① $\cancel{\mathbb{R}\setminus\mathbb{Q}}$ ~ cannot! $\sqrt[4]{3}$ ~ $x^2 = \sqrt{3}$

$\{\sqrt[4]{9}, \sqrt[8]{3^{16}}, \sqrt{\ }\}$ ... $\sqrt[16]{3^8}$ //

3^2

$\{z \notin \mathbb{Q} \mid z^2 - n = 0 \text{ for some } n \in \mathbb{Z}\} = \{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \dots\}$

$z^2 = n$

$\{\sqrt[n]{n} \mid n \in \mathbb{N}\}$ ~ $\cancel{(\sqrt[3]{3})}^2 = \sqrt[3]{9}$

② $n \in \mathbb{N}$

$A_n = (-\frac{1}{n}, 2 - \frac{1}{n})$

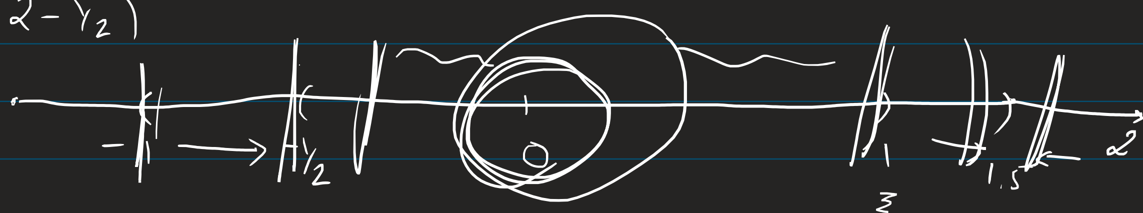
negative positive

$\bigcup_{n\in\mathbb{N}} A_n$ — $(-1, 2)$

$\bigcap_{n\in\mathbb{N}} A_n$ — $[0, 1)$ $[0, 1)$

$A_1 = (-1, 1)$

$A_2 = (-1/2, 2 - 1/2)$



③ $A \subseteq A$ true!

$\{A, \phi\}$ are partition of A

$A \cup \phi = A$

$A \cap \phi = \phi$

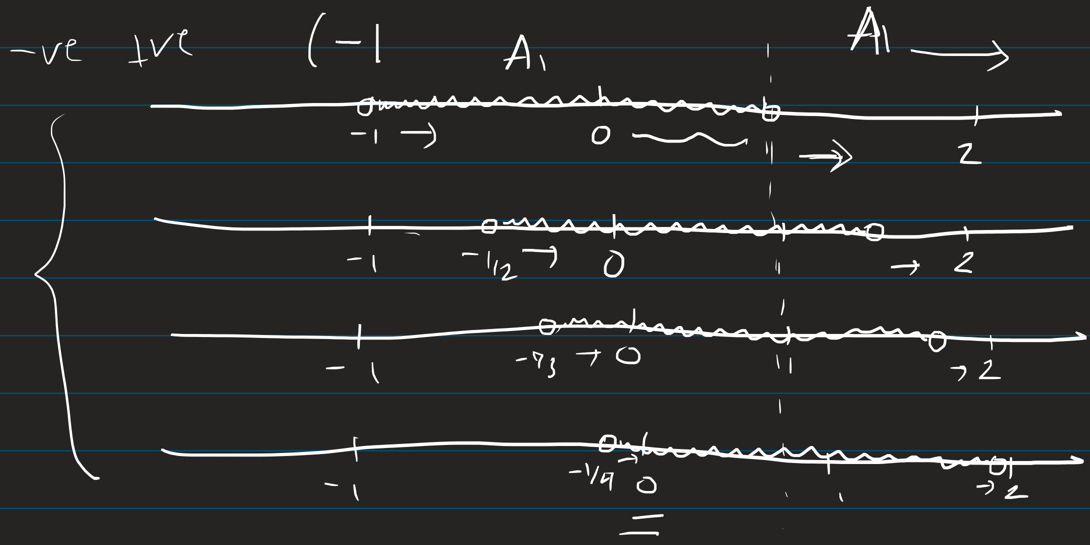
④ $S = \{\underbrace{r \in \mathbb{R}}_{\text{declare variable}} \mid \overbrace{0 \le r \le 1}^{\text{cond'l property}}\} = [0, 1]$ //

$S = \{x \mid x \in S\}$

even $= \{2k \mid k \in \mathbb{N}\}$

$= \{n \in \mathbb{N} \mid n = 2k, k \in \mathbb{N}\}$

⑤ $A_n = (-\frac{1}{n}, 2 - \frac{1}{n})$ $A_1, A_2, A_3, A_4, \dots$



$\cap A_n$

$= B$

No one sayin th

$[0, 1)$

union

take them all together ~ guess.

$\underbrace{\bigcup_n A_n = B}$ $\qquad$ $\bigcup A_n \subseteq B$ $\qquad$ $B = (-1, 2)$

$B \subseteq \bigcup_n A_n$

$A_n \subseteq B$ for any n

$B \subseteq A_m$ for some m
$\subseteq \bigcup A_n$

candidate for $\bigcup A_n$

$(-1, 2)$

" $\quad$ " $\cap A_n$ $\quad [0, 1) = C$

① $A_n \subseteq B$ $\quad$ for any n

$B \subseteq \underline{A_m}$ $\quad$ for some m
$\subseteq \bigcup A_n$

② $\cap A_n \subseteq \underline{A_m} \subseteq C$ $\quad$ for some m

$C \subseteq A_n$ $\quad$ for every n

$\{p\} \cap \{q\} = \underline{\phi}$
$p \neq q$

$$\bigcap \left(-\frac{1}{n}, 2-\frac{1}{n}\right) = [0,1)$$

$x \in [0,1)$ to show $x \in A_n$ for every $n \in \mathbb{N}$.

