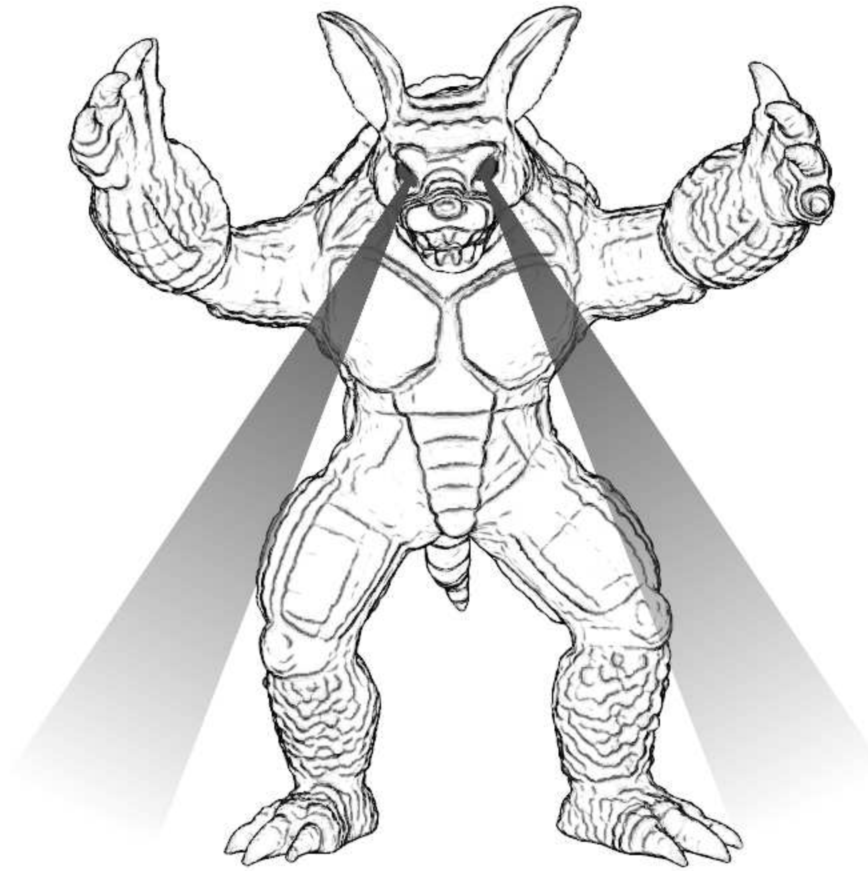
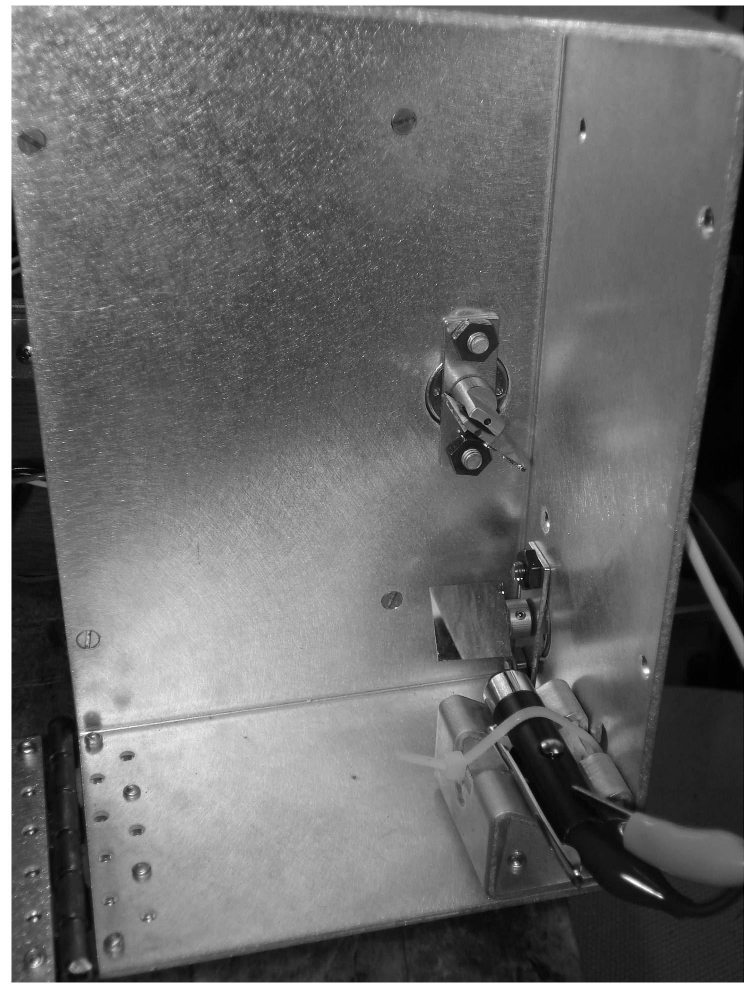
Lazerdillo

"To Project and Serve…"



The Lazerdillo

- Simple laser projector
- R31JP interface
- x, y position controls
- on/off toggle for laser

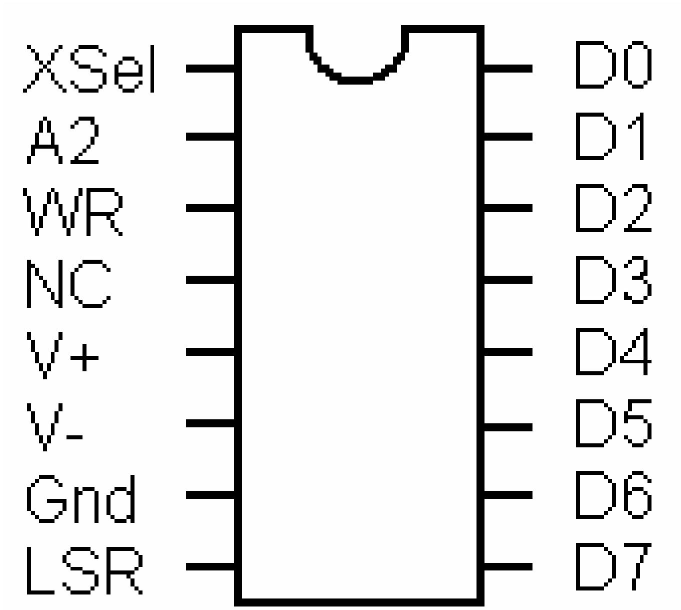


Operation

- Two steppers control the X and Y position
- X and Y DACs are 8 bits
- Buffered voltage signals control stepper motors

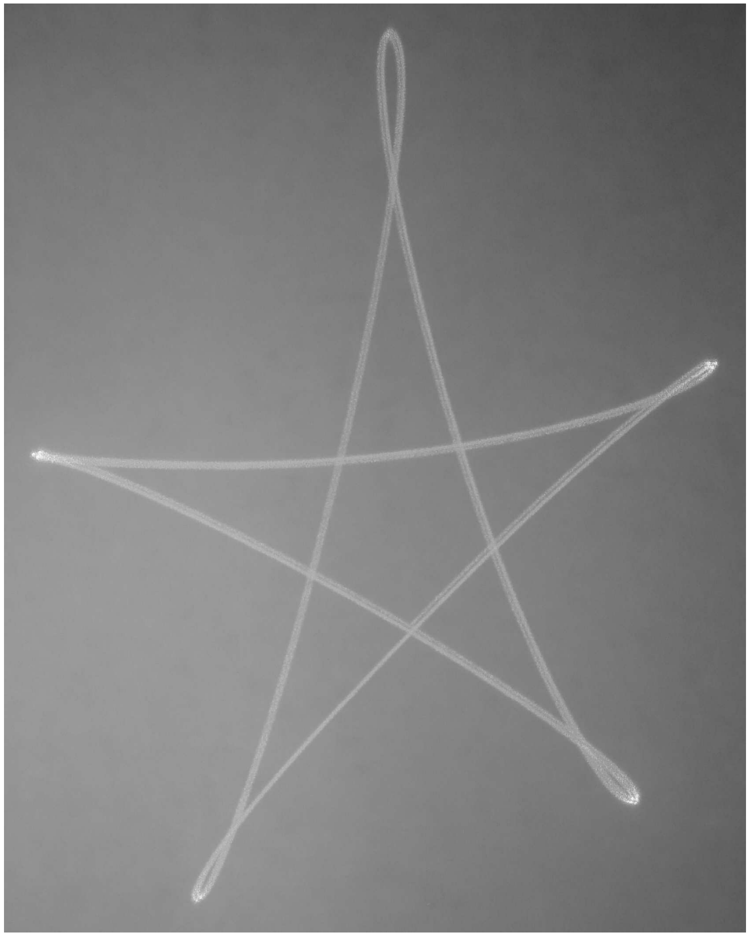
Pin-Out

- A “microcontroller-compatible” peripheral
- A2 selects X or Y DACs
- XSel selects the Lazerdillo
- WR triggers the write to the selected DAC
- LSR triggers the laser



Drawing Objects with the Lazerdillo

Figure generated by sending the laser dot through a sequence of (x,y) points.



Inertial Limitation

- Bandwidth limited.
- Lazerdillo can handle sinusoidal signals up to about 80 Hz.

Optical Limitation

- Lazerdillo works within and around the response and blending limitations of the eye
- Movie theaters run at 24 frames per second
- Fusion fails below 20 frames per second

Linear Limitation

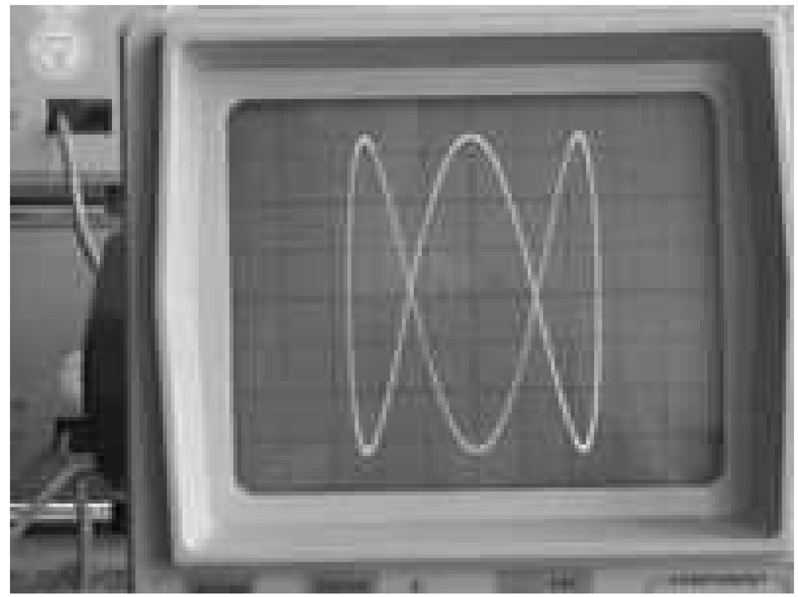
- 20 degree range of motion in x and y axes
- Voltage/Position is not linear near the edges of its range
- Rough linearity if position bytes kept between #40h and #C0h

Lissajous Figures

- Use Lazerdillo to generate Lissajous figures

- Defined by:

$$x = A sin 2\pi a t$$
$$y = B sin 2\pi (bt + \phi)$$



Interpretation of Variables

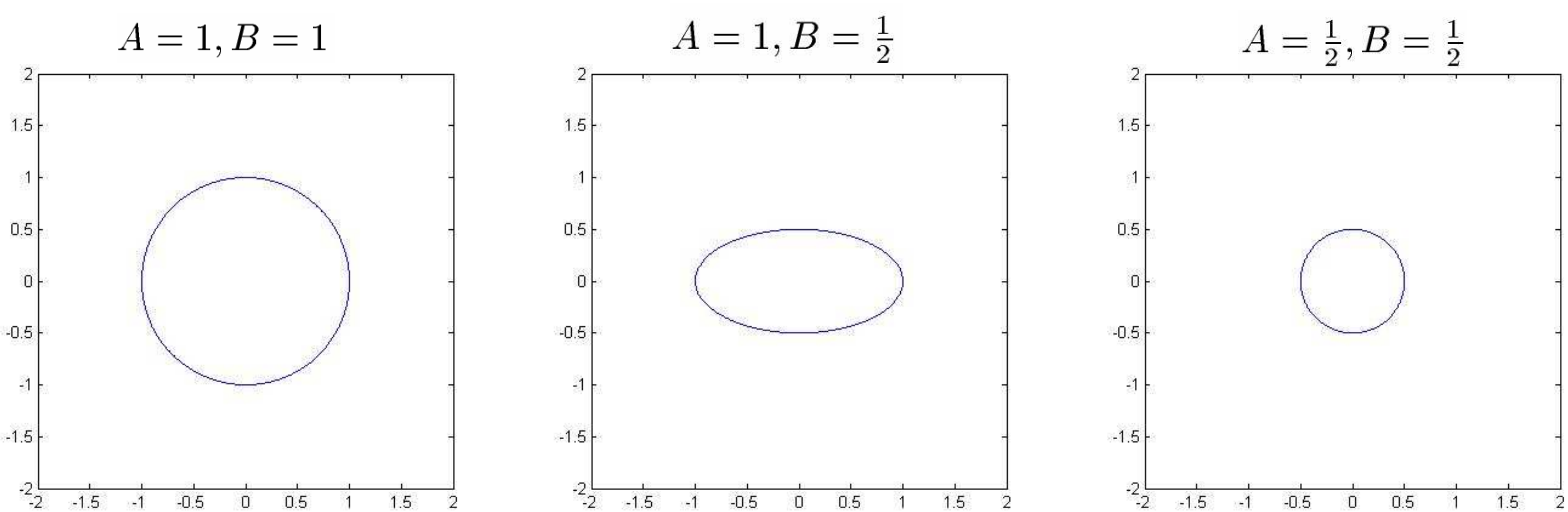
$$x = A sin 2\pi a t \qquad y = B sin 2\pi (bt + \phi)$$

- A and B represent amplitude
- Variable ϕ represents phase difference
- *a* and *b* represent oscillation frequencies

Amplitude

$$x = Asin2\pi at \qquad y = Bsin2\pi(bt + \phi)$$

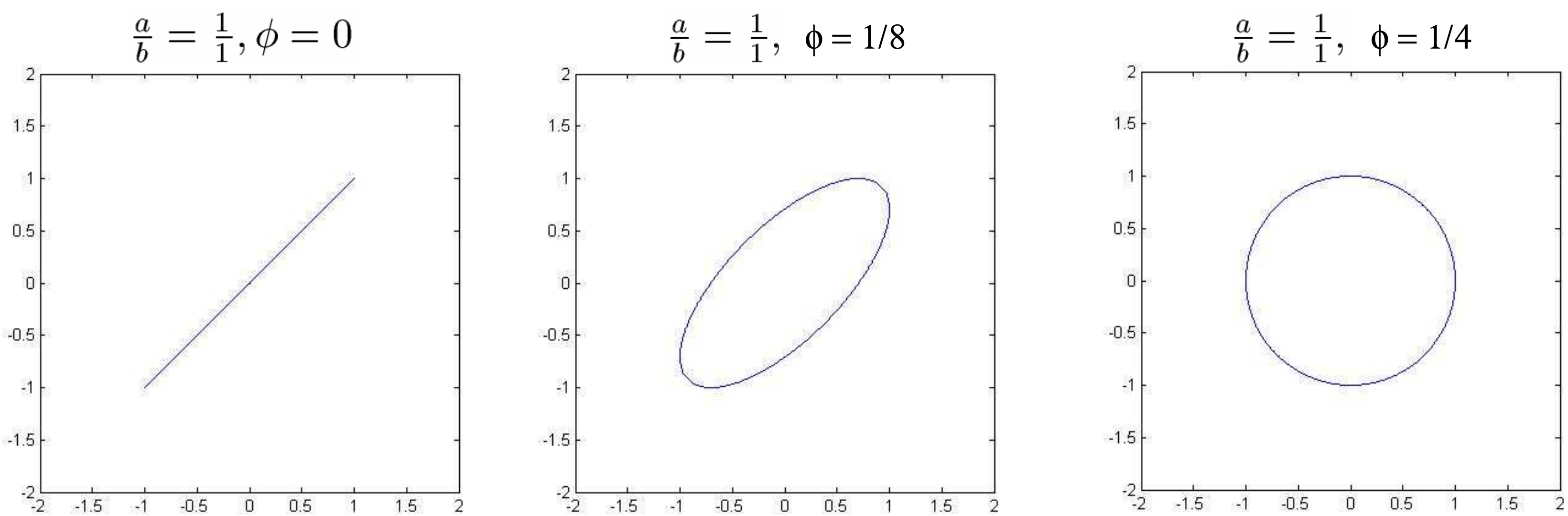
- The effect of varying just amplitude variables *A* and *B*:



Phase

$$x = A sin 2\pi a t \qquad y = B sin 2\pi (bt + \phi)$$

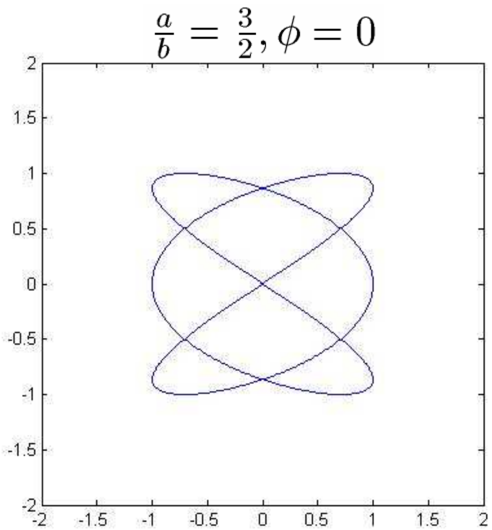
- The phase angle ϕ influences the orientation and the shape of the figure
- Consider the figures below generated with *a*/*b*=1 and different values of ϕ

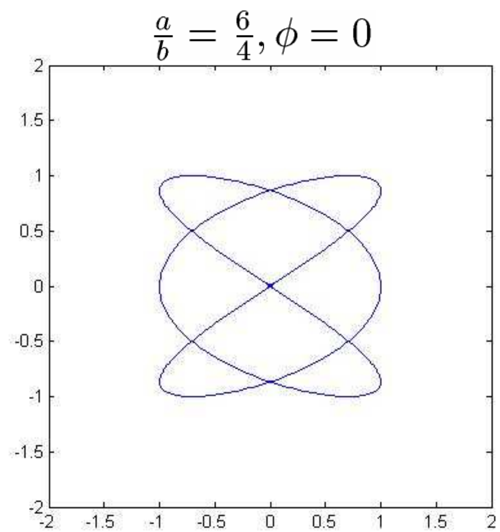


Frequency

$$x = A sin 2\pi a t \qquad y = B sin 2\pi (bt + \phi)$$

- Frequencies *a* and *b* control oscillation independently
- The ratio of frequencies *a*/*b* defines the structure
- We define *a*/*b* to be the absolute frequency ratio, and *α*/*β* to be the frequency ratio in simplest terms

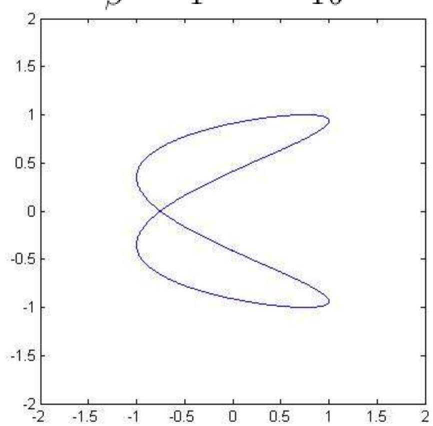




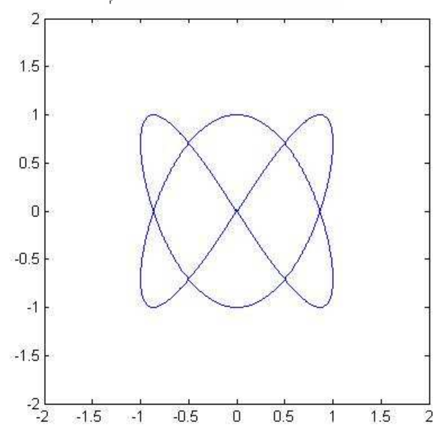
Frequency Ratio

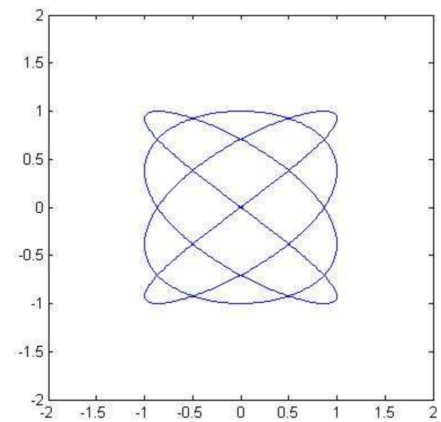
- *α* and *β:* oscillations about their respective axes per frame
- *α/β*: ratio of oscillations about the x axis versus y axis
- Frequency ratio also sets maxima along the x axis versus y
- The frequency ratio indicates the complexity of the figure

$\frac{\alpha}{\beta} = \frac{2}{1}, \phi = \frac{\pi}{16}$



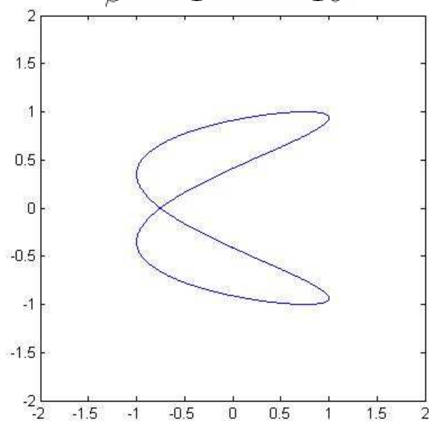
$\frac{\alpha}{\beta} = \frac{2}{3}, \phi = 0$



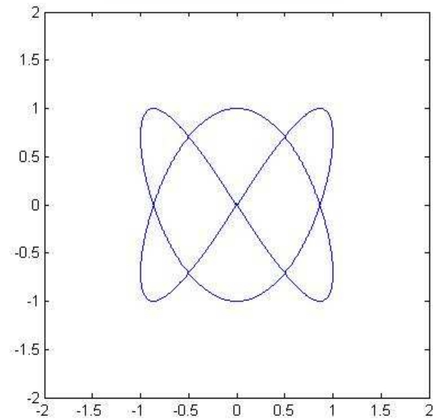


Determine mystery ratio:

$\frac{\alpha}{\beta} = \frac{2}{1}, \phi = \frac{\pi}{16}$



$\frac{\alpha}{\beta} = \frac{2}{3}, \phi = 0$



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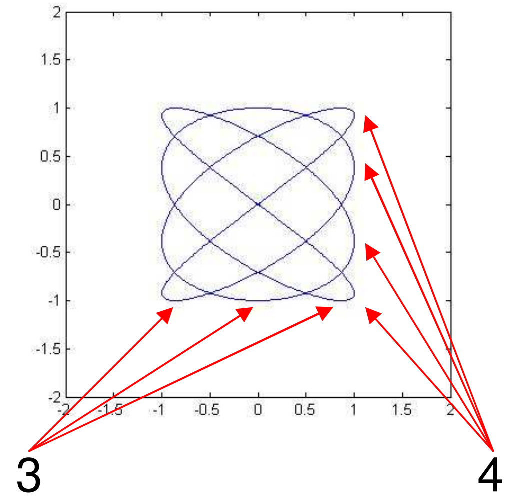

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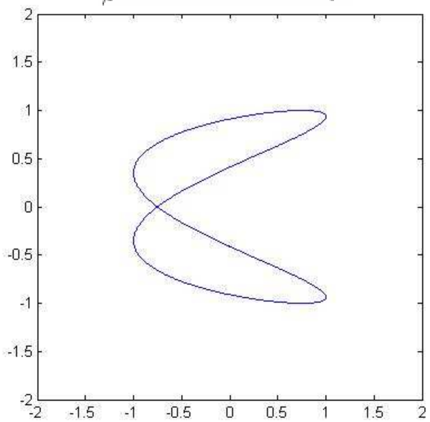
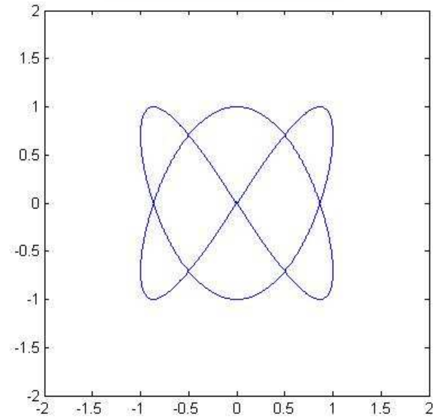
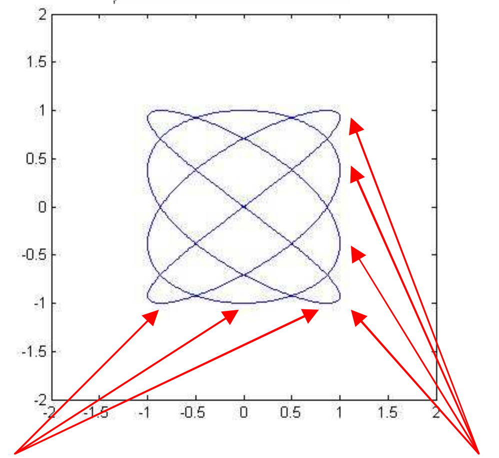


