Testing a Difference in Population Proportions

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Hypotheses

$H_0 : p_1 - p_2 = 0$

$H_a : p_1 - p_2 \neq 0$

$\alpha = 0.10$

Best Estimate of the Parameter

$\hat{p}_1 = 91/247 = 0.37$

$\hat{p}_2 = 120/308 = 0.39$

$\hat{p}_1 - \hat{p}_2 = 0.37 - 0.39 = -0.02$

1 = black

2 = Hispanic

Test Statistic

Best estimate - Hypothesized estimate

Standard error of estimate

$$\frac{\hat{p}_1 - \hat{p}_2 - 0}{se(\hat{p})}$$

where $se(\hat{p}) = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

Test Statistic

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where $se(\hat{p}) = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$$z = -0.02/0.041 = -0.48$$

Test Statistic Interpretation

$$z = -0.48$$

That means that our observed difference in sample proportions is 0.48 estimated standard errors below our hypothesized mean of equal population proportions.

Test Statistic Distribution & P-value

Standard Normal Distribution

Test Statistic Distribution & P-value

Normal (0,1)

Decision & Conclusion

p-val = 0.63 > 0.10 = α → fail to reject null hypothesis

→ don't have evidence against equal population proportions

Formally, based on our sample and our p-value, we fail to reject the null hypothesis. We conclude that there is **no significant difference** between the population proportion of parents of black and Hispanic children who report their child has had swimming lessons.

Alternative Approaches

	Swim Lessons	No Swim Lessons	Total
Black	91	156	247
Hispanic	120	188	308
Total	211	344	555

Chi-Square (X^2) Test

different hypotheses
require two-sided hypothesis
same conclusion*
*as two-sided hypothesis with proportions

Fisher's Exact Test

allows one-sided hypothesis
typically for small sample sizes
calculates different p-values*
*compared to same setup for proportions