

Spin-dependent $\mu \rightarrow e$ conversionVincenzo Cirigliano^a, Sacha Davidson^{b,*}, Yoshitaka Kuno^c^a Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA^b IPNL, CNRS/IN2P3, Université Lyon 1, Univ. Lyon, 69622 Villeurbanne, France^c Department of Physics, Osaka University, 1-1 Machikaneyama, Toyonaka, Osaka 560-0043, Japan

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ABSTRACT

The experimental sensitivity to $\mu \rightarrow e$ conversion on nuclei is expected to improve by four orders of magnitude in coming years. We consider the impact of $\mu \rightarrow e$ flavour-changing tensor and axial-vector four-fermion operators which couple to the spin of nucleons. Such operators, which have not previously been considered, contribute to $\mu \rightarrow e$ conversion in three ways: in nuclei with spin they mediate a spin-dependent transition; in all nuclei they contribute to the coherent (A^2 -enhanced) spin-independent conversion via finite recoil effects and via loop mixing with dipole, scalar, and vector operators. We estimate the spin-dependent rate in Aluminium (the target of the upcoming COMET and Mu2e experiments), show that the loop effects give the greatest sensitivity to tensor and axial-vector operators involving first-generation quarks, and discuss the complementarity of the spin-dependent and independent contributions to $\mu \rightarrow e$ conversion.

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1. Introduction

New particles and interactions beyond the Standard Model of particle physics are required to explain neutrino masses and mixing angles. The search for traces of this New Physics (NP) is pursued on many fronts. One possibility is to look directly for the new particles implicated in neutrino mass generation, for instance at the LHC [1] or SHiP [2]. A complementary approach seeks new interactions among known particles, such as neutrinoless double beta decay [3] or Charged Lepton Flavour Violation (CLFV) [4].

CLFV transitions of charged leptons are induced by the observed massive neutrinos, at unobservable rates suppressed by $(m_\nu/m_W)^4 \sim 10^{-48}$. A detectable rate would point to the existence of new heavy particles, as may arise in models that generate neutrino masses, or that address other puzzles of the Standard Model such as the hierarchy problem. Observations of CLFV are therefore crucial to identifying the NP of the lepton sector, providing information complementary to direct searches.

From a theoretical perspective, at energy scales well below the masses of the new particles, CLFV can be parametrised with effective operators (see e.g. [5]), constructed out of the kinematically accessible Standard Model (SM) fields, and respecting the relevant gauge symmetries. In this effective field theory (EFT) description,

information about the underlying new dynamics is encoded in the operator coefficients, calculable in any given model.

The experimental sensitivity to a wide variety of CLFV processes is systematically improving. Current bounds on branching ratios of τ flavour changing decays such as $\tau \rightarrow \mu\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow 3\ell$ [6–8] are $\mathcal{O}(10^{-8})$, and Belle-II is expected to improve the sensitivity by an order of magnitude [9]. The bounds on the $\mu \leftrightarrow e$ flavour changing processes are currently of order $\sim 10^{-12}$ [10,11], with the most restrictive constraint from the MEG collaboration: $BR(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13}$ [12]. Future experimental sensitivities should improve by several orders of magnitude, in particular, the COMET [13] and Mu2e [14] experiments aim to reach a sensitivity to $\mu \rightarrow e$ conversion on nuclei of $\sim 10^{-16}$, and the PRISM/PRIME proposal [15] could reach the unprecedented level of 10^{-18} .

In searches for $\mu \rightarrow e$ conversion, a μ^- from the beam is captured by a nucleus in the target, and tumbles down to the 1s state. The muon will be closer to the nucleus than an electron ($r \sim \alpha Z/m$), due to its larger mass. In the presence of a CLFV interaction with the quarks that compose the nucleus, or with its electric field, the muon can transform into an electron. This electron, emitted with an energy $E_e \simeq m_\mu$, is the signature of $\mu \rightarrow e$ conversion.