$\frac{\alpha}{\beta} = \frac{2}{3}, \phi = 0$



$\frac{\alpha}{\beta} = \frac{4}{3}, \phi = 0$



Period

$$x = Asin2\pi at \qquad y = Bsin2\pi(bt + \phi)$$

- Consider $\phi = 0$.
- Frequency ratio does not fully constrain the picture
- Ratio 2/2 will be drawn in half the time as 1/1
- Period T: time it takes to return to the starting point

Calculating the Period

- Period:

$$T = \frac{\alpha}{a} = \frac{\beta}{b}$$

- Examining units:

$$\frac{\alpha \, \frac{cycles}{frame}}{a \, \frac{cycles}{second}} = T \, \frac{seconds}{frame}$$

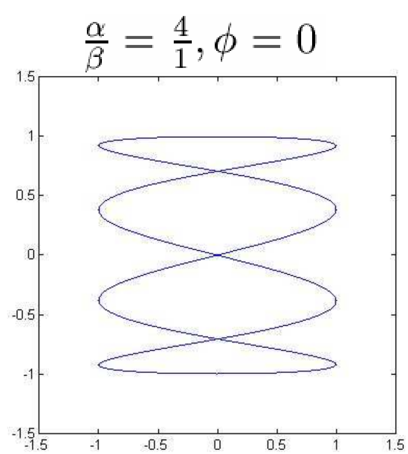
- To meet the optical refresh rate limitation:

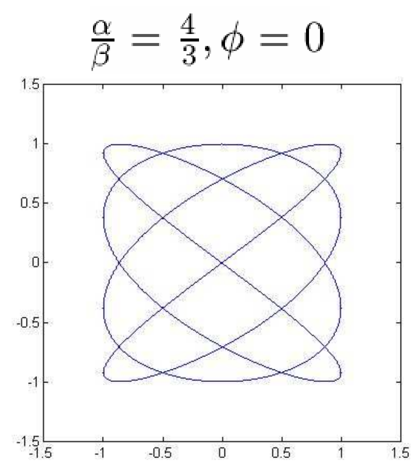
$$\frac{1}{T} > 20Hz$$

Limits of the Lazerdillo

- Lazerdillo limits the figures that can be displayed
- Neither *α* nor *β* in the simplified frequency ratio can be above 4

$$T = \frac{\alpha}{a} = \frac{4\,\frac{cycles}{frame}}{80Hz}$$
$$= \frac{1}{20} second$$

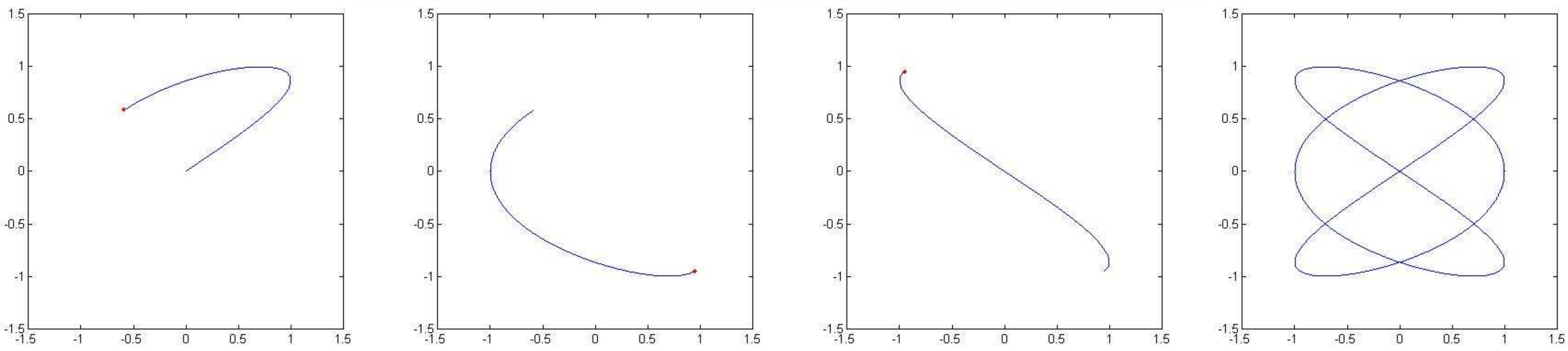




Fusion Failure

- For large T, “frames” are not full figures
- Snapshots below for T>>1/20 sec
- Full figure shown for comparison

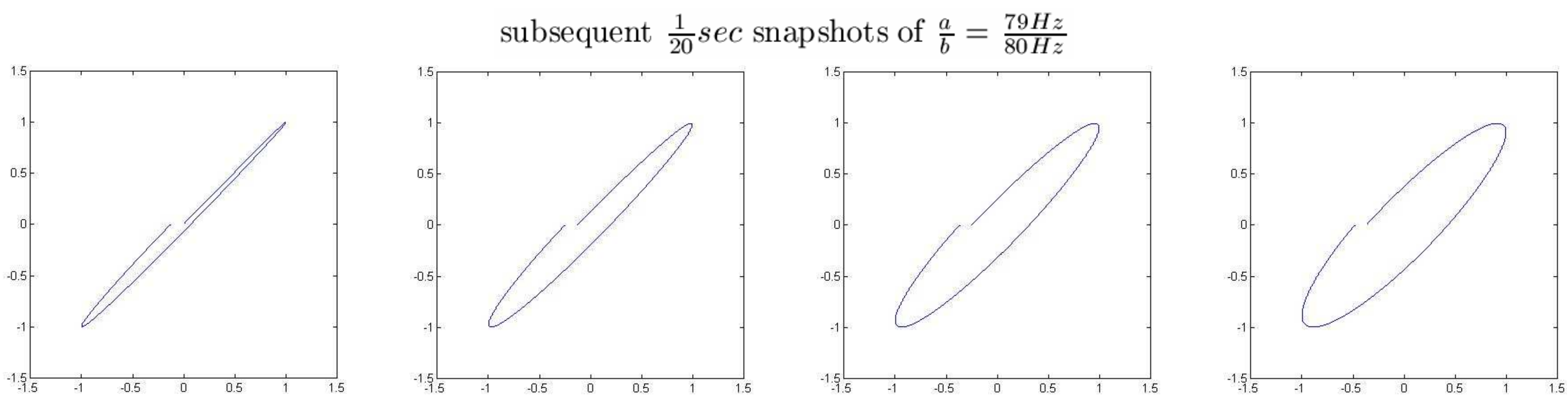
subsequent $\frac{1}{20}sec$ snapshots of $\frac{a}{b} = \frac{12Hz}{8Hz}, \phi = 0$ $(T = \frac{1}{4}sec)$



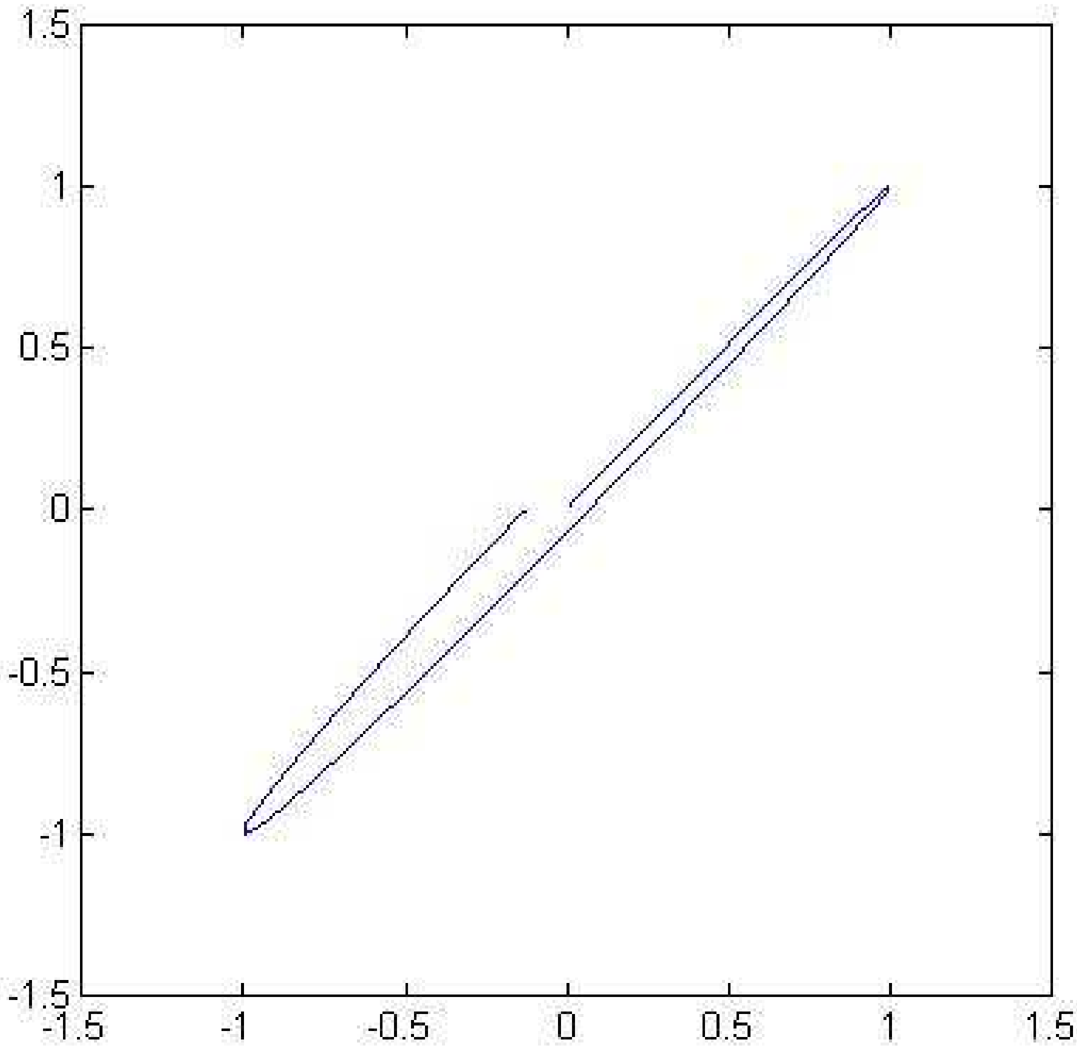
Rotation

- Below: successive images from Lissajous figure with ratio 79/80 (T = 1).

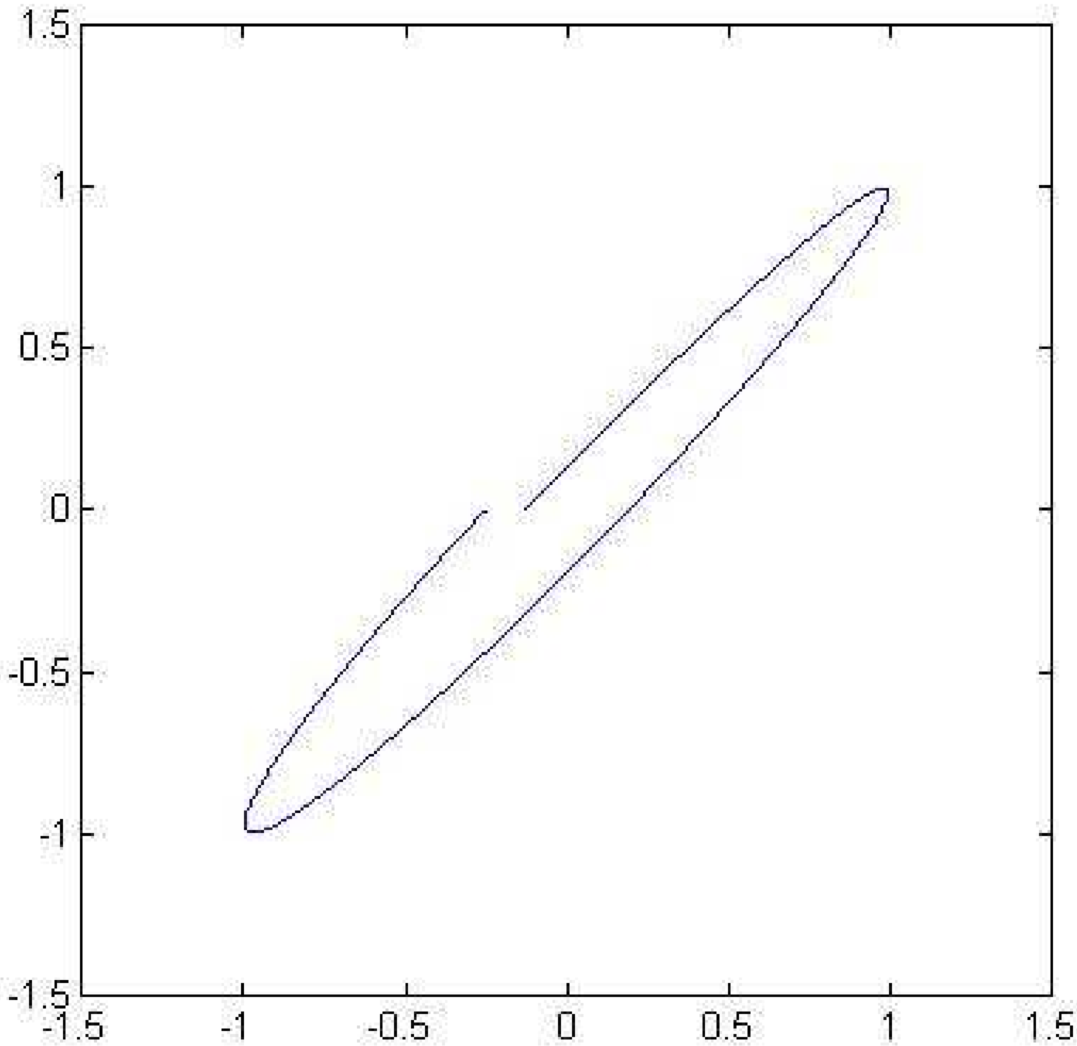
- The frames move smoothly and the figure appears to rotate!



