Problem 3.7 list all of the subsets of $A = \{1, 2, 3\}$

$\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$

$\blacksquare$

Theorem 3.8 Let A be a set. Then

(a) $A \subseteq A$

Assume that set $A = \{1, 2, 3\}$
according to definition 3.6 A is a subset of A because every element within set A is in itself, which therefore makes it a subset.

$\blacksquare$

(b) $\emptyset \subseteq A$.

Assume that set $A = \{1, 2, 3\}$
Every set also contains the contains the empty set.
Therefore A can be rewritten as $A = \{\{\}, 1, 2, 3\}$
This means that according to definition 3.6 the empty set is a subset of set A.

$\blacksquare$

Problem 3.9 Suppose A and B are sets. Describe a skeleton proof for proving that $A \subseteq B$.

Assume that we have two sets A and B.
For A to be a subset of B every element in A must be in B
Let x be an arbitrary element in A
Since A is a subset of B, every element in A must also be in B
Therefore, x must be an element in B
Because x was chosen arbitrarily we have shown that every element in A is also in B, and therefore, $A \subseteq B$

$\blacksquare$

Problem 3.10 (Transitivity of Subsets). Suppose that A, B, and C are sets. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

To prove that $A \subseteq C$ we must show that every element in A is also in C
Let x be an arbitrary element in A
Because $A \subseteq B$, x is in B
Furthermore, because $B \subseteq C$, x is in C
Therefore, x is an element of both A and C which implies that $A \subseteq C$.

$\blacksquare$

Problem 3.18 Suppose that the universe of discourse is $U = \{x, y, z, \{y\}, \{x, z\}\}$. Let $S = \{x, y, z\}$ and $T = \{x, \{y\}\}$. Find each of the following.

(a) $S \cap T$

$$S \cap T = \{x, y, z\} \cap \{x, \{y\}\} = \{x\}$$

(b) $(S \cup T)^c$

$$(S \cup T)^c = (\{x, y, z\} \cup \{x, \{y\}\})^c = \{\{x, z\}\}$$

(c) $T \setminus S$

$$T \setminus S = \{\{y\}\}$$

■

Theorem 3.19 If A and B are sets such that $A \subseteq B$, then $B^c \subseteq A^c$.
To prove this theorem we need to show that every element in B^c is in A^c
Let x be an arbitrary element in B^c. Because $x \in B^c$, $x \notin B$
This means that x is also not an element of A
Therefore, x is an element of A^c
This shows that x is an element of B^c and A^c, which proves that $B^c \subseteq A^c$.

■

Theorem 3.20 If A and B are sets, then $A \setminus B = A \cap B^c$
Let $x \in A \setminus B$,
This implies $x \in A$ and $x \notin B$
which also implies $x \in B$ and $x \in B^c$
Therefore showing that $x \in A \cap B^c$ (i)

Now let $y \in A \cap B^c$
this implies $y \in A$ and $y \in B^c$
this then implies $y \in A$ and $y \notin B$
therefore $y \in A \setminus B$ (ii)

from (i) and (ii) we have now proven that $A \setminus B = A \cap B^c$

■

Theorem 3.21 (De Morgan's Law). If A and B are sets, then

(a) $(A \cup B)^c = A^c \cap B^c$
Let $x \in (A \cup B)^c$
This means $x \notin (A \cup B)$
which implies $x \notin A$ and $x \notin B$
Therefore, $x \in A^c$ and $x \in B^c$
Hence, $x \in A^c \cup B^c$ (i)

Now let $y \in A^c \cap B^c$
this means $y \in A^c$ or $y \in B^c$
which implies $y \notin A$ or $y \notin B$
Therefore, $y \notin (A \cup B)$
Hence, $y \in (A \cup B)^c$ (ii)

From (i) and (ii) we have proven that $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$
we can use a similar proof for this theorem.
Let $x \in (A \cap B)^c$
this means $x \notin (A \cap B)$
which implies $x \notin A$ or $x \notin B$
therefore, $x \in A^c$ or $x \in B^c$
Hence, $x \in A^c \cap B^c$ (i)

Now, let $y \in (A^c \cup B^c)$
this means $y \in A^c$ and $y \in B^c$
which implies $y \notin A$ and $y \notin B$
therefore, $y \notin (A \cap B)$
hence $y \in (A \cap B)^c$ (ii)

from (i) and (ii) we have proven that $(A \cap B)^c = A^c \cup B^c$

■

Theorem 3.22 (Distribution of Union and Intersection). If A, B, and C are sets, then

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Let x be any element of $A \cup (B \cap C)$
Then, $x \in A \cup (B \cap C)$
this shows that $x \in A$ or $x \in (B \cap C)$
which implies that $x \in A$ or ($x \in B$ and $x \in C$)
which then implies that ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)
therefore, $x \in A \cup B$ and $x \in A \cup C$

hence, $x \in (A \cup B) \cap (A \cup C)$ (i)

Now let y be any element of $(A \cup B) \cap (A \cup C)$
Then $y \in (A \cup B) \cap (A \cup C)$
this shows that $y \in (A \cup B)$ and $y \in (A \cup C)$
which implies ($y \in A$ or $y \in B$) and ($y \in$ or $y \in C$)
which also implies $y \in A$ or ($y \in B$ and $y \in C$)
therefore $y \in A$ or $y \in (B \cap C)$
hence, $y \in A \cup (B \cap C)$ (ii)

From (i) and (ii) we have now proven $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Let x be any element of $A \cap (B \cup C)$
Then $x \in A \cap (B \cup C)$
this shows that $x \in A$ and $x \in (B \cup C)$
which implies $x \in A$ and ($x \in B$ or $y \in C$)
which also implies ($x \in A$ and $y \in B$) or ($x \in A$ and $x \in C$)
therefore $x \in (A \cap B)$ or $x \in (A \cap C)$
hence $x \in (A \cap B) \cup (A \cup C)$ (i)

Now let y be any element of $(A \cap B) \cup (A \cap C)$
Then, $y \in (A \cap B) \cup (A \cap C)$
this shows that ($y \in A$ and $y \in B$) or ($y \in A$ and $y \in C$)
which implies that $y \in A$ and ($y \in B$ or $y \in C$)
which also implies that $y \in A$ and $y \in (B \cup C)$
therefore, $y \in A \cap (B \cup C)$ (ii)

From (i) and (ii) we have now proven $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\blacksquare$

Problem 3.23 For each of the statements (a)-(d) on the left, find an equivalent symbolic proposition chosen from the list (i)-(v) on the right. Note that not every statement on the right will get used.
(a) $A \subsetneq B =$ V
(b) $A \cap B = \emptyset =$ (ii)
(c) $(A \cup B)^c = \emptyset =$ (iii)
(d) $(A \cap B)^c = \emptyset =$ (iV)

$\blacksquare$