



Setting up a Test for a Population Proportion

Reed Coots

Statistics Course Developer



Hypothesis Testing

Why do we do Hypothesis Tests?

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Could the value of the parameter be _____?

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Could the value of the parameter be _____?

Use data to help support that claim

C.S. Mott Children's Hospital Poll

C.S. Mott Children's Hospital conducted a national poll on an issue in children's health, sleep habits. We will be looking at an example about lack of sleep in teens.

Research Question

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Parameter of Interest - p

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Population - Parents with a teenager (age 13-18)

Parameter of Interest - p

Test for a significant increase in the proportion of parents with a teenager who believe that electronics and social media is the cause for lack of sleep.

Hypotheses

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$$H_0 : p = 0.52$$

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$$H_0 : p = 0.52$$

$$H_a : p \neq 0.52$$

IVQ

What symbol should replace the ?

Hypotheses

$$H_0 : p = 0.52$$

$$H_a : p > 0.52$$

“Significant Increase”

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Where p is the population proportion of parents with a teenager who believe that electronics and social media is the cause of their teenager's lack of sleep

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Where p is the population proportion of parents with a teenager who believe that electronics and social media is the cause of their teenager's lack of sleep

$$\alpha = 0.05$$

Survey Results

A random sample of **1018** parents with a teenager was taken and **56%** said they believe electronics and social media was the cause of their teenager's lack of sleep.

Assumptions

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Assumptions

We need a **random sample** of parents

We also need a **large enough sample size** to ensure our distribution of sample proportions is normal

That is: $n \cdot p$ be at least 10 $\rightarrow n \cdot p_o$
 $n \cdot (1-p)$ be at least 10 $\rightarrow n \cdot (1-p_o)$

Checking Assumptions

Random Sample

Checking Assumptions

Random Sample ✓

In background
information

Checking Assumptions

Random Sample ✓

$$n \cdot p_o$$

$$n \cdot (1 - p_o)$$

$$p_o = 0.52$$

Checking Assumptions

Random Sample ✓

$$n \cdot p_o = 1018 \cdot (0.52) = 529 \quad \checkmark$$

$$n \cdot (1-p_o) = 1018 \cdot (1-0.52) = 489 \quad \checkmark$$





Testing a One Population Proportion

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Hypotheses

$$H_0 : p = 0.52$$

$$H_a : p > 0.52$$

Best Estimate of p is $\hat{p} = 0.56$

Where p is the population proportion of parents with a teenager who believe that electronics and social media is the cause of their teenager's lack of sleep

$$\alpha = 0.05$$

Test Statistic

Best estimate - Hypothesized estimate

Standard error of estimate

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$$\frac{\hat{p} - p_o}{s.e.}$$

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$$s.e.(\hat{p}) = \sqrt{\frac{p \cdot (1-p)}{n}}$$

Test Statistic

Best estimate - Hypothesized estimate

Standard error of estimate

$$\frac{\hat{p} - p_o}{s.e.}$$

$$s.e. (\hat{p}) = \sqrt{\frac{p \cdot (1-p)}{n}} \quad \longrightarrow \quad s.e. (\hat{p}) = \sqrt{\frac{p_o \cdot (1-p_o)}{n}}$$

Test Statistic

$$\frac{\hat{p} - p_o}{s.e.}$$

$$\text{Null } s.e. (\hat{p}) = \sqrt{\frac{p_o \cdot (1 - p_o)}{n}}$$

Test Statistic

$$\frac{\hat{p} - p_o}{s.e.} \quad \text{Null } s.e. (\hat{p}) = \sqrt{\frac{p_o \cdot (1 - p_o)}{n}}$$

$$Z = \frac{0.56 - 0.52}{0.0157}$$

$$\mathbf{Z = 2.555}$$

Test Statistic Interpretation

$$Z = 2.555$$

That means that our observed sample proportion is 2.555
null standard errors above our hypothesized population
proportion

Test Statistic Distribution

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Test Statistic Distribution

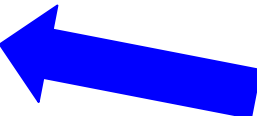
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$$\frac{\hat{p} - p_o}{s.e.(\hat{p})}$$

