PHYSICAL REVIEW VOLUME 98, NUMBER 5 JUNE 1, 1955

Internal Pair Production Associated with the Emission of High-Energy Gamma Rays

NORMAN M. KROLL, *Columbia University, New York, New York*

AND

WALTER WADA, *Naval Research Laboratories, Washington, D. C.*

(Received January 10, 1955)

The theory of inner pair production associated with the radiative capture of π^- mesons and with the decay of the π^0 meson is discussed. Appropriate distribution functions are derived and compared with recently obtained experimental results. The weak dependence of the theoretical predictions upon the details of meson theory is emphasized. The possible utility of the double conversion process, in which the π^0 meson decays into two electron-positron pairs, for the determination of the π^0 parity is also discussed.

A. INTRODUCTION

THE process of inner pair production (i.e., the emission of an electron positron pair by an excited system in place of a γ ray) has been extensively studied in connection with the decay of excited nuclei, and the dependence of what will be referred to as the conversion coefficient on the multipolarity of the γ ray has had some utility in the classification of transitions. It has been pointed out that as the energy of the transitions exceeds the necessary threshold energy by amounts large compared to mc^2 the dependence of the process on the multipolarity of the γ ray decreases, and, in fact, the conversion coefficient becomes essentially independent of the source of the γ ray in the high-energy limit.[1–3] Radiative processes accompanying the annihilation of a π meson are good examples of this situation. These have been studied experimentally in connection with π^0 decay and the radiative capture of π^- mesons.[4,5]

The main purpose of this note is to compare the predictions of the theory with the available experimental results on the mesonic processes mentioned above. In view of the present unsatisfactory state of meson theory the weak dependence of the theoretical predictions upon this theory is of importance. We shall therefore in Sec. 1 briefly rederive expressions for the conversion coefficient and appropriate distribution functions in a manner which makes this weak dependence manifest. In subsequent sections specific formulas for the processes of interest will be obtained and compared with the experimental results.

1. INTERNAL CONVERSION FORMULA

The internal conversion process may be described as follows. A given reaction produces, instead of a real photon, a virtual photon, which subsequently produces an electron positron pair. If the photon momentum is large compared to mc, then for pairs emitted with small transverse momentum the energy of the intermediate state containing the photon is nearly the same as that of the final state. This mode of production is highly favored by the associated small energy denominator and as a consequence the emitted pairs do tend to have small transverse momentum. It is the predominance of "nearly real" photons in the process which tends to dissociate the conversion of the photon from its emission.

Consider a physical system of zero total momentum which undergoes a transformation from a state A to a state B of energy difference E_{AB}, recoiling with momentum k. If the transition takes place by virtue of photon emission, then a current $J_\mu(k_\mu)$ may be associated with the process. [$k_\mu=(\mathbf{k},k_0)$ with $k_0=-(M^2+k^2)^{\frac{1}{2}}+M+E_{AB}$ where M is the mass of the recoil system.] The photon is then generated by the interaction energy $(1/c)J_\mu A_\mu$, yielding the matrix element for emission of a photon of polarization λ:

$$M_\gamma=(1/c)J_\mu(k)\epsilon_\mu{}^\lambda(\hbar c/2k)^{\frac{1}{2}}, \tag{1}$$

from which one obtains the transition probability

$$W_\gamma=\frac{k}{8\pi^2\hbar c^4}\frac{2(E+M)^2}{(E+M)^2+M^2}\int(J_{(1)}{}^2+J_{(2)}{}^2)d\Omega, \tag{2}$$

where $J_{(\lambda)}=J_\mu\epsilon_\mu{}^\lambda$.

