

# Open Problems about the Simplex Method

Sophie Huiberts  
CNRS, LIMOS

# Linear programming

maximize  $c^T x$

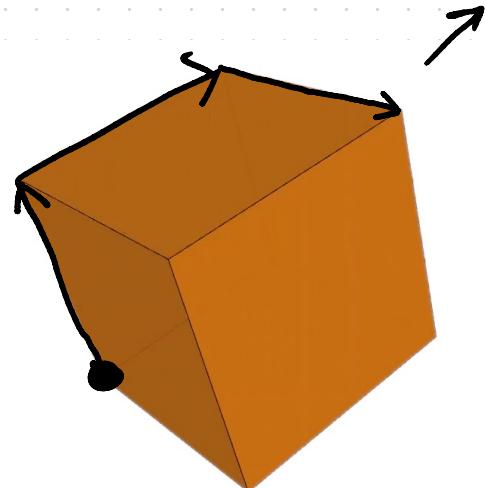
subject to  $Ax \leq b$

we get  $A \in \mathbb{R}^{n \times d}$

$b \in \mathbb{R}^n$

$c \in \mathbb{R}^d$

We compute  $x \in \mathbb{R}^d$



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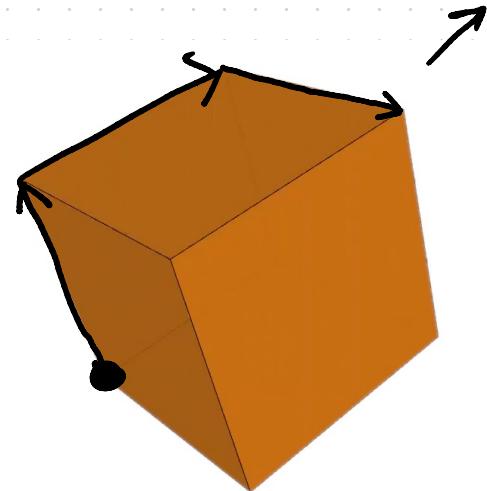
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how many pivot steps?

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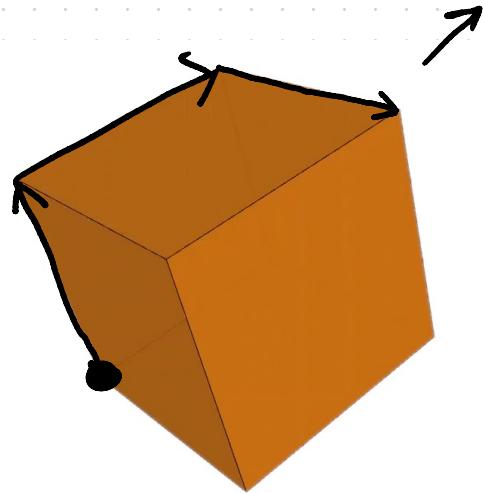
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how many pivot steps?  
(assuming non-degeneracy)

In practice

The simplex method takes  $2(n+d)$  pivot steps to solve an LP.

# Worst-case complexity

Theorem The simplex method  
has exponential worst-case  
complexity\*

\*terms and conditions apply

Simplex method is  
good in practice  
bad in theory

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this is a question for science

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many sophisticated papers  
are written about this

# Average-case analysis

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Average-case  
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Slack ratios

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lattice polytopes

Is our theory any good?