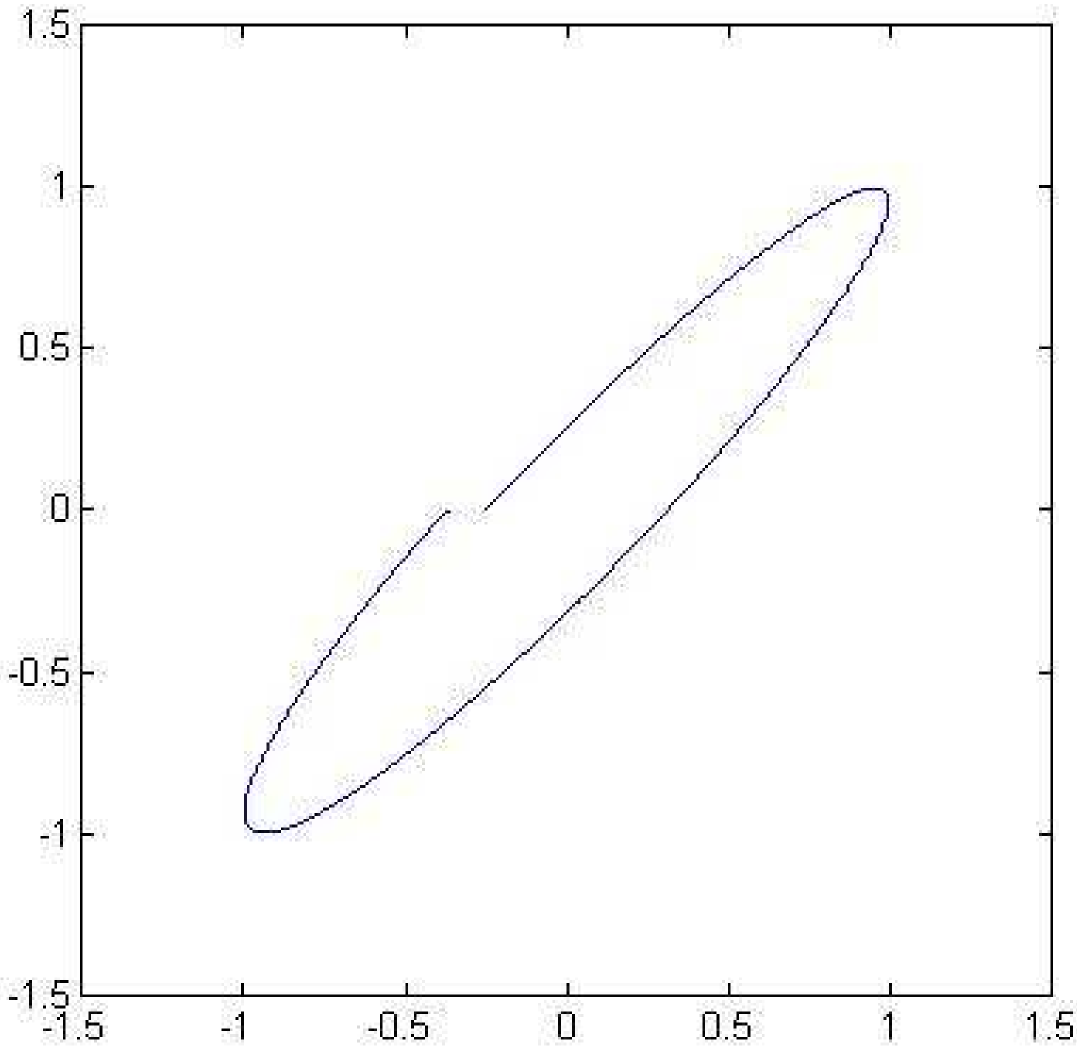
Rotation Example



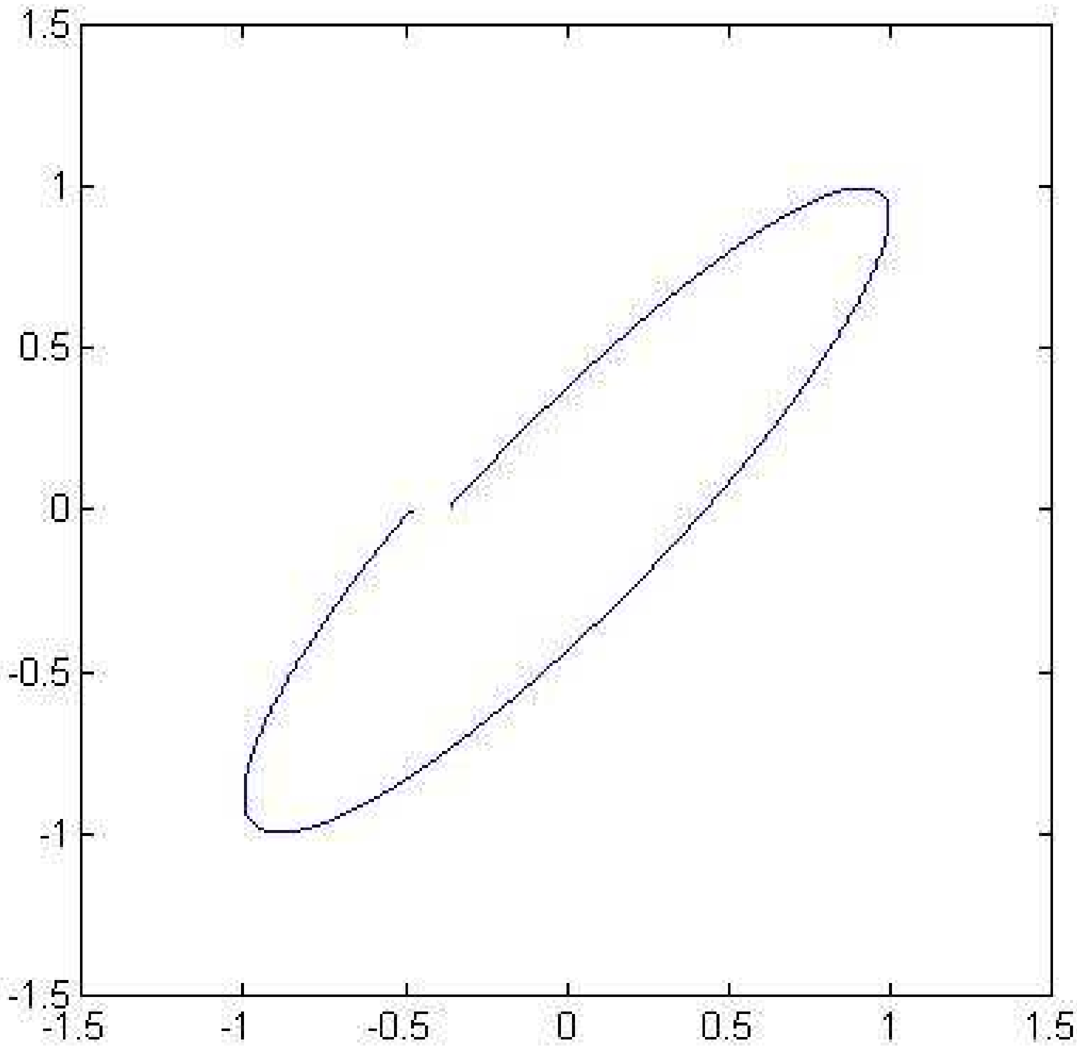
Rotation Example



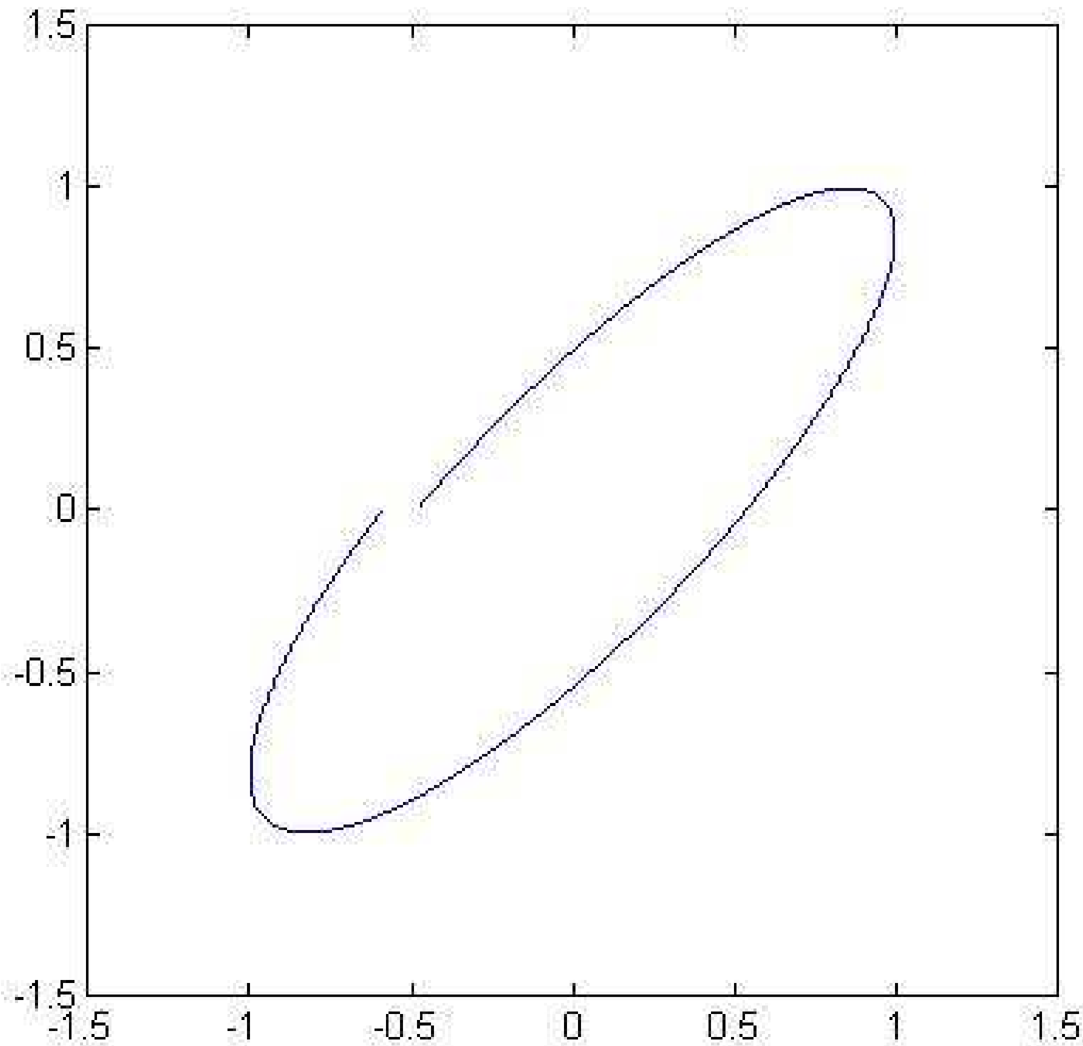
Rotation Example



Rotation Example



Rotation Example



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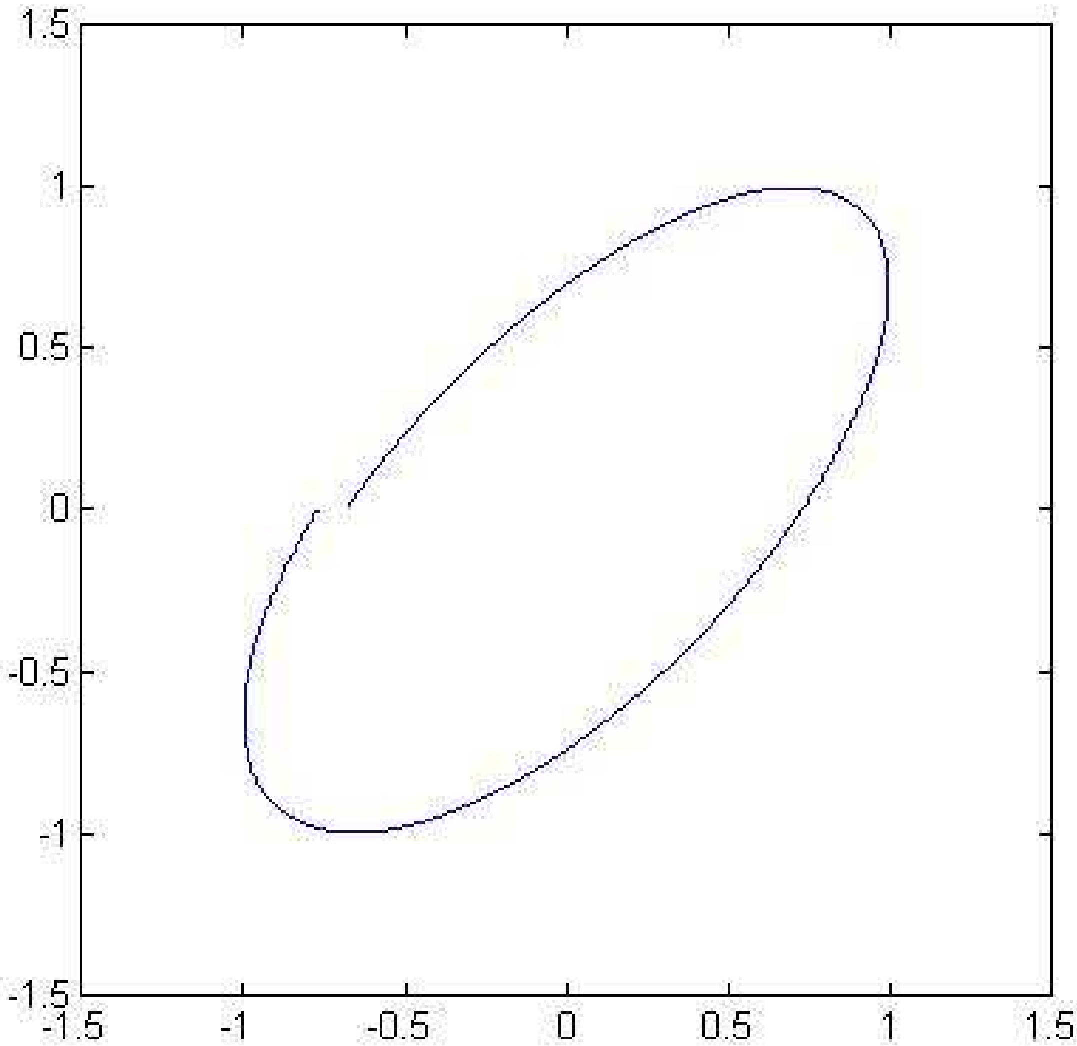
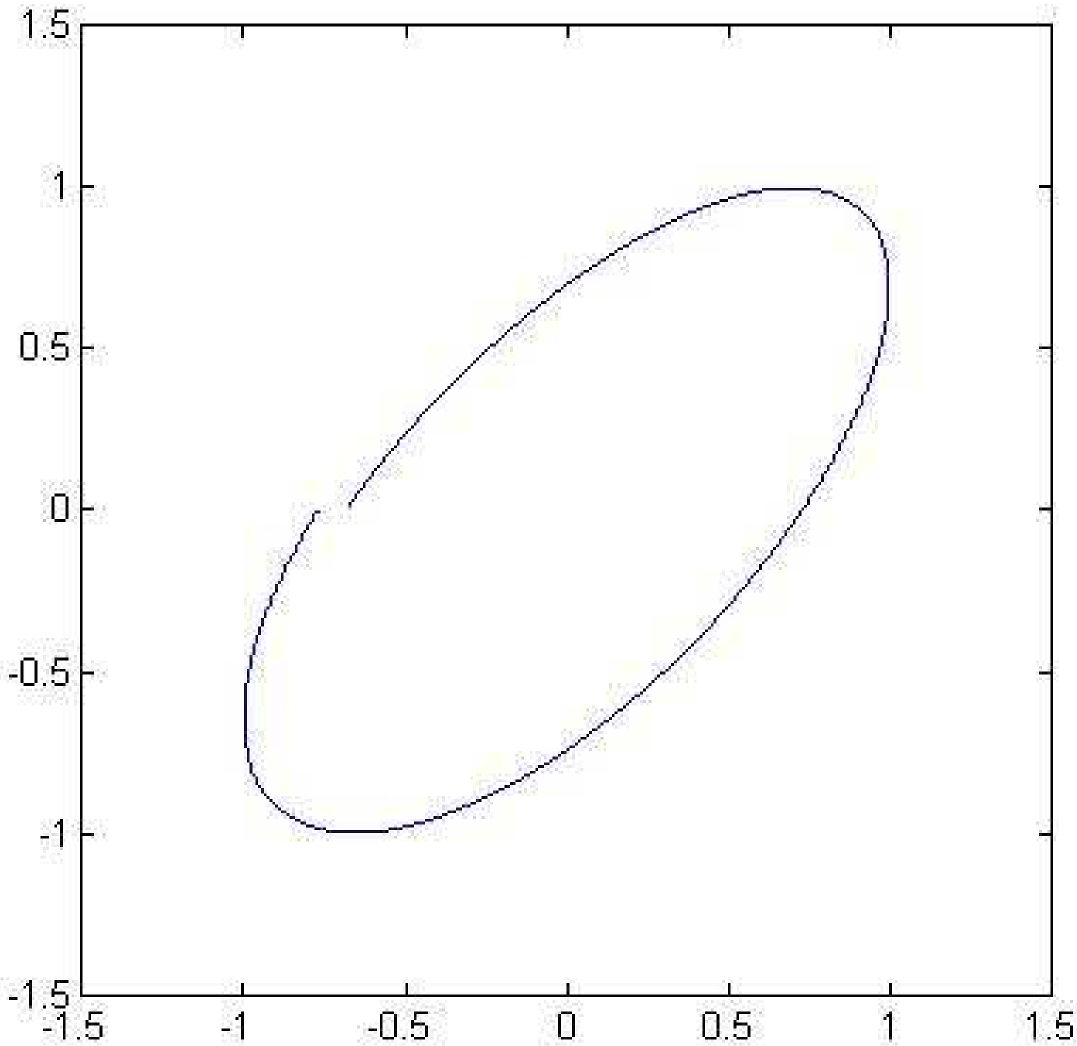
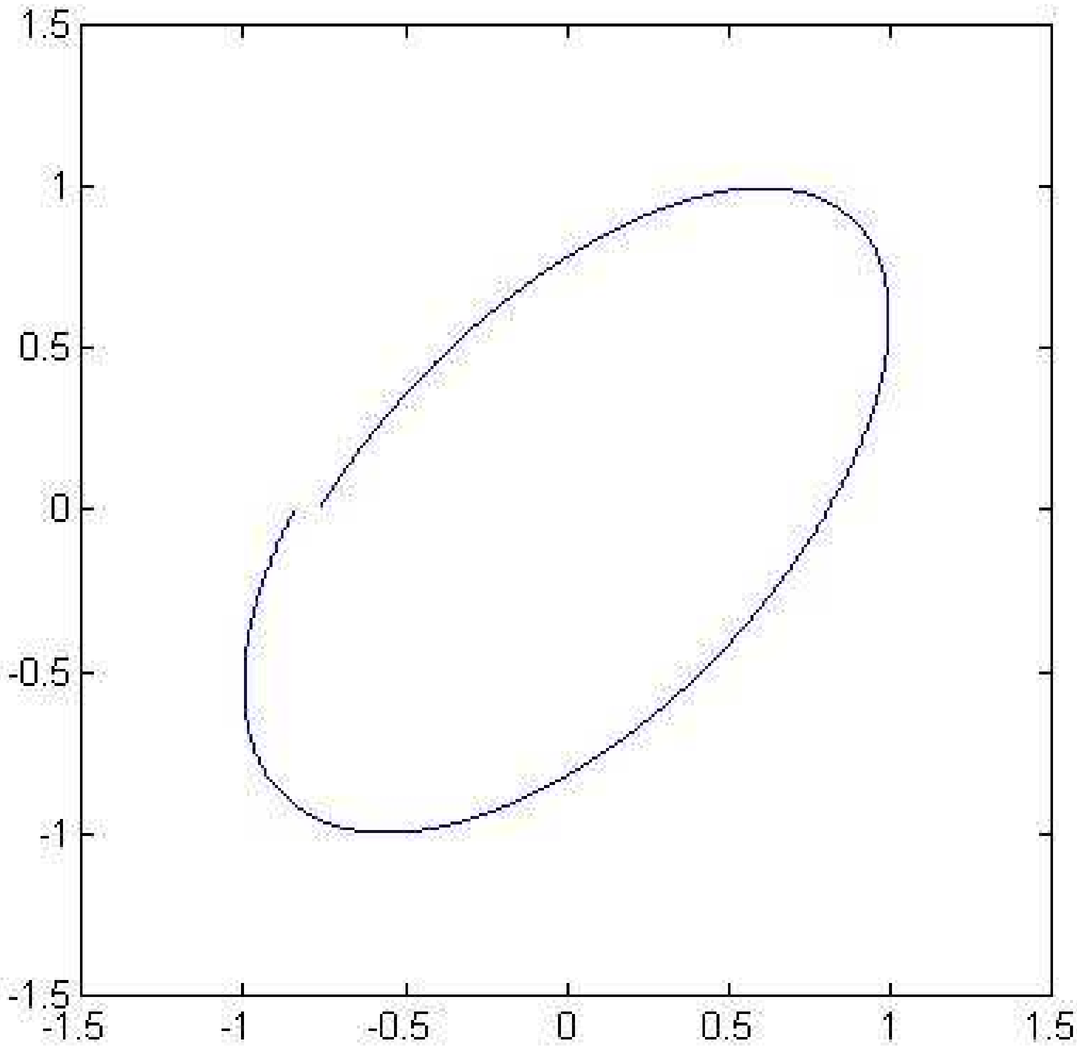