The transition probability for emission of a pair by the same current with electron momentum $p_{1\mu}$ and positron momentum $p_{2\mu}$ is given by

$$W_{\text{pair}}=\frac{\alpha}{\pi^2}\frac{1}{8\pi^2\hbar c^4}\int J_\mu(k')J_\nu(k')\left(\frac{1}{k_\mu'^2}\right)^2 \times\sum_{\text{spins}}(\bar\psi(p_1)\gamma_\mu\psi(-p_2))(\bar\psi(-p_2)\gamma_\nu\psi(p_1)) \times d\mathbf{p}_1d\mathbf{p}_2\delta(p_{10}+p_{20}-k_0'), \tag{3}$$

with $k_\mu'=p_{1\mu}+p_{2\mu}$ and $k_0'=(M^2+k'^2)^{\frac{1}{2}}-M+E_{AB}$. Now letting $\mathbf{p}_1=\mathbf{q}+\frac{1}{2}(1+\lambda)\mathbf{k}'$, $\mathbf{p}_2=-\mathbf{q}+\frac{1}{2}(1-\lambda)\mathbf{k}'$, with $\mathbf{q}\cdot\mathbf{k}'=0$, and taking Ω for the angular variables associated with k and ϕ the azimuthal angle of q, we transform the differential elements $d\mathbf{p}_1d\mathbf{p}_2$ to $(\frac{1}{2}qk'dqd\lambda d\phi)(k'^2dk'd\Omega)$. The δ function may now conveniently be

[1] J. R. Oppenheimer and L. Nedelsky, Phys. Rev. **44**, 948 (1933).
[2] M. E. Rose and G. E. Uhlenbeck, Phys. Rev. **48**, 211 (1935).
[3] M. E. Rose, Phys. Rev. **76**, 678 (1939).
[4] Lindenfeld, Sachs, and Steinberger, Phys. Rev. **89**, 531 (1953).
[5] Sargent, Cornelius, Rinehart, Lederman, and Rogers, preceding paper [Phys. Rev. **98**, 1349 (1955)].

eliminated by integrating over q, the factor qdq becoming $p_{10}p_{20}/k_0'$. Now writing

$$\sum_{\text{spins}} (\bar{\psi}(p_1)\gamma_\mu\psi(-p_2))(\bar{\psi}(p_2)\gamma_\nu\psi(p_1))$$
$$=\frac{1}{4p_{10}p_{20}}\operatorname{Tr}(-i\gamma\cdot p_1+m)\gamma_\mu(i\gamma\cdot p_2+m)\gamma_\nu=\frac{1}{p_{10}p_{20}}T_{\mu\nu},$$

Eq. (3) becomes

$$W_{\text{pair}}=\frac{\alpha}{2\pi^2}\frac{1}{8\pi^2\hbar c^4}\int J_\mu(k')J_\nu(k')\left(\frac{1}{k_\mu'^2}\right)^2 \times T_{\mu\nu}\frac{k'^3dk'}{k_0'}d\lambda d\phi d\Omega. \quad (4)$$

The conservation laws yield

$$q^2=-\tfrac{1}{4}k_\mu'^2(1-\lambda^2k'^2/k_0'^2)-m^2.$$

The requirement that q^2 be positive now determines the domain of λ and k'. It is convenient at this point to resolve $J_\mu(k)$ into transverse and longitudinal components. Thus we write

$$J_\mu=(\mathbf{J};J_0)=(J_{(1)}\boldsymbol{\varepsilon}_1+J_{(2)}\boldsymbol{\varepsilon}_2+J_{(3)}\mathbf{k}'/k';\,J_{(3)}k'/k_0'),$$

since $k_\mu J_\mu=0$. Taking $\boldsymbol{\varepsilon}_1$ as the polar axis for ϕ, Eq. (4) becomes

$$\begin{aligned}W_{\text{pair}}=\frac{\alpha}{2\pi^2}\frac{1}{8\pi^2\hbar c^4}\int \frac{k'^3}{k_0'(k_\mu'^2)^2}\{&J_{(1)}^2[-\tfrac{1}{2}k_\mu'^2\\ &\times(\sin^2\phi+\lambda^2\cos^2\phi k'^2/k_0'^2)+2m^2\cos^2\phi]\\ &+J_{(2)}^2[-\tfrac{1}{2}k_\mu'^2(\cos^2\phi+\lambda^2\sin^2\phi k'^2/k_0'^2)+2m^2\sin^2\phi]\\ &+\mathrm{Re}[J_{(1)}J_{(2)}k_\mu'^2(1-\lambda^2k'^2/k_0'^2)\sin\phi\cos\phi]\\ &+\mathrm{Re}[J_{(3)}\lambda qk'(J_{(1)}\cos\phi+J_{(2)}\sin\phi)k_\mu'^2/k_0'^2]\\ &+J_{(3)}^2(k_0'^2-\lambda^2k'^2)(k_\mu'^2)^2/2k_0'^4\}dk'd\lambda d\phi d\Omega. \quad (5)\end{aligned}$$

