Gov 2006: Formal Political Theory II
Section 9

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Today

- PSET practice
    - The basic legislative bargaining model from Baron & Ferejohn
    - Finishing up part (d) from PSET 6 Q4!
- Multiparty spatial modelling
    - Theory: Cox (1990)
    - Empirics: Ezrow (2008); Tsebelis & Crosson (2017)

Legislative bargaining

Next week we will read Baron & Ferejohn (1989), a seminal paper on legislative bargaining.

A version of this model may (?) end up on the next problem set, so let's go through a simple practice problem today!

Legislative bargaining

Set-up:

- Legislature with n members, who divide rents r (normalized to 1) among themselves
- They only care about rents: $w^i(r) = r$
- Discount rate $= \delta \in [0, 1]$

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Timing:

1. A member has proposed an allocation of r.
2. Legislature votes $\{yay, nay\}$ on the proposed allocation under simple majority rule.
3. If the proposition is not passed, then a new member is randomly chosen (with $p = \frac{1}{n}$) to propose an allocation of r.
4. This proposal is voted on, and if it does not pass, then all rents are dissipated.

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Solution: Solve by backwards induction. What allocation would be chosen in the 2nd proposal?

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What is the continuation value of the game in the first stage?

> **Def'n**: The *continuation value* is the present value of the equilibrium payoff stream for a given player.

Legislative bargaining

Solution (cont'd): The continuation value for a given member is the probability of being chosen in the 2nd stage $\times$ the discounted value of the equilibrium rents:

$$V = \frac{1}{n} \cdot \delta \cdot 1 = \frac{\delta}{n}$$

So, what does the stage 1 proposer p_1 choose?

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$$a_i^* = \begin{cases} 1 - \frac{\delta}{n}\left(\frac{n-1}{2}\right) & \text{if } i = p_1 \\ \frac{\delta}{n} & \text{for } \frac{n-1}{2} \text{ randomly-chosen members } \in \mathcal{N} \setminus \{p_1\} \\ 0 & \text{for } \frac{n-1}{2} \text{ randomly-chosen members } \in \mathcal{N} \setminus \{p_1\} \end{cases}$$

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- What is the expected payoff for the other members? $\frac{\delta}{2n}$
- Both expected payoffs are decreasing in n
- What about the difference (i.e. the advantage of the agenda-setter)? It has range $[1 - \frac{\delta}{3}, 1 - \frac{\delta}{2}]$, at its maximum when $n = 3$ and decreasing as $n \to \infty$.

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Solution: We solve using backward induction:

- Let the continuation value for member i be V_i
- Member i will support any proposal giving her at least δV_i
- So the proposer offers δV_i to her winning coalition, 0 to everyone else, and keeps $1 - \frac{n-1}{2}\delta V_i$ for herself

Legislative bargaining

The continution value is thus the weighted combination:

$$
\begin{aligned}
V_i &= \frac{1}{n}\left[1 - \frac{n-1}{2}\delta V_i\right] + \left(\frac{n-1}{2n}\right)\delta V_i \\
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So the proposer offers $\delta V_i = \frac{\delta}{n}$ to members of the minimum winning coalition, 0 to the others, and keeps the rest for herself. This is identical to the equilibrium allocation found in part (a).

Multiparty spatial modelling: motivation

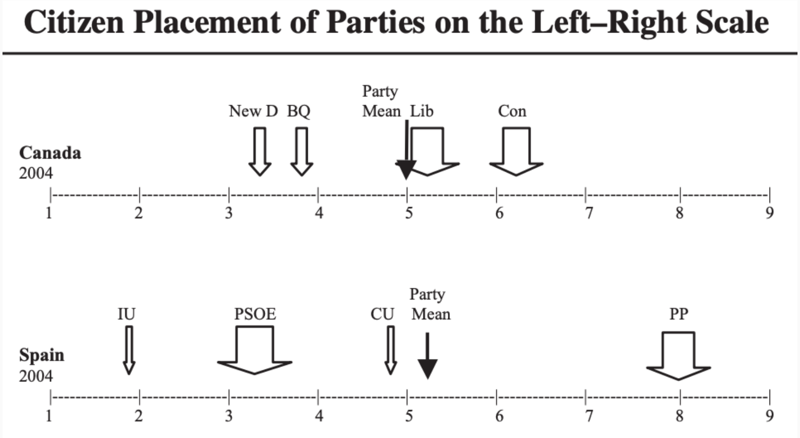

Figure 1: From Dalton (2008), "The quantity and the quality of party systems: Party system polarization, its measurement, and its consequences." *Comparative Political Studies*

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Party polarization index	2.06	4.33

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Party polarization index	2.06	4.33
Electoral system	Single member plurality	Proportional representation
Votes per voter	1	1
Partial abstention	N/A	N/A
District magnitude	1	1-35 (median=5)

Multiparty spatial modelling: Cox (1990)

Cox (1990) identifies three key factors that produce *centripetal* incentives (central clustering), at least in non-cumulative systems.

