Testing a Population Mean Difference

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Home Renovations

20 homes remodeling their kitchens, requesting cabinet quotes from 2 suppliers

Is there an average difference in cabinet quotes from these two suppliers?

Variable: Difference in cabinet quotes
(Supplier A – Supplier B)

Research Question

Is there an average difference between the cabinet quotes from the suppliers?

Populations - All houses

Parameter of Interest - Population mean difference of cabinet quotes μ_d

(Supplier A - Supplier B)

Test for a significant mean difference in cabinet quotes at the 5% significance level.

Hypotheses

$H_0 : \mu_d = 0$

$H_a : \mu_d \neq 0$

$\alpha = 0.05$

Cabinet Data

Supplier A	Supplier B	Difference
$380	$325	$55
$560	$470	$90
$425	$420	$5
$389	$375	$14
$568	$574	-$6
$651	$595	$56

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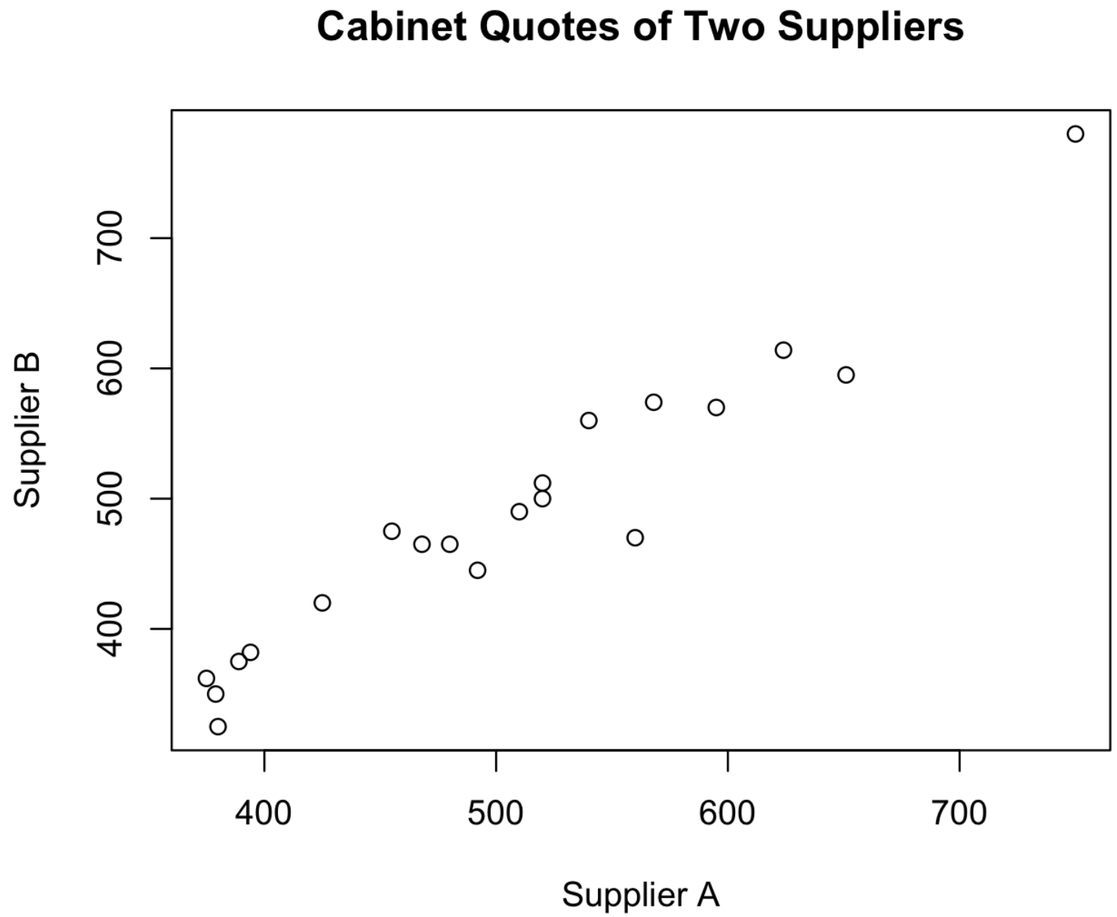

Assumptions

We need to assume that we have a **random sample of differences**, i.e. a random sample of houses.