# At a conference

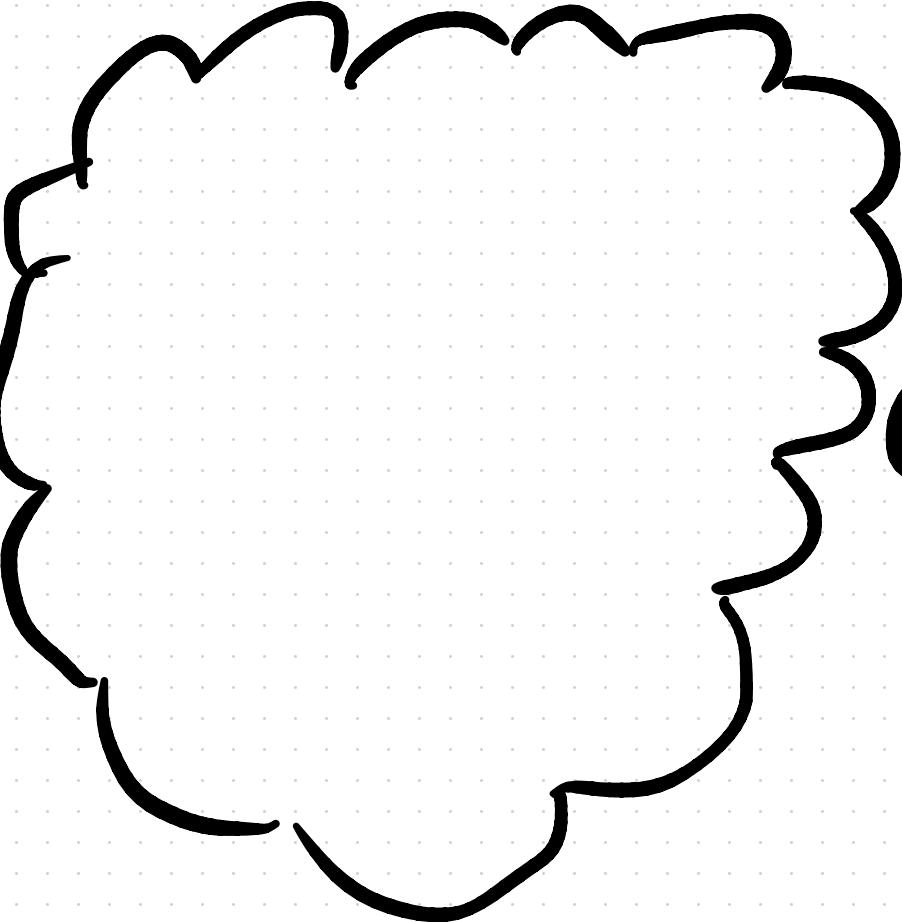


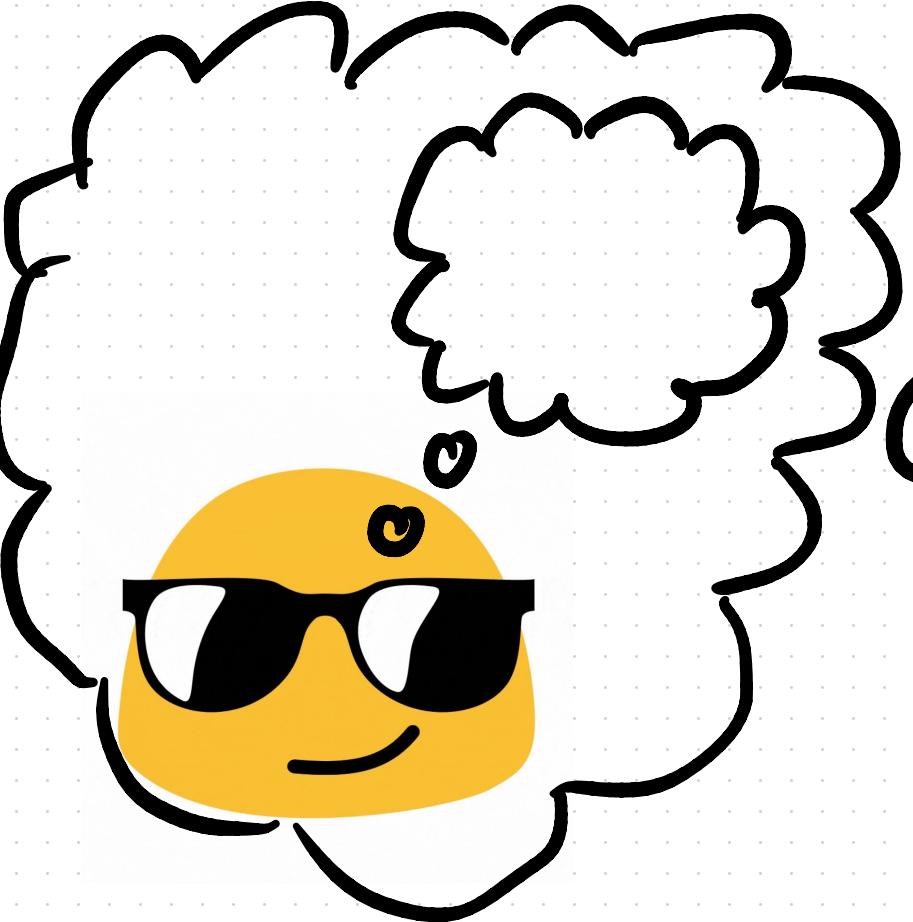
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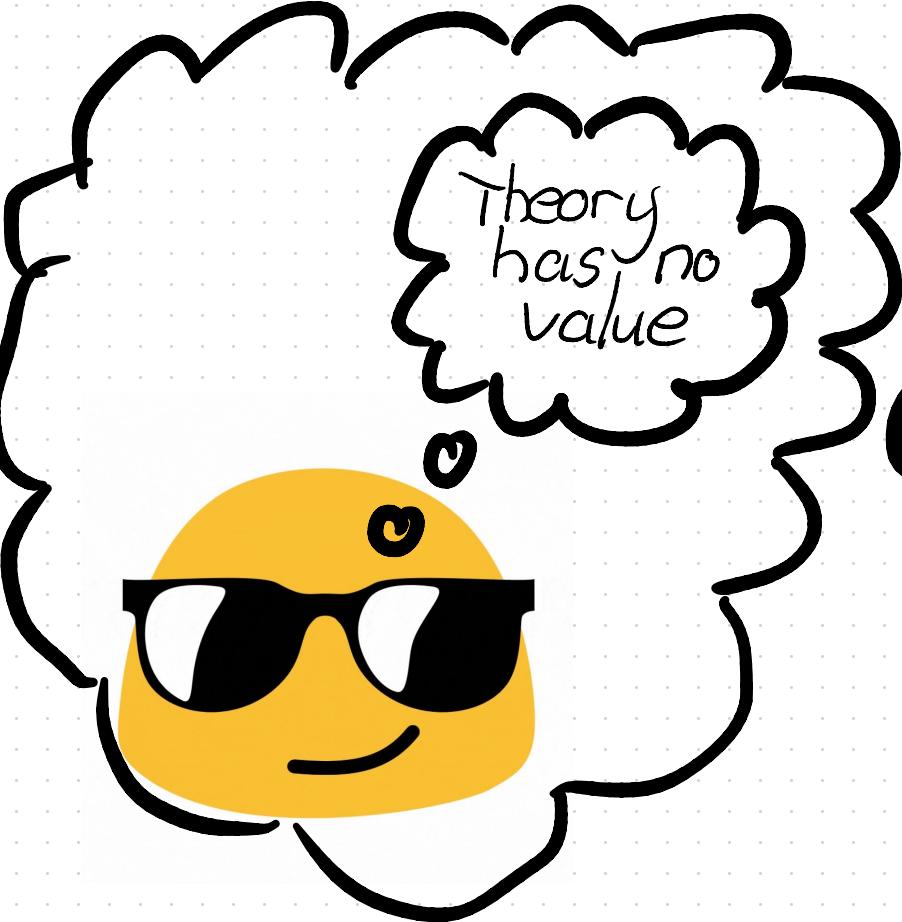


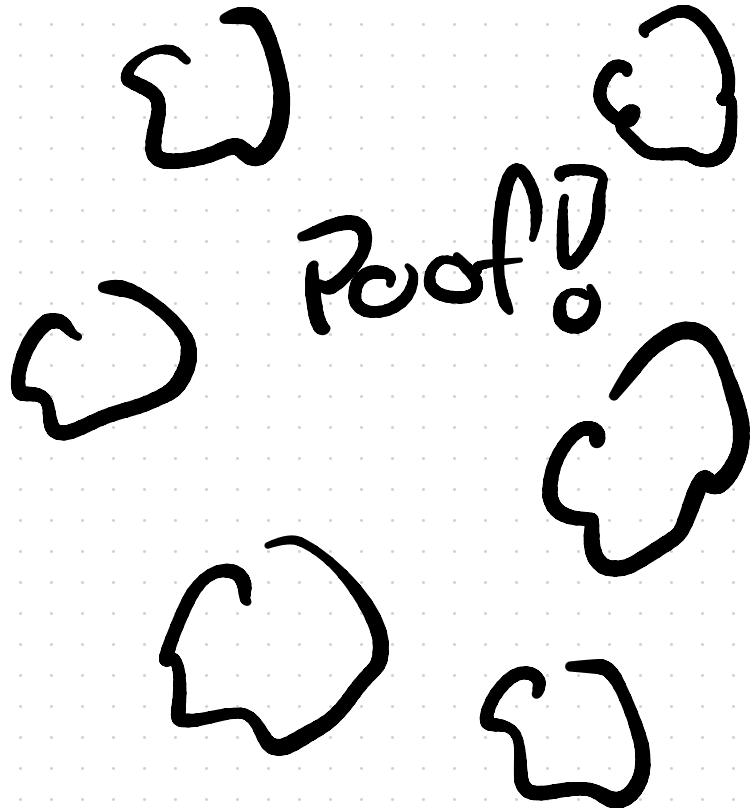
# At a conference











Theory  
has no  
value? . . .



Does theory have any bearing on reality?

Instancewise assumptions  
slack ratios

Distributional assumptions  
smoothed analysis

# Slack ratios

Suppose  $V$  is the vertex set of feasible region

$$\begin{aligned} & \text{maximize } c^T x \\ & \text{subject to } Ax \leq b \end{aligned}$$

$$\text{Let } \kappa = \frac{\min \{(b - Ax)_i : x \in V, i=1, \dots, n, (b - Ax)_i > 0\}}{\max \{ \|b - Ax\|_\infty : x \in V\}}$$

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Optimal solution after  $\frac{nd}{\kappa} \log(n/\kappa)$  steps

This  $e^{-\frac{kT}{d}}$  was Dantzig's intuition  
for developing the simplex method.  
never published?