$-\frac{1}{n} < 0 \leq x < 1 \leq 2-\frac{1}{n}$

↳ negative.

$\frac{1}{n} \leq 1$

$-\frac{1}{n} \geq -1$

$x \in \left(-\frac{1}{n}, 2-\frac{1}{n}\right)$ for any n

$2-\frac{1}{n} \geq 2-1 = 1$

$[0,1) \subseteq \left(-\frac{1}{n}, 2-\frac{1}{n}\right)$ for every n

$[0,1) \subseteq \bigcap \left(-\frac{1}{n}, 2-\frac{1}{n}\right)$

$x \in \left(-\frac{1}{n}, 2-\frac{1}{n}\right)$ for every natural number n.

$-\frac{1}{n} < x < 2-\frac{1}{n}$.

$\bigcap \left(-\frac{1}{n}, 2-\frac{1}{n}\right) \subseteq [0,1)$

assume this isn't true

$x \in \bigcap \left(-\frac{1}{n}, 2-\frac{1}{n}\right)$ that is NOT in $[0,1)$

$-\frac{1}{n} < x < 2-\frac{1}{n}$ for every n

$0 < x < 1$

$\frac{1}{n} \to 0$ (limit)

left $\longrightarrow$

ring $\longleftarrow$

$-\frac{1}{n}$ $\quad -1 \quad -\frac{1}{2} \quad -\frac{1}{3} \dashrightarrow 0$

$2-\frac{1}{n}$. $\quad 1 \quad 1.5$

Sentence break it down into compound

if, then $\Rightarrow$
only if $\Leftarrow$

$$P \Rightarrow Q$$

$$\neg (P \Rightarrow Q)$$

$$P \wedge \neg Q$$

---

open sentence : statement

sentence as variable : number

$x \rightarrow x = 3$ property

S(P): P is a man open sentence

{bank of things that P can take} ~ domain

S(P) over the domain of ~~algae~~ fungi

F

S(F): Fungus F is a man False

D = {male-identifying people} $\ni$ A

S(A): A is a man T

H = {all human beings} $\ni$ A, B

P(A) true
P(B) is false

$P(n)$: n is even ; domain $\mathbb{Z}$

$P(3)$ false

$P(-2)$ true

$A = \{x^2 \mid x \in \mathbb{Z}\} = \{1, 4, 9, 16, 25, 36, 49, \ldots\}$

$B = \{3, 7, 11, 15, \ldots\} = \{4k+3 \mid k \in \mathbb{Z}, k \geq 0\}$

$=$ $\quad +4 \; +4 \; +4$

$A \cap B = \phi$ TRUE true or not

Question: are squares of the form $4k+3$ NO

consider any integers — even $\sim 2k$; odd $\sim 2l+1$

$(2k)^2 = 4k^2 = 2(2k^2) = 4m \qquad m = k^2$

$(2l+1)^2 = 4l^2 + 1 + 4l = 4(l^2 + l) + 1$

$= 4b + 1 \qquad b = 2l^2 + 2l$

$A \subseteq \{4k, 4k+1\}$ NOT $4k+3$ $\qquad +3, +2$ don't occur...

when

$P(x,y) \implies Q(x,y)$ is true?

It's true if $P(x,y)$ is false. $\left[\begin{matrix} F \implies ? \\ F \implies ? \end{matrix} \text{ is } T\right]$

and when $P(x,y)$ & $Q(x,y)$ are true $[T \implies T \quad T]$

① when is $P(x,y)$ false

② when are $P(x,y)$ & $Q(x,y)$ true together.

(i) $P(x,y): x^2+y^2 \neq 1$ // when false?

$Q(x,y): y = x$

$$S = \{ (\underbrace{1/\sqrt{2}, 1/\sqrt{2}}_{\frac{1}{2}+\frac{1}{2}=1 \; \otimes}), (\underbrace{1,1}_{1+1=2}), (\underbrace{\sqrt{2},-\sqrt{2}}_{2+2=4}), (\underbrace{1,0}_{1+0=1 \; \otimes}) \}$$

$P(x,y) \implies Q(x,y)$ is true for $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ & $(\sqrt{2},-\sqrt{2})$

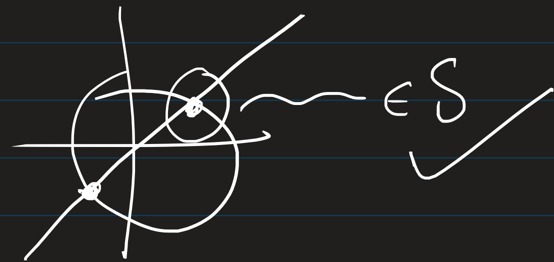
b/c $P(x,y)$ is false for these.

$P(x,y)$ & $Q(x,y)$ are true

$x^2+y^2=1$ and $y=x$

$x^2+x^2=1, \quad 2x^2=1, \quad x^2=1/2,$ $\quad x=\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

$P(x,y)$



$Q(x,y)$

P & Q tautology using atleast 3 of .

$\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow$

implication $(\underset{\text{false}}{\sim\sim} \Rightarrow \sim\sim) \sim \underline{\text{True}}$

$P \wedge \sim P$ always false

$\boxed{P \wedge (\sim P) \Rightarrow Q}$ a tautology //..

P	$\sim P$	Q	$(P \wedge \sim P)$	$(P \wedge \sim P) \Rightarrow Q$
T //	F	T	F	T
F //	T	F	F	T

a tautology