We also need the **population of differences to be normally distributed**. We can get around this assumption if we have a large sample size (about 25+).

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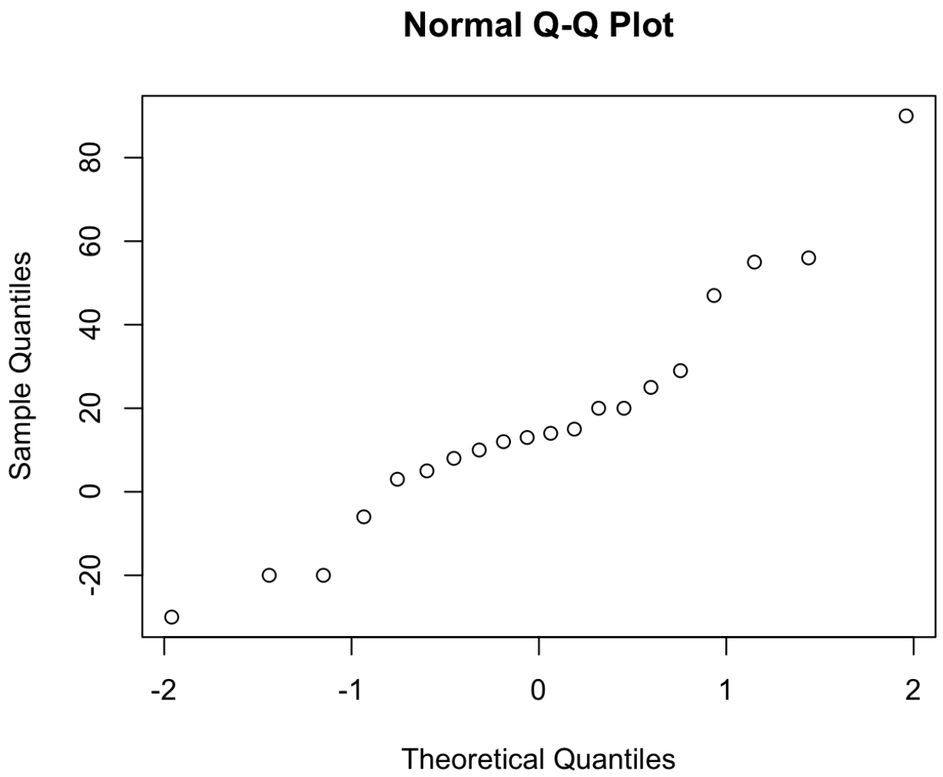
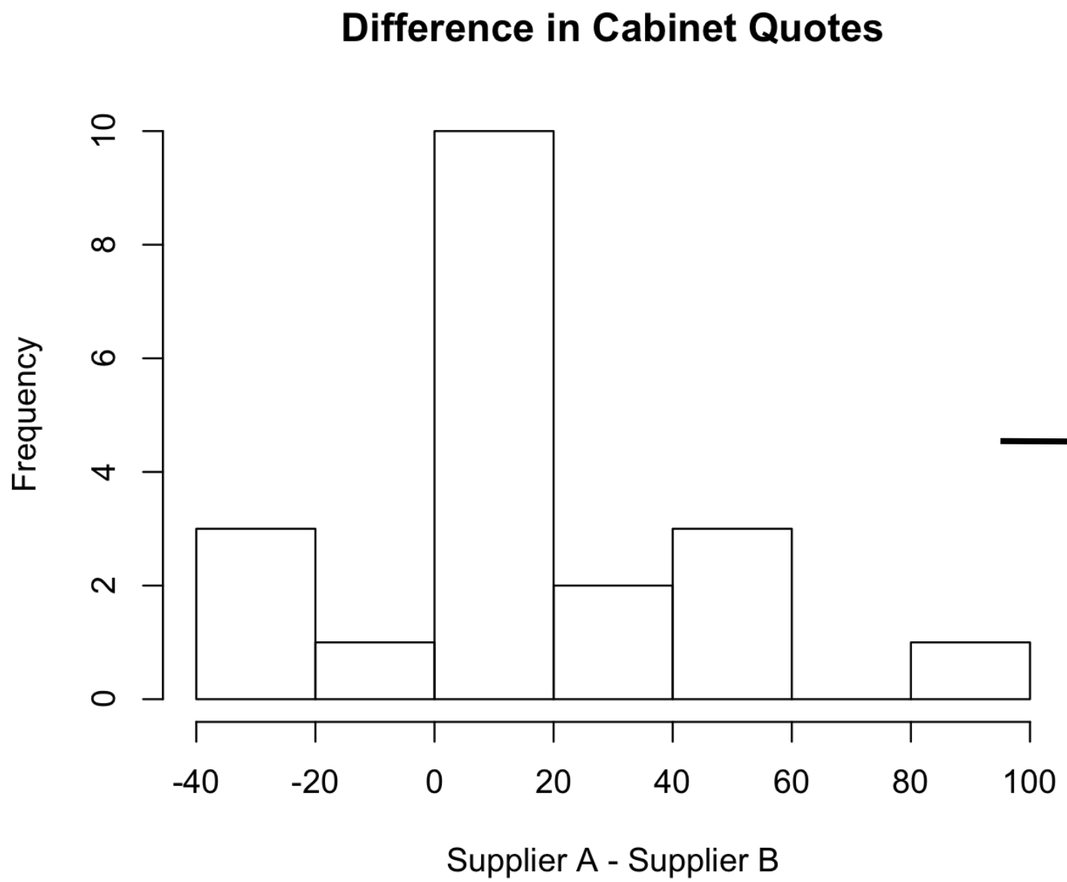