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Kuno, Sano & Tsuruda proved computing  $\kappa$  is NP-Hard

A posteriori  $k$ : run simplex, observe slacks

Name	$m \times n$	pivots	$\text{est } s$	$\text{est } r$	$\text{est bound}$
afiro	$27 \times 59$	16	2.615	440.4	$3.260 \times 10^6$
kb2	$43 \times 84$	61	$2.478 \times 10^{-2}$	72.15	$1.781 \times 10^8$
sc50b	$50 \times 98$	49	$7.226 \times 10^{-1}$	324.9	$3.185 \times 10^7$
blend	$74 \times 157$	92	$1.497 \times 10^{-2}$	38.27	$5.208 \times 10^8$
sc105	$105 \times 208$	103	$2.050 \times 10^{-1}$	708.9	$1.395 \times 10^9$
scagr7	$129 \times 269$	140	3.304	6552	$1.236 \times 10^9$
sc205	$205 \times 408$	236	$5.187 \times 10^{-2}$	2380	$8.891 \times 10^{10}$
beaconfd	$173 \times 435$	102	$3.716 \times 10^{-2}$	5607	$2.798 \times 10^{11}$
lotfi	$153 \times 461$	239	$5.474 \times 10^{-3}$	4179	$1.443 \times 10^{12}$
bore3d	$233 \times 548$	200	$2.091 \times 10^{-4}$	301.2	$5.209 \times 10^{12}$
sctap1	$300 \times 780$	328	$1.154 \times 10^{-8}$	2.000	$1.444 \times 10^{15}$
agg2	$516 \times 818$	218	$3.047 \times 10^{-2}$	$1.676 \times 10^6$	$8.060 \times 10^{14}$
scagr25	$471 \times 971$	742	$3.007 \times 10^{-2}$	$1.118 \times 10^4$	$4.964 \times 10^{12}$
standmps	$467 \times 1542$	279	$1.998 \times 10^{-2}$	101.1	$7.718 \times 10^{10}$

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conclude bound  
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A posteriori  $\kappa$ : run simplex, observe slacks

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CAN  
conclude bound  
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CAN NOT  
conclude  
much else

Kuno  
Sano  
Tsuruda

## Open Questions:

Is  $k$  also NP-Hard to approximate?

can a MIP solver do it anyway?

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does presolve improve  $k$ ?

Are these the right questions to ask?

# Smoothed analysis

Let  $\bar{A} \in \mathbb{R}^{n \times d}$  have rows of norm  $\leq 1$ .

$\bar{b} \in [-1, 1]^n$ ,  $c \in \mathbb{R}^d$

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## Theorem

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↑  
much effort spent to  
get stronger bounds

Why smoothed analysis?

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independent measurement/numerical errors  
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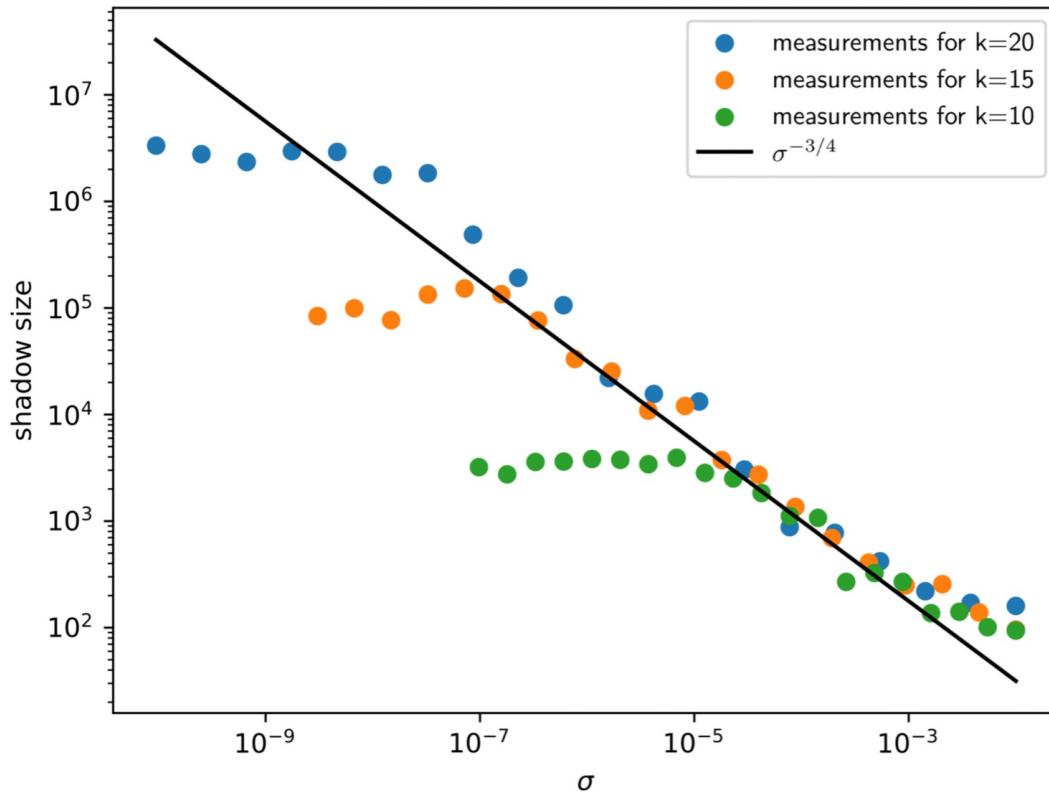
interpolates between worst case and  
average case analysis.

shows algorithm is fast on average  
in every large enough neighborhood

Can any of the resulting  
insights be tested  
experimentally?

