Lab 1: ATLAS Data Analysis

Investigations of the mass measurements of the Z^0 boson

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I. Introduction

The A Toroidal Lhc Apparatus (ATLAS) experiment at CERN in Geneva smashes high energy protons to create byproducts like the Z^0-boson. This particle is the neutral carrier of the weak force and facilitates many nuclear interactions in the universe. About 10% of the time, the Z^0 decays into a pair of charged leptons as a particle-antiparticle pair. Because charge and matter-energy are conserved, the two particles have opposite charges and their total energy must sum to the mass of the Z^0. Thus, the energy of double-lepton events in the detector is expected to peak at the mass of the Z^0. In light of the measurements made at ATLAS, my colleagues and I have conducted thorough investigations of the invariant mass distribution and fit the data to the Breit-Wigner distribution, which models the decay distribution at a reconstructed mass, to calculate the mass of the Z^0 boson. It is crucial that we understand how to take measurements of particle properties from the ATLAS detector and analyze them to estimate invariant masses. I have used total energy, transverse-momentum, pseudorapidity, and azimuthal angle of two leptons to calculate the invariant mass. Then, I used that distribution to find the Z^0 mass and the expected width parameter Γ_{exp}. I statistically analyzed the data using a chi-squared test, p-value, and 2D parameter scan to find agreement between the measured mass-distribution and the fit.

II. The Invariant Mass Distribution and its Fit

The first calculation my colleagues and I modeled was the invariant mass distribution of a particle which decayed to produce a lepton pair. I used the four measurements that came out of the ATLAS detector: total energy E, transverse-momentum p_T, pseudorapidity η, and azimuthal angle ϕ. All of these values give the four momentum of the particle (1):

$$p = (E, p_x, p_y, p_z) \quad (1)$$

where p_x, p_y, and p_z are components of lepton momentum. I calculated the components of momentum with the following equations (2), where the speed of light $c = 1$ using natural units:

$$p_x = p_T cos(\phi),\ p_y = p_T sin(\phi),\ p_z = p_T sinh(\eta) \quad (2)$$

A particle's invariant mass is the difference between the three-momentum and the energy, given in the following equation (3):

$$M = \sqrt{E^2 - (p_x^2 + p_y^2 + p_z^2)} \quad (3)$$

where M is the invariant mass with units of GeV.

To calculate the total momentum of the two-particle system, I summed the four momenta using the following equation (4):

$$p_{tot} = p_1 + p_2 \quad (4)$$

where p_1 and p_2 are the respective four momenta of each particle.

My colleagues and I collected data on the transverse momenta, pseudorapidity, azimuthal angles, and total energies of 5000 lepton pairs. For each lepton pair, we used Eq.2 to calculate the components of p_T, and then used Eq.4 to calculate the combined four-momenta. Finally, we used Eq.3 and the combined four-momenta to calculate the invariant mass of the hypothetical particle that decayed to produce that lepton pair. Repeating this process for all 5000 pairs, we found a distribution of invariant masses, represented as a histogram. Approximating this process as a Poisson counting experiment, we calculated the error on the number of events in each bin N with the following equation (5):

$$\sigma = \sqrt{N} \quad (5)$$

Next, my colleagues and I used the Breit-Wigner equation to model the distribution of decays D at a reconstructed mass m using the following equation (6):

$$D(m; m_0, \Gamma) = \frac{1}{\pi}\frac{\Gamma/2}{(m-m_0)^2+(\Gamma/2)^2} \quad (6)$$

