* Permutations ⇝ related to symmetry.

$$\begin{tikzcd} & S_4 \arrow[d, leftrightarrow] \\ S_3 \arrow[ur] \arrow[r] & D_8 \text{ (dihedral group)} \end{tikzcd}$$

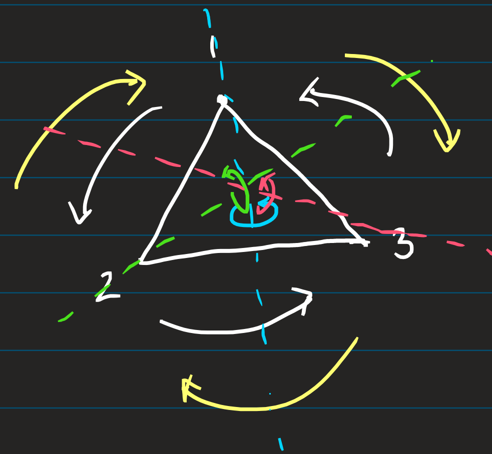
S_3

$(1\ 2\ 3)$ $\quad e$

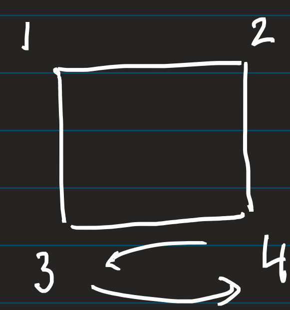
$(1\ 3\ 2)$ $\quad (1\ 2)$

$(2\ 3)$

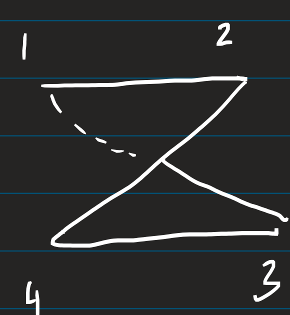
$(1\ 3)$

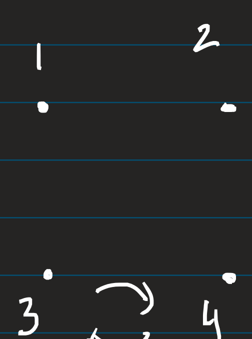


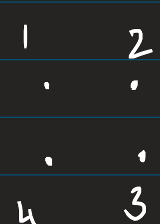
$(3\ 4)$



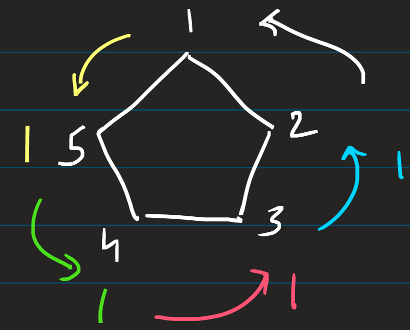
S_4







Cyclic group ~~is the~~ of order 5



$\langle e, x, x^2, x^3, x^4 \rangle$

