Electron Pair Creation in $\pi^- + p$ Capture Reactions from Rest (*).

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Summary. — Electromagnetic corrections are calculated for the processes $\pi^- + p \to n + e^+ + e^-$ and $\pi^0 \to \gamma + e^+ + e^-$. These corrections depend on the c.m. energy of the pair. For very low c.m. energy the corrections are relatively large, and due mainly to the electron-positron Coulomb attraction; they fall rapidly with increasing c.m. energy to a minimum of 0.5%, then rise slowly to a value of 1.7% at the highest c.m. energy. For both processes, the correction to the total rate amounts to about 1%. In terms of these total rates, the Panofsky ratio is found to be

$$R_p = 0.594\, P[\pi^- + p \to n + \pi^0,\ \pi^0 \to \gamma + e^+ + e^-]/P[\pi^- + p \to n + e^+ + e^-]\,.$$

For the process $\pi^- + p \to n + e^+ + e^-$, the problem of empirically identifying the contributions arising from interaction via the longitudinal component of the electromagnetic field and from size effects is discussed. These contributions are expected to amount to only 2.0% and 0.8%, respectively, of the total rate. The process $\pi^- + p \to n + 2\gamma$ is also discussed briefly.

1. – Introduction.

The processes to be studied here are:

$$\pi^- + p \to n + e^+ + e^- \tag{1.1}$$

and

$$\pi^- + p \to n + \pi^0\,, \qquad \pi^0 \to \gamma + e^+ + e^-\,. \tag{1.2}$$

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These are of interest because, first, the ratio of the two processes is directly related to the Panofsky ratio, $P[\pi^- + \mathrm{p} \to \mathrm{n} + \pi^0]/P[\pi^- + \mathrm{p} \to \mathrm{n} + \gamma]$, and, second, a detailed empirical analysis of process (1.1) would yield new information on the photon-pion, nucleon interaction, as will now be discussed.

A process closely related to (1.1) is

$$\mathrm{e}^- + \mathrm{p} \to \mathrm{e}^- + \mathrm{n} + \pi^+ \,. \tag{1.3}$$

This process has been investigated both theoretically ([1,2]) and experimentally ([3]). It may be considered as the combination of *a*) emission by the electron of a virtual photon for which $x^2 = k_0^2 - \boldsymbol{k}^2 < 0$, and *b*) photoproduction of a pion. The electromagnetic process *a*) may be assumed to be well understood so that in this way process *b*) can be investigated for negative values of x^2 (yielding, in particular, information on the nucleon electromagnetic form factors as functions of x^2). The process (1.1) to be considered here may be looked upon as *b'*) π^- capture yielding a virtual photon which *a'*) creates an electron pair. The process *b'*) differs from *b*) only in that x^2 is now positive, ranging from 0 to μ^2, where μ is the pion mass. Thus process (1.1) is complementary to process (1.3) in the sense that the former can yield information on the electromagnetic form factors for a range of positive values of the argument x^2, whereas (1.3) relates to negative values of x^2.

Since the photons involved in (1.1) are virtual rather than real, they may have longitudinal as well as transverse polarizations. It will be shown below that separation of the longitudinal contributions can be performed rather neatly by consideration of the distribution in θ, the angle between the directions of the electron and virtual photon in the c.m. system of the electron pair (see Fig. 1). Thus process (1.1) allows a direct check on the size of the longitudinal matrix element for $\pi^- + \mathrm{p} \to \mathrm{n} +$ virtual γ; although such a check could also be obtained from (1.3), this has not yet been done.

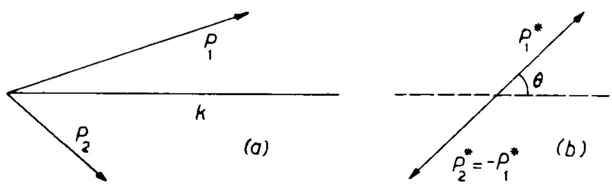

Fig. 1. – Kinematics of pair production (*a*) in the laboratory system, where $\boldsymbol{p}_1 + \boldsymbol{p}_2 = \boldsymbol{k} \neq 0$, $p_{10} + p_{20} = (\boldsymbol{k}^2 + x^2)^{\frac{1}{2}}$, and (*b*) in the pair c.m. system, where $\boldsymbol{p}_1^* + \boldsymbol{p}_2^* = \boldsymbol{k}^* = 0$ and $p_{10}^* + p_{20}^* = x = 2(\boldsymbol{p}_1^{*2} + m^2)^{\frac{1}{2}}$.

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(1) R. H. DALITZ and D. R. YENNIE: *Phys. Rev.*, **105**, 1598 (1957).

(2) S. FUBINI, Y. NAMBU and V. WATAGHIN: *Phys. Rev.*, **111**, 329 (1958).

(3) W. K. H. PANOFSKY, W. M. WOODWARD and G. B. YODH: *Phys. Rev.*, **102**, 1392 (1956).

It is unfortunate for the measurement of these contributions (size and longitudinal) to the $\pi^-+p \to n + \text{virtual } \gamma$ matrix element that they are small, amounting to about 1% and 2%, respectively, of the total rate; on the other hand, this is advantageous for the determination of the Panofsky ratio. Since there is no longitudinal contribution to (1.2) and the size effect there is negligible, it means that the ratio of (1.1) to (1.2) is related to the Panofsky ratio by a factor determined principally by electromagnetic theory and only weakly dependent on pion physics.

Since effects of the order of one per cent are of importance to any of the above measurements, electromagnetic corrections, of relative order α, need to be considered. The determination of these corrections is the main purpose of the present paper (¹).

2. – The process $\pi^-+p \to n+e^++e^-$.

Figs. 2*b*, *c*, *d*, and *e* show the diagrams which can contribute to this process in order α relative to the lowest order process, shown in Fig. 2*a*. The open circles indicate the strong-interaction vertices, the p, n, and π^- lines being omitted. It is possible for two photon lines to emerge from the strong-interaction vertex; Figs. 2*d* and 2*e* show two graphs which also contribute in relative order α. However, the probability for emission of two real photons ($\pi^-+p \to n+2\gamma$) is only 0.019α of that for the emission of one (see Appendix A), which implies that the processes of Figs. 2*d* and 2*e* contribute much less than those of Figs. 2*b* and 2*c* (which together contribute about one per cent; see below). Also, Fig. 2*a* leads to a factor $1/x^2$ favoring low-mass pairs, while Fig. 2*e* does not; thus the interference term between these two will be further reduced.

