① Unique root / zero

② Unique minimum

IVT, $f(a)<0$, $f(b)>0$

$\exists c \in (a,b)$ st $f(c)=0$

$f(a)<3$, $f(b)>3$

$\exists c \in (a,b)$ $f(c)=3$

$g(x)=f(x)-3$

$g(a)<0 \Leftrightarrow f(a)<3$

$x^5+2x-5=0$ has a unique root b/w $x=1$ & $x=2$.

* show there's a root ✓
* show it's unique — if there were 2, say c_1 & c_2 then $c_1=c_2$

By IVT, there's a root c.

Suppose there's another one b/w 1 & 2, say k

Assume not, $k \neq c$

$k<c$ or $c<k$

without loss of generality

$k<c$

$k^5 < c^5$, $2k < 2c$

$k^5 + 2k < c^5 + 2c$

$0 = k^5 + 2k - 5 < c^5 + 2c - 5 = 0$

$0 < 0$ NOT True, a contradiction.

$= f(x)$

$x^3 - 6x^2 + 11x - 6$

$1 - 6 + 11 - 6 = 0$

Is there a unique root b/w

0 and 2.5

IVT $f(0) = -6 < 0$

$f(2.5) = -0.375 < 0$

$(P \Rightarrow Q)$

↳ P is true

$f(a) > 0$ $f(b) < 0$

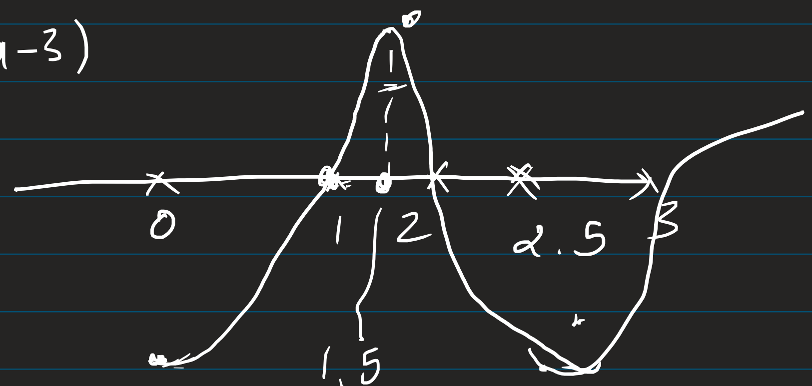
$f(\underline{x}) = a_n \underline{x}^n + a_{n-1} \underline{x}^{n-1} + \cdots + a_1 \underline{x} + a_0$

If $\underline{\underline{x=1}}$ is a zero $\Leftrightarrow$ $\underbrace{a_n + \cdots + a_0}$ $= 0$

