Where Am I on the Map?

A Comparison between the Mercator, azimuthal and conic map projection

By: Sophy Li

Candidate # 000130-0019

Math 20/30 IB Internal Assessment

Teacher: Ms. Chung

Submitted March 17, 2016

Table of Contents

How I Came Up With This Idea

While doing homework one day, I came across a video on YouTube. This video is comedic, and it questioned why a map of the world today is portrayed as it is. Such questions raised were, why is Africa 14 times larger than Greenland, but on some maps it is portrayed the same size as Greenland? Why is Canada positioned at the top of the map when it could have been drawn at the bottom? I was immediately interested, and decided to investigate further. Watching this video made me wonder if the maps I had grown up with were incorrect, and where I really live on Earth is not where I think it is. This video inspired me to dig deeper into the distortions of map projections, which became the topic I chose for my Math internal assessment.

A map projection is a systematic representation of all or part of the surface of a round body (a globe), onto a plane (a map) (2015). The problem that arises is the accuracy of transferring a 3D object into a 2D image. Often proportions are skewed. It is up to the cartographer to choose a map projection that will most accurately depict the area of the world that is projected. Figure 1 illustrates 3 types of 2D map projections: Mercator, azimuthal and conic. It is clear in Figure 1, that each projection has favored areas in terms of accuracy when portrayed onto a map.

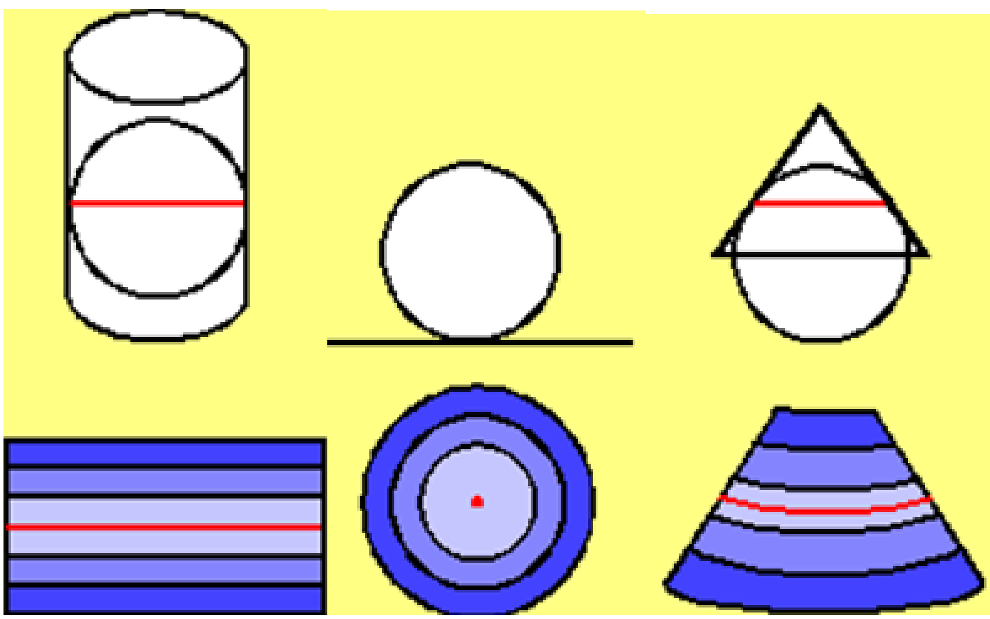

Figure 1. Areas of Distortion on a Mercator, azimuthal and conic map projection. (Geo.hunter.cuny.edu, 2015) From left to right: Mercator, azimuthal and conic. Red lines represent areas projected most accurately and black lines represent areas of equal distortion. Blue shading demonstrates degree of distortion, the darkest being most distorted.

Theoretical Accuracy

The 3 map projections in Figure 1 projects Earth's 3D shape onto a flat surface. To envision this, imagine the map as a sheet of flat paper.

On a Mercator map, the piece of paper is wrapped around the equator of the globe, forming a cylindrical shape and then opened up. The least distortion occurs where the sheet of paper was tangent to the globe, and the most distortion occurs at the North and South poles (farthest away from the equator).

Similarly, an azimuthal map projection is most accurate at the point where the piece of paper is tangent to the sphere. If for example, the paper is tangent to the sphere at the North Pole, the North Pole will receive least distortion while areas propagating farther and farther from the North Pole loses accuracy. Because there are many different types of azimuthal projections, this IA will focus on the azimuthal equidistant projection. (Figure 2) Greatest distortion on this projection occurs at the equator, near the edges of the map. The equidistant projection shows maximum latitude of 60˚S.

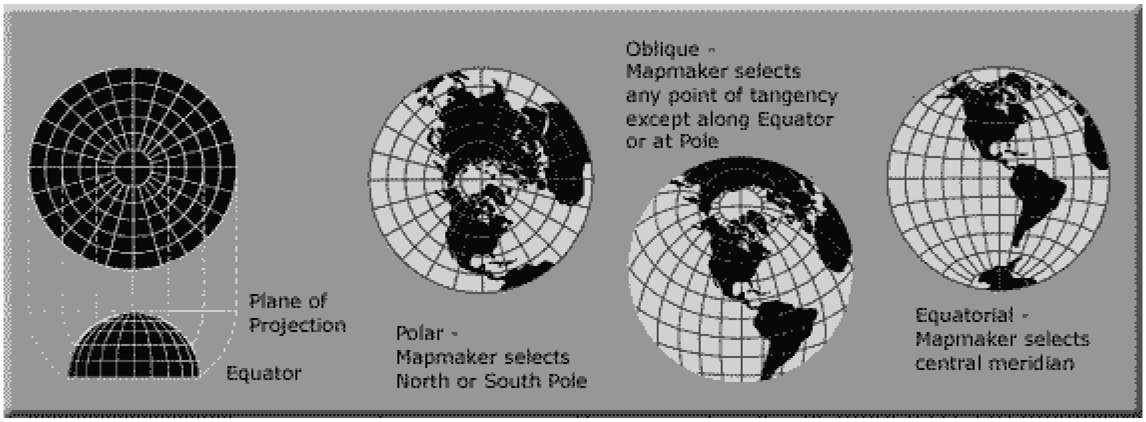

Figure 2. Equidistant Azimuthal Projection (Eastern Region Geography, 2016)

The tangency of a conic map projection exists at mid-latitudes (45 ˚), since this area is where the map is tangent to the globe. Such countries projected in mid-latitudes include the United States of America and Australia. Because there are many types of conic projections, this IA will narrow down to the Lambert Conformal projection illustrated in Figure 3.

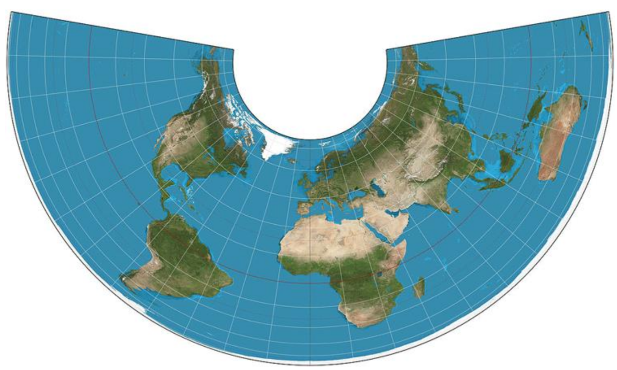

Figure 3. Lambert Conformal Map Projection (Geolounge, 2016)

Purpose

After investigating a bit further, I found each map design had its specific use. The Mercator projection for example is often used for travel and world maps. I realized that the function of each map in our lives today is dependent upon the limitations (distortions) of each projection, since it is the limitation that gives each map its uniqueness. In response, I decided that the purpose of my IA is to prove the theoretical accuracies of each map projection illustrated in Figure 1 through modelling mathematical formulae. This IA allows me to prove theoretical distortions, and further explain why they occur. More importantly, I will able to bring mathematics outside the classroom, and apply it in a real world scenario.