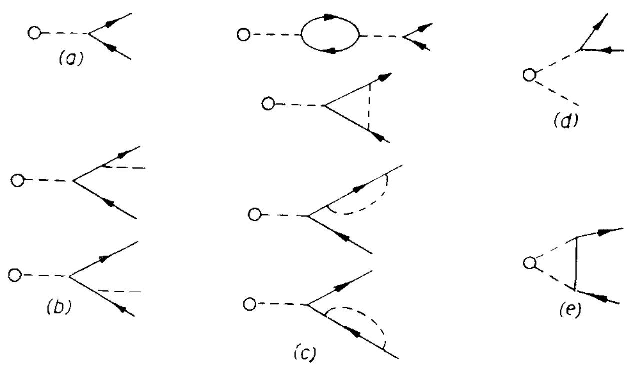

Fig. 2. – Feynman diagrams for the process $\pi^-+p \to n+ +e^++e^-(+\gamma)$. The small circles indicate the vertex at which $\pi^-+p \to n+\text{virtual } \gamma$. Dashed lines denote photons; solid lines, electrons.

(¹) Calculations to lowest order in α for the processes (1.1) and (1.2) have been performed by R. H. DALITZ: *Proc. Phys. Soc.* (*London*), A **64**, 667 (1951), and by N. KROLL and W. WADA: *Phys. Rev.*, **98**, 1355 (1955). Size and longitudinal contributions to (1.1) have been briefly considered by R. ROCKMORE and J. TAYLOR: *Phys. Rev.*, **112**, 992 (1958).

First we must evaluate the lowest order electromagnetic contribution, Fig. 2a, taking account of the small corrections to the vertex $\pi^- + p \to n +$ virtual γ. Considering a system of mass $M+E$ at rest which emits a photon to become a recoiling system of mass M, KROLL and WADA [4] have obtained the following general expression [5]:

$$(2.1)\qquad \varrho_0 = \frac{\alpha}{4\pi}\int_{-\eta}^{\eta} \mathrm{d}y \int_{2m}^{E} \frac{\mathrm{d}x}{x}\frac{k}{\varkappa}\left[1 - \frac{x^2}{(E+M)^2+M^2}\right]\left[\left(1+y^2+\frac{4m^2}{x^2}\right)R_T(x) + (1-y^2)\frac{8(E+M)^2x^2}{(2EM+E^2+x^2)^2}R_L(x)\right],$$

for the internal conversion coefficient (the rate of pair creation divided by the rate of photon emission), to lowest order in α. The symbols $\varkappa$ and k here denote the momenta of the real and virtual photons, respectively, and m is the electron mass. The quantity $x \equiv (k_0^2 - \boldsymbol{k}^2)^{\frac{1}{2}}$ is the mass of the virtual photon, while $y \equiv (p_{10} - p_{20})/|\boldsymbol{p}_1 + \boldsymbol{p}_2|$ is a measure of the energy sharing between the electron ($p_{1\mu}$) and the positron ($p_{2\mu}$). It can be shown that $y = \eta \cos\theta$, where $\eta = (1 - 4m^2/x^2)^{\frac{1}{2}}$ and θ is the angle between $\boldsymbol{p}_1$ and $\boldsymbol{k}$ in the c.m. system of the electron-positron pair (see Fig. 1). The quantities R_T and R_L depend on the current $J_\mu(\boldsymbol{k})$ which produces the virtual photons:

$$(2.2)\qquad \begin{cases} R_T(x) = \sum\int \mathrm{d}\Omega_k\left[|J_1(\boldsymbol{k})|^2 + |J_2(\boldsymbol{k})|^2\right] \Big/ \sum\int \mathrm{d}\Omega_\varkappa\left[|J_1(\boldsymbol{\varkappa})|^2 + |J_2(\boldsymbol{\varkappa})|^2\right], \\ R_L(x) = \sum\int \mathrm{d}\Omega_k\left[|J_3(\boldsymbol{k})|^2\right] \Big/ \sum\int \mathrm{d}\Omega_\varkappa\left[|J_1(\boldsymbol{\varkappa})|^2 + |J_2(\boldsymbol{\varkappa})|^2\right]. \end{cases}$$

Here J_1 and J_2 are the components transverse to $\boldsymbol{k}$, and J_3 is the component in the direction of $\boldsymbol{k}$. The indicated summations are the usual sum-averages over nucleon spin states. For the present process, $M = M_n = 1839\,m$ and $E = M_p + M_\pi - M_n = 270.3\,m$, where m denotes the electron mass. It can be shown that $k/\varkappa = (1 - 0.0023\,x^2/E^2)(1 - x^2/E^2)^{\frac{1}{2}}$ to within 0.001%, so eq. (2.1) can be written as

$$(2.3)\qquad \varrho_0 = (\alpha/4\pi)\int_{-\eta}^{\eta}\mathrm{d}y\int_{2m}^{E}(\mathrm{d}x/x)(1-x^2)^{\frac{1}{2}}(1-0.0117\,x^2)\cdot \left[(1+y^2+4m^2/x^2)R_T + 2.284(1-y^2)\,x^2R_L/(1+0.0684\,x^2)^2\right] =$$

$$= (2\alpha/3\pi)\int_{2m}^{E}(\mathrm{d}x/x)(1-x^2)^{\frac{1}{2}}(1-0.0117\,x^2)\cdot \left[R_T + 1.142\,x^2R_L/(1+0.0684\,x^2)^2\right](1+2m^2/x^2)(1-4m^2/x^2)^{\frac{1}{2}},$$

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[5] After correction of a numerical error. Note that throughout this paper we choose units such that $\hbar = c = 1$.

where the unit of energy has been chosen so that $E=1$. To obtain expressions for $J_\mu(\boldsymbol{k})$ we shall use eqs. (14) and (14′) of Ref (2), making use of the fact that the matrix element for $\pi^-+p\to n+[\gamma]$ is the same as that for $[\gamma]+p\to n+\pi^+$ but for a change of sign of k_μ and q_μ. The result is,

$$(2.4)\qquad \begin{cases} |J_1(k)|=|J_2(k)|=C[e^V+k^2\mu^S]\,, \\ |J_3(k)|=C\{e^V-ek^2/(2k_0-x^2)+[(e^V-e)/x^2+\mu^S]\,k^2\}, \end{cases}$$

for the appropriate nucleon spin combinations, where C is a constant and some non-significant $1/M$ terms have been dropped. The quantities μ^S and e^V are given by $\mu^S=\mu'_p F_2^p+\mu'_n F_2^n+e^S(x^2)/2M$ and $e^{S,V}=e[F_1^p\pm F_1^n]$ (the « + » sign being associated with « S »), where $\mu'_{p,n}$ are the proton and neutron anomalous moments and $F_{1,2}^{p,n}$ are the corresponding form factors. Since the magnetic moment term is small, we can replace $\mu^S(x^2)$ by $\mu_0^S=\mu^S(0)=0.87e/2M=0.064\,e$. The vector charge can be written in the form $e^V=e(1+r_V^2x^2/6+\dots)$: electron scattering experiments indicate that $r_V=0.80f$, or 0.57 in the present units (6,7). Eqs. (2.2) and (2.4) yield

