Gov 2006: Formal Political Theory II
Section 1

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February 5, 2019

Agenda

- Introductions / logistics

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How to...

- write up problem sets
- read a formal theory paper
- come up with your own research idea!

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Review:

- Definitions
- Probabilistic voting model

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- We posted a summary of today's lecture on Canvas – these bullet points should help to guide your reading each week.

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- review tricky parts of the PSETs
- review anything that was unclear from lecture
- brainstorm ideas for the final paper

How to... do problem sets!

- You can work in groups on the PSETS, but you must write up your own solutions and state who you worked with at the top. It is strongly recommended that you read through the PSET on your own before discussing with others.

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- Your solutions should be typeset and submitted via Canvas as a PDF.
- Help me (and your future self) out by showing your working!

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Unconstrained optimization → read the paper thoroughly, work through each step of the proof with pen & paper

Constrained optimization → look for key elements:

- Motivation
- Model set-up
- Timing of the game
- Solving the model
- Comparative statics
- Intuitions?

How to... come up with a research idea

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- Start thinking about ideas now. We will also do “brainstorms” in section. Group meetings to discuss preliminary ideas in Weeks 3/4.

Review: Definitions

Given an ordering of policies, the preferences of voter i are **single-peaked** if:

If $q'' \leq q' \leq q(\alpha^i)$ or, if $q'' \geq q' \geq q(\alpha^i)$, then $W(q''; \alpha^i) \leq W(q'; \alpha^i)$,

where W is the indirect utility function and $q(\alpha^i)$ is voter i's bliss point.

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Given an ordering of policies and voters, the preferences of a set of voters satisfy the **single-crossing property** if:

If $q > q'$ and $\alpha^{i''} > \alpha^i$, or if $q < q'$ and $\alpha^{i''} < \alpha^i$, then

$W(q; \alpha^i) \geq W(q'; \alpha^i) \implies W(q; \alpha^{i''}) \geq W(q'; \alpha^{i''})$.

What about multi-dimensional policy spaces?

Let $\boldsymbol{q}$ be a *vector* of policies and, as before, let α^i be a scalar. Then voters have **intermediate preferences** if their indirect utility function $W(\boldsymbol{q};\alpha^i)$ can be written:

$$W(\boldsymbol{q};\alpha^i) = J(\boldsymbol{q}) + K(\alpha^i)H(\boldsymbol{q})$$

where $K(\alpha^i)$ is monotonic in α^i, for any $J(\boldsymbol{q})$ and $H(\boldsymbol{q})$ common to all voters.

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- Key advantage (compared to, say, the Median Voter Theorem) is that you can deal with multidimensional policy spaces in a tractable way
- There are many "versions" of the probabilistic voting model – this exposition is based on the simple model in PT, which is in turn based on Lindbeck & Weibull (1987)

**This section is based on Prof. Larreguy's lecture slides from 2018.*

Basic idea of probabilistic voting

- Let π_P^i be the probability perceived by the candidates that voter i votes for party P, where $P = A, B$, and suppose that these probabilities refer to independent events for different voters.

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- Since, there are I voters, the expected vote share of party P is then

$$\pi_P = \frac{1}{I} \sum_{i=1}^{I} \pi_P^i.$$

- Under Downsian competition with two identical parties, π_P^i jumps discontinuously from 0 to 1 as voter i always votes with certainty for the party that promises the better policy.

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- Under Downsian competition with two identical parties, π_P^i jumps discontinuously from 0 to 1 as voter i always votes with certainty for the party that promises the better policy.
- Because of these discontinuous jumps, a Nash equilibrium may fail to exist.

Smoothing out discontinuities

- Probabilistic voting models instead develop a model where

$$\pi_A^i = F^i(V(\mathbf{q}_A; \alpha i), V(\mathbf{q}_B; \alpha^i)),$$

where $F^i(\cdot)$ is a smooth function, increasing in the first argument and decreasing in the second.

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- Note that $\mathbf{q}_A$ and $\mathbf{q}_B$ can now be n dimensional vectors.
- This smoothness implies that a small unilateral deviation by one party does not lead to jumps in its expected vote share and thus gives rise to well-defined equilibria.

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- In fact, we often go further and consider the special case where all $F^i(\cdot)$'s are **uniform**.

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- Clearly, party B faces a symmetric problem:

$$\pi_B = 1 - \pi_A \equiv 1 - \frac{1}{I} \sum_{i=1}^{I} F^i(V(\mathbf{q}_A; \alpha^i) - V(\mathbf{q}_B; \alpha^i)). \tag{2}$$

First-Order Conditions and Nash Equilibrium

- The first-order conditions for the two parties can be written as

$$\begin{aligned} \text{Party } A \text{ - } \sum_{i=1}^{I} f^i(V(\mathbf{q}_A;\alpha^i) - V(\mathbf{q}_B;\alpha^i))\frac{\partial V(\mathbf{q}_A;\alpha^i)}{\partial q_{jA}} &= 0, \\ \text{Party } B \text{ - } \sum_{i=1}^{I} f^i(V(\mathbf{q}_A;\alpha^i) - V(\mathbf{q}_B;\alpha^i))\frac{\partial V(\mathbf{q}_B;\alpha^i)}{\partial q_{jB}} &= 0, \end{aligned}$$

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  each for j=1,..,n.
- It must be that in a Nash equilibrium both parties will choose: $\mathbf{q}_A = \mathbf{q}_B$.
- We are then back to policy convergence!

Optimal Choices

- The FOCs for a maximum of (1), evaluated at the equilibrium policy $\mathbf{q}_A$, and taking $\mathbf{q}_B$ as given, can be written as

$$\sum_{i=1}^{I} f^i(0)\frac{\partial V(\mathbf{q}_A;\alpha^i)}{\partial q_{jA}} = 0 \text{ for } j = 1, .., n$$

where $\mathbf{q}_A = (q_{1A}, .., q_{jA}, ..)$ for all j, and $f^i(0)$ denotes the density of the c.d.f. $F^i(\cdot)$, evaluated at 0 (that is, in equilibrium) when $V(\mathbf{q}_A;\alpha^i) = V(\mathbf{q}_B;\alpha^i)$.

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- Thus, the equilibrium under this form of electoral competition implements the maximum of a particular weighted social welfare function, where voter i receives weight $f^i(0)$.
- In other words, we have that the equilibrium policies are determined as

$$\mathbf{q}^* \in \arg\max_{\mathbf{q}} \sum_{i=1}^{I} f^i(0) V(\mathbf{q}; \alpha^i)$$

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- In other words, more "responsive" voters, who have a higher density $f^i(0)$, receive a better treatment under electoral competition.
- Clearly, if all voters are equally responsive (if they all have the same value of $f^i(0)$), this form of electoral competition implements the utilitarian optimum.

Extensions of the basic PV model

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Examples:

- Divide voters into 3 classes: poor, middle class, rich. (Persson Tabellini – simplified version in PT pp. 53-57)
- Characterize voters as “ideological” or “swing” voters. Allow the incumbent to use repression/violence against certain groups as well as proposing a policy platform (Robinson & Torvik, 2009)