Initial analytic estimates of the $\mu \rightarrow e$ conversion rate were obtained by Feinberg and Weinberg [16], a wider range of nuclei were studied numerically by Shankar [17], and relativistic effects relevant in heavier nuclei were included in Ref. [18]. State of the art conversion rates for a broad range of nuclei induced by CLFV

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operators which can contribute coherently to $\mu \rightarrow e$ conversion were obtained in Ref. [19], while some missing operators were included in Ref. [20].

The calculation has some similarities with dark matter scattering on nuclei [21–23], where the cross-section can be classified as spin-dependent (SD) or spin-independent (SI). Previous analyses of $\mu \rightarrow e$ conversion [19,20] focused on CLFV interactions involving a scalar or vector nucleon current, because, similarly to SI dark matter scattering, these sum coherently across the nucleus at the amplitude level, giving an amplification $\sim A^2$ in the rate, where A is the atomic number. However, other processes are possible, such as spin-dependent conversion on the ground state nucleus, which we explore here, or incoherent $\mu \rightarrow e$ conversion, where the final-state nucleus is in an excited state [17,24].

The upcoming exceptional experimental sensitivities motivate our study of new contributions to $\mu \rightarrow e$ conversion induced by tensor and axial vector operators,¹ which were not considered in Refs. [19,20]. These operators couple to the spin of the nucleus and can induce “spin-dependent” $\mu \rightarrow e$ conversion in nuclei with spin (such as Aluminium, the proposed target of COMET and Mu2e), not enhanced by A^2 . In addition, the tensor and axial operators will contribute to “spin-independent” conversion via finite-momentum-transfer corrections [25,26], and Renormalisation Group mixing [27,28].² In an EFT framework, our analysis shows new sensitivities to previously unconstrained combinations of dimension-six operator coefficients, as we illustrate below. In the absence of CLFV, this gives new constraints on the coefficients, and when CLFV is observed, it could assist in determining its origin.

2. Estimating the $\mu \rightarrow e$ conversion rate

Our starting point is the effective Lagrangian [4]

$$\delta\mathcal{L} = -2\sqrt{2}G_F \sum_Y \left(C_{D,Y} \mathcal{O}_{D,Y} + C_{GG,Y} \mathcal{O}_{GG,Y} + \sum_{q=u,d,s} \sum_O C_{O,Y}^{qq} \mathcal{O}_{O,Y}^{qq} + h.c. \right) \quad (1)$$

where $Y \in \{L, R\}$ and $O \in \{V, A, S, T\}$ and the operators are explicitly given by ($P_{L,R} = 1/2(1 \mp \gamma_5)$)

$$\begin{aligned} \mathcal{O}_{D,Y} &= m_\mu (\bar{e} \sigma^{\alpha\beta} P_Y \mu) F_{\alpha\beta} \\ \mathcal{O}_{GG,Y} &= \frac{9}{32\pi^2 m_t} (\bar{e} P_Y \mu) \text{Tr}[G_{\alpha\beta} G^{\alpha\beta}] \\ \mathcal{O}_{V,Y}^{qq} &= (\bar{e} \gamma^\alpha P_Y \mu) (\bar{q} \gamma_\alpha q) \\ \mathcal{O}_{A,Y}^{qq} &= (\bar{e} \gamma^\alpha P_Y \mu) (\bar{q} \gamma_\alpha \gamma_5 q) \\ \mathcal{O}_{S,Y}^{qq} &= (\bar{e} P_Y \mu) (\bar{q} q) \\ \mathcal{O}_{T,Y}^{qq} &= (\bar{e} \sigma^{\alpha\beta} P_Y \mu) (\bar{q} \sigma_{\alpha\beta} q). \end{aligned} \quad (2)$$

While our primary focus is on the tensor ($\mathcal{O}_{T,Y}^{qq}$) and axial ($\mathcal{O}_{A,Y}^{qq}$) operators, we include the vector, scalar, dipole and gluon operators because the first three are induced by loops, and the last arises by integrating out heavy quarks.

¹ We leave out the light-quark pseudoscalar operators and gluon operators such as $G\tilde{G}$ that can be induced by heavy-quark pseudoscalar operators at the heavy quark thresholds. The effect of this class of operators in a nucleus is suppressed both by spin and momentum transfer.

