteq A \}$

$|A|$, $\underbrace{|P(A)| = 2^{|A|}}$

$(1+x)^n$

$= \sum_{r=0}^{n} \binom{n}{r} x^r$

↳ $\frac{n!}{r!(n-r)!}$

↳ no. of ways you choose r things from n things without repetition

$|A| = n$

0 elements — ϕ → 1

1 element — $\binom{n}{1}$

2 elements — $\binom{n}{2}$

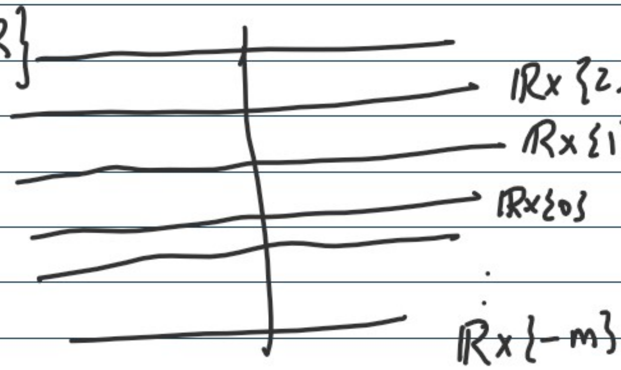
⋮

n elements — $\binom{n}{n} = 1 \to A$

$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$

$= (1+1)^n = 2^n$

(2) $\mathbb{R}^2 = \{ (x,y) \mid x, y \in \mathbb{R} \}$



$$\mathbb{R}\times\mathbb{Z} = \{(x,n) \mid x\in\mathbb{R},\ n\in\mathbb{Z}\}$$

$$\bigcup_{n\in\mathbb{Z}} \mathbb{R}\times\{n\} \quad ''$$

~~B = {x bottles)~~

$\mathcal{F}$ = set of all flowers

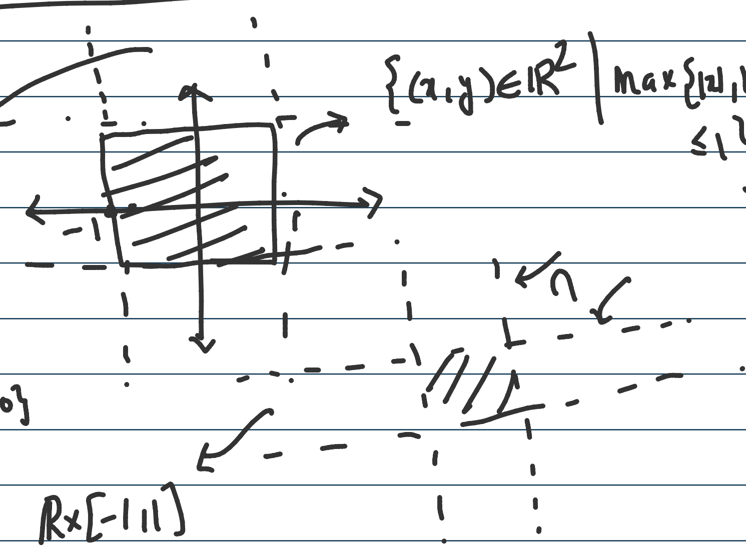
C = set of all colours

thorny red rose = (red, rose) $\in C\times\mathcal{F}$

rose red $\quad$ $\mathcal{F}\times C$ $\stackrel{?}{=}$

$\hookrightarrow A$

$A\times C\times\mathcal{F}$



$[-1,1]\times\mathbb{R}$

$\{(x,y)\in\mathbb{R}^2 \mid \max\{|x|,|y|\} \le 1\}$

$[-1,1]\times\{0\}$

$\mathbb{R}\times[-1,1]$

$$\mathbb{R}\times[-1,1] \cap [-1,1]\times\mathbb{R} \neq \phi$$

$$= [-1,1] \times [-1,1]$$

$A = B \qquad A\subseteq B \ \&\ B\subseteq A.$

$$[-1,1]\times[-1,1] \subseteq \mathbb{R}\times[-1,1]$$
$$\cap\ [-1,1]\times\mathbb{R}$$

$$(x,y)\in \mathbb{R}\times[-1,1]\ \cap\ [-1,1]\times\mathbb{R}$$

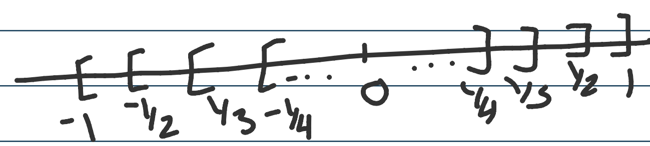
$$(x,y)\in \mathbb{R}\times[-1,1]\ \&\ (x,y)\in[-1,1]\times\mathbb{R}$$