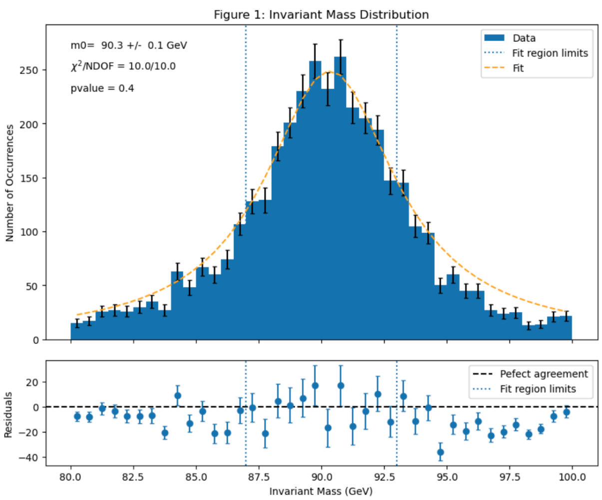

Fig.1: Invariant mass distribution of Z^0 with histogram from 80 to 100 GeV with 41 bins, and Breit-Wigner fit from 87 to 93 GeV

where m_0 is the true rest-mass of the Z^0 in GeV, and Γ is a width parameter in GeV. In the real world, the Heisenberg uncertainty principle relates the lifetime of the particle to the true width parameter Γ_0. However, in the detector, we are restrained by experimental uncertainties, so the width we measure will be greater than the true width parameter, $\Gamma_{exp} > \Gamma_0$.

To fit the mass-distribution with the Breit-Wigner function, I used the middle 47.6% of the data with bins centered between 87 to 93 GeV, and I added an overall normalization to fit the mass-distribution to $\frac{5000}{2} \times D$. This fit can be seen in Figure 1.

The fitted mass of the Z^0 is found to be $m_0 = 90.3$ GeV, the fitted uncertainty on the Z^0 mass is 0.1 GeV, the chi-square is 10.0, the number of degrees of freedom is 10, and the p-value is 0.4. To calculate the best fit mass m_0, I used the Breit-Wigner function and found the best fitting parameter for m_0 that gives this fit. To calculate the uncertainties of the mass of Z^0 and the expected width parameter, I used the covariance matrix from the Breit-Wigner fit, which takes into account the relationship between m_0 and Γ in Eq.6 and their respective uncertainties. A p-value of 0.4 means that if we assume the null hypothesis is true, then 40% of the time we will get the observed experimental result. This implies that the mass-distribution is consistent with the Breit-Wigner equation. In summary, using the Breit-Wigner function, I found $m_0 = 90.3 \pm 0.1$ GeV, and $\Gamma_{exp} = 6.4 \pm 0.2$ GeV.

III. The 2D Parameter Scan

The next analysis my colleagues and I performed was a 2D chi-square scan of the mass-width parameter space. Because the Breit-Wigner fit is a 2 parameter fit, m_0 and Γ_{exp} can not be determined independently, so we used a 2D chi-square scan to visualize the joint probability space as seen in Figure 2.

In this plot, we scanned mass values from 89 to 91 GeV and width values from 5 to 8 GeV. Using this, we performed a chi-squared test of Eq.6 with the scanned mass and width values. We then made a filled contour plot of $\Delta\square^2 = \square^2 - \square^2_{min}$ to make a $\Delta\square^2$ map. I found the $\Delta\square^2$ corresponding to the 1σ and 3σ confidence levels when there are two free parameters in the fit. Repeated experiments would yield results of m_0 and Γ_{exp} that fall within the 1σ enclosed range about 68% of the time, and within the 3σ enclosed range about 99% of the time.I placed a

