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# The $\mu - e$ Conversion in Nuclei, $\mu \rightarrow e\gamma$ , $\mu \rightarrow 3e$ Decays and TeV Scale See-Saw Scenarios of Neutrino Mass Generation

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## Abstract

We perform a detailed analysis of lepton flavour violation (LFV) within minimal see-saw type extensions of the Standard Model (SM), which give a viable mechanism of neutrino mass generation and provide new particle content at the electroweak scale. We focus, mainly, on predictions and constraints set on each scenario from  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$  and  $\mu - e$  conversion in the nuclei. In this class of models, the flavour structure of the Yukawa couplings between the additional scalar and fermion representations and the SM leptons is highly constrained by neutrino oscillation measurements. In particular, we show that in some regions of the parameters space of type I and type II see-saw models, the Dirac and Majorana phases of the neutrino mixing matrix, the ordering and hierarchy of the active neutrino mass spectrum as well as the value of the reactor mixing angle  $\theta_{13}$  may considerably affect the size of the LFV observables. The interplay of the latter clearly allows to discriminate among the different low energy see-saw possibilities.

## 1 Introduction

After several decades of neutrino experiments, a clear quantitative picture of the neutrino oscillation parameters is gradually emerging (see, e.g. [1]). The Super-Kamiokande collaboration established that the atmospheric neutrino mass squared splitting is  $|\Delta m_A^2| \sim \mathcal{O}(10^{-3} \text{ eV}^2)$  and that the corresponding mixing angle is large, possibly maximal  $\theta_{23} \cong \pi/4$  [2]. The data from SNO, Super-Kamiokande and KamLAND experiments [3, 4, 5] allowed to establish the large mixing angle solution as a unique solution of the long standing solar neutrino problem, with a solar neutrino mass squared splitting  $\Delta m_\odot^2 \sim \mathcal{O}(10^{-5} \text{ eV}^2)$

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and mixing angle  $\theta_{12} \cong \arcsin(\sqrt{0.3})$ . A series of subsequent experiments, using reactor and accelerator neutrinos, have pinned down the atmospheric and solar neutrino oscillation parameters with a few to several percent accuracy, as summarized in Table 1.

Furthermore, in June of 2011 the T2K collaboration reported [6] evidence at  $2.5\sigma$  for a non-zero value of the angle  $\theta_{13}$ . Subsequently the MINOS [7] and Double Chooz [8] collaborations also reported evidence for  $\theta_{13} \neq 0$ , although with a smaller statistical significance. Global analyses of the neutrino oscillation data, including the data from the T2K and MINOS experiments, performed in [9, 10] showed that actually  $\sin \theta_{13} \neq 0$  at  $\geq 3\sigma$ . The results of the analysis [9], in which  $\Delta m_{21}^2 \equiv \Delta m_\odot^2$  and  $|\Delta m_{31}^2| \equiv |\Delta m_A^2|$  were determined as well, are shown in Table 1.

Recently, the first data of the Daya Bay reactor antineutrino experiment on  $\theta_{13}$  was published [11]. The value of  $\sin^2 2\theta_{13}$  was measured with a rather high precision and was found to be different from zero at  $5.2\sigma$ :

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005, \quad 0.04 \leq \sin^2 2\theta_{13} \leq 0.14, \quad 3\sigma, \quad (1)$$

where we have given also the  $3\sigma$  interval of allowed values of  $\sin^2 2\theta_{13}$ . Subsequently, the RENO experiment reported a  $4.9\sigma$  evidence for a non-zero value of  $\theta_{13}$  [12]:

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019. \quad (2)$$

The value of  $\theta_{13}$  determined in the RENO experiment is compatible with that measured in the Daya Bay experiment. It is interesting to note also that the mean value of  $\sin^2 \theta_{13}$  found in the global analysis of the neutrino oscillation data in [9] differs very little from the mean values found in the Daya Bay and RENO experiments.

The results on  $\theta_{13}$  described above will have far reaching implications for the program of research in neutrino physics. A relatively large value of  $\sin \theta_{13} \sim 0.15$  opens up the possibilities, in particular, i) for searching for CP violation effects in neutrino oscillations experiments with high intensity accelerator neutrino beams (like T2K, NO $\nu$ A, etc.); ii) for determining the sign of  $\Delta m_{32}^2$ , and thus the type of neutrino mass spectrum, which can be with normal or inverted ordering (see, e.g. [1]), in the long baseline neutrino oscillation experiments at accelerators (NO $\nu$ A, etc.), in the experiments studying the oscillations of atmospheric neutrinos (see, e.g. [15]), as well as in experiments with reactor antineutrinos [16]. It has important implications for the neutrinoless double beta ( $(\beta\beta)_{0\nu}-$ ) decay phenomenology in the case of neutrino mass spectrum with normal ordering (NO) [17]. A value of  $\sin \theta_{13} \gtrsim 0.09$  is a necessary condition for a successful “flavoured” leptogenesis when the CP violation required for the generation of the matter-antimatter asymmetry of the Universe is provided entirely by the Dirac CP violating phase in the neutrino mixing matrix [18].<sup>2</sup> As was already discussed to some extent in the literature and we will see further, in certain specific cases a value of  $\sin \theta_{13} \sim 0.15$  can have important implications also for the phenomenology of the lepton flavour violation (LFV) processes involving the charged leptons in theories incorporating one of the possible TeV scale see-saw mechanisms of neutrino mass generation.

Despite the compelling evidence for the nonconservation of the leptonic flavour in neutrino oscillations, all searches for lepton flavour violation (LFV) in the charged lepton sector have

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<sup>2</sup>If indeed  $\sin \theta_{13} \cong 0.15$  and the neutrino mass spectrum is with inverted ordering (IO), further important implications for “flavoured” leptogenesis are possible [19].

Table 1: The best-fit values and  $3\sigma$  allowed ranges of the 3-neutrino oscillation parameters, derived from a global fit of the current neutrino oscillation data, including the T2K and MINOS (but not the Daya Bay) results (from [9]). The Daya Bay data [11] on  $\sin^2 \theta_{13}$  is given in the last line. The values (values in brackets) of  $\sin^2 \theta_{12}$  are obtained using the “old” [13] (“new” [14]) reactor  $\bar{\nu}_e$  fluxes in the analysis.

Parameter	best-fit ( $\pm 1\sigma$ )	$3\sigma$
$\Delta m_\odot^2$ [ $10^{-5}$ eV $^2$ ]	$7.58^{+0.22}_{-0.26}$	6.99 - 8.18
$ \Delta m_A^2 $ [ $10^{-3}$ eV $^2$ ]	$2.35^{+0.12}_{-0.09}$	2.06 - 2.67
$\sin^2 \theta_{12}$	$0.306^{+0.018}_{-0.015}$	0.259(0.265) - 0.359(0.364)
$\sin^2 \theta_{23}$	$0.42^{+0.08}_{-0.03}$	0.34 - 0.64
$\sin^2 \theta_{13}$ [9]	$0.021(0.025)^{+0.007}_{-0.007}$	0.001(0.005) - 0.044(0.050)
$\sin^2 \theta_{13}$ [11]	$0.0236 \pm 0.0042$	0.010 - 0.036

been so far negative. The best limits follow from the non-observation of the LFV muon decays  $\mu^+ \rightarrow e^+ \gamma$  and  $\mu^+ \rightarrow e^+ e^- e^+$ ,

$$\text{BR}(\mu^+ \rightarrow e^+ \gamma) < 2.4 \times 10^{-12} \quad [20], \quad (3)$$

$$\text{BR}(\mu^+ \rightarrow e^+ e^- e^+) < 1.0 \times 10^{-12} \quad [21], \quad (4)$$

and from the non-observation of conversion of muons into electrons in Titanium,

$$\text{CR}(\mu \text{Ti} \rightarrow e \text{Ti}) < 4.3 \times 10^{-12} \quad [22]. \quad (5)$$

Besides, there are stringent constraints on the tau-muon and tau-electron flavour violation from the non-observation of LFV radiative tau decays [23]:

$$\text{BR}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}, \quad (6)$$

$$\text{BR}(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}. \quad (7)$$

The indicated stringent upper limits on the rates of the LFV processes involving the charged leptons lead to severe constraints on models of new physics which predict new particles at the electroweak scale coupled to the charged leptons. Indeed, the dipole operator which leads to the process  $\mu \rightarrow e \gamma$  has the form:

$$-\mathcal{L} = m_\mu \bar{\mu} (f_{M1}^{\mu e} + \gamma_5 f_{E1}^{\mu e}) \sigma^{\nu\rho} e F_{\nu\rho} + \text{h.c.} \quad (8)$$

where  $f_{M1}^{\mu e}$  and  $f_{E1}^{\mu e}$  are, respectively, the transition magnetic and electric dipole moment form factors. This operator is generated at the quantum level through particles with masses  $\Lambda$  which couple to the charged leptons, hence the form factors can be parameterised as  $f^{\mu e} = \frac{\theta_{\mu e}^2}{16\pi^2 \Lambda^2}$ ,  $\theta_{\mu e}$  being a parameter which measures the strength of the coupling of the new particles to the electron and the muon. The present experimental limit on  $\text{BR}(\mu \rightarrow e \gamma)$  sets the following upper limit on the two form factors:  $|f_{E1}^{\mu e}|, |f_{M1}^{\mu e}| \lesssim 10^{-12} \text{ GeV}^{-2}$ . The latter in turn translates into  $\Lambda \gtrsim 20 \text{ TeV}$  if  $\theta_{\mu e} \sim 1/\sqrt{2}$ , or in  $\theta_{\mu e} \lesssim 0.01$  if  $\Lambda \sim 300 \text{ GeV}$ . It is then apparent that experiments searching for lepton flavour violation can probe models of new physics which cannot be tested in collider experiments, either because the new particles are

not kinematically accessible with the available collider energies, or because the couplings of the new particles to the Standard Model (SM) particles are too feeble to produce the former with rates necessary for their observation given the luminosity of the present colliders.

Low scale see-saw models are a particularly interesting class of models of new physics which are severely constrained by experiments searching for lepton flavour violation. In this class of models the flavour structure of the couplings of the new particles to the charged leptons is basically determined by the requirement of reproducing the data on the neutrino oscillation parameters [24, 25, 26, 27, 28]. Hence, the rates for the lepton flavour violating processes in the charged lepton sector can be calculated in terms of a few parameters, the predicted rates being possibly within the reach of the future experiments searching for lepton flavour violation, even when the parameters of the model do not allow production of the new particles with observable rates at the LHC [28].

The role of the experiments searching for lepton flavour violation to constrain low scale see-saw models will be significantly strengthened in the next years. Searches for  $\mu - e$  conversion at the planned COMET experiment at KEK [29] and Mu2e experiment at Fermilab [30] aim to reach sensitivity to  $\text{CR}(\mu \text{ Al} \rightarrow e \text{ Al}) \approx 10^{-16}$ , while, in the longer run, the PRISM/PRIME experiment in KEK [31] and the project-X experiment in Fermilab [32] are being designed to probe values of the  $\mu - e$  conversion rate on Ti, which are by 2 orders of magnitude smaller,  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \approx 10^{-18}$  [31]. If these experiments reach the projected sensitivity without observing a signal, the upper limits on the form factors  $f_{M1}$ ,  $f_{E1}$  will improve by two orders of magnitude. There are also plans to perform a new search for the  $\mu^+ \rightarrow e^+ e^- e^+$  decay [33], which will probe values of the corresponding branching ratio down to  $\text{BR}(\mu^+ \rightarrow e^+ e^- e^+) \approx 10^{-15}$ , i.e., by 3 orders of magnitude smaller than the best current upper limit eq. (4). Furthermore, searches for tau lepton flavour violation at superB factories aim to reach a sensitivity to  $\text{BR}(\tau \rightarrow (\mu, e)\gamma) \approx 10^{-9}$  [34, 35].

In this paper we will study the constraints on low (TeV) scale see-saw models of neutrino mass generation from present and future experiments searching for lepton flavour violation, with especial emphasis on  $\mu - e$  conversion in nuclei, which is among all search strategies the one with brightest perspectives. The paper is organized as follows: in Sections 2, 3 and 4 we review the main features of the three types of see-saw mechanisms. We discuss for each scenario the predictions and experimental constraints on the relevant parameter space arising from LFV processes. The results are summarized and discussed in the concluding Section 5.

## 2 TeV Scale Type I See-Saw Model

We consider in detail in this Section LFV processes emerging in type I see-saw extensions of the SM [36]. We denote the light and heavy Majorana mass eigenstates of the type I see-saw model as  $\chi_i$  and  $N_k$ , respectively.<sup>3</sup> The charged and neutral current weak interactions involving the light Majorana neutrinos have the form:

$$\mathcal{L}_{CC}^\nu = -\frac{g}{\sqrt{2}} \bar{\ell} \gamma_\alpha \nu_{\ell L} W^\alpha + \text{h.c.} = -\frac{g}{\sqrt{2}} \bar{\ell} \gamma_\alpha ((1+\eta)U)_{\ell i} \chi_{iL} W^\alpha + \text{h.c.}, \quad (9)$$

$$\mathcal{L}_{NC}^\nu = -\frac{g}{2c_w} \overline{\nu_{\ell L}} \gamma_\alpha \nu_{\ell L} Z^\alpha = -\frac{g}{2c_w} \overline{\chi_{iL}} \gamma_\alpha (U^\dagger (1+\eta+\eta^\dagger) U)_{ij} \chi_{jL} Z^\alpha, \quad (10)$$

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<sup>3</sup>We use the same notations as in [28, 37].

where  $(1 + \eta)U = U_{\text{PMNS}}$  is the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix [38, 39],  $U$  is a  $3 \times 3$  unitary matrix which diagonalises the Majorana mass matrix of the left-handed (LH) flavour neutrinos  $\nu_{\ell L}$  (generated by the see-saw mechanism), and the matrix  $\eta$  characterises the deviations from unitarity of the PMNS matrix. The elements of  $U_{\text{PMNS}}$  are determined in experiments studying the oscillations of the flavour neutrinos and antineutrinos,  $\nu_\ell$  and  $\bar{\nu}_\ell$ ,  $\ell = e, \mu, \tau$ , at relatively low energies. In these experiments the initial states of the flavour neutrinos, produced in weak processes, are coherent superpositions of the states of the light massive Majorana neutrino  $\chi_i$  only. The states of the heavy Majorana neutrino  $N_j$  are not present in the superpositions representing the initial flavour neutrino states and this leads to deviations from unitarity of the PMNS matrix.

The matrix  $\eta$  can be expressed in terms of a matrix  $RV$  whose elements  $(RV)_{\ell k}$  determine the strength of the charged current (CC) and neutral current (NC) weak interaction couplings of the heavy Majorana neutrinos  $N_k$  to the  $W^\pm$ -boson and the charged lepton  $\ell$ , and to the  $Z^0$  boson and the left-handed (LH) flavour neutrino  $\nu_{\ell L}$ ,  $\ell = e, \mu, \tau$ :

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}} \bar{\ell} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k W^\alpha + \text{h.c.}, \quad (11)$$

$$\mathcal{L}_{NC}^N = -\frac{g}{4c_w} \bar{\nu}_{\ell L} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k Z^\alpha + \text{h.c..} \quad (12)$$

Here  $V$  is the unitary matrix which diagonalises the Majorana mass matrix of the heavy RH neutrinos and the matrix  $R$  is determined by (see [37])  $R^* \cong M_D M_N^{-1}$ ,  $M_D$  and  $M_N$  being the neutrino Dirac and the RH neutrino Majorana mass matrices, respectively,  $|M_D| \ll |M_N|$ . We have:

$$\eta \equiv -\frac{1}{2} R R^\dagger = -\frac{1}{2} (RV)(RV)^\dagger = \eta^\dagger. \quad (13)$$

The elements of the matrices  $RV$  and  $\eta$  can be constrained by using the existing neutrino oscillation data, data on electroweak (EW) processes, etc. [40, 41]. They should satisfy also the constraint which is characteristic of the type I see-saw mechanism under discussion:

$$|\sum_k (RV)_{\ell' k}^* M_k (RV)_{k \ell}^\dagger| \cong |(m_\nu)_{\ell' \ell}| \lesssim 1 \text{ eV}, \quad \ell', \ell = e, \mu, \tau. \quad (14)$$

Here  $m_\nu$  is the Majorana mass matrix of the LH flavour neutrinos generated by the see-saw mechanism. The upper limit  $|(m_\nu)_{\ell' \ell}| \lesssim 1 \text{ eV}$ ,  $\ell, \ell' = e, \mu, \tau$ , follows from the existing data on the neutrino masses and on the neutrino mixing [42]. For the values of the masses  $M_k$  of the heavy Majorana neutrinos  $N_k$  of interest for the present study,  $M_k = \mathcal{O}(100 - 1000)$  GeV, the simplest scheme in which the constraint (14) can be satisfied is [28] the scheme with two heavy Majorana neutrinos (see, e.g., [43, 44, 45]),  $N_1$  and  $N_2$ , which form a pseudo-Dirac neutrino pair [46, 47]:  $M_2 = M_1(1 + z)$ , where  $z \ll 1$ , which naturally arises in type I see-saw models with a mildly broken lepton number symmetry [25] and in the inverse see-saw model [48, 49]. In the scenario where the CC and NC couplings of  $N_{1,2}$  are sizable, the requirement of reproducing the correct low energy neutrino oscillation parameters constrains significantly [25, 26] and in certain cases determines the Yukawa couplings [24, 27, 28]. Correspondingly, the elements  $(RV)_{\ell 1}$  and  $(RV)_{\ell 2}$  in eqs. (11) and (12) are also determined

and in the case of interest take the form [28]:

$$|(RV)_{\ell 1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_3}{m_2 + m_3} \left| U_{\ell 3} + i\sqrt{m_2/m_3} U_{\ell 2} \right|^2, \quad \text{NH}, \quad (15)$$

$$|(RV)_{\ell 1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_2}{m_1 + m_2} \left| U_{\ell 2} + i\sqrt{m_1/m_2} U_{\ell 1} \right|^2 \cong \frac{1}{4} \frac{y^2 v^2}{M_1^2} |U_{\ell 2} + iU_{\ell 1}|^2, \quad \text{IH}, \quad (16)$$

$$(RV)_{\ell 2} = \pm i (RV)_{\ell 1} \sqrt{\frac{M_1}{M_2}}, \quad \ell = e, \mu, \tau, \quad (17)$$

where  $y$  represents the maximum eigenvalue of the neutrino Yukawa matrix and  $v \simeq 174$  GeV. In eq. (16) we have used the fact that for the IH spectrum one has  $m_1 \cong m_2$ . Due to the relation (17) between  $(RV)_{\ell 1}$  and  $(RV)_{\ell 2}$ , eq. (14) is automatically satisfied.

