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Notes about the model

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1 Breaking down the model equation

We want to describe the number of individuals at time t. We first recall our notation.

Subscripts We refer to the sexes by their initials ('m', 'f' or 'h'). We use "fert" to denote the event of becoming fertile, and "die" for the death of an individual. Thus an individual of sex s is born with age zero, becomes fertile at age $a_{s,\text{fert}}$, and dies at age $a_{s,\text{die}}$.

Birth rates We write $b_{\text{h}\to\text{f}}(a)$ to denote the rate of birth of females from hermaphrodites aged a. In general, we denote by $b_{sp\to so}(a)$ the rate of birth of offspring of sex so from a parent of sex sp and age a. We obtained these rates experimentally (see section YYYY).

Deaths We assume all individuals have the same adult lifespan.

The number of individuals of age a at time t is denoted $n_s(a,t)$, where s is a sex (either 'm', 'f' or 'h').
Equation (1) below

$$n_s(a,t+1) = \begin{cases} \displaystyle\sum_{i=a_{\text{f,fert}}}^{a_{\text{f,die}}} n_{\text{f}}(t,i)\cdot b_{\text{f}\to s}(i) + \sum_{j=a_{\text{h,fert}}}^{a_{\text{h,die}}} n_{\text{h}}(t,j)\cdot b_{\text{h}\to s}(j) & \text{if } a=0 \text{ (eggs produced)} \\ n_s(a-1,t) & \text{if } 0<a<a_{s,\text{die}} \text{ (ageing individuals)} \\ 0 & \text{if } a\geq a_{s,\text{die}} \text{ (dead individuals)} \end{cases} \tag{1}$$

is a (very terse) way of describing the changes on the population at any step.

Eggs are laid by fertile mothers. Example: how many *male* eggs are laid from time t to time $t+1$?

This is precisely the number of males of age zero at time $t+1$, which we denote $n_{\text{m}}(0,t+1)$.

To obtain this quantity, we break the total in two parts: the eggs laid by females and the eggs laid by hermaphrodites.

The fertile ages for *females* range from $a_{\text{f,fert}}$ to $a_{\text{f,die}}$. In a time-unit, a female mother of age i, (between $a_{\text{f,fert}}$ and $a_{\text{f,die}}$) produces

- $b_{\text{f}\to\text{m}}(i)$ male eggs,
- $b_{\text{f}\to\text{f}}(i)$ female eggs, and
- $b_{\text{f}\to\text{h}}(i)$ hermaphrodite eggs.

To know how many *male* eggs were laid by *all* female mothers of age i at the time t, we multiply the number of mothers at that time and age (which is $n_{\text{f}}(i,t)$) by the number of male eggs each one lays (which is $b_{\text{f}\to\text{m}}(i)$). That is $n_{\text{f}}(i,t)\cdot b_{\text{f}\to\text{m}}(i)$. Note that this considers only female mothers of a particular age (i).

The total number of male eggs laid by females is the sum of the number male eggs laid by all females in fertile age: that is, we sum $n_{\text{f}}(i,t)\cdot b_{\text{f}\to\text{m}}(i)$ for all ages (values of i) between $a_{\text{f,fert}}$ and $a_{\text{f,die}}$. In symbols,

$$\#\text{ of male eggs laid by females at time } t = \sum_{i=a_{\text{f,fert}}}^{a_{f,die}} n_{\text{f}}(i,t)\cdot b_{\text{f}\to\text{m}}(i),$$

which is read "the number of male eggs laid by females at time t is the sum, for i ranging from $a_{\text{f,fert}}$ to $a_{\text{f,die}}$, of $n_{\text{f}}(i,t)\cdot b_{\text{f}\to\text{m}}(i)$."

Since hermaphrodites also lay male eggs, we have a similar expression to account for their contribution

$$\#\text{ of male eggs laid by hermaphrodites at time } t = \sum_{j=a_{\text{h,fert}}}^{a_{h,die}} n_{\text{h}}(j,t)\cdot b_{\text{h}\to\text{m}}(j),$$

which is read "the number of male eggs laid by hermaphrodites at time t is the sum, for j ranging from $a_{\mathrm{h,fert}}$ to $a_{\mathrm{h,die}}$, of $n_{\mathrm{h}}(j,t) \cdot b_{\mathrm{h}\to\mathrm{m}}(j)$." (Note we used letter j for the hermaphrodite's age; this is to avoid confusion.)

We are finally able to assemble the total of *male* eggs at time $t+1$. We aggregate the contributions of female and hermaphrodite mothers which are in some fertile age:

$$n_{\mathrm{m}}(0,t+1) = \underbrace{\sum_{i=a_{\mathrm{f,fert}}}^{a_{f,die}} n_{\mathrm{f}}(i,t) \cdot b_{\mathrm{f}\to\mathrm{m}}(i)}_{\text{eggs from females}} + \underbrace{\sum_{j=a_{\mathrm{h,fert}}}^{a_{h,die}} n_{\mathrm{h}}(j,t) \cdot b_{\mathrm{h}\to\mathrm{m}}(j)}_{\text{eggs from females}}.$$

which can be read as "the number of 0-aged males at time $t+1$ is the sum of the male eggs laid by fertile females and the sum of the male eggs laid by fertile hermaphrodites."

We make the expression more general replacing 'm' for s, where s is the egg's sex. Finally, the number of zero-aged individuals of sex s at time $t+1$ is

$$n_s(0,t+1) = \sum_{i=a_{\mathrm{f,fert}}}^{a_{f,die}} n_{\mathrm{f}}(i,t) \cdot b_{\mathrm{f}\to s}(i) + \sum_{j=a_{\mathrm{h,fert}}}^{a_{h,die}} n_{\mathrm{h}}(j,t) \cdot b_{\mathrm{h}\to s}(j).$$

Individuals age. We suppose the only cause of death is old age: a female can only attain age a at time $t+1$ if $a < a_{\mathrm{f,die}}$. So if a is a "possible" age, the total number of females of age a at time $t+1$ (which we denote by $n_{\mathrm{f}}(a,t+1)$) is precisely the number of females which had age $a-1$ at time t. In symbols

$$n_{\mathrm{f}}(a,t+1) = n_{\mathrm{f}}(a-1,t) \quad \text{if } a < a_{\mathrm{f,die}}.$$

This remains true if we replace f by the "generic" sex s:

$$n_s(a,t+1) = n_s(a-1,t) \quad \text{if } a < a_{s,\mathrm{die}}.$$

One final note: it is impossible to reach age 0 by becoming older (individuals reach that age by being *born*), so we also require $0 < a$ in equation (1).

Deaths. For any sex s, no individual is alive at age $a_{s,\mathrm{die}}$. We state that by writing that at time $t+1$, the number of s-sexed individuals of age $a_{s,\mathrm{die}}$ (or older) is zero. In symbols

$$n_s(a,t+1) = 0 \quad \text{if } a \geq a_{s,\mathrm{die}}.$$