MFE I - Homework 3

To upload on Gradescope by ***Wednesday, October 19th, midnight****.*

The below homework specifications will be enforced. If the specifications are not respected, points might be deducted, or the homework assignment may not be accepted for grading. Each exercise is worth 10 points.

Guidelines for your work

- Write your name (as on the roster) and NetID on the first page.
- If you write on paper, use clean and new sheets of paper and take as much space as necessary.
- Number your pages in the top-right corner, such as 1/3, 2/3, 3/3.
- Use a draft and hand in your final version. Make sure that it
  - is clean and legible;
  - has each problem clearly indicated;
  - does not have anything crossed out or contain notes in the margins;
  - has solutions in which all steps are clearly shown and explained, including all steps of the computations;
  - has grammatically correct complete sentences, including punctuation and spelling;
  - is written using correct mathematical terminology and notation;
  - has final answers in exact forms (do not approximate unless otherwise stated).
- You may consult your classmates or other resources (including Campuswire and office hours) for ideas on the problems; however, the solutions you turn in must be in your own words and must reflect your own understanding. Your solutions and write-ups will be checked for textual similarities. You may not copy from, reword, or paraphrase another student's work or any other resource material; such conduct will be treated as a violation of academic integrity. Remember that you will not learn anything by simply copying, rewording or paraphrasing another person's work.

Guidelines for Gradescope

- You can either write on blank or lined paper, use a tablet, or type your assignment in LaTeX.
- Your work should be uploaded as a single PDF file (not as separate photos).
- If you write down on paper, scan your work using a scanner or an app. Make sure that the scans are not blurry and are in portrait mode.
- When you upload this file, match each exercise with the corresponding pages.

Exercise I: False or false (2.1, 2.2)

Explain why each of the following statements is false using words, specific counterexamples, and/or graphs.

1. If f is defined on (a, b) and $f(c) = 0$ at some point $c \in (a, b)$, then $f'(c) = 0$.
2. If
$$f(x) = \begin{cases} 2x + 1 & x \leq 0 \\ x^2 + 2x & x > 0, \end{cases}$$
then $f'(0) = 2$.
3. The tangent line to the graph of f at a point intersects the graph of f at exactly one point.
4. If $f'(x) > g'(x)$ for all $x \in (a, b)$, then $f(x) > g(x)$ for all $x \in (a, b)$.
5. If f is a function and f^2 is differentiable everywhere, then f is differentiable everywhere.

Exercise II: Costs (2.1, 2.2, 2.3)

The cost (in dollars) to produce x hundred toys is given by

$$C(x) = 1000 + 100x - 0.25x^2.$$

Make sure to use correct units in your answers.

1. What are the fixed costs? What can they represent?
2. What is the cost to produce 4000 toys?
3. Find the marginal cost. What is its interpretation?
4. Use the marginal cost to estimate the cost to produce 4100 toys, and compare to the real value.
5. Compute the average cost $A(x)$, the marginal average cost $A'(x)$, and give their interpretation.
6. Compute $A'(40)$ and give its interpretation.

Exercise III: Some economics functions (2.4)

1. Consider a cost function C, and the average cost function

$$A(x) = \frac{C(x)}{x}.$$

   (a) Compute the derivative of A in terms of A, C', x only (there should be no C in your formula).

   (b) If the marginal cost is greater than the average cost, what happens to the average cost? Explain using Question 1, and then explain in your own words why this makes sense.

2. If $h(y)$ denotes an individual's happiness (called "utility" in economics) when having income y, then

$$R(y) = -\frac{yh''(y)}{h'(y)}$$

   is called the coefficient of relative risk aversion, which measures how much an individual is willing to take risks. Compute R for the following utility functions, where a, b, c, d are constants

$$h(y) = (ay + b)^c, \qquad h(y) = \frac{ay + b}{cy + d}.$$

   In the computations, you can assume that no denominator is 0. Simplify as much as possible.

Exercise IV: Functions of functions (2.4, 2.5)

If an appliance is used for a total of N hours, then its value V (in hundreds of dollars) is given by the function

$$V(N) = \left(\frac{2N + 820}{N + 2}\right)^{2/3}.$$

Furthermore, suppose that the total number of hours N that the appliance has been used is based on the number of years t it has been in operation, in such a way that $N(t) = 120t - 16t^{3/2}$.

1. What is the initial value of the appliance? Give an approximation with two significant digits.

2. We stop using the appliance after a total of 1000 hours. How many years does this correspond to? You can use a calculator to find this, but give the exact number. What is the value of the appliance at this point? Give an approximation with two significant digits.

3. Find $V'(N)$ in terms of N only. Simplify as much as possible.

4. Find $N'(t)$ in terms of t.

5. Find the instantaneous rate of change of the value function V in terms of t and $N(t)$.

6. At what rate is the value of the appliance changing when we have been using it for 9 years? Give an approximation with two significant digits. Interpret your result.