Estimating a Population Mean with Confidence

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Cartwheel Study

- 25 team members/colleagues (all adults) asked to perform a cartwheel
- **Variable**: Cartwheel Distance (in inches)

Research Question

What is the **average** cartwheel distance (in inches) for adults?

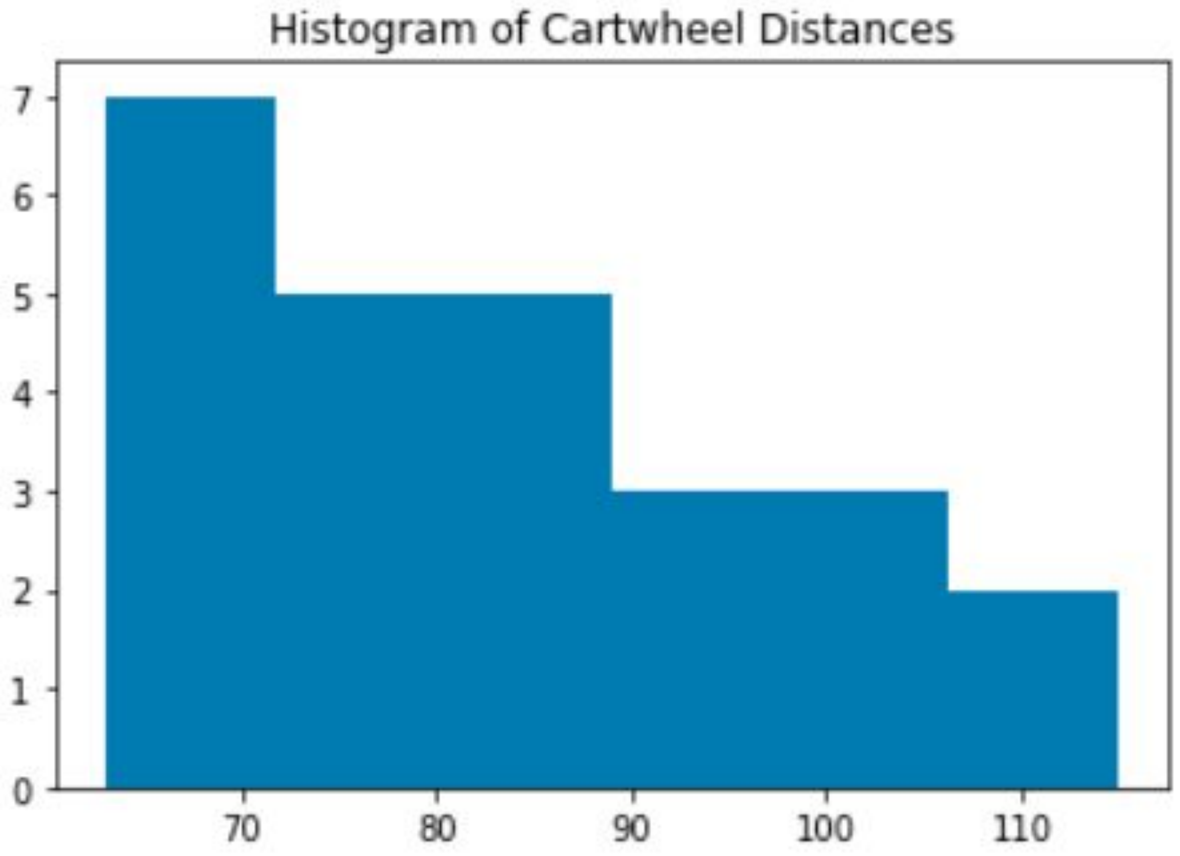
- **Population**: All adults
- **Parameter of Interest**: population mean cartwheel distance μ

Construct a 95% confidence interval for the mean cartwheel distance for the population of all such adults.