$$(2.5)\qquad \begin{cases} R_T\doteq 1+r_V^2/3-2(\mu^S/e)x^2\,, \\ R_L=0.142\,(1+\mu^S/e)^{-2}\{(1-0.466\,x^2)^{-1}+1.88\,[r_V^2/6+(\mu^S/e)(1-x^2)]\}^2\,, \end{cases}$$

where the static approximation $k^2\approx 1-x^2$, $k_0\approx 1$ has been used in the small terms. Substituting eqs. (2.5) into eq. (2.3) and integrating, one obtains

$$(2.6)\qquad \begin{aligned} \varrho_0 &= \varrho_{0T0}+\varrho_{0TM}+\varrho_{0TS}+\varrho_{0L}= \\ &= (69.00-0.65+0.55+1.42)\cdot 10^{-4}=0.007\,03\,, \end{aligned}$$

where ϱ_{0T0} is the coefficient for conversion of transverse photons without magnetic moment or size effect contributions, ϱ_{0TM} and ϱ_{0TS} are the corrections to this rate arising from the magnetic moment and size effect terms, and ϱ_{0L} is the longitudinal contribution; all are calculated to lowest order in α at the virtual photon-electron pair vertex (denoted by the first subscript 0). The corresponding distributions in x are shown in Fig. 3. The size effects have been evaluated with the experimental value $r_V=0.80f$ (8).

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(6) D. R. YENNIE, M. LÉVY and D. G. RAVENHALL: *Rev. Mod. Phys.*, **29**, 144 (1957).

(7) R. HOFSTADTER, F. BUMILLER and M. R. YEARIAN: *Rev. Mod. Phys.*, **30**, 482 (1958).

(8) It is to be noted that the transverse size effect of eq. (2.6) is about half that given by ROCKMORE and TAYLOR (4), due to an error of a factor 3 in their expressions combined with the fact that they used the earlier experimental value $r_V\doteq\frac{1}{2}\,\mu$.

To evaluate the contribution from the diagrams of Figs. 2b (corresponding to the emission of real radiation) and 2c (virtual radiative corrections), we will make use of Källén and Sabry's fourth-order vacuum polarization calculation [9].

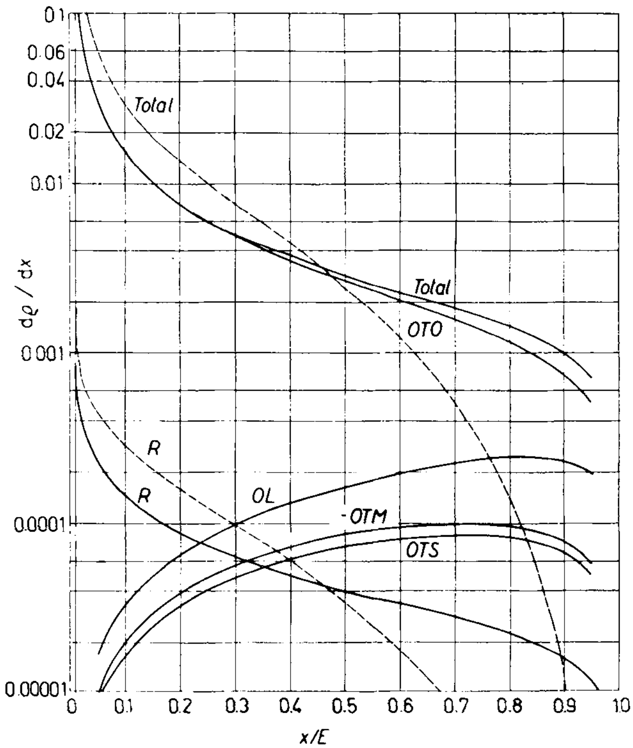

Fig. 3. – Distribution of events with respect to the electron pair mass x for the processes $\pi^0 \to \gamma + e^+ + e^-$ $(+\gamma)$ (dashed curves) and $\pi^- + p \to n + e^+ + e^-$ $(+\gamma)$ (solid curves). OTO: contribution of transverse (virtual) photons without magnetic moment and size contributions, to lowest order in α; OL: contribution of longitudinal photons, to lowest order in α; OTM, OTS: magnetic moment and size effect contributions, respectively, to the transverse rate (note sign of OTM; OTS, proportional to r_V^2, is here evaluated for $r_V = 0.80f$); R: radiative corrections (*i.e.*, corrections of relative order α due to both real and virtual radiation). Note that all curves fall to zero at $x = 1/2m = 0.00739$; the radiative corrections reach peaks of 0.0072 (π^0) and 0.0036 (capture).

If the circles and intermediate photon lines were omitted from Figs. 2b, the remaining portions would correspond to $\langle 0|j^{(1)}|z\rangle$ in Källén's notation

([9]) G. Källén and A. Sabry: *Dan. Mat. Fys. Medd.*, **29**, no. 17 (1955).

(where $|z\rangle$ denotes the (e^+, e^-, γ) state), so that these contributions amount to [10]

$$(2.7)\quad \left\{\begin{aligned} \varrho(2b) &= D' \sum\int \mathrm{d}\Omega_k \int k^2\,\mathrm{d}k\, J^\mu(\boldsymbol{k}) J^{\nu *}(\boldsymbol{k}) x^{-2} \sum_{z:k_\lambda} \langle 0|j_\mu^{(1)}|z\rangle\langle z|j_\nu^{(1)}|0\rangle \\ &= D' \sum\int \mathrm{d}\Omega_k \int k^2\,\mathrm{d}k\, J^\mu(\boldsymbol{k}) J^{\nu *}(\boldsymbol{k}) x^{-2}(x^2 g_{\mu\nu} - k_\mu k_\nu)\pi_a^{(1)}(-x^2) \\ &= D\int(\mathrm{d}x/x)(1-x^2)^{\frac{1}{2}} \sum\int \mathrm{d}\Omega_k[|J_1(\boldsymbol{k})|^2 + |J_2(\boldsymbol{k})|^2 + x^2|J_3(\boldsymbol{k})|^2]\pi_a^{(1)}(-x^2) \end{aligned}\right.$$

where D and D' are constants and $\sum\limits_{z:k\lambda}$ indicates summation over all electron-positron-photon states $|z\rangle$ having four-momentum k_λ; the other summation is the usual sum-average over nucleon spin states. We have made use of the approximations $k_0 \approx E = 1$ and $k \approx (1-x^2)^{\frac{1}{2}}$ (recoil effects neglected), and of $k_\mu J^\mu = 0$.