Modelling

I have chosen 14 pairs of cities across the Northern Hemisphere as the sample of my modelling process. Using a standard formula, I will calculate the theoretical distance between each pair of cities. Then using other formulae specific to each map projection, I will calculate the experimental distance of each pair of cities. This will be followed by a percent error calculation between the standard and experimental distances for each city of each map projection. Doing so, I will be able to analyze which areas of each map projection were the most accurate, and make a conclusion whether or not my calculated distortions matched those of the theoretical distortions. The following pairs of cities are shown below:

1. Calgary (Canada) and Las Vegas (United States)
2. Paris (France) and Berlin (Germany)
3. Rangoon (Myanmar) and Bangkok (Thailand)
4. Calgary (Canada), Vancouver (Canada)
5. Montreal (Canada), Quebec City (Canada)
6. Phoenix (United States), El Paso City (United States)
7. Ottawa (Canada), Toronto (Canada)
8. Berlin (Germany), Vienna (Austria)
9. Bangkok (Thailand), Singapore
10. Atlanta (United States), Miami (United States)
11. Bangalore (India), Colombo (Sri Lanka)
12. Tokyo (Japan), Seoul (Korea)
13. Oslo (Norway), Helsinki (Finland)
14. Paris (France), Budapest (Hungary)

Inferring that each data set will have unique strength and weaknesses in terms of accuracy when projected onto a Mercator, azimuthal and conic map projection, a percent error calculation is necessary for comparison. The formula is represented below:

$$\text{Percentage error} = \left|\frac{(\text{experimental value} - \text{theoretical value})}{\text{theoretical value}}\right| \text{ x } 100\%$$

To suite the purpose of my IA, the percentage error equation is modified:

$$\boxed{\text{Percentage error} = \left|\frac{(\text{map projected distance} - \text{standard distance})}{\text{standard distance}}\right| \text{ x } 100\%}$$

Justifications for the Model

1. The 14 pairs of cities are considered pool data, increasing the accuracy of results.
2. The pairs of cities chosen are positioned vertically, horizontally or oblique on a plane, while scattered across the Northern Hemisphere. This range provides variety to the IA, and therefore accommodates and accounts for the different areas of distortion suggested.
3. The cities I chose are located in the Northern Hemisphere of Earth for 2 reasons; firstly, most of the cities in the world are located in the Northern Hemisphere. Secondly, according to Figure 1 there is not a noticeable difference in distortion between the Northern/ Southern Hemisphere regions of each map projection.

Limitations

1. Each pair of cities is very close together, with a separation distance no greater than 3000 km. This was intended because many of the formulae used in this IA are limited to short distances.
2. The IA cannot propose a “best map projection” since each projection has its own strengths and weaknesses.
3. Conclusions would be drawn by SAMPLING from the pool of data. (For example, only the top 3 pairs of cities will determine the general trend in accuracy of a map projection.)

Assumptions

1. Earth is perfectly spherical. Using a spherical model instead of an elliptical model will give error up to 0.3%. (Chris Veness, 2015)
2. Greenwich is at the centre of the map with a longitude of 0.0015˚ W.
3. North Pole has latitude of 90.0000˚N.
4. All final values are rounded to 4 significant figures behind the decimal. This will ensure consistency with the original latitude and longitude values.
5. Since the formulae I adapted into my IA are theoretical and proven, it is assumed any error lies within the projections themselves.

NOTE: the following sample calculations adapting formulae for standard, Mercator, azimuthal and conic projections is to find the distance between Calgary and Las Vegas.

Standard Map Projection

Sample Calculation: Distance Calculation between Calgary and Las Vegas

The greatest assumption in this IA is that Earth is perfectly spherical. This assumption makes calculations easier to compare, and will yield an error at most 0.3%. Considering Earth in its real form, the distance between Calgary and Las Vegas is 1663.763 km, but since this IA assumes that Earth is spherical, a separate calculation attends to this assumption. This is demonstrated in the following steps.

The coordinates of downtown Calgary is 51.05234786 N˚ (Latitude), 114.0888547˚ W (Longitude). The coordinates of Las Vegas, US is 36.114647˚ N (Latitude), 115.172813˚ W (Longitude). (Viewphotos.org, 2015)

NOTE: when using the latitude and longitude values in the formula, north is positive, south is negative, east is positive and west is negative.

The following equation is a modified Pythagoras’ Theorem, to calculate the distance between 2 points on a standard projection. The math to prove this formula is beyond the scope of this IA, but the formula will be briefly explained. The Pythagoras formula allows X and Y coordinates from the 3D Earth model to be transformed onto a 2D plane. This formula is limited in that it can only be used for distances that are close together. The formulae to be used for Mercator, azimuthal and conic are all extensions of the Pythagoras’ Theorem.

$$x = \Delta\lambda \cdot \cos \varphi_m$$
$$y = \Delta\varphi$$
$$d = R \cdot \sqrt{x^2 + y^2}$$

(Chris Veness, 2016)

Where:
φ_m = maximum latitude
$\Delta\varphi$ = change in latitude
$\Delta\lambda$= change in longitude
R= Earth radius= 6, 371 km

<u>Calculating Latitudes:</u>
φ_1 = Calgary latitude= 51.05234786˚
Converted to radians: $\frac{(\varphi 1 \text{ x } \boldsymbol{\pi})}{180}$
$= \frac{(51.05234786^\circ \text{ x } \boldsymbol{\pi})}{180}$
= 0.891031561

φ_2= Las Vegas latitude= 36.114647˚
Converted to radians: $\frac{(\varphi 2 \text{ x } \boldsymbol{\pi})}{180}$
$= \frac{(36.114647^\circ \text{ x } \boldsymbol{\pi})}{180}$
= 0.63031949

<u>Calculating Longitudes:</u>
$\lambda_{1=}$ Calgary longitude= -114.0888547˚
Converted to radians: $\frac{(\lambda 1 \text{ x } \boldsymbol{\pi})}{180}$
$= \frac{(-114.0888547^\circ \text{ x } \boldsymbol{\pi})}{180}$
= -1.99122615

$\lambda_{2=}$ Las Vegas longitude= -115.172813˚
Converted to radians: $\frac{(\lambda 2 \text{ x } \boldsymbol{\pi})}{180}$
$= \frac{(-115.172813^\circ \text{ x } \boldsymbol{\pi})}{180}$
= -2.010144796

<u>Calculating Δ Latitude:</u>
$\Delta\varphi = \varphi_2 - \varphi_1$ = 0.63031949- 0.891031561 = -0.260712

<u>Calculating</u> Δ <u>Longitude:</u>
$\Delta\lambda = \lambda_2 - \lambda_1 =$ -2.010144796– (-1.99122615) = -0.018918642

<u>Calculating x</u>
$x = \Delta\lambda \cdot \cos \varphi_m$
x = -0.018918642 · cos 0.891031561
x = -0.0118924492

y= Δφ
y= -0.260712

<u>Calculating d</u>

d = R · $\sqrt{x^2 + y^2}$

d = 6, 371 · $\sqrt{(-0.0118924492) + (-0.260712)^2}$

d = 6, 371 · 0.2609831604

d = 1662.7237

The theoretical distance between Calgary and Las Vegas, assuming Earth is not perfectly spherical is 1663. 763 km. The experimental distance is 1662.7237 km. The percent error is -0.0624%, which matches the assumption, that the error in the spherical formulae is less than 0.3%. Since error is proven to be minimal, I will be using 1662.7237 km as the reference value for the distance between Calgary and Las Vegas and the standard distance formula for the other 13 pairs of cities.