² The analogous mixing of SD to SI dark matter interactions was discussed in [29,30].

At zero momentum transfer, the quark bilinears can be matched onto nucleon bilinears

$$\bar{q}(x) \Gamma_O q(x) \rightarrow G_O^{N,q} \bar{N}(x) \Gamma_O N(x) \quad (3)$$

where the vector charges are $G_V^{p,u} = G_V^{n,d} = 2$ and $G_V^{p,d} = G_V^{n,u} = 1$, and for the axial charges we use the results inferred in Ref. [22] by using the HERMES measurements [31], namely $G_A^{p,u} = G_A^{n,d} = 0.84(1)$, $G_A^{p,d} = G_A^{n,u} = -0.43(1)$, and $G_A^{p,s} = G_A^{n,s} = -0.085(18)$. For the tensor charges we use the lattice QCD results [32] in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV, namely $G_T^{p,u} = G_T^{n,d} = 0.77(7)$, $G_T^{p,d} = G_T^{n,u} = -0.23(3)$, and $G_T^{p,s} = G_T^{n,s} = 0.008(9)$. Finally, for the scalar charges induced by light quarks we use a precise dispersive determination [33], $G_S^{p,u} = \frac{m_N}{m_u} 0.021(2)$, $G_S^{p,d} = \frac{m_N}{m_d} 0.041(3)$, $G_S^{n,u} = \frac{m_N}{m_u} 0.019(2)$, and $G_S^{n,d} = \frac{m_N}{m_d} 0.045(3)$, and an average of lattice results [34] for the strange charge: $G_S^{p,s} = G_S^{n,s} = \frac{m_N}{m_s} 0.043(11)$. In all cases, we take central values of the $\overline{\text{MS}}$ quark masses at $\mu = 2$ GeV, namely $m_u = 2.2$ MeV, $m_d = 4.7$ MeV, and $m_s = 96$ MeV [35].

Taking the above matching into account, the nucleon-level effective Lagrangian has the same structure of (1) with the replacements $\bar{q} \Gamma_O q \rightarrow \bar{N} \Gamma_O N$ and with effective couplings given by³

$$\tilde{C}_{O,Y}^{NN} = \sum_{q=u,d,s} G_O^{N,q} C_{O,Y}^{qq}. \quad (4)$$

However, we remove the tensor operators, because their effects can be reabsorbed into shifts to the axial-vector and scalar operator coefficients. In fact, to leading order in a non-relativistic expansion $\bar{N} \sigma^{ij} N = \epsilon^{ijk} \bar{N} \gamma^k \gamma_5 N$, so that the spin-dependent nucleon effective Lagrangian for $\mu \rightarrow e$ conversion reads

$$-2\sqrt{2}G_F \sum_N \sum_Y \left(\tilde{C}_{A,Y}^{NN} (\bar{e} \gamma^\alpha P_Y \mu) (\bar{N} \gamma_\alpha \gamma_5 N) + h.c. \right) \quad (5)$$

where $N \in \{n, p\}$, $Y \in \{L, R\}$, and

$$\tilde{C}_{A,X}^{NN} = \sum_q \left(G_A^{N,q} C_{A,X}^{qq} + 2G_T^{N,q} C_{T,X}^{qq} \right). \quad (6)$$

Furthermore, at finite recoil the tensor operator induces a contribution to the SI amplitude, since $\bar{u}_N(p) \sigma^{0i} u_N(p-q)$ contains a term proportional to q^i/m_N [25,26], which contracts, in the amplitude, with the spin of the helicity-eigenstate electron. The net effect is tantamount to replacing the coefficient of the scalar operator with

$$\tilde{C}_{S,Y}^{NN} \rightarrow \tilde{C}_{S,Y}^{NN} + \frac{m_\mu}{m_N} \tilde{C}_{T,Y}^{NN}. \quad (7)$$