The diagrams of Fig. 2c contribute to relative order α only through their interference with those of Fig. 2a; this contribution is

$$(2.8)\quad \left\{\begin{aligned} \varrho(2c) &= D' \sum\int \mathrm{d}\Omega_k \int k^2\,\mathrm{d}k\, J^\mu(\boldsymbol{k}) J^{\nu *}(\boldsymbol{k}) x^{-2} \sum_{z:k\lambda} [\langle 0|j_\mu^{(2)}|z\rangle\langle z|j_\nu^{(0)}|0\rangle + \text{c.c.}] \\ &= D\int(\mathrm{d}x/x)(1-x^2)^{\frac{1}{2}} \sum\int \mathrm{d}\Omega_k[|J_1(\boldsymbol{k})|^2 + |J_2(\boldsymbol{k})|^2 + x^2|J_3(\boldsymbol{k})|^2]\pi_b^{(1)}(-x^2)\,, \end{aligned}\right.$$

where the summation $\sum\limits_{z:k\lambda}$ is here over states $|z\rangle$ containing an electron-positron pair. The constant D appearing in eqs. (2.7) and (2.8) can easily be obtained by noting that the contribution of Fig. 2a alone differs from $\varrho(2b)$ only in having $\pi_a^{(1)}(-x^2)$ replaced by $\pi^{(0)}(-x^2)$, with the latter given by [11]

$$(2.9)\qquad \pi^{(0)}(-x^2) = (\alpha/3\pi)(1+2m^2/x^2)(1-4m^2/x^2)^{\frac{1}{2}} \qquad \text{for } x^2 \geqslant 4m^2.$$

Comparison with eqs. (2.3) and (2.2) now shows that

$$(2.10)\qquad D = 2/\sum\int \mathrm{d}\Omega_k[|J_1(\boldsymbol{\varkappa})|^2 + |J_2(\boldsymbol{\varkappa})|^2]\,.$$

Thus the electromagnetic corrections to the basic process of Fig. 2a (neglecting recoil) are

$$(2.11)\qquad \varrho_R = 2\int(\mathrm{d}x/x)(1-x^2)^{\frac{1}{2}}(R_T + x^2 R_L)\pi^{(1)}(-x^2)\,,$$

([10]) G. KÄLLÉN: *Helv. Phys. Acta*, **25**, 417 (1952). See eqs. (43) for the definition of $\pi_a^{(1)}(-x^2)$.

([11]) See ref. ([9]), eq. (6).

where

$$\pi^{(1)}(-x^2) = \pi_a^{(1)}(-x^2) + \pi_b^{(1)}(-x^2)\,. \tag{2.12}$$

The function $\pi^{(1)}(-x^2)$ has been calculated by Källén and Sabry ([12]), and is plotted in Fig. 4. It will be noted that $\pi^{(1)}(-x^2)$ has a sharp peak at $x = 2m$ (at which point the e^+-e^- relative velocity vanishes); this peak arises from the

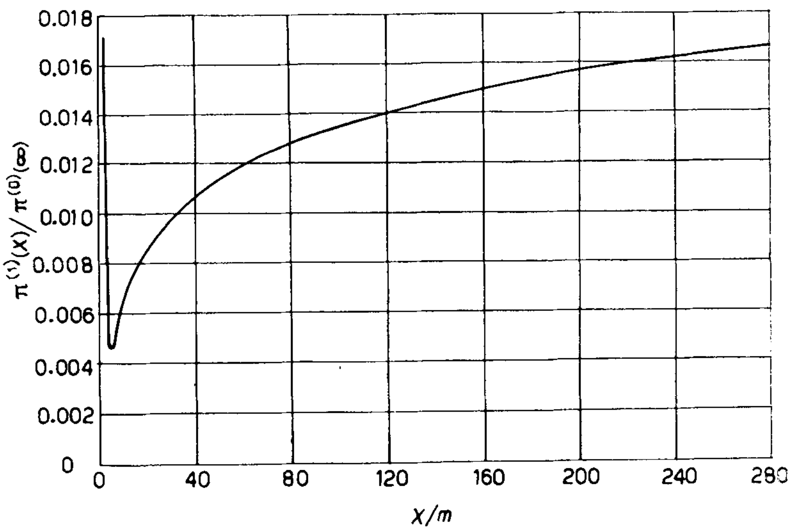

Fig. 4. – The function $\pi^{(1)}(x)$.

competition between an infinity in the matrix element and the vanishing of phase space for the pair. Such an infinity is real, and is shown in Appendix B to result from Coulomb effects (although the manner of approach to infinity at extremely small velocities is not correctly given by this order of perturbation theory). The distribution of $\varrho_R(x)$ is given in Fig. 3. Numerical integration yields

$$\varrho_R = \varrho_{RT} + \varrho_{RL} = (0.66 + 0.02)\cdot 10^{-4}\,. \tag{2.13}$$

Combination of this with eq. (2.6) yields the internal conversion coefficient, $P[\pi^- + p \to n + e^+ + e^- (+\gamma)]/P[\pi^- + p \to n + \gamma]$:

$$\varrho = \varrho_0 + \varrho_R = 0.00710\,. \tag{2.14}$$

([12]) See ref. ([9]), eq. (49). The separate terms $\pi_a^{(1)}$ and $\pi_b^{(1)}$ are not well defined because of the infrared divergence.

3. – The process $\pi^-+p\to n+\pi^0$, $\pi^0\to\gamma+e^++e^-$.

For the process $\pi^0\to\gamma+e^++e^-$, $M=0$ and $k/\varkappa=1-x^2/\mu^2$, where μ denotes the mass of the π^0 meson ($=264.3\,m$). Eq. (2.1) yields

$$\varrho_0=\frac{4\alpha}{3\pi}\int\frac{dx}{x}\left[1-\frac{x^2}{\mu^2}\right]^2\left[R_T+\frac{4R_L}{(1+x^2/\mu^2)^2}\frac{x^2}{\mu^2}\right]\left[1+\frac{2m^2}{x^2}\right]\left[1-\frac{4m^2}{x^2}\right]^{\frac{1}{2}},\tag{3.1}$$

for the contribution of lowest order in α, after doubling to allow for conversion of either photon. Invariance arguments indicate that $R_T=(k/\varkappa)(1+\varepsilon^2/x^2+...)$ where ε is a «size» associated with the intermediate states occurring in the process (the first approximation $R_T\approx k/\varkappa$ is, in fact, a result of assuming the simplest possible phenomenological form $\varphi\varepsilon^{\mu\nu\varrho\sigma}F_{\mu\nu}F_{\varrho\sigma}$ for the π^0-2γ interaction). Since these states are assumed always to include a proton-antiproton pair, it is expected that $\varepsilon\sim1/2M_p$; and perturbation theory indicates that it is actually smaller. Thus $\varepsilon^2x^2\leqslant(1/2M_p)^2x^2=0.005\,x^2/\mu^2$; since most events are associated with values of x *much* less than μ, it should be an excellent approximation to set $R_T=k/\varkappa$. Angular momentum conservation requires that $R_L=0$, since a real photon carries unit angular momentum about its direction of motion, while a longitudinal virtual photon carries none. Thus we find

$$\varrho_0=\frac{4\alpha}{3\pi}\int\frac{dx}{x}\left[1-\frac{x^2}{\mu^2}\right]^3\left[1+\frac{2m^2}{x^2}\right]\left[1-\frac{4m^2}{x^2}\right]^{\frac{1}{2}}=0.01186\,.\tag{3.2}$$