The integrand in (5) is then the distribution function in the variables k', λ, ϕ, and Ω. The angular correlations described by (5) are not generally accessible to observation due to a lack of knowledge of the direction of J_μ, although an exception to this remark will be noted in connection with the decay of the π_0 meson.[6] Integrating over angles yields

$$W_{\text{pair}}=\frac{\alpha}{2\pi}\frac{1}{8\pi^2\hbar c^4}\int\frac{k'^3dk'd\lambda}{k_0'}\left\{\left[\frac{k_0'^2+\lambda^2k'^2}{2k_0'^2(-k_\mu'^2)}+\frac{2m^2}{(k_\mu'^2)^2}\right]\times\int(J_{(1)}^2+J_{(2)}^2)d\Omega+\frac{k_0'^2-\lambda^2k'^2}{2k_0'^4}\int J_{(3)}^2d\Omega\right\} \quad (6)$$

[6] Some applications of these correlations in connection with nuclear transitions have been pointed out by G. Goldring, Proc. Phys. Soc. (London) **A66**, 391 (1953).

from which we obtain, for the conversion coefficient,

$$\rho=\frac{\alpha}{2\pi}\frac{2(E+M)^2}{(E+M)^2+M^2}\int\frac{k'^3dk'd\lambda}{k_0'k}\left\{\left[\frac{k_0'^2+\lambda^2k'^2}{2k_0'^2(-k_\mu'^2)}+\frac{2m^2}{(k_\mu^2)^2}\right]R_T(k')+\frac{k_0'^2-\lambda^2k'^2}{2k_0'^4}R_L(k')\right\} \quad (7)$$

with

$$R_T=\frac{\int(J_{(1)}^2(k')+J_{(2)}^2(k'))d\Omega}{\int(J_{(1)}^2(k)+J_{(2)}^2(k))d\Omega};$$

$$R_L=\frac{\int J_{(3)}^2(k')d\Omega}{\int(J_{(1)}^2(k)+J_{(2)}^2(k))d\Omega}.$$

For the discussion of distribution functions it is somewhat more convenient to use $x^2=-k_\mu'^2$ instead of k' and $y=\lambda k/k_0$ instead of λ as variables,[7] in which case ρ is given by

$$\rho=\frac{\alpha}{4\pi}\int_{2m}^{E}dx\int_{-\eta}^{\eta}dy\left(\frac{k'}{k}\right)\frac{(E+M)^2+M^2-x^2}{(E+M)^2+M^2}\times\left\{\left[\frac{1+y^2}{x}+\frac{4m^2}{x^3}\right]R_T(x)+(1-y^2)\frac{4(E+M)^2x}{(2EM+E^2+x^2)^2}R_L(x)\right\}, \quad (8)$$

with

$$(k'/k)^2=\frac{(2EM+E^2)^2-2x^2(2M^2+2EM+E^2)+x^4}{(2EM+E^2)^2},$$

$$\eta=[1-(2m/x)^2]^{\frac{1}{2}}.$$

The y integration can also be performed without specifying R_T and R_L, to yield

$$\rho=\frac{2\alpha}{3\pi}\int_{2m}^{E}dx\left(\frac{k'}{k}\right)\frac{(E+M)^2+M^2-x^2}{(E+M)^2+M^2}\left(1-\frac{4m^2}{x^2}\right)^{\frac{1}{2}}\times\left(1+\frac{2m^2}{x^2}\right)\left[\frac{R_T}{x}+\frac{2(E+M)^2x}{(2EM+E^2+x^2)^2}R_L\right]. \quad (9)$$

