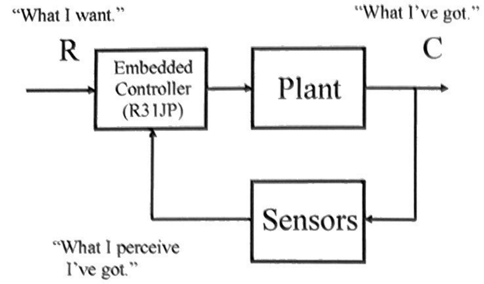
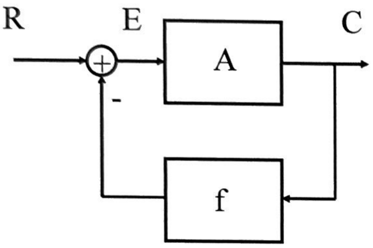
Often, we may find an embedded controller in an over all system structure that looks like this:



This is a feedback or "error-driven" structure. Let's try to understand how it might behave! We'll surely depend on the characteristics of all 3 blocks.

Before we get to the digital controller, let's look at an analog example:



A little arithmetic:

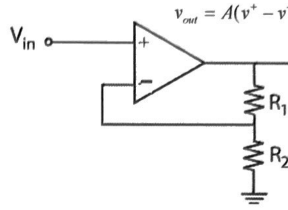
$$E = R - fC$$

$$C = AE = A(R - fC)$$

Collecting terms:

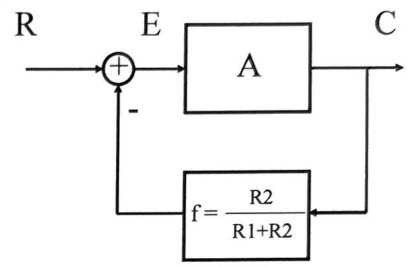
$$C + AfC = AR \text{ or } \boxed{\frac{C}{R} = \frac{A}{1+Af}} \leftarrow \text{BLACK'S FORMULA}$$

Example: Non-inverting op-amp gain block:



We know that $\frac{C}{R} = \frac{R_1 + R_2}{R_2}$ for an ideal op-amp

In block diagram form:



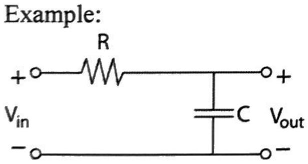
According to Black's formula: $\frac{C}{R} = \frac{A}{1 + A\frac{R_2}{R_1 + R_2}} \rightarrow \frac{R_1 + R_2}{R_2}$ as $A \rightarrow \infty$

Aha! But what do we do if one of the blocks (or more) is not simply a gain, but rather a "dynamic" block described by a differential equation?

Answer: Find a way to "pretend" that the differential equation is an algebraic equation!

Laplace Operator Calculus: $\frac{d}{dt} \Leftrightarrow s$

Example:



$$i(t) = C\frac{dv_{out}(t)}{dt} \Leftrightarrow i(s) = sCv_{out}(s)$$

$$Z = \frac{1}{sC}$$

Differential Equation:

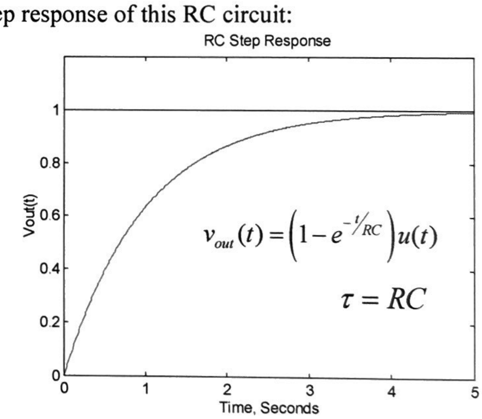
$$v_{in} = iR + v_{out} = RC\frac{dv_{out}}{dt} + v_{out}$$

Laplace: "impedance" divider:

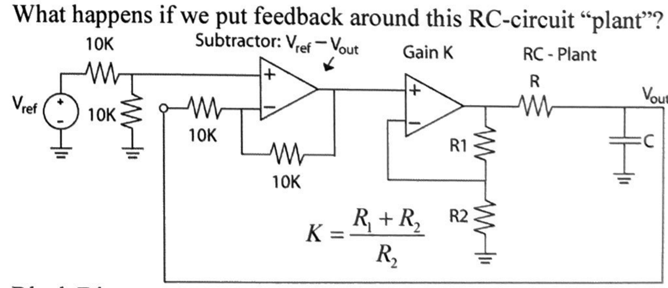
$$\frac{v_{out}}{v_{in}}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{sRC + 1}$$

or $v_{out} \cdot [sRC + 1] = v_{in} \Leftrightarrow RC\frac{dv_{out}}{dt} + v_{out} = v_{in}$

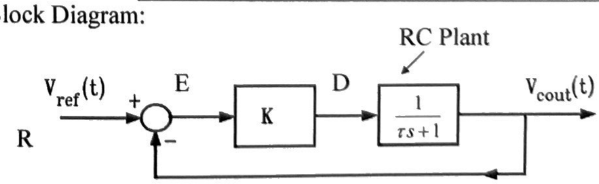
Unit step response of this RC circuit:



What happens if we put feedback around this RC-circuit "plant"?