Upper bounds on the couplings of RH neutrinos with SM particles can be obtained from the low energy electroweak precision data on lepton number conserving processes like  $\pi \rightarrow \ell \bar{\nu}_\ell$ ,  $Z \rightarrow \nu \bar{\nu}$  and other tree-level processes involving light neutrinos in the final state [40]. These bounds read:

$$|(RV)_{e 1}|^2 \lesssim 2 \times 10^{-3}, \quad (18)$$

$$|(RV)_{\mu 1}|^2 \lesssim 0.8 \times 10^{-3}, \quad (19)$$

$$|(RV)_{\tau 1}|^2 \lesssim 2.6 \times 10^{-3}. \quad (20)$$

Let us add that in the class of type I see-saw models with two heavy Majorana neutrinos we are considering (see, e.g., [43, 44, 45]), one of the three light (Majorana) neutrinos is massless and hence the neutrino mass spectrum is hierarchical. Two possible types of hierarchical spectrum are allowed by the current neutrino data (see, e.g., [1]): *i*) normal hierarchical (NH),  $m_1 = 0$ ,  $m_2 = \sqrt{\Delta m_\odot^2}$  and  $m_3 = \sqrt{\Delta m_A^2}$ , where  $\Delta m_\odot^2 \equiv m_2^2 - m_1^2 > 0$  and  $\Delta m_A^2 \equiv m_3^2 - m_1^2$ ; *ii*) inverted hierarchical (IH),  $m_3 = 0$ ,  $m_2 = \sqrt{|\Delta m_A^2|}$  and  $m_1 = \sqrt{|\Delta m_A^2| - \Delta m_\odot^2} \cong \sqrt{|\Delta m_A^2|}$ , where  $\Delta m_\odot^2 \equiv m_2^2 - m_1^2 > 0$  and  $\Delta m_A^2 = m_3^2 - m_2^2 < 0$ . In both cases we have:  $\Delta m_\odot^2 / |\Delta m_A^2| \cong 0.03 \ll 1$ .

The numerical results we will present further will be obtained employing the standard parametrisation for the unitary matrix  $U$ :

$$U = V(\theta_{12}, \theta_{23}, \theta_{13}, \delta) Q(\alpha_{21}, \alpha_{31}). \quad (21)$$

Here (see, e.g., [1])

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (22)$$

where we have used the standard notation  $c_{ij} \equiv \cos \theta_{ij}$ ,  $s_{ij} \equiv \sin \theta_{ij}$ ,  $\delta$  is the Dirac CP violation (CPV) phase and the matrix  $Q$  contains the two Majorana CPV phases <sup>4</sup> [50],

$$Q = \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}). \quad (23)$$

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<sup>4</sup> In the case of the type II see-saw mechanism, to be discussed in Section 3, we have  $\eta = 0$  and thus the neutrino mixing matrix coincides with  $U$ :  $U_{\text{PMNS}} = U$ . We will employ the parametrisation (21) - (23) for  $U$  also in that case.

We recall that  $U_{\text{PMNS}} = (1 + \eta)U$ . Thus, up to corrections which depend on the elements of the matrix  $\eta$  whose absolute values, however, do not exceed approximately  $5 \times 10^{-3}$  [40], the values of the angles  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{13}$  coincide with the values of the solar neutrino, atmospheric neutrino and the 1-3 (or “reactor”) mixing angles, determined in the 3-neutrino mixing analyses of the neutrino oscillation data and reported in Table 1.

Given the neutrino masses and mixing angles, the TeV scale type I see-saw scenario we are considering is characterised by four parameters [28]: the mass (scale)  $M_1$ , the Yukawa coupling  $y$ , the parameter  $z$  of the splitting between the masses of the two heavy Majorana neutrinos and a phase  $\omega$ . The mass  $M_1$  and the Yukawa coupling  $y$  can be determined, in principle, from the measured rates of two lepton flavour violating (LFV) processes, the  $\mu \rightarrow e\gamma$  decay and the  $\mu - e$  conversion in nuclei, for instance. The mass splitting parameter  $z$  and the phase  $\omega$ , together with  $M_1$  and  $y$ , enter, e.g., into the expression for the rate of  $(\beta\beta)_{0\nu}$ -decay, predicted by the model. The latter was discussed in detail in [28].

## 2.1 The $\mu \rightarrow e\gamma$ Decay

In this subsection we update briefly the discussion of the limits on the parameters of the TeV scale type I see-saw model, derived in [28] using the experimental upper bound on the  $\mu \rightarrow e\gamma$  decay rate obtained in 1999 in the MEGA experiment [51]. After the publication of [28], the MEG collaboration reported a new more stringent upper bound on the  $\mu \rightarrow e\gamma$  decay rate [20] given in eq. (3). Such an update is also necessary in view of the relatively large nonzero value of the reactor angle  $\theta_{13}$  measured in the Daya Bay and RENO experiments [11, 12] and reported in eqs. (1) and (2). As was discussed in [28], in particular, the rate of the  $\mu \rightarrow e\gamma$  decay in the type I see-saw scheme considered can be strongly suppressed for certain values of  $\theta_{13}$ .

The  $\mu \rightarrow e\gamma$  decay branching ratio in the scenario under discussion is given by [52, 53]:

$$\text{BR}(\mu \rightarrow e\gamma) = \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e + \nu_\mu + \bar{\nu}_e)} = \frac{3\alpha_{\text{em}}}{32\pi} |T|^2, \quad (24)$$

where  $\alpha_{\text{em}}$  is the fine structure constant and [28]

$$|T| \cong \frac{2+z}{1+z} |(RV)_{\mu 1}^* (RV)_{e 1}| |G(X) - G(0)|. \quad (25)$$

In eqs. (24) and (25) the loop integration function  $G(x)$  has the form:

$$G(x) = \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \log(x)}{3(x-1)^4}, \quad (26)$$

where  $X \equiv (M_1/M_W)^2$ . In deriving the expression for the matrix element  $T$ , eq. (25), we have assumed that the difference between  $M_1$  and  $M_2$  is negligibly small and used  $M_1 \cong M_2$ . It is easy to verify that  $G(x)$  is a monotonic function which takes values in the interval  $[4/3, 10/3]$ , with  $G(x) \cong \frac{10}{3} - x$  for  $x \ll 1$ .

Using the expressions of  $|RV_{\mu 1}|^2$  and  $|RV_{e 1}|^2$  in terms of neutrino parameters, eqs. (15)

and (16), we obtain the  $\mu \rightarrow e\gamma$  decay branching ratio for the NH and IH spectra:

$$\text{NH : } \text{BR}(\mu \rightarrow e\gamma) \cong \frac{3\alpha_{\text{em}}}{32\pi} \left( \frac{y^2 v^2}{M_1^2} \frac{m_3}{m_2 + m_3} \right)^2 \left| U_{\mu 3} + i\sqrt{\frac{m_2}{m_3}} U_{\mu 2} \right|^2 \left| U_{e 3} + i\sqrt{\frac{m_2}{m_3}} U_{e 2} \right|^2 [G(X) - G(0)]^2 , \quad (27)$$

$$\text{IH : } \text{BR}(\mu \rightarrow e\gamma) \cong \frac{3\alpha_{\text{em}}}{32\pi} \left( \frac{y^2 v^2}{M_1^2} \frac{1}{2} \right)^2 |U_{\mu 2} + iU_{\mu 1}|^2 |U_{e 2} + iU_{e 1}|^2 [G(X) - G(0)]^2 . \quad (28)$$

The data on the process  $\mu \rightarrow e\gamma$  set very stringent constraints on the TeV scale type I see-saw mechanism. The current upper bound on  $\text{BR}(\mu \rightarrow e\gamma)$  was obtained in the MEG experiment at PSI [20] and is given in eq. (3). It is an improvement by a factor of 5 of the previous best upper limit of the MEGA experiment, published in 1999 [51]. The projected sensitivity of the MEG experiment is  $\text{BR}(\mu \rightarrow e\gamma) \sim 10^{-13}$  [20]. For  $M_1 = 100 \text{ GeV}$  ( $M_1 = 1 \text{ TeV}$ ) and  $z \ll 1$  we get the following upper limit on the product  $|RV_{\mu 1}^*(RV)_{e 1}|$  of the heavy Majorana neutrino couplings to the muon (electron) and the  $W^\pm$  boson and to the  $Z^0$  boson from the the upper limit eq. (3):

$$|(RV)_{\mu 1}^* (RV)_{e 1}| < 0.8 \times 10^{-4} (0.3 \times 10^{-4}) , \quad (29)$$

where we have used eqs. (24) and (25). This can be recast as an upper bound on the neutrino Yukawa coupling  $y$ . Taking, *e.g.*, the best fit values of the solar and atmospheric oscillation parameters given in Table 1, we get:

$$y \lesssim 0.035 \text{ (0.21) for NH with } M_1 = 100 \text{ GeV (1000 GeV)} \text{ and } \sin \theta_{13} = 0.1 , \quad (30)$$

$$y \lesssim 0.025 \text{ (0.15) for IH with } M_1 = 100 \text{ GeV (1000 GeV)} \text{ and } \sin \theta_{13} = 0.1 . \quad (31)$$

The constraints which follow from the current MEG upper bound on  $\text{BR}(\mu \rightarrow e\gamma)$  will not be valid in the case of a cancellation between the different terms in one of the factors  $|U_{\ell 3} + i\sqrt{m_2/m_3} U_{\ell 2}|^2$  and  $|U_{\ell 2} + iU_{\ell 1}|^2$ ,  $\ell = e, \mu$ , in the expressions (27) and (28) for  $\text{BR}(\mu \rightarrow e\gamma)$ . Employing the standard parametrisation of  $U$ , eqs. (21) - (23), one can show that in the case of NH spectrum we can have  $|U_{e 3} + i\sqrt{m_2/m_3} U_{e 2}| = 0$  if [28] (see also [24])  $\sin(\delta + (\alpha_{21} - \alpha_{31})/2) = 1$  and  $\tan \theta_{13} = (\Delta m_\odot^2 / \Delta m_A^2)^{1/4} \sin \theta_{12}$ . Using the  $3\sigma$  allowed ranges of  $\Delta m_\odot^2$ ,  $\Delta m_A^2$  and  $\sin^2 \theta_{12}$  given in Table 1, we find that the second condition can be satisfied provided  $\sin^2 \theta_{13} \gtrsim 0.04$ , which lies outside the  $3\sigma$  range of allowed values of  $\sin^2 \theta_{13}$  found in the Daya Bay experiment [11] (see eq. (1)).

In the case of IH spectrum, the factor  $|U_{e 2} + iU_{e 1}|^2$  can be rather small for  $\sin(\alpha_{21}/2) = -1$ :  $|U_{e 2} + iU_{e 1}|^2 = c_{13}^2 (1 - \sin 2\theta_{12}) \cong 0.0765$ , where we have used the best fit values of  $\sin^2 \theta_{12} = 0.306$  and  $\sin^2 \theta_{13} = 0.0236$ . It is also possible to have a strong suppression of the factor  $|U_{\mu 2} + iU_{\mu 1}|^2$  [28]. Indeed, using the standard parametrisation of the matrix  $U$  it is not difficult to show that for fixed values of the angles  $\theta_{12}$ ,  $\theta_{23}$  and of the phases  $\alpha_{21}$  and  $\delta$ ,  $|U_{\mu 2} + iU_{\mu 1}|^2$  has a minimum for

$$\sin \theta_{13} = \frac{c_{23}}{s_{23}} \frac{\cos 2\theta_{12} \cos \delta \sin \frac{\alpha_{21}}{2} - \cos \frac{\alpha_{21}}{2} \sin \delta}{1 + 2c_{12} s_{12} \sin \frac{\alpha_{21}}{2}} . \quad (32)$$

At the minimum we get:

$$\min(|U_{\mu 2} + iU_{\mu 1}|^2) = c_{23}^2 \frac{\left(\cos \delta \cos \frac{\alpha_{21}}{2} + \cos 2\theta_{12} \sin \delta \sin \frac{\alpha_{21}}{2}\right)^2}{1 + 2c_{12} s_{12} \sin \frac{\alpha_{21}}{2}}. \quad (33)$$

Notice that, from the equation above, the  $\mu \rightarrow e\gamma$  branching ratio is highly suppressed if the Dirac and Majorana phases take CP conserving values, mainly:  $\delta \simeq 0$  and  $\alpha_{21} \simeq \pi$ . In this case, from eq. (32) we get the lower bound  $\sin(\theta_{13}) \gtrsim 0.13$ , which is in agreement with the Daya Bay measurement reported in Tab. 1. On the other hand, assuming CPV phases, we still may have  $\min(|U_{\mu 2} + iU_{\mu 1}|^2) = 0$ , provided  $\theta_{12}$  and the Dirac and Majorana phases  $\delta$  and  $\alpha_{21}$  satisfy the following conditions:  $\cos \delta \cos(\alpha_{21}/2) + \cos 2\theta_{12} \sin \delta \sin(\alpha_{21}/2) = 0$  and  $\text{sgn}(\cos \delta \cos \frac{\alpha_{21}}{2}) = -\text{sgn}(\sin \delta \sin \frac{\alpha_{21}}{2})$ . Taking  $\cos \delta > 0$  ( $\cos \delta < 0$ ) and using  $\tan \delta = -\tan(\alpha_{21}/2)/\cos 2\theta_{12}$  in eq. (32), we get the relation between  $s_{13}$ ,  $\delta$  and  $\cos 2\theta_{12}$ , for which  $\min(|U_{\mu 2} + iU_{\mu 1}|^2) = 0$ :

$$\sin \theta_{13} = \frac{c_{23}}{s_{23}} \frac{\sqrt{1 + \tan^2 \delta} \cos 2\theta_{12}}{\sqrt{1 + \cos^2 2\theta_{12} \tan^2 \delta} + 2c_{12} s_{12} \text{sgn}(\cos \delta)}. \quad (34)$$

Using the  $3\sigma$  intervals of allowed values of  $\sin^2 \theta_{12}$  and  $\sin^2 \theta_{23}$  (found with the “new” reactor  $\bar{\nu}_e$  fluxes, see Table 1) and allowing  $\delta$  to vary in the interval  $[0, 2\pi]$ , we find that the values of  $\sin \theta_{13}$  obtained using eq. (34) lie in the interval  $\sin \theta_{13} \gtrsim 0.11$ . As it follows from eq. (1), we have at  $3\sigma$ :  $0.10 \lesssim \sin \theta_{13} \lesssim 0.19$ . The values of  $0.11 \lesssim \sin \theta_{13} \lesssim 0.19$  correspond to  $0 \leq \delta \lesssim 0.7$ . These conclusions are illustrated in Fig. 1. For  $\sin \theta_{13}$  and  $\delta$  lying in the indicated intervals we can have  $|U_{\mu 2} + iU_{\mu 1}|^2 = 0$  and thus a strong suppression of the  $\mu \rightarrow e\gamma$  decay rate. As we will see in subsections 2.2 and 2.3, in the model we are considering, the predicted  $\mu - e$  conversion rate in a given nucleus and  $\mu \rightarrow 3e$  decay rate are also proportional to  $|(RV)_{\mu 1}^*(RV)_{e 1}|^2$ , as like the  $\mu \rightarrow e\gamma$  decay rate. This implies that in the case of the TeV scale type I see-saw mechanism and IH light neutrino mass spectrum, if, e.g.,  $\text{BR}(\mu \rightarrow e\gamma)$  is strongly suppressed due to  $|U_{\mu 2} + iU_{\mu 1}|^2 \cong 0$ , the  $\mu - e$  conversion and the  $\mu \rightarrow 3e$  decay rates will also be strongly suppressed<sup>5</sup>. The suppression under discussion cannot hold if, for instance, it is experimentally established that  $\delta$  is definitely bigger than 1.0. That would be the case if the existing indications [9] that  $\cos \delta < 0$  receive unambiguous confirmation.

The limits on the parameters  $|(RV)_{\mu 1}|$  and  $|(RV)_{e 1}|$ , implied by the electroweak precision data, eqs. (18) - (20), and the upper bound on  $\text{BR}(\mu \rightarrow e\gamma)$ , eq. (3), are illustrated in Fig. 2. The results shown are obtained for the best fit values of  $\sin \theta_{13} = 0.156$  and of the other neutrino oscillation parameters given in Table 1.

## 2.2 The $\mu - e$ Conversion in Nuclei

We will discuss next the predictions of the TeV scale type I see-saw extension of the SM for the rate of the  $\mu - e$  conversion in nuclei, as well as the experimental constraints that can

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<sup>5</sup> Let us note that in the case of IH spectrum we are discussing actually one has  $|(RV)_{\mu 1}|^2 \propto |U_{\mu 2} + i\sqrt{m_1/m_2}U_{\mu 1}|^2$  (see eq. (16)), with  $m_2 = \sqrt{|\Delta m_A^2|}$  and  $m_1 = \sqrt{|\Delta m_A^2| - \Delta m_\odot^2} \cong \sqrt{|\Delta m_A^2|}(1 - 0.5\Delta m_\odot^2/|\Delta m_A^2|)$ . Therefore when  $|U_{\mu 2} + iU_{\mu 1}| = 0$  we still have  $|U_{\mu 2} + i\sqrt{m_1/m_2}U_{\mu 1}|^2 \neq 0$ . However, in this case  $|U_{\mu 2} + i\sqrt{m_1/m_2}U_{\mu 1}|^2 \cong (\Delta m_\odot^2/(4|\Delta m_A^2|))^2|U_{\mu 1}|^2 \lesssim 1.7 \times 10^{-5}$ , where we have used  $\delta = 0$  (which maximises  $|U_{\mu 1}|^2$ ) and the best fit values of the other neutrino oscillation parameters. Thus, our conclusions about the suppression of  $\text{BR}(\mu \rightarrow e\gamma)$ , the  $\mu - e$  conversion and the  $\mu \rightarrow 3e$  decay rates are still valid.