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- number of votes per voter is $\{high, low\}$
- partial abstention $\{permitted, not\ permitted\}$
- district magnitude $\{high, low\}$

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- partial abstention not permitted
- district magnitude low

Multiparty spatial modelling: empirical tests

TABLE 1 *Estimating Weighted Average Party Policy Extremism*

	Left–right party placements based on:								
	Experts (1980–83)			Citizens (1987–90)			Manifestos (1980–83)		
Variable	Full	Bivariate	Bivariate	Full	Bivariate	Bivariate	Full	Bivariate	Bivariate
Degree of proportionality	**−0.009 (0.01)**	**−0.004 (0.01)**		**−0.02** (0.007)**	**−0.02** (0.006)**		**−0.001 (0.01)**	**0.001 (0.008)**	
Effective Number of Parliamentary Parties	0.03 (0.06)		0.01 (0.05)	0.01 (0.03)		−0.03 (0.04)	0.02 (0.05)		0.02 (0.04)
Constant	0.78*** (0.20)	0.85*** (0.15)	0.76*** (0.19)	1.09*** (0.12)	1.12*** (0.09)	0.96*** (0.15)	0.39** (0.15)	0.43*** (0.11)	0.38** (0.14)
N	15	15	15	12	12	12	15	15	15
Adjusted R^2	−0.12	−0.06	−0.07	0.38	0.44	−0.03	−0.15	−0.07	−0.06

Notes: Parameters are ordinary least squares (OLS) coefficients. Estimated standard errors are in parentheses. $*p < 0.10$, $**p < 0.05$, $***p < 0.01$, two-tailed test. The dependent variable is the average party's policy distance from the left–right position of the mean voter weighted by its relative share of the vote, divided by the standard deviation of voters left–right self-placements (refer to Equation 1 in the text). The definitions of the independent variables are given in the text. The ideological scales based on experts and manifestos have been recalibrated so that these placements are also on the 1–10 scale that is used in the Eurobarometer surveys. Each country is observed only once for each set of analyses. The specific countries included in each set of analyses are presented in Appendix Table B.

Figure 2: From Ezrow (2008). "Parties' policy programmes and the dog that didn't bark: No evidence that proportional systems promote extreme party positioning." British Journal of Political Science 38.3 (2008): 479-497.

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"The conventional understanding developed in the influential spatial modelling study by Cox is that proportional electoral rules exert centrifugal incentives that motivate parties to present non-centrist policy programmes." (Ezrow, 2008)

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Is the "conventional understanding" correct?

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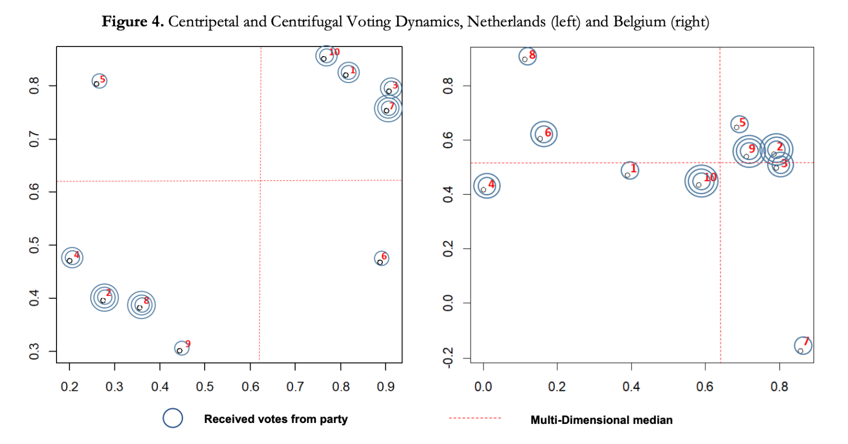

Figure 3: From Tsebelis & Crosson (2017). "MULTIPLE VOTE SYSTEMS: Toxin or Tonic for Political Polarization?"