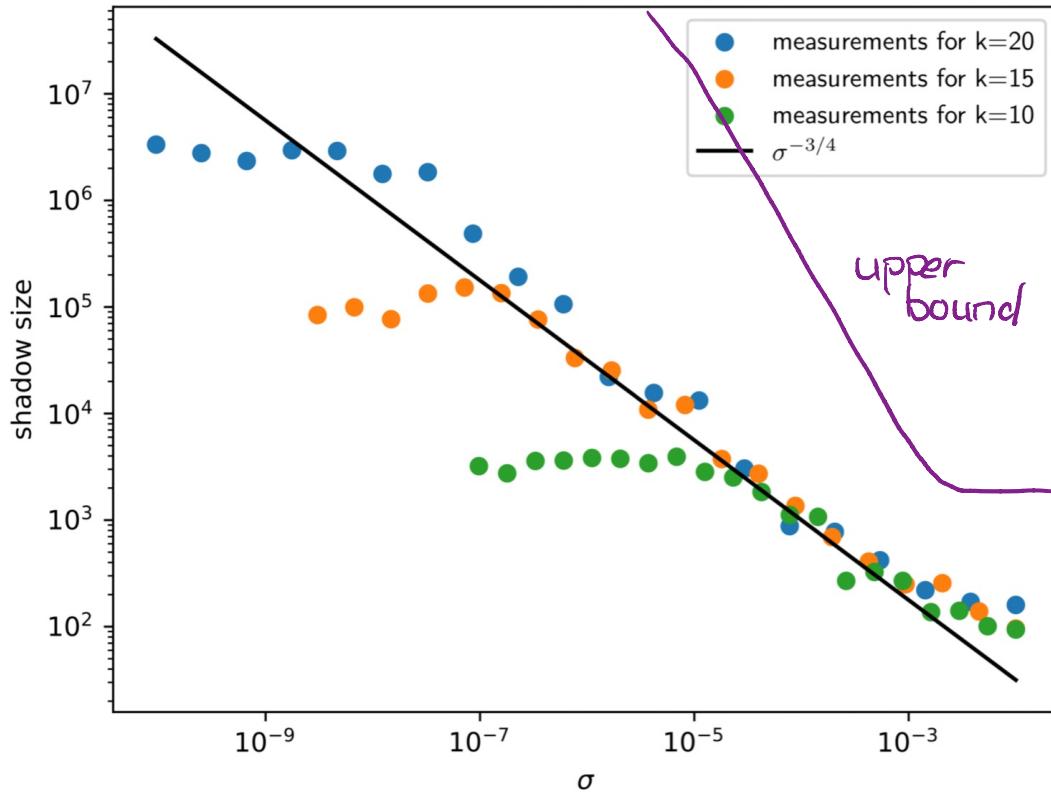
# Synthetic data

Measured shadow sizes



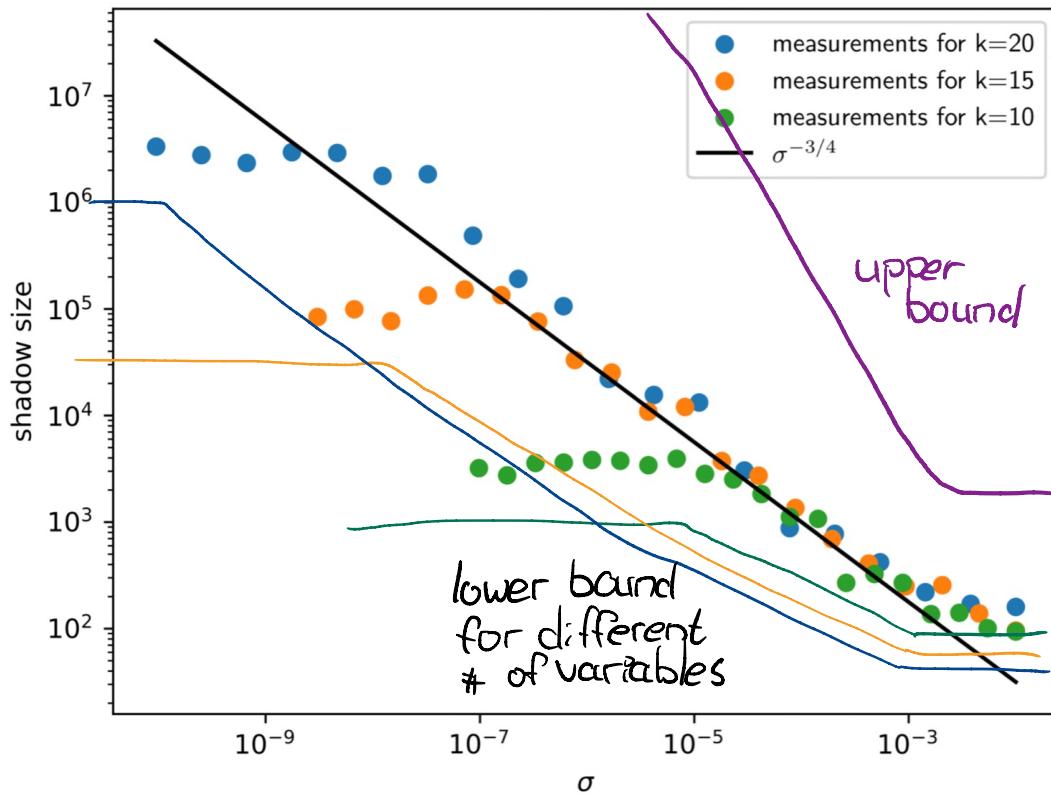
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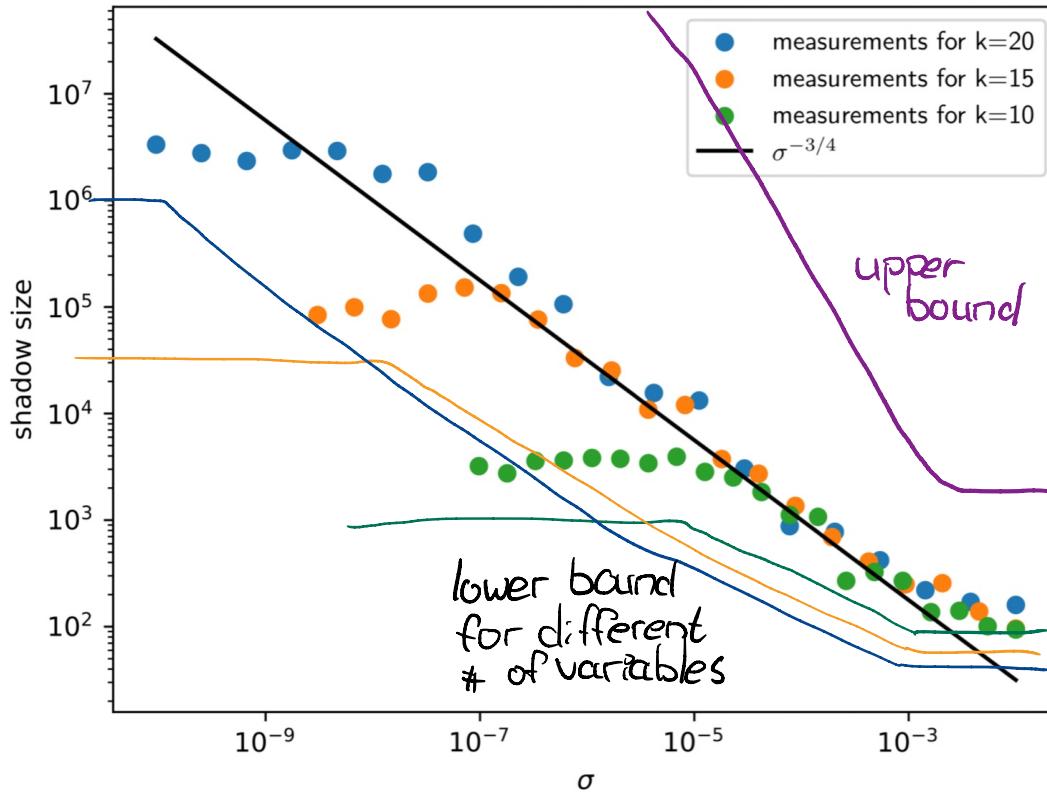
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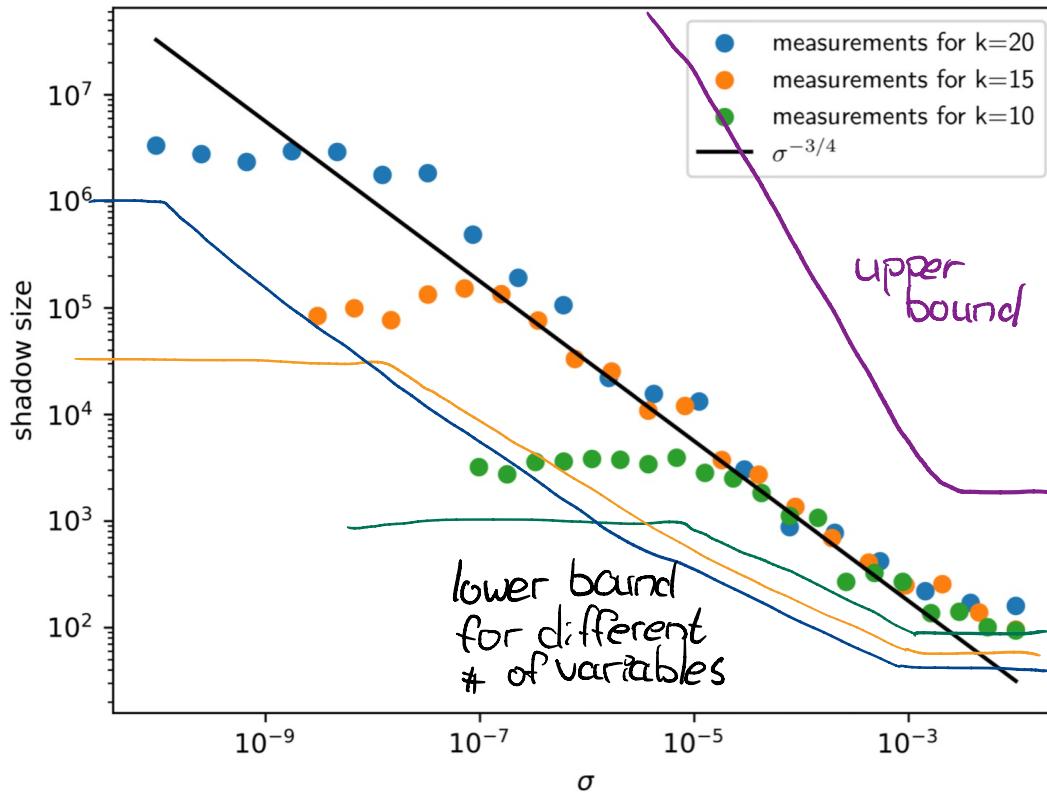
Measured shadow sizes



