

Midterm (4)

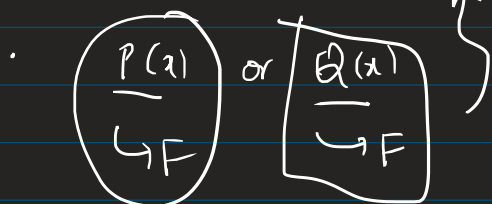
$P(x) \Rightarrow Q(x)$ to be true for all $x \in T$

① $P(x)$ is false for all $x \in T$ ✓

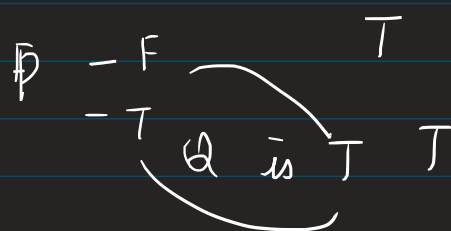
② $P(x)$ and $Q(x)$ is true for all $x \in T$

(1) $P \wedge Q$ are false for all $x \in T$

• at least one of them is false //



(2) $Q(x)$ is true for all $x \in T$ ✓



(3) $P(x)$ is false ✓

T
 $Q(x)$ is true

(4) $P(x)$ is true α

$Q(x)$ is false

F

★

(\Rightarrow)

T

$=$

\wedge

T

$=$

\vee

T

F for P

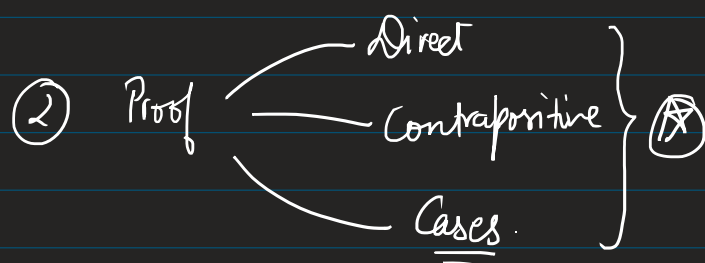
$T \wedge T$ for $P \wedge Q$

both are T

at least is T

Quiz Topics

① Logical Equivalence ✓✓



When a particular method should be used?

Judgement call. No problems to develop of judgement.

Prove if $\underline{n > 0}$, then $n^2 > 0$

Proof: Consider an arbitrary $\underline{n > 0}$

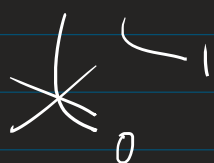
$$n^2 = n \cdot n > n \cdot 0 = 0$$

(Fact about inequality - multiplying by a positive number doesn't change the sign)

Contrapositive: if $\overset{\text{smaller } \leq}{\underline{n^2 \leq 0}}$, then $n \leq 0$
(can $n^2 < 0$ happen?)

$n^2 = 0$, then $n = 0$ (tautology)

* \mathbb{N} first natural number?



$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$
~~$$= \{0, 1, 2, \dots\}$$~~

$\mathbb{N} := \mathbb{Z}_{>0}$ positive integers

$\mathbb{Z}_{\geq 0}$ non-negative integers.

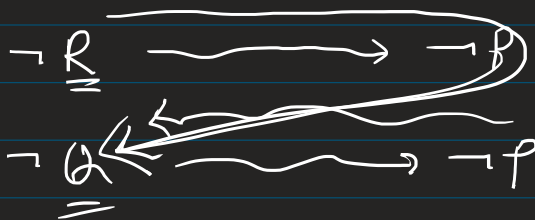
* $P: x$ is odd

$Q: 3x - 1$ is even

$R: 5x - 2$ is odd

$$P \Rightarrow Q \quad P \Rightarrow R \equiv \neg R \Rightarrow \neg P$$

$$\equiv \neg Q \Rightarrow \neg P$$



$$\underline{P \Rightarrow Q}$$

$$\neg R \Rightarrow \neg P$$

$$\underline{P \Rightarrow Q}$$

"Transitivity"

ends of one thing
to begin

Q: $3x-1$ is odd

\rightarrow P: about x

R: $5x-2$ is even

⑤

Q



R: x is even

R

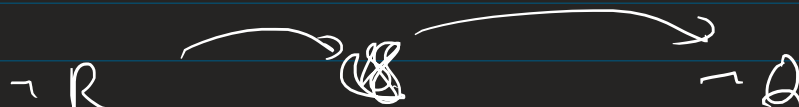
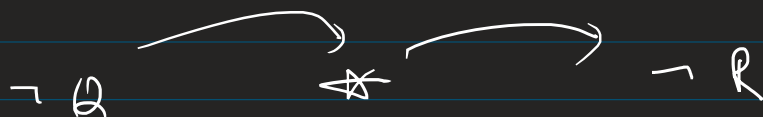


d

$3x-1$ is odd

$3x-1$ is even

x is even



$\mathbb{N} = \mathbb{Z}_{>0}$

* suppose $n \leq 5$

what can n be?

$n \in \{1, 2, 3, 4, 5\}$ ✓

* $ab = 6$

$c+d = 6$

what can a, b, c, d be?

<u>a</u>	<u>b</u>
1	6
2	3
6	1
3	2

<u>c</u>	<u>d</u>
1	5
3	3
5	1
4	2
2	4
3	3

a	b	c	d
1	6	1	5
2	3	2	4
		3	3

$a=1, b=-1$ (1,1)

* $a, b \in \mathbb{Z}$, $ab = 1$

what can a, b be?

a	b
1	1
-1	-1

HW

$$ab=1 \Rightarrow a, b \in \{\pm 1\}$$

Let's assume that $a, b \notin \{\pm 1\}$ i.e. $a, b \in \mathbb{Z}$ st $|a| > 1$

$$\text{if } |a| > 1 \text{ and } ab=1 \\ \text{then } b = 1/a \notin \mathbb{Z}$$

(Proof by contradiction)

$$a = \underline{2}, \quad b = \underline{1/2}$$

$$* \quad a \equiv b \pmod{n} \text{ iff } n | a-b \rightsquigarrow \underline{a = b + nl}, \quad l \in \mathbb{Z}$$

a special kind of equality

$$\text{with one rule } \underline{\underline{kn \equiv 0 \pmod{n}}}$$

$$\begin{aligned} \underline{4} \pmod{3} & \quad \underline{\underline{=}} \quad \underline{3+1} \pmod{3} \\ & \quad = 0+1 \pmod{3} \\ & \quad = 1 \pmod{3} \end{aligned}$$

$$\text{mod } n \rightarrow \underline{0, \dots, n-1} \quad \checkmark$$

* Writing Proofs

↳ Indicate method of proof $P \Rightarrow Q$.
 (how you're "directly" Assume $\neg Q \dots$
 \sim
 \sim
 inducti...)

↳ Written proof is not for you or about you
 it's written for the reader

The audience are your peers

$$\underline{\underline{\text{Solving the problem}}} \quad \underline{\underline{=}} \quad \underline{\underline{\text{writing proof}}}$$

↳ Don't abbreviate words

↳ You can use "symbols" — \mathbb{Z} "
— $=$
— $a \in \mathbb{R}$ fine

↳ NOT with symbols are logical symbols

{ \wedge \vee \Rightarrow \exists \forall
and or implies there exists for all.

$A = \{$

$A \cap B = \{ x \in A \text{ ~~and~~ } x \in B \}$

logical symbols — develop math thinking

— NOT in math writing