* Problem 7

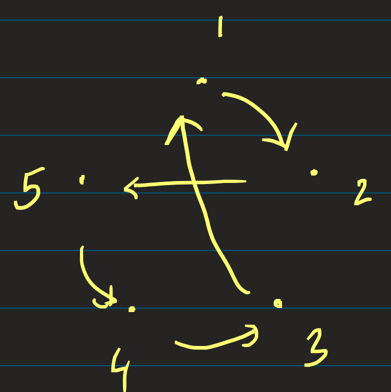
$(21)(23)(24)(25)$

$= (12543)$

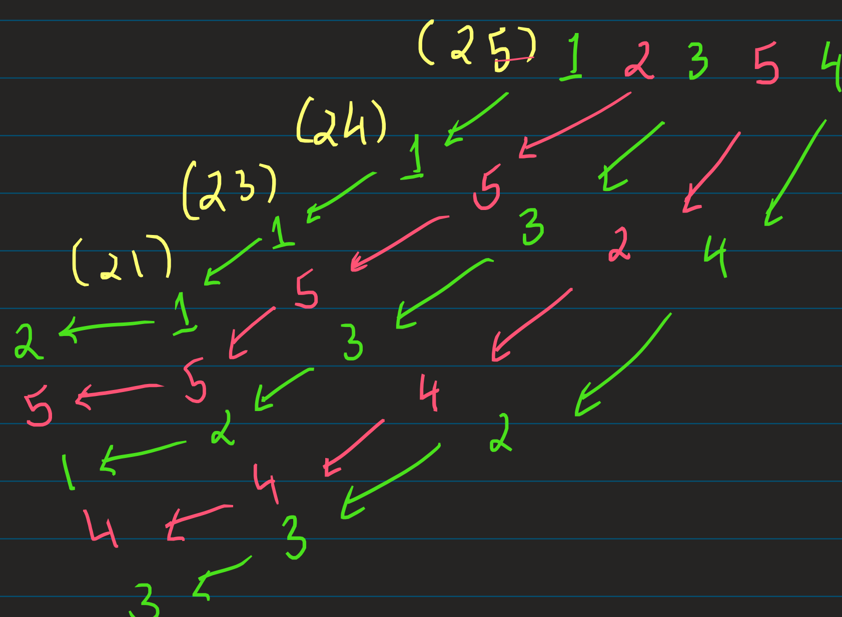
S_5

$(f \circ g \circ h)(x)$

$f(g(h(x)))$



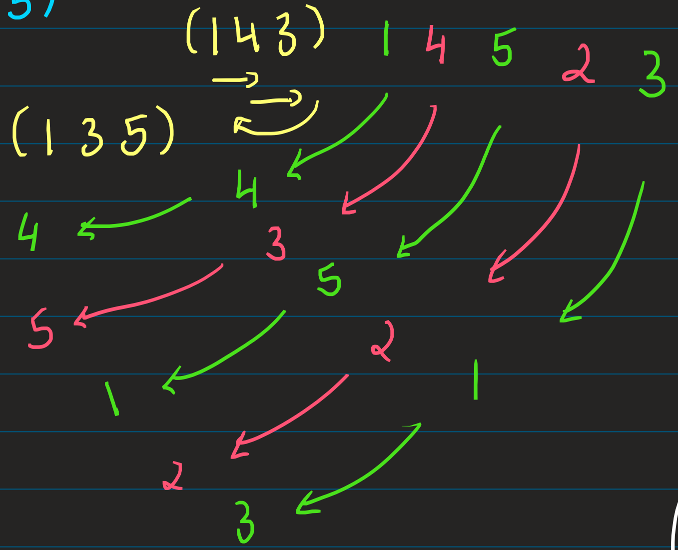
$\boxed{(1\ 2\ 5\ 4\ 3)}$

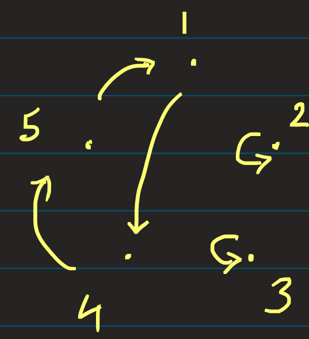


• $(1\ 3\ 5)(1\ 4\ 3)$

$= (1\ 4\ 5)$

diff from $(1\ 3\ 4)$ $\qquad$ $(2\ 3)$



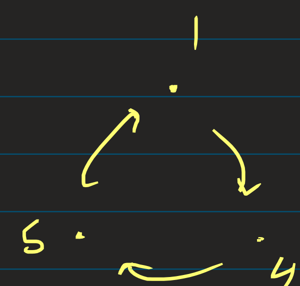


$$\left((f\circ g)^{-1} = g^{-1}\circ f^{-1}\right)$$

• $\underbrace{((1\ 3\ 5)(1\ 4\ 3))}_{=\,(1\ 4\ 5)^{-1}}^{-1} = (1\ 4\ 3)^{-1}(1\ 3\ 5)^{-1}$