CAN tell if  
theorem is  
tight

# Synthetic data

Measured shadow sizes



CAN tell if theorem is tight

CAN NOT tell if theorem is useful

# The first linear program

Given 77 ingredients,

find the cheapest diet

that meets all 9 nutrient needs

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Side Question:

is this normal to want  
and possible to achieve?

George Stigler (1911 - 1991)

a quintessential conservative

opposed to rent-control  
& price controls

ON THINKING ABOUT GEORGE STIGLER\*

C. R. McCann, Jr. and Mark Perlman

opposed to minimum wage

THE ECONOMICS OF MINIMUM WAGE LEGISLATION

By GEORGE J. STIGLER\*

# THE ECONOMICS OF MINIMUM WAGE LEGISLATION

*By GEORGE J. STIGLER\**

One final point: We seek to abolish poverty in good part because it leads to undernourishment. In this connection, dietary appraisals show that in any income class, no matter how low, a portion of the families secure adequate diets, and in any income class, as high as the studies go, a

2  
1946

# THE ECONOMICS OF MINIMUM WAGE LEGISLATION

By GEORGE J. STIGLER\*

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## THE COST OF SUBSISTENCE

GEORGE J. STIGLER  
*University of Minnesota*

1945

would have cost about \$115 in 1939.<sup>25</sup>

These low-cost diets of the professional dieticians thus cost about two or three times as much as a minimum cost diet.

Why do these conventional diets cost so much? The answer is

Calculating a  
“minimal justified resource need”

is justifying reducing

resource availability



**INTERNATIONAL COVENANT  
ON ECONOMIC, SOCIAL  
AND CULTURAL RIGHTS**

1967 human  
rights treaty



INTERNATIONAL COVENANT  
ON ECONOMIC, SOCIAL  
AND CULTURAL RIGHTS



OFFICE OF THE HIGH COMMISSIONER  
FOR HUMAN RIGHTS



CESCR General Comment No. 12: The Right to Adequate Food (Art. 11)

1967 human  
rights treaty

1999 explanation  
of these rights



# INTERNATIONAL COVENANT ON ECONOMIC, SOCIAL AND CULTURAL RIGHTS



OFFICE OF THE HIGH COMMISSIONER  
FOR HUMAN RIGHTS



CESCR General Comment No. 12: The Right to Adequate Food (Art. 11)

The right to adequate food shall therefore not be interpreted in a narrow or restrictive sense which equates it with a minimum package of calories, proteins and other specific nutrients.

1967 human rights treaty

1999 explanation of these rights

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you know who else made  
a list of 77 foods and  
nutrients to compute a  
minimal resource allowance  
preventing malnutrition?

SLIDE 1

Ministry of Defense  
Coordination of Government Activities in the Territories

## **Food Consumption in the Gaza Strip – Red Lines**

**1 January 2008**

## SLIDE 2

### Goals of Analysis

- As part of the policy formulated by the Security Cabinet on September 19, 2007, Israel will limit the entry of goods into the Gaza Strip.
- In order to allow for a basic fabric of life in the Gaza Strip, the deputy defense minister approved allowing 106 trucks carrying basic humanitarian products into the Gaza Strip, mostly food (all products are specified in the appendices). In addition, food in seed form was approved for entry via the aggregate conveyor belt located near the Karni crossing.
- This research examines the main food component.
- The goal of the analysis – to identify the point of intervention for prevention of malnutrition in the Gaza Strip.
- The basis for the analysis is a model formulated by the Ministry of Health (at this point, according to average Israeli consumption) and a model formulated by the Palestinian Ministry of Economy.
- The Ministry of Health is conducting work for calculating the minimal subsistence basket based on the Arab sector in Israel. The “minimum basket” allows nutrition that is sufficient for subsistence without the development of malnutrition.

## SLIDE 2

### Background

- The security situation in the Gaza Strip and, on the other hand, the interest in preventing a humanitarian crisis have created a need for a solution to the issue of bringing essential goods into the Gaza Strip.
- The issue became more pressing following the Security Cabinet decision of September 19, 2007, according to which Israel would limit the entry of goods into the Gaza Strip.
- In order to allow for a basic fabric of life in the Gaza Strip, the deputy defense minister approved allowing 106 trucks carrying basic humanitarian products into the Gaza Strip, including 77 basic food products. In addition, food in grain form was approved for entry via the aggregate conveyor belt located near the Karni crossing.

In order to review the composition of food required by the population and in order to validate the “working assumption” (“106”), work was undertaken in cooperation with Ministry of Health officials in order to analyze the food basket required by the population and, as a derivative, the scope of food that enters.

### Main Working Assumptions

- The work that was undertaken analyzed the situation in terms of the food that enters the Gaza Strip and did not take into account distribution/division inside the area.
- There is internal food production in the Gaza Strip (vegetable and chicken farming), which was taken into account as a component of the food basket and needs to be addressed in terms of inputs.
- The figures used in the consumption models were “converted” into supply over five days and translated, in some of the sections, into truckloads, taking into account packaging weight.