It is clear that the coefficient of R_T is strongly peaked at small values of x, and always exceeds that of $R_L(x)$. Therefore, unless R_L is much greater than unity, the transverse conversion coefficient (i.e., the contribution

[7] Note that $y=(p_{01}-p_{02})|/|\mathbf{p}_1+\mathbf{p}_2|$ is essentially a measure of the energy partition, while x may be thought of as the "rest mass" of the virtual photon. The smallness of x compared to k corresponds to the predominance of nearly real intermediate photons as discussed at the beginning of this section.

from R_T) is much larger than the longitudinal conversion coefficient. Furthermore, the strong peaking at small values of x means that a good estimate of ρ_T can be obtained by setting $R_T(x)=R_T(0)=1$. One finds in this way that

$$\rho_T=2\alpha/3\pi[\ln(2E/m)-11/6-g(E/M)], \tag{10}$$

with $g(E/M)\approx(5/24)(E/M)^2$ for small (E/M) and increasing to $\ln 2-\frac{1}{4}$ for $E/M=\infty$. The predominance of small values of x is reflected in the appearance of the logarithm in (10), and (10) is a good approximation whenever the logarithm is dominant. Clearly ρ_T is also very insensitive to the ratio E/M. Some idea of the sensitivity to variation in R_T can be had by noting that the replacement of R_T by k'/k instead of unity has the effect of changing the fraction $-11/6$ to $-(\ln 2+4/3)$.

In view of the weak dependence of ρ_T on the form of R_T, it is also worthwhile to obtain an approximate universal function for the distribution in y by replacing R_T by unity in (8) and integrating over x first. This yields

$$\rho_T=\frac{\alpha}{4\pi}\int_{-(1-4m^2/E^2)^{\frac{1}{2}}}^{(1-4m^2/E^2)^{\frac{1}{2}}} dy\left\{(1+y^2)\left[\ln\frac{E(1-y^2)^{\frac{1}{2}}}{m}-\left(1-\frac{4m^2}{E^2(1-y^2)}\right)^{\frac{1}{2}}+\ln\tfrac{1}{2}\left[1+\left(1-\frac{4m^2}{E^2(1-y^2)}\right)^{\frac{1}{2}}\right]\right]+\tfrac{1}{2}\left(1-\frac{4m^2}{E^2}-y^2\right)\right\} \tag{11}$$

for $E/M=0$. For $E/m\cong 270$ the distribution function is rather flat, with a relative minimum at $y=0$, a maximum at $y=\pm 0.88$, and falling to zero at the limits. The ratio of the maximum to the relative minimum is 1.3.

2. APPLICATION TO PHOTOMESONIC PROCESSES

(a) Reaction $\pi^-+p\rightarrow n+e^++e^-$

We now apply the theory described in Sec. 1 to the conversion of the γ ray emitted in the radiative capture of a π^- meson by a proton. The weak dependence of the conversion process on $R_T(x)$ and $R_L(x)$ has been emphasized in the preceding section. It is possible, however, to obtain approximate forms for these quantities from meson theory. Indeed the Kroll-Ruderman theorem[8] on photomeson production shows that the expressions for R_T and R_L obtained from the second order perturbation theory are valid to terms of order μ/M, where μ is the meson mass, even when all orders of perturbation theory are taken into account. The results from second order perturbation theory are simply $R_T=1$ and $R_L=2\mu^4/(2\mu^2-x^2)^2$, evaluated for $\mu/M=0$. Thus the results for ρ_T obtained in the preceding section may be applied unchanged. For ρ_L one finds, again taking $\mu/M=0$,

$$\rho_L=\frac{2\alpha}{3\pi}\int_{2m}^{E}dx\left(1-\frac{x^2}{\mu^2}\right)^{\frac{1}{2}}\left(1-\frac{4m^2}{x^2}\right)^{\frac{1}{2}}\times\left(1+\frac{2m^2}{x^2}\right)\frac{x\mu^2}{(2\mu^2-x^2)^2}$$

$$\cong\frac{2\alpha}{3\pi}\int_0^{\mu}dx\left(1-\frac{x^2}{\mu^2}\right)^{\frac{1}{2}}\frac{x\mu^2}{(2\mu^2-x^2)}=\frac{\alpha}{6\pi}\left(\frac{\pi}{2}-1\right)$$

$$=\frac{\alpha}{8\pi}\left(\frac{\pi}{2}-1\right)\int_{-1}^{1}(1-y^2)dy. \tag{12}$$

