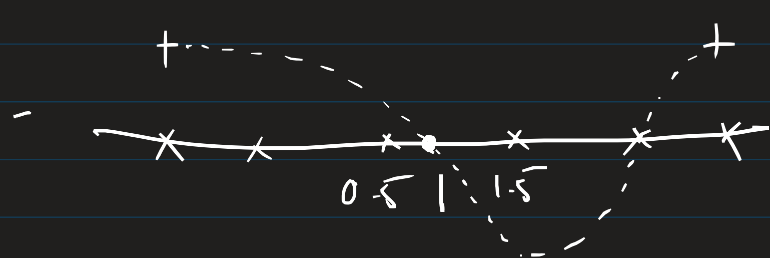
$p(x) = a_n x^n + \cdots + a_1 x + a_0$

1 is a zero

$p(1) = 0$

$a_n + a_{n-1} + \cdots + a_1 + a_0 = 0$

$x^5 - x + x - 1$



$f(0.5) < 0$
or
$f(1.5) < 0$

$\boxed{4.13}$ if $c^2 = a^2 + b^2$, then $3 \mid ab$

If $a^2 + b^2$ is a square, then $3 \mid ab$

Lemma: If $x \in \mathbb{Z}$, then $x \equiv 0 \bmod 3$ or $x \equiv 1 \bmod 3$

Case I: $x = 3k$, then $x^2 = 3(3k^2) \equiv 0 \bmod 3$

II: $x = 3k+1$, then $x^2 = 3(3k^2 + 2k) + 1 \equiv 1 \bmod 3$

$\lfloor$ II : $x = 3k+2$, then $x^2 = 3(3k^2+4k+1)+1$

$\equiv 1 \mod 3$ ✓

$x^2 - 1 \equiv 0 \mod 3$

$\perp$

Contrapositive: $3 \nmid ab$ then a^2+b^2 is NOT a square

Proof If $3 \nmid ab$, then $\overset{\text{show}}{}$ $a^2+b^2 \equiv 2 \mod 3$ ← Lemma

result follows by lemma.

So, since $3 \nmid ab$, then $3 \nmid a$ & $3 \nmid b$

$\left[3 \mid a \text{ or } 3 \mid b, \text{ then } 3 \mid ab \right]$ ✓

$a = 3k$ $\quad$ $b = 3l$

$ab = 3(bk)$ $\quad$ $ab = 3(al)$

$a^2 \equiv \underbrace{4 \mod 3}_{3+1} \equiv 1 \mod 3$

$b^2 \equiv 4 \mod 3 \equiv 1 \mod 3$

Then $a \equiv 1 \mod 3$ or $a \equiv 2 \mod 3$

$b \equiv 1 \mod 3$ or $b \equiv 2 \mod 3$

→ Lemma

Therefore $a^2 \equiv 1 \mod 3$ and $b^2 \equiv 1 \mod 3$

Hence $a^2+b^2 \equiv 2 \mod 3$ ▨

$3 \mod 3 \equiv 0 \mod 3$.

1:00 PM 13 hours.

clock arithmetic

$13 \bmod 12 \equiv 1 \bmod 12$

$\hookrightarrow 12+1$

$\boxed{23 \bmod 12 \equiv 11 \bmod 12}$

$a \in \mathbb{Z}$ $a^2 \equiv 0, 1 \bmod 4$.

$(2k)^2 = 4k^2$

$(2k+1)^2 = 4k^2 + 4k + 1$

$3k$

$3k+1$

$3k+2$

mod 9

$\boxed{\text{mod } 3}$

Base Case: 1

$\boxed{(k) \Rightarrow (k+1)}$

SCRATCH [Important at 2

How $\boxed{1 \Rightarrow 2}$].