## SLIDE 6

### General Daily Food Consumption in the Gaza Strip per Ministry of Health Scale (in tons)

Age/Type of food	Male/Female			Female			Male					Total for general population (minus 6- 12 month age bracket)	Food additive for -12 month age bracket)	Total quantity required for general population
	2-3	4-6	7-10	11-24	24-50	51+	11-14	15-18	19-24	24-50	51+			
Grains	11.94	37.15	40.43	63.94	53.52	14.65	25.66	25.71	25.71	68.33	15.23	382.28	3.98	386.26
Vegetables	12.62	37.00	40.52	60.03	50.25	14.64	24.64	24.68	24.68	65.61	14.85	369.53	4.21	373.74
Fruit	16.99	58.80	67.42	102.65	85.92	23.84	43.69	45.02	45.02	119.68	25.80	636.86	6.33	643.19
Milk	39.49	70.18	68.53	140.88	78.61	26.18	51.40	26.34	46.34	82.13	22.13	672.22	13.16	685.38
Meat	14.09	39.83	62.61	60.93	51.00	15.57	24.08	23.38	23.38	62.15	14.95	371.98	4.70	376.67
Oil	0.00	1.18	1.81	0.00	3.39	0.56	0.82	2.22	1.85	7.87	1.06	20.75	0.00	20.75
Sugar	4.35	5.04	5.58	4.95	5.27	2.01	2.87	5.18	4.07	12.78	1.85	53.95	1.45	55.40

- The figures are in tons per calendar day (consumption over seven days per week, unlike supply which is calculated based on five days per week).
- The portion of consumption is measured by the Health Ministry in Israel and provides for 2,000-2,500 calories per adult and 1,550 calories per child.
- The quantities in this table are average consumption according to Israeli standards and are not minimal subsistence portions.
- The Ministry of Health has been requested to calculate the minimal subsistence basket according to the Arab sector in Israel. The “minimal basket” allows for nutrition that is sufficient for subsistence without the development of malnutrition.

**SLIDE 13**

### Additives in Wheat

Number	Added Vitamin/Mineral	Quantity	
1	Thiamine (Vitamin B1)	4.4	Milligram per Kilogram
2	Vitamin B2	2.6	Milligram per Kilogram
3	Niacin	35	Milligram per Kilogram
4	Folic Acid	0.4	Milligram per Kilogram
5	Iron	25	Milligram per Kilogram
6	Folato	1	Milligram per Kilogram
7	Vitamin B6	2.5	Milligram per Kilogram
8	Zinc	15	Milligram per Kilogram
9	Vitamin A	1	Milligram per Kilogram
10	Vitamin B3	0.02	Milligram per Kilogram

## SLIDE 14

### Summary and Conclusions

- According to the model supplied by the Israeli Ministry of Health, there is a need for a daily supply of 104 food trucks (5 days a week).
- The model takes into account an exaggerated consumption of milk (3 times the known consumption in the Gaza Strip). Thus, on decreasing the milk component, the working assumption of 106 trucks (+ Karni conveyor belt) which includes about 90 truckloads of basic food, certainly meets nutritional needs in the Gaza Strip.
- The Ministry of Health Model assumes lower consumption of flour than what is known to be in effect.
- The Ministry of Health model is based on the average Israeli consumption, rather than a minimalist basket according to consumption habits in the Arab sector (the Ministry of Health is currently analyzing this).
- Following receipt of the new basket, it will be possible to define a red line as a warning sign.
- The Ministry of Health estimates that the new basket will be 20% lower than the current basket.

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noisy  
measurement

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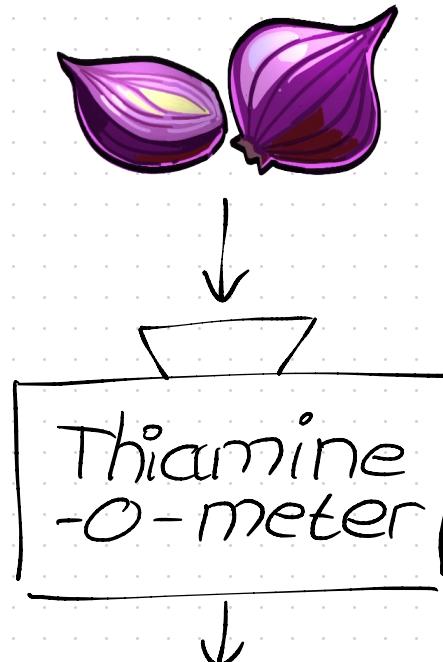
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Type of Operations	No. of repetitions
Multiplication	15,315
Division	1,234
Addition of two numbers	14,561
Addition of 77 numbers	190
Addition of 9 numbers	85

1948 ↙



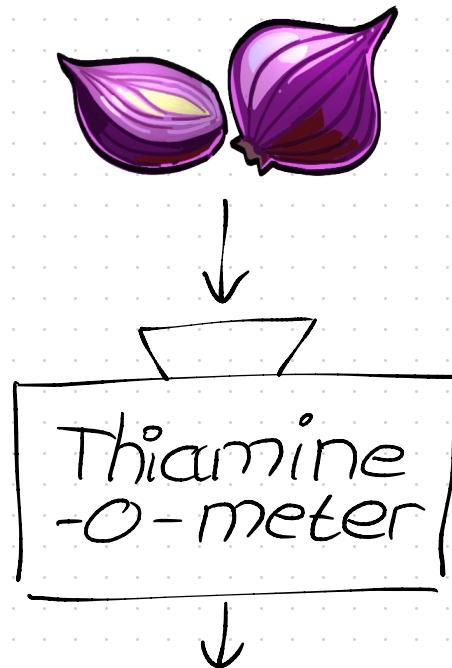
noisy  
measurement

# The first linear program

Given 77 ingredients,  
find the cheapest diet  
that meets all 9 nutrient needs

Type of Operations	No. of repetitions
Multiplication	15,315
Division	1,234
Addition of two numbers	14,561
Addition of 77 numbers	190
Addition of 9 numbers	85

←  
1948



Open question: how many pivot steps is that?

noisy  
measurement

# Final Intermission



↗ New York City

History  
Lesson

# Mathematical Tables Project 1938 - 1948

450 computers employed



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To perform these computations with desk machines required 5 computers for 21 days, with 4 hours per day supervision by a mathematician.

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# Linear Programming and Extensions

George B. Dantzig

Dantzig's  
famous  
book

## *STIGLER'S NUTRITION MODEL: AN EXAMPLE OF FORMULATION AND SOLUTION*

One of the first applications of the simplex algorithm was to the determination of an adequate diet that was of least cost.<sup>1</sup> In the fall of 1947, J. Laderman of the Mathematical Tables Project of the National Bureau of Standards undertook, as a test of the newly proposed simplex method, the first large-scale computation in this field. It was a system with nine equations in seventy-seven unknowns. Using hand-operated desk calculators, approximately 120 man-days were required to obtain a solution.