Numerically, $\rho_T=0.00693$, $\rho_L=0.00022$,[9] and one notes that ρ_L is indeed much smaller than ρ_T. The result for ρ is thus in agreement with the value 0.0062 ± 0.0013 obtained by Sargent *et al.*[5]

The distribution functions in x and y can be conveniently compared with the few experimental events by comparing the integral of the distribution function with the cumulative number of events. Thus, in Fig. 1,

$$G(x)=\int_{2m}^{x}g(x')dx'\bigg/\int_{2m}^{E}g(x')dx'$$

is compared with the fraction of the observed events for which x is less than the specified value. Similarly, in Fig. 2,

$$H(y)=\int_0^{y}h(y')dy'\bigg/\int_0^{(1-4m^2/E^2)^{\frac{1}{2}}}h(y')dy'$$

is compared with the fraction of the observed events for which $|y|$ is less than the specified value.[10] Both dis-

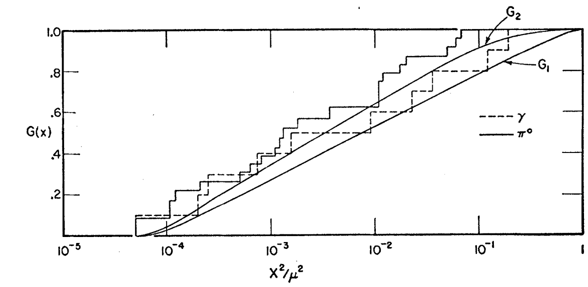

FIG. 1. Distribution in x for the inner pairs arising from the process $\pi^-+p\rightarrow n+e^++e^-$ (designated by γ - - - and G_1) and from the process $\pi^0\rightarrow\gamma+e^++e^-$ (designated by π_0 —— and G_2). The variable x is defined by $x^2=(p_1+p_2)_\mu(p_1+p_2)_\mu$. The functions $G_1(x)$ and $G_2(x)$ are, according to theory, the fractional number of events for which x is smaller than the value appearing in the argument. The step curves are the corresponding experimental results.

[8] N. Kroll and M. Ruderman, Phys. Rev. **93**, 233 (1954).

[9] These results are in agreement with those reported by W. Wada, Phys. Rev. **95**, 618(A) (1953), but in disagreement with that of W. Thirring, Proc. Roy. Irish Acad. **A54**, 205 (1951). In this latter work an expression for the current operator matrix element is used which fails to satisfy the conservation law.

[10] $g(x)$ and $h(y)$ are the integrands in Eqs. (9) and (11) respectively, modified to include the effects R_L.

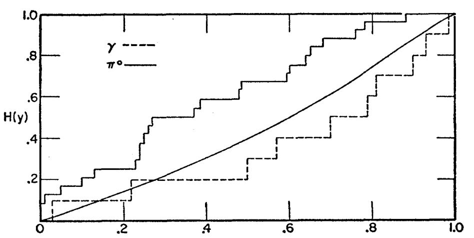

FIG. 2. Energy partition of inner pairs arising from the process $\pi^- + p \to n + e^+ + e^-$ (designated by γ ----) and from the process $\pi^0 \to \gamma + e^+ + e^-$ (designated by π^0 ——). The variable y is defined by $y = (p_{01} - p_{02})/|\mathbf{p}_1 + \mathbf{p}_2|$. The function $H(y)$ is, according to theory, the fractional number of events for which $|y|$ is smaller than the value appearing in the argument. The step curves are the corresponding experimental results.

tributions are in satisfactory agreement with the experimental results.

