6.44, 6.46

Induction — talking about "inducting" over a set

e.g. ① $\mathbb{N} = \mathbb{Z}_{>0}$ 1

② $\mathbb{Z}_{\geq 0}$ 0

3

What do you want $S \subseteq \mathbb{R}$ to have so that you can induct over S?

① S should have a min → M says

Base case: $P(\min S)$

$S = \mathbb{Z}, \mathbb{Q}$ work ; well-ordered

② "There should be a notion of "next number"" → Y says

$\boxed{k \Rightarrow k+1}$

min S should exist

first step → J says pattern/ sequence

* Base step → Ja "complete an elementary step"

* Induction Step . →

$a_n = a_{n-1} + a_{n-2}$

$a_1 = \;,\; a_2 =$

$\boxed{n \geq 3}$

$\boxed{S = \{ n \in \mathbb{Z} \mid n \geq 3 \}}$

S working for induction!

Induction step: assume a step

prove the next step

So $\mathbb{Z}$, $\mathbb{Q}$ ① doesn't hold for them

There is (+1) Does ② ?

"I think so?" - A. No. - S.

"Don't think so" - M. Yes - D.

No - J. No.

No - J.

No — WHY IS THAT

Say $a \in \mathbb{Q}$ and b is the "next rational number".

What about $\frac{a+b}{2}$?

$P(k) \Rightarrow P(l)$ where l is the next number after k

1

2 ✓

3 ✓

⋮ ✓

in $\mathbb{Q}$ no next number

Assuming we're trying list $\mathbb{Q}$ wrt $<$

There's a $\mathbb{Q}$ has a "next number"
way

② $\mathbb{Q}$ no w/ a caveat

$\mathbb{R}$ — ① & ② NO!

you have a place to start

you have a place to go next

In theory, you can do induction on $\mathbb{Q}$ with a certain ordering

$\{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots\}$

$(k \Rightarrow k+\frac{1}{2})$

If you look at a question

Base — where to start? 0, 1, 25

Induction — $k \Rightarrow k+1$

assume k — do I just have to assume one step or can I assume more than one step.

$1 \wedge \cdots \wedge k-2 \wedge k-1 \wedge k \Rightarrow k+1$ ✓ "Strong induction"

$\underbrace{a_k}$ $\underbrace{a_{k-1}}$ $\underbrace{a_{k-2}}$

3 or more consecutive indices.

$P(k) \Rightarrow P(k+1)$ diff.

$P(1) \wedge \cdots \wedge P(k) \Rightarrow P(k+1)$

46 $S = \{ i \in \mathbb{Z} \mid i \geq 2 \}$

$P \subseteq S$ w/ property $\underline{2, 3} \in P$

and $\boxed{n \in S \text{ then } n \in P \text{ or } n = ab,\ a, b \in S}$

$n \in S$

P

product elements from S.

$P(n)$: $\underline{n \in P}$ or n is a product of elements from P.

So base case is $P(2)$ i.e. to show $2 \in P$ or 2 is a product of elements in P.

Given $2 \in P$. So $P(2)$ is true.

$\boxed{(k \Rightarrow k+1) \quad k=2 \\ (2 \Rightarrow 3)}$

So assume $P(k)$ is $\underbrace{\text{true for all } k < n}_{}$ $\boxed{n-1}$

Let's prove $P(n)$ is true

$n \in S$ so $n \in P$ or $n = ab$ where $\underbrace{a, b \in S}$

If $n \in P$, we're done

$\boxed{\begin{matrix} n-2 & n-6 & \\ & 2 & n-8 \end{matrix}}$

So assume $n \notin P$, so $n = ab$ where $a, b \in S$.

so $a, b < n$ → $\underbrace{P(a) \text{ \& } P(b)}$ are true

So the induction hypothesis tells us that

$a \in P$ or a is a product of elements in P

$b \in P$ or b is a product of elements in P

$n = ab$, n is a product of elements in P

$\blacksquare$

So $\boxed{2 \mid 2^n}$

assume for $\boxed{2^k}$ let's prove for 2^{k+1}

$2 \mid 2^k$

But we didn't have to assume that 2^{k-1} was true

$2^k = 2l$

$2^{k+1} = 2(2l)$, $2 \mid 2^{k+1}$

$2 \mid 2^{k-1}$

$k \Rightarrow k+1$

if k, then $k+1$

k is true ×

2 $\boxed{2 \text{ is true}}$ 3 is true

$k \Rightarrow k+1$ is true

2 is true

then 3

2, 3 true $\Rightarrow$ 4 true

$\boxed{2, 3, 4, \text{ true} \Rightarrow 5 \text{ true}}$

$\boxed{4 \text{ true} \Rightarrow 5 \text{ true}}$

$a_{n+3} \equiv a_n \bmod 2 \quad \forall n \geq 1$

$\boxed{k = n+3}$ ✓

Proof: Using $\boxed{a_{n+3} = a_{n+2} + a_{n+1}}$ — this is the recursive relation

$= (a_{n+1} + a_n) + a_{n+1}$

$\boxed{k = n+2}$ ✓

$= 2a_{n+1} + a_n$

Formula is called a recursive relation

$\rightarrow \boxed{a_k = a_{k-1} + a_{k-2}}$

$3 \mid 2.$

$S = \{3k \mid k \geq 1\}$

44 $\boxed{a_{n+3} \equiv a_n \bmod 2}$ this for 44

$n \Rightarrow n+3$ i.s.