The particular problem solved was one which had been studied earlier by G. J. Stigler [1945-1], who had proposed a solution based on the substitution of certain foods by others which gave more nutrition per dollar. He then examined a "handful" of the possible 510 ways to combine the selected foods. He did not claim the solution to be the cheapest, but gave good

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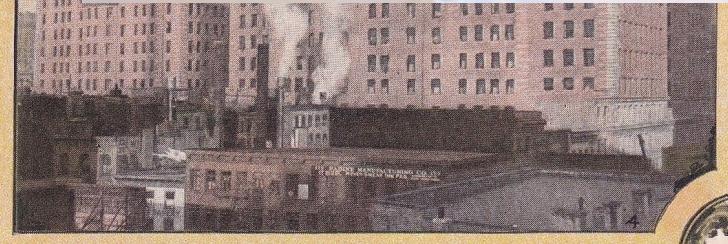
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38. Mathematical Tables Project computers with adding machines

# Historical takeaways



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Human computers  
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Their contributions  
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# Historical takeaways



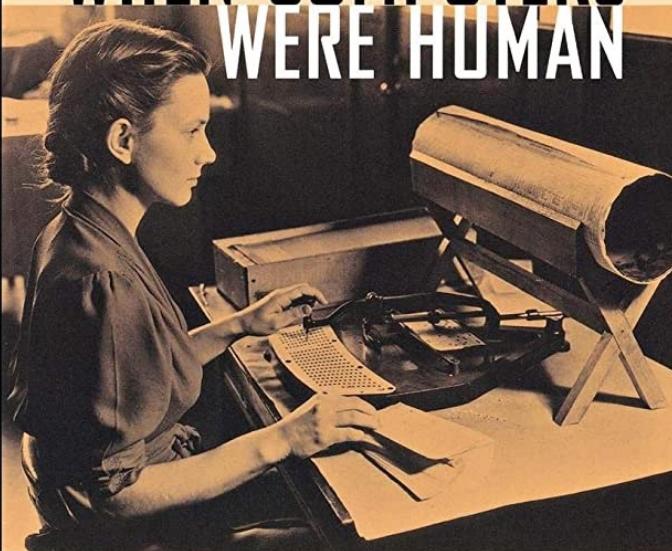
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Human computers played an important role in early comb. opt. history

Their contributions were made invisible by contemporary white men

Their demographics are exactly those underrepresented in our field today

# WHEN COMPUTERS WERE HUMAN



*David Alan Grier*

Consider including  
this history in  
your lectures

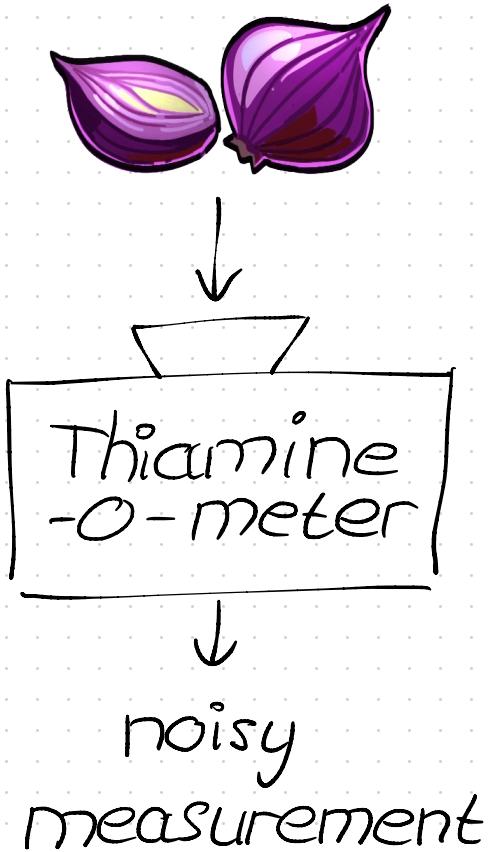


today's source



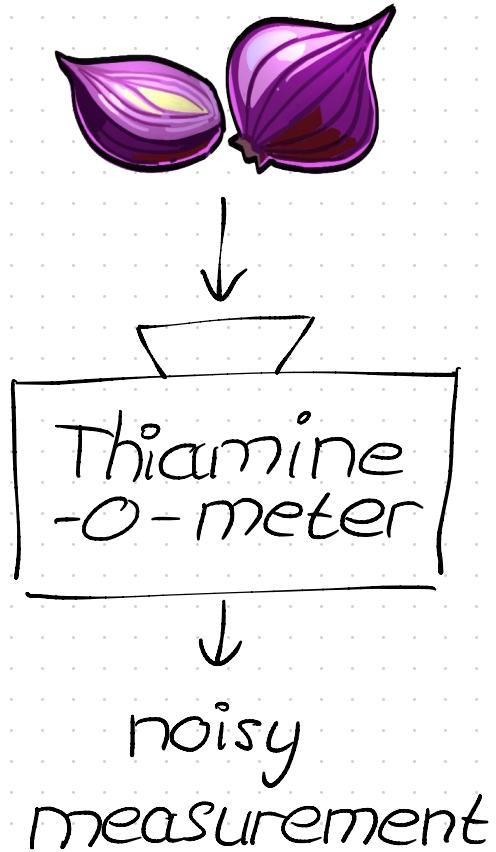
↓  
noisy  
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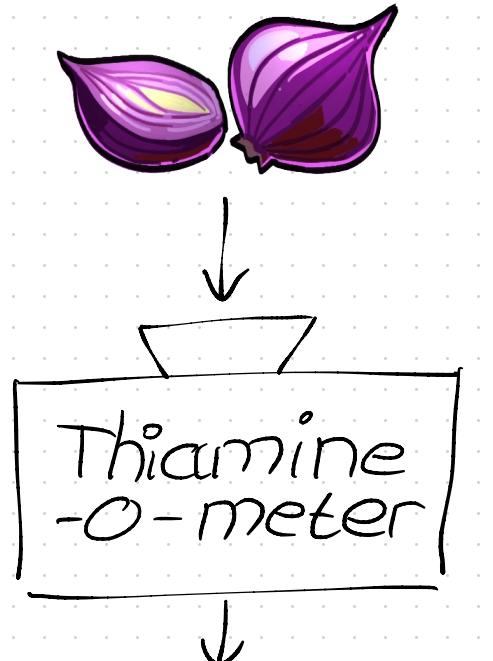
Probably not.



Could smoothed analysis  
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Probably not.

1. different pivot rule

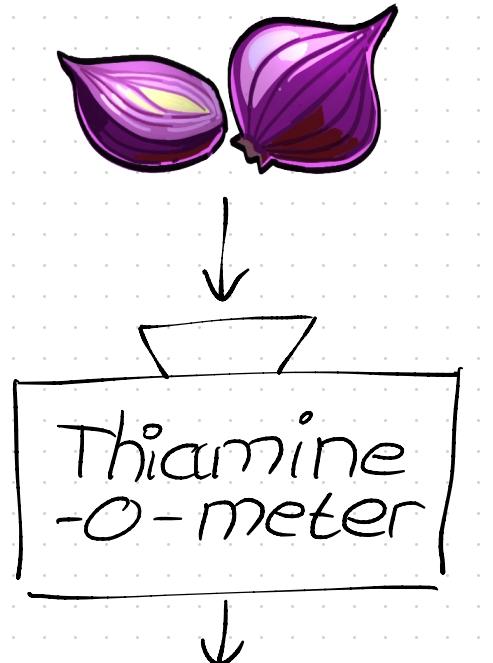


noisy  
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1. different pivot rule
2. different phase 1.