We write the conversion rate $\Gamma = \Gamma_{SI} + \Gamma_{SD}$, where Γ_{SI} is the A^2 -enhanced rate occurring in any nucleus, and Γ_{SD} is only relevant in nuclei with spin. The usual SI branching ratio reads [4,19]

$$\begin{aligned} \text{BR}_{SI} &= 2B_0 \left| \left[\tilde{C}_{V,R}^{pp} + \tilde{C}_{S,L}^{pp} \right] Z F_p(m_\mu) \right. \\ &\quad + \left[\tilde{C}_{V,R}^{nn} + \tilde{C}_{S,L}^{nn} \right] [A - Z] F_n(m_\mu) \\ &\quad \left. + 2C_{D,L} Z e F_p(m_\mu) \right|^2 + \{L \leftrightarrow R\}, \end{aligned} \quad (8)$$

where $B_0 = G_F^2 m_\mu^5 (\alpha Z)^3 / (\pi^2 \Gamma_{cap})$, Γ_{cap} is the rate for the muon to transform to a neutrino by capture on the nucleus ($0.7054 \times$

³ The gluon operators $\mathcal{O}_{GG,Y}$ induce a shift in the coefficient of the nucleon scalar density $\tilde{C}_{S,Y}^{NN}$, as discussed in Ref. [20]. We do not explicitly include this effect as it is not relevant to our discussion.

$10^6/\text{sec}$ in Aluminium [36]), and the form factors $F_{p,n}(|\vec{k}|) = \int d^3x e^{-i\vec{k}\cdot\vec{x}} \rho_{p,n}(x)$ can be found in Eq. (30) of Ref. [19].

In the evaluation of Γ_{SD} from (5) we treat the muon as non-relativistic and the electron as a plane wave. Both are good approximations for low- Z nuclei; for definiteness we focus on Aluminium ($Z = 13$, $A = 27$, $J = 5/2$) the proposed target for the COMET and Mu2e experiments. After approximating the muon wavefunction in the nucleus to its value at the origin and taking it outside the integral over the nucleus [16], the nuclear part of the spin-dependent $\mu \rightarrow e$ amplitude corresponds to that of “standard” spin-dependent WIMP nucleus scattering. At momentum transfer \vec{q} , this is

$$\int d^3x e^{-i\vec{q}\cdot\vec{x}} \langle A | \bar{N}(x) \gamma^k \gamma_5 N(x) | A \rangle. \quad (9)$$

The $\mu \rightarrow e$ amplitude is then obtained by multiplying by the appropriate lepton current and coefficients.⁴ By analogy with WIMP scattering [22,23,37], we obtain:

$$\text{BR}_{SD} = 8B_0 \frac{J_{AI} + 1}{J_{AI}} \left| S_p^{AI} \tilde{C}_{A,L}^{pp} + S_n^{AI} \tilde{C}_{A,L}^{nn} \right|^2 \frac{S_A(m_\mu)}{S_A(0)} + \{L \leftrightarrow R\}. \quad (10)$$

The spin expectation values S_N^{AI} are defined as $S_N^{AI} = \langle J_{AI}, J_z = J_{AI} | S_N^z | J_{AI}, J_z = J_{AI} \rangle$, where S_N^z is the z component of the total nucleon spin, and the expectation value is over the nuclear ground state. They can be implemented in our QFT notation (with relativistic state normalisation for $A|$) by setting Eqn. (9) at $|\vec{q}| = 0$ to

$$2S_N^{AI} \frac{(J_{AI})^k}{|J_{AI}|} \times 2m_{AI} (2\pi)^3 \delta^{(3)}(p_{AI,out} - p_{AI,in}).$$

The axial structure factor $S_A(|\vec{q}|)$ [23,37] reads

$$S_A(q) = a_{L,+}^2 S_{00}(q) + a_{L,+} a_{L,-} S_{01}(q) + a_{L,-}^2 S_{11}(q)$$

where $a_{L,\pm} = \tilde{C}_{A,L}^{pp} \pm \tilde{C}_{A,L}^{nn}$. The S_N^{AI} and $S_{ij}(q)$ have been calculated in the shell model in Refs. [37,38]. At $|\vec{q}| \equiv q = 0$ the conversion rate is controlled by the spin expectation values; we use $S_n^{AI} = 0.030$ and $S_p^{AI} = 0.34$ [38]. At finite momentum transfer $q = m_\mu$, the structure factors provide a non-trivial correction. Using dominance of the proton contribution ($S_p^{AI} \gg S_n^{AI}$) we find from Ref. [38] $S_{AI}(m_\mu)/S_{AI}(0) \simeq 0.29$.