For completeness we note the result obtained for scalar mesons. The Kroll-Ruderman theorem applies also in this case provided one includes the anomalous moments phenomenologically. One finds that ρ_T is given essentially by (9) while for ρ_L one obtains

$$= (2\alpha/3\pi)(2M/\mu)^2(5/4 - 3\pi/8)(g_p + g_n)^{-2},$$

where g_p and g_n are gyromagnetic ratios for the proton and neutron respectively in nuclear magnetons. One notes that the large enhancement of ρ_L arising from the factor $(2M/\mu)^2$ is compensated by the factor $(g_p + g_n)^2$ in the denominator. Consequently ρ_L is small compared to ρ_T for scalar mesons also, and ρ is thus only weakly dependent upon the parity of the captured meson.

(b) Reaction $\pi^0 \to \gamma + e^+ + e^-$

The theory may also be applied to the conversion of the γ rays emitted in the decay of a neutral meson. We consider first the case in which only one of the γ rays is converted. In this case the recoiling system is, of course, the other photon, so that $M = 0$. Furthermore, R_L vanishes for both scalar and pseudoscalar mesons. The form of R_T which one obtains from the lowest order of perturbation theory is, again for both scalar and pseudoscalar mesons, $(k'/k)f(x/2M)$ with $f(0) = 1$ and varying only a small amount from this value over the available range of its argument. The factor k'/k is a

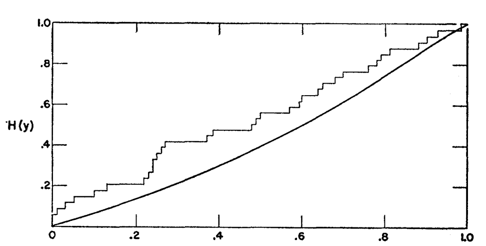

FIG. 3. Energy partition for the two processes considered together. The experimental results represented in Fig. 2, for the two processes $\pi^- + P \to n + e^+ + e^-$ and $\pi^0 \to \gamma + e^+ + e^-$ are combined. The step curve gives the fractional number of events of either type for which $|y|$ is less than the value appearing in the argument.

consequence of gauge invariance, relativistic invariance, and the form of the photon emission matrix elements, while the behavior of $f(x/2M)$ can be attributed to the fact that x is always small compared to the energy of the intermediate states. (These always involve the presence of a nucleon-antinucleon pair.) It is therefore reasonable to take $R_T = k'/k$ (in spite of the inadequacy of perturbation theory) and thus obtain

$$\rho = \frac{4\alpha}{3\pi}\int_{2m}^{\mu}\left(1 - \frac{x^2}{\mu^2}\right)^3\left(1 - \frac{4m^2}{x^2}\right)^{\frac{1}{2}}\left(1 + \frac{2m^2}{x^2}\right)\frac{dx}{x}$$

$$= \frac{\alpha}{2\pi}\int_0^{(1-4m^2/\mu^2)^{\frac{1}{2}}} dy\left\{(1+y^2)\left[\ln\frac{\mu^2(1-y^2)}{4m^2} - \frac{11}{6} + 3\left(\frac{4m^2}{\mu^2(1-y^2)}\right) - \frac{3}{2}\left(\frac{4m^2}{\mu^2(1-y^2)}\right)^2 + \frac{1}{3}\left(\frac{4m^2}{\mu^2(1-y^2)}\right)^3\right] + 1 - \frac{4m^2}{\mu^2} - y^2\right\}$$

$$= \frac{2\alpha}{3\pi}\left(\ln\frac{\mu^2}{m^2} - \frac{7}{2}\right) = 0.0118. \quad (13)$$

In obtaining (13), (8) has been doubled to take into account the fact that either photon can convert.[11]

In Figs. 1 and 2 the distributions in x and y are compared to the experimental results in a manner similar to that used for the pairs associated with radiative capture. The theoretical distributions in y are so similar for the two situations that a single plot serves for both. In view of the fact that the π^0 meson is not at rest when it decays, it is worth noting that x is an invariant and therefore independent of the motion of the π^0. The variable y is essentially independent of the motion of the π^0 as long as both pair particles are highly relativistic and the angle between them is small. Since this is the case for virtually all of the pairs, no correction has been made. One notes that the x distribution is in good agreement with the theory. The y distribution, on the other hand, appears to be in some disagreement in a manner suggesting a marked preference for equipartition.[12] Considerably more data would be required, however, to establish a real contradiction. In view of the fact that the theoretical y distribution functions are essentially identical for the capture and the decay pairs it is worth noting that the combined distributions are in better agreement with the theory (Fig. 3).