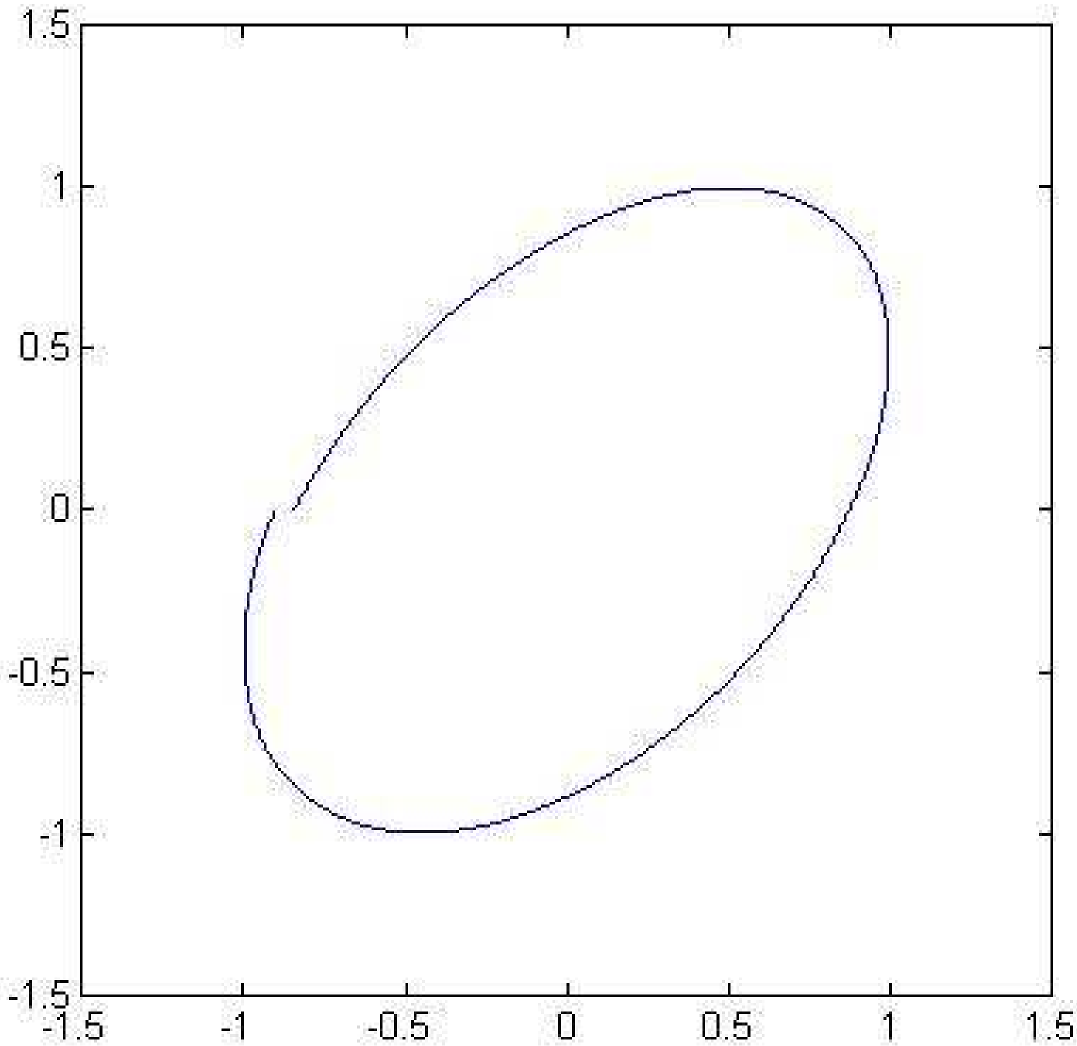
Rotation Example



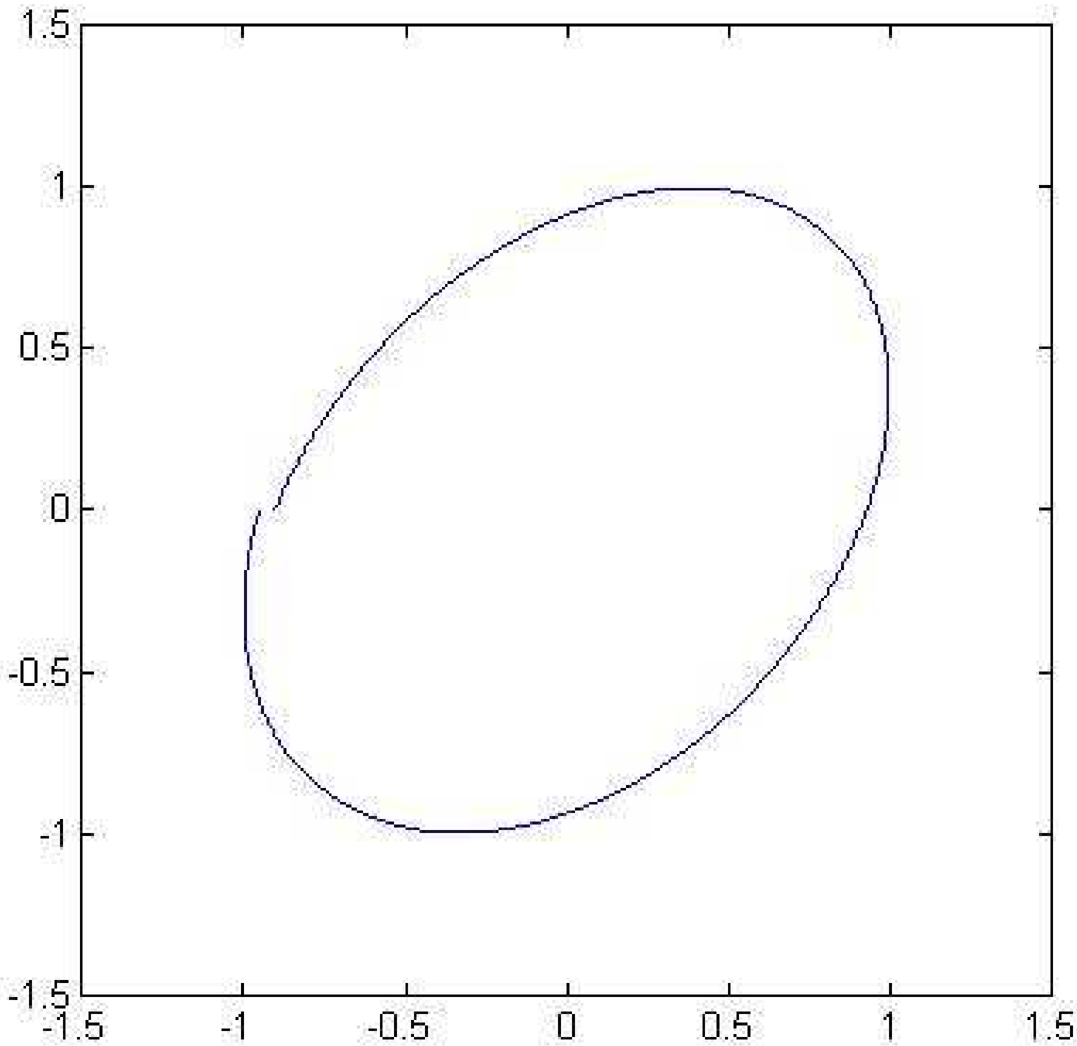
Rotation Example



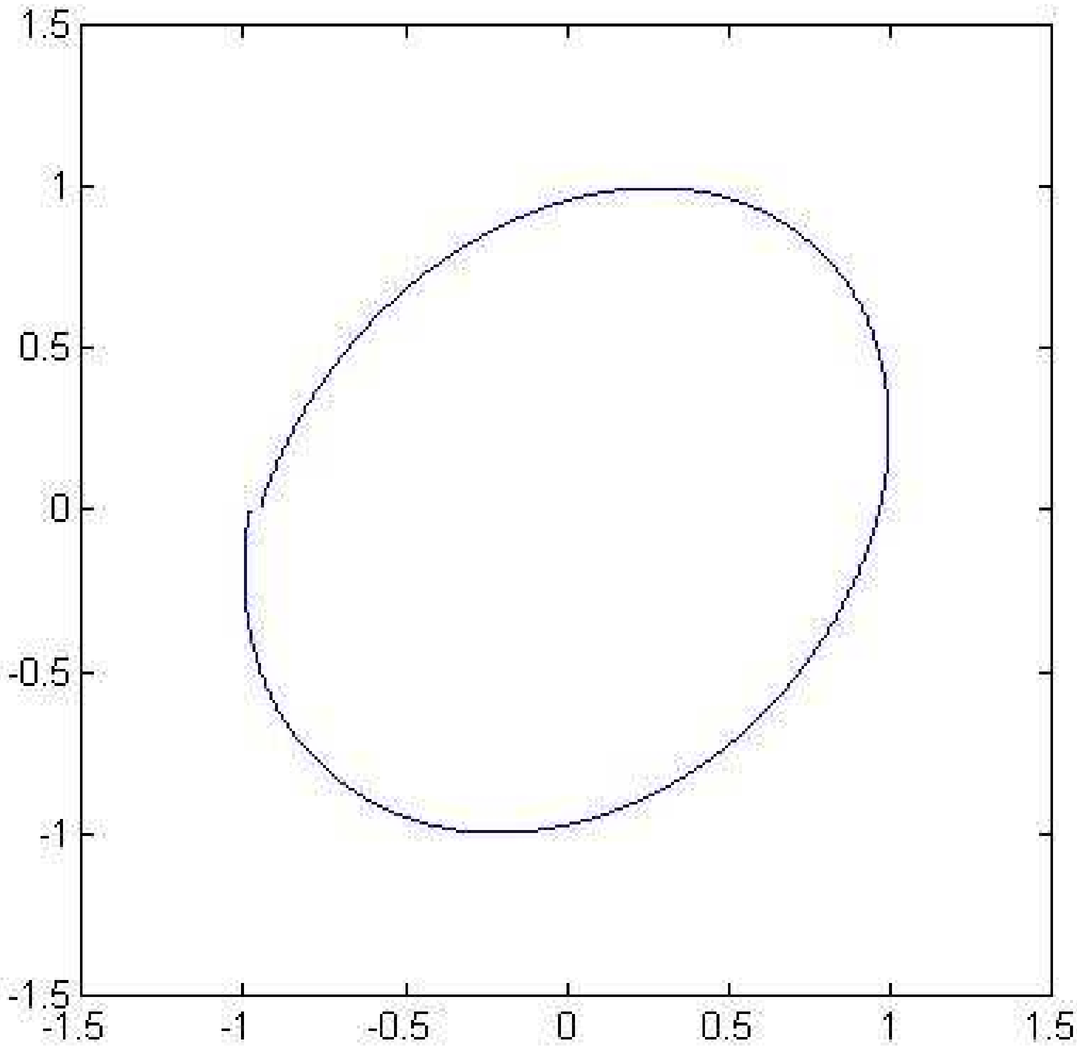
Rotation Example



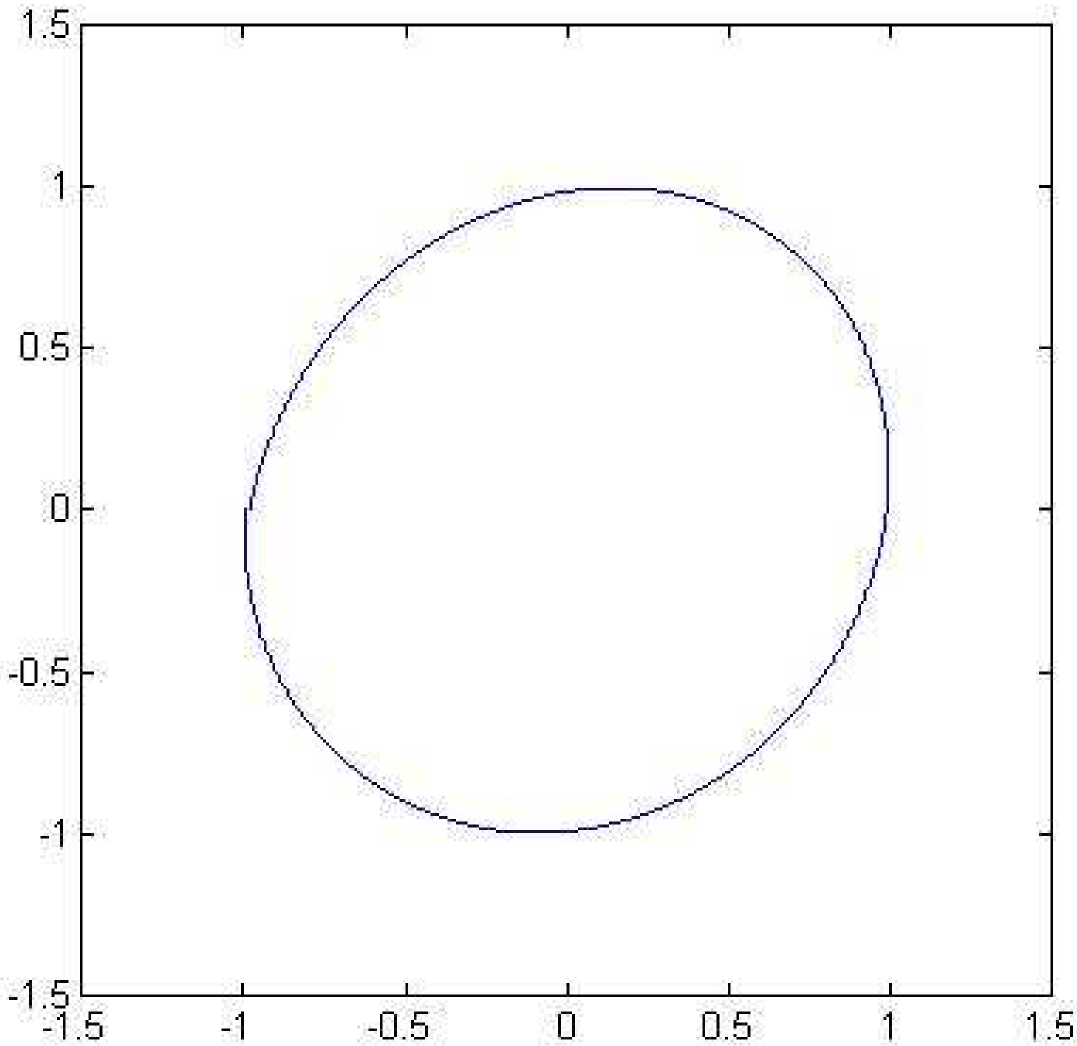
Rotation Example



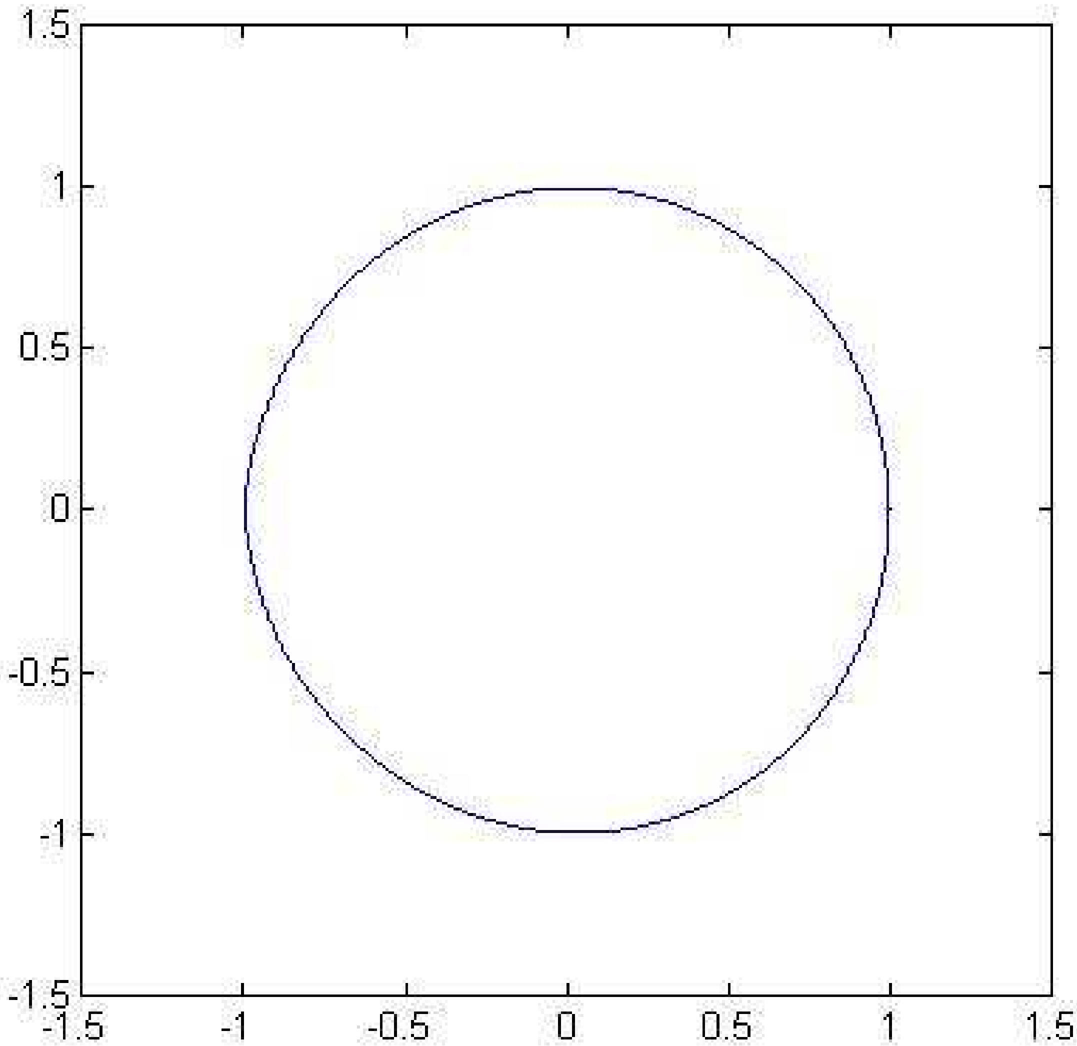
Rotation Example



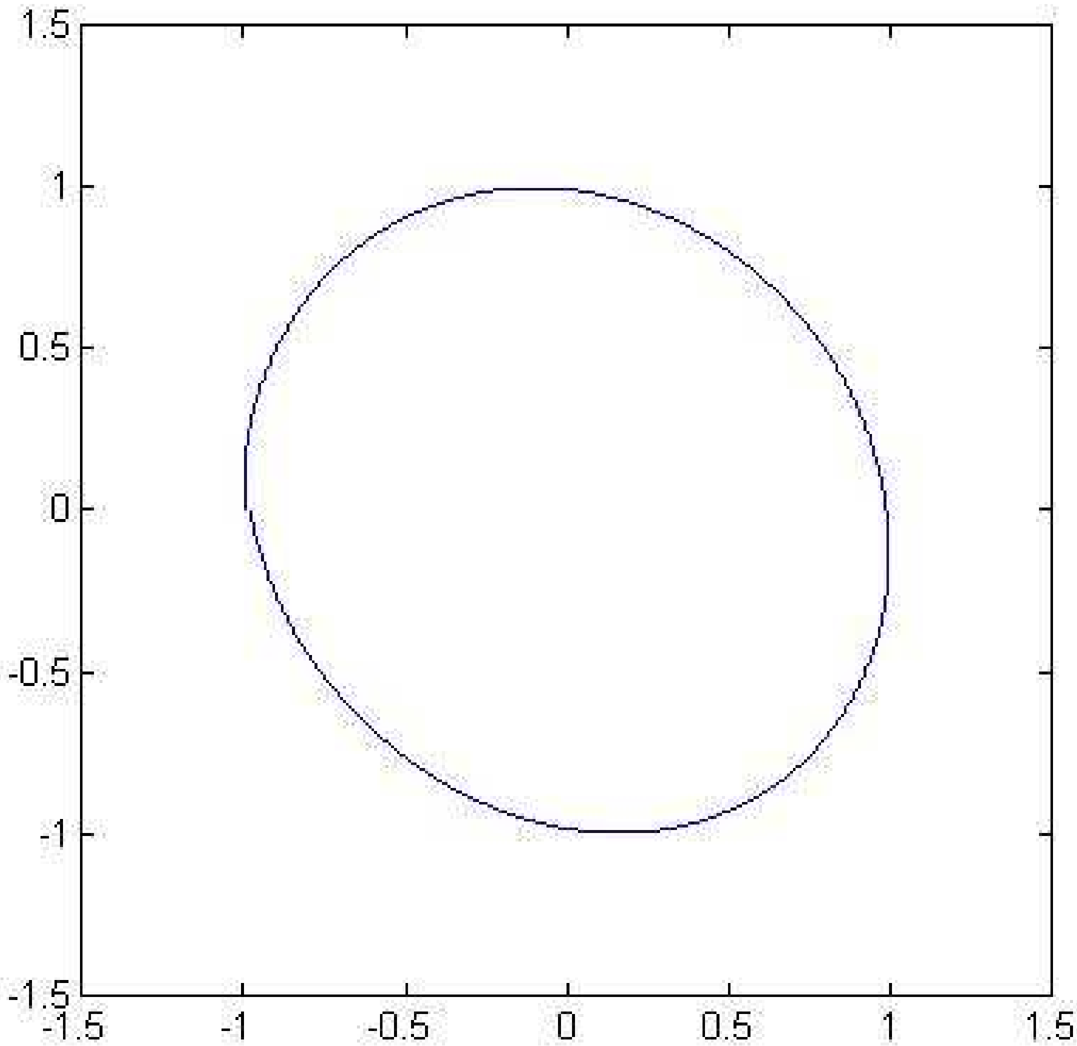
Rotation Example



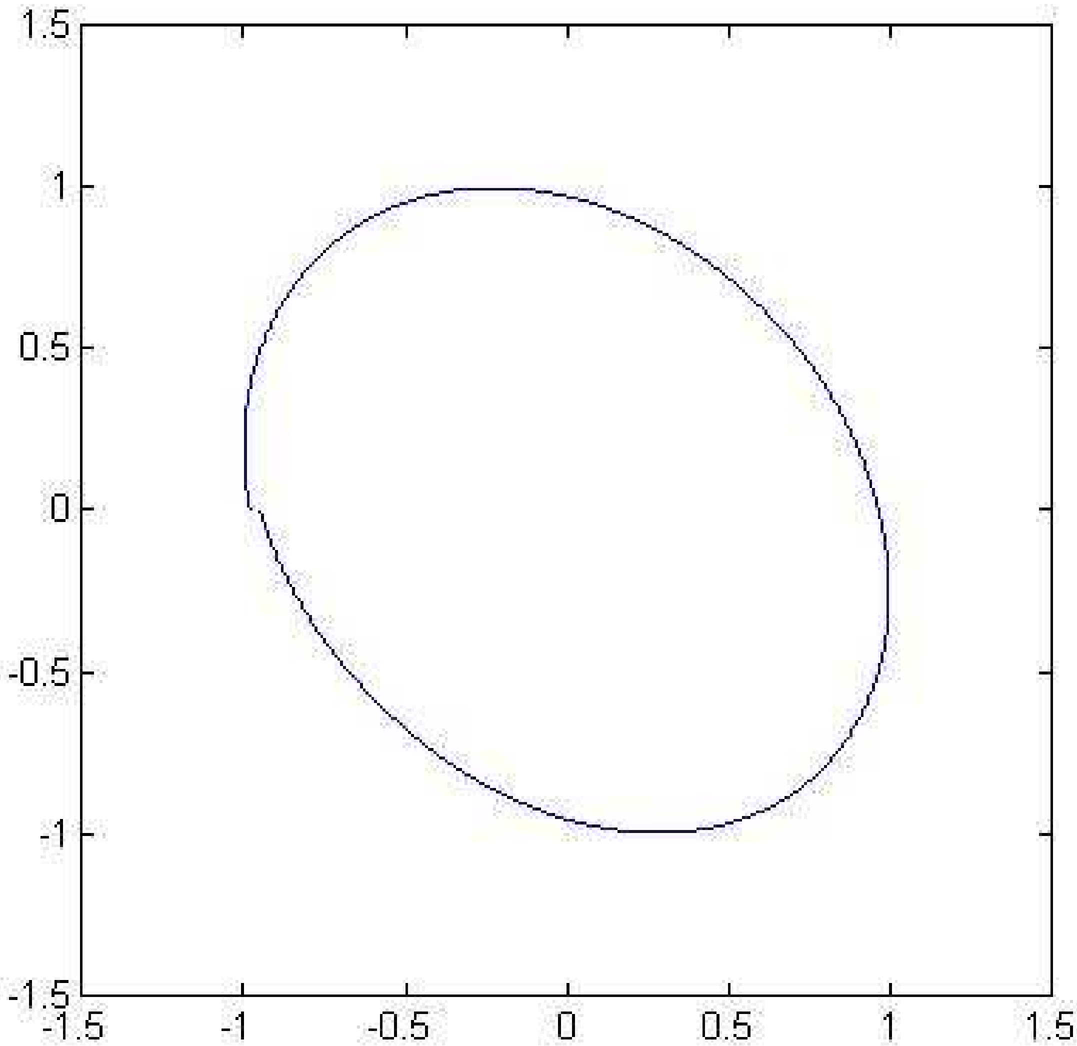
Rotation Example



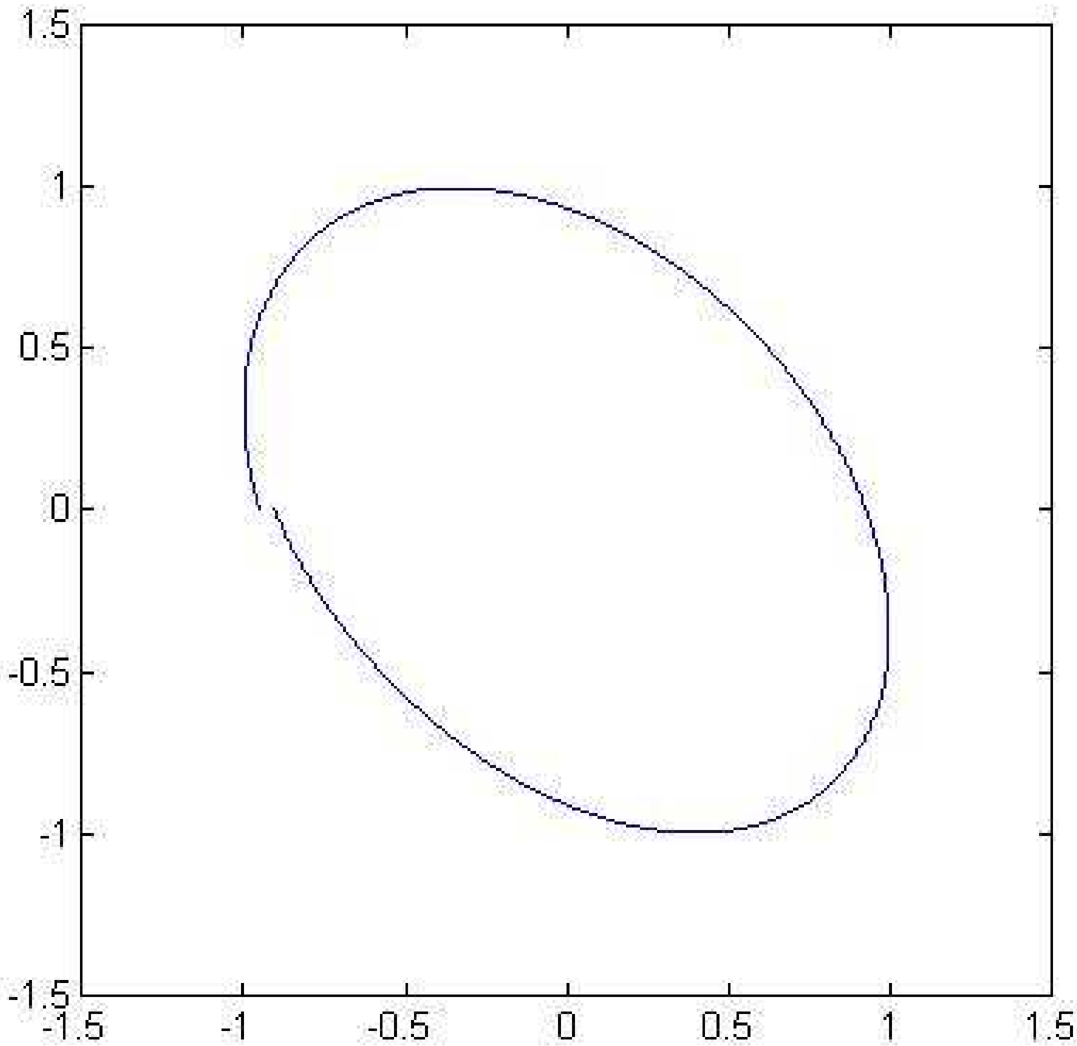
Rotation Example



Rotation Example



Rotation Example



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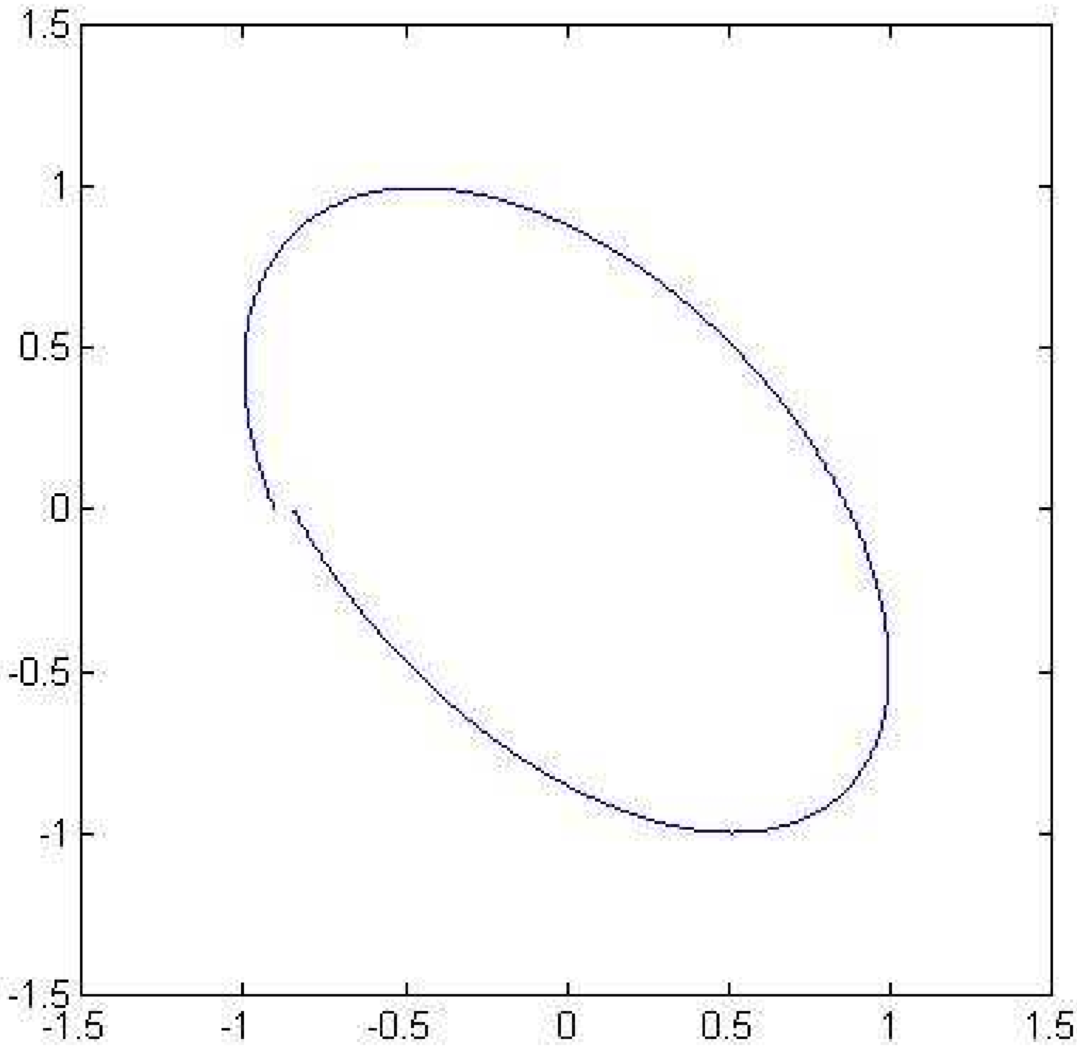
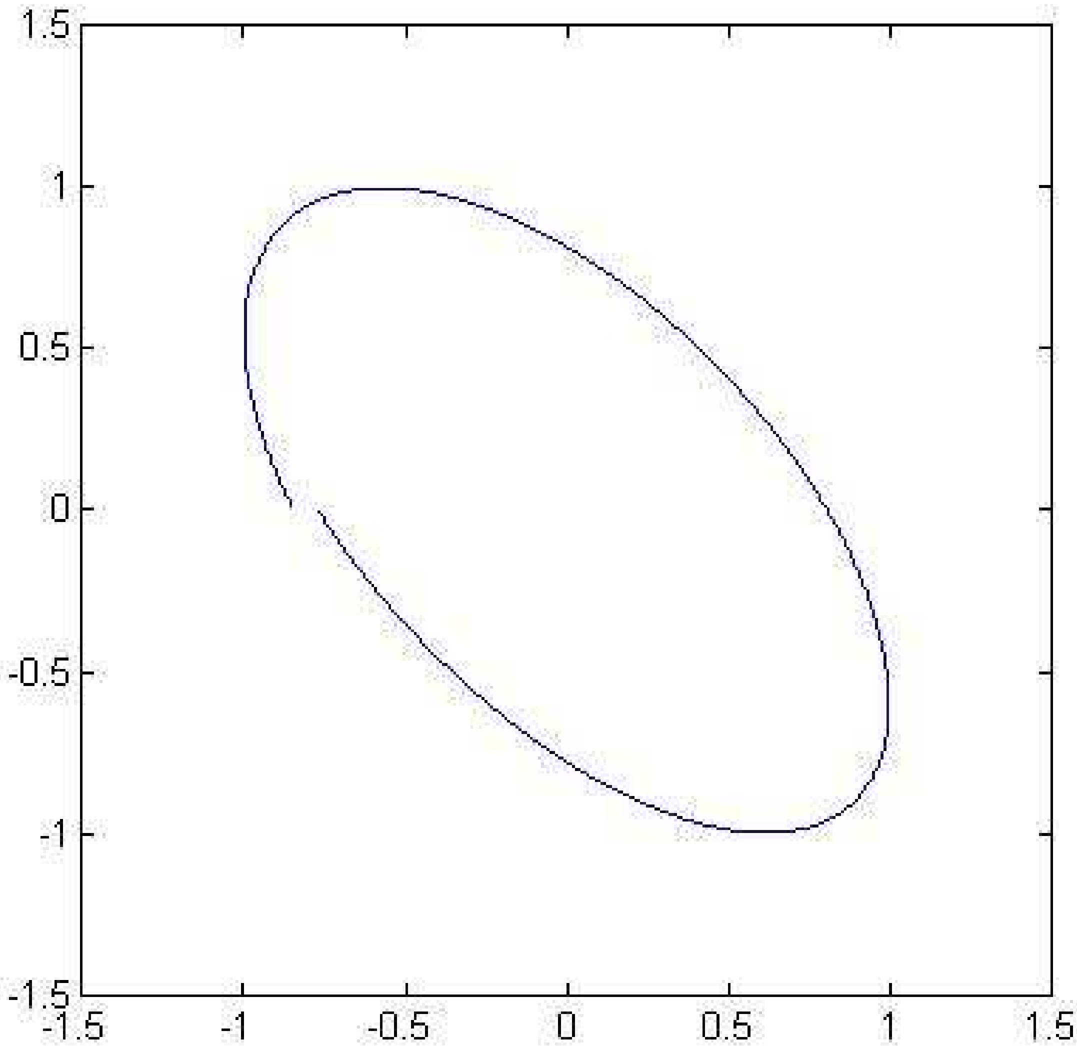
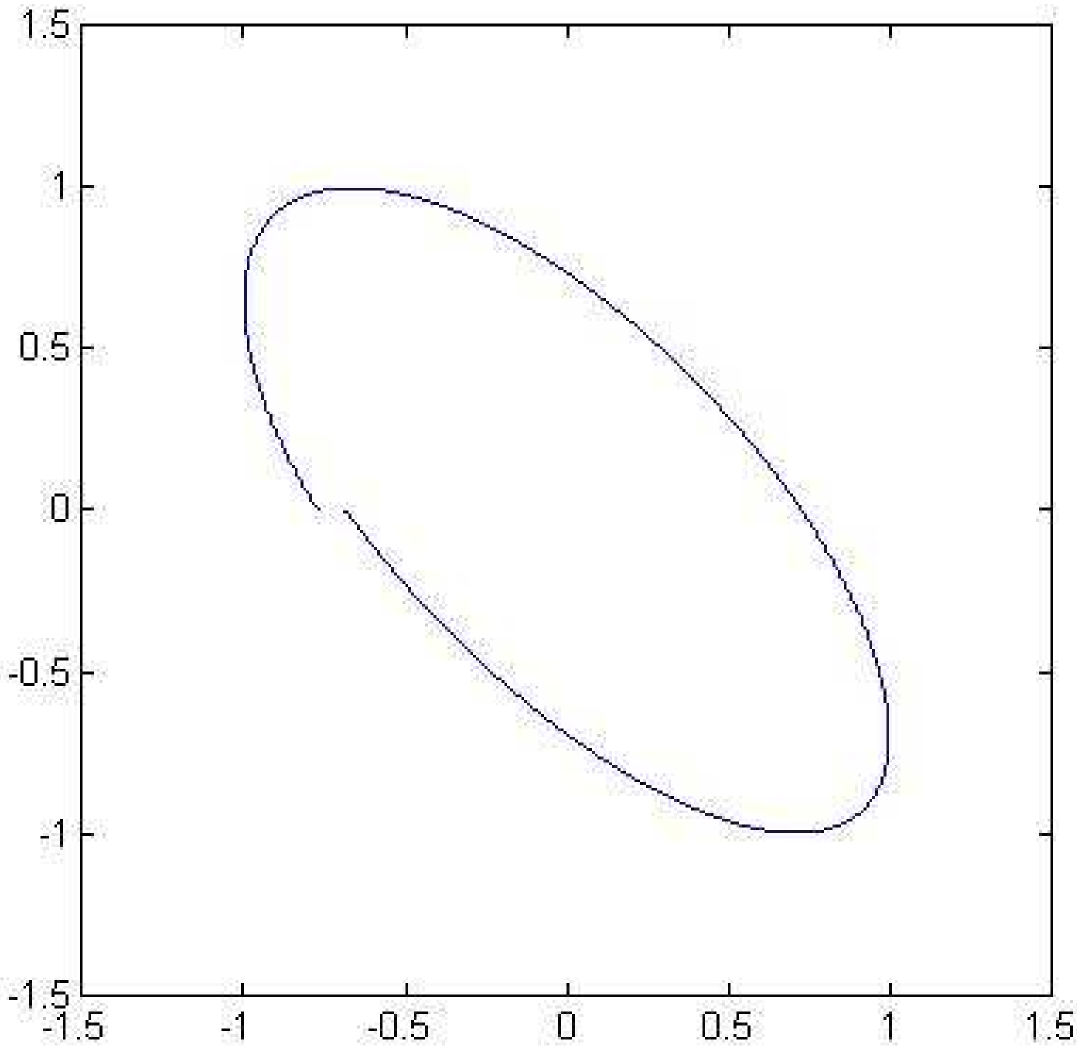


