2x2 MIMO Link with Channel Estimation

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***Abstract*—The purpose of this project was to simulate a 2x2 MIMO link in MATLAB. In this paper, BER tables for different MIMO equalization techniques are computed and the data rate is discussed. The simulation passed 1000 symbols through two transmit antenna through a flat-fading Rayleigh channel with white noise to two receive antenna. The simulation was repeated with three channel estimation techniques; zero-forcing to cancel ISI, MMSE to mitigate ISI and account for the effect of noise, and pre-coding to mitigate the effect of the channel before the signal is passed through it. The Rayleigh flat-fading channel was created using Smith's method for maximum Doppler frequency shifts of 10, 100, an 1000 Hz. The simulation was iterated 10,000 times and the results were averaged for each combination of modulation and Doppler frequency shift. The results show that pre-coding is the most equalization technique, followed by MMSE, then zero-forcing.**

***Index Terms*—MIMO, MMSE, Zero-forcing, SVD, pre-coding, Rayleigh, Smith's fading**

I. Introduction

The multiple-input, multiple-output (MIMO) technique is popular in today's communications standards like LTE and 5G because it provides the high speed data transmissions demanded by the modern market. A 2x2 MIMO link sends data from two transmit antennas to two receive antennas. In a MIMO system, multiplexing increases the data rate and diversity improves performance. Specifically, multiplexing exploits the channel gain matrix structure to obtain independent signaling paths through which independent data is sent. A 2x2 MIMO system has a 2x2 channel gain matrix.

$$\begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix}$$

When combined with two transmitted signals and noise, two received signals have traveled along independent signaling paths.

r1 = h11s1 + h21s2 + n1
r2 = h12s1 + h22s2 + n2

This can be expressed in matrix form, and generalized for more receive/transmit antenna.

$$R = H_{R_x X T_x} S+N$$

If the data is sent from a mobile unit, the frequency experiences a Doppler frequency shift before reaching the receive antenna. This is modelled in the channel, with a larger Doppler shift making a worse channel.

When the channel is known that the receiver or at the transmitter, interference can be reduced. When there is accurate channel state information at the receiver (CSIR), the equalization techniques zero-forcing and MMSE (minimum mean squared error) can be used to decouple the effects of the channel. Zero-forcing nulls interference signals, and is implemented by applying the inverse of the frequency response of the channel to the received signal. Zero-forcing suffers from noise enhancement, decreasing MIMO performance.

$$W_{ZF} = H^{-1}$$

An equalizer that better optimizes between ISI mitigation and noise enhancement is the MMSE equalizer. The MMSE equalizer takes into account effects of noise. It is implemented by applying the following detector matrix to the received signal before demodulation.

$$W_{MMSE} = H^H(HH^H + \sigma_n^2 I)^{-1}y$$

When there is accurate channel state information at the transmitter (CSIT), pre-coding or beamforming is used to decrease the corruption of the signal caused by the channel. Precoding aligns the transmit signal vector with the eigenvector of the channel. To implement, the Singular Value Decomposition (SVD) of the channel gain matrix is obtained. The signal is both pre- and post-filtered with the orthogonal vectors V and U obtained from the SVD.

II. Implementation

A. Generate Rayleigh Fading Channel

To simulate the MIMO channel, Rayleigh fading coefficients were generated using Smith's fading simulator. First, complex

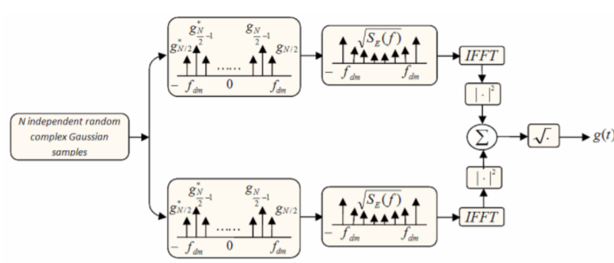

Fig. 1. Smith's Fading Simulator

Gaussian noise variables of length N/2 were generated. They were reflected and conjugated to create the negative frequency components. The positive and negative components were concatenated to make a noise vector of length N. This was repeated for both the in-phase and quadrature variables. These were each multiplied by the fading spectrum $\sqrt{S_E(f)}$). (The

Doppler spreading spectrum $S_E(f)$ was created for frequencies from the negative of the maximum Doppler frequency to the maximum Dppler frequency, however their endpoints were not included because the PSD goes to infinity there.) Next, an IFFT was performed on the frequency domain signals to get two time series of length N. The square root of the sum of these components was taken to obtain the simulated Rayleigh fading signal. This process was put in a function that took the length of the series N and the maximum Doppler frequency shift f_m as inputs. To generate a single channel gain matrix, this function was called four times with N = 1 and fm = 10, 100, or 1000 depending on the trial. The four outputs were then arranged in a 2x2 channel gain matrix.

