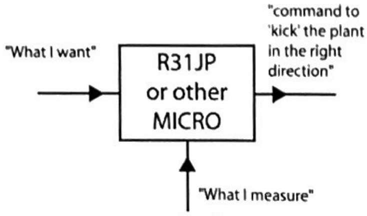
Remember our goal:

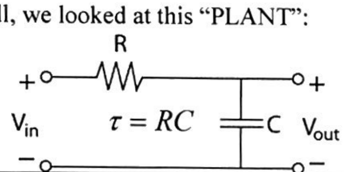
Configure the R31JP as the means of a feedback system



In a feedback system, we are typically concerned with 3 system properties:

- STABILITY
- TRANSIENT RESPONSE
- STEADY-STATE ERROR

Last time, recall, we looked at this "PLANT":



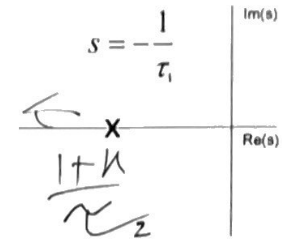
Using Laplace $\frac{d}{dt} \Leftrightarrow s$	Differential Equation	Step Response
$\frac{v_{out}}{v_{in}}(s) = \frac{1}{\tau s + 1}$ transfer function	$\tau \frac{dv_{out}}{dt} + v_{out} = v_{in}$	$v_{out}(t) = \left(1 - e^{-t/\tau}\right)u(t)$

relate these two on a pole-zero map

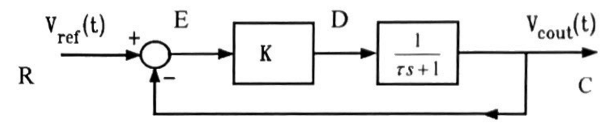
s = "natural frequency"

Pole-zero map:

- τ must be positive, s must be negative for stability.
- So this plot tells us about stability and response speed.



Feedback controller for the RC-plant:

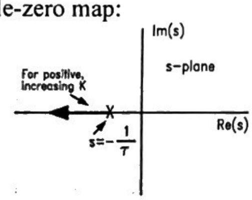


$$\frac{C}{R} = \frac{\frac{K}{\tau s + 1}}{1 + \frac{K}{\tau s + 1}} = \frac{K}{\tau s + 1 + K} = \frac{\frac{K}{1+K}}{\frac{\tau}{1+K}s + 1}$$

Step response:

$$v_{out} = \frac{K}{1+K} \cdot \left[1 - e^{-\frac{t}{\tau/1+K}}\right] u(t)$$

Pole-zero map:



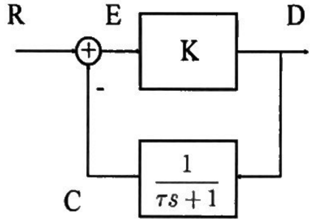
So, under feedback:

- Apparently always stable
- Improved transient response with increasing K
- Some steady state error, but decreases as K increases

Why does the overall system appear to get "faster" if we use feedback with high gain?

Let's see how the loop "kicks" the plant.

That is what's $\frac{D}{R}(s)$?



$\frac{1}{\tau_1 s + 1}$

$\tau_1 = \frac{\tau_2}{1+K}$

τ_1 = placeholder value

τ_2 = motor property

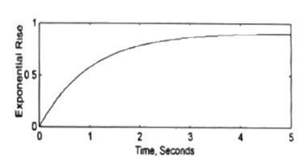
$$\frac{D}{R}(s) = \frac{K}{1 + \frac{K}{\tau s + 1}} = \frac{\frac{K(\tau s+1)}{1+K}}{\frac{s\tau}{1+K} + 1} = \frac{K}{1+K} \cdot (\tau s + 1) \cdot \frac{1}{\frac{\tau}{1+K}s + 1}$$

Takes a derivative and scales by tau

Scales by $\frac{K}{1+K}$; Takes the rise "as is"

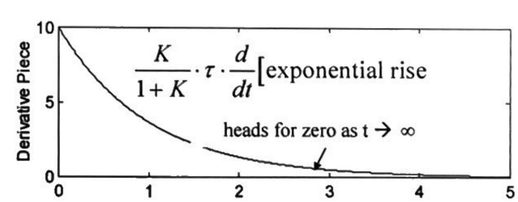
This is our actual exponential rise

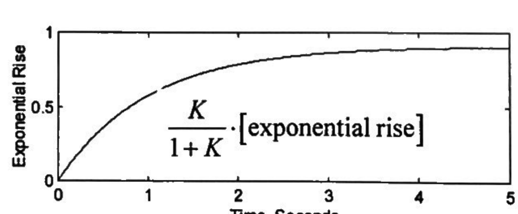
Here is the exponential rise:

