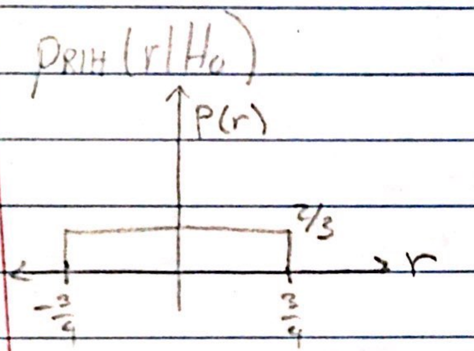
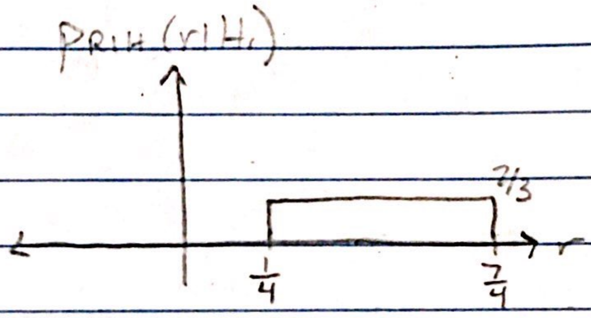
a. $p_{R|H}(r|H_0) = 0 + N = \begin{cases} 2/3 & -3/4 < r < 3/4 \\ 0 & \text{else} \end{cases}$

$p_{R|H}(r|H_1) = 1 + N = \begin{cases} 2/3 & 1/4 < r < 7/4 \\ 0 & \text{else} \end{cases}$

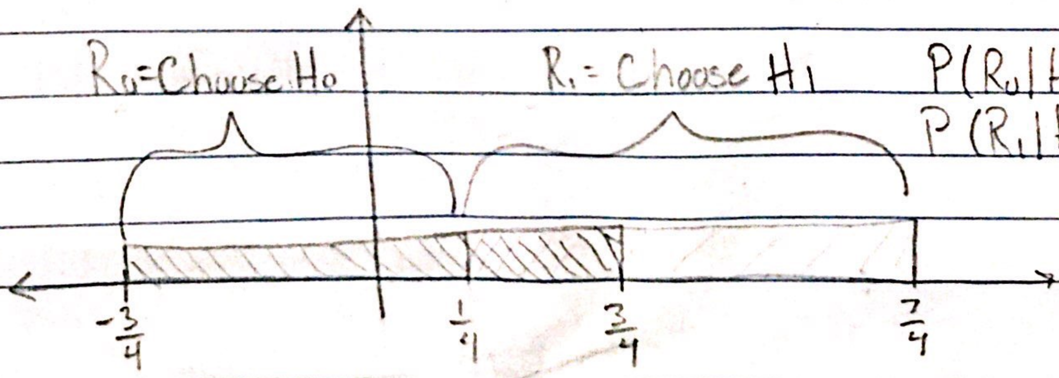




b. $L(r) = \left[\dfrac{p_{R|H}(r|H_1)}{p_{R|H}(r|H_0)}\right] \underset{H_0}{\overset{H_1}{\gtrless}} \dfrac{(C_{10} - C_{00})P_0}{(C_{01} - C_{11})P_1} \equiv \eta$

$L(r) = \dfrac{\begin{pmatrix} 2/3 & 1/4 < r < 7/4 \\ 0 & \text{else} \end{pmatrix}}{\begin{pmatrix} 2/3 & -3/4 < r < 3/4 \\ 0 & \text{else} \end{pmatrix}} \underset{H_0}{\overset{H_1}{\gtrless}} \dfrac{(1-0)\,1/4}{(1-0)\,3/4} \equiv \eta$

$= \begin{cases} 0 & -3/4 < r < 1/4 \\ 1 & 1/4 < r < 3/4 \\ \infty & 3/4 < r < 7/4 \\ 0 & \text{else} \end{cases} \underset{H_0}{\overset{H_1}{\gtrless}} \dfrac{1}{3}$



$P(R_0|H_1) = 0$

$P(R_1|H_0) = \int_{1/4}^{3/4} \frac{2}{3}\, dr$

$= \frac{1}{2} \cdot \frac{2}{3}$

$= \frac{1}{3}$

c. Pr[Decide H_0, H_1 true] + Pr[Decide H_1, H_0 true]

$= Pr(R_0 | H_1)\, P(H_1) + Pr(R_1 | H_0)\, P(H_0)$

$= 0 \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{4}$

$= \frac{1}{12}$

$= P[error]$

$P[error] = \frac{1}{12}$

d. ROC