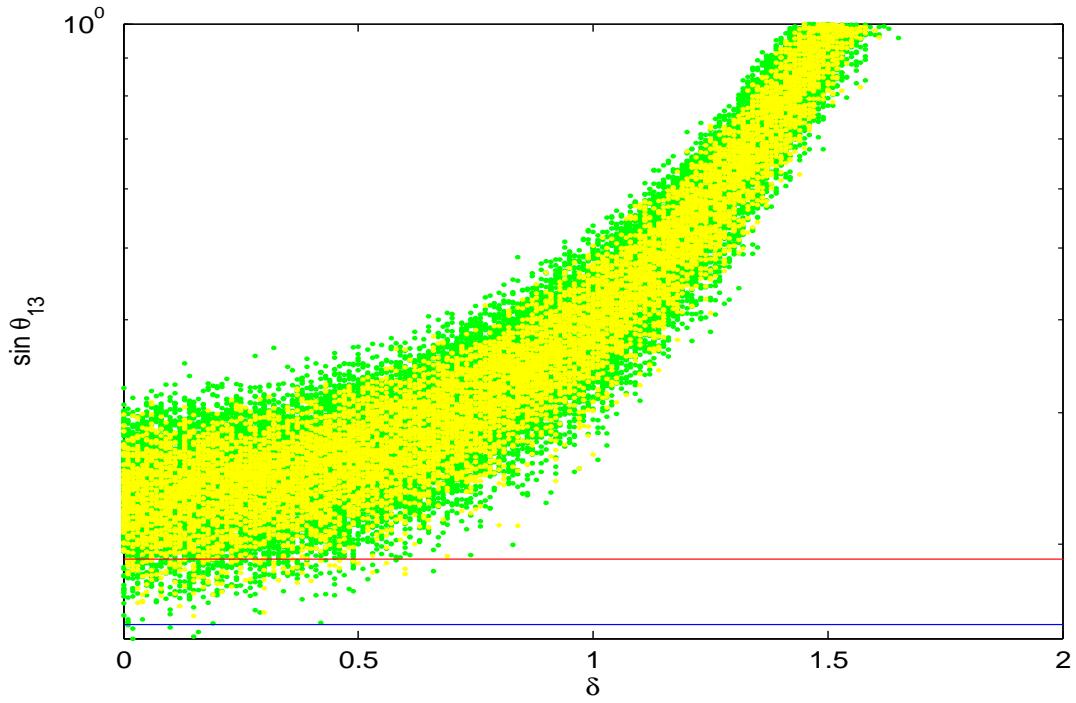


Figure 1: Values of  $\sin \theta_{13}$ , as a function of the phase  $\delta$ , which yield a suppressed rate of the process  $\mu \rightarrow e\gamma$ . The values are obtained using eq. (34), the  $2\sigma$  ( $3\sigma$ ) intervals of allowed values of  $\sin^2 \theta_{12}$  and  $\sin^2 \theta_{23}$ , yellow (green) points (found with the “new” reactor  $\bar{\nu}_e$  fluxes, see Table 1) and allowing  $\delta$  to vary in the interval  $[0, 2\pi]$ . The red and blue horizontal lines correspond to the  $3\sigma$  upper limit  $\sin \theta_{13} = 0.191$  and the best fit value  $\sin \theta_{13} = 0.156$ .

be imposed on this see-saw scenario by the current and prospective  $\mu - e$  conversion data. In the type I see-saw scenario of interest, the  $\mu - e$  conversion rate in a nucleus  $\mathcal{N}$  is very well approximated by the expression [55]:

$$\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N}) \equiv \frac{\Gamma(\mu \mathcal{N} \rightarrow e \mathcal{N})}{\Gamma_{\text{capt}}} = \frac{\alpha_{\text{em}}^5}{2 \pi^4 \sin^4 \theta_W} \frac{Z_{\text{eff}}^4}{Z} |F(-m_\mu^2)|^2 \frac{G_F^2 m_\mu^5}{\Gamma_{\text{capt}}} |(RV)_{\mu 1}^*(RV)_{e 1}|^2 |\mathcal{C}_{\mu e}|^2 . \quad (35)$$

In eq. (35)  $Z$  is the proton number of the nucleus  $\mathcal{N}$ ,  $\theta_W$  is the weak mixing angle,  $\sin^2 \theta_W = 0.23$ ,  $F(-m_\mu^2)$  is the nuclear form factor at momentum transfer squared  $q^2 = -m_\mu^2$ ,  $m_\mu$  being the muon mass,  $Z_{\text{eff}}$  is an effective atomic charge and  $\Gamma_{\text{capt}}$  is the experimentally known total muon capture rate. The loop integral factor  $\mathcal{C}_{\mu e}$  receives contributions from  $\gamma$ -penguin,  $Z^0$ -penguin and box type diagrams. In the earlier version of the present article [56] we have used the expression for  $|\mathcal{C}_{\mu e}|$  found in [55] (in the notations of ref. [57]) in a model with an active heavy Majorana neutrino. It was pointed out in [58], however, that the result for  $|\mathcal{C}_{\mu e}|$  of [55] is not directly applicable to the case of TeV scale type I see-saw model we are considering. The authors of [58] performed a detailed calculation of  $|\mathcal{C}_{\mu e}|$  in the model of interest and obtained a new expression for  $|\mathcal{C}_{\mu e}|$ . We have performed an independent

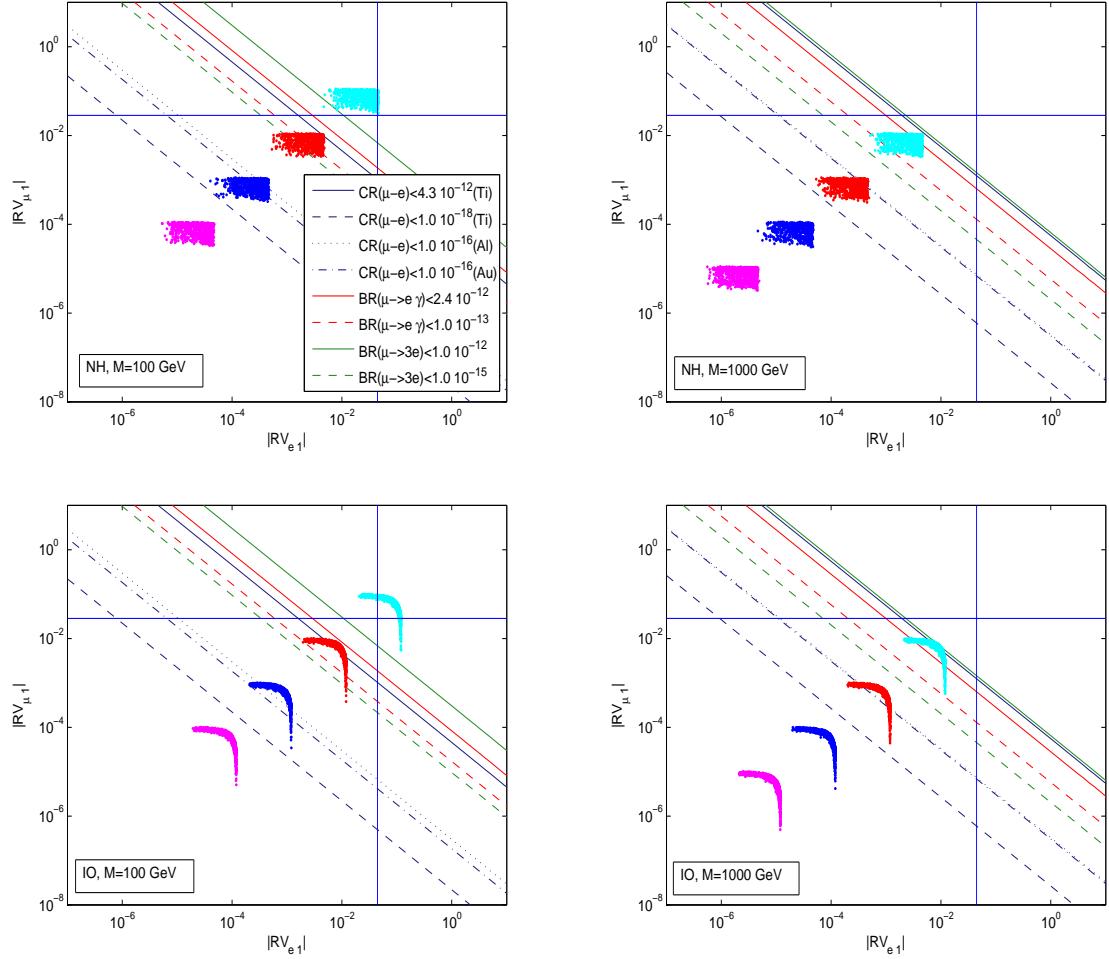


Figure 2: Correlation between  $|RV_{e1}|$  and  $|RV_{\mu 1}|$  in the case of NH (upper panels) and IH (lower panels) light neutrino mass spectrum, for  $M_1 = 100$  ( $1000$ ) GeV and, *i*)  $y = 0.0001$  (magenta points), *ii*)  $y = 0.001$  (blue points), *iii*)  $y = 0.01$  (red points) and *iv*)  $y = 0.1$  (cyan points). The constraints from several LFV processes discussed in the text are shown.

calculation of the factor  $|\mathcal{C}_{\mu e}|$  in the model under discussion<sup>6</sup>. Our result for  $|\mathcal{C}_{\mu e}|$  coincides with that derived in [58] and reads:

$$\mathcal{C}_{\mu e} \cong Z [2F_u^{\mu e}(X) + F_d^{\mu e}(X)] + N [F_u^{\mu e}(X) + 2F_d^{\mu e}(X)] , \quad (36)$$

$$F_q^{\mu e}(X) = Q_q \sin^2 \theta_W [F_\gamma(X) - F_z^{\mu e}(X) + G_\gamma(X)] + \frac{1}{4} [2I_3 F_z^{\mu e}(X) + F_B^{\mu eqq}(X)] . \quad (37)$$

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<sup>6</sup>The new results are published as an erratum to [56].

Here  $N$  is the neutron number of the nucleus  $\mathcal{N}$ ,  $X = M_1^2/M_W^2$ , and

$$F_\gamma(x) = \frac{x(7x^2 - x - 12)}{12(1-x)^3} - \frac{x^2(12 - 10x + x^2)}{6(1-x)^4} \log x, , \quad (38)$$

$$G_\gamma(x) = -\frac{x(2x^2 + 5x - 1)}{4(1-x)^3} - \frac{3x^3}{2(1-x)^4} \log x, \quad (39)$$

$$F_z(x) = -\frac{5x}{2(1-x)} - \frac{5x^2}{2(1-x)^2} \log x, \quad (40)$$

$$G_z(x, y) = -\frac{1}{2(x-y)} \left[ \frac{x^2(1-y)}{(1-x)} \log x - \frac{y^2(1-x)}{(1-y)} \log y \right], \quad (41)$$

$$\begin{aligned} F_{Box}(x, y) = & \frac{1}{x-y} \left\{ (4 + \frac{xy}{4}) \left[ \frac{1}{1-x} + \frac{x^2}{(1-x)^2} \log x - \frac{1}{1-y} - \frac{y^2}{(1-y)^2} \log y \right] \right. \\ & \left. - 2xy \left[ \frac{1}{1-x} + \frac{x}{(1-x)^2} \log x - \frac{1}{1-y} - \frac{y}{(1-y)^2} \log y \right] \right\}, \end{aligned} \quad (42)$$

$$\begin{aligned} F_{XBox}(x, y) = & -\frac{1}{x-y} \left\{ (1 + \frac{xy}{4}) \left[ \frac{1}{1-x} + \frac{x^2}{(1-x)^2} \log x - \frac{1}{1-y} - \frac{y^2}{(1-y)^2} \log y \right] \right. \\ & \left. - 2xy \left[ \frac{1}{1-x} + \frac{x}{(1-x)^2} \log x - \frac{1}{1-y} - \frac{y}{(1-y)^2} \log y \right] \right\}, \end{aligned} \quad (43)$$

$$F_z^{\mu e}(x) = F_z(x) + 2G_z(0, x), \quad F_{Box}^{\mu euu}(x) = F_{Box}(x, 0) - F_{Box}(0, 0), \quad (44)$$

$$F_{Box}^{\mu edd}(x) = F_{XBox}(x, 0) - F_{XBox}(0, 0). \quad (45)$$

In what follows we will present results for three nuclei which were used in the past, and are of interest for possible future  $\mu - e$  conversion experiments:  $^{48}_{22}\text{Ti}$ ,  $^{27}_{13}\text{Al}$  and  $^{197}_{79}\text{Au}$ . For these nuclei one has, respectively: i)  $Z_{eff} = 17.6$ ;  $11.62$ ;  $33.64$ , ii)  $F(q^2 = -m_\mu^2) \approx 0.54$ ;  $0.64$ ;  $0.20$ , and iii)  $\Gamma_{capt} = 2.59$ ;  $0.69$ ;  $13.07 \times 10^6 \text{ sec}^{-1}$  [31].

The dependence of the loop integration factor  $\mathcal{C}_{\mu e}$  on the see-saw mass scale  $M_1$  for the three nuclei of interest is shown in Fig. 3. The first feature to notice is that  $|\mathcal{C}_{\mu e}|$  for  $^{48}_{22}\text{Ti}$ ,  $^{27}_{13}\text{Al}$  and  $^{197}_{79}\text{Au}$  has maxima  $|\mathcal{C}_{\mu e}| = 34.4$ ;  $20.4$ ;  $124$  at  $M_1 = 250$ ;  $267$ ;  $214$  GeV, respectively. At  $M_1 = 250$  GeV,  $|\mathcal{C}_{\mu e}|$  for  $^{27}_{13}\text{Al}$  and  $^{197}_{79}\text{Au}$  takes the values  $|\mathcal{C}_{\mu e}(\text{Al})| \cong 20.4$  and  $|\mathcal{C}_{\mu e}(\text{Au})| \cong 123.1$ ; at  $M_1 = 267$  GeV, we have  $|\mathcal{C}_{\mu e}(\text{Ti})| \cong 34.4$  and  $|\mathcal{C}_{\mu e}(\text{Au})| \cong 122.4$ ; and finally, at  $M_1 = 214$  GeV, we find  $|\mathcal{C}_{\mu e}(\text{Ti})| \cong 34.3$  and  $|\mathcal{C}_{\mu e}(\text{Al})| \cong 20.2$ . These maxima of  $|\mathcal{C}_{\mu e}|$  give the biggest enhancement factors for the conversion rate when  $M_1 \leq 1000$  GeV. Beside the maxima,  $|\mathcal{C}_{\mu e}|$  goes through zero at  $M_1 = 4595$ ;  $6215$ ;  $2470$  GeV for  $^{48}_{22}\text{Ti}$ ,  $^{27}_{13}\text{Al}$  and  $^{197}_{79}\text{Au}$ , respectively, as was noticed also in [58].

Qualitatively, the dependence of the factor  $|\mathcal{C}_{\mu e}|$  defined in eqs. (37)-(42) on  $M_1$  exhibits the same features as the factor  $|\mathcal{C}_{\mu e}|$  derived in [55], namely [56], at goes through zero at a certain value of  $M_1 = M_1^0(\mathcal{N})$  which depends on the nucleus  $\mathcal{N}$  and is a monotonically increasing function of  $M_1$  in the interval  $[50 \text{ GeV}, 10^4 \text{ GeV}]$  when  $M_1$  decreases (increases) starting from the value  $M_1 = M_1^0(\mathcal{N})$ . The values of  $M_1^0(\mathcal{N})$  at which  $|\mathcal{C}_{\mu e}|$  given in eqs. (37)-(42) and that obtained in [55] are zero differ roughly by a factor of 10 to 20, depending on the nucleus  $\mathcal{N}$ .

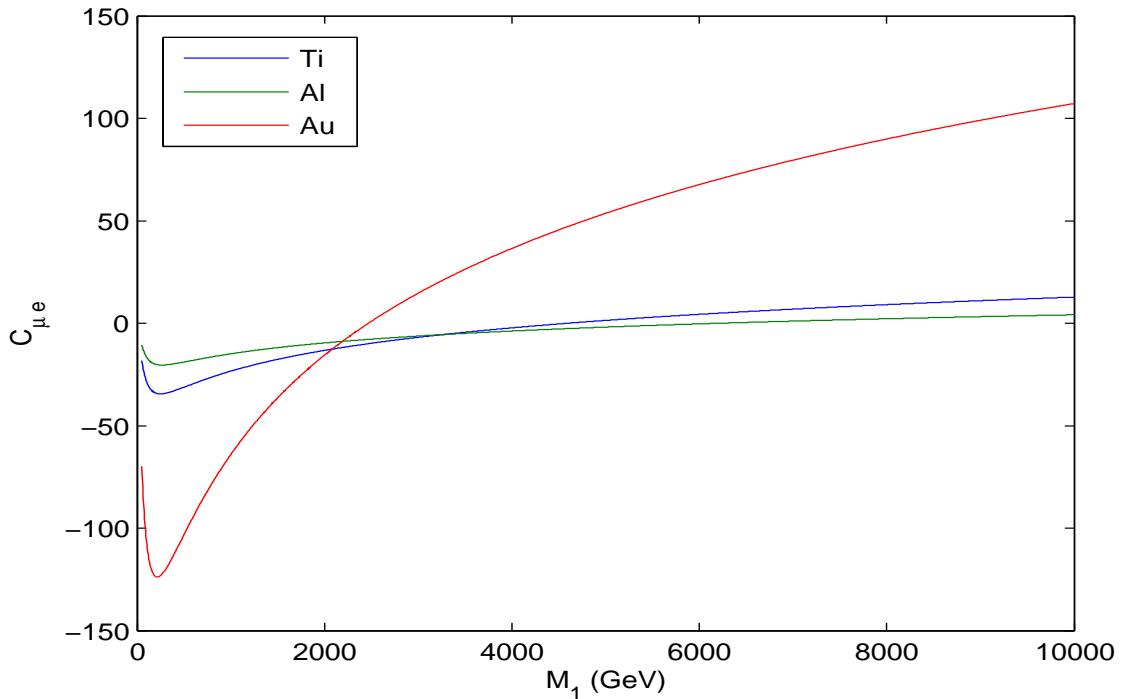


Figure 3: The  $\mu - e$  conversion loop integration factor  $C_{\mu e}$  versus the see-saw mass scale  $M_1$ , for three different nuclei: *i*)  $^{48}_{22}\text{Ti}$  (blue line), *ii*)  $^{27}_{13}\text{Al}$ , (green line), and *iii*)  $^{197}_{79}\text{Au}$  (red line).

