Likelihood Ratio Tests for Fixed Effects and Variance Components

Hypothesis Testing for Multilevel Regression Models

- When fitting multilevel models, many hypothesis tests regarding the parameters are based on comparisons of *competing* models
- **Reference model**: A "full" or "saturated" model containing all parameters of interest; different from the null model where only intercepts are included
- **Nested model**: A "reduced" model, where some of the parameters in the reference model are constrained to zero

Likelihood Ratio Testing

- Compared to the reference model, does fitting the nested model substantially drop the likelihood (or make the observed data seem less likely)?
- Likelihood Ratio Tests: -2 **ML** log-likelihoods

Fixed Effects

Covariance Parameters

Likelihood Ratio Tests for Fixed Effects

- Simple idea, but assumes "large" sample of clusters and "large" samples within clusters
- **Null hypothesis**: selected fixed effects are all equal to zero (not important predictors)
- **Test statistic**: difference in final -2 **ML** log-likelihoods between nested and reference models (nested model has some fixed effects set to zero)
- Refer difference to chi-square distribution with q degrees of freedom, where q is difference between two models in number of fixed effects estimated

Likelihood Ratio Tests for (Co)variance Parameters

- **If the null hypothesis does not specify that a covariance parameter is equal to the boundary of its parameter space (e.g., a variance is equal to zero):**

Same approach as for fixed effects**, using restricted maximum likelihood**

- **If testing that a variance is equal to zero (e.g., the variance of a given random effect):**

Compute test statistics, but refer to approximate mixture of chi-square distributions

See below for an example using the ESS data

Not widely implemented in software...

Likelihood Ratio Test Example: Testing for Random Slopes in the ESS Case Study

- Null hypothesis: The variance of the random interviewer effects on the slope of interest is zero (in other words, these random effects on the slope are not needed in the model)
- Alternative hypothesis: The variance of the random interviewer effects on the slope of interest is greater than zero
- First, fit the model WITH random interviewer effects on the slope of interest, using restricted maximum likelihood estimation

-2 REML log-likelihood = 7143.3

- Next, fit the nested model WITHOUT the random interviewer effects on the slope:

-2 REML log-likelihood = 7166.8 (higher value = worse fit!)

- Compute the positive difference in the -2 REML log-likelihood values ("REML criterion") for the models:

Test Statistic (TS) = 7166.8 – 7143.3 = 23.5

- Refer the TS to a mixture of chi-square distributions with 1 and 2 DF, and equal weight 0.5:

```
> 0.5*(1-pchisq(23.5,1)) + 0.5*(1-pchisq(23.5,2))

[1] 4.569231e-06
```

$p < 0.001$ (Reject the null hypothesis that the variance of the random interviewer effects on the slope is zero; strong evidence of interviewer variance in the slopes!)

- We would follow the same approach when testing the variance of the random interviewer effects on the intercept
- If we fit a model that ONLY includes random effects of the clusters on the intercept (a random intercept model), we would test that variance by removing the random effects, and referring the same test statistic to a mixture of chi-square distributions with 0 and 1 DF, and equal weight 0.5