04/13

$S+T=\{s+t \mid s\in S,\ t\in T\}$

$3\mathbb{N}+5\mathbb{N} = m\mathbb{N}$, $\underline{m}$ is some number
?

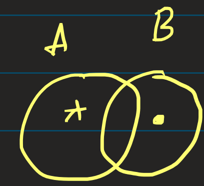
* $A, B \le \mathbb{Z}$, then $A\cup B \le \mathbb{Z} \iff A\subseteq B$ or $B\subseteq A$

Ans

($\Leftarrow$) ✓

($\Rightarrow$) $A\cup B \le \mathbb{Z} \Rightarrow A\subseteq B$ or $B \subseteq A$.

$A\not\subseteq B$ & $B\not\subseteq A$ (by Emily P.)



$\exists\, \underline{a\in A},\ \underline{b\in B}$ $\quad a+b\in A\cup B$

$\underbrace{a\notin B,\ b\notin A}$

$a\in A\subseteq A\cup B$
$+$
$b\in B\subseteq A\cup B$

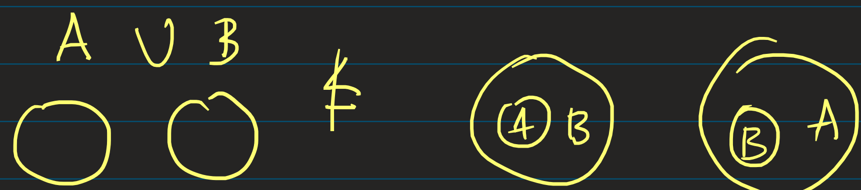
$\underline{a+b}\in A$ or

$a+b\in B$

$\underline{-a}\in A$, $-b\in B$

$(a+b)-a\in A$ & $(a+b)-b\in B$

$b\in A$ & $a\in B$



Q1. (Homomorphisms)

$(\mathbb{R}, +)$ is a group

$\mathbb{R}^{\times}$ or $\mathbb{R}^{*}$ $= (\mathbb{R} \setminus \{0\}, \cdot)$

Subgroup of $\mathbb{R}^{\times}$ is $(\mathbb{R}_{>0}, \cdot)$

Give a homomorphism $\phi : (\mathbb{R}, +) \longrightarrow (\mathbb{R}_{>0}, \cdot)$

↳ isomorphism (think pre-calculus) → a familiar function

↳ not an isomorphism (you can give an easy answer)

Q2. (subgroups)

$\mathbb{R}^{\times}$ a above (a) Is $(\mathbb{R}_{<0}, \cdot)$ a subgroup?

(b) $\mathbb{C}^{\times} = (\mathbb{C} \setminus \{0\}, \cdot)$

$$\frac{1}{a+ib} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$$

$$S = \{ z \in \mathbb{C} \mid |z| = 1 \}$$

Show $S \leq \mathbb{C}^{\times}$

1 (a) $\phi : (\mathbb{R}, +) \longrightarrow (\mathbb{R}_{>0}, \cdot)$

$\phi(x+y) = \phi(x)\,\phi(y) \longrightarrow$ exponential rule

$\phi(x) = e^{x}$

$\phi^{-1} : (\mathbb{R}_{>0}, \cdot) \longrightarrow (\mathbb{R}, +)$

$x \longmapsto \ln x$

[ln is a group homomorphism]

(b) $\phi : G \longrightarrow H$

e_H

$\phi(g) = e_H \quad \forall g \in G$

$\phi(g_1 + g_2) = e_H = e_H + e_H = \phi(g_1) + \phi(g_2)$

$\phi : (\mathbb{R}, +) \longrightarrow (\mathbb{R}_{>0}, \cdot)$

$\phi(x) = 1 \quad ; \quad \phi(2) = 1 = \phi(3$

$\phi(x+y) = 1 = 1 \cdot 1 = \phi(x) \cdot \phi(y)$

2 (a) $(\mathbb{R}_{<0}, \cdot)$ a subgroup?

No!

$-1, -1 \in \mathbb{R}_{<0}$

$(-1)^2 = 1 \notin \mathbb{R}_{<0}$

odd numbers are not a subgroup

$3+7=10$ not odd!

eg. $\mathbb{Z} = n\mathbb{Z} \cup n\mathbb{Z}+1 \cup \cdots \cup n\mathbb{Z}+(n-1)$

✓ ✗ ✗

(b) $S = \{ z \in \mathbb{C} \mid |z|=1 \} \leq \mathbb{C}^\times$

* $S \neq \phi$, $1 \in S$ ✓

* $z, w \in S$, is $zw^{-1} \in S$? ✓

$|zw^{-1}| = \left|\frac{z}{w}\right| = \frac{|z|}{|w|} = 1$, so $zw^{-1} \in S$

$S = \{ z \in \mathbb{C} \mid |z|=1 \}$

$= \{ x+iy \mid |x+iy|=1 \}$

$= \{ (x,y) \in \mathbb{R}^2 \mid x^2+y^2=1 \}$ — CIRCLE OF RADIUS 1.