Mercator Map Projection

Background

Today there is the GPS, but imagine how difficult road travel would be without the invention of the Mercator projection? The Mercator map projection was invented by Gerardus Mercator in the 1500s. It takes the form of a rectangular grid and parallel longitudinal lines (Figure 4). Due to the relatively simplistic grid lines, the Mercator map projection was an easy tool for navigation and travel. Additionally, its rectangular shape enables efficient storage. However, the Mercator projection also had its down sides. For example the straight, parallel lines did not accommodate for the curvature of the Earth, giving the impression that Earth has sharp corners and straight edges.

The Mercator map still hangs on our walls today. Despite distortions, it is a simple map that completes its duty to map the world. Since the Mercator projection was invented at the beginning of the colonial period, many historians wish to get rid of this projection as its distortions enlarges imperialistic countries. This leads us to question, should we tear down the Mercator projection still used today in our atlases and textbooks, and replace it with a more accurate and ethical map projection?

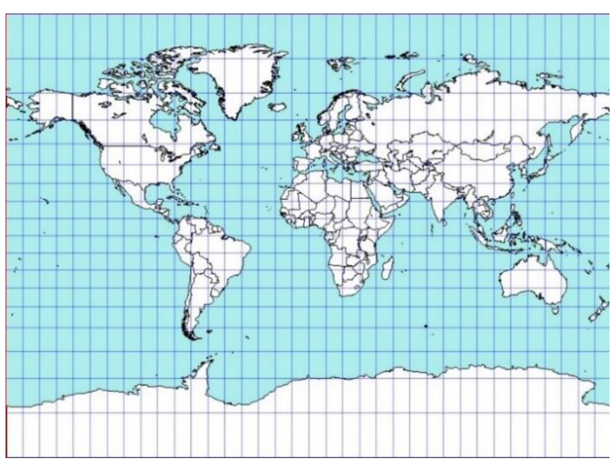

Figure 4. Mercator Map Projection (Mgaqua.net, 2016)

Sample Calculations: Distance Calculation between Calgary and Las Vegas

$$x = R\,(\lambda - \lambda_0)$$
$$y = R\,(\mathit{ln}\,(\mathit{tan}\,(\frac{\pi}{4} + \frac{\varphi}{2}))$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Kartoweb.itc.nl, 2016)

Where:

λ_0 = primary longitude (0.0015˚W is the longitude of Greenwich. This is used as reference in the formula as it is assumed Greenwich is the centre of the projected map.)

$= -0.0015^{\circ} \times \frac{\pi}{180}$

$= -2.617993878 \times 10^{-5}$

R= Earth radius= 6, 371 km

Known (in radians):

φ_1 = Calgary latitude= 0.891031561

φ_2= Las Vegas latitude= 0.63031949

λ_1= Calgary longitude= -1.99122615

λ_2=Las Vegas longitude= -2.010144796

<u>Calculating x_1</u>

$x_1 = R(\lambda_1 - \lambda_0)$

$x_1 = 6,371\,(-1.99122615 - (-2.617993878 \times 10^{-5}))$

$x_1 = -12685.9348..$

<u>Calculating x2</u>

$x_2 = R(\lambda_2 - \lambda_0)$

$x_2 = 6,371\,(-2.010144796 - (-2.617993878 \times 10^{-5}))$

$x_2 = -12806.4655..$

<u>Calculating y1</u>

$y_1 = R\,(\ln\,(\tan\,(\frac{\pi}{4} + \frac{\varphi 1}{2}))$

$y_1 = 6,371(\ln\,(\tan\,(\frac{\pi}{4} + \frac{0.891031561}{2}))$

$y_1 = 6623.1391\ldots$

<u>Calculating y2</u>

$y_2 = R\,(\ln\,(\tan\,(\frac{\pi}{4} + \frac{\varphi 2}{2}))$

$y_2 = 6,371(\ln\,(\tan\,(\frac{\pi}{4} + \frac{0.63031949}{2}))$

$y_2 = 4311.5781\ldots$

Calculating Distance

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(-12806.4655 \ldots - (-12685.9348 \ldots))^2 + (4311.5781 \ldots - 6623.1391 \ldots)^2}$

d= 2314.7013 km

Sample Calculations for percent error of Mercator projected distance between Calgary and Las Vegas and Standard projected distance

Percentage error = $\left|\frac{(\text{map projected distance} - \text{standard distance})}{\text{standard distance}}\right|$ x 100%

Percentage error= $\left|\frac{(2314.7013 \ldots \text{km} - 1662.7237 \ldots \text{km})}{1662.7237 \ldots \text{km}}\right|$ x 100%

Percentage error= 39.2114%

Table 1: Percent Error of Mercator Projected Distance of Calgary and Las Vegas vs. Standard distance of Calgary and Las Vegas

Cities	Area of Latitude	Area of Longitude	Standard	Mercator	Percent Error
Calgary and Las Vegas	43.5835	-114.6308	1662.7237	2314.7013	39.2114
Paris and Berlin	50.6867	7.8671	850.2461	1384.9622	62.8896
Rangoon and Bangkok	15.2782	98.3259	573.7075	597.7305	4.1873
Calgary and Vancouver	50.1675	-118.6048	661.2665	1050.2431	58.8230
Montreal and Quebec City	46.1592	-72.3920	231.0289	335.9867	45.4306
Pheonix and El Paso City	32.6202	-109.2450	555.1529	664.6762	19.7285
Ottowa and Toronto	44.5607	-77.5460	346.9762	492.1331	41.8348
Berlin and Vienna	50.3584	14.8750	520.7188	822.8913	58.0299
Bangkok and Singapore	7.5282	102.1509	1430.1519	1447.2158	1.1932
Atlanta and Miami	29.7652	-82.2995	967.8428	1124.1695	16.1521
Bangalore and Colombo	9.9506	78.7048	714.6680	726.8276	1.7014
Tokyo and Seoul	36.6250	133.3250	1140.2134	1437.9021	26.1082
Oslo and Helsinki	60.0604	17.8438	790.3624	1578.3448	99.6989
Paris and Budapest	48.1746	10.7011	1231.1980	1870.9041	51.9580

Azimuthal Map Projection

Background

The azimuthal equidistant projection is often seen today, for example it is the symbol of the United Nations! Of course the UN symbol is different than the projection used in this IA, as it maps both the Northern and Southern Hemispheres. According to ‘Map Projections: Summary 2016’, an equidistant azimuthal projection means that the scale is constant when mapping from 3D to 2D. In other words, if you were to find the distance between 2 sets of points A and B, the distance between the points on the sphere are proportional to it being mapped on paper. The azimuthal equidistant projection is useful for maps of airlines, atlases and polar hemispheres. Even NASA uses azimuthal projections to take pictures of planets in space!

For the purpose of this IA, it is assumed the azimuthal projection will expand from the North Pole out to the equator (Figure 5). The formula that will be used requires primary latitude, φ_0 and primary longitude, λ_0. Similar to the Mercator calculations, primary longitude is the longitude of Greenwich to use as reference. The primary latitude will be the latitude of the North Pole, (90.0000˚). This is because the map is set as tangent to the globe at the North Pole.