3. Loop effects and the RGEs

QED and QCD loops change the magnitude of some operator coefficients, and QED loops can transform one operator into another. Such Standard Model loops are necessarily present, and their dominant (log-enhanced) effects are included in the evolution with scale of the operator coefficients, as described by the Renormalisation Group Equations (RGEs) of QED and QCD (see [5] for an introduction to the RG running of operators with the scale μ). If the New Physics scale is well above m_W , loops involving the W , Z , and h could also be relevant. However, we focus here on the RGE evolution from the experimental scale μ_N up to the weak scale m_W . Since any UV model can be mapped into a set of operator coefficients at $\mu = m_W$, our calculation does not lose generality while remaining quite simple.

⁴ At finite recoil, the vector or scalar operators can also contribute to the spin-dependent amplitude [26]. We neglect these contributions, because we estimate their interference with the axial vector is suppressed by $\mathcal{O}(m_\mu/m_N)$.

We consider the one-loop RGEs of QED and QCD for $\mu \leftrightarrow e$ flavour-changing operators [27,28]. Defining $\lambda = \frac{\alpha_s(m_W)}{\alpha_s(\mu_N)}$, their solution can be approximated as

$$C_I(\mu_N) \simeq C_J(m_W) \lambda^{a_I} \left(\delta_{JI} - \frac{\alpha_e \tilde{\Gamma}_{JI}^e}{4\pi} \log \frac{m_W}{\mu_N} \right) \quad (11)$$

where I, J represent the super- and subscripts which label operator coefficients. The a_I describe the QCD running and are only non-zero for scalars and tensors: for $N_f = 5$ one has $a_I = \frac{\Gamma_{II}^s}{2\beta_0} = \{-\frac{12}{23}, \frac{4}{23}\}$ for $I = S, T, ..$. We use this scaling to always give results in terms of coefficients at the low scale $\mu_N = 2$ GeV, where we match quarks to nucleons. Γ^e is the one-loop QED anomalous dimension matrix, rescaled [39,40] for $J, I \in T, S$ to account for the QCD running:

$$\tilde{\Gamma}_{JI}^e = \Gamma_{JI}^e f_{JI}, \quad f_{JI} = \frac{1}{1 + a_J - a_I} \frac{\lambda^{a_I - a_J} - \lambda}{1 - \lambda}. \quad (12)$$

In the estimates presented here, we focus on the effects of the off-diagonal elements of $\tilde{\Gamma}_{JI}^e$, which mix one operator into another, and neglect the QED running of individual coefficients.

In RG evolution down to μ_N , photon exchange between the external legs of a tensor operator can mix it to a scalar operator. This contribution to the scalar coefficient is

$$\Delta \tilde{C}_{S,X}^{NN}(\mu_N) \sim \sum_q G_S^{N,q} f_{TS} 24 Q_q \frac{\alpha_e}{\pi} \log \frac{m_W}{\mu_N} C_{T,X}^{qq}(\mu_N) \quad (13)$$

where f_{TS} is from Eq. (12).

The tensor operator also mixes to the dipole, when the quark lines are closed and an external photon is attached. This gives a contribution to the dipole coefficient

$$\left| \Delta C_{D,X}^{e\mu}(\mu_N) \right| \sim \frac{2 Q_q N_c m_q}{e m_\mu} \frac{\alpha_e}{\pi} \log \frac{m_W}{\mu_N} C_{T,X}^{qq}(\mu_N) \quad (14)$$

which is suppressed by m_q/m_μ , due to a mass insertion on the quark line. For tensor operators involving u, d or s quark bilinears, the mixing to the scalar operator described in Eq. (13) gives a larger contribution to SI $\mu \rightarrow e$ conversion than this mixing to the dipole. So for the remainder of this letter, we do not discuss the contribution of Eq. (14) to $\mu \rightarrow e$ conversion. We will discuss heavier quarks⁵ in a later publication [41].

