



Estimating a Difference in Population Proportions with Confidence

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C.S. Mott Children's Hospital Poll

C.S. Mott Children's Hospital conducted a national poll on an issue in children's health, water safety. We will be looking at an example about swimming lessons.



Research Question

What is the difference in population proportions of parents reporting that their children age 6-18 have had some swimming lessons between white children and black children?

Populations - All parents of white children age 6-18 and all parents of black children age 6-18

Parameter of Interest - Difference in population proportions ($p_1 - p_2$)

We'll let 1 = white and 2 = black

Research Question

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Construct a 95% confidence interval for the difference in population proportions of parents reporting that their child has had swimming lessons.

Survey Results

- A sample of 247 parents of black children age 6-18 was taken with 91 saying that their child has had some swimming lessons.
- A sample of 988 parents of white children age 6-18 was taken with 543 saying that their child has had some swimming lessons.

Difference in Proportion Confidence Interval

Best Estimate \pm Margin of Error

$\hat{p}_1 - \hat{p}_2 \pm \text{Margin of Error}$

$\hat{p}_1 - \hat{p}_2 \pm \text{“a few”} \cdot \text{se}(\hat{p}_1 - \hat{p}_2)$

$$\hat{p}_1 - \hat{p}_2 \pm 1.96 \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Best Estimate of the Parameter

$$\hat{p}_1 = 543/988 = 0.55$$

1 = white

$$\hat{p}_2 = 91/247 = 0.37$$

2 = black

$$\hat{p}_1 - \hat{p}_2 = 0.55 - 0.37 = 0.18$$

Difference in Proportion Confidence Interval

$$\hat{p}_1 - \hat{p}_2 \pm 1.96 \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$0.18 \pm 1.96 \cdot 0.0345$$

$$0.18 \pm 0.0677$$

$$(0.1123, 0.2477)$$

Interpreting the Confidence Interval

“range of reasonable values for our parameter”

With 95% confidence, the population proportion of parents with white children who have taken swimming lessons is 11.23 to 24.77% higher than the population proportion of parents with black children who have taken swimming lessons.