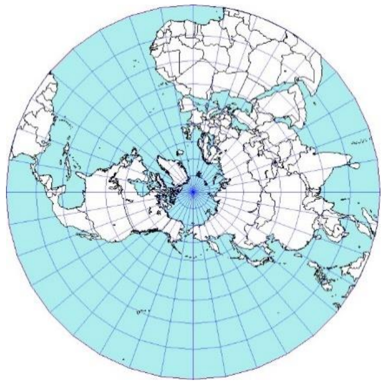

Figure 5. Azimuthal projection viewed from the North Pole (Mgaqua.net, 2016)

Sample Calculations: Distance Calculation between Calgary and Las Vegas

$\cos c = \sin(\varphi_0)\cdot\sin(\varphi_1) + \cos(\varphi_0)\cdot\cos(\varphi_1)\cdot\cos(\lambda_1 - \lambda_0)$
$k = \frac{c}{\sin c}$
$x = k\cdot \cos(\Phi_1)\cdot\sin(\lambda_1 - \lambda_0)$
$y = k\cdot \cos(\Phi_0)\cdot\sin(\Phi_1) - \sin(\Phi_0)\cdot\cos(\Phi_1)\cdot\cos(\lambda_1 - \lambda_0)$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(Mathworld.wolfram.com, 2016)

Known (in radians):

Φ_0 (primary latitude, North Pole) = $\frac{(90.0000^\circ \text{ x } \boldsymbol{\pi})}{180}$ =1.570796…
Φ_1 = Calgary latitude= 0.891031561
φ_2= Las Vegas latitude= 0.63031949
λ_0 (primary longitude, Greenwich) = $\frac{(-0.0015^\circ \text{x } \boldsymbol{\pi})}{180}$ = -2.617993878 x 10^{-5}
λ_1= Calgary longitude= -1.99122615
λ_2= Las Vegas longitude= -2.010144796

<u>Calculating c 1</u>
$\cos c = \sin(\varphi_0) \cdot \sin(\varphi_1) + \cos(\varphi_0) \cdot \cos(\varphi_1) \cdot \cos(\lambda_1 - \lambda_0)$
cos c= sin(1.570796…) · sin(0.891031561) + cos(1.570796…) · cos(0.891031561) · cos(-1.99122615-(-2.617993878 x 10^{-5}))
cos c= 0.7777
c= 0.6798…

Calculating k1

$k=\frac{c}{\sin c}$

$k= (\frac{0.6798...}{\sin(0.6798...)})$

$k= 1.0814...$

Calculating x1

$x= k \cdot \cos(\Phi_1) \cdot \sin(\lambda_{1-}\lambda_0)$

$x_1= 1.0814... \cdot \cos (0.891031561) \cdot \sin(-1.99122615-(-2.617993878 \times 10^{-5}))$

$x_1= -0.6206...$

Calculating y1

$y= k \cdot \cos(\Phi_0) \cdot \sin(\Phi_1) - \sin(\Phi_0) \cdot \cos(\Phi_1) \cdot \cos(\lambda_{1-}\lambda_0)$

$y_1= 1.0814... \cdot \cos (1.570796...) \cdot \sin (0.891031561) - \sin (1.570796...) \cdot \cos (0.891031561) \cdot \cos (-1.99122615-(-2.617993878 \times 10^{-5}))$

$y_{1=} 0.2774...$

Note: x_2 and y_2 were calculated the same way as shown above. They are respectively -0.8512… and 0.4000…

Calculating d

$d = R \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = 6,371 \cdot \sqrt{(-0.8512 ... - (-0.6206 ...))^2 + (0.4000 ... - (0.2774 ...))^2}$

$d = 6,371 \cdot 0.2612...$

$d = 1663.7884$ km

Note: Calculating percent error follows the same process as the Mercator sample calculations, being ±0.0640%.

Table 2: Percent Error of Azimuthal Equidistant Projected Distance of Calgary and Las Vegas vs. Standard distance of Calgary and Las Vegas

Cities	Area of Latitude	Area of Longitude	Standard	Azimuthal	Percent Error
Calgary and Las Vegas	43.5835	-114.6308	1662.7237	1663.7884	0.0640
Paris and Berlin	50.6867	7.8671	850.2461	932.9685	9.7292
Rangoon and Bangkok	15.2782	98.3259	573.7075	715.8477	24.7757
Calgary and Vancouver	50.1675	-118.6048	661.2665	724.5320	9.5673
Montreal and Quebec City	46.1592	-72.3920	231.0289	247.7177	7.2237
Pheonix and El Paso City	32.6202	-109.2450	555.1529	654.6695	17.9260
Ottowa and Toronto	44.5607	-77.5460	346.9762	378.8009	9.1720
Berlin and Vienna	50.3584	14.8750	520.7188	531.8915	2.1456
Bangkok and Singapore	7.5282	102.1509	1430.1519	1481.6997	3.6044
Atlanta and Miami	29.7652	-82.2995	967.8428	1012.4456	4.6085
Bangalore and Colombo	9.9506	78.7048	714.6680	758.1281	6.0812
Tokyo and Seoul	36.6250	133.3250	1140.2134	1330.9321	16.7266
Oslo and Helsinki	60.0604	17.8438	790.3624	822.6113	4.0803
Paris and Budapest	48.1746	10.7011	1231.1980	1359.1275	10.3907

Conic Map Projection

Background

Conic map projections differ from Mercator and azimuthal maps in that they focus on depicting moderate, mid latitude zones of the Earth rather than the extremes. As an individual living in Alberta, this projection can be extremely accurate. But can it be envisioned that world maps would one day be a conic projection instead of a rectangular projection? The Lambert conformal projection has a simplistic design and permanent distortion making it rarely used as a world map, but instead used to visualize temperate regions, climate projections and weather maps (Borneman, 2015).

Sample Calculations: Distance Calculation between Calgary and Las Vegas

$$n= \frac{\ln(\cos\Phi 1 \sec\varphi 2)}{\ln[\tan\left(\frac{1}{4}\pi+\frac{1}{2}\varphi 2\right)\cot(\frac{1}{4}\pi+\frac{1}{2}\varphi 1)]}$$

$$F= \frac{\cos\Phi 1 \tan^{n}(\frac{1}{4}\pi+\frac{1}{2}\varphi 1)}{n}$$

$$p = F\cot^{n}(\frac{1}{4}\pi + \frac{1}{2}\varphi)$$

$$p_0 = F\cot^{n}(\frac{1}{4}\pi + \frac{1}{2}\varphi_0)$$

$$x = p \sin[n(\lambda - \lambda_0)]$$

$$y= p_0 - p\cos[n(\lambda - \lambda_0)]$$

(Mathworld.wolfram.com, 2016)

Known (in radians):

Φ_0 (primary latitude, North Pole) = 1.570796…
Φ_1 = Calgary latitude= 0.891031561
φ_2= Las Vegas latitude= 0.63031949
λ_0 (primary longitude, Greenwich) =-2.617993878 x 10^{-5}
λ_1=Calgary longitude= -1.99122615
λ_2= Las Vegas longitude= -2.010144796

<u>Calculating n</u>

$$n= \frac{\ln(\cos\Phi 1 \sec\varphi 2)}{\ln[\tan\left(\frac{1}{4}\boldsymbol{\pi}+\frac{1}{2}\varphi 2\right)\cot(\frac{1}{4}\boldsymbol{\pi}+\frac{1}{2}\varphi 1)]}$$

$$n= \frac{\ln(\cos(0.891031561)\sec(0.63031949))}{\ln[\tan\left(\frac{1}{4}\boldsymbol{\pi}+\frac{1}{2}0.63031949\right)\cot(\frac{1}{4}\boldsymbol{\pi}+\frac{1}{2}0.891031561)]}$$

$$n= \frac{-0.2508\ldots}{-0.3628\ldots}$$

n= 0.6913…

Calculating F

$$F = \frac{cos\Phi 1 \tan^{n}(\frac{1}{4}\boldsymbol{\pi}+\frac{1}{2}\varphi 1)}{n}$$

$$F = \frac{\cos(0.891031561) \tan^{0.6913\ldots}(\frac{1}{4}\boldsymbol{\pi}+\frac{1}{2}0.891031561)}{0.6913\ldots}$$

$$F = \frac{1.2898\ldots}{0.6913\ldots}$$

F= 1.8655…

Calculating p_1

$p_1 = \mathrm{Fcot}^{n}(\frac{1}{4}\boldsymbol{\pi} + \frac{1}{2}\varphi 1)$

$p_1 = 1.8655\ldots\cot^{0.6913\ldots}(\frac{1}{4}\boldsymbol{\pi} + \frac{1}{2}0.891031561)$