Block Diagram:



Apply Black's formula: $A = \dfrac{K}{\tau s + 1}$ $\quad f = 1$

$$\frac{C}{R} = \frac{\dfrac{K}{\tau s+1}}{1+\dfrac{K}{\tau s+1}} = \frac{K}{\tau s + 1 + K} = \frac{\dfrac{K}{1+K}}{\dfrac{\tau}{1+K}s+1}$$

Pole-zero plot:

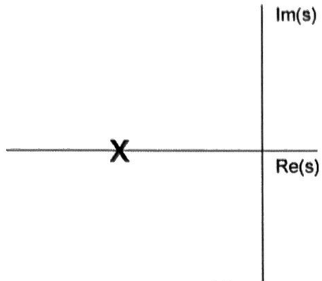
ORIGINAL RC CIRCUIT:

Recall Solution:

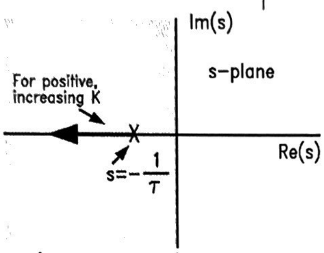
$$v_{out}(t) = C = \left[1 - e^{-t/RC}\right]u(t)$$

"natural frequency"

$$s = -\frac{1}{RC}$$



$$v_{out} = \frac{K}{1+K}\cdot\left[1 - e^{-\frac{t}{\tau/1+K}}\right]u(t)$$



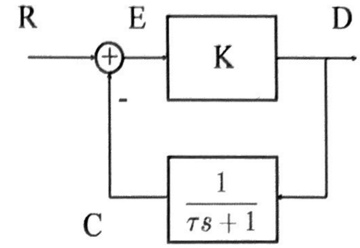
starts here near small K ~0

Always stable: steady state error, transient response improve as K gets large.

Why does the overall system appear to get "faster" if we use feedback with high gain?

Let's see how the loop "kicks" the plant.

That is what's $\dfrac{D}{R}(s)$?



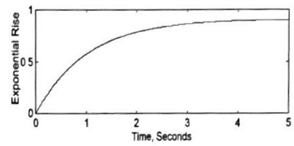
Takes a derivative and scales by tau

$$\frac{D}{R}(s) = \frac{K}{1+\dfrac{K}{\tau s+1}} = \frac{\dfrac{K(\tau s+1)}{1+K}}{\dfrac{s\tau}{1+K}+1} = \frac{K}{1+K}\cdot(\tau s+1)\cdot\frac{1}{\dfrac{\tau}{1+K}s+1}$$

Here is the exponential rise:

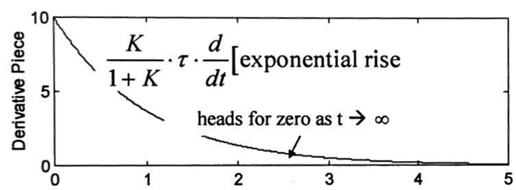
Scales by $\dfrac{K}{1+K}$ takes the rise "as is"

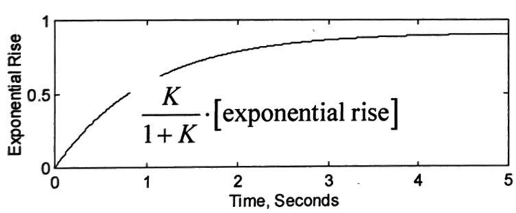
This is our actual exponential rise

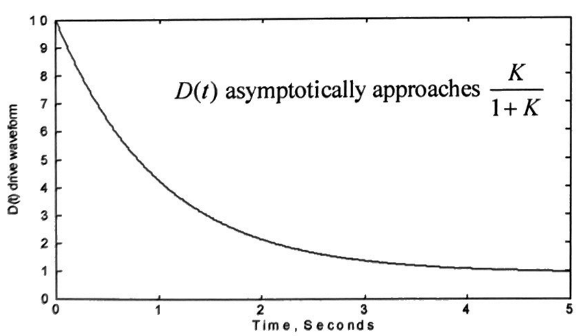


$$\left[1 - e^{-t/\tau/1+K}\right]u(t)$$

So D(t) is the sum of two pieces:







Initially, it "kicks" the RC plant very hard (relative to steady state) to get it "moving"...