f is one-one $\qquad f: A \longrightarrow B$

To show $\quad f(C \cap D) = f(C) \cap f(D)$

Set equality means $\quad f(C \cap D) \subseteq f(C) \cap f(D)$

$$f(C) \cap f(D) \subseteq f(C \cap D)$$

Proof: $\quad x \in f(C \cap D)$ i.e. $x = f(y)$, $y \in C \cap D$

since $y \in C \cap D \Rightarrow y \in C$ & $y \in D$

therefore $\quad f(y) \in f(C)$ & $f(y) \in f(D)$

But $x = f(y)$ so $x \in f(C)$ & $x \in f(D)$

$$\Rightarrow x \in f(C) \cap f(D)$$

$$f(C \cap D) \subseteq f(C) \cap f(D)$$

$x \in f(C) \cap f(D)$ i.e. $x \in f(C)$ & $x \in f(D)$

$x = f(y)$, $y \in C$

$x = f(z)$, $z \in D$

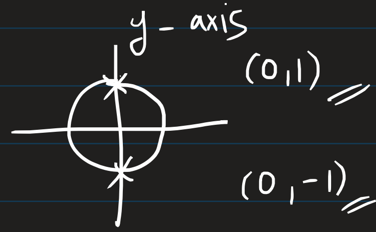
i.e. $\quad f(y) = x = f(z)$ i.e. $f(y) = f(z)$

Hence $y = z$, since f is one-one.

$C \ni y = z \in D \qquad y, z \in C \cap D$

$$x = f(y) = f(z) \in f(C \cap D).$$

* $x^2+y^2=1$



i.e. 0 R 1 & 0 R -1

but for a function 0 R a unique number

NOT a function.

Let $x=0$, if y has 2 values, then a problem!

Countability

$|A|$, $|B| < \infty$

Fact: There exists a bijection b/w A & B iff $|A|=|B|$

injection (one-one)

surjection (onto)

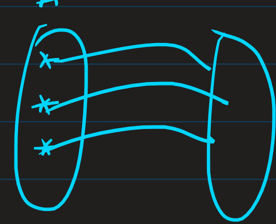
$f: A \longrightarrow B$ injectivity

↳ every distinct element ~~here~~ in A goes to a distinct element in B

$A \longleftrightarrow f(A) \subset B$

$|A| = |f(A)|$

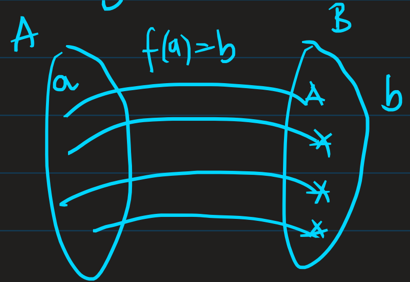
A



$|A| \leq |B|$ ✓

$f: A \longrightarrow B$

for ever $b \in B$ there's an $a \in A$ st $\underline{b = f(A)}$

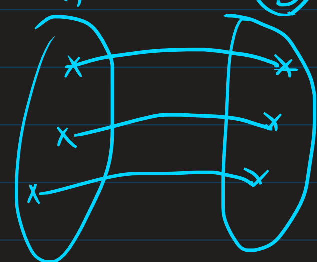


$|A| \geq |B|$

$\boxed{|A| = |B|}$

inj $\longrightarrow$ surj $\longrightarrow$

$A \longrightarrow B$



$\longrightarrow$ Having the $\underline{\text{same size}}$ $\longleftrightarrow$ $|A| = |B|$

$f: A \longrightarrow B$ bij

there exists a bijective

A, B infinite $\underline{|A| = |B|}$ by def^n if $\underline{f: A \longrightarrow B}$

$f: A \longrightarrow B$ inj

$A \longleftrightarrow f(A)$. bij

$f: \bar{\mathbb{R}} \longrightarrow \bar{\{0, 1\}}$

inj $f(x) = 0$

f $\mathbb{R} \subseteq \{0, 1\}$

$\underline{\mathbb{R} \longleftrightarrow f(\mathbb{R})} \subseteq \{0, 1\}$

$(a,b) \longrightarrow 2^a 3^b$

$f: \mathbb{Q}^+ \longrightarrow \mathbb{N}\times\mathbb{N} \xrightarrow{g \text{ inj}} \mathbb{N} \ni 5$

$\mathbb{Q}^+ \longleftrightarrow f(\mathbb{Q}^+) \subseteq \underbrace{\mathbb{N}\times\mathbb{N}}_{\text{countable}}$

countable

$f: A \longrightarrow f(A)$ bijective.