Curiously, one-loop QED corrections to the axial operator generate the vector [28].⁶ If a New Physics model induces a non-zero coefficient $C_{A,Y}^{qq}(m_W)$, then photon exchange between the external legs induces a contribution to the vector coefficient at the experimental scale:

$$\Delta C_{V,Y}^{qq}(\mu_N) \simeq -3 Q_q \frac{\alpha_e}{\pi} \log \frac{m_W}{\mu_N} C_{A,Y}^{qq}(\mu_N) \quad (15)$$

As a result, the SI and SD processes will have comparable sensitivities to axial vector operators.

4. Results

To interpret our results, we first estimate the sensitivity of SD and SI $\mu \rightarrow e$ conversion to the coefficients of the tensor and axial operators of Eqn. (2). We allow a single operator coefficient to be non-zero at m_W , and consider its various contributions to SD

⁵ The heavy quark scalar contribution to $\mu \rightarrow e$ conversion [20] is suppressed $\propto 1/m_Q$, so the tensor mixing to the dipole could dominate.

⁶ If the lepton current contained γ^μ , rather than $\gamma^\mu \gamma_5$, this would not occur.

and SI $\mu \rightarrow e$ conversion (sometimes referred to as setting bounds “one-operator-at-a-time”).

Suppose first that only the tensor coefficient $C_{T,L}^{uu}$ is present at m_W . Recall that $C_{T,L}^{uu}(m_W)$ can contribute to $\mu \rightarrow e$ conversion in three ways: to the SI rate via the finite momentum transfer effects of Eqn. (7), to the SI rate via the RG mixing to the scalar given in Eqn. (13), and directly to the SD rate as given in Eqn. (10). It is easy to check that the RG mixing contribution to $\tilde{C}_{S,N}^{NN}(\mu_N)$ is an order of magnitude larger than the finite recoil contribution. Furthermore, the RG mixing effect is dominant contribution of $C_{T,L}^{uu}(m_W)$ to $\mu \rightarrow e$ conversion, as can be seen numerically by calculating the SD and SI contributions to the branching ratio:

$$BR(\mu\text{Al} \rightarrow e\text{Al}) \sim .12|1.54C_{T,L}^{uu}|^2 + .27|47C_{T,L}^{uu}|^2 \quad (16)$$

where the coefficients are at the experimental scale, and the second term is the A^2 -enhanced SI contribution.

The RG mixing is the largest contribution of $C_{T,L}^{uu}(m_W)$ to $\mu \rightarrow e$ conversion due to three enhancements: first, the anomalous dimension Γ_{TS}^e is large, and second, the $G_S^{N,q}$ coefficients of Eqn. (3) are an order of magnitude larger than $G_T^{N,q}$. The combination of these gives $\Delta\tilde{C}_{S,X}^{NN}(\mu_N) \gtrsim \tilde{C}_{T,X}^{pp}(\mu_N)$, which respectively contribute to the SI and SD rates. Finally, the scalar coefficient benefits from a further A^2 enhancement in the SI conversion rate. This shows that including the RG effects can change the branching ratio by orders of magnitude.

A similar estimate for the axial operator $\mathcal{O}_{A,L}^{uu}$ gives

$$BR(\mu\text{Al} \rightarrow e\text{Al}) \sim .12|0.84C_{A,L}^{uu}|^2 + .27|.69C_{A,L}^{uu}|^2. \quad (17)$$

We see that the RG mixing of $\mathcal{O}_{A,L}^{uu}$ into $\mathcal{O}_{V,L}^{uu}$, whose coefficient contributes to SI $\mu \rightarrow e$ conversion, also gives the best sensitivity to $C_{A,L}^{uu}$. However, the ratio of SI to SD contributions is smaller than in the tensor case, due to the smaller anomalous dimension in Eqn. (15).