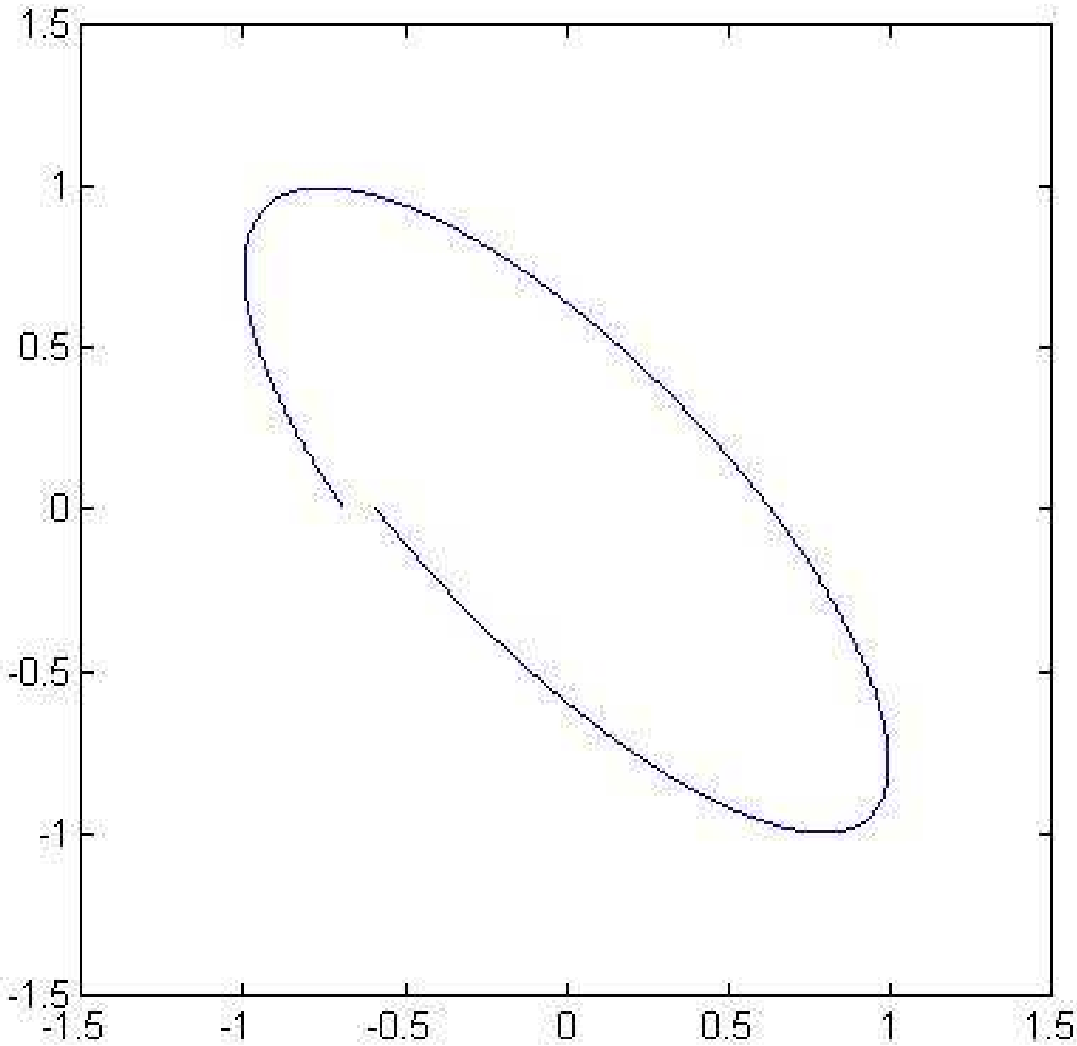
Rotation Example



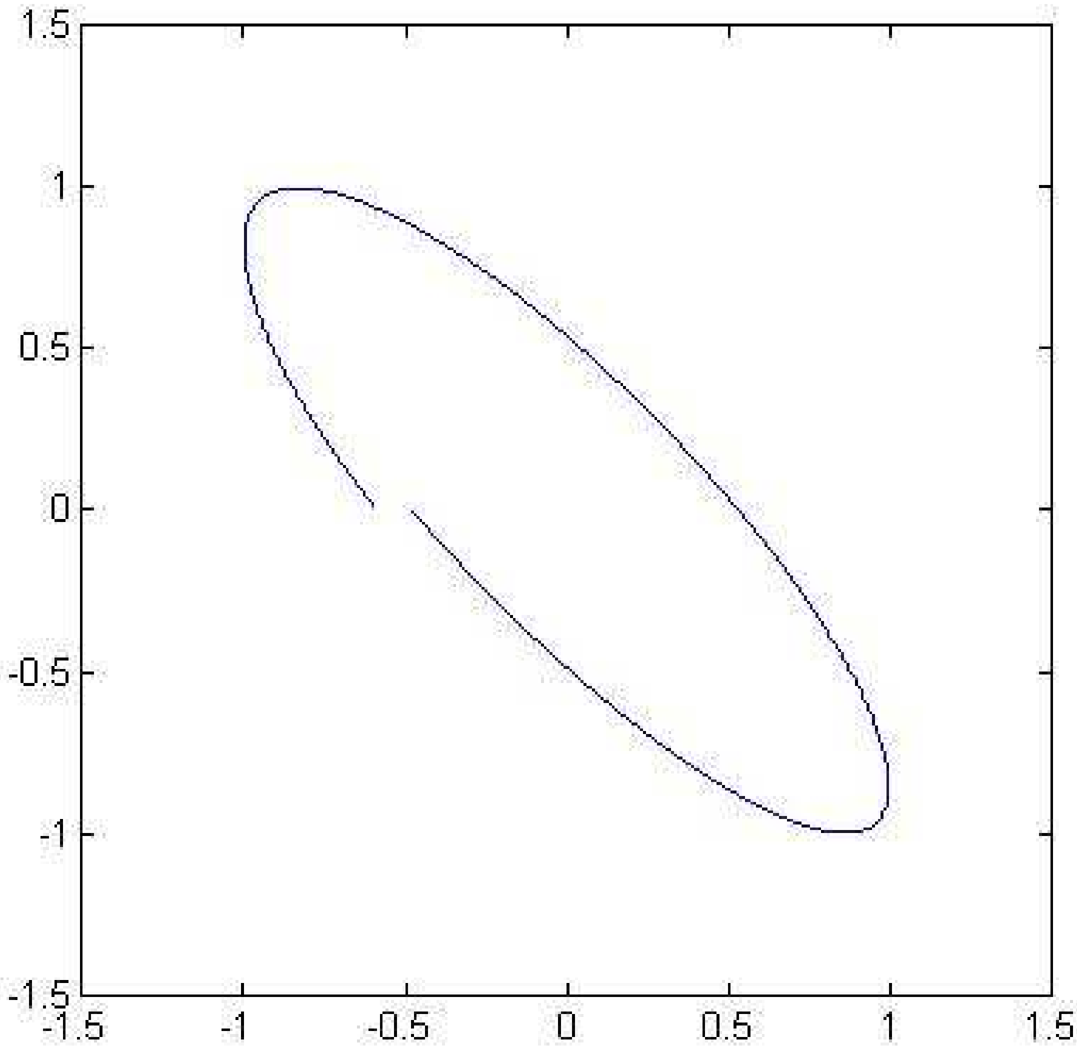
Rotation Example



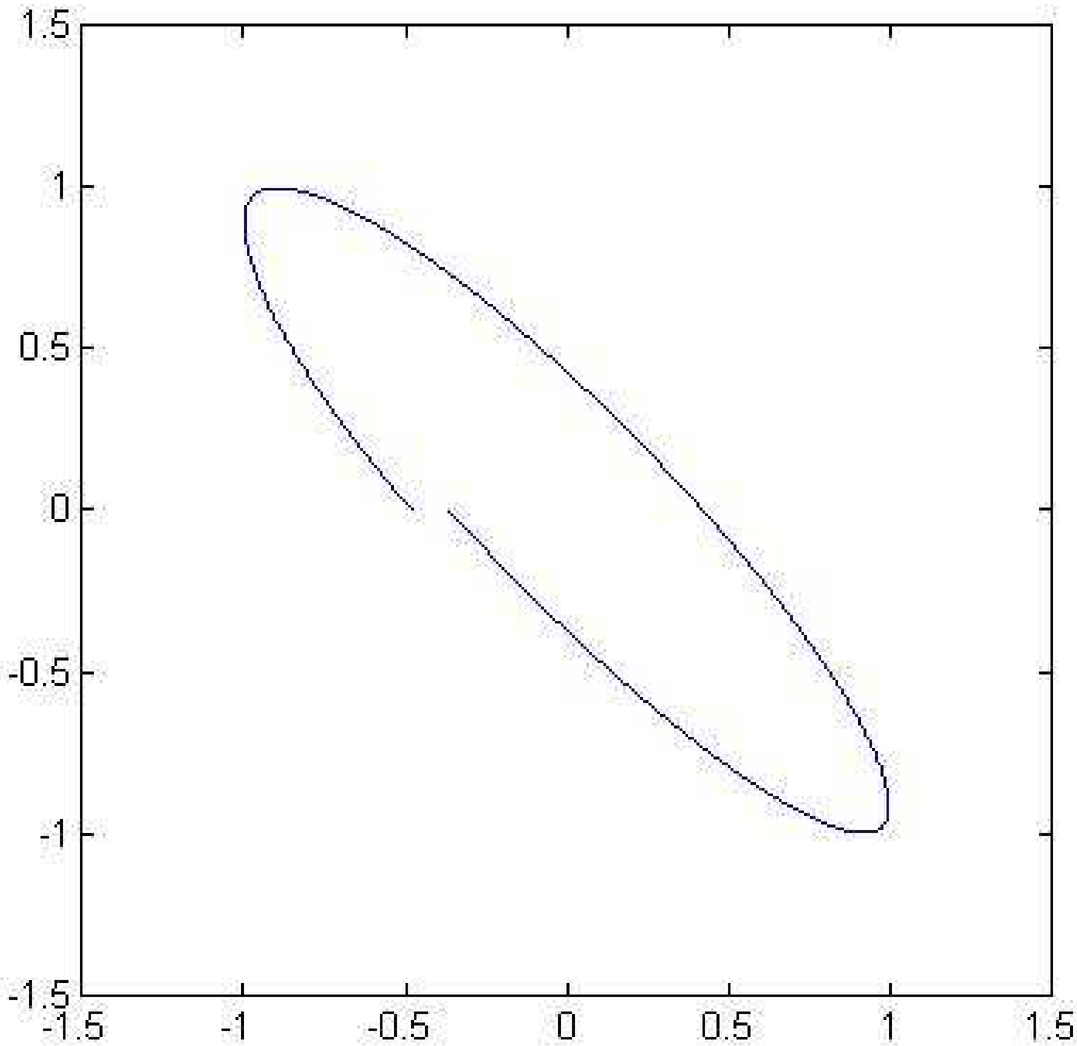
Rotation Example



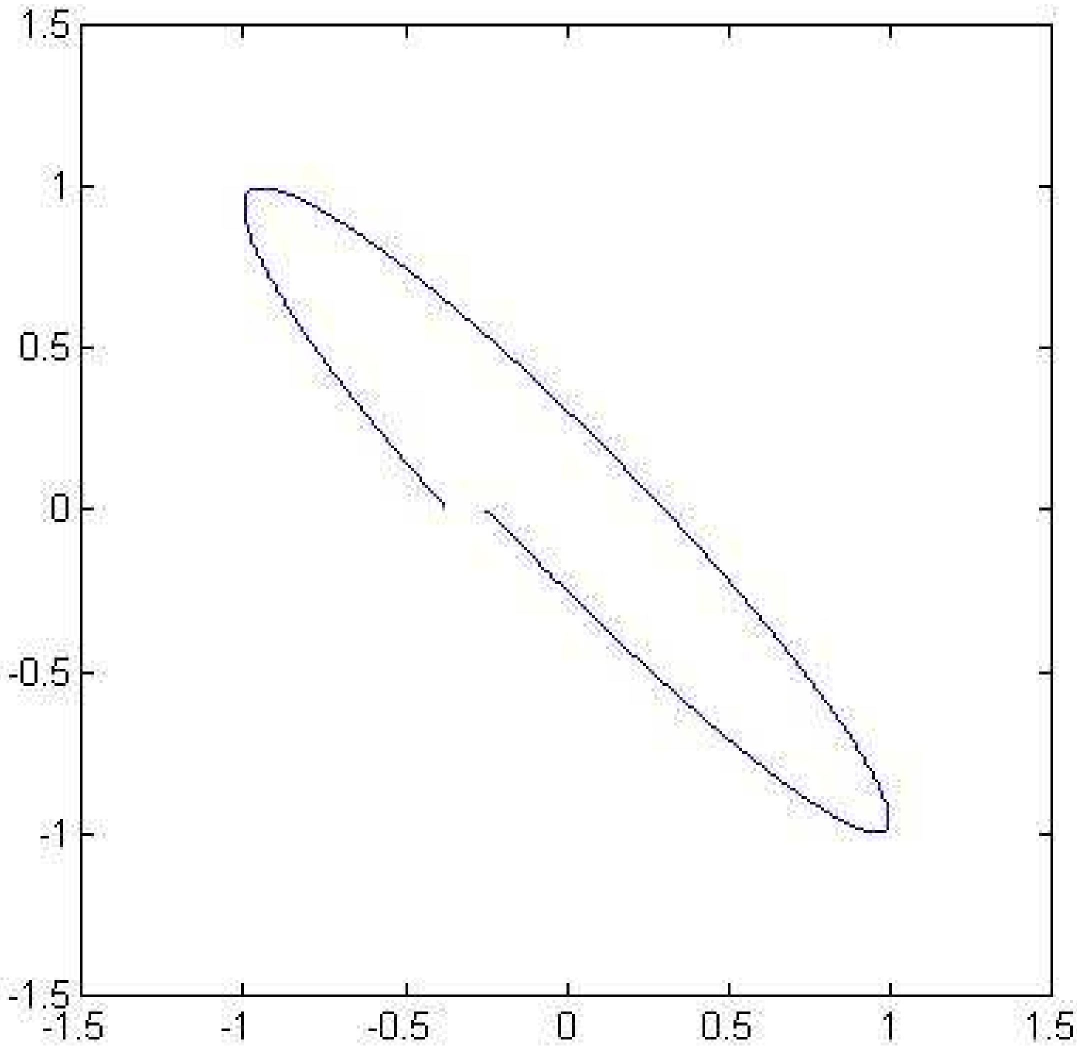
Rotation Example



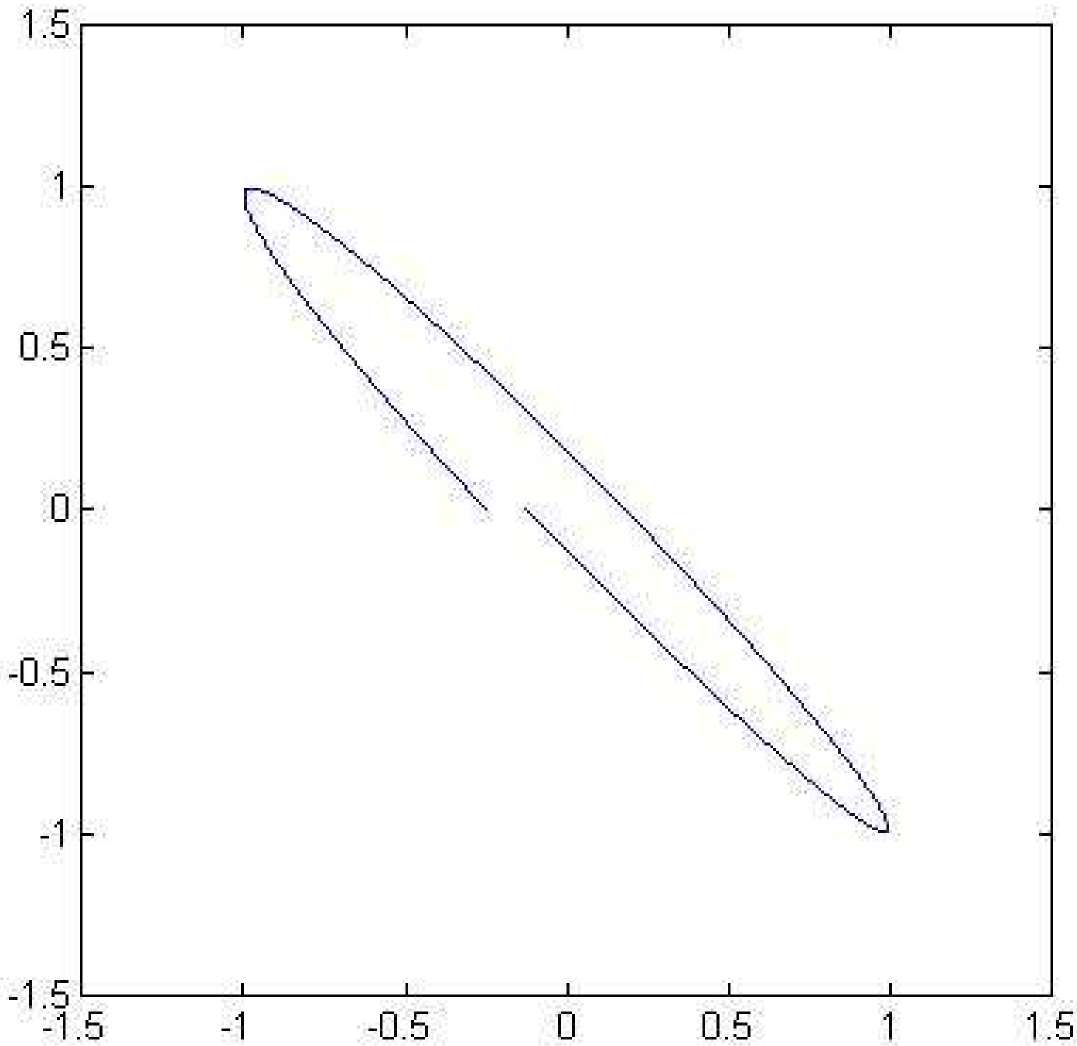
Rotation Example



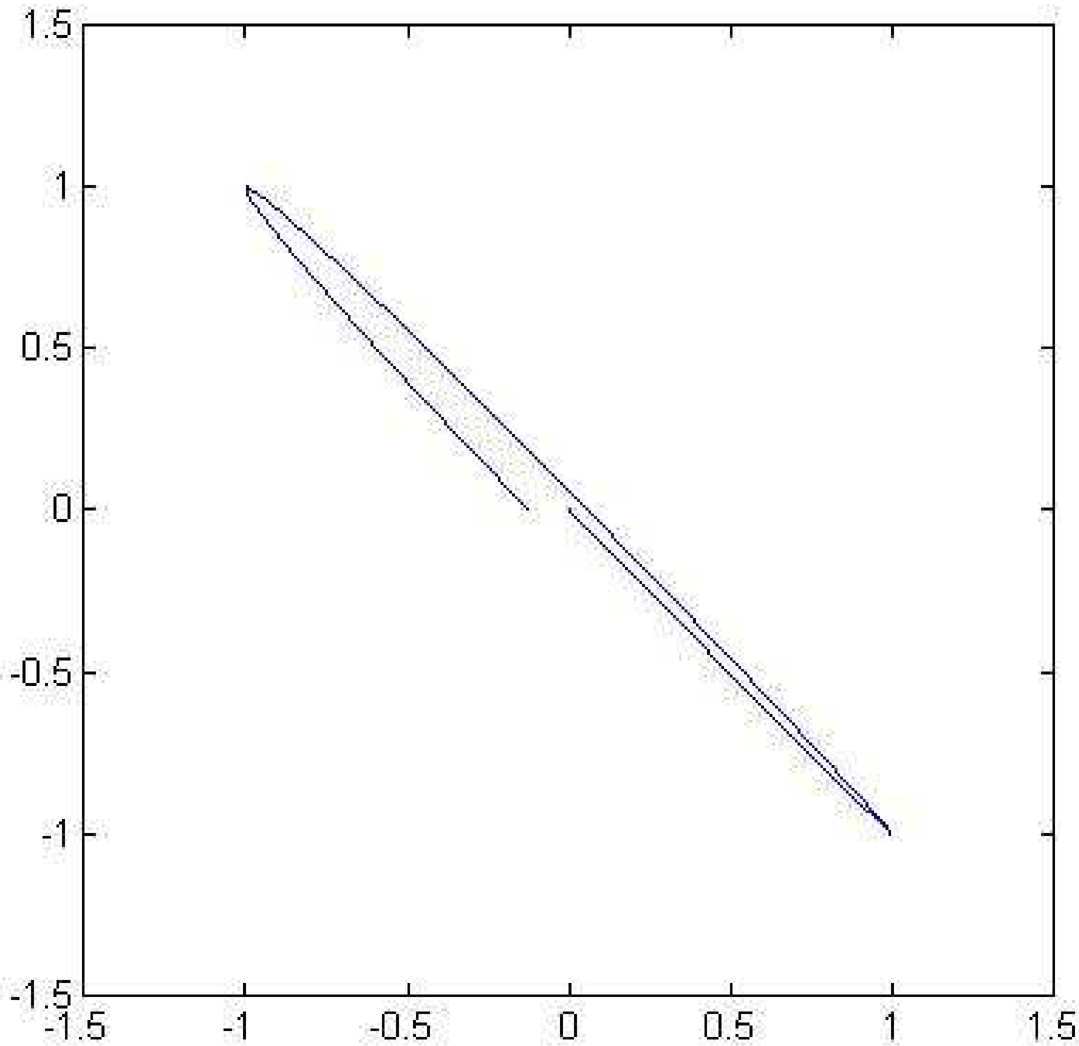
Rotation Example



Rotation Example



Rotation Example



Rotation: Another Look

- Why does rotation occur?

- If we look at the equations of the figure in the previous example:

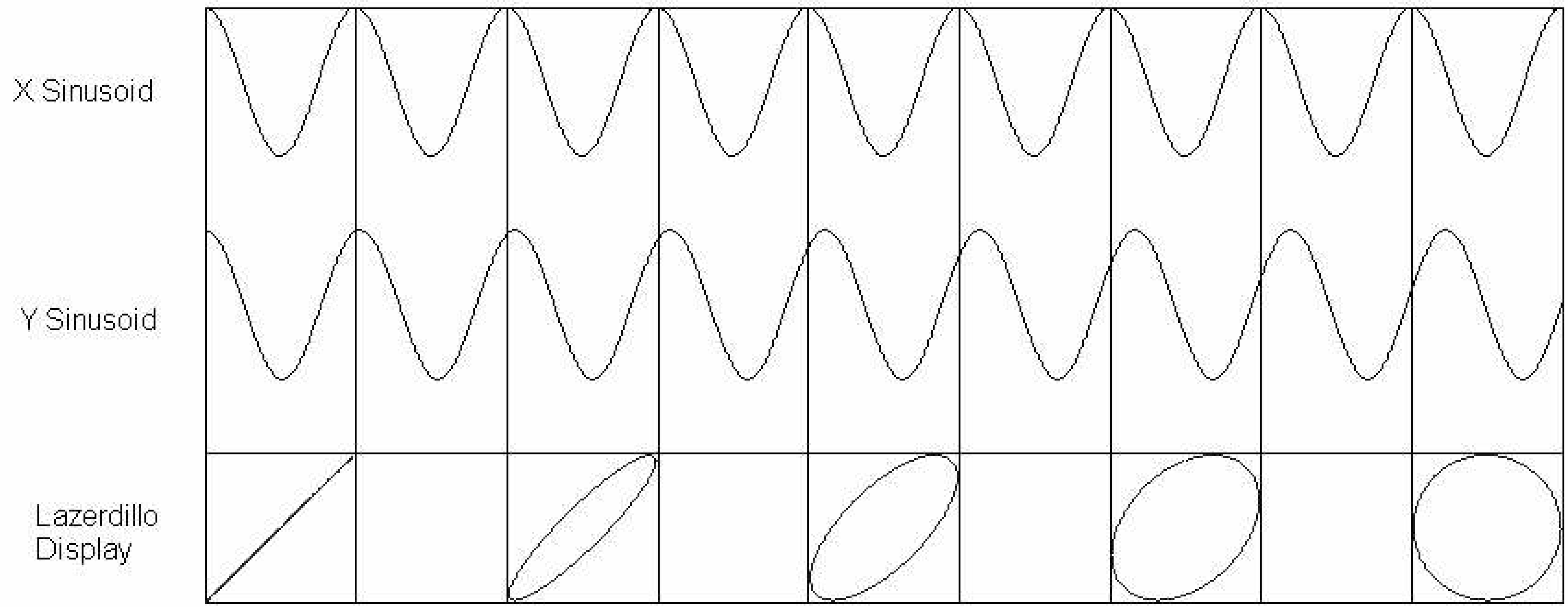
$$x = \sin 2\pi 79t \qquad y = \sin 2\pi 80t$$

$$x = \sin 2\pi 79t \qquad y = \sin 2\pi(79t + t)$$

$$x = \sin 2\pi at \qquad y = \sin 2\pi(bt + \phi(t))$$

- Small frequency offset becomes time-varying phase!