For  $M_1$  lying inside the interested interval (100 - 1000) GeV, the loop integration factor  $|C_{\mu e}|$  takes rather large values for each of the three nuclei. As our calculations show,  $|C_{\mu e}|$  is not smaller than 23.4 for the Ti and 14.9 for the Al, while for the Au nucleus it exceeds 64.1. Since the  $\mu - e$  conversion rate is enhanced by the factor  $|C_{\mu e}|^2$ , it is very sensitive to the product  $|(RV)_{\mu 1}^*(RV)_{e 1}|$  of CC couplings of the heavy Majorana neutrinos to the electron and muon for the values of  $M_1$  in the interval of interest.

The best experimental upper bound on the conversion rate is [22]:  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \lesssim 4.3 \times 10^{-12}$ . This bound implies a constraint on  $|(RV)_{\mu 1}^*(RV)_{e 1}|$ , which is shown in Fig. 2 for  $M_1 = 100; 1000$  GeV. It is quite remarkable that, as Fig. 2 shows, the constraint on the product of couplings  $|(RV)_{\mu 1}^*(RV)_{e 1}|$  implied by the best experimental upper limit on  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti})$  is more stringent than the constraint following from the best experimental upper limit on  $\text{BR}(\mu \rightarrow e\gamma)$  although the two experimental upper limits are very similar quantitatively and the expression for  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti})$  has an additional factor of  $\alpha = 1/137$  with respect to the expression for  $\text{BR}(\mu \rightarrow e\gamma)$ .

Future experimental searches for  $\mu - e$  conversion in  $^{48}_{22}\text{Ti}$  can reach the sensitivity of  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \approx 10^{-18}$  [31]. Therefore, for values of  $M_1$  outside the narrow intervals quoted above for which the loop integration factor  $|C_{\mu e}|$  is strongly suppressed, an upper bound on the  $\mu - e$  conversion ratio of  $\mathcal{O}(10^{-18})$  can be translated into the following stringent constraint on the heavy Majorana neutrino CC couplings to the muon and electron:

$$|(RV)_{\mu 1}^*(RV)_{e 1}| \lesssim 2.17 \times 10^{-8} \quad (2.63 \times 10^{-8}) \quad \text{for } M_1 \approx 100 \text{ (1000) GeV}. \quad (46)$$

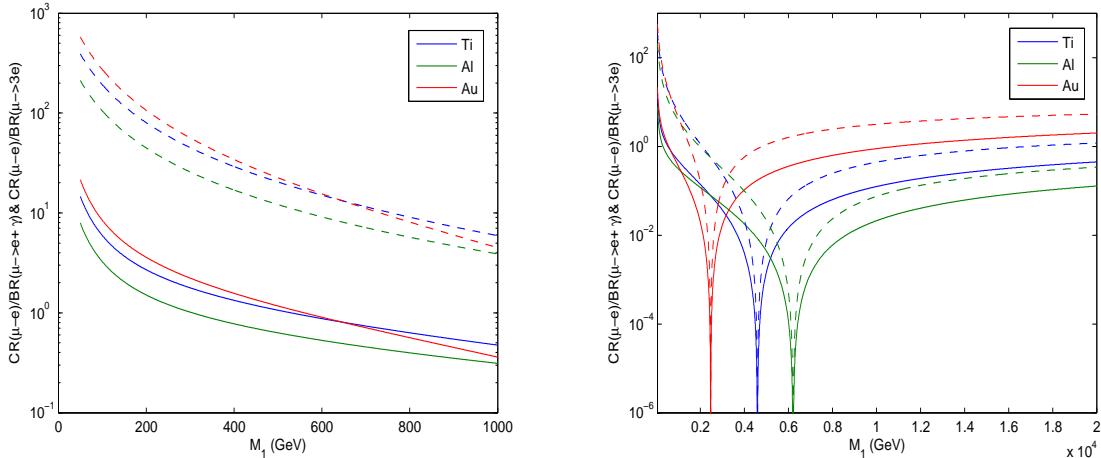


Figure 4: The ratio of the  $\mu - e$  relative conversion rate and the branching ratio of the i)  $\mu \rightarrow e\gamma$  decay (solid lines), ii)  $\mu \rightarrow 3e$  decay (dashed lines), versus the type I see-saw mass scale  $M_1$ , for three different nuclei:  $^{48}\text{Ti}$  (blue lines),  $^{27}\text{Al}$  (green lines) and  $^{197}\text{Au}$  (red lines).

As was noticed earlier, the two parameters of the type I see-saw model considered, the mass scale  $M_1$  and the Yukawa coupling  $y$ , can be determined, in principle, from data on  $\text{BR}(\mu \rightarrow e\gamma)$  (or  $\text{BR}(\mu \rightarrow 3e)$ ) and  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti})$  if the two processes will be observed. Actually, the ratio of the rates of  $\mu - e$  conversion in any given nucleus  $\mathcal{N}$ ,  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N})$ , and of the  $\mu \rightarrow e\gamma$  decay, depends only on the mass (scale)  $M_1$  and can be used, in principle, to determine the latter. In the case of  $\mu - e$  conversion on titanium, for instance, we find:

$$R \left( \frac{\mu - e}{\mu \rightarrow e\gamma} \right) \equiv \frac{\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti})}{\text{BR}(\mu \rightarrow e\gamma)} \approx 5.95 \text{ (0.48)} \quad \text{for } M_1 \approx 100 \text{ (1000) GeV}. \quad (47)$$

The correlation between  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N})$  and  $\text{BR}(\mu \rightarrow e\gamma)$  in the model considered is illustrated in Fig. 4. The type I see-saw mass scale  $M_1$  would be uniquely determined if  $\mu - e$  conversion is observed in two different nuclei or if, e.g., the  $\mu \rightarrow e\gamma$  decay or  $\mu - e$  conversion in a given nucleus is observed and it is experimentally established that  $R(\frac{\mu - e}{\mu \rightarrow e\gamma}) \lesssim 10^{-3}$ . In the latter case  $M_1$  could be determined with a relatively high precision. Furthermore, as Fig. 4 indicates, if the RH neutrino mass  $M_1$  lies in the interval  $(50 - 1000)$  GeV,  $M_1$  would be uniquely determined provided  $R(\frac{\mu - e}{\mu \rightarrow e\gamma})$  is measured with a sufficiently high precision.

We note also that the correlation between  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N})$  and  $\text{BR}(\mu \rightarrow e\gamma)$  in the type I see-saw model considered is qualitatively and quantitatively very different from the correlation in models where the  $\mu - e$  conversion is dominated by the photon penguin diagram, e.g., the supersymmetric high-scale see-saw model which predicts approximately [59]  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \approx 5 \times 10^{-3} \text{ BR}(\mu \rightarrow e\gamma)$ .

### 2.3 The $\mu \rightarrow 3e$ decay

The  $\mu \rightarrow 3e$  decay branching ratio has been calculated in [54] in a type I seesaw mechanism of neutrino mass generation with arbitrary fixed number of heavy RH neutrinos. After

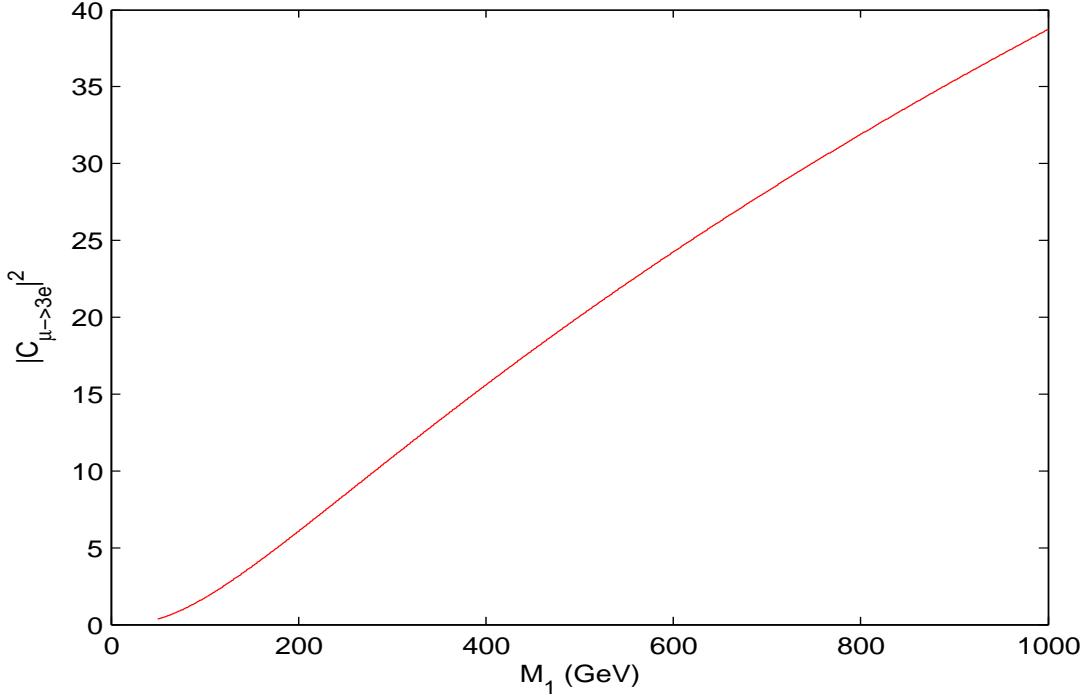


Figure 5: The  $\mu \rightarrow 3e$  decay rate factor  $|C_{\mu 3e}|^2$  as a function of the see-saw mass scale  $M_1$ .

recalculating the form factors and neglecting the effects of mass difference between  $N_1$  and  $N_2$ , we find in the model of interest to leading order in the small parameters  $|(RV)_{l1}|$ :<sup>7</sup>

$$\text{BR}(\mu \rightarrow 3e) = \frac{\alpha_{em}^2}{16\pi^2 \sin^4 \theta_W} |(RV)_{\mu 1}^*(RV)_{e 1}|^2 |C_{\mu 3e}(x)|^2, \quad (48)$$

$$\begin{aligned} |C_{\mu 3e}(x)|^2 &= 2 \left| \frac{1}{2} F_B^{\mu 3e} + F_z^{\mu 3e} - 2 \sin^2 \theta_W (F_z^{\mu 3e} - F_\gamma) \right|^2 + 4 \sin^4 \theta_W |F_z^{\mu 3e} - F_\gamma|^2 \\ &\quad + 16 \sin^2 \theta_W \left[ (F_z^{\mu 3e} + \frac{1}{2} F_B^{\mu 3e}) G_\gamma \right] - 48 \sin^4 \theta_W [(F_z^{\mu 3e} - F_\gamma^{\mu 3e}) G_\gamma] \\ &\quad + 32 \sin^4 \theta_W |G_\gamma|^2 \left( \log \frac{m_\mu^2}{m_e^2} - \frac{11}{4} \right), \end{aligned} \quad (49)$$

where  $F_\gamma(x)$ ,  $G_\gamma(x)$ ,  $F_z(x)$ ,  $G_z(x, y)$ ,  $F_{XBox}(x, y)$  are defined in (38), (39), (40), (41), (43), and

$$F_z^{\mu 3e}(x) = F_z(x) + 2G_z(0, x), \quad F_B^{\mu 3e}(x) = -2(F_{XBox}(0, x) - F_{XBox}(0, 0)). \quad (50)$$

The dependence of the  $\mu \rightarrow 3e$  decay rate factor  $|C_{\mu 3e}|^2$  on the type I see-saw mass scale  $M_1$  is shown in Fig. 5. At  $M_1 = 100$  (1000) GeV we have:  $|C_{\mu 3e}|^2 \cong 1.75$  (38.73), i.e.,  $|C_{\mu 3e}|^2$  increases by a factor of 22 when  $M_1$  changes from 100 GeV to 1000 GeV. Using the quoted

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<sup>7</sup>The new results are published as an erratum to [56].

values of  $|C_{\mu 3e}|^2$  we get the following constraint from the current limit on  $\text{BR}(\mu \rightarrow 3e)$ , eq. (4):

$$|(RV)_{\mu 1}^*(RV)_{e 1}| \lesssim 3.01 \times 10^{-4} (6.39 \times 10^{-6}) \quad \text{for } M_1 = 100 \text{ (1000) GeV}. \quad (51)$$

Thus, for  $M_1 = 100$  GeV the constraint on  $|(RV)_{\mu 1}^*(RV)_{e 1}|$  obtained using the current experimental upper limit on  $\text{BR}(\mu \rightarrow 3e)$  is by a factor of 3.90 less stringent than that obtained from the current upper limit on  $\text{BR}(\mu \rightarrow e\gamma)$  (see eq. (29)), while for  $M_1 = 1000$  GeV it is by a factor of 4.7 more stringent. However, for the two values of  $M_1$  considered, the upper limit on  $|(RV)_{\mu 1}^*(RV)_{e 1}|$  from the current experimental bound on the  $\mu - e$  conversion rate, eq. (5), is the most stringent. This is clearly seen in Fig. 2. It follows also from Fig. 2 that an experiment sensitive to a  $\mu - e$  conversion rate  $\text{CR}(\mu \text{ Al} \rightarrow e \text{ Al}) \approx 10^{-16}$ , will probe smaller values of the product of couplings  $|(RV)_{\mu 1}^*(RV)_{e 1}|$  than an experiment sensitive to  $\text{BR}(\mu \rightarrow 3e) = 10^{-15}$ .

In Fig. 4 we show the correlation between  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N})$  and  $\text{BR}(\mu \rightarrow 3e)$  in the TeV scale see-saw model considered. As it follows from Fig. 4, the observation of the  $\mu \rightarrow 3e$  decay or of the  $\mu - e$  conversion in a given nucleus, combined with data on the ratio  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N})/\text{BR}(\mu \rightarrow 3e)$  would lead either to a unique determination of the type I see-saw scale  $M_1$ , or to two values, or else to a relatively narrow interval of values, of  $M_1$  compatible with the data. One can get the same type of information on the scale  $M_1$  from data on the ratio  $\text{BR}(\mu \rightarrow 3e)/\text{BR}(\mu \rightarrow e\gamma)$ , provided at least one of the two decays  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  is observed.

It should be added finally that for  $M_1 \gtrsim 100$  GeV we have:  $\text{BR}(\mu \rightarrow 3e)/\text{BR}(\mu \rightarrow e\gamma) \gtrsim 0.031$ . Thus, if it is experimentally established that  $\text{BR}(\mu \rightarrow 3e)/\text{BR}(\mu \rightarrow e\gamma)$  is definitely smaller than the quoted lower bound, the model considered with  $M_1 \gtrsim 100$  GeV will be ruled out. Such a result would be consistent also just with a see-saw scale  $M_1 < 100$  GeV.

### 3 TeV Scale Type II See-Saw Model

We will consider in this section the type II see-saw [60] extension of the SM for the generation of the light neutrino masses. In its minimal formulation it includes one additional  $SU(2)_L$  triplet Higgs field  $\Delta$ , which has weak hypercharge  $Y_W = 2$ :

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \quad (52)$$

The Lagrangian of the type II see-saw scenario, which is sometimes called also the “Higgs Triplet Model” (HTM), reads <sup>8</sup>:

$$\mathcal{L}_{\text{seesaw}}^{\text{II}} = -M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) - \left( h_{\ell\ell'} \overline{\psi^C}_{\ell L} i\tau_2 \Delta \psi_{\ell' L} + \mu_\Delta H^T i\tau_2 \Delta^\dagger H + \text{h.c.} \right), \quad (53)$$

where  $(\psi_{\ell L})^T \equiv (\nu_{\ell L}^T \quad \ell_L^T)$ ,  $\overline{\psi^C}_{\ell L} \equiv (-\nu_{\ell L}^T C^{-1} \quad -\ell_L^T C^{-1})$ , and  $H$  are, respectively, the SM lepton and Higgs doublets,  $C$  being the charge conjugation matrix, and  $\mu_\Delta$  is a real

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<sup>8</sup>We do not give here, for simplicity, all the quadratic and quartic terms present in the scalar potential (see, *e.g.*, [61]).

parameter characterising the soft explicit breaking of the total lepton charge conservation. We are interested in the low energy see-saw scenario, where the new physics scale  $M_\Delta$  associated with the mass of  $\Delta$  takes values  $100 \text{ GeV} \lesssim M_\Delta \lesssim 1 \text{ TeV}$ , which, in principle, can be probed by LHC [62].

The flavour structure of the Yukawa coupling matrix  $h$  and the size of the lepton charge soft breaking parameter  $\mu_\Delta$  are related to the light neutrino mass matrix  $m_\nu$ , which is generated when the neutral component of  $\Delta$  develops a “small” vev  $v_\Delta \propto \mu_\Delta$ . Indeed, setting  $\Delta^0 = v_\Delta$  and  $H^T = (0 \ v)^T$  with  $v \simeq 174 \text{ GeV}$ , from Lagrangian (53) one obtains:

$$(m_\nu)_{\ell\ell'} \equiv m_{\ell\ell'} \simeq 2 h_{\ell\ell'} v_\Delta. \quad (54)$$

The matrix of Yukawa couplings  $h_{\ell\ell'}$  is directly related to the PMNS neutrino mixing matrix  $U_{\text{PMNS}} \equiv U$ , which is unitary in this case:

$$h_{\ell\ell'} \equiv \frac{1}{2v_\Delta} (U^* \text{diag}(m_1, m_2, m_3) U^\dagger)_{\ell\ell'}. \quad (55)$$

An upper limit on  $v_\Delta$  can be obtained from considering its effect on the parameter  $\rho = M_W^2/M_Z^2 \cos^2 \theta_W$ . In the SM,  $\rho = 1$  at tree-level, while in the HTM one has

$$\rho \equiv 1 + \delta\rho = \frac{1 + 2x^2}{1 + 4x^2}, \quad x \equiv v_\Delta/v. \quad (56)$$

The measurement  $\rho \approx 1$  leads to the bound  $v_\Delta/v \lesssim 0.03$ , or  $v_\Delta < 5 \text{ GeV}$  (see, e.g., [63]).