$p_1 = 0.9092\ldots$

Calculating p_0

$p_0 = \mathrm{Fcot}^{n}(\frac{1}{4}\boldsymbol{\pi} + \frac{1}{2}\varphi_0)$

$p_0 = 1.8655\ldots\cot^{0.6913\ldots}(\frac{1}{4}\boldsymbol{\pi} + \frac{1}{2}1.570796 \ldots)$

$p_0 = 0.0000\ldots$

Calculating x_1

$x_1 = p_1 \sin[n(\lambda_1 - \lambda_0)]$

$x_1 = 0.9092\ldots\sin[0.6913\ldots(-1.99122615 - (-2.617993878 \times 10^{-5}))]$

$x_1 = -0.8921\ldots$

Calculating y_1

$y_1 = p_0 - p_1 \cos[n(\lambda_1 - \lambda_0)]$

$y_1 = 0.0000\ldots - 0.9092\ldots\cos[0.6913\ldots(-1.99122615 - (-2.617993878 \times 10^{-5}))]$

$y_1 = -0.7420\ldots$

Note: p2, x2 and y_2 were calculated the same way as shown above. They are respectively 1.1684…, -1.1493… and -1.0592…

Calculating d

$d = R \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = 6,371 \cdot \sqrt{(1.1684 \ldots - (-0.8921 \ldots))^2 + (-1.0592 \ldots - (-0.7420 \ldots))^2}$

$d = 6,371 \cdot 0.4083\ldots$

d = 2601.6179… km

Note: Calculating percent error uses the same process as the Mercator projection. The result is ±56.4672%.

Table 3: Percent Error of Lambert Conformal Projected Distance of Calgary and Las Vegas vs. Standard distance of Calgary and Las Vegas

Cities	Area of Latitude	Area of Longitude	Standard	Conic	Percent Error
Calgary and Las Vegas	43.5835	-114.6308	1662.7237	2601.6172	56.4672
Paris and Berlin	50.6867	7.8671	850.2461	871.5126	2.5012
Rangoon and Bangkok	15.2782	98.3259	573.7075	672.3431	17.1927
Calgary and Vancouver	50.1675	-118.6048	661.2665	296.5632	55.1522
Montreal and Quebec City	46.1592	-72.3920	231.0289	287.9177	24.6241
Pheonix and El Paso City	32.6202	-109.2450	555.1529	250.8088	54.8217
Ottowa and Toronto	44.5607	-77.5460	346.9762	401.9364	15.8398
Berlin and Vienna	50.3584	14.8750	520.7188	641.3239	23.1613
Bangkok and Singapore	7.5282	102.1509	1430.1519	1555.2591	8.7478
Atlanta and Miami	29.7652	-82.2995	967.8428	1087.5634	12.3698
Bangalore and Colombo	9.9506	78.7048	714.6680	797.0233	11.5236
Tokyo and Seoul	36.6250	133.3250	1140.2134	484.9403	57.4693
Oslo and Helsinki	60.0604	17.8438	790.3624	751.2992	4.9424
Paris and Budapest	48.1746	10.7011	1231.1980	1258.0877	2.1840

Analysis

Table 4: Top 3 pairs of cities with the most accurate distances projected on a Mercator, azimuthal equidistant and Lambert Conformal map projection.

Map Projections	Rank	Top 3 Cities with the most accurate distances	Percent Error comparing projected and standard distance (%)	Area
Mercator	1	Bangkok and Singapore	1.1932	Equator
	2	Bangalore and Colombo	1.7014	Equator
	3	Rangoon and Bangkok	4.1873	Equator
Azimuthal	1	Calgary and Las Vegas	0.0640	Middle of Northern Hemisphere
	2	Berlin and Vienna	2.1456	Middle of Northern Hemisphere, closer towards North pole
	3	Bangkok and Singapore	3.6044	Equator
Conic	1	Paris and Budapest	2.1840	Middle of Northern Hemisphere, closer towards North pole
	2	Paris and Berlin	2.5012	Middle of Northern Hemisphere, closer towards North pole
	3	Oslo and Helsinki	4.9424	Middle of Northern Hemisphere/ Edge of map

Table 5: Top 3 pairs of cities with the most inaccurate distances projected on a Mercator, azimuthal equidistant and Lambert Conformal map projection.

Map Projections	Rank	Top 3 Cities with the most inaccurate distances	Percent Error (%Percent Error comparing projected and standard distance (%)	Area
Mercator	11	Calgary and Vancouver	58.8230	Middle of Northern Hemisphere
	12	Paris and Berlin	62.8896	Middle of Northern Hemisphere, closer towards North pole
	13	Oslo and Helsinki	99.6989	Middle of Northern Hemisphere/ Edge of map
Azimuthal	11	Tokyo and Seoul	16.7266	Middle of Northern Hemisphere/ Edge of map
	12	Phoenix and El Paso City	17.9260	Lower Northern Hemisphere
	13	Rangoon and Bangkok	24.7757	Equator
Conic	11	Calgary and Vancouver	55.1522	Middle of Northern Hemisphere
	12	Calgary and Las Vegas	56.4672	Middle of Northern Hemisphere
	13	Tokyo and Seoul	57.4693	Middle of Northern Hemisphere/ Edge of Map

Mercator

Theoretically, areas near the equator receive least distortion as that area is tangent to the Earth's 3D shape. From Table 4, it is perfectly clear that the top 3 most accurate distances are nearest to the equator. This supports the theoretical distortion. To double check in Table 5, the 3 least accurate distances are cities closest to the poles, away from the equator. Therefore through mathematical representation in my modelling examples, the accuracy of a Mercator map projection is accurate near the equator, and least accurate near the poles, confirming the theoretical distortions represented in Figure 1.

Azimuthal

Theoretically, the most accurate areas projected on an azimuthal projection are nearest to the North Pole. The least accurate areas are nearest the equator. Tables 4 and 5 supported the theoretical relatively well with the exception of the percent error of Bangkok and Singapore. These 2 cities are considered near the equator, which according to predictions should have an inaccurate representation. Then why are they the 3rd most accurate set of data in my model?

When researching to find the answer to this problem, I came upon an article that explained in further detail about equidistant azimuthal projections. It wrote "Distances are correct between points along straight lines through the center." (Eastern Region Geography 2016) In other words, all longitude lines drawn from the center (North Pole) would be most accurate on the projected map! This matched my results of Bangkok and Singapore. Since the cities are aligned quite close to the longitude of 105˚, they are considered points "along straight lines through the center", explaining the unexpected accuracy. To check, I introduced 2 more sets of data points. The first is Singapore and Cambodia. Singapore and

Cambodia are almost perfectly aligned with the longitude of 105 ˚, resulting in a small percent error of 0.6548%. This supports the argument that distances are accurate along straight lines through the center of the North Pole. To verify, I also calculated the percent error between Singapore and Kuala Lumpur. This alignment is far off the longitude of 105 ˚, and not surprisingly, the percent error was big (33.0438%).

This shows that for an azimuthal equidistant projection: not only does accuracy decrease with points farther from the North Pole, but also when distances are far from the longitudinal lines drawn from the North Pole.

Conic

Assuming the cone is pointed towards the North Pole, mid-latitudes in the northern hemisphere should receive least distortion. This matches with the results in Table 4, in which the top 3 data sets were all considered in the "middle of the Northern Hemisphere, closer towards North Pole."

