Conjugacy as an equiv relation (G)

① Reflexive $g \sim g$

i.e. can you find $h \in G$?

st $g = hgh^{-1}$

② Symmetry

$g \sim g'$

$\boxed{(a^{-1})^{-1} = a}$

you found some $h \in G$ st

$g = hg'h^{-1}$

$g' = \square g \square^{-1}$

③ Transitivity

$g \sim g'$, $g' \sim g''$

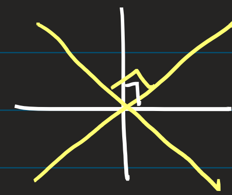
$h, k \in G$

$g = hg'h^{-1}$ & $g' = kg''k^{-1}$

$g = \square g'' \square^{-1}$

(☆) $T: \mathbb{R}^n \longrightarrow \mathbb{R}^n$

T can be written as a matrix



$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$(x, y) \longmapsto (x+y, y)$

$B = \begin{matrix}(1,0)\\(0,1)\end{matrix} \qquad \begin{matrix}(1,0)\\(0,1)\end{matrix} \quad , \quad e = \begin{matrix}(1,1)\\(1,-1)\end{matrix} \qquad \begin{matrix}(1,1)\\(1,-1)\end{matrix}$

$$T(1,0) = (1+0, 0) = (1,0)$$
$$= 1\cdot(1,0) + 0\cdot(0,1)$$

$$T(0,1) = (0+1, 1) = (1,1)$$
$$= 1\cdot(1,0) + 1\cdot(0,1)$$

$$[T]_B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$T(1,1) = (1+1, 1) = (2,1) = \frac{3}{2}\cdot(1,1) + \frac{1}{2}(1,-1)$$

$$T(1,-1) = (1-1, -1) = (0,1) = \frac{1}{2}(1,1) - \frac{1}{2}(1,-1)$$

$$[T]_e = \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix}$$

$\exists$ an invertible matrix P st $\xrightarrow{}$ change of basis

$$\underbrace{[T]_{\mathcal{B}} = P[T]_{e}P^{-1}}_{\text{conjugates}}$$

$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$\mathcal{B} \qquad e$

$T(1,0) = a(1,1) + b(1,-1)$

$T(0,1) = c(1,1) + d(1,-1)$

$P = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

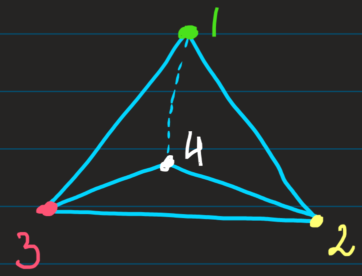
Matrix representations of T are conjugates

Classwork 2

Q1. Consider the group $G = \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z}$.

Find an n and subgroup $H \leq S_n$ st $G \cong H$.

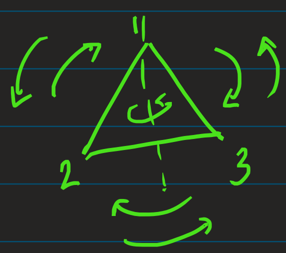
Q2. Consider a regular tetrahedron T.



T

Carefully write down the cycles that correspond to the rotations of T. Which subgroup of S_4 can you identify it with?

eg



$$\boxed{\begin{matrix}(1\,3\,2)\\(1\,2\,3)\end{matrix}}$$

(23), (13), (12)

$$\boxed{\begin{matrix}\text{Prob 6}\\ \text{in HW4}\end{matrix}}$$

Ans 1 Sun Tzu's thm

$$\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z} \cong \underbrace{\mathbb{Z}/15\mathbb{Z}}_{\text{cyclic gp of order 15}}$$

so we need to find an n st S_n has an element of order 15

$n = 15$, b/c $(1\,2\,3 \cdots 15) \in S_{15}$ ✓

and $o(1\,2\,3 \cdots 15) = 15$

$n \geq 15$

$\langle (1\,2\,3 \cdots 15) \rangle \cong \mathbb{Z}/15\mathbb{Z}$ ∎

Smaller n st it works, $\underline{n = 8}$

$$(1\,2\,3)(\underline{4\,5\,6\,7\,8}) = (4\,5\,6\,7\,8)(1\,\underline{2}\,3) \checkmark$$

3 5

$$\sigma\tau = \tau\sigma$$

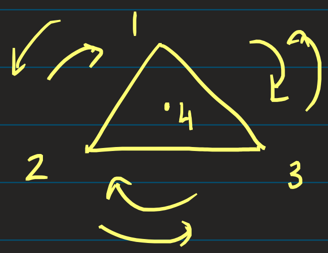
& $\gcd(o(\sigma), o(\tau)) = 1$

Then $o(\sigma\tau) = o(\sigma) \cdot o(\tau)$

So, $o((1\,2\,3)(4\,5\,6\,7\,8)) = 3 \cdot 5 = 15$

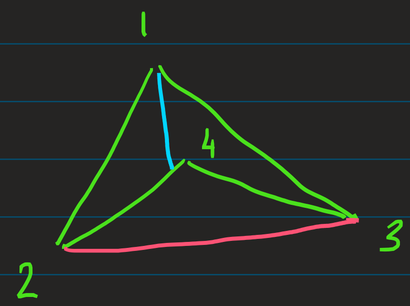
Cayley's Thm : For any finit grp G, $\exists\, n$ & $H \leq S_n$ st $G \cong H$

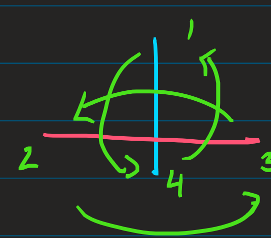
Ans 2



fix 4 $\{$ (123) (132)　　(124) (142) $\}$ fix 3

fix 2 $\{$ (134) (143)　　(234) (243) $\}$ fix 1

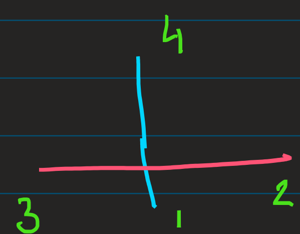




(14)(23) ✓

(12)(34)

(13)(24)



( )

$A_4 = \{ (\,),$ disjoint 2-cycles, 3-cycles $\}$.

Sgn $= 1$

12 elements $= \frac{1}{2}(24)$ → S_4