Lab 1: The Apollo Mission

Investigations of the Apollo Command Module and Saturn V Capsule Rocket

Written By: Sophiya Mehra

I. Introduction

In preparation for sending a manned flight to the Moon as part of the Apollo program, my team and I have conducted thorough investigations on physical influences that might affect the performance of the rocket in the Earth-Moon system. It is crucial that we understand what potentials and forces are at play as well as the specifics of the new Saturn V Rocket before launching. We have quantified the gravitational potential and force in the Earth-Moon system which will allow for charting the most optimal trajectory to the Moon. We have also quantified the projected performance of the altitude and burn time of the Saturn V Stage 1 Rocket, with very optimistic results.

II. The gravitational potential of the Earth-Moon system

The first calculation my team modeled was the gravitational potential of the Earth-Moon system. Gravitational potential is a scalar quantity associated with how much work per unit mass it would take to move an object from one point to another in a gravitational field. The gravitational potential at a distance r from any mass M is given by the following equation:

$$\phi(r) = -\frac{GM}{r} \quad (1)$$

where G is the gravitational constant of $6.67 * 10^{-11}$ m^3 kg^{-1} s^{-2}.

My team modeled the gravitational potential of the Earth-Moon system by adding the separate gravitational potentials of the Earth and the Moon, using masses of $M_{earth} = 5.9*10^{24}$ kg and $M_{moon} = 7.3*10^{22}$ kg respectively. We also computed the distance from an arbitrary point to the Earth and the Moon by placing the Earth at (0,0) meters and the moon at $(2.69*10^8, 2.69*10^8)$ meters, and put those values into the above equation 1 to create a plot of total gravitational potential as a function of distance from the Earth in Figure 1 and a contour plot of the same function in Figure 2.

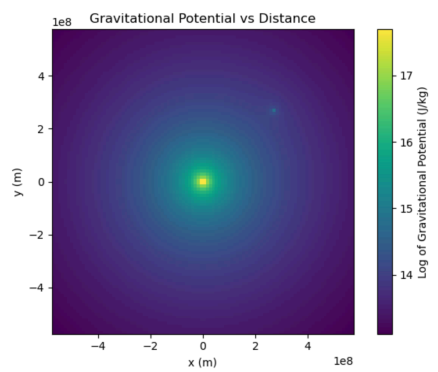

Figure 1: Magnitude of gravitational potential of Earth-Moon system with Earth at the (0,0) and Moon at (2.69e8,2.69e8)

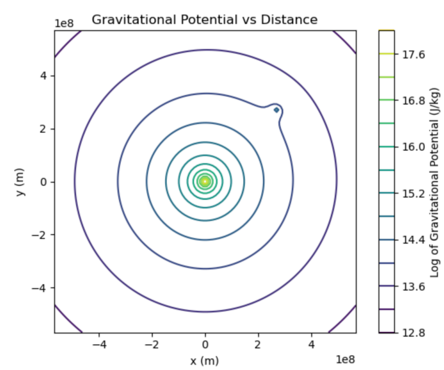

Figure 2: Contour plot of magnitude of gravitational potential of Earth-Moon system as function of distance from Earth

Because Earth is very massive compared to the moon, its gravitational potential is much more influential to rockets in the Earth-Moon system by almost 100 times because potential is directly proportional to mass. This means that the rocket will be pulled towards the Earth more than toward the Moon except for when it is very close to the Moon. The contour plot helps us recognize areas with equal gravitational potential, which means there is no extra work required to move along the contours, which can be useful for future trajectory and fuel planning.

III. The gravitational force of the Earth-Moon system

The next calculation my team modeled was the gravitational force a rocket would feel in the Earth-Moon system. Gravitational force is the attractive force between objects with mass. The gravitational force $\vec{F}$ that a mass M_1 exerts on mass m_2 is given by the following equation:

$$\vec{F} = -\frac{GM_1 m_2}{\left|\vec{r}_{21}\right|}\widehat{r_{21}} \quad (2)$$

where $\vec{r}_{21}$ is the displacement vector from M_1 to m_2.