$2 \Rightarrow 3$

$\boxed{6\text{-}31}$ $n=2$

$$(a_1+a_2)\left(\frac{1}{a_1}+\frac{1}{a_2}\right) \geq 4$$

$\Downarrow$

$n=3$

$$(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right) \geq 9$$

$$(\underbrace{a_1+a_2}_{\star}+a_3)\left(\underbrace{\frac{1}{a_1}+\frac{1}{a_2}}_{\star}+\frac{1}{a_3}\right) = \underbrace{\boxed{(a_1+a_2)\left(\frac{1}{a_1}+\frac{1}{a_2}\right)}}_{\geq 4}$$

$$+ \underbrace{\boxed{a_3 \cdot \frac{1}{a_3}}}_{1} + \boxed{a_3\left(\frac{1}{a_1}+\frac{1}{a_2}\right)}$$

$$+ (a_1+a_2)\frac{1}{a_3}$$

$$(\underbrace{a_1+\cdots+a_k}+a_{k+1})\left(\underbrace{\frac{1}{a_1}+\cdots+\frac{1}{a_k}}+\frac{1}{a_{k+1}}\right)$$

$n=2$

$$\geq \underbrace{4+1}_{5} + \underbrace{\frac{a_3}{a_1}+\frac{a_3}{a_2}+\frac{a_1}{a_3}+\frac{a_2}{a_3}}_{\geq 4}$$

$+4$

$$\underbrace{\frac{a_3}{a_1}}_{\geq 1}+\underbrace{\frac{a_3}{a_2}}_{\geq 1}+\underbrace{\frac{a_1}{a_3}}_{\geq 1}+\underbrace{\frac{a_2}{a_3}}_{\geq 1} \geq 4$$

HW

$$\boxed{\frac{a+b}{2} \geq \sqrt{ab}}$$ Remember:

$$a+b \geq 2\sqrt{ab}$$

$$\underbrace{\frac{a_3}{a_1}} + \underbrace{\frac{a_1}{a_3}} \geq 2\sqrt{\underbrace{\frac{a_1}{a_3} \cdot \frac{a_3}{a_1}}} = 2$$

$$\frac{a_2}{a_3} + \frac{a_3}{a_2} \geq 2\sqrt{\frac{a_2}{a_3} \cdot \frac{a_3}{a_2}} = 2$$

AM $\geq$ GM for length k.

$$a_{k+1}\left(\frac{1}{a_1} + \cdots + \frac{1}{a_k}\right)$$

$$\frac{a+b+c+d}{4} \geq \sqrt[4]{\underbrace{abcd}}$$

$\frac{a_1}{a_2}, \frac{a_2}{a_1}$

$=1$

$\left(\frac{a_1}{a_2} + \frac{a_2}{a_1} \geq 2\right.$

$\geq$

$$\sum a_i \qquad \sum \frac{1}{a_i}$$

$$\prod a_i$$

$AM \geq GM \geq HM$

$$\left(\sum a_i\right)\left(\sum \frac{1}{a_i}\right) \geq n^2$$

$$\frac{\sum a_i}{n} \geq \boxed{\frac{n}{\sum \frac{1}{a_i}}}$$

HM

harmonic mean.

$$\boxed{AM \geq HM}$$

Let//.

$a_n = \underbrace{1+3+5+\cdots+(2n-1)}$

$=$

What's the formula; $\boxed{a_n = n^2}$ $\boxed{a_n = f(n)}$ $\boxed{f(x) = x^2}$

$a_n = \underline{f(n)}$

$a_1 = 1$, $a_2 = 2$, $a_n = \underbrace{a_{n-1}} + 2\underbrace{a_{n-2}}$

$a_3 = 2 + 2\cdot 1 = 4$

$a_4 = a_3 + 2\cdot a_2 = 4 + 4 = 8$

$a_5 = a_4 + 2 \cdot a_3 = 8 + 8 = 16$

$a_6 = a_5 + 2 \cdot \underline{\underline{a_4}} = 16 + 2 \cdot 8 = 32$

Conjecture $\boxed{a_n = 2^{n-1}}$ $\qquad P(n): a_n = 2^{n-1}$

Prove by induction

* $P(1): a_1 = 1$ ✓

  $P(2): a_2 = 2$ ✓

* Strong induction $\quad$ assume $P(k)$ is true for $k \le \underline{\underline{n}}$

$a_n = a_{\underline{n-1}} + 2 a_{\underline{n-2}} = 2^{n-2} + 2 \cdot 2^{n-3}$

$= 2^{n-2} + 2^{n-2}$

$= 2 \cdot 2^{n-2} = 2^{n-1}$ ∎