$x\in\mathbb{R}$ & $x\in[-1,1]$

$y\in[-1,1]$ & $y\in\mathbb{R}$

$$x\in\mathbb{R}\cap[-1,1]=[-1,1]$$
$$y\in[-1,1]\cap\mathbb{R}=[-1,1]$$

$$\bigcap_{n\in\mathbb{N}}\left[-\frac{1}{n},\frac{1}{n}\right]$$



$= \{0\}$ //

$$x\in\bigcap_{n\in\mathbb{N}}\left[-\frac{1}{n},\frac{1}{n}\right] \text{ iff } x\in\left[-\frac{1}{n},\frac{1}{n}\right]\ \forall n\in\mathbb{N}\ldots$$

$$\mathbb{R}=\bigcup_{x\in\mathbb{R}}\{x\}$$

$$A=\bigcup_{x\in A}\{x\}=\{x\mid x\in A\}.$$

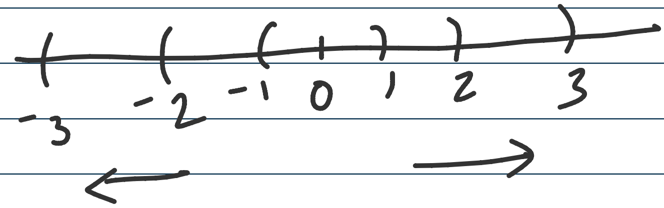
$x\in\bigcup$ $\boxed{x\in \text{ one of them}}$ ~~atleast~~

$x \in \bigcup$ $\boxed{x \in \text{one of them}}$

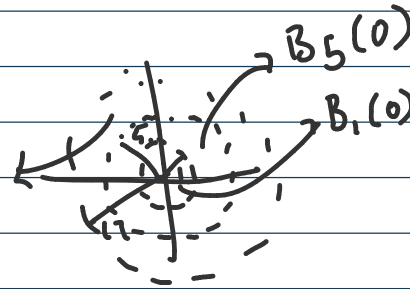
$x \in \bigcap$ $\boxed{x \in \text{all of them}}$

$$\mathbb{R} = \bigcup_{n \in \mathbb{N}} (-n, n)$$

Archimedean property ↪ 105A Real analysis

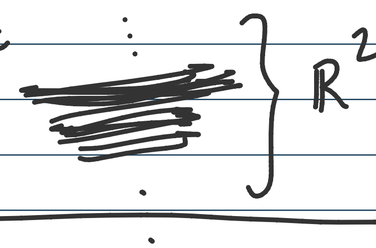


$$\mathbb{R}^2 = \bigcup_{n \in \mathbb{N}} B_n(0)$$



$$\mathbb{R}^2 = \bigcup_{a \in \mathbb{R}} \mathbb{R} \times \{a\}$$

↪ $f(x) = a$



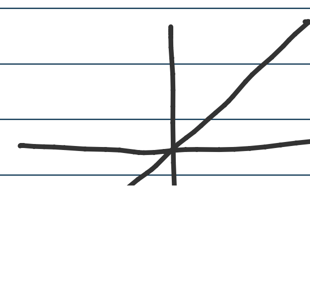
A, B

$A \times B \supseteq S$

~~do~~ are there subsets $U \subseteq A$, $V \subseteq B$

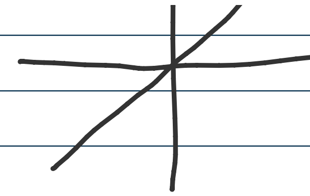
such that $S = U \times V$?

NO! diagonal



$\mathbb{R} \times \mathbb{R}$

$\mathbb{R} \times \mathbb{R}$



$\Delta = \{(x,x) \mid x \in \mathbb{R}\}$

↳ graph of the identity function

↳ line $y = x$

$\Delta \neq U \times V$

for any $U, V \subseteq \mathbb{R}$

assume that such a thing happens

$\Delta = U \times V \qquad U, V \subseteq \mathbb{R}$.

$(x,y) \in U \times V = \Delta$

$\boxed{y = x}$ for every $x \in U$, $y \in V$

$\rightarrow \boxed{U = V}$

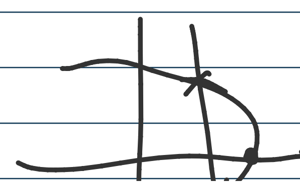
$x \in U$; then $(x,y) \in \Delta$, for any $y \in V$

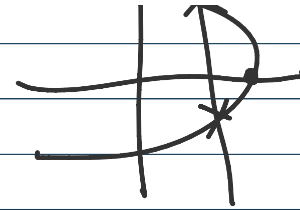
$x = y \in V$, $U \subseteq V$

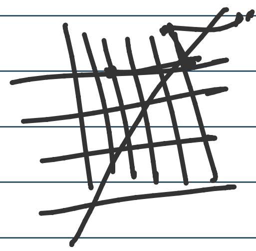
$V \subseteq U$, $\qquad \boxed{U = V} \neq \mathbb{R} \qquad \Delta \neq \mathbb{R}^2$

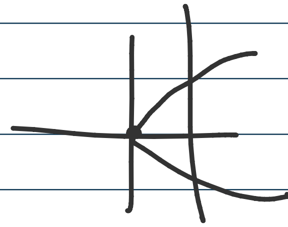
$f(x) =$ polynomials (x).

$\cancel{U \times V}$









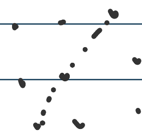
$= \text{graph of } \sqrt{x}$

$\cup \text{ graph of } -\sqrt{x}$

$A \times A, \quad \Delta_A = \{(a,a) \mid a \in A\}$ //

$\Delta_A \neq B \times C$

$B, C \subseteq A$



$A = \{1,2,3\}$

$$|P(A \times A)| = 2^{|A \times A|} = 2^{|A|^2}$$

$$\left|\{\underbrace{B}_{P(A)} \times \underbrace{C}_{P(A)} \mid B, C \subseteq A\}\right| = |P(A)| \cdot |P(A)|$$

$$= 2^{|A|} \cdot 2^{|A|}$$

$$= 2^{2|A|}$$