There's a root $\underline{x=1}$ in $0, 2.5$

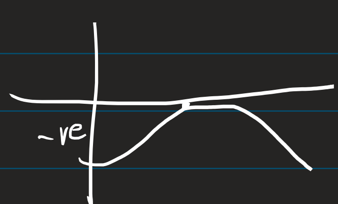
0.5 1.5

$(x-1)(x-2)(x-3)$

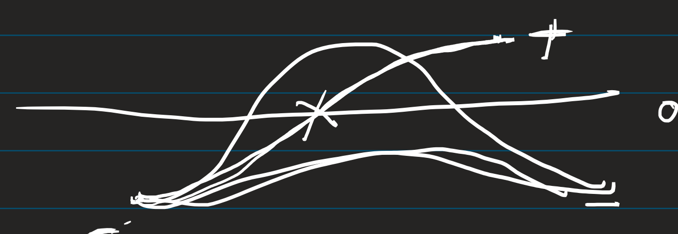


$f(1.5) > 0$ $f(2.5) < 0$

2



IVT



$0 \;\&\; \underline{\underline{2.5}}$ ✓

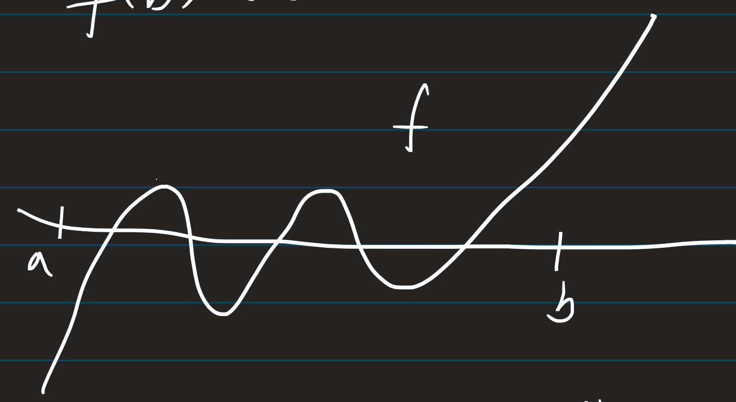
$1 < 0$

1 was a zero

0.5, 1.5

$f(0.5) \;\&\; f(1.5) > 0$ ✓

$f(a) > 0 \qquad f(b) < 0$



$f(a) < 0 \qquad f(b) > 0$ "a root" b/w a & b

S a finite set of natural numbers

S has a minimum — $s \in S$ st

$s \leq a \quad \forall a \in S$

Prove the minimum s is unique!

well, suppo there's another $t \in S$ $\boxed{\text{minimum}}$

$t \leq a \quad \forall a \in S$

In particular $s \in S$, so $\boxed{t \leq s}$

i.e. since s is <u>also</u> the minimum and $t \in S$

$\boxed{s \leq t}$

so $s = t$, therefore s is unique

* "Closure property"

$S \subseteq \mathbb{R}$, we say $\boxed{S \text{ is closed under multiplication}}$ if $\boxed{x, y \in S, \text{ then } x \cdot y \in S}$

or

" " add" $[x, y \in S, \text{ then } x+y \in S]$

$\mathbb{Z}$ is closed under both

* $S = \{x \in \mathbb{R} \mid |x| < 1\}$

Claim that $\underline{\underline{S}}$ is closed under multiplication.

Proof. To show: for any $x, y \in S$, $x \cdot y \in S$

Consider $x, y \in S$, then $|x| < 1$ and $|y| < 1$

To show $xy \in S$, we need to show $\underline{|xy| < 1}$

Note

$$\underline{|xy|} \overset{\text{HW}}{=} |x| \cdot |y| < 1 \cdot 1 = 1$$ ▨

Therefore $xy \in S$.

$a < 1$
$b < 1$ $\quad a \cdot b < 1 \cdot 1$

Non-example Is $S = \{x \mid |x| \leq 2\}$ closed under multiplication?

[$\underline{\underline{\text{No}}}$, $\underline{\text{give a countrexample}}$]

[Yes, think of a proof]

$x = y = 1.5$

$x \cdot y = 2.25 > 2$

$xy \notin S$

* Suppose A is a set w/ 4 distinct integers

$$A = \{ a_1, a_2, a_3, a_4 \}$$

Then [are there 2 numbers in A st their difference is 0 mod 3 (i.e. divisible by 3)]?

"Not always", "not necessarily" $\boxed{\text{"Yes"}}$

"

"

"

$A = \{ a_1, a_2, a_3 \}$ $\emptyset$ replace 3 by 2

$a_1 - a_2$ $a_2 - a_3$

one of these is 0 mod 2.

$3a$, $3a+1$, $3a+2$

$A = \{ a_1, a_2, a_3, a_4 \}$

$3k$ $3l+1$ $3m+2$

$3k - 3a$
$= 3(k-a)$

$3l - 3a$

$a_4 - a_3$

0 mod 3 1 mod 3 2 mod 3

4

1 mod 3

x xx

$a \equiv 1 \bmod 3$ 4 $b \equiv 1 \bmod 3$

and more $a - b \equiv 0 \bmod 3$

A − 5 distinct element

mod 4

0 mod 4 1 m 2 m 3 m

>5

xx