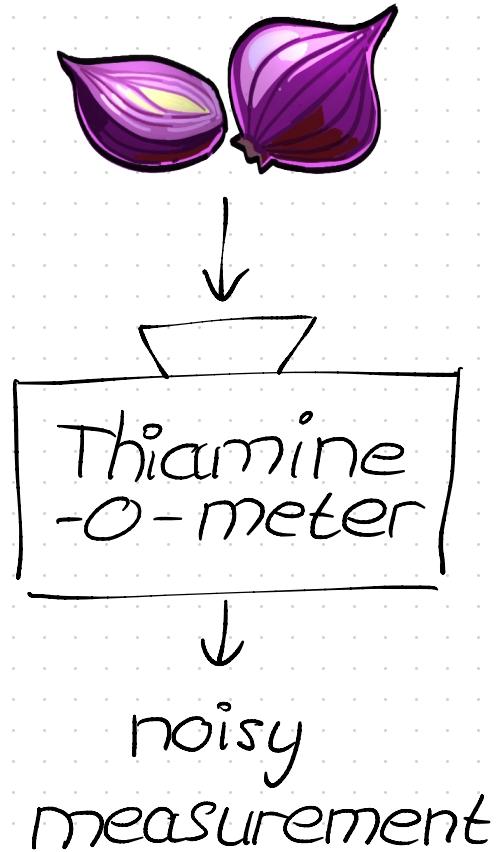


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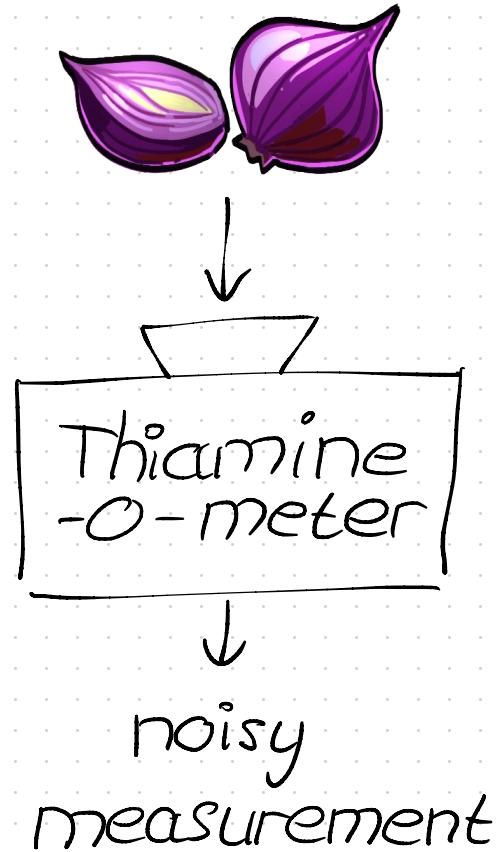
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2. different phase 1
3. non-negativity constraints



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1. different pivot rule
2. different phase 1
3. non-negativity constraints
4. multiplicative error  $\geq 15\%$ ,  
but need additive error



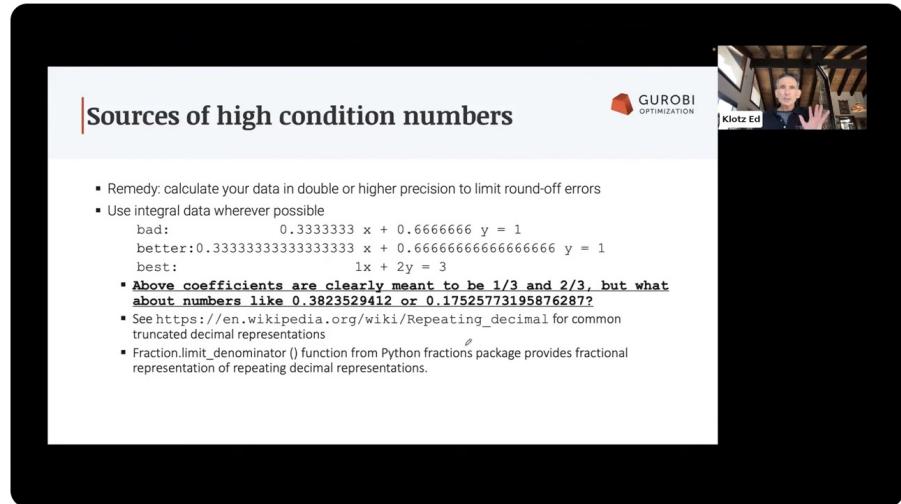
A more fundamental issue?

Smoothed analysis is based on the notion  
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# A more fundamental issue?

Smoothed analysis is based on the notion  
"more noise is better"

Practitioners say  
"less noise is better"



Sources of high condition numbers

GUROBI  
OPTIMIZATION

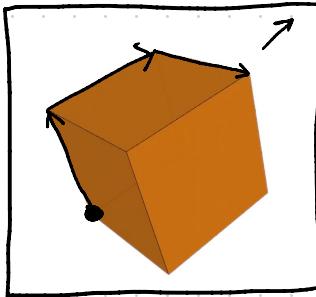
Klotz Ed

- Remedy: calculate your data in double or higher precision to limit round-off errors
- Use integral data wherever possible
  - bad:  $0.3333333 x + 0.6666666 y = 1$
  - better:  $0.333333333333333 x + 0.666666666666666 y = 1$
  - best:  $1x + 2y = 3$
- Above coefficients are clearly meant to be 1/3 and 2/3, but what about numbers like 0.3823529412 or 0.175257731958762877?
- See [https://en.wikipedia.org/wiki/Repeating\\_decimal](https://en.wikipedia.org/wiki/Repeating_decimal) for common truncated decimal representations
- Fraction.limit\_denominator () function from Python fractions package provides fractional representation of repeating decimal representations.

Are we studying the simplex method?

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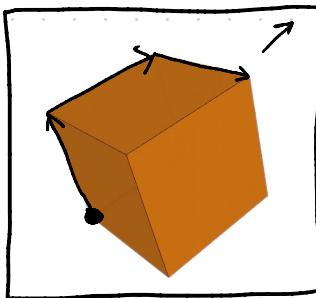
This geometry



doesn't exist.

Are we studying the simplex method?

This geometry



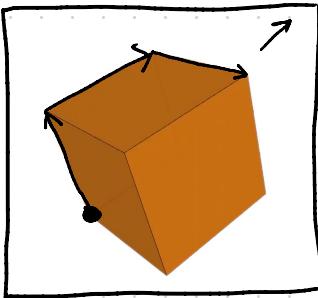
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What does exist :

- { linear algebra
- bound shifting
- bound perturbations
- Harris ratio test

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Our theory are untested  
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Are we looking at the wrong algorithm?