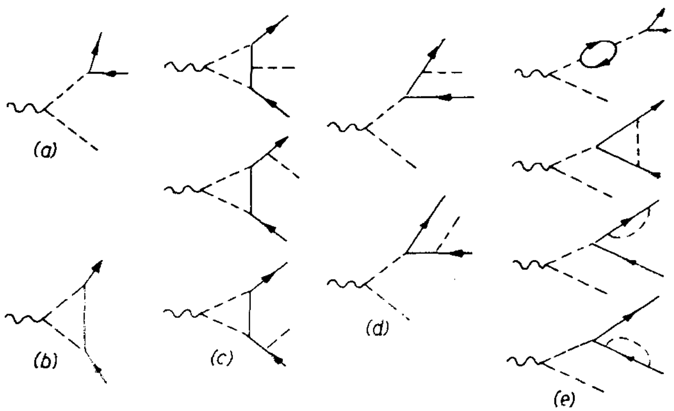

Fig. 5. – Feynman diagrams for the process $\pi^0\to\gamma+e^++e^-(+\gamma)$. Wavy lines denote the π^0; dashed lines, photons; and solid lines, electrons.

The above contribution corresponds to Fig. 5a; we will now investigate those contributions which are of order α relative to Fig. 5a. With the observation that the emission of a photon from the vertex producing the π^0 need not be considered because of the low kinetic energy, and hence phase space, available, it is seen that the terms which may contribute to relative order α are those corresponding to Figs. 5*b*, *c*, *d*, and *e*. Since the *total* process here is $\pi^- + p \to n + \gamma + e^+ + e^-$, it might be thought at first that there would be interference between the processes of Figs. 2 and 5; however, the intermediate π^0 causes the (γ, e^+, e^-) system here to have combined « mass » extremely close to the π^0 mass, while for Fig. 2 the « mass » is generally much smaller.

Although Fig. 5*b* would appear to contribute to relative order α, the matrix element is actually down by an additional factor m/μ [13], so that its contribution is of relative order $(m/\mu)^2\alpha$ and negligible.

The graphs of Fig. 5*c* would appear to contribute to relative order α through interference with those of Fig. 5*a*. But these, too, cannot contribute appreciable since i) they must contribute negligibly for low energy of the real photon, where they reduce to Fig. 5*b*, and ii) although for large electron-pair mass (where the main contribution 5*a* is very small) they may contribute in relative order α, for small electron-pair mass they do not share the enhancement which occurs for 5*a*, *d* and *e* due to the large photon propagator.

In the radiative terms given by Fig. 5*d*, we will neglect the symmetrization with respect to the two photons, since the bremsstrahlung photon will generally have much lower energy than the other, and be emitted in a different direction. With this simplification, the contribution from 5*d* and the interference term between 5*e* and 5*a* can be evaluated (as for Figs. 2*b* and 2*c*, for π^- capture) from Ref. [9], yielding

$$\varrho_R = 4\int\frac{\mathrm{d}x}{x}\left[1-\frac{x^2}{\mu^2}\right]^3\pi^{(1)}(-x^2) = 0.000105\,. \tag{(3.2)}$$

Thus the internal conversion coefficient $P[\pi^0 \to \gamma + e^+e^-(+\gamma)]/P[\pi^0 \to \gamma + \gamma]$ is

$$\varrho = \varrho_0 + \varrho_R = 0.01196\,. \tag{3.3}$$

The x-distributions are shown in Fig. 3.

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[13] H. MIYAZAWA and R. OEHME: *Phys. Rev.*, **99**, 315 (1955). This factor may be obtained as follows. This diagram must yield a contribution of the form $C(m/\mu)\varphi\bar{u}\gamma^5 u$, where $C(m/\mu)$ is a polynomial in m/μ. The transformation $u \to \gamma^5 u$ carries this expression into its negative; while the same transformation applied to the (unintegrated) expression corresponding to the diagram has precisely the effect of inverting the sign of m. Thus $C(-m/\mu) = -C(m/\mu)$, so that C must have m/μ as a factor. Actually, there is a logarithmic divergence, but the contribution is still negligible for any reasonable size of the intermediate state of $\pi^0 \to 2\gamma$; see S. DRELL: *Nuovo Cimento*, **11**, 693 (1959).

4. – Conclusions.

From eqs. (2.14) and (3.3) it follows that the Panofsky ratio is given by

$$R_p = 0.594 \frac{P(\pi^- + p \to n + \pi^0, \; \pi^0 \to \gamma + e^+ + e^-)}{P(\pi^- + p \to n + e^+ + e^-)} . \tag{4.1}$$

The coefficient 0.594 should suffer significant error neither from the approximations made above of neglecting certain diagrams and symmetrization nor from the uncertainty in the size and magnetic moment contributions. The principal uncertainty is due to our lack of knowledge of the longitudinal matrix elements and (in principle) can be removed by direct measurement of this contribution (see below). On the other hand, since the photons will not generally be observed experimentally, there is some danger of confusing events of the π^0 type with those of the capture type which have lost energy through inner bremsstrahlung; Fig. 6 indicates in a graphical way the possible extent