As we will see, the amplitudes of the LFV processes  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$  and  $\mu + \mathcal{N} \rightarrow e + \mathcal{N}$  in the model under discussion are proportional, to leading order, to a product of 2 elements of the Yukawa coupling matrix  $h$ . This implies that in order for the rates of the indicated LFV processes to be close to the existing upper limits and within the sensitivity of the ongoing MEG and the planned future experiments for  $M_\Delta \sim (100 - 1000) \text{ GeV}$ , the Higgs triplet vacuum expectation value  $v_\Delta$  must be relatively small, roughly  $v_\Delta \sim (1 - 100) \text{ eV}$ . In the case of  $M_\Delta \sim v = 174 \text{ GeV}$  we have  $v_\Delta \cong \mu_\Delta$ , while if  $M_\Delta^2 >> v^2$ , then  $v_\Delta \cong \mu_\Delta v^2/(2M_\Delta^2)$  (see, e.g., [61, 63]) with  $v^2/(2M_\Delta^2) \cong 0.015$  for  $M_\Delta = 1000 \text{ GeV}$ . Thus, in both cases a relatively small value of  $v_\Delta$  implies that  $\mu_\Delta$  has also to be small. A nonzero but relatively small value of  $\mu_\Delta$  can be generated, e.g., at higher orders in perturbation theory [64] or in the context of theories with extra dimensions (see, e.g., [65]).

The physical singly-charged Higgs scalar field practically coincides with the triplet scalar field  $\Delta^+$ , the admixture of the doublet charged scalar field being suppressed by the factor  $v_\Delta/v$ . The singly- and doubly- charged Higgs scalars  $\Delta^+$  and  $\Delta^{++}$  have, in general, different masses [64, 66]:  $m_{\Delta^+} \neq m_{\Delta^{++}}$ . Both situations  $m_{\Delta^+} > m_{\Delta^{++}}$  and  $m_{\Delta^+} < m_{\Delta^{++}}$  are possible. In some cases, for simplicity, we will present numerical results for  $m_{\Delta^+} \cong m_{\Delta^{++}} \equiv M_\Delta$ , but one must keep in mind that  $m_{\Delta^+}$  and  $m_{\Delta^{++}}$  can have different values.

In the mass eigenstate basis, the effective charged lepton flavour changing operators arise at one-loop order from the exchange of the singly- and doubly-charged physical Higgs scalar fields. The corresponding effective low energy LFV Lagrangian, which contributes to the  $\mu - e$  transition processes we are interested in, can be written in the form:

$$\begin{aligned} \mathcal{L}^{eff} = & -4 \frac{e G_F}{\sqrt{2}} (m_\mu A_R \bar{e} \sigma^{\alpha\beta} P_R \mu F_{\beta\alpha} + \text{h.c.}) \\ & - \frac{e^2 G_F}{\sqrt{2}} \left( A_L (-m_\mu^2) \bar{e} \gamma^\alpha P_L \mu \sum_{Q=u,d} q_Q \bar{Q} \gamma_\alpha Q + \text{h.c.} \right), \end{aligned} \quad (57)$$

where  $e$  is the proton charge, and  $q_u = 2/3$  and  $q_d = -1/3$  are the electric charges of the up and down quarks (in units of the proton charge). We obtain for the form factors  $A_{R,L}$ :

$$A_R = -\frac{1}{\sqrt{2}G_F} \frac{(h^\dagger h)_{e\mu}}{48\pi^2} \left( \frac{1}{8m_{\Delta^+}^2} + \frac{1}{m_{\Delta^{++}}^2} \right), \quad (58)$$

$$A_L(q^2) = -\frac{1}{\sqrt{2}G_F} \frac{h_{le}^* h_{l\mu}}{6\pi^2} \left( \frac{1}{12m_{\Delta^+}^2} + \frac{1}{m_{\Delta^{++}}^2} f\left(\frac{-q^2}{m_{\Delta^{++}}^2}, \frac{m_l^2}{m_{\Delta^{++}}^2}\right) \right), \quad (59)$$

$m_l$  being the mass of the charged lepton  $l$ ,  $l = e, \mu, \tau$ . In the limit where the transition is dominated by the exchange of a virtual doubly charged scalar  $\Delta^{++}$ , these expressions reduce to those obtained in [67, 68]; to the best of our knowledge the expression of  $A_L(q^2)$  for the general case is a new result. The term with the form factor  $A_R$  in eq. (57) generates the  $\mu \rightarrow e\gamma$  decay amplitude. It corresponds to the contribution of the one loop diagrams with virtual neutrino and  $\Delta^+$  [69] and with virtual charged lepton and  $\Delta^{++}$  [67] (see also [71]). The second term involving the form factor  $A_L$ , together with  $A_R$ , generates the  $\mu - e$  conversion amplitude. The loop function  $f(r, s_l)$  is well known [68]:

$$f(r, s_l) = \frac{4s_l}{r} + \log(s_l) + \left(1 - \frac{2s_l}{r}\right) \sqrt{1 + \frac{4s_l}{r}} \log \frac{\sqrt{r} + \sqrt{r + 4s_l}}{\sqrt{r} - \sqrt{r + 4s_l}}. \quad (60)$$

Notice that in the limit in which the charged lepton masses  $m_l$  are much smaller than the doubly-charged scalar mass  $m_{\Delta^{++}}$ , one has  $f(r, s_l) \simeq \log(r) = \log(m_\mu^2/m_{\Delta^{++}}^2)$ . For  $m_{\Delta^{++}} = (100 - 1000)$  GeV, this is an excellent approximation for  $f(r, s_e)$ , but cannot be used for  $f(r, s_\mu)$  and  $f(r, s_\tau)$ . The ratios  $f(r, s_e)/f(r, s_\mu)$  and  $f(r, s_e)/f(r, s_\tau)$  change relatively little when  $m_{\Delta^{++}}$  increases from 100 GeV to 1000 GeV, and at  $m_{\Delta^{++}} = 100$  (1000) GeV take the values:  $f(r, s_e)/f(r, s_\mu) \cong 1.2$  (1.1) and  $f(r, s_e)/f(r, s_\tau) \cong 2.1$  (1.7). More generally,  $f(r, s_l)$ ,  $l = e, \mu, \tau$ , are monotonically (slowly) decreasing functions of  $m_{\Delta^{++}}$ <sup>9</sup>: for  $m_{\Delta^{++}} = 100$  (1000) GeV we have, e.g.,  $f(r, s_e) \cong -13.7$  ( $-18.3$ ).

### 3.1 The $\mu \rightarrow e\gamma$ Decay

Using eqs. (57) and (58) we get for the  $\mu \rightarrow e\gamma$  decay branching ratio in the case under discussion [69, 61]:

$$\text{BR}(\mu \rightarrow e\gamma) \cong 384\pi^2 (4\pi\alpha_{\text{em}}) |A_R|^2 = \frac{\alpha_{\text{em}}}{192\pi} \frac{|(h^\dagger h)_{e\mu}|^2}{G_F^2} \left( \frac{1}{m_{\Delta^+}^2} + \frac{8}{m_{\Delta^{++}}^2} \right)^2. \quad (61)$$

For  $m_{\Delta^+} \cong m_{\Delta^{++}} \equiv M_\Delta$ , the upper limit on  $\text{BR}(\mu \rightarrow e\gamma)$  reported by the MEG experiment, eq. (3), implies the following upper bound on  $|(h^\dagger h)_{e\mu}|$ :

$$|(h^\dagger h)_{e\mu}| < 5.8 \times 10^{-6} \left( \frac{M_\Delta}{100 \text{ GeV}} \right)^2. \quad (62)$$

One can use this upper bound, in particular, to obtain a lower bound on the vacuum expectation value of  $\Delta^0$ ,  $v_\Delta$ <sup>10</sup>. Indeed, from eq. (55) it is not difficult to get:

$$|(h^\dagger h)_{e\mu}| = \frac{1}{4v_\Delta^2} \left| U_{e2} U_{2\mu}^\dagger \Delta m_{21}^2 + U_{e3} U_{3\mu}^\dagger \Delta m_{31}^2 \right|, \quad (63)$$

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<sup>9</sup>Note that we have  $f(r, s_l) < 0$ ,  $l = e, \mu, \tau$ .

<sup>10</sup>This was noticed also in [70].

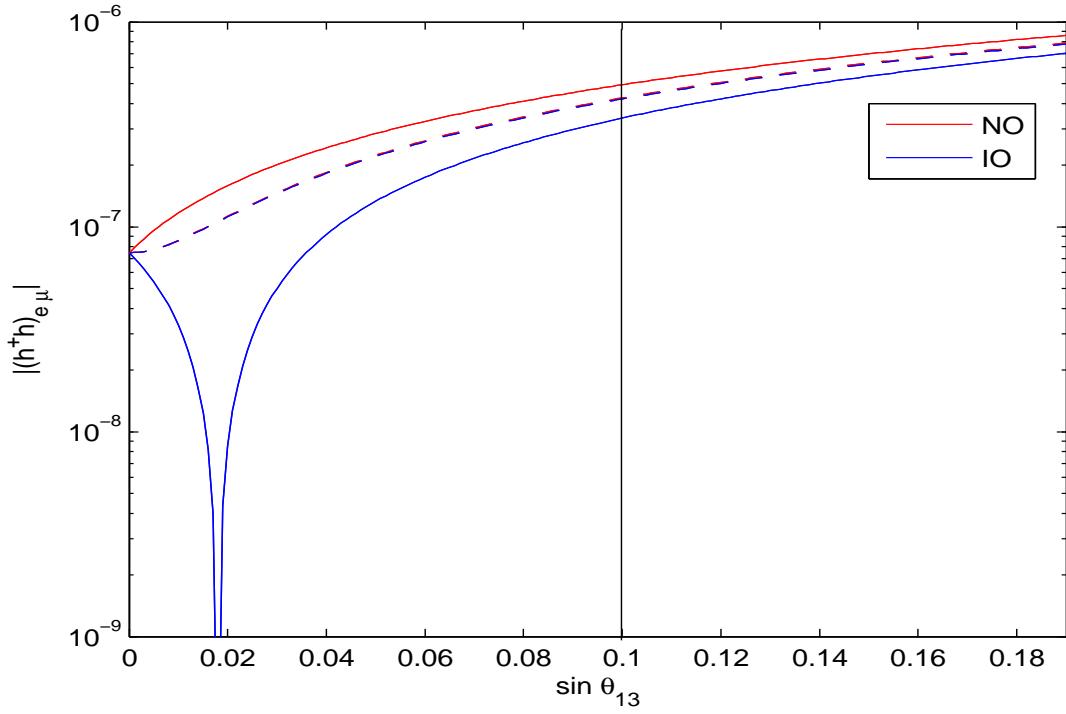


Figure 6: The dependence of  $|(h^\dagger h)_{e\mu}|$  on  $\sin \theta_{13}$  for  $v_\Delta = 9.5$  eV and  $\delta = 0$  (solid lines) and  $\delta = \pi/2$  (dashed line). The other neutrino oscillation parameters are set to their best fit values given in Table 1. The vertical line corresponds to the current  $3\sigma$  allowed minimal value of  $\sin \theta_{13}$  (see eq. (1)).

where we have used the unitarity of  $U$ . The above expression for  $|(h^\dagger h)_{e\mu}|$  is exact. It follows from eq. (63) that the prediction for  $|(h^\dagger h)_{e\mu}|$ , and thus for  $\text{BR}(\mu \rightarrow e\gamma)$ , depends, in general, on the Dirac CPV phase  $\delta$  of the standard parametrisation of the PMNS matrix  $U$  (see eq. (21)). For the best fit values of  $\sin^2 \theta_{13} = 0.0236$  and of the other neutrino oscillation parameters listed in Table 1, the term  $\propto \Delta m_{21}^2$  in eq. (63) is approximately a factor of 10 smaller than the term  $\propto \Delta m_{31}^2$ . In this case  $\text{BR}(\mu \rightarrow e\gamma)$  exhibits a relatively weak dependence on the type of the neutrino mass spectrum and on the Dirac phase  $\delta$ . Neglecting the term  $\propto \Delta m_{21}^2$ , we obtain from (62) and (63):

$$v_\Delta > 2.1 \times 10^2 \left| s_{13} s_{23} \Delta m_{31}^2 \right|^{\frac{1}{2}} \left( \frac{100 \text{ GeV}}{M_\Delta} \right) \cong 3.0 \text{ eV} \left( \frac{100 \text{ GeV}}{M_\Delta} \right). \quad (64)$$

For the  $3\sigma$  allowed ranges of values of  $\sin^2 2\theta_{13}$  given in eq. (1) and of the other neutrino oscillation parameters quoted in Table 1, the absolute lower bound on  $v_\Delta$  corresponds approximately to  $v_\Delta > 1.5$  eV ( $100 \text{ GeV}/M_\Delta$ ) and is reached in the case of  $\Delta m_{31}^2 > 0$  ( $\Delta m_{31}^2 < 0$ ) for  $\delta = \pi$  (0).

We note further that if  $\delta \cong \pi/2$  ( $3\pi/2$ ), the term  $\propto \Delta m_{21}^2$  in the expression for  $|(h^\dagger h)_{e\mu}|$  (and thus for  $\text{BR}(\mu \rightarrow e\gamma)$ ) always plays a subdominant role as long as the other neutrino oscillation parameters lie in their currently allowed  $3\sigma$  ranges. Therefore in this case the dependence of  $\text{BR}(\mu \rightarrow e\gamma)$  on the type of neutrino mass spectrum is negligible. The specific

features of the predictions for  $|(h^\dagger h)_{e\mu}|$  discussed above are illustrated in Fig. 6.

Exploiting the fact that  $v_\Delta^2 |(h^\dagger h)_{e\mu}|$  is known with a rather good precision, we can write:

$$\text{BR}(\mu \rightarrow e\gamma) \cong 2.7 \times 10^{-10} \left( \frac{1 \text{ eV}}{v_\Delta} \right)^4 \left( \frac{100 \text{ GeV}}{M_\Delta} \right)^4, \quad (65)$$

where we have used eq. (61) and the best fit values of the neutrino oscillation parameters. It follows from eq. (65) that for the values of  $v_\Delta$  and  $M_\Delta$  (or  $m_{\Delta^+}$  and/or  $m_{\Delta^{++}}$ ) of interest,  $\text{BR}(\mu \rightarrow e\gamma)$  can have a value within the projected sensitivity of the ongoing MEG experiment.

### 3.2 The $\mu \rightarrow 3e$ decay

In the TeV scale type II see-saw scenario, the  $\mu \rightarrow 3e$  decay amplitude is generated at the tree level by the diagram with exchange of a virtual doubly-charged Higgs boson  $\Delta^{++}$ . The branching ratio can be easily calculated (see, e.g., [67, 71]):

$$\text{BR}(\mu \rightarrow 3e) = \frac{1}{G_F^2} \frac{|(h^\dagger)_{ee}(h)_{\mu e}|^2}{m_{\Delta^{++}}^4} = \frac{1}{G_F^2 m_{\Delta^{++}}^4} \frac{|m_{ee}^* m_{\mu e}|^2}{16 v_\Delta^4}, \quad (66)$$

where we have used eq. (54). From the present limit  $\text{BR}(\mu \rightarrow 3e) < 10^{-12}$ , one can obtain the following constraint on  $|(h^\dagger)_{ee}(h)_{\mu e}|$ :

$$|(h^\dagger)_{ee}(h)_{\mu e}| < 1.2 \times 10^{-7} \left( \frac{m_{\Delta^{++}}}{100 \text{ GeV}} \right)^2. \quad (67)$$

In the model under discussion,  $\text{BR}(\mu \rightarrow 3e)$  depends on the factor  $|m_{ee}^* m_{\mu e}|$ , which involves the product of two elements of the neutrino Majorana mass matrix, on the neutrino mass spectrum and on the Majorana and Dirac CPV phases in the PMNS matrix  $U$ . For the values of  $m_{\Delta^+}$  and  $m_{\Delta^{++}}$  in the range of  $\sim (100 - 1000)$  GeV and of  $v_\Delta \ll 1$  MeV of interest,  $m_{ee}$  practically coincides with the effective Majorana mass in neutrinoless double beta ( $(\beta\beta)_{0\nu^-}$ ) decay (see, e.g., [71, 72, 73]),  $|\langle m \rangle|$ :

$$|m_{ee}| = \left| \sum_{j=1}^3 m_j U_{ej}^2 \right| \cong |\langle m \rangle|. \quad (68)$$

Depending on the type of neutrino mass spectrum, the value of the lightest neutrino mass and on the values of the CPV Majorana and Dirac phases in the PMNS matrix,  $|m_{ee}|$  can take any value between 0 and  $m_0$ , where  $m_0 = m_1 \cong m_2 \cong m_3$  is the value of the neutrino masses in the case of quasi-degenerate (QD) spectrum,  $m_0 \gtrsim 0.1$  eV (see, e.g., [72]). It follows from the searches for the  $(\beta\beta)_{0\nu^-}$ -decay that  $|m_{ee}| \lesssim m_0 \lesssim 1$  eV, while the cosmological constraints on the sum of the neutrino masses imply  $m_0 \lesssim 0.3$  eV (see, e.g., [1]). As is well known, the  $(\beta\beta)_{0\nu^-}$ -decay is claimed to have been observed in [74, 75], with the reported half-life corresponding to [75]  $|m_{ee}| = 0.32 \pm 0.03$  eV. This claim will be tested in a new generation of  $(\beta\beta)_{0\nu^-}$ -decay experiments which either are already taking data or are in preparation at present (see, e.g., [1, 76]).