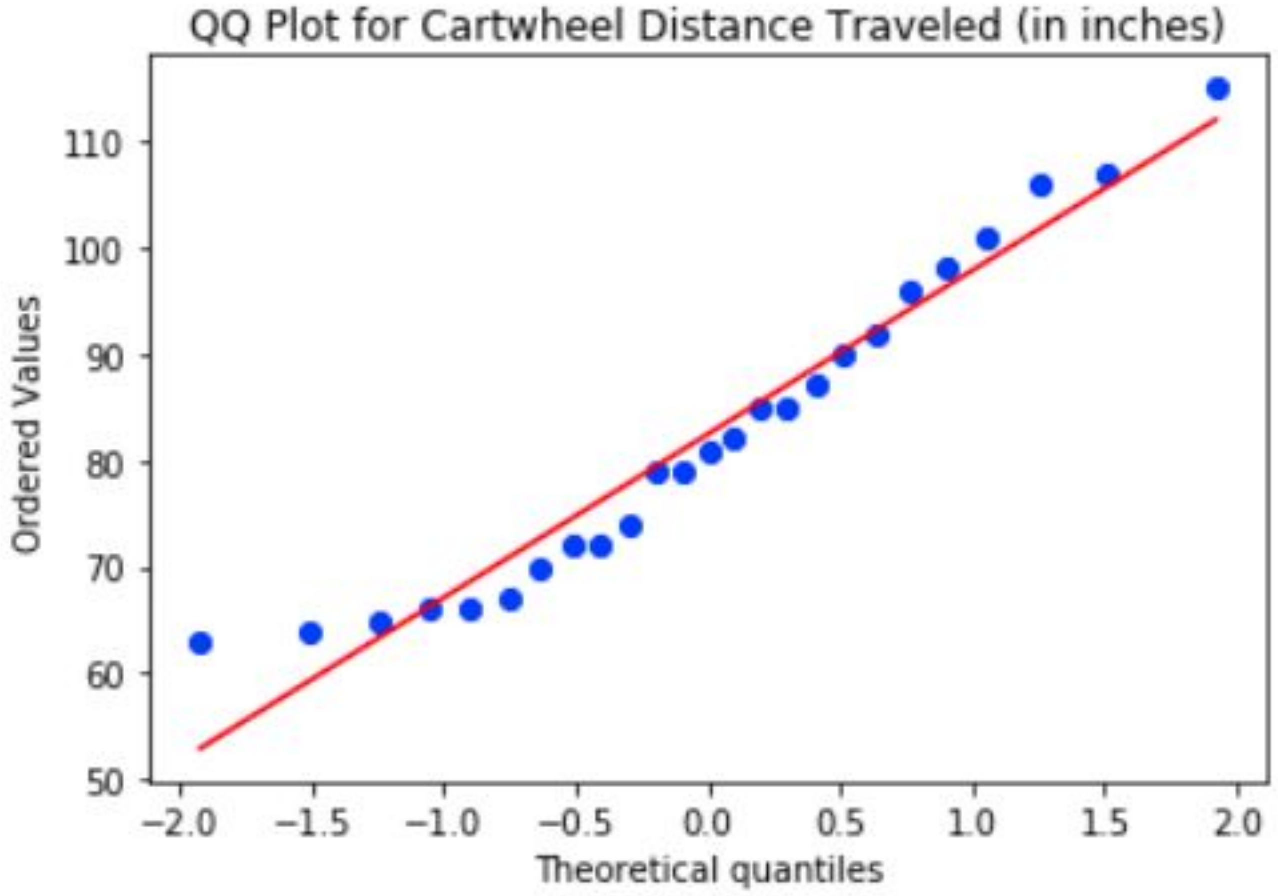
Cartwheel Study Data

	ID	Age	Gender	GenderGroup	Glasses	GlassesGroup	Height	Wingspan	CWDistance	Complete	CompleteGroup	Score
0	1	56	F	1	Y	1	62.0	61.0	79	Y	1	7
1	2	26	F	1	Y	1	62.0	60.0	70	Y	1	8
2	3	33	F	1	Y	1	66.0	64.0	85	Y	1	7
3	4	39	F	1	N	0	64.0	63.0	87	Y	1	10
4	5	27	M	2	N	0	73.0	75.0	72	N	0	4

Cartwheel Distance Summary



Cartwheel Distance Traveled (in inches)



Cartwheel Distance Summary

```
df.describe()["CWDistance"]
```

```
count     25.000000
mean      82.480000
std       15.058552
min       63.000000
25%       70.000000
50%       81.000000
75%       92.000000
max      115.000000
Name: CWDistance, dtype: float64
```