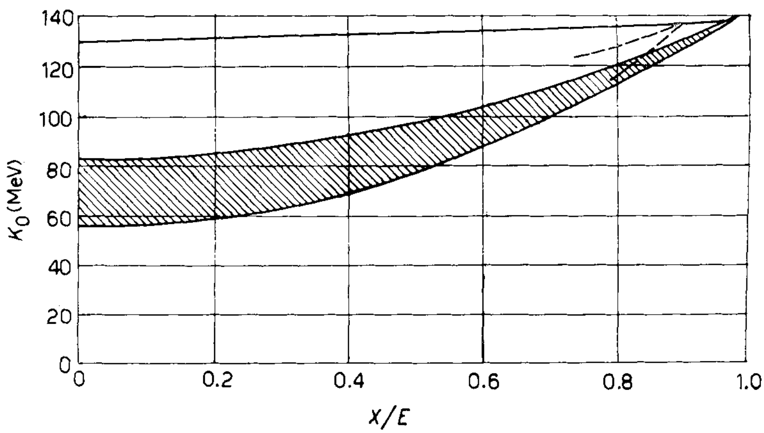

Fig. 6. – Possible combinations of mass and energy for pairs from $\pi^-+p \to n+e^++e^-$ (upper curve) and $\pi^-+p \to n+\pi^0$, $\pi^0 \to \gamma+e^++e^-$ (shaded area). Inner bremsstrahlung will move pairs slightly down and to the left; the dashed curves indicate the boundary of the region to which one point of the upper curve may be shifted in this way.

of this confusion. Fortunately, less than 0.3% of the π^0-type events have $x \geqslant 0.8$ (see Fig. 3), so that an energy uncertainty of as much as 10 MeV due to inner bremsstrahlung loss and uncertainty of measurement should still allow good separation of the two types of events.

A measurement of the contribution of longitudinal photons to process (1.1) is possible; and it would be significant, as the size of the longitudinal matrix element has never been checked, although such a check could also be obtained from process (1.3). Separation of the longitudinal contribution is easiest if attention is restricted to events such that $x \gg 2m$, for eq. (2.1) shows that in this case the transverse and longitudinal contributions have the distributions

$$
\left\{
\begin{aligned}
&\varrho_T(y) \propto 1 + y^2 = 1 + \cos^2\theta\,, \\
&\varrho_L(y) \propto 1 - y^2 = \sin^2\theta\,,
\end{aligned}
\right.
\tag{4.2}
$$

independently of x, where θ is defined in Fig. 1. The restriction $x \gg 2m$ results, in fact, in no loss of information, since the ratio $w_L(x_0)/[w(x_0)]^{\frac{1}{2}}$, where $w_L(x_0)$ and $w(x_0)$ are the cumulative distributions $w_{(L)}(x_0) = \int_{x_0}^{\mu} \mathrm{d}x\, \varrho_{(L)}(x)$ for longitudinal and total events, respectively, shows a broad maximum for x_0 between 0.5 and 0.6 (recall that $\mu \approx 1$ in the units being used, so that $2m = 0.007$). To reduce the effect of uncertainties for high-mass pairs (*e.g.*, due to the magnitude of the size effect), one might choose $x_0 = 0.4$; this would include 17% of all events, of which 10% are expected to be longitudinal. Thus a few thousand events of the type (1.1) should yield a rough measurement of the longitudinal contribution. The distributions (4.2) have been obtained only to lowest order in α; the question arises of whether the higher order radiative corrections (*i.e.*, those from the graphs of Figs. 2 and 5 other than $2a$ and $5a$) may modify them appreciably. As shown in Appendix C, photon-electron interaction via a magnetic moment term does not significantly modify the distributions, for $x \gg 2m$. On the other hand, the effect of *real*-radiative corrections (inner bremsstrahlung) may warrant further investigation when accurate experiments are carried out, although it would be expected to be small.

In principle, at least, the size effect contribution can be checked by observing the distribution of pair events in x, since this distribution is dependent on the nucleon vector charge radius r_V. The magnitude of the size effect contribution as a function of x is shown in Fig. 3 for the present empirical value of $r_V = 0.80f$; it is proportional to r_V^2. Unfortunately, the effect is so small that its measurement is exceedingly difficult; the contribution to the total rate (for $r_V = 0.80f$) is only 0.8% (cf. eq. (2.6)). This smallness is partly due to the preference for low-mass pairs, which results from the factor $1/x$ in (2.3); the contribution actually amounts to about 10% for $x \approx \mu$. Thus if some automatic means were available for locating high-mass pairs, a measurement of the size effect might become feasible. Of course, accurate knowledge of the magnetic moment and longitudinal photon contribution would be necessary for this measurement, since these have a similar and larger effect on the x-distribution.

* * *

The author wishes to express his thanks to Professor R. H. DALITZ for suggesting this problem and for much helpful discussion concerning it.

APPENDIX A

Estimation of the matrix element for $\pi^-+p\to n+2\gamma$.

The matrix element $M_{2\gamma}$ for the process $\pi^-+p\to n+2\gamma$ is related to the matrix element M_γ for the process $\pi^-+p\to n+\gamma$ by gauge invariance. In the limit that the photon energies vanish, at least, we have

$$M_{2\gamma}=\Big[ie\sum_i\varepsilon_{2i}\sum_j\partial M_\gamma/\partial q_{ji}\Big]+[\boldsymbol{\epsilon}_1\leftrightarrow\boldsymbol{\epsilon}_2] \tag{A.1}$$

where $[\boldsymbol{\epsilon}_1\leftrightarrow\boldsymbol{\epsilon}_2]$ denotes the quantity resulting from interchange of the polarization vectors $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ in the preceding expression, and the index j distinguishes the various charged particles of momenta $\boldsymbol{q}_j$ involved in the process. Whether the expression so obtained is accurate here depends upon whether the photon energies are small compared with an appropriate characteristic mass.

The only charged particle momentum appearing in eqs. (14) and (14′) of ref. ([2]) is the meson momentum $\boldsymbol{q}$; since we desire the matrix element $M_{2\gamma}$ only for the case of vanishing meson momentum, only the terms of M_γ linear in $\boldsymbol{q}$ are of interest. Referring also to eqs. (13) and (13′) of ref. ([2]), it is seen that the terms linear in $\boldsymbol{q}$ are

$$+ie\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}\boldsymbol{q}\cdot\boldsymbol{\epsilon}_1}{k}-\frac{\mu^V}{6f^2}\left(\frac{\delta_{33}}{q^3}\right)_0[2\boldsymbol{q}\cdot\boldsymbol{k}\times\boldsymbol{\epsilon}_1+i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_1\boldsymbol{q}\cdot\boldsymbol{k}-i\boldsymbol{\sigma}\cdot\boldsymbol{k}\boldsymbol{q}\cdot\boldsymbol{\epsilon}_1]+i\mu^S\boldsymbol{\sigma}\cdot\boldsymbol{q}\times(\boldsymbol{k}\times\boldsymbol{\epsilon}_1), \tag{A.2}$$

