Problem Set #1

OSM
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Exercise 1

1. The state variables are the number of barrels B and the price p_t of the barrels.

2. The control variable is the number of barrels the owner chooses to sell (s).

3. The transition equation is B' = B - s where B' is the number of barrels left in future periods, B is the number of barrels in the current period, and s is the number of sales in the current period.

4. The sequence problem of the owner is

$$V_t(B,p) = max \sum_{t=1}^{\infty} (s_t)_{t=1}^{\infty} (\frac{1}{1+r}_{t-1}) p_t s_t$$

The Bellman equation is

$$V(B) = max_{0 \leq s \leq B}(ps) + \frac{1}{1+r} V(B')$$

5. The owner's Euler equation is

$$p_t = \frac{1}{1+r} p_{t+1}$$

because

$$(\frac{1}{1+r}_{t-1}) p_t s_t + \lambda(B - s)$$

and taking this with respect to sales becomes

$$(\frac{1}{1+r}_{t-1}) p_t + \lambda = 0$$

which equals

$$(\frac{1}{1+r}_t) p_{t+1} + \lambda = 0$$

which becomes

$$(\frac{1}{1+r}_{t-1}) p_t = \frac{1}{1+r}_t) p_{t+1}$$

which becomes the owner's Euler equation.

6. If $p_{t+1} = p_t$, the owner would sell all of her barrels of oil during the first period. If $p_{t+1} > p_t$, the owner would never sell any of her barrels of oil - she would continuously holding onto the barrels because selling her barrels during future periods give her more value. The condition on the path of prices necessary for an interior solution is if p_t is between p_{t+1} and $\frac{1}{1+r} p_{t+1}$

Exercise 2

1. The state variables are y_t, k_t, and z_t.
2. The control variables are c_t and i_t.
3. The Bellman Equation that represents this sequence problem is

$$V(k_t) = max_{c_t} u(c_t) + \beta EV(k')$$

   where $c_t = (1-\delta)k_t + z(k_t)^\alpha - k_{t+1}$

4. The solution is in the jupyter notebook.

Exercise 3

1. The Bellman Equation that represents the planner's problem is

$$V(k_t, z_t) = max_{c_t} u(c_t) + \beta EV(k', z')$$

   where $c_t = (1-\delta)k_t + z(k_t)^\alpha - k_{t+1}$.

2. The solution is in the jupyter notebook.

Exercise 4

1. The Bellman Equation that represents this optimal stopping problem is

$$V(w_t) = max\{V^a(w_t), V^d(w_t)\}$$

   where

$$V^a(w_t) = w \sum_{n=0}^{\infty} \beta^n$$

$$V^d(w_t) = b + \beta EV(w')$$

2. The solution is in the jupyter notebook.