~~inj~~

f has an inverse if it's bijective

$\boxed{f: A \longrightarrow B}$ was only injective

then f doesn't have an inverse

$f: \boxed{A} \longrightarrow \boxed{f(A)}$

bijective

$f^{-1}: \boxed{f(A)} \longrightarrow \boxed{A}$

$f: \mathbb{R} \longrightarrow \mathbb{R}$ not surjective

$x \longmapsto x^2$ not injective

$-1 \longmapsto x^2$

$f: [0,\infty) \longrightarrow \mathbb{R}$;

$x \longmapsto x^2$

$f: [0,\infty) \longrightarrow [0,\infty)$

$x \longmapsto x^2$

$f^{-1}: [0,\infty) \longrightarrow [0,\infty)$

$x \longmapsto \sqrt{x}$

$f([0,\infty)) = [0,\infty)$

$= \{f(x) \mid x \in [0,\infty)\} = \{x^2 \mid x \in [0,\infty)\}$

$= [0,\infty)$

Definition of a function:

Relation $R \subseteq A \times B$

* $\operatorname{dom} R = A$ //

* $(a;b), (a;c) \in R$ then $b = c$
$f(a)$

$f: A \to B$ is a function

if $f(a)$ is a thing

$p \Rightarrow q$
$\neq q \Rightarrow p$

if $a_1 = a_2$:
then $f(a_1) = f(a_2)$

$\boxed{x = y \Rightarrow f(x) = f(y)}$

→ well-defined

$[a] = [b]$

Prove $f([a]) = f([b])$.

* $[f(x) = f(y) \Rightarrow x = y]$

$x \neq y \Rightarrow f(x) \neq f(y)$

well-definedness [same elements go to same elements]
[distinct " " " distinct]

$[a] = [b]$ $f([a]) \neq f([b])$

$$f: \mathbb{Z}_4 \longrightarrow \mathbb{Z}_4 \quad \text{(Grp theory)}$$

$$[a] \longmapsto [2a]$$

$$[0] \longmapsto [0]$$

$$[1] \longmapsto [2]$$

$$[2] \longmapsto [2 \cdot 2] = [4] = [0]$$

$$[3] \longmapsto [6] = [2]$$

IIIA (Group Theory) "

* $[a] = [b]$

Prove $[2a+3] = [2b+3]$.

$$f: \text{lines in } \mathbb{R}^3 \longrightarrow \text{lines in } \mathbb{R}^3$$

$$\ell \longmapsto \ell^{\perp}$$

not-well defined

y-axis

x-axis

z-axis

* $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto -x$ $\qquad f \circ f = id_{\mathbb{R}}$

$$f(f(x)) = f(-x) = -(-x) = x$$

* $f: \mathbb{C} \to \mathbb{C}$, $z \mapsto \bar{z}$, $a+bi \mapsto a-bi$ $\qquad f \circ f = id_{\mathbb{C}}$

Def^n: If f is st $f \circ f = id$, then f is called an involution

(58)

$$A \xrightarrow{f} \underbrace{B \xrightarrow{g} A}$$

$$f \circ g = id_B,$$

If g is surj, then $g \circ f = id_A$ //

To show $\boxed{g(f(a)) = a}$ for any $a \in A$.

Proof: Consider any $a \in A$

Since g is surj, $\exists b \in B$ st $\boxed{g(b) = a}$

$$\boxed{b} = id_B(b) = f \circ g(b) = f(g(b)) = \boxed{f(a)}$$

$$a = g(b) = g(f(a))$$

$\blacksquare$