To find the gravitational force in the Earth-Moon system, my team used M_1 as the mass of the Earth and Moon separately and m_2 as the mass of the Apollo Command Module, which is 5500 kg, before adding them together to get the total force on the Apollo Command Module. In Figure 3, we created a vector field to represent the direction that the gravitational force would pull the rocket.

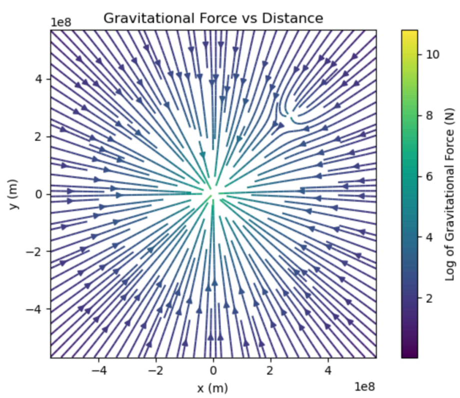

Figure 3: Vector field plot of gravitational force vs distance from Earth's center of the Earth-Moon system

As is evident from Figure 3, most of the gravitational force felt by the Apollo Command Module is towards the Earth due to the mass difference between the Earth and the Moon. However, very close to the Moon, the field lines are distorted due to the gravitational influence of the Moon, meaning the closer the rocket is to the Moon, it will feel a pull towards the Moon. This is imperative when calculating trajectories to find the optimal path to reach the Moon.

IV. Projected performance of the Saturn V Stage 1

The last calculation my team modeled was the specifics of the trajectory and burn time of the Saturn V Stage 1 rocket. One of the basic laws of physics that governs rockets is the conservation of momentum, which means that the backwards ejection of fuel pushes the rocket forwards. My team calculated the change in the rocket's velocity Δv as a function of time using the Tsiolkovsky rocket equation:

$$\Delta v(t) = v_e ln(\frac{m_0}{m(t)}) - gt \quad (3)$$

where we used a fuel exhaust velocity of $v_e = 2.4*10^6$ m/s, wet mass of $m_0 = 2.8*10^6$ kg, dry mass of $m_f = 7.5*10^5$ kg, burn rate of $m = 1.3*10^4$ kg/s, and gravitational acceleration of $g = 9.81$ m/s^2. In equation 3, $m(t) = m_0 - mt$ is the mass of the rocket at time t.

First, to find the total burn time of the rocket, we used the equation:

$$T = \frac{m_0 - m_f}{m} \quad (4)$$

where T is the total burn time in seconds. We found the burn time to be approximately 157.7 seconds.

Next, we calculated the altitude of the rocket at burnout when it used up all of its fuel supply using the following equation:

$$h = \int_0^T \Delta v(t) dt \quad (5)$$

where h is the altitude in meters. We calculated the altitude at burnout to be 74.09 km. This is important in order to know if the rocket is carrying enough fuel to escape Earth's gravity and make it into space.

V. Discussion and Future Work

Through these calculations, my team is optimistic about the chances of getting a man on the Moon. However, we must continue iterating on our models to make sure that all approximations are taken into account to avoid any catastrophic events. While creating our models in Section II and III, my team approximated the Earth and Moon as point masses, which works at great distances but not immediately next to the surface where inhomogeneities can cause differences in gravitational potentials and forces. Newer models should take these issues into consideration. Additionally, the Tsiolkovsky rocket equation had many simplifying assumptions that we must understand if we are to continue using that equation to model our rocket. Finally, we assumed the burn rate m to be constant although that might not be the case for real rockets.

We were recently informed about the statistics of the prototype of Saturn V, where the burn time was 160 sec and altitude reached was 70 km. The burn time is consistent with our calculated value of 157 sec. However, our calculated altitude was about 4 km greater than the prototypes. This could be from neglecting the drag of air resistance, which could slow the rocket down and not allow it to reach a higher altitude. Thus, future models should include air resistance as part of their calculations.

My team along with others will continue iterating our models of gravitational potential, force, and performance of the Saturn V Stage 1 rocket to be as accurate as possible to contribute to the larger Apollo mission of safely landing a manned mission on the Moon.