However in Table 5, the top 3 most inaccurate pair of cities was similarly categorized as "middle of the Northern Hemisphere." Here I was confused, according to the theoretical prediction, shouldn't cities in the middle of the Northern Hemisphere always be most accurate?

According to Geolounge, "Lambert Conformal map projections are designed to be able to be wrapped around a cone on of a sphere but aren't supposed to be geometrically accurate." My data fluctuated in response, making it difficult to reach a conclusion.

Researching for weeks, I could not find an answer to the variance in my data. If the formula I used was theoretical and proven, the result of my data should support predicted distortions. This troubled me until I found another type of conic map projection. It was still Lambert Conformal, but rather than being a tanget conic projection it was a secant conic projection (Figure 6). On a secant map projection, the cone touches Earth at 2 places around the globe, where latitudes are at 30˚ and latitudes at 60˚. This matched my results in Table 3. Areas in the "Middle of the Northern Hemisphere" such as Calgary and Vancouver, Ottowa and Tortont, Montreal and Quebec City, etc… had the most distortion because their latitutudes were around 45˚ on a secant map projection. Rather, areas that were closer to the pole (latitude of 60˚), such as Paris and Berlin, Paris and Budapest and Oslo and Helsinki were much more accurate. Similarly, areas of 30˚ latitude such as Atlanta and Miami and Bangkok and Singapore were also more accurate. Since the secant conic map projection represented my modelling much more accurately than the tangent conic projection, it is safe to conclude that the modelling formula I incorporated for the conic projection represented a secant projection, and have proved the theoretical accuracy of a secant map projection to be correct.

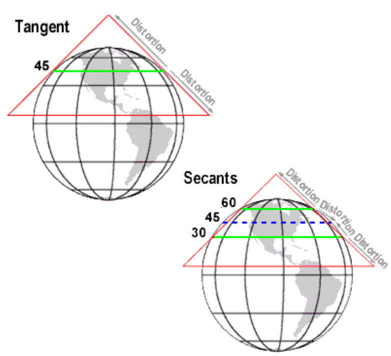

Figure 6. Tangent Vs. Secant Conic Projection (Tangents and Secants, 2016)

Conclusion/ Reflection

The modelling of the Mercator map projection proved to be very successful as it matched the theoretical predictions. Distances between cities are most accurate near the equator, and decrease with accuracy when closer to the poles. This proved consistent with all of my modelling results.

To focus this IA, I decided to only calculate distances on an equidistant azimuthal projection. This projection is significant in that it scaled proportionally to the longitudinal lines on a globe. The bulk of my data results matched with the theoretical prediction; that distances near the North Pole receive most distortion while distortion increases near the equator with the exception of the data set for Bangkok and Singapore. These 2 cities are extremely close to the equator, but their distance was extremely accurate. This was puzzling until I discovered that the equidistant azimuthal projection had a special characteristic. All lines drawn from the center of the North Pole to the equator, also known as longitudinal lines on the globe receive no distortion. And because Bangkok and Singapore were aligned close to the longitude of 105˚, it received very little distortion when projected in 2D form.

The conic map projection was my third choice of projection to investigate. I focused on the Lambert Conformal projection for the same reasons as the azimuthal projection, which was to narrow down my IA. The theoretical accuracy for a conic projection is most accurate where the map is tangent to the globe. In the beginning of the IA I assumed this was to be at one point, 45˚ latitude around the globe. Through deeper research, I found this was not necessarily the case as my data supported a secant conic projection. This meant that the map was tangent to 2 places around the globe, 30˚ and 60˚. It was then concluded that there is most distortion at the North Pole and at 45˚ latitude. There is least distortion at 30˚ latitude and 60˚ latitude.

After completing this IA, I felt accomplished to bring math outside the classroom and apply it in the real world. I chose a topic that was of my interest and critical to our world today. The Mercator map projection, developed hundreds of years ago are still hanging in our classrooms, folded in our atlases and stored in our car for road trips. The equidistant azimuthal map projection is ideal for pilots, seismic and audio work. Its importance goes so far to represent the emblem of the United Nations! And the conic map projection is used by weathermen to show weather patterns and predict future climate change. This IA proved the uniqueness of each map projection, not to conclude which one is better or should be used most often, but to conclude that each map projection is gifted in its own way. With the continuous advancement of map projections, it would be quite easy to answer the question, “Where am I on the map?”

References

About.com Geography,. (2016). *Mercator Projection*. Retrieved 26 January 2016, from http://geography.about.com/library/weekly/aa030201b.htm

Borneman, E. (2015). *Types of Map Projections - Geolounge*. *Geolounge*. Retrieved 31 January 2016, from http://www.geolounge.com/types-map-projections/

Chris Veness, w. (2015). *Calculate distance and bearing between two Latitude/Longitude points using haversine formula in JavaScript*. *Movable-type.co.uk*. Retrieved 13 December 2015, from http://www.movable-type.co.uk/scripts/latlong.html

Eastern Region Geography, I. (2016). *Map Projections Poster*. *Egsc.usgs.gov*. Retrieved 29 January 2016, from http://egsc.usgs.gov/isb//pubs/MapProjections/projections.html

Geo.hunter.cuny.edu,. (2015). *How to choose a projection*. Retrieved 17 December 2015, from http://www.geo.hunter.cuny.edu/~jochen/gtech201/lectures/lec6concepts/map%20coordinate%20systems/how%20to%20choose%20a%20projection.htm

Kartoweb.itc.nl,. (2016). *Geometric aspects of mapping: map projections*. Retrieved 26 January 2016, from http://kartoweb.itc.nl/geometrics/map%20projections/body.htm

"Map Projections: Perspective Cylindrical Projections". 2016. *Progonos.Com*. http://www.progonos.com/furuti/MapProj/Normal/ProjCyl/ProjCEA/projCEA.html.

Mathworld.wolfram.com,. (2016). *Azimuthal Equidistant Projection -- from Wolfram MathWorld*. Retrieved 28 January 2016, from http://mathworld.wolfram.com/AzimuthalEquidistantProjection.html

Mathworld.wolfram.com,. (2016). *Lambert Conformal Conic Projection -- from Wolfram MathWorld*. Retrieved 28 January 2016, from http://mathworld.wolfram.com/LambertConformalConicProjection.html

Mgaqua.net,. (2016). *Aquariu.NET Documentation*. Retrieved 29 January 2016, from http://www.mgaqua.net/AquaDoc/Projections/Projections_Cylindrical.aspx

Progonos.com,. (2016). *Map Projections: Azimuthal Projections*. Retrieved 29 January 2016, from http://www.progonos.com/furuti/MapProj/Normal/ProjAz/projAz.html

Stockton, Nick. 2016. "Get To Know A Projection: Azimuthal Orthographic". *WIRED*. http://www.wired.com/2014/11/get-to-know-a-projection-azimuthal-orthographic/.

"Tangents And Secants". 2016. *Geography.Hunter.Cuny.Edu*. http://www.geography.hunter.cuny.edu/~jochen/GTECH361/lectures/lecture04/concepts/Map%20coordinate%20systems/Tangents%20and%20secants.htm.