B. Create 2x2 MIMO Link

To simulate the 2x2 MIMO link, bits were generated. The number of bits generated depended on the M-ary of the modulation. This bit generation affects data rate, as each type of QAM allows a different number of bits per symbol ($log2(M)$ bits per symbol). For example, 4 QAM sends 2 bits per symbol and 64 QAM sends 8 bits per symbol. If each symbol takes the same amount of time to send, then 64 QAM sends more bits per unit time than 4 QAM does. The bits were then shaped into symbols and modulated. The symbol vector was divided into two vectors, s1 and s2, one for each transmitter. The signals were arranged in a 2x1 column vector. Noise vectors n1 and n2 were then generated and arranged in a 2x1 column vector. The data was then "transmitted" through the channel and noise by multiplying the gain matrix H with the signal matrix and adding the noise matrix. This resulted in the received matrix, a single column with r1 and r2. The CSIR equalizers were implemented at this stage. The data was demodulated, reshaped, and BER was computed.

	BSPK	4 QAM	16 QAM	64 QAM
fm = 10Hz	2.4855e-01	3.2864e-01	4.0766e-01	4.3824e-01
fm = 100Hz	2.4793e-01	3.2900e-01	3.9330e-01	4.2994e-01
fm = 1kHz	2.5814e-01	3.3791e-01	4.2201e-01	4.5238e-01

Fig. 2. BER Table for with no channel estimation (SNR = 20 dB). Repeated for BPSK, 4 QAM, 16 QAM, and 64 QAM modulation and maximum Doppler frequency shifts of 10, 100, and 1000 Hz.

C. Zero-Forcing

After transmission, zero-forcing was implemented by multiplying the inverse of the channel gain matrix with the received vector. A BER with magnitude of e-3 was obtained for 4 QAM with SNR of 20dB. BER increased with higher orders of QAM and greater maximum Doppler frequency shift.

D. MMSE

Alternately, MMSE equalization was implemented by multiplying the received signal by the W_MMSE above. The noise matrix was obtained by taking the variance of n1 and the variance of n2 and arranging them in a column vector. The results show that BER reached an order of magnitude of e-4 for 4 QAM with SNR of 20 dB. Like zero-forcing, BER increased with higher orders of QAM and greater maximum Doppler frequency shift.

	BSPK	4 QAM	16 QAM	64 QAM
fm = 10Hz	5.9340e-04	1.2361e-03	4.2480e-03	1.3442e-02
fm = 100Hz	5.8780e-03	1.1262e-02	3.8554e-02	9.1267e-02
fm = 1kHz	5.1872e-02	9.0321e-02	1.9092e-01	2.7394e-01

Fig. 3. BER Table for zero-forcing channel estimator (SNR = 20 dB). Repeated for BPSK, 4 QAM, 16 QAM, and 64 QAM modulation and maximum doppler frequency shifts of 10, 100, and 1000 Hz.

	BSPK	4 QAM	16 QAM	64 QAM
fm = 10Hz	5.0140e-04	8.4175e-04	4.2188e-03	1.3270e-02
fm = 100Hz	5.2515e-03	9.3595e-03	3.4465e-02	8.9799e-02
fm = 1kHz	4.4910e-02	7.6773e-02	1.8116e-01	2.6454e-01

Fig. 4. BER Table for MMSE channel estimator (SNR = 20 dB). Repeated for BPSK, 4 QAM, 16 QAM, and 64 QAM modulation and maximum Doppler frequency shifts of 10, 100, and 1000 Hz.

E. Pre-coding with SVD

Precoding was implemented by following the method described in "Broadband MIMO-OFDM Wireless Communications" by Stuber et al. The orthogonal SVD decomposition of the channel gain matrix is obtained, then V was used to prefilter the signal immediately before transmission and U was used to postfilter the received signal vector. The results show that BER reached an order of magnitude of e-4 for 4 QAM with SNR of 20 dB. Like zero-forcing and MMSE, BER increased with higher orders of QAM and greater maximum Doppler frequency shift.

	BSPK	4 QAM	16 QAM	64 QAM
fm = 10Hz	3.5270e-04	4.4575e-04	1.7158e-01	2.8478e-01
fm = 100Hz	4.6822e-03	4.9124e-03	1.0923e-01	2.0616e-01
fm = 1kHz	3.8445e-02	3.8058e-02	2.7068e-01	3.6180e-01

Fig. 5. BER Table for pre-coding (SNR = 20 dB). Repeated for BPSK, 4 QAM, 16 QAM, and 64 QAM modulation and maximum Doppler frequency shifts of 10, 100, and 1000 Hz.

III. Discussion of Results

All three channel estimation improved upon BER performance compared to MIMO without a channel estimator. Also, in all three techniques, higher maximum Doppler shift and higher orders of QAM resulted in inferior BER performance. Only lower orders of QAM, or a lower data rate, provided acceptable BER performance. By incorporating noise in the equalization, MMSE improved BER versus zero-forcing.

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