Gov 2006: Formal Political Theory II
Section 6

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Reminder

Final Paper Progress Check

Published

Edit

To make sure you are making progress on your final papers, the following is due on **March 26**.

Write down:

(1) the key assumptions you are making

(2) the notation you are using

(3) the solution concept you are using

(4) any special definitions

(5) the types of results you hope to obtain

For example, if you are studying an extensive-form non-cooperative game, then describe the actors, the game tree (strategy sets at each node and information sets) the payoffs, and the solution concept you will use. If you have any initial results, then state them. Otherwise, state some results you are hoping to be able to prove.

This should take only a few pages, probably 3-5.

Reminder

Section feedback form: `https://goo.gl/forms/ZYFgevfdeLNEpsnz2`

My office hours are Mondays 2-4pm. However, I will be traveling Monday 25th March so I will hold extra OH later this week, or via slack.

Today

Multidimensional electoral competition

- Intermediate preferences
- Recap main results from Plott & McKelvey
- Discussion: how should we interpret these results?

New definitions

- The **top-cycle set** is the set of all x such that for each $y \in X$ there exists a number k and a sequence of alternatives $x_1, ..., x_k$ with $x_1 = x,\ x_k = y$, and $x_i T x_{i+1}$ for all $i = 1, ..., k-1$.

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- Problem: the top-cycle set is often very big!

New definitions

- Let $n(x, y)$ be the number of voters who (weakly) prefer x to y.
- Then **minmax set** is the set of policies whose maximal opposition is minimal. Formally: $minmax(X) = arg\ min_x max_{y \neq x} n(y, x)$.

New definitions

- The **uncovered set** is the set of alternatives each of which can defeat every other alternative in the space either directly or indirectly at one remove.
- Formally: $x \in X$ is *covered* whenever there exists some other policy $y \in X$ such that yPx and $\{z : xPz\} \subseteq \{z : yPz\}$

Intermediate preferences

Voters have **intermediate preferences**, if their indirect utility function $U(p;\alpha^i)$ can be written as:

$$U(p;\alpha^i) = J(p) + K(\alpha^i)H(p),$$

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Intermediate preferences $\implies$ there is "a single source of disagreement among different individuals" (PT, p.26)

(Median Voter Theorem with Intermediate Preferences)
Suppose that voters vote sincerely and have intermediate preferences. Then a Condorcet winner always exists and coincides with the bliss point of the voter with the median value of α^i, $p(\alpha^m)$.

Proof

The proof is analogous to the proof of the median-voter theorem. Since $p(\alpha^m)$ is the maximum for agent α^m, we have that

$$U(p(\alpha^m);\alpha^m) = J(p(\alpha^m)) + K(\alpha^m)H(p(\alpha^m)) \geq U(p;\alpha^m) = J(p) + K(\alpha^m)H(p),$$

for all p. Therefore,

$$K(\alpha^m) \quad \lesseqqgtr \quad \frac{J(p(\alpha^m)) - J(p)}{H(p) - H(p(\alpha^m))} \text{ as } H(p) \gtreqqless H(p(\alpha^m)),$$

$$\text{for any } p \quad \neq \quad p(\alpha^m).$$

Proof (cont'd)

Suppose that $H(p) < H(p(\alpha^m))$ and $K(\alpha^i)$ is monotonically increasing (the case of monotonically decreasing is analogous). Then $K(\alpha^i) \geq K(\alpha^m)$ for all $\alpha^i > \alpha^m$, and these will all satisfy

$$K(\alpha^i) > \frac{J(p(\alpha^m)) - J(p)}{H(p) - H(p(\alpha^m))}$$

and therefore

$$U(p(\alpha^m); \alpha^i) = J(p(\alpha^m)) + K(\alpha^i)H(p(\alpha^m)) \geq U(p; \alpha^i) = J(p) + K(\alpha^i)H(p),$$

so they will support $p(\alpha^m)$ against p. The other cases are proved similarly. This shows that the policy $p(\alpha^m)$ always collects at least half the votes against any alternative policy.