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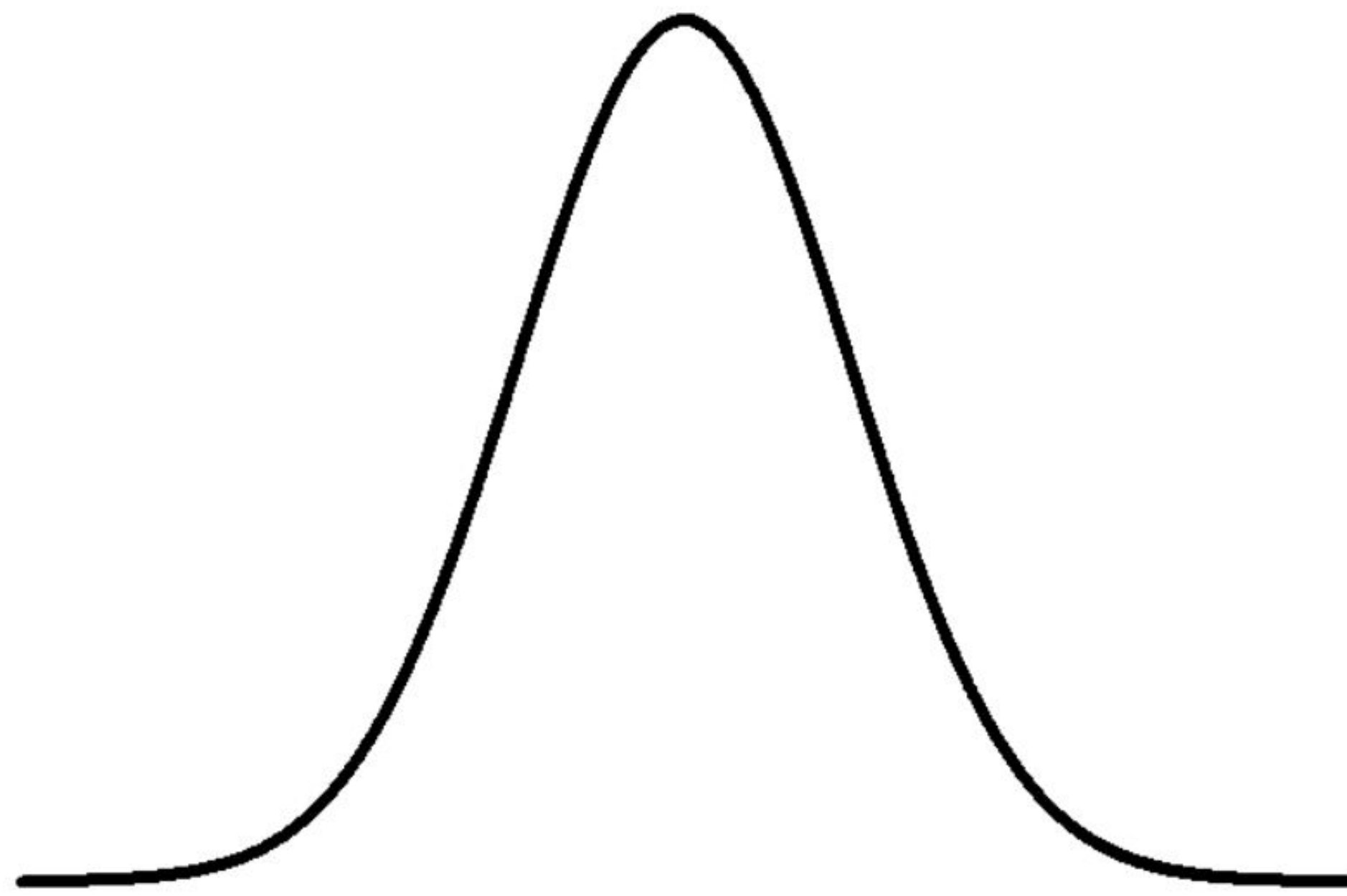
 **Centers Data**

Test Statistic Distribution

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Scales Data $\frac{\hat{p} - p_o}{s.e.(\hat{p})}$ Centers Data

The P-Value



Conclusions

$$p\text{-value} = 0.0053 < \alpha = 0.05$$

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Reject the null hypothesis ($H_0: p = 0.52$)

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$p\text{-value} = 0.0053 < \alpha = 0.05$

Reject the null hypothesis ($H_0: p = 0.52$)

There is sufficient evidence to conclude that the population proportion of parents with a teenager who believe that electronics and social media is the cause for lack of sleep is greater than 52%.

Summary

- **4 main steps to a hypothesis test**
 - Stating hypothesis & select significance level (α)
 - Checking assumptions
 - Calculating a test statistic and getting a p-value from the test statistic
 - Drawing a conclusions from the p-value

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 - Stating hypothesis & select significance level (α)
 - Checking assumptions
 - Calculating a test statistic and getting a p-value from the test statistic
 - Drawing a conclusions from the p-value
- The Z test statistic distribution is $N(0, 1)$