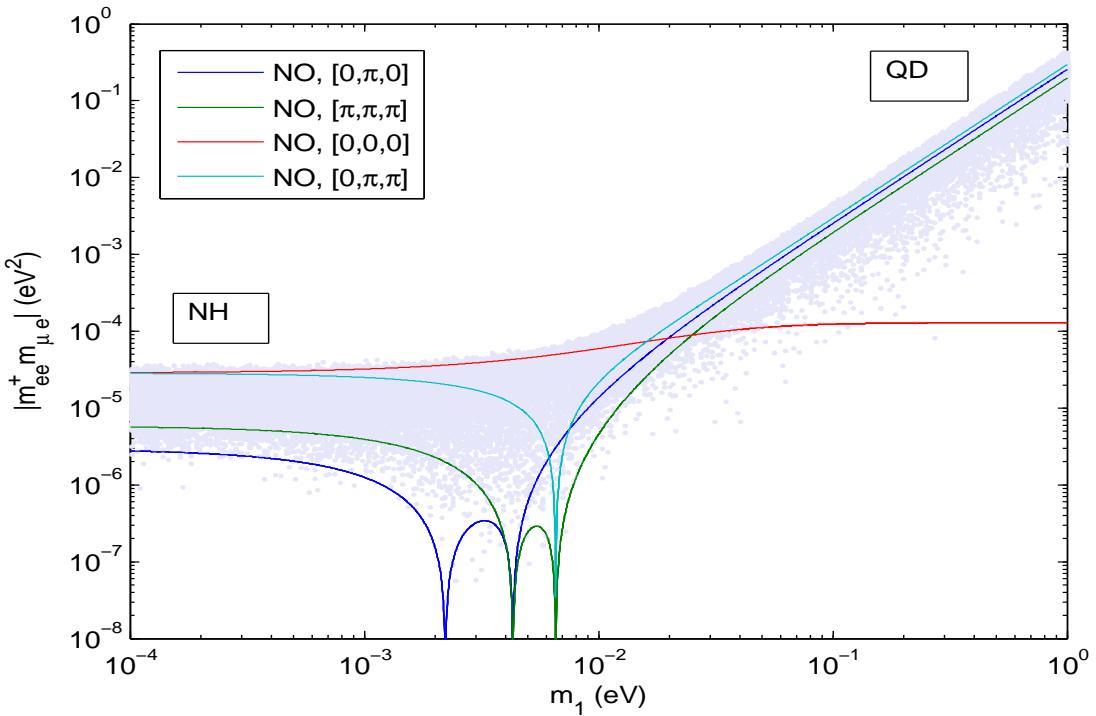


Figure 7: The dependence of  $|m_{ee}^* m_{\mu e}|$  on the lightest neutrino mass  $m_1$  in the case of neutrino mass spectrum with normal ordering ( $\Delta m_A^2 > 0$ ), for four sets of values of the Dirac and the two Majorana CPV phases,  $[\delta, \alpha_{21}, \alpha_{31}]$ . The depicted curves correspond to the best fit values of  $\sin \theta_{13}$  (eq. (1)) and of the other neutrino oscillation parameters given in Table 1. The scattered points are obtained by varying the neutrino oscillation parameters within their corresponding  $3\sigma$  intervals and giving random values to the CPV Dirac and Majorana phases.

In the case of NH light neutrino mass spectrum with  $m_1 \ll 10^{-4}$  eV,  $|m_{ee}|$  lies in the interval  $3.6 \times 10^{-4}$  eV  $\lesssim |m_{ee}| \lesssim 5.2 \times 10^{-3}$  eV. This interval was obtained by taking into account the  $3\sigma$  allowed ranges of values of  $\sin^2 \theta_{13}$  (eq. (1)),  $\sin^2 \theta_{12}$ ,  $\sin^2 \theta_{23}$  and  $\Delta m_A^2$  and  $\Delta m_{32}^2$ . For the best fit values (b.f.v.) of the latter we get <sup>11</sup>:  $1.45 \times 10^{-3}$  eV  $\lesssim |m_{ee}| \lesssim 3.75 \times 10^{-3}$  eV. The minimal and the maximal values correspond to the combination of the CPV phases  $(\alpha_{21} - \alpha_{31} + 2\delta) = \pi$  and 0, respectively. However, for  $m_1 \gtrsim 10^{-4}$  eV, one can have  $|m_{ee}| = 0$  for specific values of  $m_1$  if the CPV phases  $\alpha_{21}$  and  $\alpha_{31} - 2\delta$  possess the CP conserving values  $\alpha_{21} = \pi$  and  $(\alpha_{31} - 2\delta) = 0, \pi$  (see, e.g., [17]): for the  $[\pi, 0]$  combination this occurs at  $m_1 \cong 2.3 \times 10^{-3}$  eV, while in the case of the  $[\pi, \pi]$  one we have  $|m_{ee}| = 0$  at  $m_1 \cong 6.5 \times 10^{-3}$  eV. If the light neutrino mass spectrum is with inverted ordering ( $\Delta m_A^2 \equiv \Delta m_{32}^2 < 0$ ,  $m_3 < m_1 < m_2$ ) or of inverted hierarchical (IH) type ( $m_3 \ll m_1 < m_2$ ), we have [77]  $|m_{ee}| \gtrsim \sqrt{|\Delta m_A^2| + m_3^2} \cos 2\theta_{12} \gtrsim 1.27 \times 10^{-2}$  eV, while in the case of QD spectrum,  $|m_{ee}| \gtrsim m_0 \cos 2\theta_{12} \gtrsim 2.8 \times 10^{-2}$  eV, where we used the  $3\sigma$  minimal allowed values of  $|\Delta m_A^2|$  and  $\cos 2\theta_{12}$ .

<sup>11</sup>The numerical values quoted further in this subsection are obtained for the indicated best fit values of the neutrino oscillation parameters, unless otherwise stated.

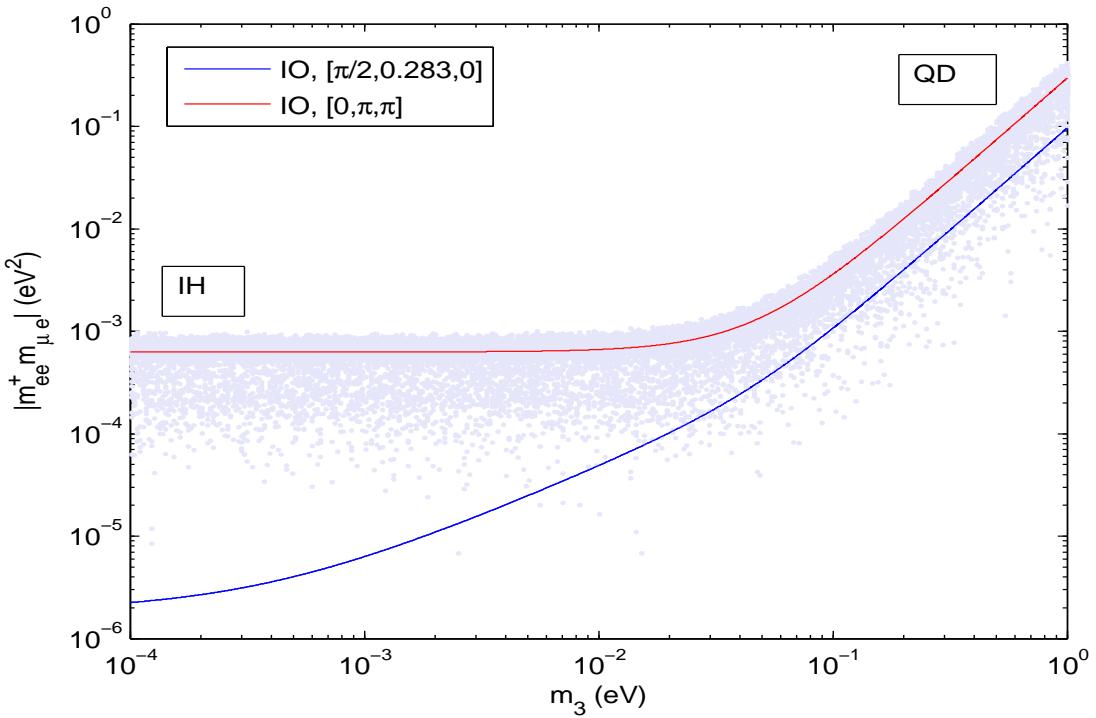


Figure 8: The same as in Fig. 7 in the case of a light neutrino mass spectrum with inverted ordering ( $\Delta m_A^2 < 0$ ) (see text for details).

We consider next briefly the dependence of the neutrino mass matrix element  $|m_{\mu e}|$  on the type of the neutrino mass spectrum and on the CPV Majorana and Dirac phases. In the case of NH spectrum with  $m_1 = 0$ , the maximal value of  $|m_{\mu e}|$  is obtained for  $\alpha_{31} - \alpha_{21} = \delta$ ,  $\delta = \pi$ , and reads:  $\max(|m_{\mu e}|) \cong 8.1 \times 10^{-3}$  eV. We get  $|m_{\mu e}| = 0$  for  $\alpha_{21} = \pi$ ,  $\delta = 0$  ( $\pi$ ) and  $\alpha_{31} = 0$  ( $\pi$ ). As can be shown, for each of these two sets of values of the CPV phases, the zero takes place at essentially the same value of  $m_1 \cong 4.3 \times 10^{-3}$  eV (Fig. 7). If the neutrino mass spectrum is of the IH type with negligible  $m_3 \cong 0$ , the maximal value of  $|m_{\mu e}|$  corresponds to  $\delta = 0$  and  $\alpha_{21} = \pi$  and is given by  $\max(|m_{\mu e}|) \cong \sqrt{|\Delta m_A^2|} c_{13} (c_{23} \sin 2\theta_{12} + s_{23} s_{13} \cos 2\theta_{12})$ . The element  $|m_{\mu e}|$  is strongly suppressed, i.e., we have  $|m_{\mu e}| \ll \max(|m_{\mu e}|)$ , for  $\delta \cong \pi/2$  and a value of the Majorana phase  $\alpha_{21}$  which is determined by the equation:

$$c_{23} c_{12} s_{12} \sin \alpha_{21} \cong (c_{12}^2 + s_{12}^2 \cos \alpha_{21}) s_{23} s_{13}. \quad (69)$$

For the b.f.v. of the neutrino mixing angles this equation is satisfied for  $\alpha_{21} \cong 0.283$ .

The properties of  $|m_{ee}|$  and  $|m_{\mu e}|$  described above allow us to understand most of the specific features of the dependence of the quantity  $|m_{ee}^* m_{\mu e}|$  of interest on the the neutrino mass spectrum and the leptonic CPV phases. For NH spectrum and negligible  $m_1 \cong 0$ , the maximum of the latter is obtained for  $\alpha_{31} - \alpha_{21} = \delta = 0$  and is given by:

$$\max(|m_{ee}^* m_{\mu e}|) = \left| (m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2) c_{13} (m_2 s_{12} (c_{12} c_{23} - s_{12} s_{23} s_{13}) + m_3 s_{23} s_{13}) \right|, \quad (70)$$

with  $m_2 = \sqrt{\Delta m_\odot^2}$  and  $m_3 = \sqrt{\Delta m_A^2}$ . Using the b.f.v. of the neutrino oscillation parameters we get  $\max(|m_{ee}^* m_{\mu e}|) \cong 2.9 \times 10^{-5}$  eV<sup>2</sup> (see Fig. 7). This implies  $\text{BR}(\mu \rightarrow 3e) \lesssim 6 \times$

$10^{-9}(1 \text{ eV}/v_\Delta)^4(100 \text{ GeV}/m_{\Delta++})^4$ . In the case of NH spectrum and non-negligible  $m_1$  we have  $|m_{ee}^* m_{\mu e}| = 0$  for the values of the CPV phases and  $m_1$  discussed above, for which either  $|m_{ee}| = 0$  or  $|m_{\mu e}| = 0$ . The scattered points in Fig. 7 correspond to the possible values the quantity  $|m_{ee}^* m_{\mu e}|$  can assume when varying the neutrino oscillation parameters within their corresponding  $3\sigma$  intervals and giving random values to the CPV Dirac and Majorana phases from the interval  $[0, 2\pi]$ .

The maximum of  $|m_{ee} m_{\mu e}|$  for the IH spectrum with a negligible  $m_3$  is reached for  $\delta = 0$  and  $\alpha_{21} = \pi$ , and reads:

$$\max(|m_{ee}^* m_{\mu e}|) \cong |\Delta m_A^2| c_{13}^3 \left( \frac{1}{2} c_{23} \sin 4\theta_{12} + s_{23} s_{13} \cos^2 2\theta_{12} \right). \quad (71)$$

Numerically this gives  $\max(|m_{ee} m_{\mu e}|) \cong 6.1 \times 10^{-4} \text{ eV}^2$  (Fig. 8). For  $\text{BR}(\mu \rightarrow 3e)$  we thus obtain:  $\text{BR}(\mu \rightarrow 3e) \lesssim 2.4 \times 10^{-6}(1 \text{ eV}/v_\Delta)^4(100 \text{ GeV}/m_{\Delta++})^4$ . One can have  $|m_{ee} m_{\mu e}| \ll \max(|m_{ee} m_{\mu e}|)$  in the case of IH spectrum with  $m_3 = 0$  for, e.g.,  $\delta \cong \pi/2$  and  $\alpha_{21} \cong 0.283$ , for which  $|m_{\mu e}|$  has a minimum. For the indicated values of the phases we find:  $|m_{ee} m_{\mu e}| \cong 1.2 \times 10^{-6} \text{ eV}^2$  (see Fig. 8). Similarly to the case of a neutrino mass spectrum with normal ordering discussed above, we show in Fig. 8 the range of values the LFV term  $|m_{ee} m_{\mu e}|$  can assume (scattered points).

Finally, in the case of QD spectrum,  $|m_{ee} m_{\mu e}|$  will be relatively strongly suppressed with respect to its possible maximal value for this spectrum (i.e., we will have  $|m_{ee} m_{\mu e}| \ll \max(|m_{ee}^* m_{\mu e}|)$ ) if, e.g., the Majorana and Dirac phases are zero, thus conserving the CP symmetry:  $\alpha_{21} = \alpha_{31} = \delta = 0$ . Then one has:  $|m_{ee} m_{\mu e}| \cong |\Delta m_A^2| s_{13} s_{23} c_{13}/2 \cong 1.2 \times 10^{-4} \text{ eV}^2$ . Note that this value is still larger than the maximal value of  $|m_{ee} m_{\mu e}|$  for the NH neutrino mass spectrum with a negligible  $m_1$  (see Fig. 7). The maximum of  $|m_{ee} m_{\mu e}|$  takes place for another set of CP conserving values of the Majorana and Dirac phases:  $\alpha_{21} = \alpha_{31} = \pi$  and  $\delta = 0$ . At the maximum we have:

$$\max(|m_{ee}^* m_{\mu e}|) \cong m_0^2 (c_{13}^3 \cos 2\theta_{12} - s_{13}^2) c_{13} (c_{23} \sin 2\theta_{12} + 2 c_{12}^2 s_{23} s_{13}), \quad m_0 \gtrsim 0.1 \text{ eV}. \quad (72)$$

For the b.f.v. of the neutrino mixing angles we get  $\max(|m_{ee}^* m_{\mu e}|) \cong 0.3 m_0^2$ . For  $m_0 \lesssim 0.3 \text{ eV}$  this implies  $\max(|m_{ee}^* m_{\mu e}|) \lesssim 2.7 \times 10^{-2} \text{ eV}^2$ , leading to an upper bound on  $\text{BR}(\mu \rightarrow 3e)$ , which is by a factor approximately of  $4.1 \times 10^3$  larger than in the case of IH spectrum.

The features of  $|m_{ee} m_{\mu e}|$  discussed above are illustrated in Figs. 7 and 8.

It should be clear from the preceding discussion that in the case of the type II seesaw model considered, the value of the quantity  $|(h^\dagger)_{ee}(h)_{\mu e}|^2 \propto |m_{ee}^* m_{\mu e}|^2$ , and thus the prediction for  $\text{BR}(\mu \rightarrow 3e)$ , depends very strongly on the type of neutrino mass spectrum. For a given spectrum, it exhibits also a very strong dependence on the values of the Majorana and Dirac CPV phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\delta$ , as well as on the value of the lightest neutrino mass,  $\min(m_j)$ . As a consequence, the prediction for  $\text{BR}(\mu \rightarrow 3e)$  for given  $v_\Delta$  and  $m_{\Delta++}$  can vary by a few to several orders of magnitude when one varies the values of  $\min(m_j)$  and of the CPV phases. Nevertheless, for all possible types of neutrino mass spectrum - NH, IH, QD, etc., there are relatively large regions of the parameter space of the model where  $\text{BR}(\mu \rightarrow 3e)$  has a value within the sensitivity of the planned experimental searches for the  $\mu \rightarrow 3e$  decay [33]. The region of interest for the NH spectrum is considerably smaller than those for the IH and QD spectra. In the case NO spectrum ( $\Delta m_A^2 > 0$ ),  $\text{BR}(\mu \rightarrow 3e)$  can

$\mathcal{N}$	$D m_\mu^{-5/2}$	$V^{(p)} m_\mu^{-5/2}$	$V^{(n)} m_\mu^{-5/2}$	$\Gamma_{\text{capt}} (10^6 \text{s}^{-1})$
$^{48}_{22}\text{Ti}$	0.0864	0.0396	0.0468	2.590
$^{27}_{13}\text{Al}$	0.0362	0.0161	0.0173	0.7054
$^{197}_{79}\text{Au}$	0.189	0.0974	0.146	13.07

Table 2: Nuclear parameters related to  $\mu - e$  conversion in  $^{48}_{22}\text{Ti}$ ,  $^{27}_{13}\text{Al}$  and  $^{197}_{79}\text{Au}$ . The numerical values of the overlap integrals  $D$ ,  $V^{(p)}$  and  $V^{(n)}$  are taken from [78].

be strongly suppressed for certain values of the lightest neutrino mass  $m_1$  from the interval  $\sim (2 \times 10^{-3} - 10^{-2})$  eV (Fig. 7). For the IO spectrum ( $\Delta m_A^2 < 0$ ), a similar suppression can take place for  $m_3 \ll 10^{-2}$  eV (Fig. 8). In the cases when  $|m_{ee}^* m_{\mu e}|^2$  is very strongly suppressed, the one-loop corrections to the  $\mu \rightarrow 3e$  decay amplitude should be taken into account since they might give a larger contribution than that of the tree level diagram we are considering. The analysis of this case, however, is beyond the scope of the present investigation.

### 3.3 The $\mu - e$ Conversion in Nuclei

Consider next the  $\mu - e$  conversion in a generic nucleus  $\mathcal{N}$ . We parametrise the corresponding conversion rate following the effective field theory approach developed in [78]. Taking into account the interaction Lagrangian (57), we get in the type II see-saw scenario

$$\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N}) \cong (4\pi\alpha_{\text{em}})^2 \frac{2G_F^2}{\Gamma_{\text{capt}}} \left| A_R \frac{D}{\sqrt{4\pi\alpha_{\text{em}}}} + (2q_u + q_d) A_L V^{(p)} \right|^2. \quad (73)$$

The parameters  $D$  and  $V^{(p)}$  represent overlap integrals of the muon and electron wave functions and are related to the effective dipole and vector type operators in the interaction Lagrangian, respectively (see, *e.g.* [78]).