Viewphotos.org,. (2015). *Calgary Map (Alberta), Images viewphotos.org*. Retrieved 13 December 2015, from http://www.viewphotos.org/canada/coordinates-of-Calgary-59.html

(2015). Retrieved 13 December 2015, from http://pubs.usgs.gov/pp/1395/report.pdf

2015. Video. https://www.youtube.com/watch?v=vVX-PrBRtTY

Appendices

A: Excel Spread Sheet Calculations of the Standard Distances of the 14 pairs of cities

City 1	Lat 1	Long 1	City 2	Lat 2	Long 2	Lat 1 Radian	Long 1 Radian	Lng Delta	Lat 2 Radian	Long 2 Radian	Lat Delta	Maximum Lat	x	y	Dist
Calgary	51.0523	-114.0889	Las Vegas	36.1146	-115.1728	0.8910	-1.9912	-0.0189	0.6303	-2.0101	-0.2607	0.8910	-0.0119	-0.2607	1662.7237
Paris	48.8567	2.3508	Berlin	52.5167	13.3833	0.8527	0.0410	0.1926	0.9166	0.2336	0.0639	0.9166	0.1172	0.0639	850.2461
Rangoon	16.8000	96.1500	Bangkok	13.7563	100.5018	0.2932	1.6781	0.0760	0.2401	1.7541	-0.0531	0.2932	0.0727	-0.0531	573.7075
Calgary	51.0523	-114.0889	Vancouver	49.2827	-123.1207	0.8910	-1.9912	-0.1576	0.8601	-2.1489	-0.0309	0.8910	-0.0991	-0.0309	661.2665
Montreal	45.5017	-73.5673	Quebec City	46.8167	-71.2167	0.7942	-1.2840	0.0410	0.8171	-1.2430	0.0230	0.8171	0.0281	0.0230	231.0289
Pheonix	33.4500	-112.0667	El Paso City	31.7903	-106.4233	0.5838	-1.9559	0.0985	0.5548	-1.8574	-0.0290	0.5838	0.0822	-0.0290	555.1529
Ottowa	45.4214	-75.6919	Toronto	43.7000	-79.4000	0.7928	-1.3211	-0.0647	0.7627	-1.3858	-0.0300	0.7928	-0.0454	-0.0300	346.9762
Berlin	52.5167	13.3833	Vienna	48.2000	16.3667	0.9166	0.2336	0.0521	0.8412	0.2857	-0.0753	0.9166	0.0317	-0.0753	520.7188
Bangkok	13.7563	100.5018	Singapore	1.3000	103.8000	0.2401	1.7541	0.0576	0.0227	1.8117	-0.2174	0.2401	0.0559	-0.2174	1430.1519
Atlanta	33.7550	-84.3900	Miami	25.7753	-80.2089	0.5891	-1.4729	0.0730	0.4499	-1.3999	-0.1393	0.5891	0.0607	-0.1393	967.8428
Bangalore	12.9667	77.5667	Colombo	6.9344	79.8428	0.2263	1.3538	0.0397	0.1210	1.3935	-0.1053	0.2263	0.0387	-0.1053	714.6680
Tokyo	35.6833	139.6833	Seoul	37.5667	126.9667	0.6228	2.4379	-0.2219	0.6557	2.2160	0.0329	0.6557	-0.1759	0.0329	1140.2134
Oslo	59.9500	10.7500	Helsinki	60.1708	24.9375	1.0463	0.1876	0.2476	1.0502	0.4352	0.0039	1.0463	0.1240	0.0039	790.3624
Paris	48.8567	2.3508	Budapest	47.4925	19.0514	0.8527	0.0410	0.2915	0.8289	0.3325	-0.0238	0.8527	0.1918	-0.0238	1231.1980

B: Excel Spread Sheet Calculations of the Mercator Map Projected Distances of the 14 pairs of cities

City 1	Lat 1	Long 1	City 2	Lat 2	Long 2	Lat 1 Radian	Long 1 Radian	Lng Delta 1	x1	y1	Lat 2 Radian	Long 2 Radian	Lng Delta 2	x2	y2	Dist
Calgary	51.0523	-114.0889	Las Vegas	36.1146	-115.1728	0.8910	-1.9912	-1.9912	-12685.9348	6623.1391	0.6303	-2.0101	-2.0101	-12806.4655	4311.5781	2314.7013
Paris	48.8567	2.3508	Berlin	52.5167	13.3833	0.8527	0.0410	0.0411	261.5638	6243.5869	0.9166	0.2336	0.2336	1488.3218	6886.3816	1384.9622
Rangoon	16.8000	96.1500	Bangkok	13.7563	100.5018	0.2932	1.6781	1.6782	10691.5588	1895.4329	0.2401	1.7541	1.7541	11175.4569	1544.5419	597.7305
Calgary	51.0523	-114.0889	Vancouver	49.2827	-123.1207	0.8910	-1.9912	-1.9912	-12685.9348	6623.1391	0.8601	-2.1489	-2.1488	-13690.2302	6315.8911	1050.2431
Montreal	45.5017	-73.5673	Quebec City	46.8167	-71.2167	0.7942	-1.2840	-1.2840	-8180.1436	5694.4735	0.8171	-1.2430	-1.2429	-7918.7688	5905.5902	335.9867
Pheonix	33.4500	-112.0667	El Paso City	31.7903	-106.4233	0.5838	-1.9559	-1.9559	-12461.0815	3950.7616	0.5548	-1.8574	-1.8574	-11833.5640	3731.6352	664.6762
Ottowa	45.4214	-75.6919	Toronto	43.7000	-79.4000	0.7928	-1.3211	-1.3210	-8416.3883	5681.7431	0.7627	-1.3858	-1.3858	-8828.7102	5413.0692	492.1331
Berlin	52.5167	13.3833	Vienna	48.2000	16.3667	0.9166	0.2336	0.2336	1488.3218	6886.3816	0.8412	0.2857	0.2857	1820.0608	6133.3216	822.8913
Bangkok	13.7563	100.5018	Singapore	1.3000	103.8000	0.2401	1.7541	1.7541	11175.4569	1544.5419	0.0227	1.8117	1.8117	11542.2000	144.5658	1447.2158
Atlanta	33.7550	-84.3900	Miami	25.7753	-80.2089	0.5891	-1.4729	-1.4729	-9383.5729	3991.4803	0.4499	-1.3999	-1.3999	-8918.6558	2967.9525	1124.1695
Bangalore	12.9667	77.5667	Colombo	6.9344	79.8428	0.2263	1.3538	1.3538	8625.1902	1454.2989	0.1210	1.3935	1.3935	8878.2809	772.9594	726.8276
Tokyo	35.6833	139.6833	Seoul	37.5667	126.9667	0.6228	2.4379	2.4380	15532.2408	4252.3673	0.6557	2.2160	2.2160	14118.2194	4513.3385	1437.9021
Oslo	59.9500	10.7500	Helsinki	60.1708	24.9375	1.0463	0.1876	0.1876	1195.5122	8379.2274	1.0502	0.4352	0.4353	2773.0902	8428.4212	1578.3448
Paris	48.8567	2.3508	Budapest	47.4925	19.0514	0.8527	0.0410	0.0411	261.5638	6243.5869	0.8289	0.3325	0.3325	2118.5858	6016.0968	1870.9041

C: Excel Spread Sheet Calculations of the Azimuthal Equidistant Map Projected Distances of the 14 pairs of cities