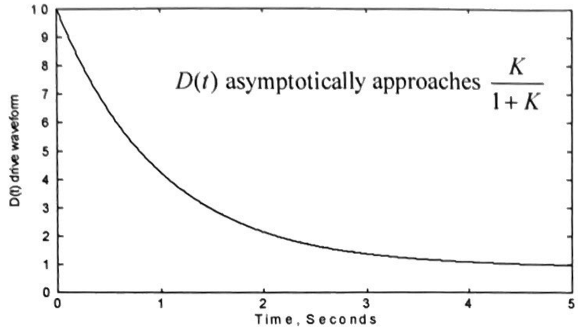


$$\left[1 - e^{-t/\tau/1+K}\right]u(t)$$

So D(t) is the sum of two pieces:





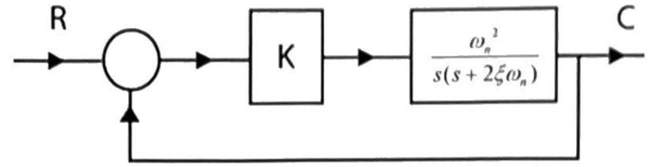


D(t) asymptotically approaches K/(1+K).

But initially, it "kicks" the RC plant very hard (relative to steady state) to get it "moving"…

- Why are we focusing on step responses? Good input to examine transient response and steady error!
- Why are we focusing on 1st order systems? Well, they're easy to understand. But they are obviously not the only system type in the world.

Second order systems:



$$\frac{C}{R} = \frac{\dfrac{K\omega_n^2}{s^2+2\xi\omega_n s}}{1+\dfrac{K\omega_n^2}{s^2+2\xi\omega_n s}} = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+K\omega_n^2}$$

Why this 2nd order system? There are many slight variations. But this one is fine for exploring the behavior of a 2nd order denominator.

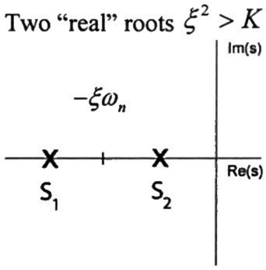
What are the natural frequencies? Roots of the denominator polynomial.

Let's find roots of the denominator polynomial using quadratic formula:

$$S_{1,2} = -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - 4K\omega_n^2/4} = -\xi\omega_n \pm \omega_n\cdot\sqrt{(\xi^2 - K)}$$

3 interesting cases:

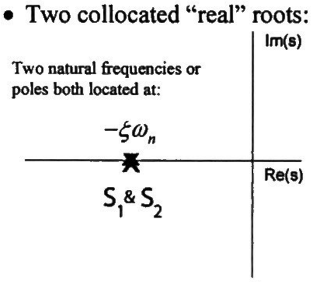
- Two "real" roots $\xi^2 > K$



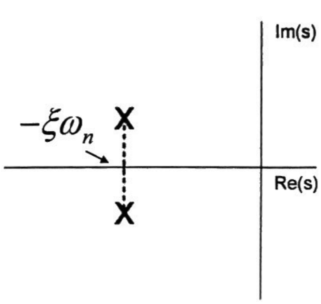
Two negative real natural frequencies

Response to a step will involve the sum of two exponentials may "appear" 1st order, denominated by slowest natural frequency S_2

- Two collocated "real" roots: $\xi^2 = K$



- Two complex roots: $\xi^2 < K$



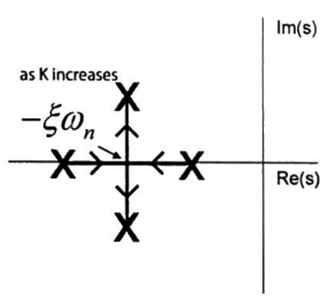
Poles:

$$S_{1,2} = -\xi\omega_n \pm j\,\overbrace{\omega_n\sqrt{k-\xi^2}}^{\omega_d}$$

Step response for two complex roots with K = 1 :

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

Root Locus (Pole-zero map) as we vary K: $0 \le K \le \infty$



Plot below shows plots of step responses of the closed loop system for lots of different K's.

$\alpha = \xi\omega_n$ is constant in the picture.

Finally: Why all the fuss about 1st and 2nd order systems? Many systems have a dominant 1st order pole or a complex conjugate pair of poles, and appear approximately as 1st and 2nd order.