In the case of a light nucleus, *i.e.* for  $Z \lesssim 30$ , one has with a good approximation  $D \simeq 8\sqrt{4\pi\alpha_{\text{em}}} V^{(p)}$ , with the vector type overlap integral of the proton,  $V^{(p)}$ , given by [78]:

$$V^{(p)} \simeq \frac{1}{4\pi} m_\mu^{5/2} \alpha_{\text{em}}^{3/2} Z_{\text{eff}}^2 Z^{1/2} F(-m_\mu^2), \quad (74)$$

where  $F(q^2)$  is the form factor of the nucleus. The parameters  $D m_\mu^{-5/2}$ ,  $V^{(p)} m_\mu^{-5/2}$  and  $\Gamma_{\text{capt}}$  for  $^{48}_{22}\text{Ti}$ ,  $^{27}_{13}\text{Al}$  and  $^{197}_{79}\text{Au}$  are given in Table 2. Using the result for  $D$  quoted above, eqs. (58), (59) and (74), the conversion rate (73) can be written as

$$\begin{aligned} \text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N}) \cong & \frac{\alpha_{\text{em}}^5}{36\pi^4} \frac{m_\mu^5}{\Gamma_{\text{capt}}} Z_{\text{eff}}^4 Z F^2(-m_\mu^2) \left| (h^\dagger h)_{e\mu} \left[ \frac{5}{24m_{\Delta^+}^2} + \frac{1}{m_{\Delta^{++}}^2} \right] \right. \\ & \left. + \frac{1}{m_{\Delta^{++}}^2} \sum_{l=e,\mu,\tau} h_{el}^\dagger f(r, s_l) h_{l\mu} \right|^2. \end{aligned} \quad (75)$$

In contrast to previous studies, which assume that the  $\mu - e$  conversion is dominated by the  $\Delta^{++}$  exchange [68], we will consider in this work a scenario where both scalars contribute to

the transition amplitude. Thus, assuming  $m_{\Delta^+} \cong m_{\Delta^{++}} \equiv M_\Delta$ , we have  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N}) \propto |C_{\mu e}^{(II)}|^2$ , where

$$C_{\mu e}^{(II)} \equiv \frac{1}{4v_\Delta^2} \left[ \frac{29}{24} (m^\dagger m)_{e\mu} + \sum_{l=e,\mu,\tau} m_{el}^\dagger f(r, s_l) m_{l\mu} \right], \quad (76)$$

and we have used eq. (54). The upper limit on the  $\mu - e$  conversion rate in Ti, eq. (5), leads to the following upper limit on  $|C_{\mu e}^{(II)}|$ :

$$|C_{\mu e}^{(II)}| < 1.24 \times 10^{-4} \left( \frac{M_\Delta}{100 \text{ GeV}} \right)^2. \quad (77)$$

In obtaining it we have used the values of  $\Gamma_{\text{capt}}$ ,  $Z_{\text{eff}}$ ,  $Z$  and  $F(-m_\mu^2)$  for Ti given in subsection 2.2. An experiment sensitive to  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \approx 10^{-18}$  [31] will be able to probe values of  $|C_{\mu e}^{(II)}| \gtrsim 5.8 \times 10^{-8} (M_\Delta/(100 \text{ GeV}))^2$ .

The  $\mu - e$  conversion rate in a given nucleus depends through the quantity  $C_{\mu e}^{(II)}$ , on the type of neutrino mass spectrum and the Majorana and Dirac CPV phases in the PMNS matrix. Using the b.f.v. of the neutrino oscillation parameters and performing a scan over the values of the CPV phases and the lightest neutrino mass, which in the cases of NO ( $\Delta m_A^2 > 0$ ) and IO ( $\Delta m_A^2 < 0$ ) spectra was varied in the intervals  $(10^{-4} - 1) \text{ eV}$  and  $(10^{-5} - 1) \text{ eV}$ , respectively, we have identified the possible ranges of values of  $4v_\Delta^2 |C_{\mu e}^{(II)}|$ . The latter are shown in Figs. (9) and (10).

For  $M_\Delta = 200$  (1000) GeV and NH spectrum with negligible  $m_1$  ( $m_1 \ll 10^{-3} \text{ eV}$ ), the maximal value of  $4v_\Delta^2 |C_{\mu e}^{(II)}|$  occurs for  $[\delta, \alpha_{21}, \alpha_{31}] = [0, 0, 0]$  and at the maximum we have  $4v_\Delta^2 |C_{\mu e}^{(II)}| \cong 2.9$  (3.8)  $\times 10^{-3} \text{ eV}^2$ . For values of the CPV phases  $[\delta, \alpha_{21}, \alpha_{31}] = [0, \pi, 0]$  and  $M_\Delta = 200 \text{ GeV}$ ,  $4v_\Delta^2 |C_{\mu e}^{(II)}|$  goes through zero at  $m_1 \cong 2 \times 10^{-2} \text{ eV}$  (Fig. 9). In the case of a larger charged scalar mass, *i.e.*  $M_\Delta = 1000 \text{ GeV}$ , such cancellation occurs at a different value of the lightest neutrino mass, mainly  $m_1 = 0.025 \text{ eV}$ .

The maximum of  $4v_\Delta^2 |C_{\mu e}^{(II)}|$  in the case of IH spectrum with negligible  $m_3$ , occurs for maximal CPV phases:  $[\delta, \alpha_{21}, \alpha_{31}] = [\pi/2, 3\pi/2, 0]$ . At the maximum in this case one has  $4v_\Delta^2 |C_{\mu e}^{(II)}| \cong 6$  (7)  $\times 10^{-3} \text{ eV}^2$  for  $M_\Delta = 200$  (1000) GeV. As Fig. 10 shows, for other sets of values of the CPV phases,  $4v_\Delta^2 |C_{\mu e}^{(II)}|$  can be much smaller. Taking again CP conserving phases, *e.g.*  $[\pi, \pi, 0]$ , one can get a strong suppression of the branching ratio for  $m_3 = 7.2$  (15)  $\times 10^{-3} \text{ eV}$  and  $M_\Delta = 200$  (1000) GeV. Allowing  $\sin \theta_{13}$  to take values other than the best fit one, we find that  $4v_\Delta^2 |C_{\mu e}^{(II)}|$  can even go through zero at, *e.g.*,  $[\delta, \alpha_{21}, \alpha_{31}] = [\pi, \pi, \pi/2]$  for  $\sin \theta_{13} \cong 0.137$ , which lies within the  $2\sigma$  allowed region. In Fig. 10 we report other examples in which the CPV phases in the PMNS matrix take different sets of CP violating values and the quantity  $4v_\Delta^2 |C_{\mu e}^{(II)}|$  (and the conversion rate) can vary by several orders of magnitude for specific values of the lightest neutrino mass  $m_3$  and the see-saw mass scale  $M_\Delta$ .

If the neutrino mass spectrum is quasi-degenerate,  $m_{1,2,3} \cong m_0 \gtrsim 0.1 \text{ eV}$ , we have for  $m_0 \lesssim 0.3 \text{ eV}$ :  $2.8 \times 10^{-3} \text{ eV}^2 \lesssim 4v_\Delta^2 |C_{\mu e}^{(II)}| \lesssim 0.4 \text{ eV}^2$ . The minimal value corresponds to  $\Delta m_A^2 > 0$  (NO spectrum) and  $[\delta, \alpha_{21}, \alpha_{31}] = [\pi, 0, 0]$ ; for *e.g.*  $[\delta, \alpha_{21}, \alpha_{31}] = [0, 0, 0]$  and  $M_\Delta = 200 \text{ GeV}$  we get in the QD region  $4v_\Delta^2 |C_{\mu e}^{(II)}| \cong 3.3 \times 10^{-3} \text{ eV}^2$  (Fig. 9).

Finally, the scattered points in Figs. 9 and 10 are obtained by varying all the neutrino oscillation parameters within the corresponding  $3\sigma$  intervals and allowing for arbitrary values of the Dirac and Majorana phases in the interval  $[0, 2\pi]$ .

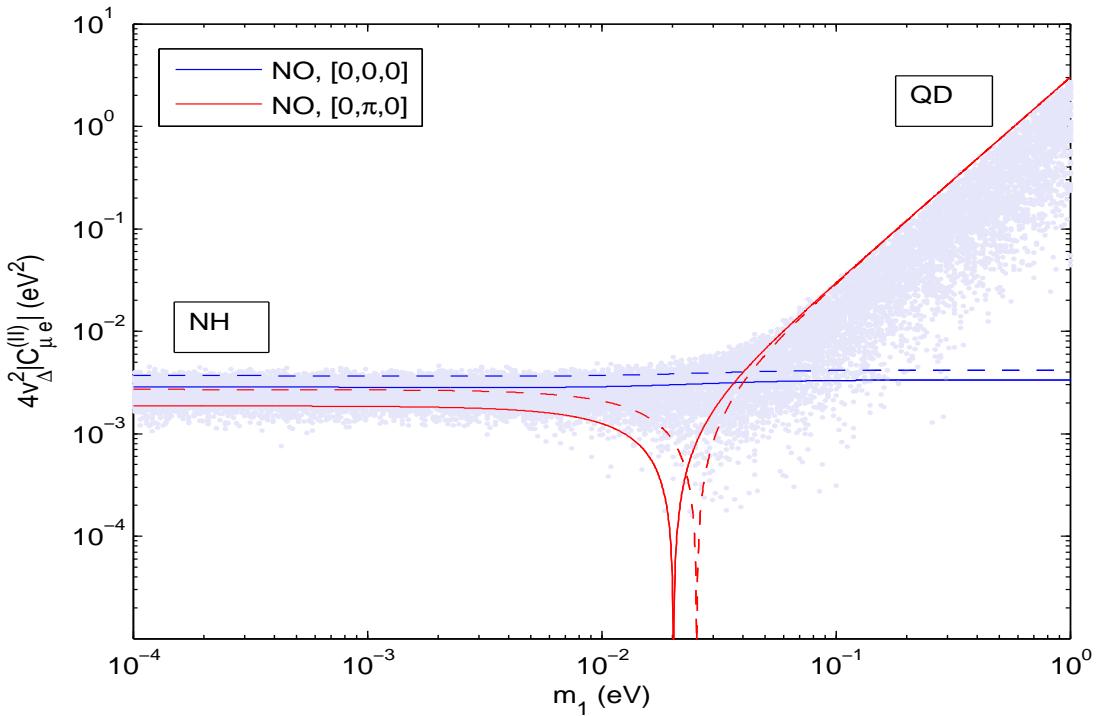


Figure 9: The dependence of  $4v_\Delta^2 |C_{\mu e}^{(II)}|$  (given in  $\text{eV}^2$ ) on the lightest neutrino mass  $m_1$  in the case of neutrino mass spectrum with normal ordering ( $\Delta m_A^2 > 0$ ), for two sets of values of the Dirac and the two Majorana CPV phases,  $[\delta, \alpha_{21}, \alpha_{31}]$  and  $M_\Delta = 200$  (1000) GeV, plain (dashed) curves. The figure is obtained for the best fit values of  $\sin \theta_{13}$  (eq. (1)) and of the other neutrino oscillation parameters given in Table 1 (see text for details).

We remark that the previous estimates, as well as Figs 9 and 10, were realized under the assumption that the singly- and doubly-charged scalars have masses of the same order, *i.e.*  $m_{\Delta^+} \cong m_{\Delta^{++}} \equiv M_\Delta$ . The case in which the dominant contribution to the conversion amplitude is provided by the exchange of  $\Delta^{++}$ , *i.e.* for  $m_{\Delta^+} \gg m_{\Delta^{++}} \gtrsim 100$  GeV, shows similar features: the upper limits of the conversion ratio in the cases of NO and IO spectra are unchanged and a strong suppression can occur for specific values of the CPV phases and  $\min(m_j)$ . Taking, instead, the opposite limit  $m_{\Delta^{++}} \gg m_{\Delta^+}$ , with  $m_{\Delta^+} = (100 - 1000)$  GeV, the dominant contribution to the  $\mu - e$  conversion amplitude is given by the exchange of the singly-charged scalar, therefore we have:  $|C_{\mu e}^{(II)}| \propto |(h^\dagger h)_{e\mu}|$ . As it was pointed out in subsection 3.1,  $|(h^\dagger h)_{e\mu}|$  shows a relative weak dependance on the type of neutrino mass spectrum and on the CPV phases in the PMNS matrix. Moreover, no suppression of the conversion amplitude occurs if  $\sin(\theta_{13})$  is taken within the current  $3\sigma$  experimental bound (see Fig. 6). In this case, from the best experimental upper bound on the conversion rate in Ti,  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) < 4.3 \times 10^{-12}$ , we get the constraint:

$$|(h^\dagger h)_{e\mu}| < 6 \times 10^{-4} \left( \frac{m_{\Delta^+}}{100 \text{ GeV}} \right)^2, \quad (78)$$

which provides a weaker bound with respect to that obtained from the  $\mu \rightarrow e\gamma$  decay (see eq. (62)). A  $\mu - e$  conversion experiment sensitive to *i.e.*  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \approx 10^{-18}$ , can

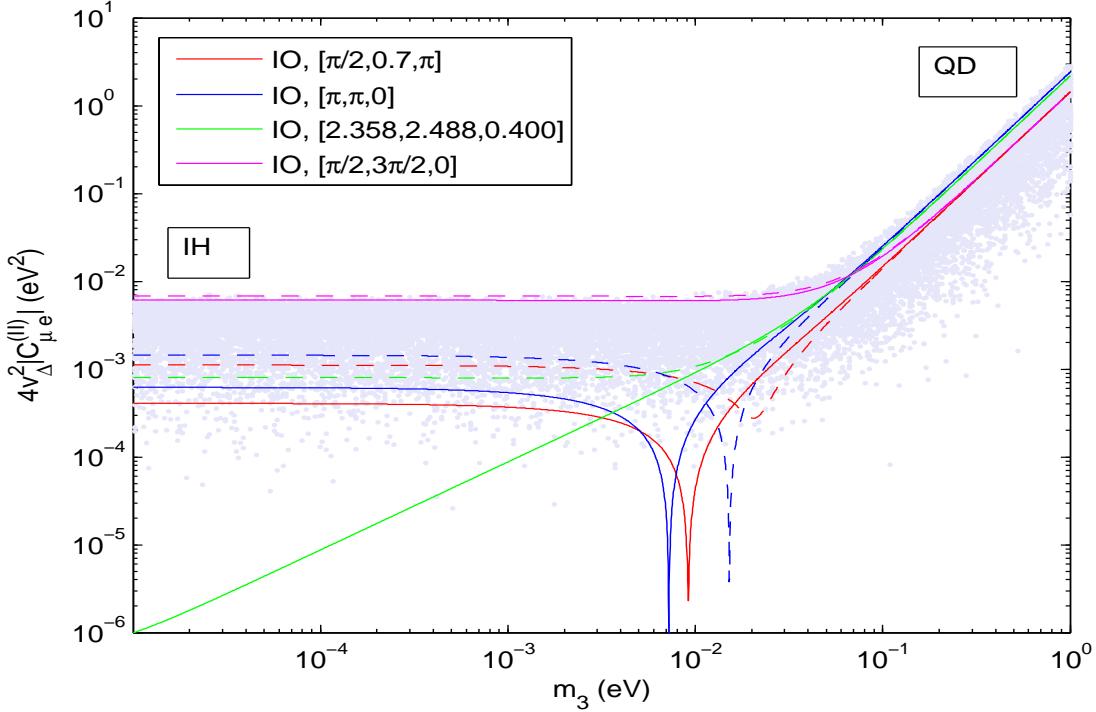


Figure 10: The same as in Fig. 9 in the case of a light neutrino mass spectrum with inverted ordering.

probe values of  $|(\bar{h}^\dagger h)_{e\mu}|$  which are by a factor  $2 \times 10^3$  smaller and could set the limit:

$$|(\bar{h}^\dagger h)_{e\mu}| < 3 \times 10^{-7} \left( \frac{m_{\Delta^+}}{100 \text{ GeV}} \right)^2. \quad (79)$$

## 4 TeV Scale Type III See-Saw Model

We turn in this section to the study of lepton flavour violating processes in type III see-saw [79] extensions of the SM. In the scenarios under discussion, the SM particle content is enlarged by adding  $SU(2)_L$ -triplets of fermions,  $\mathbf{F}_{jR} \equiv (F_{jR}^1, F_{jR}^2, F_{jR}^3)$ ,  $j \geq 2$ , possessing zero weak hypercharge and a mass  $M_k$  at the electroweak scale:  $M_k \approx (100 - 1000)$  GeV. The corresponding interaction and mass terms in the see-saw Lagrangian read:

$$\mathcal{L}_{\text{seesaw}}^{\text{III}} = -\lambda_{\ell j} \bar{\psi}_{\ell L} \boldsymbol{\tau} \tilde{H} \cdot \mathbf{F}_{jR} - \frac{1}{2} (M_R)_{ij} \overline{\mathbf{F}_{iL}^C} \cdot \mathbf{F}_{jR} + \text{h.c.}, \quad (80)$$

where  $\boldsymbol{\tau} \equiv (\tau^1, \tau^2, \tau^3)$ ,  $\tau^a$  being the usual  $SU(2)_L$  generators in the fundamental representation.

It is convenient in the following discussion to work with the charge eigenstates  $F_{jR}^\pm \equiv (F_{jR}^1 \mp iF_{jR}^2)$  and  $F_{jR}^0 \equiv F_{jR}^3$ . Then, the physical states in the above Lagrangian correspond to electrically charged Dirac and neutral Majorana fermions, which are denoted as  $E_j$  and

$N_j$ , respectively:<sup>12</sup>

$$E_j \equiv F_{jR}^- + F_{jL}^{+C} \quad N_j \equiv F_{jL}^{0C} + F_{jR}^0. \quad (81)$$

In the basis in which the charged lepton mass matrix is diagonal, the CC and NC weak interaction Lagrangian of the light Majorana neutrino mass eigenstates  $\chi_j$  read:

$$\mathcal{L}_{CC}^\nu = -\frac{g}{\sqrt{2}} \bar{\ell} \gamma_\alpha ((1-\eta)U)_{\ell i} \chi_{iL} W^\alpha + \text{h.c.}, \quad (82)$$

$$\mathcal{L}_{NC}^\nu = -\frac{g}{2c_w} \overline{\chi_{iL}} \gamma_\alpha (U^\dagger (1+2\eta)U)_{ij} \chi_{jL} Z^\alpha. \quad (83)$$

Similarly to the type I see-saw scenario discussed earlier, the heavy Majorana mass eigenstates  $N_k$  might acquire a sizable coupling to the weak gauge bosons through the mixing with the light Majorana neutrinos:

$$\mathcal{L}_{CC}^N = \frac{g}{2\sqrt{2}} \bar{\ell} \gamma_\alpha (RV)_{\ell k} (1-\gamma_5) N_k W^\alpha + \text{h.c.}, \quad (84)$$

$$\mathcal{L}_{NC}^N = -\frac{g}{4c_w} \overline{\nu}_\ell \gamma_\alpha (RV)_{\ell j} (1-\gamma_5) N_j Z^\alpha + \text{h.c..} \quad (85)$$

In the expressions given above, the non-unitary part of the neutrino mixing matrix, *i.e.* the matrix  $\eta$ , and the matrix  $R$  are defined as in the type I see-saw scenario discussed in Section 2 (see eq. (13)), while  $V$  in this case diagonalizes the symmetric mass matrix  $M_R$  in eq. (80):  $M_R \cong V^* \text{diag}(M_1, M_2, \dots) V^\dagger$ .