(c) Reaction $\pi^0 \to 2e^+ + 2e^-$

The case in which both γ rays are internally converted, while a very rare process, may be of interest in connection with the determination of the parity of the

[11] This result has been obtained previously by R. H. Dalitz, Proc. Phys. Soc. (London) **A64**, 667 (1951).

[12] A much stronger effect of the same kind has been observed by A. M. Anand, Proc. Roy. Soc. (London) **A220**, 183 (1953).

π^0. This comes about as a result of the angular correlations described in Eq. (5). It has been pointed out that the polarizations of the two photons emitted in the decay of the π^0 are perpendicular or parallel as the π^0 is pseudoscalar or scalar.[13] According to (5), there is a strong correlation between the plane determined by the propagation and polarization vectors of the photon and the plane of the pair into which it converts. Thus the two pairs should tend to lie in perpendicular planes or the same plane accordingly as the meson is pseudoscalar or scalar.

Using the same form for the meson decay matrix element as was used in the derivation of (13), and neglecting exchange, one finds for the transverse conversion coefficient

$$\rho_T=\frac{1}{\pi}\left(\frac{\alpha}{4\pi}\right)^2\int_{2m}^{\mu-2m}dx_1\int_{2m}^{\mu-x_1}dx_2\int_{-\eta_1}^{\eta_1}dy_1\int_{-\eta_2}^{\eta_2}dy_2$$
$$\int_0^{2\pi}d\phi\left(1-\frac{2(x_1^2+x_2^2)}{\mu^2}+\frac{(x_1^2-x_2^2)^2}{\mu^4}\right)^{\frac{3}{2}}$$
$$\times\left\{\left[\frac{1}{x_1x_2}+\left(\frac{y_1^2}{x_1}+\frac{4m^2}{x_1^3}\right)\left(\frac{y_2^2}{x_2}+\frac{4m^2}{x_2^3}\right)\right]\sin^2\phi\right.$$
$$\left.+\left[\frac{y_1^2+y_2^2}{x_1x_2}+\frac{4m^2}{x_2x_1^3}+\frac{4m^2}{x_1x_2^3}\right]\cos^2\phi\right\} \quad (14)$$

for a pseudoscalar π^0, the $\sin^2\phi$ and $\cos^2\phi$ factors being interchanged for the scalar case. From the form of (14) it is apparent that the process can be regarded as resulting from nearly independent conversions of the two γ rays. Cross terms involving the longitudinal conversion of one photon and the transverse conversion of the other vanish for both the scalar and pseudoscalar case, while the longitudinal conversion of both photons contributes a negligible amount in the scalar case and vanishes in the pseudoscalar case. For any individual case of two pair decay there are two sets of values for the variables, x_1, y_1, x_2, y_2 corresponding to the two possible pairings of the electrons with the positrons. The neglect of exchange corresponds to the neglect of interference between the amplitudes arising from the two possible pairings. For the great majority of cases, the product x_1x_2 will be much smaller for one pairing than for the other, a fact which makes the exchange effect negligible. In comparing (14) with experiment, the pairing which yields the smaller value should be used.

The correlation function for the planes of the two pairs may be obtained by integrating ρ_T over the variables $x_1\cdots y_2$, yielding

$$\rho_T=\frac{1}{\pi}\rho_T\int_0^{2\pi}(0.59\sin^2\phi+0.41\cos^2\phi)d\phi$$

for the pseudoscalar case, with $\sin^2\phi$ and $\cos^2\phi$ interchanged for the scalar case. The numerical value of ρ_T is 3.47×10^{-5} so that only one out of 29 000 π^0's decay into two pairs. In spite of this small value, the process does offer some possibility of yielding information on the parity of the π^0.

[13] C. N. Yang, Phys. Rev. **77**, 292 (1950), and 722 (1950).