City 1	Lat 1	Long 1	City 2	Lat 2	Long 2	Lat 1 Radian	Long 1 Radian	Lat 2 Radian	Long 2 Radian	cos c 1	c 1	k 1	cos c2	C2	k2	x1	x2	y1	y2	Distance
Calgary	51.0523	-114.0889	Las Vegas	36.1146	-115.1728	0.8910	-1.9912	0.6303	-2.0101	0.7777	0.6798	1.0814	0.5894	0.9405	1.1642	-0.6206	-0.8512	0.2774	0.4000	1663.7884
Paris	48.8567	2.3508	Berlin	52.5167	13.3833	0.8527	0.0410	0.9166	0.2336	0.7531	0.7181	1.0914	0.7935	0.6542	1.0751	0.0295	0.1514	-0.7175	-0.6364	932.9685
Rangoon	16.8000	96.1500	Bangkok	13.7563	100.5018	0.2932	1.6781	0.2401	1.7541	0.2890	1.2776	1.3345	0.2378	1.3307	1.3700	1.2702	1.3084	0.1369	0.2426	715.8477
Calgary	51.0523	-114.0889	Vancouver	49.2827	-123.1207	0.8910	-1.9912	0.8601	-2.1489	0.7777	0.6798	1.0814	0.7579	0.7107	1.0894	-0.6206	-0.5952	0.2774	0.3883	724.5320
Montreal	45.5017	-73.5673	Quebec City	46.8167	-71.2167	0.7942	-1.2840	0.8171	-1.2430	0.7133	0.7766	1.1081	0.7292	0.7537	1.1013	-0.7449	-0.7135	-0.2197	-0.2427	247.7177
Pheonix	33.4500	-112.0667	El Paso City	31.7903	-106.4233	0.5838	-1.9559	0.5548	-1.8574	0.5512	0.9870	1.1829	0.5268	1.0160	1.1953	-0.9147	-0.9745	0.3708	0.2872	654.6695
Ottowa	45.4214	-75.6919	Toronto	43.7000	-79.4000	0.7928	-1.3211	0.7627	-1.3858	0.7123	0.7780	1.1085	0.6909	0.8081	1.1177	-0.7539	-0.7943	-0.1923	-0.1487	378.8009
Berlin	52.5167	13.3833	Vienna	48.2000	16.3667	0.9166	0.2336	0.8412	0.2857	0.7935	0.6542	1.0751	0.7455	0.7295	1.0945	0.1514	0.2056	-0.6364	-0.7000	531.8915
Bangkok	13.7563	100.5018	Singapore	1.3000	103.8000	0.2401	1.7541	0.0227	1.8117	0.2378	1.3307	1.3700	0.0227	1.5481	1.5485	1.3084	1.5034	0.2426	0.3693	1481.6997
Atlanta	33.7550	-84.3900	Miami	25.7753	-80.2089	0.5891	-1.4729	0.4499	-1.3999	0.5556	0.9817	1.1807	0.4348	1.1209	1.2448	-0.9770	-1.1046	-0.0960	-0.1906	1012.4456
Bangalore	12.9667	77.5667	Colombo	6.9344	79.8428	0.2263	1.3538	0.1210	1.3935	0.2244	1.3445	1.3797	0.1207	1.4498	1.4605	1.3130	1.4271	-0.2894	-0.2556	758.1281
Tokyo	35.6833	139.6833	Seoul	37.5667	126.9667	0.6228	2.4379	0.6557	2.2160	0.5833	0.9480	1.1671	0.6097	0.9151	1.1545	0.6134	0.7312	0.7229	0.5503	1330.9321
Oslo	59.9500	10.7500	Helsinki	60.1708	24.9375	1.0463	0.1876	1.0502	0.4352	0.8656	0.5245	1.0474	0.8675	0.5206	1.0466	0.0978	0.2195	-0.5153	-0.4721	822.6113
Paris	48.8567	2.3508	Budapest	47.4925	19.0514	0.8527	0.0410	0.8289	0.3325	0.7531	0.7181	1.0914	0.7372	0.7419	1.0980	0.0295	0.2422	-0.7175	-0.7013	1359.1275

D: Excel Spread Sheet Calculations of the Lambert Conformal Conic Map Projected Distances of the 14 pairs of cities

City 1	Lat 1	Long 1	City 2	Lat 2	Long 2	Lat 1 Radian	Long 1 Radian	Lat 2 Radian	Long 2 Radian	n numerator	n denominator	n	tan^n	F	p1	p2	po	x1	x2	y1	y2	Distance
Calgary	51.0523	-114.0889	Las Vegas	36.1146	-115.1728	0.8910	-1.9912	0.6303	-2.0101	-0.2509	-0.3628	0.6914	2.0519	1.8656	0.9092	1.1684	0.0000	-0.8921	-1.1493	-0.7420	-1.0592	2601.6179
Paris	48.8567	2.3508	Berlin	52.5167	13.3833	0.8527	0.0410	0.9166	0.2336	0.0781	0.1009	0.7738	2.1347	1.8151	0.8502	0.7864	0.0000	0.0270	0.1414	-0.6718	-0.5967	871.4948
Rangoon	16.8000	96.1500	Bangkok	13.7563	100.5018	0.2932	1.6781	0.2401	1.7541	-0.0145	-0.0551	0.2635	1.0816	3.9289	3.6326	3.6857	0.0330	1.5547	1.6438	-3.5887	-3.6453	672.3465
Calgary	51.0523	-114.0889	Vancouver	49.2827	-123.1207	0.8910	-1.9912	0.8601	-2.1489	-0.0370	-0.0482	0.7680	2.2219	1.8187	0.8186	0.8494	0.0000	-0.8178	-0.8468	-0.6343	-0.6708	296.5786
Montreal	45.5017	-73.5673	Quebec City	46.8167	-71.2167	0.7942	-1.2840	0.8171	-1.2430	0.0239	0.0331	0.7213	1.9054	1.8515	0.9717	0.9488	0.0000	-0.7767	-0.7411	-0.8166	-0.7887	287.9194
Pheonix	33.4500	-112.0667	El Paso City	31.7903	-106.4233	0.5838	-1.9559	0.5548	-1.8574	-0.0185	-0.0344	0.5391	1.3970	2.1621	1.5477	1.5767	0.0001	-1.3459	-1.3279	-1.4716	-1.5066	250.8183
Ottowa	45.4214	-75.6919	Toronto	43.7000	-79.4000	0.7928	-1.3211	0.7627	-1.3858	-0.0296	-0.0422	0.7017	1.8697	1.8702	1.0003	1.0303	0.0000	-0.8000	-0.8513	-0.8495	-0.8862	401.9405
Berlin	52.5167	13.3833	Vienna	48.2000	16.3667	0.9166	0.2336	0.8412	0.2857	-0.0910	-0.1182	0.7702	2.2992	1.8165	0.7901	0.8654	0.0000	0.1414	0.1889	-0.6012	-0.6900	641.3298
Bangkok	13.7563	100.5018	Singapore	1.3000	103.8000	0.2401	1.7541	0.0227	1.8117	-0.0288	-0.2197	0.1313	1.0323	7.6385	7.3992	7.6158	0.7064	1.6888	1.7942	-6.6891	-6.9093	1555.2712
Atlanta	33.7550	-84.3900	Miami	25.7753	-80.2089	0.5891	-1.4729	0.4499	-1.3999	-0.0798	-0.1607	0.4968	1.3652	2.2845	1.6734	1.8124	0.0003	-1.1182	-1.1614	-1.6019	-1.7671	1087.5703
Bangalore	12.9667	77.5667	Colombo	6.9344	79.8428	0.2263	1.3538	0.1210	1.3935	-0.0185	-0.1069	0.1729	1.0403	5.8638	5.6369	5.7421	0.2550	1.3073	1.3700	-5.3776	-5.4859	797.0310
Tokyo	35.6833	139.6833	Seoul	37.5667	126.9667	0.6228	2.4379	0.6557	2.2160	0.0244	0.0410	0.5966	1.4892	2.0274	1.3615	1.3286	0.0000	1.3523	1.2877	-1.2685	-1.2282	484.9131
Oslo	59.9500	10.7500	Helsinki	60.1708	24.9375	1.0463	0.1876	1.0502	0.4352	0.0067	0.0077	0.8666	3.1258	1.8063	0.5779	0.5740	0.0000	0.0936	0.2114	-0.3562	-0.3523	751.2887
Paris	48.8567	2.3508	Budapest	47.4925	19.0514	0.8527	0.0410	0.8289	0.3325	-0.0266	-0.0357	0.7452	2.0757	1.8327	0.8829	0.9067	0.0000	0.0270	0.2224	-0.7106	-0.7392	1258.0868