for the case of a real photon ($k_\mu^2=0$, $\boldsymbol{k}\cdot\boldsymbol{\epsilon}=0$). The subscript on the factor $(\delta_{33}/q^3)_0$ denotes evaluation for vanishing q. The factor $(1-\omega/M)^{-1}$ which multiplies the first term of (A.2) as it appears in eqs. (14) of ref. ([2]) is here absorbed into the phase space, as usual; the inverse factor which would occur with the remaining terms of (A.2) need not be retained to the accuracy of ref. ([2]). An approximate expression for the desired matrix element now results from replacing $\boldsymbol{q}$ by $e\boldsymbol{\epsilon}_2$ and symmetrizing with respect to the two photons. When the numerical values $\mu^V=(g_p-g_n)e/2M=0.35e$, $\mu^S=(g_p+g_n)e/2M=0.064e$, $f^2=0.08$, and $(\delta_{33}/q^3)_0=0.23$ are inserted, it is found that the first term is the dominant one. But this term arose from the meson-current term of ref. ([2]), and must therefore correspond to the diagram in which both photons originate at one point of the meson line: so it should have the symmetrized form: $2\boldsymbol{\sigma}\cdot(\boldsymbol{k}_1+\boldsymbol{k}_2)\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_2/[(\boldsymbol{k}_1+\boldsymbol{k}_2)^2+1]$. While the two expressions agree if the energy of either photon vanishes, they differ considerably for intermediate energies; this could be expected for this term, since the « characteristic mass » is evidently the meson mass, generally comparable with the photon momenta. It will be assumed that the other terms are given with sufficient accuracy by (A.1); in any case, their contribution is fairly small. Thus we find, finally,

$$\begin{aligned}M_{2\gamma}\propto e^2\{2/[(\boldsymbol{k}_1+\boldsymbol{k}_2)^2+1]+0.23\}\boldsymbol{\sigma}\cdot(\boldsymbol{k}_1+\boldsymbol{k}_2)\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_2-\\-0.23[\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_1\boldsymbol{k}_1\cdot\boldsymbol{\epsilon}_2+\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_2\boldsymbol{k}_2\cdot\boldsymbol{\epsilon}_1]+0.34i(\boldsymbol{k}_1-\boldsymbol{k}_2)\cdot\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_2\,.\end{aligned} \tag{A.3}$$

Upon squaring this, taking the usual sum-average over nucleon polarizations, and summing over photon polarizations, one obtains

(A.4) $$\sum |M_{2\gamma}|^2 \propto e^4\{A^2(1+c^2)[1-2(1-c)k_1k_2]+0.93Ak_1k_2c(1-c^2)\},$$

where $A=[1-(1-c)k_1k_2]^{-1}+0.23$ and $c=\boldsymbol{k}_1\cdot\boldsymbol{k}_2/k_1k_2$, to sufficient accuracy. The corresponding quantity for single-photon emission is, from eqs. (14) and (14′) of ref. (2),

(A.5) $$\sum |M_\gamma|^2 \propto 2(e+\mu^s)^2 = 2.26e^2,$$

so that the ratio of two-photon emission to single-photon emission is

(A.6) $$\frac{P(2\gamma)}{P(\gamma)}=\frac{1}{(2\pi)^3}\,\frac{\frac{1}{2}\int(\mathrm{d}^3k_1/2k_1)(\mathrm{d}^3k_2/2k_2)\sum|M_{2\gamma}|^2\delta(E-k_1-k_2)}{\int(\mathrm{d}^3k_1/2k_1)\sum|M_\gamma|^2\delta(E-k_1)}=0.019\alpha.$$

for the capture by protons of negative pions at rest.

Appendix B

Electromagnetic corrections to the production of pairs having extremely low relative velocity.

The matrix element for pair production can be written in the form

(A.7) $$M=\langle e^+, e^-|\int \mathrm{d}^4r\,F_\mu(r)\,\overline{\psi}(r)\gamma^\mu\psi(r)\,|0\rangle,$$

where $\psi(r)$ is the electron field operator and $F_\mu(r)$ depends on the momenta and spins of the incoming particles. Let us consider this expression in the pair c.m. system; for small electron velocity v we then find an expression of the form:

(A.8) $$M=\int \mathrm{d}^3r'\,\widetilde{u}_{-\boldsymbol{p}}(\boldsymbol{r}')\,\overline{\boldsymbol{F}}(\boldsymbol{r}')\cdot\boldsymbol{\sigma}\,u_{\boldsymbol{p}}(\boldsymbol{r}'),$$

where $u_{\pm\boldsymbol{p}}$ are non-relativistic positron and electron spinors, respectively. Neglecting the (inessential) spin dependence and making the substitution

(A.9) $$u_{\boldsymbol{p}_1}(\boldsymbol{r}_1)\,u_{\boldsymbol{p}_2}(\boldsymbol{r}_2)=\exp[i(\boldsymbol{p}_1+\boldsymbol{p}_2)\cdot(\boldsymbol{r}_1+\boldsymbol{r}_2)/2]\,\varphi_{(\boldsymbol{p}_1-\boldsymbol{p}_2)/2}(\boldsymbol{r}_1-\boldsymbol{r}_2),$$

we obtain

(A.10) $$M=\overline{G}(0)\,\varphi_{\boldsymbol{p}}(0),$$

where $\bar{G}(\boldsymbol{k})$ is essentially the Fourier transform of $\bar{F}(\boldsymbol{r}')$, and depends on the pair c.m. motion. Now the lowest-order perturbation calculation corresponds to taking φ_p to be a plane wave. At extremely low velocities, the chief electromagnetic correction is due to the Coulomb interaction between the outgoing particles, and can be accounted for by using the Coulomb function $\varphi_p^{C}(\boldsymbol{r}_1-\boldsymbol{r}_2)$. Thus at extremely low velocities the rate of pair production W^{C} is related to that obtained neglecting all electromagnetic corrections by the equation

$$W^{C}/W(1)=|\varphi_p^{C}(0)|^2/|\varphi_p^{pw}(0)|^2=\zeta/(1-\exp[-\zeta]) \,, \tag{A.11}$$

where φ^{pw} is the plane-wave function, $\zeta=2\pi\alpha/v$, and the expression for $|\varphi_p^{C}(0)|^2$ is taken from HEITLER [14]. It is interesting to compare this correction factor with that obtained from a perturbation calculation to second order in α. This latter factor is

$$W(2)/W(1)\approx\{[\pi^{(0)}+\pi^{(1)}]/\pi^{(0)}\}^2 \qquad \text{or} \qquad W(2)/W(1)=1+2\pi^{(1)}/\pi^{(0)} \,. \tag{A.12}$$