Plott (1967): Sufficient conditions for equilibrium

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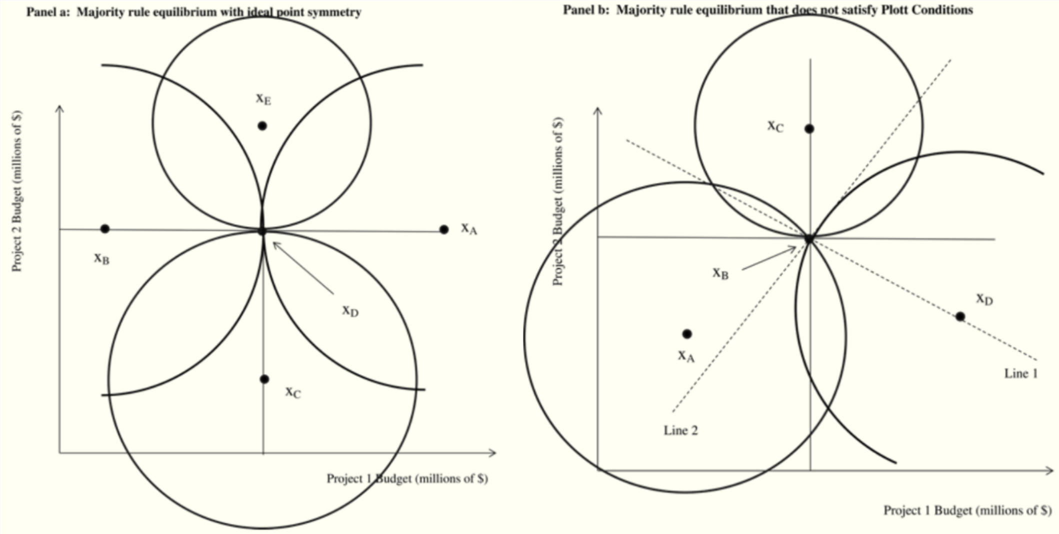
Most famous is **pairwise symmetry** = all nonzero utility gradients at the equilibrium must be divisible into pairs that point in opposite directions.

Plott (1967): Sufficient conditions for equilibrium



Plott (1967): Interpretation

"The most important point is that there is certainly nothing inherent in utility theory which would assure the existence of an equilibrium. In fact, it would only be an accident (and a highly improbable one) if an equilibrium exists at all."
(Plott, 1967, pp.791-2)

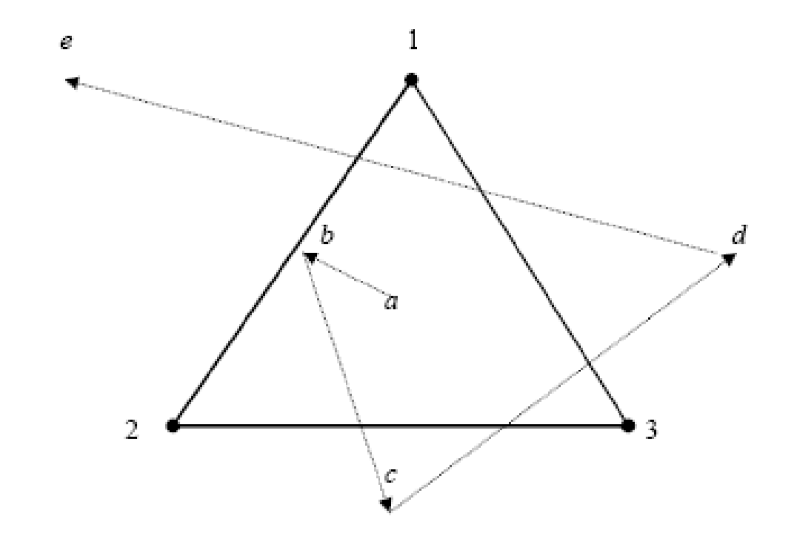
McKelvey (1976): The “chaos theorem”

Assuming:

- A multidimensional policy space
- at least 3 voters
- sincere voting
- Euclidean preferences
- simple majority rule
- Plott’s symmetry condition doesn’t hold

Then the cyclical top cycle set covers the whole policy space. That is, the agenda setter can achieve *any* policy outcome through a finite series of pairwise votes, regardless of the initial policy status quo.

McKelvey (1976): The "chaos theorem"



Interpretation

How should we interpret these results? Does McKelvey's "Chaos Theorem" imply actual chaos?

Response 1: Strategic behavior

Recall that McKelvey's theorem assumes that voters are sincere. The only strategic actor is (implicitly) the agenda-setter. Is this plausible?

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Example: The Powell Amendment*

A 1956 bill authorizing federal funding for schools is amended to require these schools to be racially desegregated. Some Republicans support the amendment strategically, knowing that Northern Democrats will not be able to justify voting against it – as a result, the amended bill fails since both Republicans and Southern Democrats vote against.

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*This example was popularized by Riker, though many scholars disagree with his interpretation! Nethertheless, it is a nice illustration of the logic...

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Definitions

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Theorem (Gibbard–Satterthwaite)

Suppose there are at least three alternatives and that for each individual any strict ranking of these alternatives is permissible. Then the only unanimous, strategyproof social choice function is a dictatorship.

Response 2: Institutions matter!

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BUT: there is a famous objection to this line of reasoning...

Riker: If certain institutions induce certain equilibria, then don't actors have preferences over institutions too (based on expected outcomes)? If so, the problem of instability returns!

Response 3: Uncertainty

Probabilistic voting models introduce uncertainty, which can either be viewed as random pre-election shocks to the candidates' favorability, or uncertainty from the candidates' perspective about the preferences of voters (or turnout).

For example, the model of Lindbeck & Weibull (1987) produces a Downsian (i.e. convergent) equilibrium, in which parties adopt identical platforms.