Summarize the Data



n = 20 observations

Minimum = -$30
Maximum = $90

Median = $13.50
Mean = $17.30
Standard Deviation = $28.49

Test Statistic

Assuming the sampling distribution of the sample mean difference is normal,

$$t = \frac{\text{best estimate - hypothesized estimate}}{\text{estimated standard error of estimate}}$$

Test Statistic

Best estimate - Hypothesized estimate

Estimated standard error of estimate

$$t = \frac{\bar{x}_d - 0}{s_d / \sqrt{n}} = \frac{17.30 - 0}{28.49 / \sqrt{20}}$$

n = 20 observations

Mean = $17.30

SD = $28.49

Test Statistic

Best estimate - Hypothesized estimate

Estimated standard error of estimate

$$t = \frac{\bar{x}_d - 0}{s_d / \sqrt{n}} = \frac{17.30 - 0}{28.49 / \sqrt{20}} = \frac{17.30}{6.37}$$

$$= 2.72$$

Test Statistic

Best estimate - Hypothesized estimate

Estimated standard error of estimate

$$t = \frac{\bar{x}_d - 0}{s_d / \sqrt{n}} = 2.72$$

Our observed mean difference is 2.72 (estimated) standard errors above our null value of 0.

Test Statistic Distribution & P-value

t (19)

Decision & Conclusion

p-val = 0.014 < 0.05 = α → reject null hypothesis

→ have evidence against mean difference in cabinet quotes is 0

Formally, based on our sample and our p-value, we reject the null hypothesis. We conclude that the mean difference of cabinet quote prices for Suppliers A less B is **significantly different** from 0.

95% Confidence Interval

Mean = \$17.30
Standard deviation = \$28.49

$n = 20 \rightarrow t^* = 2.093$

$$\bar{x}_d \pm t^* \left(\frac{s_d}{\sqrt{n}}\right)$$

\$17.30 ± 2.093 (\$28.49/√20)
\$17.30 ± 2.093 (\$6.37)
\$17.30 ± \$13.33
(\$3.97, \$30.63)

Note 0 is NOT in our range of reasonable values for mean difference in cabinet prices.

Wilcoxon Signed Rank Test

If *normality* doesn't hold, we can use the Wilcoxon Signed Rank Test to test for the median.

p-val = 0.020

Again, we reject H_0 and conclude that the median difference in the cabinet quotes, Supplier A less B, is different from 0.

Summary

Hypothesis Test allow you to assess theories about a population parameter of interest

- ~parameter = mean difference

Extension of the one mean hypothesis test

- ~with difference variable
- ~collected on same individual (house)

Can obtain similar information from Confidence Intervals and Wilcoxon Signed Rank Test