Rotating Figures



- Sinusoids have a slightly different frequency
- Frame by frame they appear to have identical frequencies with different instantaneous phase offsets

Rotation

a	80	80	80	80
b	80	79.9	79	75
α/β	1/1	1/1	1/1	1/1
$\phi(t)$	0	-0.1*t	-t	-5*t
Rotation Frequency	0 Hz	0.1 Hz	1 Hz	5 Hz
Rotation Period	0 sec	10 sec	1 sec	.2 sec

- Rotation frequencies above 1 Hz are too fast for the eye and the image blurs

Modes of Rotation

- Lets try a different example with a different base frequency ratio, say 2/3

$$x = \sin 2\pi 54t \qquad y = \sin 2\pi 80t$$

$$x = \sin 2\pi 54t \qquad y = \sin 2\pi (81t - t)$$

$$\phi_y(t) = -t \qquad frequency = 1Hz$$

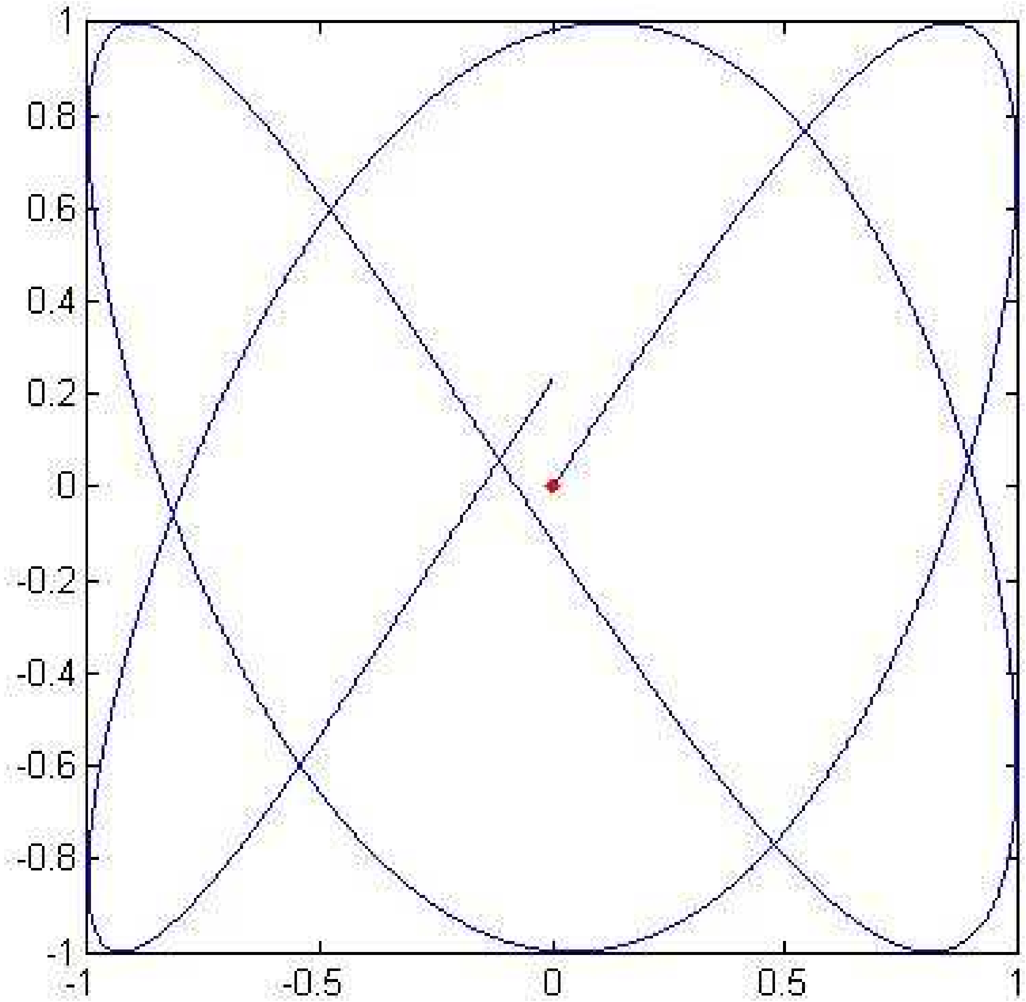
$$x = \sin 2\pi 54t \qquad y = \sin 2\pi 80t$$

$$x = \sin 2\pi (\tfrac{160}{3}t + \tfrac{2}{3}t) \qquad y = \sin 2\pi 80t$$

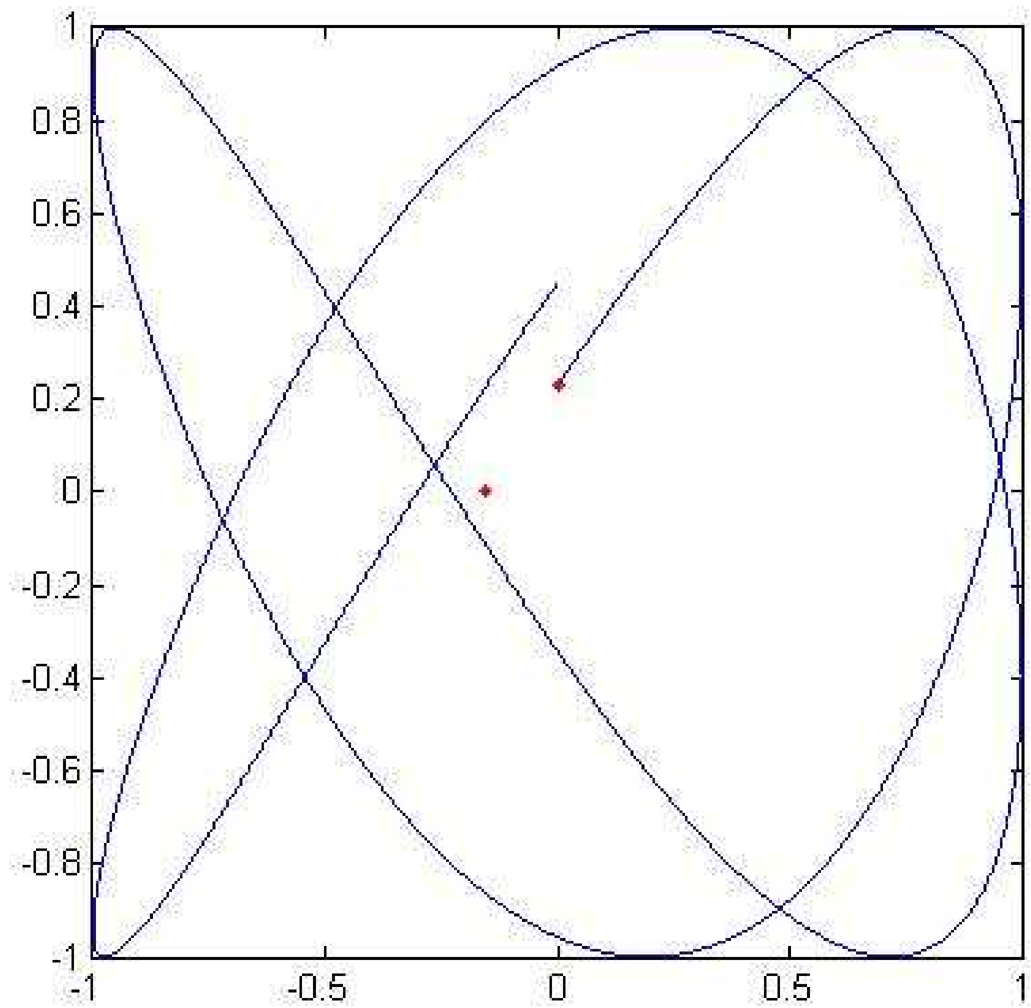
$$\phi_x(t) = \tfrac{2}{3}t \qquad frequency = 2/3Hz$$

- In this example the rotation frequency is different about the x and y axes

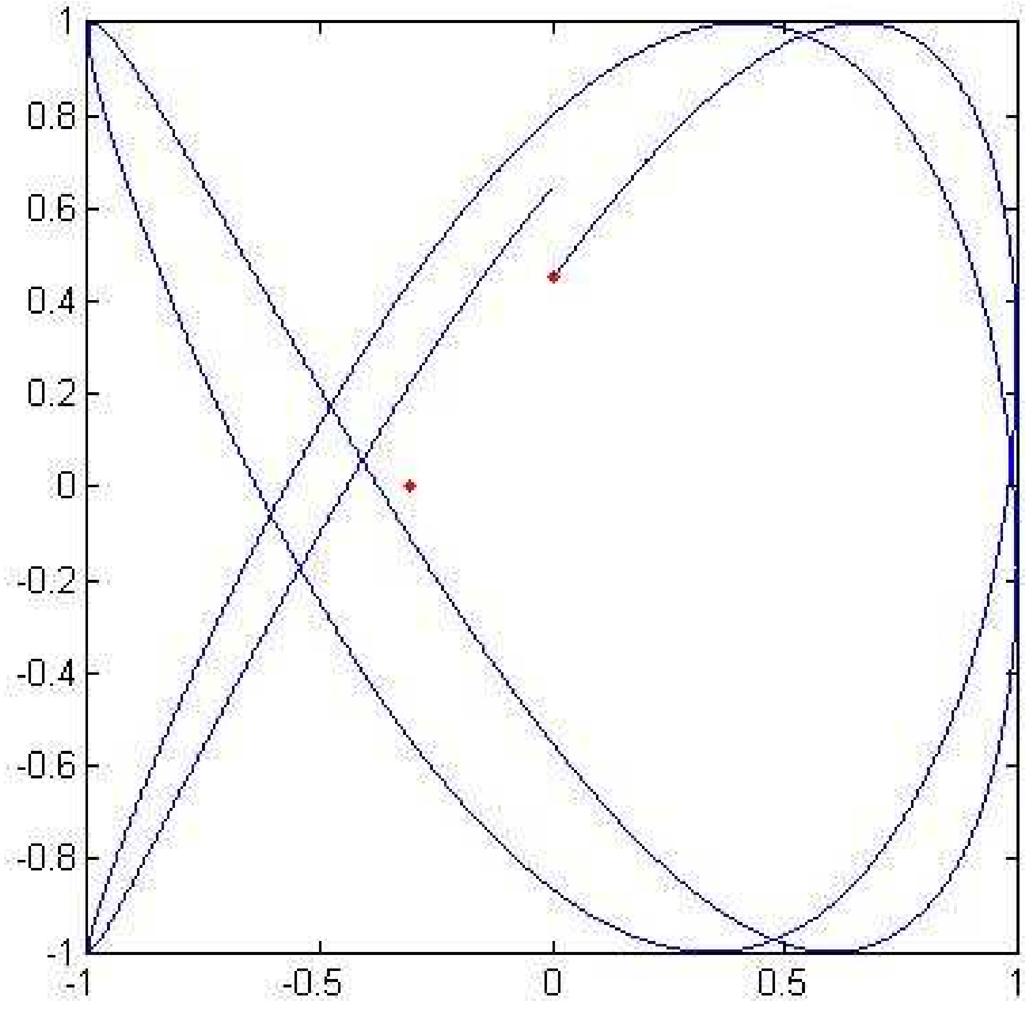
Rotation Speeds



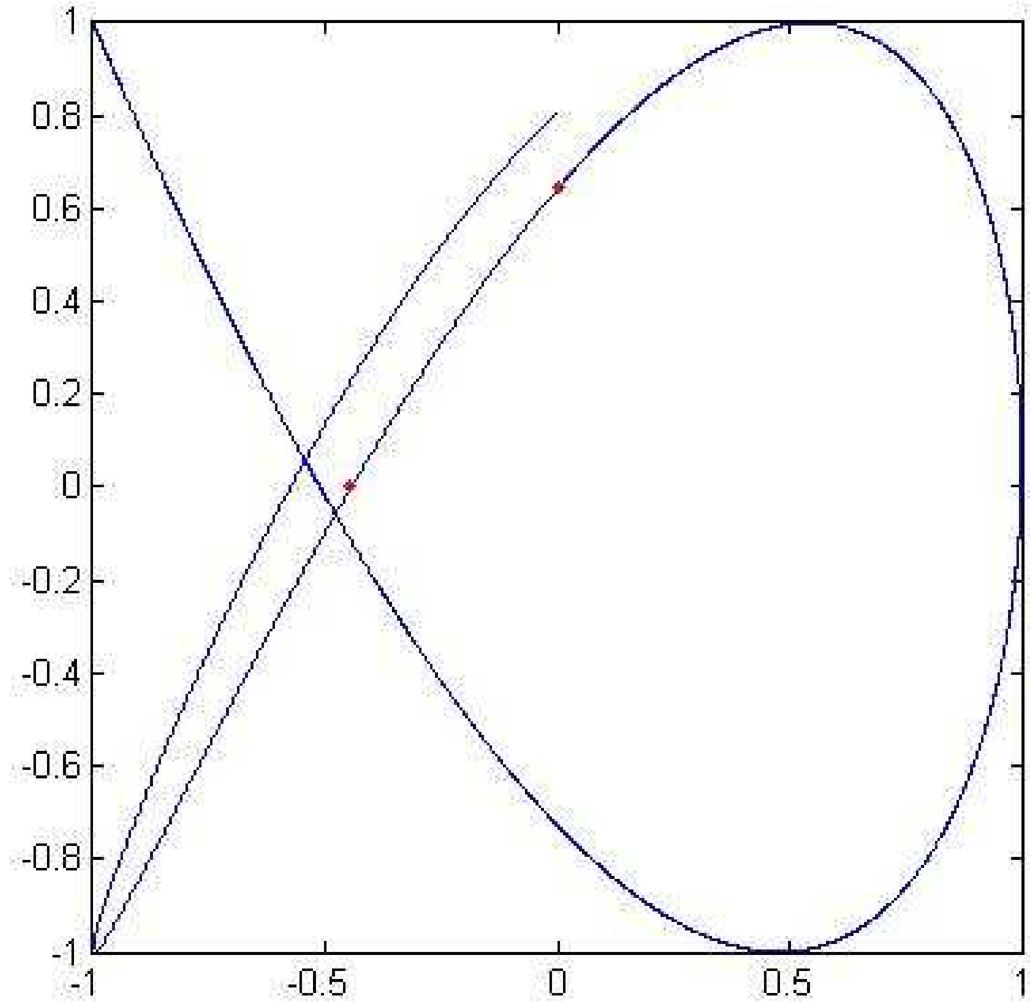
Rotation Speeds



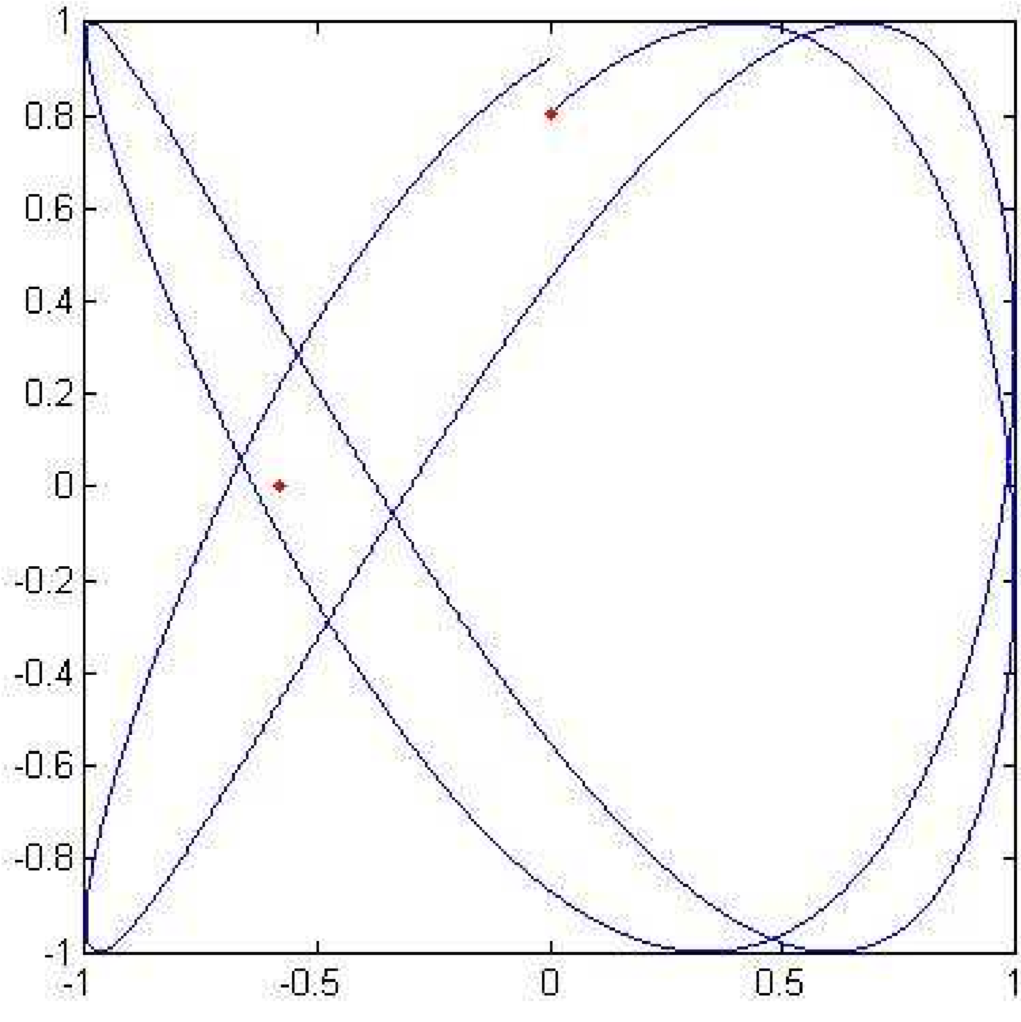
Rotation Speeds



Rotation Speeds



Rotation Speeds



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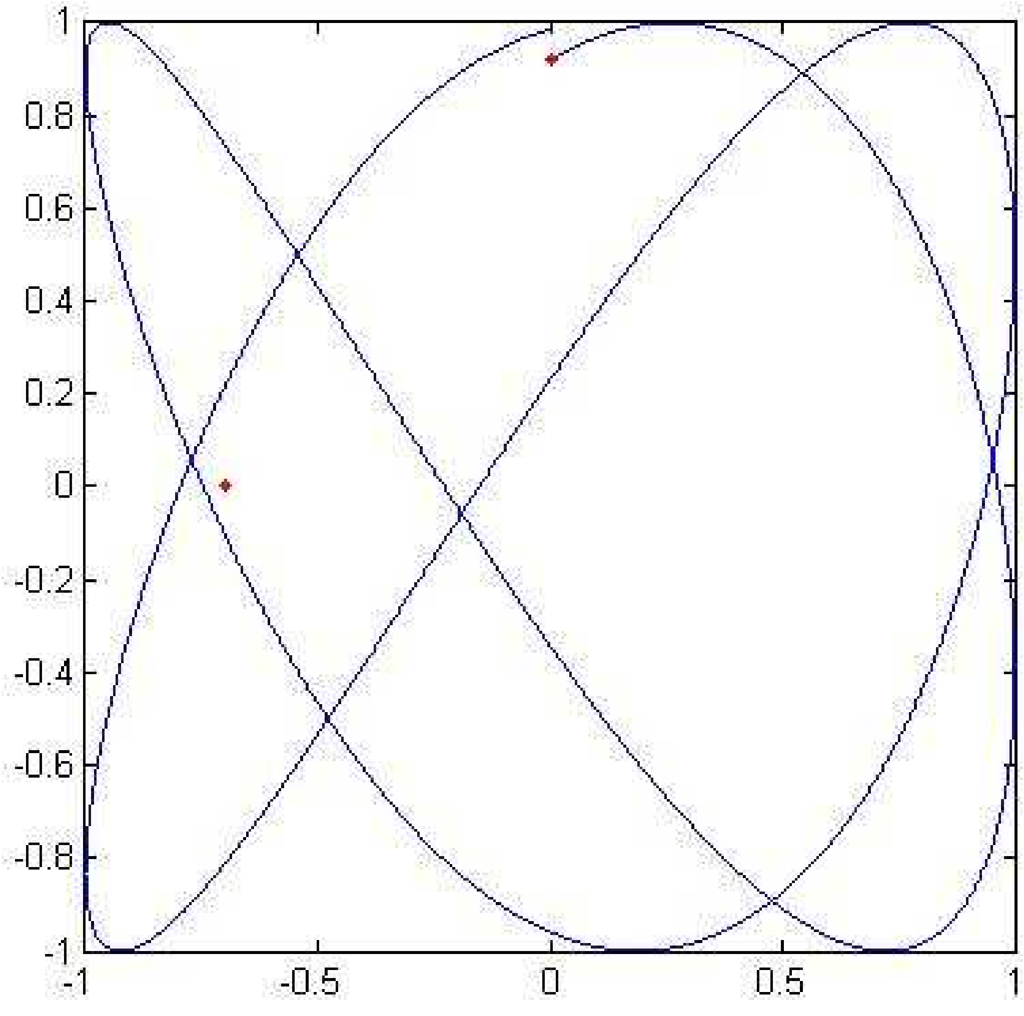

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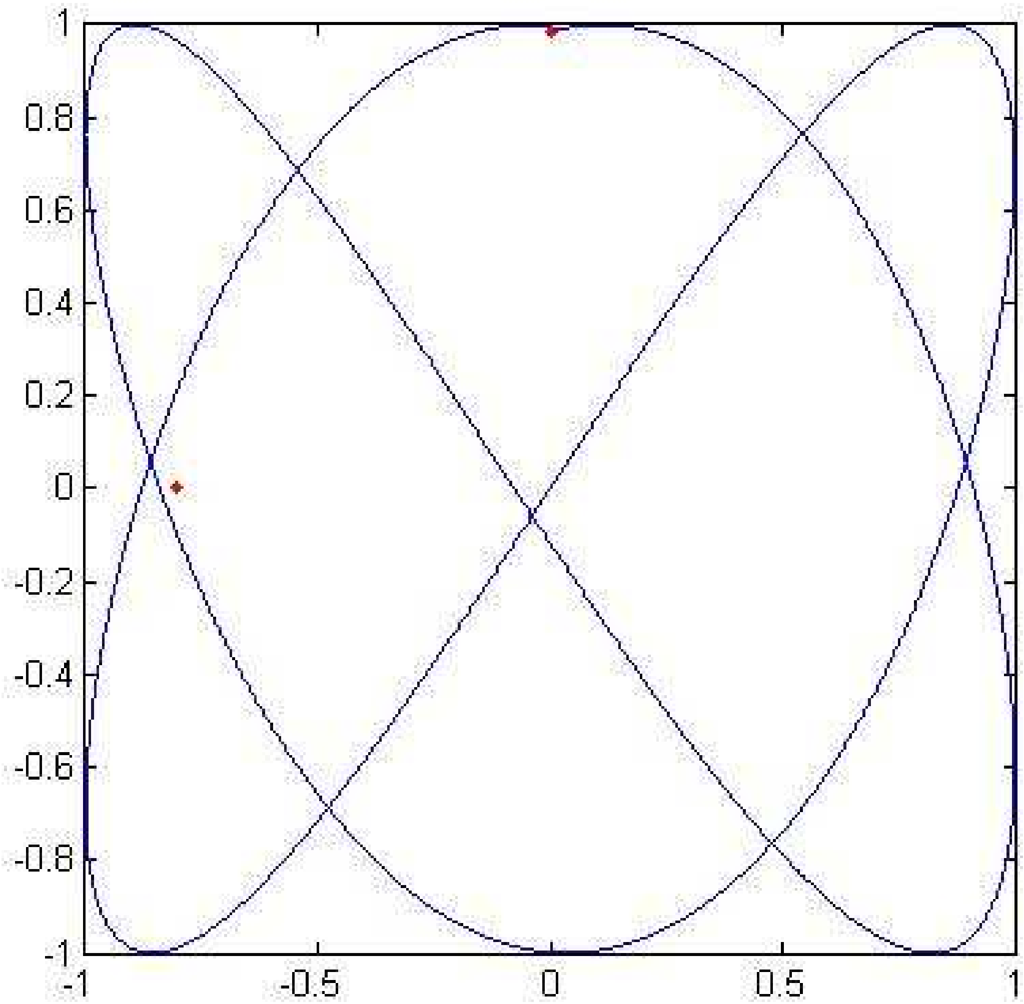