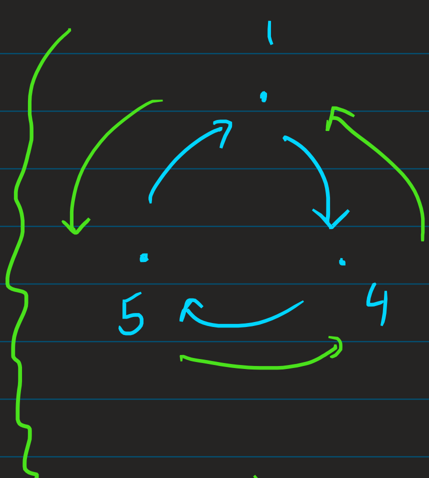
$= (1\ 3\ 4)(1\ 5\ 3)$

$= (1\ 5\ 4)$



$(1\ 4\ 5)\quad (4\ 5\ 1)$

$(5\ 1\ 4)$



$\rightarrow (1\ 5\ 4)$

* $(\mathbb{Q}^\times, \cdot)$

-1, order?

$\langle -1 \rangle = \{-1, (-1)^2, (-1)^3, (-1)^4, \dots\}$

$= \{1, -1\}$

$(-1)^2 = 1$ order of -1 is 2.

* $\{f: \mathbb{R} \to \mathbb{R}\}$ is this a group under composition

1. $\{\text{bijective}, f: \mathbb{R}^\times \to \mathbb{R}^\times\}$ is a group under composition

2. $\{\text{bijective}, g: \mathbb{C} \to \mathbb{C}\}$ is a group under composition

1. $f(x) = 1/x$

$(f \circ f)(x) = f(f(x)) = f(1/x) = 1/{1/x} = x = id(x)$

$f \circ f = id$

2. $g(z) = \bar{z}$, $(g \circ g)(z) = g(g(z)) = g(\bar{z}) = \bar{\bar{z}} = z$

g has order 2.

- How to distinguish groups that have the same size.

$\mathbb{Z}/4\mathbb{Z}$

0	1+1+1+1
1	1
2	1+1
3	1+1+1

order 4

$\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

$(0,0)$
$(1,0)$
$(0,1)$
$(1,1)$

- Sun Tzu's theorem

$$\mathbb{Z}/6\mathbb{Z} \xrightarrow{\sim} \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$$

$$x \bmod 6 \longmapsto (x \bmod 2,\ x \bmod 3)$$

$$2 \bmod 6 \longmapsto (2 \bmod 2,\ 2 \bmod 3) = (0,2)$$

$$x \bmod 6 \longmapsto (1 \bmod 2,\ 0 \bmod 3)$$

$$\shortparallel$$

$$(x \bmod 2,\ x \bmod 3)$$

$x = 3$

Sun Tzu's theorem (most general)

$$\mathbb{Z}/mn\mathbb{Z} \xrightarrow{\sim} \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$$

$\gcd(m,n)=1$

$$\mathbb{Z}/30\mathbb{Z} = \mathbb{Z}/(6\cdot 5)\mathbb{Z} \xrightarrow{\sim} \mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z}$$

$$30 = \underbrace{2\cdot 3}_{6} \cdot \underbrace{5}_{5}$$

$$\downarrow \wr$$

$$\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z}$$

$$n = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n}$$

$$\mathbb{Z}/n\mathbb{Z} \xrightarrow{\sim} \mathbb{Z}/p_1^{a_1}\mathbb{Z} \times \mathbb{Z}/p_2^{a_2}\mathbb{Z} \times \cdots \times \mathbb{Z}/p_n^{a_n}\mathbb{Z}$$

$$n = \underbrace{280} = 7 \times 40 = \underbrace{7 \times 5}_{35} \times 2^3$$

$$35 \times 8$$

$$\mathbb{Z}/280\mathbb{Z} \cong \mathbb{Z}/8\mathbb{Z} \times \underbrace{\mathbb{Z}/7\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z}}$$

$$\cong$$

$$\mathbb{Z}/8\mathbb{Z} \times \underbrace{\mathbb{Z}/35\mathbb{Z}}$$