SI $\mu \rightarrow e$ conversion will also give the best sensitivity to tensor and axial operators involving d quarks. However, in the case of strange quarks, the vector current vanishes in the nucleon, so $\mathcal{O}_{A,Y}^{ss}$ only contributes to SD $\mu \rightarrow e$ conversion. The largest contribution of the strange tensor operator is via its mixing to the scalar, with a sensitivity to $C_{T,X}^{ss}$ reduced by a factor $\sim G_T^{Ns}/2G_T^{Nu}$ with respect to $C_{T,X}^{uu}$. The strange tensor also mixes significantly to the dipole (see Eqn. (14)) which contributes to $\mu \rightarrow e\gamma$; we estimate that the sensitivity to $C_{T,Y}^{ss}$ of the MEG-II experiment with $BR \sim \text{few} \times 10^{-14}$ (as expected after their upgrade), would be comparable to that of COMET or Mu2e with $BR \sim \text{few} \times 10^{-16}$.

Let us now focus on the complementarity of SD and SI contributions to the $\mu \rightarrow e$ conversion rate, which depend on different combinations of operator coefficients. So once a signal is observed, measuring $\mu \rightarrow e$ conversion in targets with and without spin could assist in differentiating among operators or models. To illustrate this complementarity, we restrict to scalar and tensor operators involving u quarks, whose coefficients we would like to determine. Fig. 1 represents the allowed parameter space for $C_{T,L}^{uu}$ and $C_{S,L}^{uu}$ evaluated at μ_N (dotted blue) and m_W (solid red). We see that, irrespective of the operator scale, SD $\mu \rightarrow e$ conversion always gives an independent constraint. In its absence, there would be an unconstrained direction in parameter space, corresponding to $C_{T,Y}^{uu}$ at the experimental scale, or the diagonal red band at m_W . The figure also shows that the enhanced sensitivity of SI conversion illustrated in Eqn. (16) requires the (model-dependent) assumption that the model does not induce a scalar contribution which cancels the mixing of the tensor into the scalar, which would correspond to venturing along the red ellipse in the plot.

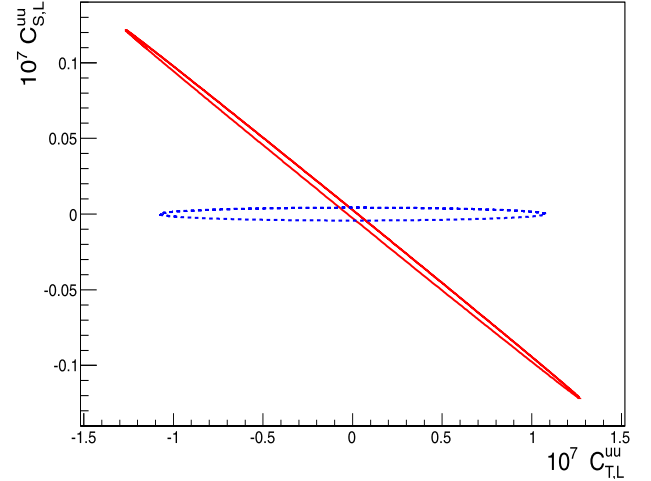


Fig. 1. The horizontal dotted blue (diagonal red) areas are the allowed parameter space at the experimental scale (at m_W), if $BR(\mu \rightarrow e \text{ conversion}) \leq 10^{-14}$. This plot assumes that CLFV only occurs in up-quark operators.

5. Prospects

In this letter, we followed the pragmatic low-energy perspective of parametrising charged Lepton Flavour Violating interactions with effective operators, and considered the contribution of axial vectors and tensors to $\mu \rightarrow e$ conversion. To our knowledge, this has not been studied previously. We found that the Spin-Dependent process depends on different operator coefficients from the Spin-Independent case, so comparing $\mu \rightarrow e$ conversion rates in targets with and without spin would give additional constraints, and could allow to identify axial or tensor operators coefficients. In future work [41], we plan to give rates for a complete set of operators, estimate their uncertainties due to higher order terms and neglected effects, and explore realistic prospects for distinguishing models/operators using targets with and without spin, such as different isotopes of Ti , a nucleus used for the past $\mu \rightarrow e$ conversion conversion searches.

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