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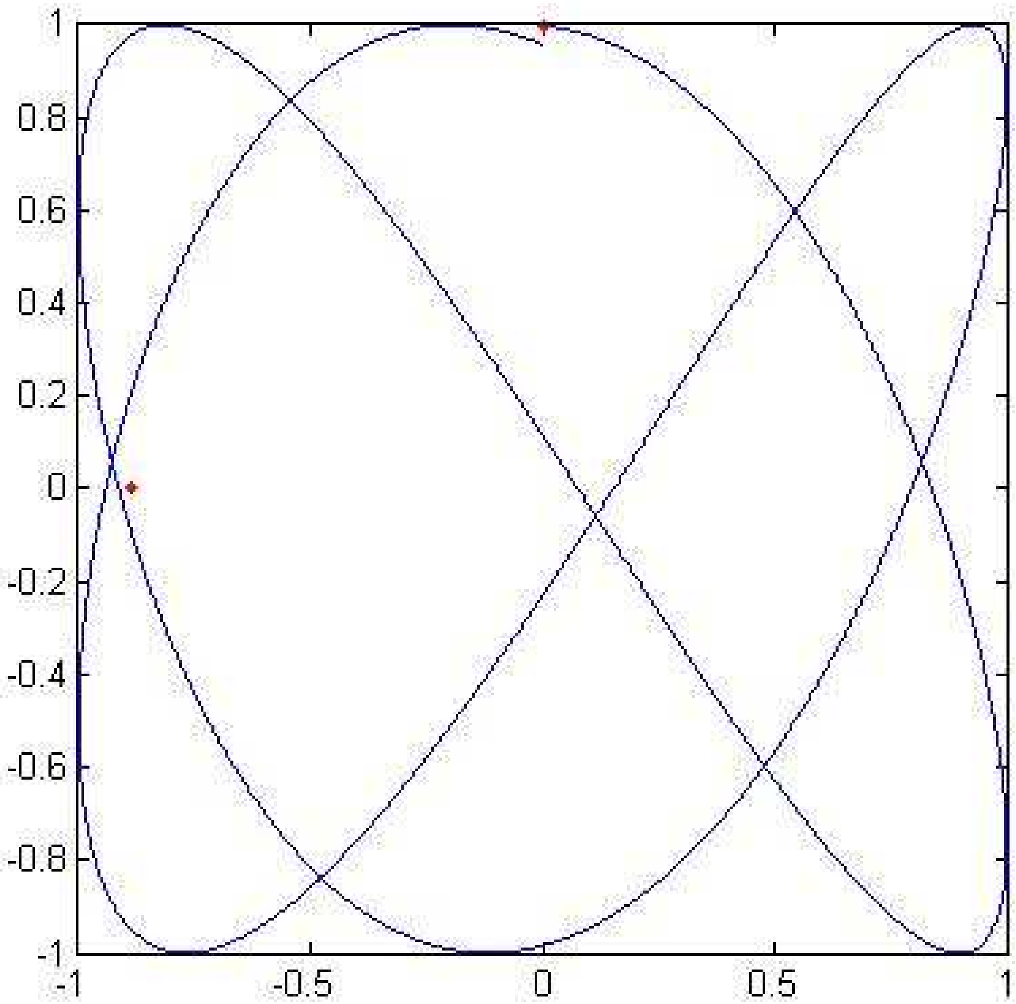
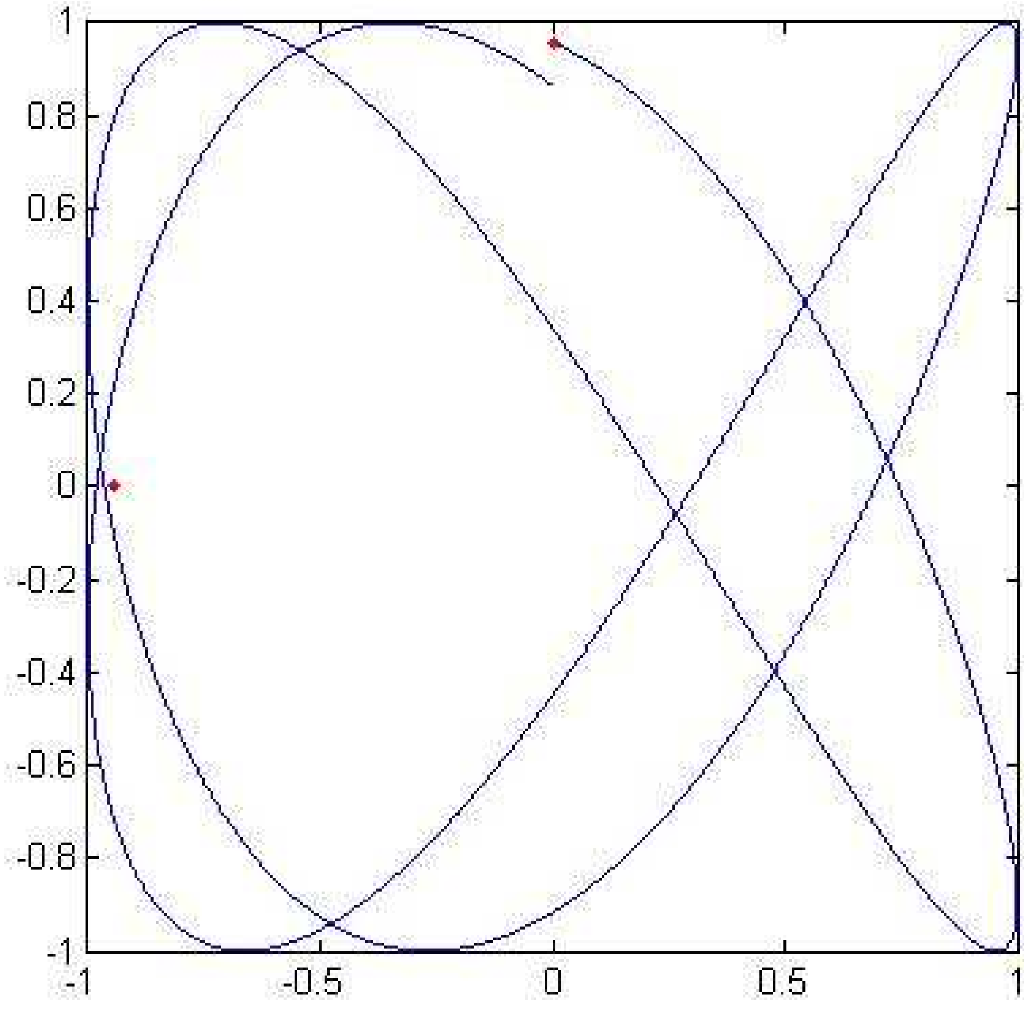
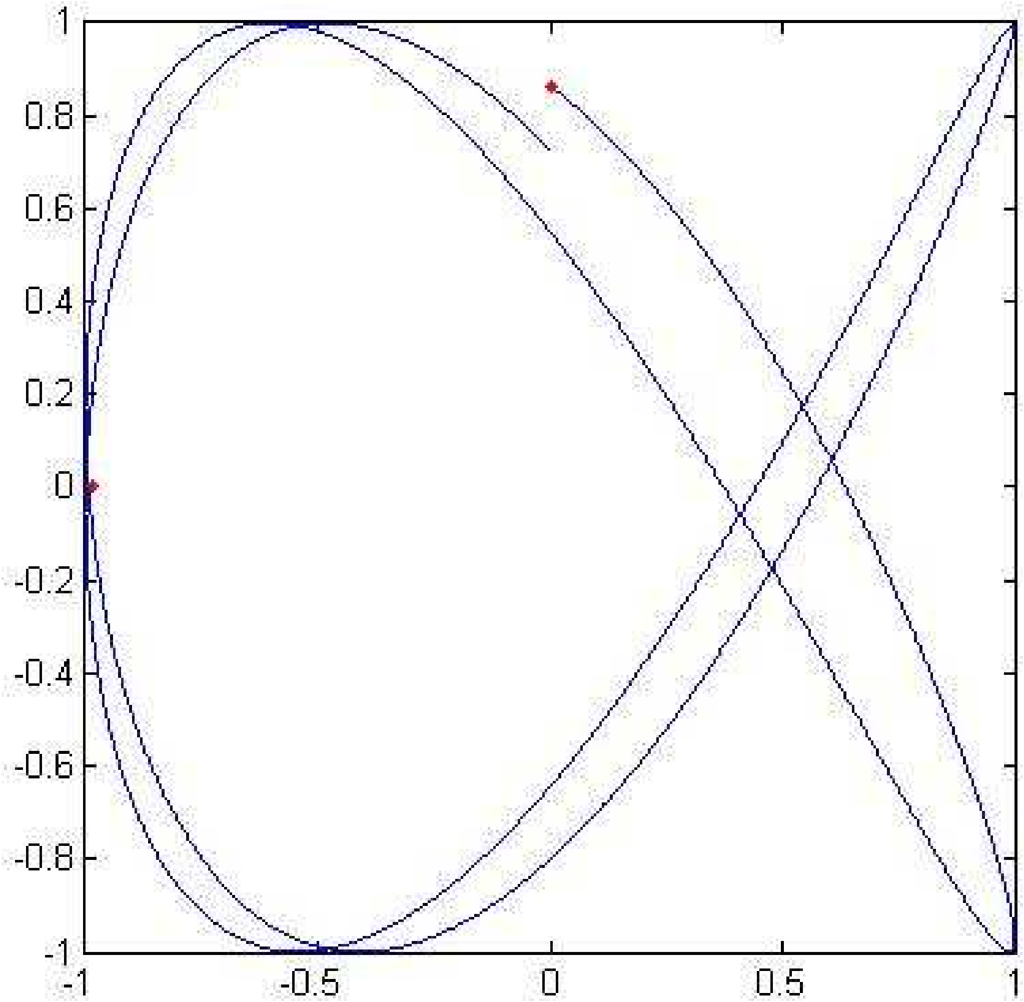


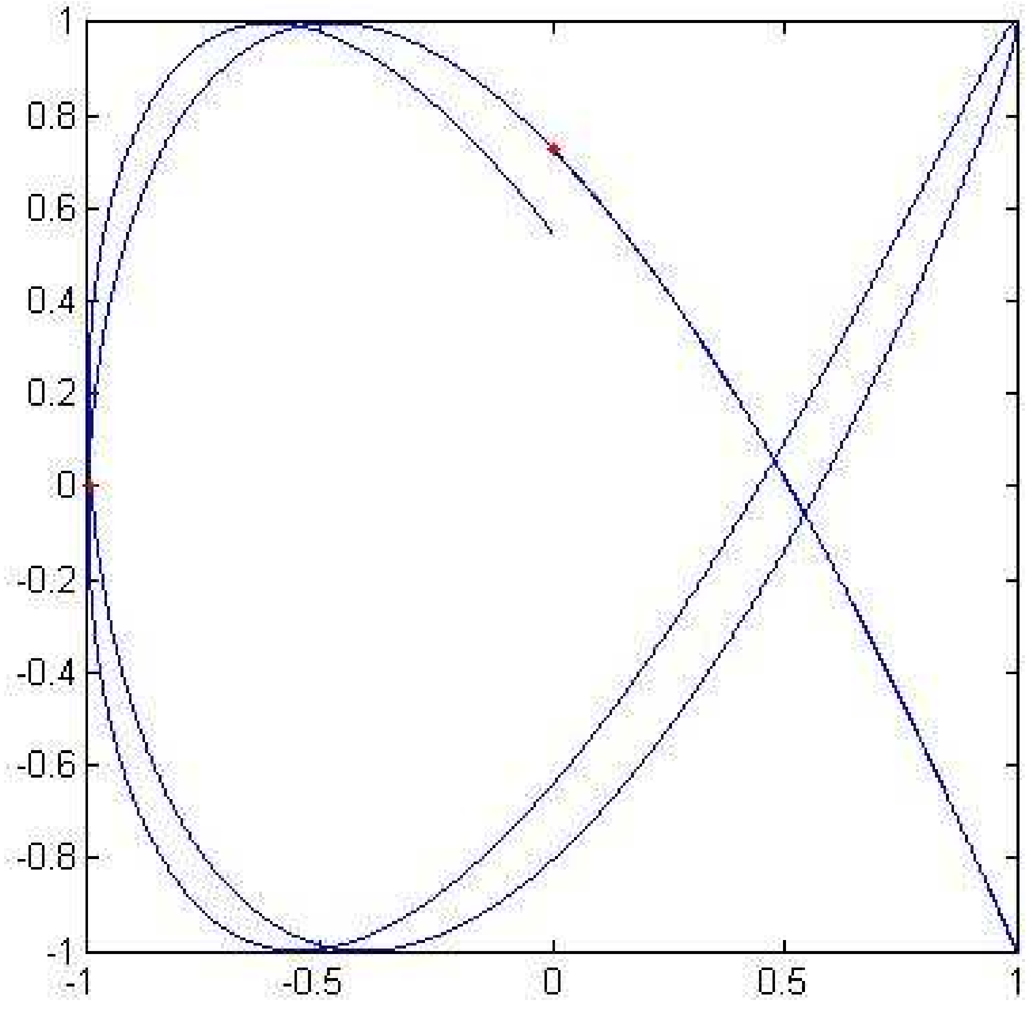
Rotation Speeds



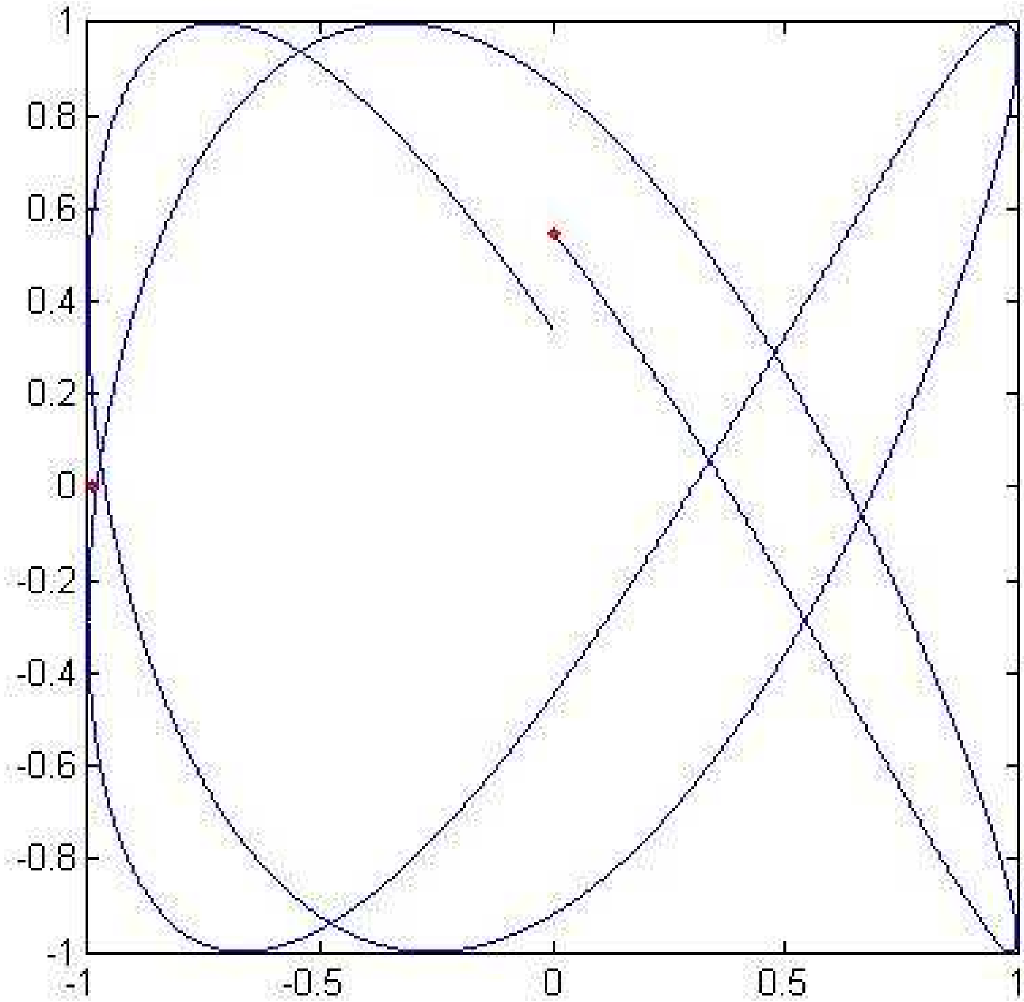
Rotation Speeds



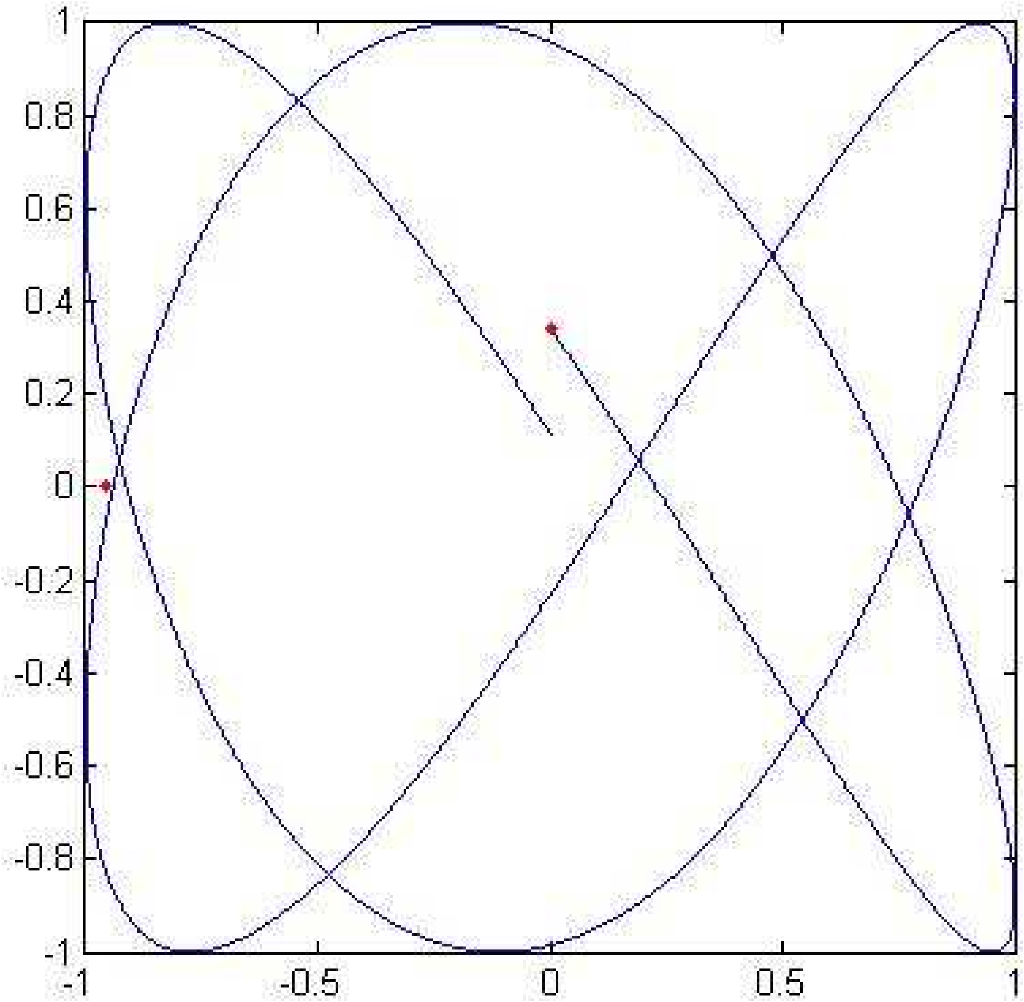
Rotation Speeds



Rotation Speeds



Rotation Speeds



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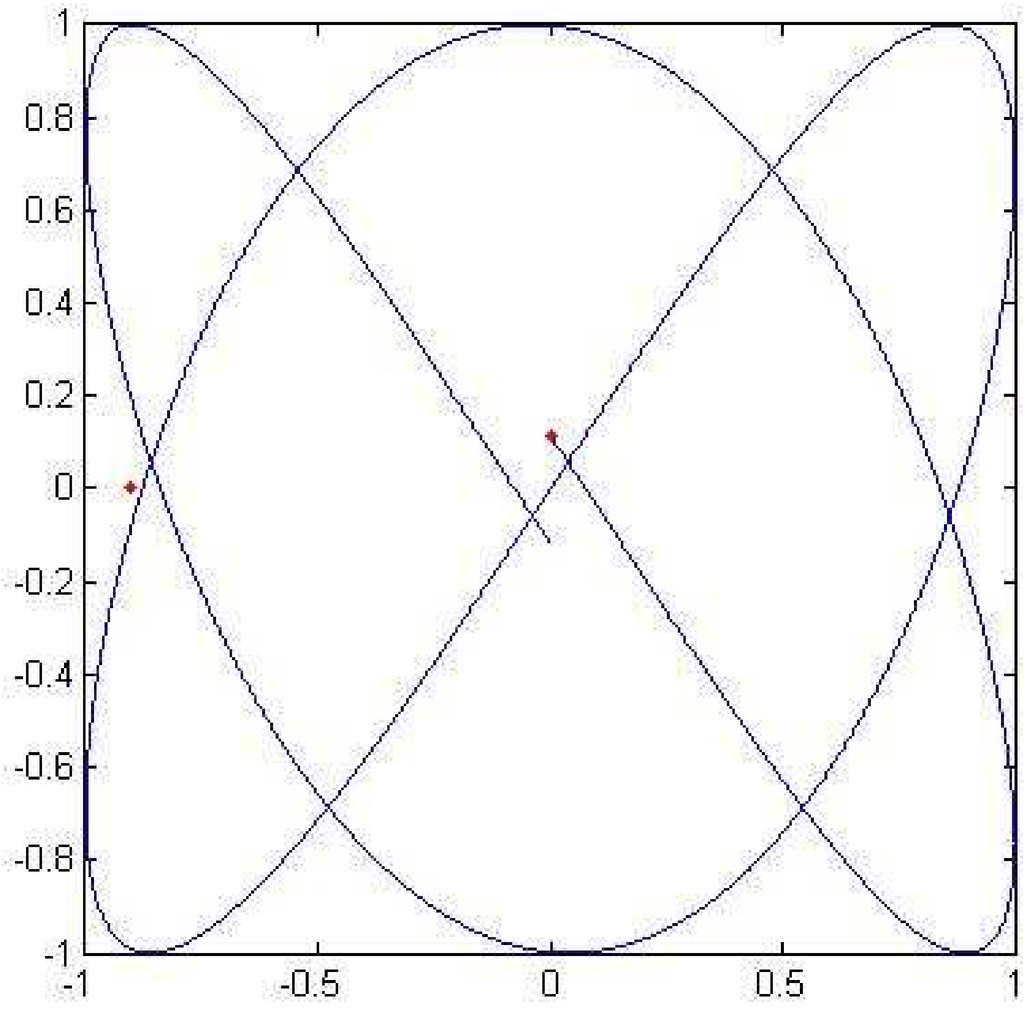
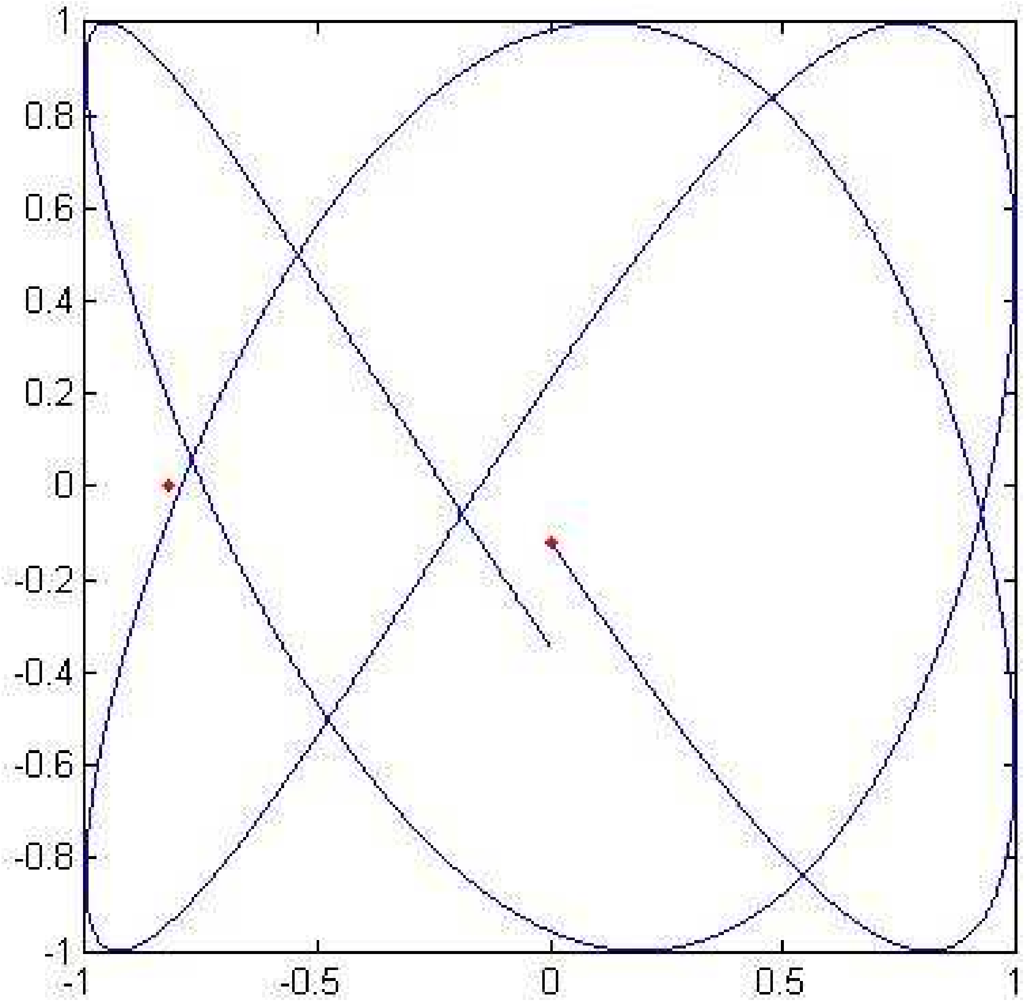
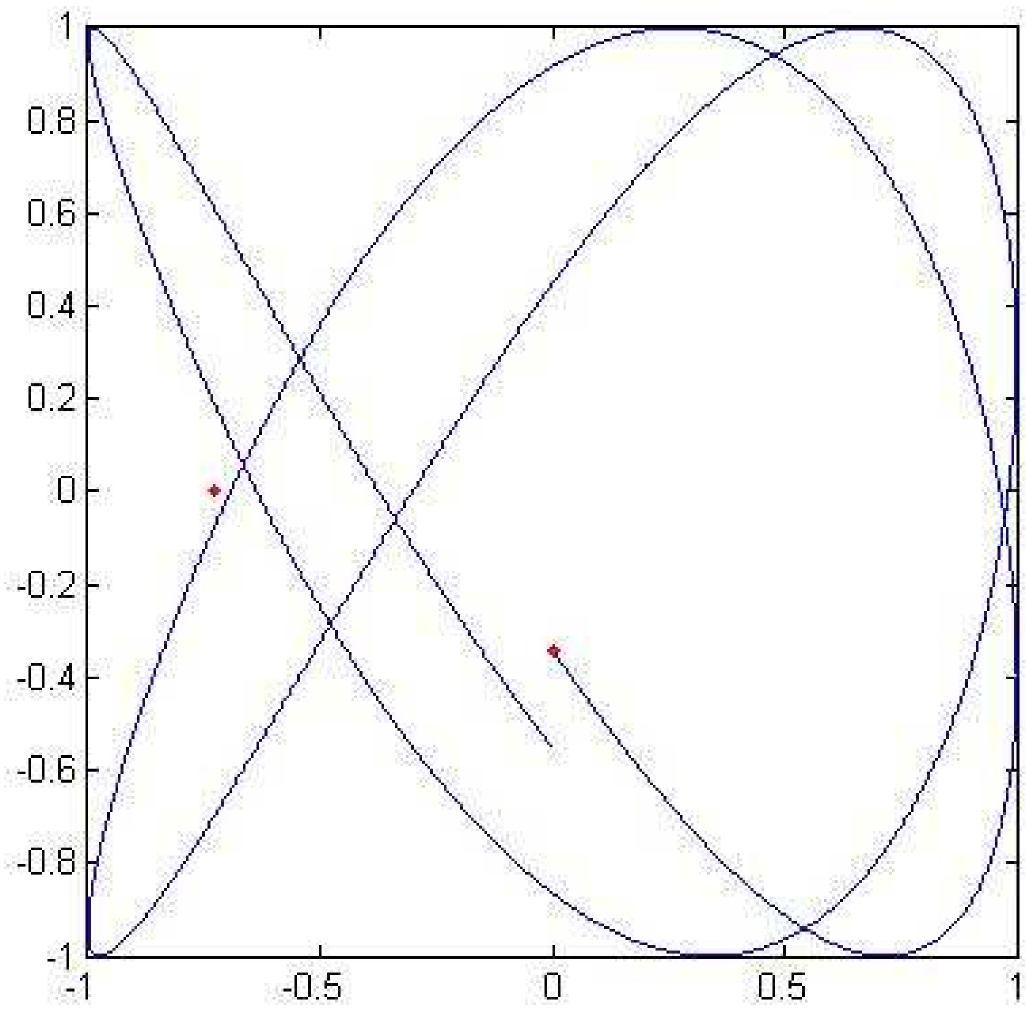