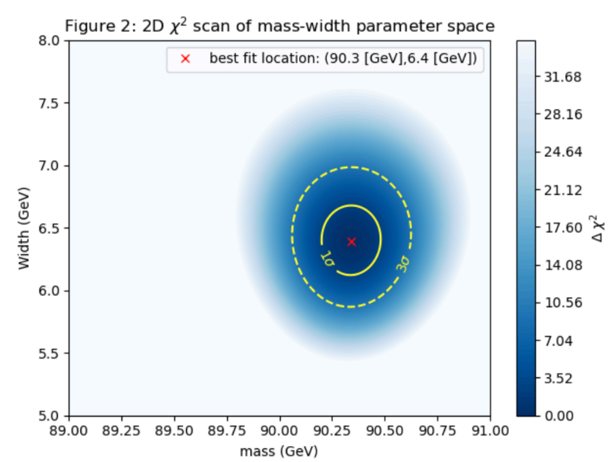

Fig. 2: 2D chi-square scan of mass-width parameter space, with a mass scan from 89 to 91 GeV, width scan from 5 to 8 GeV, and 300 bins along each dimension, plotted with 1σ=2.3 and 3σ=9.21 confidence levels. For ease of visibility, the filled contour plot was clipped at 35 units

cross at the best fit location from Section II, where m_0 = 90.3 GeV, and Γ_{exp} = 6.4 GeV.

IV. Discussion and Future Work

Through these calculations, I was able to find the best mass fit for the Z^0 boson by fitting the invariant mass distribution–of a particle that decayed to produce the lepton pairs measured–with the Breit-Wigner function. I also conducted a 2D chi-square scan of the mass-width parameter space to visually represent how the mass and width influenced each other in the joint probability space. I found that the fitted mass of the Z^0 is $m_0 = 90.3 \pm 0.1$ GeV, and the best fit width parameter is $\Gamma_{exp} = 6.4 \pm 0.2$ GeV.

However, my colleagues and I used some assumptions and simplifications when calculating these values. When getting our data from the ATLAS detector, we did not take into account any experimental or systematic uncertainties that occurred because of the measurement devices, which would affect the fit we used on our mass-distribution data. In the future, we hope to account for these uncertainties. We also did not take into account the energy resolution of the ATLAS detector. This affects how precisely we can measure the best fit mass and width parameter, so what we calculate might not be the exact true mass of Z^0 and the true width parameter Γ_0. We also approximated the invariant mass distribution as a Poisson counting experiment, which led us to assume the error on the number of events as Eq.5. Furthermore, we represented the mass-distribution using only 41 bins and applied the Breit-Wigner fit to a specific range, which led to less precisely calculated values for m_0 and Γ_0.

To conclude our analysis of the measured Z^0, we compared it to the latest accepted value from the Particle Data Group (PDG) of $m_{Z0} = 91.1880 \pm 0.0020$ GeV, where using natural units, the speed of light is taken to be 1. Using the uncertainty in the difference, I calculated whether the difference of the two values was within 2σ of 0. Using the following equation (7), I calculated the difference of the uncertainty in the difference and propagated errors appropriately:

$$diff = m_0 - m_{accepted}\ ,\ \sigma_{diff} = \sqrt{\sigma_{m_0}^{\ 2} + \sigma_{m_{accepted}}^{\ 2}} \quad (7)$$

I calculated a value of 9.1 for the ratio in Eq.7, which implies that the two values do not agree with each other. This is because the difference between the two values is 9σ away from 0, which means that the values are statistically different and quantitatively do not agree with each other. Thus, the measured value of Z^0 does not agree with the latest accepted values from the PDG.

In the future, we would like to create particle accelerators with more precise energy resolutions, so we can detect smaller peaks and energies, which can lead to detecting more elusive particles. Additionally, we can also iterate our current model with the Breit-Weigner peak by including experimental and systematic uncertainties into the mass-distribution to get a more accurate fit. We can try modeling the mass-distribution without the assumption of Eq.5. We can also use the Breit-Weigner fit over the whole mass-distribution instead of only fitting the masses between 80 to 100 GeV, change the amount of bins we use for a more precise result, and make the normalization of the fit more accurate instead of using 2500 as the normalization. Finally, we can also try using different variations of the Breit-Wigner fit that account for relativistic corrections or using other similar fits to find different estimations of the mass of the Z^0 boson.

My colleagues and I will continue iterating our models of the invariant mass distribution with added uncertainties and parameters to be as accurate as possible to contribute to the larger ATLAS experiment.