The neutrino Yukawa couplings  $\lambda_{\ell j}$  can be partially constrained by low-energy neutrino oscillation data and electroweak precision observable (see, *e.g.* [80, 81]). Notice that, unlike the type I see-saw extension of the Standard Model, now we have flavour changing neutral currents (FCNCs) in the charged lepton sector. The latter are described by the interaction Lagrangian:

$$\mathcal{L}_{NC}^\ell = \frac{g}{2c_w} (\bar{\ell}_L \gamma_\alpha (1-4\eta)_{\ell\ell'} \ell'_L - 2s_w^2 \bar{\ell} \gamma_\alpha \ell) Z^\alpha. \quad (86)$$

Finally,<sup>13</sup> the interactions of the new heavy charged leptons,  $E_j$ , with the weak gauge bosons at leading order in the mixing angle between the heavy and the light mass eigenstates read:

$$\mathcal{L}_{CC}^E = -g \overline{E}_j \gamma_\alpha N_j W^\alpha + g \overline{E}_j \gamma_\alpha (RV)_{\ell j} \nu_{\ell R}^C W^\alpha + \text{h.c.}, \quad (87)$$

$$\mathcal{L}_{NC}^E = g c_w \overline{E}_j \gamma_\alpha E_j Z^\alpha - \frac{g}{2\sqrt{2}c_w} (\bar{\ell} \gamma_\alpha (RV)_{\ell j} (1-\gamma_5) E_j Z^\alpha + \text{h.c.}). \quad (88)$$

## 4.1 The $\mu \rightarrow e\gamma$ Decay

Charged lepton radiative decays receive additional contributions with respect to the scenario with singlet RH neutrinos, due to the presence of new lepton flavour violating interactions

<sup>12</sup>In the following we will denote as  $E_j$  and  $N_j$  the mass eigenstates obtained from the diagonalization of the full charged and neutral lepton mass matrices.

<sup>13</sup>Flavour changing couplings between the charged leptons and the SM Higgs boson  $H$  arise as well in the TeV-scale type III see-saw scenarios [81] which enter at one-loop in the lepton flavour violating processes (see next subsection).

in the low energy effective Lagrangian (see eqs. (86) and (88)). Following the computation reported in [81], we have for the  $\mu \rightarrow e\gamma$  decay branching ratio in the present scenario:

$$\text{BR}(\mu \rightarrow e\gamma) = \frac{3\alpha_{\text{em}}}{32\pi} |T|^2, \quad (89)$$

where the amplitude  $T$  is given by

$$T \cong -2 \left( \frac{13}{3} + \mathcal{C} \right) \eta_{\mu e} + \sum_k (RV)_{ek} (RV)_{\mu k}^* [A(x_k) + B(y_k) + C(z_k)], \quad (90)$$

with  $x_k = (M_k/M_W)^2$ ,  $y_k = (M_k/M_Z)^2$ ,  $z_k = (M_k/M_H)^2$  and  $\mathcal{C} \simeq -6.56$ . The loop functions  $A(x_k)$ ,  $B(y_k)$  and  $C(z_k)$  read [81]:

$$A(x) = \frac{-30 + 153x - 198x^2 + 75x^3 + 18(4-3x)x^2 \log x}{3(x-1)^4}, \quad (91)$$

$$B(y) = \frac{33 - 18y - 45y^2 + 30y^3 + 18(4-3y)y \log y}{3(y-1)^4}, \quad (92)$$

$$C(z) = \frac{-7 + 12z + 3z^2 - 8z^3 + 6(3z-2)z \log z}{3(z-1)^4}. \quad (93)$$

In the simple scenario of degenerate fermion triplets with an overall mass scale  $\overline{M}$  we obtain taking  $M_H = 125$  GeV:

$$T/\eta_{\mu e} \cong 11.6 \text{ (5.2)}, \quad \text{for } \overline{M} = 100 \text{ (1000)} \text{ GeV}. \quad (94)$$

For  $\overline{M} = 100$  (1000) GeV, the current best upper limit on the  $\mu \rightarrow e\gamma$  decay branching ratio obtained in the MEG experiment, eq. (3), implies the bound:

$$|\eta_{\mu e}| < 9 \text{ (20)} \times 10^{-6}, \quad \text{for } \overline{M} = 100 \text{ (1000)} \text{ GeV}. \quad (95)$$

If no positive signal will be observed by the MEG experiment, that is if it results that  $\text{BR}(\mu \rightarrow e\gamma) < 10^{-13}$ , the following upper limit on the non-unitarity lepton flavour violating coupling  $|\eta_{\mu e}|$  can be set:

$$|\eta_{\mu e}| < 2 \text{ (4)} \times 10^{-6}, \quad \text{for } \overline{M} = 100 \text{ (1000)} \text{ GeV}. \quad (96)$$

## 4.2 The $\mu \rightarrow 3e$ and $\mu - e$ Conversion in Nuclei

The effective  $\mu - e - Z$  effective coupling in the Lagrangian (86) provides the dominant contribution (at tree-level) to the  $\mu \rightarrow 3e$  decay rate and the  $\mu - e$  conversion rate in a nucleus. In the case of the first process we have (see, *e.g.*, [80]):

$$\text{BR}(\mu \rightarrow 3e) \simeq 16 |\eta_{\mu e}|^2 \left( 3 \sin^4 \theta_W - 2 \sin^2 \theta_W + \frac{1}{2} \right). \quad (97)$$

Taking into account the experimental upper limit reported in (4), we get the following upper limit on the  $\mu - e$  effective coupling:

$$|\eta_{\mu e}| < 5.6 \times 10^{-7}. \quad (98)$$

which is a stronger constraint with respect to the one derived from the non-observation of the  $\mu \rightarrow e\gamma$  decay (see eqs. (96) and (95)), mediated (at one-loop) by an effective dipole operator.

More stringent constraints on the effective  $\mu - e - Z$  coupling can be obtained using the data from the  $\mu - e$  conversion experiments. Indeed, according to the general parametrisation given in [78] (see also [82, 81]), we have for the  $\mu - e$  conversion ratio in a nucleus  $\mathcal{N}$  with  $N$  neutrons and  $Z$  protons:

$$\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N}) \cong \frac{2 G_F^2}{\Gamma_{\text{capt}}} |\mathcal{C}_{\mu e}|^2 \left| (2 g_{LV(u)} + g_{LV(d)}) V^{(p)} + (g_{LV(u)} + 2 g_{LV(d)}) V^{(n)} \right|^2, \quad (99)$$

where in this case <sup>14</sup>

$$\mathcal{C}_{\mu e} \equiv 4 \eta_{\mu e}, \quad (100)$$

$$V^{(n)} \simeq \frac{N}{Z} V^{(p)}, \quad g_{LV(u)} = 1 - \frac{8}{3} s_w^2 \quad \text{and} \quad g_{LV(d)} = -1 + \frac{4}{3} s_w^2. \quad (101)$$

An upper bound on  $|\eta_{\mu e}|$  can be derived from the present experimental upper limit on the  $\mu - e$  conversion rate in the nucleus of  $^{48}\text{Ti}$ ,  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \lesssim 4.3 \times 10^{-12}$ . From eqs. (99)-(101) we get:

$$|\eta_{\mu e}| \lesssim 2.6 \times 10^{-7}. \quad (102)$$

If in the  $\mu - e$  conversion experiments with  $^{48}\text{Ti}$  the prospective sensitivity to  $\text{CR}(\mu \text{ Ti} \rightarrow e \text{ Ti}) \sim 10^{-18}$  will be reached, these experiments will be able to probe values of  $|\eta_{\mu e}|$  as small as  $|\eta_{\mu e}| \sim 1.3 \times 10^{-10}$ .

## 5 Discussion and Conclusions

We have performed a detailed analysis of charged lepton flavour violating (LFV) processes –  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$  and  $\mu - e$  conversion in nuclei – in the context of see-saw type extensions of the Standard Model, in which the scale of new physics  $\Lambda$  is taken in the TeV range,  $\Lambda \sim (100 - 1000)$  GeV. In this class of models, an effective Majorana mass term for the light left-handed active neutrinos is generated after electroweak symmetry breaking due to the decoupling of additional “heavy” scalar and/or fermion representations. We have analyzed in full generality the phenomenology of the three different and well-known (see-saw) mechanisms of neutrino mass generation, in their minimal formulation: *i*) type I see-saw models, in which the new particle content consist of at least 2 RH neutrinos, which are not charged under the SM gauge group; *ii*) type III see-saw models, where the RH neutrinos are taken in the adjoint representation of  $SU(2)_L$  with zero hypercharge; *iii*) type II see-saw (or Higgs triplet) models, where the scalar sector of the theory is extended with the addition of at least one scalar triplet of  $SU(2)_L$  coupled to charged leptons.

Under certain conditions the couplings of the SM charged leptons with the new fermions and/or scalars are, in principle, sizable enough to allow for their production and detection

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<sup>14</sup>The expression for  $V^{(n)}$  is valid under the assumption of equal proton and neutron number densities in the given nucleus [78]. The numerical value of the nuclear form factors for  $^{48}\text{Ti}$ ,  $^{27}\text{Al}$  and  $^{197}\text{Au}$  is reported in Table 2.

at present collider facilities, LHC included. On the other hand, remarkable indirect tests of such scenarios are also possible in ongoing and future experiments looking for charged lepton flavour violation. Indeed, the flavour structure of the interactions between the SM leptons and the new “heavy” particle states is mainly determined by the requirement of reproducing neutrino oscillation data, in such a way that the unknown parameter space can be expressed in terms of very few quantities. The latter can, therefore, be constrained by the measurement of different LFV observables. Further and complementary information is provided also by experiments which search for lepton number violating phenomena, such as neutrinoless double beta decays of even-even nuclei.

We summarize below the phenomenological implications of a possible observation of the LFV processes given above for each kind of (TeV scale) see-saw extensions of the SM.

**Type I see-saw results.** In this case the  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  decay branching ratios  $\text{BR}(\mu \rightarrow e\gamma)$  and  $\text{BR}(\mu \rightarrow 3e)$ , and the  $\mu - e$  conversion rate in a nucleus  $\mathcal{N}$ ,  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N})$ ,  $\mathcal{N} = \text{Al}, \text{Ti}, \text{Au}$ , can have values close to the existing upper limits and within the sensitivity of the ongoing MEG experiment searching for the  $\mu \rightarrow e\gamma$  decay and the future planned  $\mu - e$  conversion and  $\mu \rightarrow 3e$  decay experiments [29, 30, 31, 32, 33]. The relevant LFV observable in the minimal scenario, with the addition of only two RH neutrinos to the SM particle content, is provided by the quantity  $|(\text{RV})_{\mu 1}^*(\text{RV})_{e 1}|$ , where  $(\text{RV})_{\ell j}$  ( $j = 1, 2$ ) denote the couplings of the fermion singlets to the SM charged leptons (see eqs. (15) and (16)). If the MEG experiment reaches the projected sensitivity and no positive signal will be observed implying that  $\text{BR}(\mu \rightarrow e\gamma) < 10^{-13}$ , there still will be a relatively large interval of values of  $|(\text{RV})_{\mu 1}^*(\text{RV})_{e 1}|$ , as Fig. 2 shows, for which the  $\mu - e$  conversion and  $\mu \rightarrow 3e$  decay are predicted to have observable rates in the planned next generation of experiments.

It follows from the analysis performed by us that as a consequence of an accidental cancellation between the contributions due to the different one loop diagrams in the  $\mu - e$  conversion amplitude, the rate of  $\mu - e$  conversion in Al and Ti or in Au can be strongly suppressed for certain values of the see-saw scale  $M_1$ . As we have seen, this suppression can be efficient either for the conversion in Al and Ti or for the conversion in Au, but not for all the three nuclei, the reason being that the values of  $M_1$  for which it happens in Al and Ti differ significantly from those for which it occurs in Au. In both the cases of Al or Ti and Au, the suppression can be effective only for values of  $M_1$  lying in very narrow intervals (see Figs. 3 and 4).

In the case of IH light neutrino mass spectrum, all the three LFV observables,  $\text{BR}(\mu \rightarrow e\gamma)$ ,  $\text{BR}(\mu \rightarrow 3e)$  and  $\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N})$ , can be strongly suppressed due to the fact that the LFV factor  $|(\text{RV})_{\mu 1}|^2 \propto |U_{\mu 2} + i\sqrt{m_1/m_2}U_{\mu 1}|^2 \cong |U_{\mu 2} + iU_{\mu 1}|^2$ , in the expressions of the three rates can be exceedingly small. This requires a special relation between the Dirac and the Majorana CPV phases  $\delta$  and  $\alpha_{21}$ , as well as between the neutrino mixing angle  $\theta_{13}$  and the phase  $\delta$  (see eq. (34)). For the values of  $\sin \theta_{13}$  from the current  $3\sigma$  allowed interval, eq. (1), one can have  $|U_{\mu 2} + iU_{\mu 1}|^2 \cong 0$  provided  $0 \leq \delta \lesssim 0.7$ . *A priori* it is not clear why the relations between  $\delta$  and  $\alpha_{21}$ , and between  $\delta$  and  $\theta_{13}$ , leading to  $|U_{\mu 2} + iU_{\mu 1}|^2 = 0$ , should take place (although, in general, it might be a consequence of the existence of an approximate symmetry). The suppression under discussion cannot hold if, for instance, it is experimentally established that  $\delta$  is definitely bigger than 1.0. That would be the case if the existing indications [9] that  $\cos \delta < 0$  receive unambiguous confirmation.

We note finally that for  $M_1 \gtrsim 100$  GeV we have:  $\text{BR}(\mu \rightarrow 3e)/\text{BR}(\mu \rightarrow e\gamma) \gtrsim 0.031$ .

Thus, if it is experimentally established that  $\text{BR}(\mu \rightarrow 3e)/\text{BR}(\mu \rightarrow e\gamma)$  is definitely smaller than the quoted lower bound, the model considered with  $M_1 \gtrsim 100$  GeV will be ruled out. Such a result would be consistent also just with a see-saw scale  $M_1 < 100$  GeV.

**Type II see-saw results.** It follows from the results obtained in Section 3 that the predictions for the  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  decay branching ratios, as well as the  $\mu - e$  conversion rate in a nucleus  $\mathcal{N}$ , in the TeV scale type II see-saw scenario considered exhibit, in general, different dependence on the masses of the singly- and doubly-charged Higgs particles  $\Delta^+$  and  $\Delta^{++}$ , which mediate (to leading order) the three processes. For  $m_{\Delta^+} \cong m_{\Delta^{++}} \cong M_\Delta$ , all the three rates are proportional to  $M_\Delta^{-4}$ , i.e., they diminish as the 4th power of the see-saw scale when the latter increases.

The matrix of Yukawa couplings  $h_{\ell\ell'}$  which are responsible for the LFV processes of interest, is directly related to the neutrino Majorana mass matrix and thus to the PMNS neutrino mixing matrix  $U$ . As a consequence,  $\text{BR}(\mu \rightarrow e\gamma)$ ,  $\text{BR}(\mu \rightarrow 3e)$  and  $\text{CR}(\mu\mathcal{N} \rightarrow e\mathcal{N})$  depend, in general, on the neutrino mass and mixing parameters, including the CPV phases in  $U$ .

To be more specific,  $\text{BR}(\mu \rightarrow e\gamma)$  does not depend on the Majorana CPV phases and on  $\min(m_j)$ , and its dependence on the Dirac CPV phase and on the type of neutrino mass spectrum is insignificant. In contrast, both  $\text{BR}(\mu \rightarrow 3e)$  and  $\text{CR}(\mu\mathcal{N} \rightarrow e\mathcal{N})$  exhibit very strong dependence on the type of neutrino mass spectrum and on the values of the Majorana and Dirac CPV phases. As a consequence, the predictions for  $\text{BR}(\mu \rightarrow 3e)$  and  $\text{CR}(\mu\mathcal{N} \rightarrow e\mathcal{N})$  for given  $M_\Delta$  can vary by several orders of magnitude not only when the spectrum changes from NH (IH) to QD as a function of the lightest neutrino mass, but also when one varies only the values of the CPV phases keeping the type of the neutrino mass spectrum fixed. All the three observables under discussion can have values within the sensitivity of the currently running MEG experiment on the  $\mu \rightarrow e\gamma$  decay and the planned future experiments on the  $\mu \rightarrow 3e$  decay and  $\mu - e$  conversion. However, for a given see-saw scale in the range of  $\sim (100 - 1000)$  GeV, the planned experiments on  $\mu - e$  conversion in Al or Ti will provide the most sensitive probe of the LFV Yukawa couplings of the TeV scale type II see-saw model.

**Type III see-saw results.** Unlike the type I see-saw extension of the SM discussed in Section 2, in this scenario we have several – possibly sizable – lepton flavour violating interactions in the low energy effective Lagrangian, due to the higher  $SU(2)_L$  representation of the new fermion fields. In particular, FCNCs arise at tree-level from the non-unitarity of the PMNS matrix (see eq. (86)). Thus, the effective  $\mu - e - Z$  coupling in (86) makes it possible an enhancement of at least two orders of magnitude of the rates of  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$  and  $\mu - e$  conversion with respect to the ones predicted in the type I see-saw scenario, with RH neutrinos taken in the TeV range. Consequently, all the predicted LFV observables may be probed in the related present and future experiments. As in the previous scenarios, the strongest constraint on the flavour structure of this class of models is by far provided by the expected very high sensitivity reach of  $\mu - e$  conversion experiments.

In conclusion, the oncoming combination of data on neutrino oscillations, collider searches and lepton number/flavour violating processes represent an important opportunity to reveal

in the next future the fundamental mechanism at the basis of the generation of neutrino masses as well as the underlying physics beyond the standard theory.

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