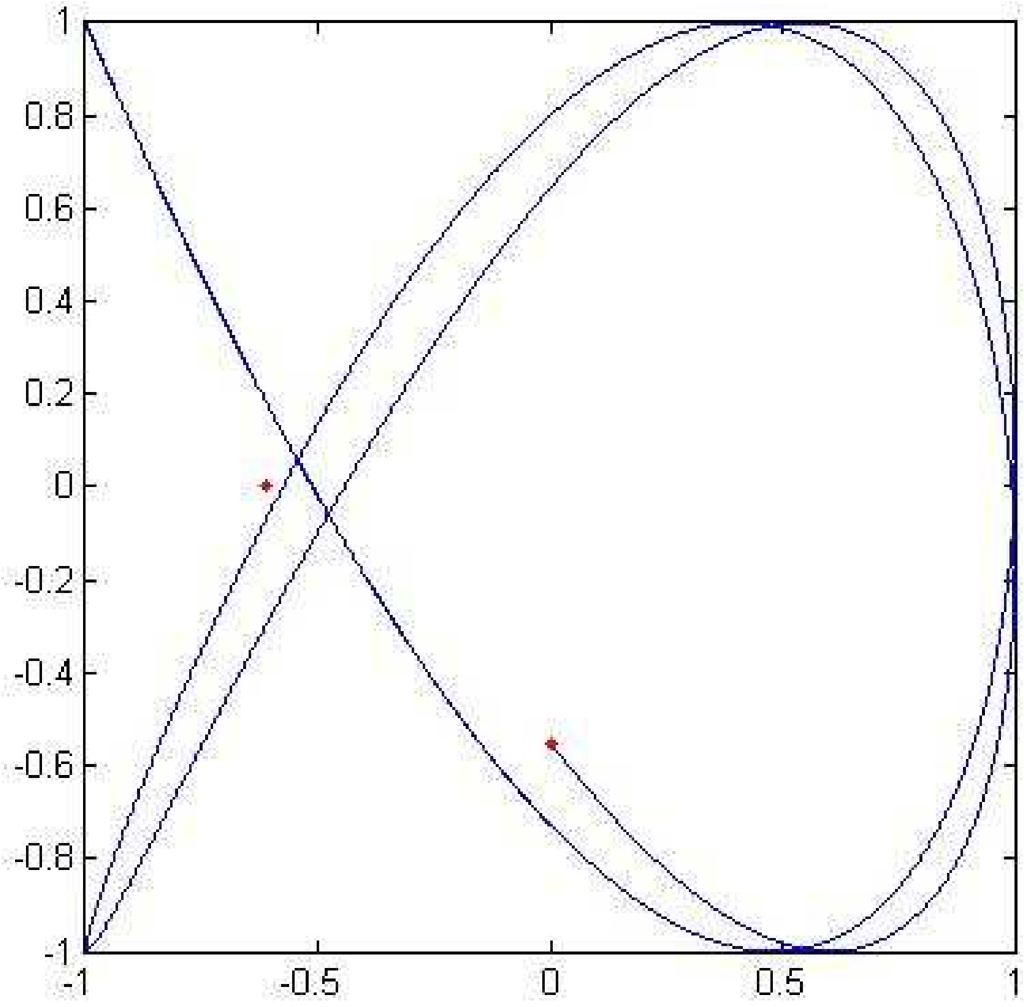
Rotation Speeds



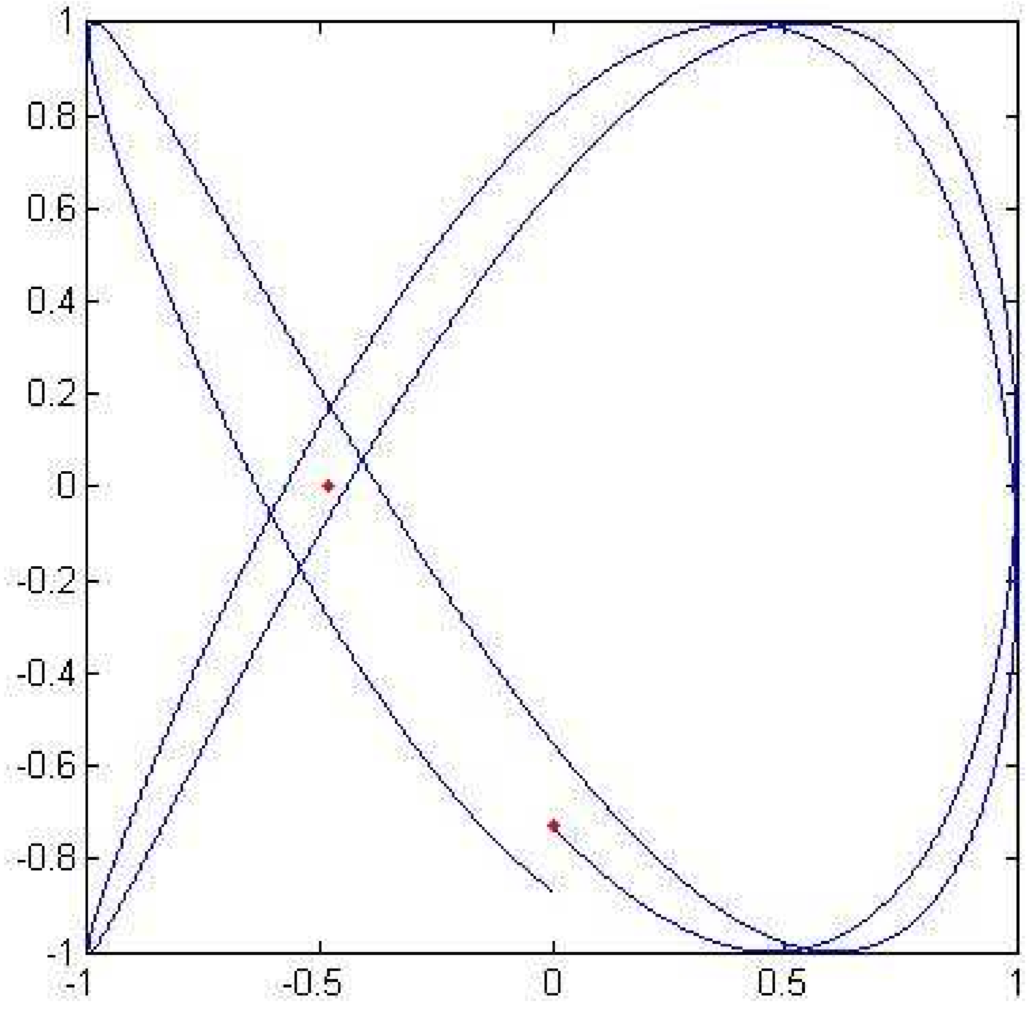
Rotation Speeds



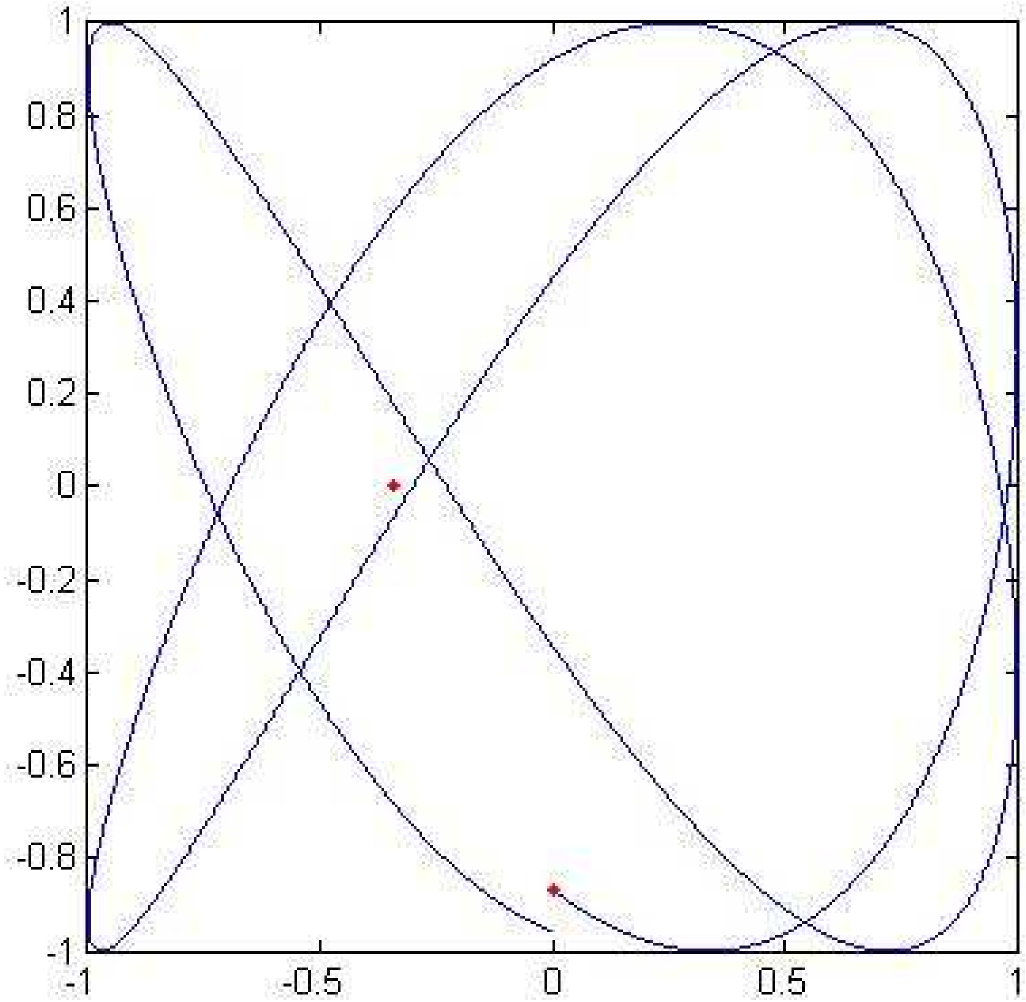
Rotation Speeds



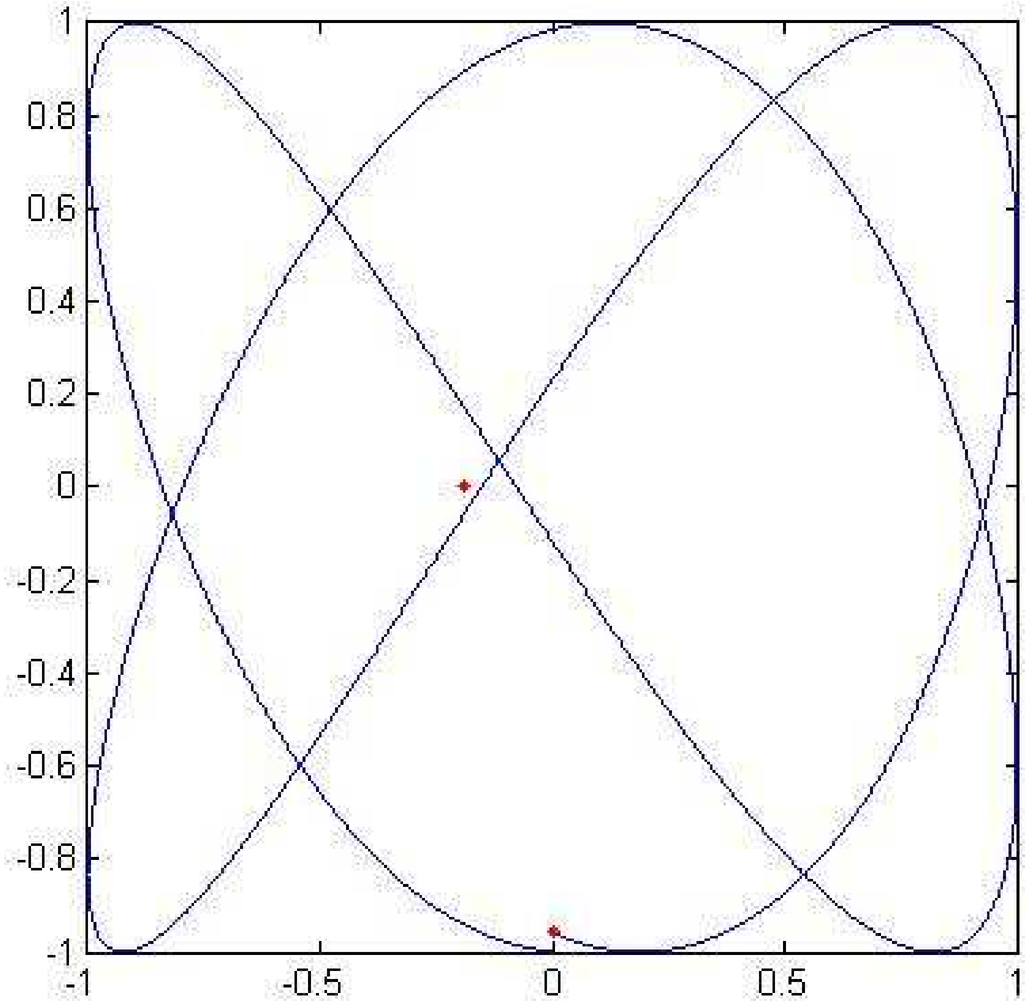
Rotation Speeds



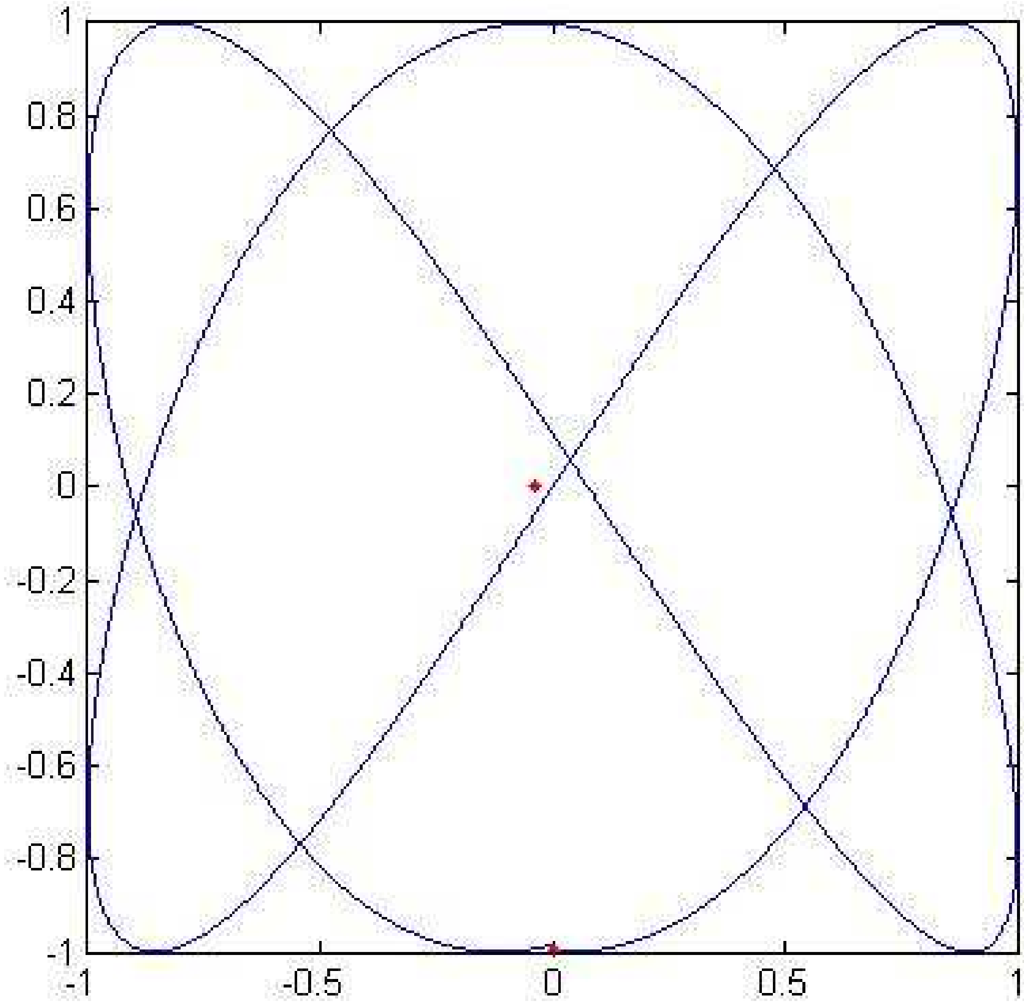
Rotation Speeds



Rotation Speeds



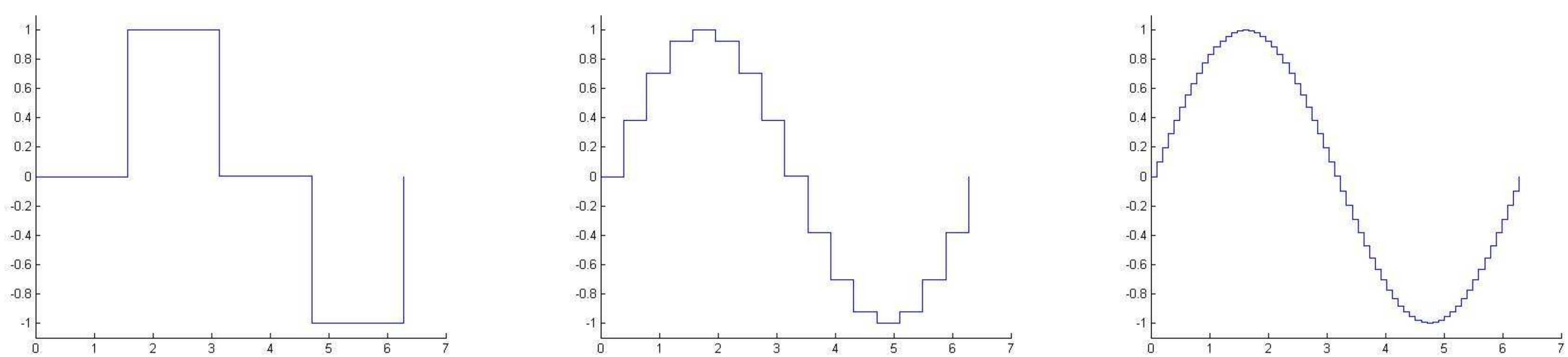
Rotation Speeds



Sine waves

- The microcontroller must make sine waves
- Generate sine waves with a lookup table
- Frequency control must be precise

Sine waves with 4, 16, 64 data points



Timing

- “Clean” sine wave requires more data points per period
- … which requires a higher interrupt frequency
- Higher interrupt frequencies give less time to execute code between interrupts

Interrupts per Second

- Suppose 180 machine cycles is adequate to service a point calculation.
- Works with 8 bit auto-reload counter.
- The R31JP runs off a 11.0592 MHz clock
- 1 machine cycle equals 12 clock cycles
- Interrupt frequency:

$$11.0592M\,\frac{\text{clock cycles}}{\text{second}} \div 12\,\frac{\text{clock cycles}}{\text{machine cycles}} \div 180\,\frac{\text{machine cycles}}{\text{interrupt}} = 5120\,\frac{\text{interrupts}}{\text{second}}$$

Table Advance

- Table advance for an 80 Hz wave:

$$256\,\frac{\text{table elements}}{\text{period}} \div 5120\,\frac{\text{interrupts}}{\text{second}} * 80\,\frac{\text{periods}}{\text{second}} = 4\,\frac{\text{table elements}}{\text{interrupt}}$$

- At Lazerdillo’s maximum frequency the table skip will be 4 elements per interrupt
- Corresponds to 64 data points per period
- Sine waves will be reasonably smooth

Precision

- Precise control of the frequency is needed to make rotating figures
- Previous example: a frequency ratio of 79.9/80 for 10 second rotation period.
- Requires frequency precision to 0.1 Hz
- That’s a range of 80/0.1=800

Table Counter Bytes

- A single byte has a factor of only 256 between the largest and smallest numbers
- Use a second byte to track the table advance
- Second byte provides fractional control over the table advance

Fractional Byte

- What is a fractional byte?

Standard Byte		Fractional Byte	
0 0 0 0 0 0 1 1	.	1 1 1 1 1 1 1 1	3.9961
		$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ $\frac{1}{128}$ $\frac{1}{256}$	

- Decimal positions provide fine control over the table advance
- Double byte table advance above converts to about 79.92 Hz
- Enables slow rotation

Poor 1-byte Table Addressing

- Suppose at each iteration we add the standard “advance” byte of the table advance to the table address and truncate the rest

Table Advance:

3.4 elements per interrupt

Interrupt	1 Byte Address
0	0
1	3
2	6
3	9

Better 2-byte Table Addressing

- Use two bytes to track our position in the table.
- Do a 16 bit add to keep track of table position.
- Use one “rounded” byte to actually address ➔ the 256-byte table .

Example Table Advance:

3.4 elements per interrupt

After 3 interrupts:

10.2

Interrupt	1 Byte Address	2 Byte Address
0	0	0➔0
1	3	3.4➔3
2	6	6.8➔6
3	9	10.2➔10

Rotation Revisited

- Lissajous Figures complete a rotation when ϕ(t) passes through an entire period
- With our example we have:

$$\frac{a}{b} = \frac{\#03.FFh}{\#04.00h} \approx \frac{79.92Hz}{80Hz} \quad , \quad \phi(t) = \#00.01h * t$$

- Converting to time:

$$256\,\frac{\text{table elements}}{\text{rotation}} * 256\,\frac{\text{fractional table elements}}{\text{table element}} \div$$

$$5120\,\frac{\text{interrupts}}{\text{second}} \div 1\,\frac{\text{fractional table element}}{\text{interrupt}} = 12.8\,\frac{\text{seconds}}{\text{rotation}}$$

Details of the Rotation Period

$$256\,\frac{\text{table elements}}{\text{rotation}} * 256\,\frac{\text{fractional table elements}}{\text{table element}} \div$$

$$5120\,\frac{\text{interrupts}}{\text{second}} \div 1\,\frac{\text{fractional table element}}{\text{interrupt}} = 12.8\,\frac{\text{seconds}}{\text{rotation}}$$

$256\,\frac{\text{table elements}}{\text{rotation}}$	• $\phi(t)$ must pass through all 256 elements of the sine table
$256\,\frac{\text{fractional table elements}}{\text{table element}}$	• The fractional byte yields 256 fractional table elements per full table element
$5120\,\frac{\text{interrupts}}{\text{second}}$	• Calculated previously
$1\,\frac{\text{fractional table element}}{\text{interrupt}}$	• Corresponds to: $\phi(t) = \#00.01h * t$
$12.8\,\frac{\text{seconds}}{\text{rotation}}$	• Rotation period!