For low v, $\eta=(1-4m^2/x^2)^{\frac{1}{2}}\to(4p_0^2-4m^2)^{\frac{1}{2}}/2p_0=v$, where p_0 and v are evaluated in the pair c.m. system. Noting that η here is the same as Källén's δ, we find from eqs. (49), (A.7), and (6) of ref. [9] that

$$\pi^{(1)}\xrightarrow[v\to 0]{}-(\alpha^2/2\pi^2)[4\varphi(-1)+2\varphi(1)+\pi^2/2]=\alpha^2/4 \quad \text{and} \quad \pi^{(0)}\xrightarrow[v\to 0]{}\alpha v/2\pi \tag{A.13}$$

Thus

$$W(2)/W(1)\xrightarrow[v\to 0]{}1+\pi\alpha/v \,. \tag{A.14}$$

This agrees with the result of expanding (A.11) in powers of α:

$$W^{C}/W(1)=1+\pi\alpha/v+\dots \,. \tag{A.15}$$

This expansion diverges for $v\leqslant\alpha$ (an extremely low velocity for present purdoses); and, in fact, the low velocity limit of (A.14) is seen to differ from the ocrrect limit

$$W^{C}/W(1)\xrightarrow[v\to 0]{}2\pi\alpha/v \,, \tag{A.16}$$

resulting from (A.11), although both go to infinity as $1/v$. It should be emphasized that events with e^+-e^- relative velocity $v\leqslant\alpha$ are of no practical importance for the processes considered here; such events are an extremely small fraction of all events, and can certainly not be distinguished experimentally.

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[14] W. HEITLER: *The Quantum Theory of Radiation*, 2nd ed. (Oxford, 1944), p. 84.

APPENDIX C

Derivation of the distributions in $\cos\theta$.

These distributions, in the approximation of vanishing electron mass, may be derived very directly as follows. We shall use the representation

$$\text{(A.17)}\qquad \boldsymbol{\gamma}=\begin{pmatrix}0 & \boldsymbol{\sigma}\\ -\boldsymbol{\sigma} & 0\end{pmatrix},\qquad \gamma_0=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

In the limit $m\to 0$, the Dirac equation then splits into two independent equations each involving 2-component spinors:

$$\text{(A.18)}\qquad (\boldsymbol{\sigma}\cdot\boldsymbol{p}\mp p_0)u=0\,.$$

The upper two components will thus correspond to a right-handed electron (mass neglected) or a left-handed positron; and the lower two components will correspond to a left-handed electron or a right-handed positron. As before, let J_μ denote the current due to the meson capture process; then the pair-production matrix element is proportional to $e\bar{\psi}J_\mu\gamma^\mu\psi$, where the usual first order term

$$\text{(A.19)}\qquad H_1=eA_\mu\bar{\psi}\gamma^\mu\psi\,,$$

has been used for the interaction between the photon and electron fields. If we consider the process in the c.m. system of the pair, where J_0 vanishes by the current conservation condition $k_\mu J_\mu=0$, and use the representation (A.17), the matrix element becomes $au^+\boldsymbol{J}\cdot\boldsymbol{\sigma}v$, where u and v are either both «upper» or both «lower» two-component spinors and a is a constant. Now let us introduce coordinates ξ, η and ζ such that the ζ-axis is in the direction of the electron momentum, and use the standard representation for $\boldsymbol{\sigma}$, with σ_ζ diagonal (so that, *e.g.*, the top component of the four corresponds to a right-handed electron, while the bottom component corresponds to a left-handed electron). The matrix element now becomes $a(J_\xi\pm J_\eta)$, since σ_ζ connects *electrons* only with *electrons*. Thus the longitudinal component of the current, J_3 (*i.e.*, the one in the direction of $\boldsymbol{k}$) gives rise to a matrix element $aJ_3\sin\theta\cdot(\cos\varphi+i\sin\varphi)$ (where, of course, θ is the angle between the electron and photon directions). The transverse contribution may be evaluated by choosing the x_1-axis to lie along the ξ-axis, whereupon that matrix element becomes $a(J_1\pm iJ_2\cos\theta)$. The corresponding transition probabilities are thus

$$\text{(A.20)}\qquad W_T\propto|J_1|^2(1+\cos^2\theta)\qquad\text{and}\qquad W_L\propto|J_3|^2\sin^2\theta\,,$$

since $|J_2|=|J_1|$.

It can now easily be seen that although the form (A.20) taken by these distributions is modified by the anomalous magnetic moment of the electron, the effect is negligible. Thus, let us assume an interaction of the form

$$H_2 = eA_\mu k_\nu \overline{\psi}\sigma^{\mu\nu}\psi \,. \tag{A.21}$$

Proceeding as before, the matrix element is again found to be $u^\dagger \boldsymbol{J}\cdot\boldsymbol{\sigma}v$, but with one of u and v being an « upper » spinor, and the other, a « lower » spinor. In this case it is only σ_ξ that connects electrons with positrons so that production of pairs occurs by matrix elements proportional to $\pm J_\xi$. The angular distributions are now found to be

$$W'_T \propto |J_1|^2 \sin^2\theta \qquad \text{and} \qquad W'_L \propto |J_3|^2 \cos^2\theta \,. \tag{A.22}$$

Since the two interactions connect different sets of states (« corresponding » two-component spinors for the first and « opposite » ones for the second), there is no interference term (in the limit $m \to 0$). Thus, since the magnetic moment interaction resulting from electromagnetic theory is of order αH_2, magnetic moment effects will modify the $y \doteq \cos\theta$ distributions only to order α^2, provided $p_0 \gg m$ (or, equivalently, provided $x = 2p_0 \gg m$).

RIASSUNTO (*)

Si calcolano le correzioni elettromagnetiche per i processi $\pi^-+p \to n+e^++e^-$ e $\pi^0 \to \gamma+e^++e^-$. Queste correzioni dipendono dall'energia nel sistema del c.m. della coppia. Nel caso di bassa energia della coppia nel sistema del centro di massa, le correzioni sono relativamente grandi e dovute principalmente all'attrazione columbiana elettrone-positrone; esse decrescono rapidamente ad un minimo del 0.5% con l'energia del c.m. crescente e poi aumentano lentamente al valore dell'1.7% per il massimo valore dell'energia del c.m. Per i due processi le correzioni al tasso totale ammontano a circa 1%. In termini di questi tassi, il rapporto di Panofsky risulta

$$P_P = 0.594\,P[\pi^-+p \to n+\pi^0,\ \pi^0 \to \gamma+e^++e^-]/P[\pi^-+p \to n+e^++e^-]\,.$$

Per il processo $\pi^-+p \to n+e^++e^-$, si discute il problema di identificare empiricamente i contributi dovuti all'interazione tramite la componente longitudinale del campo elettromagnetico e degli effetti di dimensione. Si prevede che tali contributi ammontino solo al 2% e al 0.8% rispettivamente. Si discute brevemente anche il processo $\pi^-+p \to n+2\gamma$.

(*) *Traduzione a cura della Redazione.*