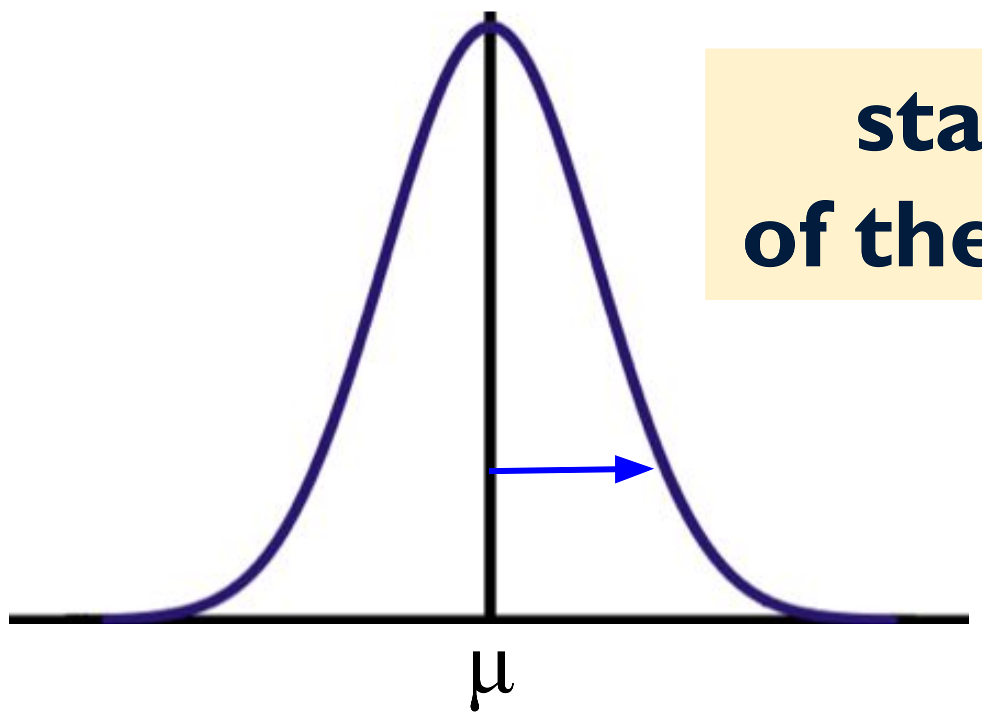
n = 25 observations

Minimum = 63 inches
Maximum = 115 inches

Mean = 82.48 inches
Standard Deviation = 15.06 inches

Sampling Distribution of Sample Mean

If model for population of responses is approximately normal (or sample size is 'large' enough), distribution of sample mean is (approx.) normal.



All possible values of <u>sample mean</u>

standard error of the sample mean $= \dfrac{\sigma}{\sqrt{n}}$

estimated **standard error of the sample mean** $= \dfrac{s}{\sqrt{n}}$

Confidence Interval Basics

Best Estimate ± Margin of Error

Best Estimate = Unbiased Point Estimate

Margin of Error = *"a few"* Estimated Standard Errors

"a few" = multiplier from appropriate distribution based on desired confidence level and sample design

95% Confidence Level ►► 0.05 Significance

95% Confidence Interval Calculations

Best Estimate ± Margin of Error

Sample mean ± "a few" · estimated standard error of sample mean

$$\bar{x} \quad \pm \quad t^* \quad \left(\frac{s}{\sqrt{n}}\right)$$

t^* multiplier comes from a t-distribution with $n - 1$ degrees of freedom

95% confidence

$n = 25 \rightarrow t^* = 2.064$

$n = 1000 \rightarrow t^* = 1.962$

95% Confidence Interval Calculations

Mean = 82.48 inches

Standard Deviation = 15.06 inches

n = 25 observations → t^* = 2.064

$$\bar{x} \quad \pm \quad t^* \left(\frac{s}{\sqrt{n}}\right)$$

$$82.48 \pm 2.064\left(\frac{15.06}{\sqrt{25}}\right)$$

$$82.48 \pm 2.064(3.012)$$

$$82.48 \pm 6.22$$

$$(76.26 \text{ inches}, 88.70 \text{ inches})$$

What if 99% Confidence?

$$\bar{x} \quad \pm \quad t^* \left(\frac{s}{\sqrt{n}}\right)$$

$$82.48 \pm 2.064\left(\frac{15.06}{\sqrt{25}}\right) \rightarrow (76.26 \text{ inches}, 88.70 \text{ inches})$$

Think about it: What parts of this 95% confidence interval will change if we want to be 99% confident instead?

Q: Will the **resulting 99% confidence interval** be **wider** or **narrower**?

Interpreting the Confidence **Interval**

"range of reasonable values for our parameter"

With 95% confidence, the population mean cartwheel distance for all adults is estimated to be between 76.26 inches and 88.70 inches.

Interpreting the Confidence **Level**

What does "with 95% confidence" mean?

If this procedure were repeated over and over,
each time producing a 95% confidence interval estimate,

we would **expect 95% of those resulting intervals**
to contain the population mean cartwheel distance.

Summary

- Confidence Intervals are used to give an *interval* estimate for our parameter of interest **~ a population mean**
- Center of the Confidence Interval is our best estimate **~ the sample mean**
- Margin of Error is "a few" (estimated) standard errors **~ for means we use t* multipliers**
- Assumptions for Confidence Interval for Population mean
  ~ data considered a random sample
  ~ population of responses is normal (else n large helps)
- Know how to interpret the **interval** and the **level**