IN5270 Mandatory exercise 1

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Problem 1: Use linear/quadratic functions for verification

Consider the ODE problem:

$$u'' + \omega^2 u = f(t), \quad u(0) = I, \quad u'(0) = V, \quad t \in (0, T]$$

Discretize this equation according to: $[D_t D_t t]^n = 0$

a)

Using Backward Euler on $u'(0) = V$ to find u^{-1}:

$$\frac{u^1 - u^{-1}}{2\Delta t} = V \rightarrow u^{-1} = u^1 - 2\Delta t V$$

rewriting $u'' \approx \frac{u(t_{n+1} - 2u(t_n) + u(t_{n-1})}{\Delta t^2}$ This gives us the approx equation:

$$\frac{u(t_{n+1} - 2u(t_n) + u(t_{n-1})}{\Delta t^2} + \omega u(t_n) = f(t)$$

I use this yo find u^1 with the new u^{-1} given above and u(0) = I, when n = 0:

$$u^1 = \frac{1}{2}\Delta t^2 f(t) + I - \frac{1}{2}\Delta t^2 \omega^2 I + \Delta t^2 V$$

b)

Using the initial conditions: $u(0) = I, u'(0) = V$, to compute c and d in the exact linear function: $u_e = ct + d$

$$u(0) = c * 0 + d = I \rightarrow d = I$$
$$u'(0) = c = V \rightarrow c = V$$

Using this to find f: $u'' + \omega^2 u = f \rightarrow 0 + \omega^2(Vt + I) = f \rightarrow f = \omega^2(Vt + I)$
$[D_t D_t t]^n = [D_t(D_t t)]^n = [D_t(1)]^n = 0^n = 0$ and using the fact that $D_t D_t$ operator is linear to show:

$$
\begin{aligned}
[D_t D_t(Vt + I)]^n &= [D_t D_t(Vt)]^n + [D_t D_t(d)]^n = V[D_t D_t(t)]^n + 0 \\
&= V * 0 + 0 = 0
\end{aligned}
$$

Showing that u_e is a perfect solution: $u'' + \omega^2 u = f \rightarrow 0 + \omega^2(Vt + I) = \omega^2(Vt + I)$ This gives 0.

c)

This is done in the file 'vib_undamped_verify_mms.py'. Using $u'' + \omega^2 u = f(t) \rightarrow u'' + \omega^2 u - f(t) = 0 \rightarrow R = u'' + \omega^2 u - f(t)$

d)

Adding a new function to the file named 'quadratic' were I test if the exact quadratic function : $u_e = bt^2 + ct + d$. The quadratic function fulfills the discrete equation.

e)

Adding a new function to the file named 'polynomial_3degree', were I test if the exact polynomial of degree three : $u_e = at^3 + bt^2 + ct + d$ The quadratic function dosent fulfill the discrete equation, it gets a residual step1: a*dt**3, error.

```
1x-193-157-183-247:Oblig1 soranhussein$ python vib_undamped_verify_mms.py
=== Testing exact solution: <function <lambda> at 0x10527f668> ===
Initial conditions u(0)=I, u'(0)=V:
residual step1: 0
residual: 0
=== Testing exact solution: <function <lambda> at 0x10527f668> ===
Initial conditions u(0)=I, u'(0)=V:
residual step1: 0
residual: 0
=== Testing exact solution: <function <lambda> at 0x10527f668> ===
Initial conditions u(0)=I, u'(0)=V:
residual step1: a*dt**3
residual: 0
```

.png

Figure 1: Run of all the diffrent functions

f)

Adding a solver function to the file, with the numerical equation and intial contitions:

$$
\begin{aligned}
u[0] =& I \\
u[1] =& u[0] - \frac{1}{2}dt^2 w^2 u[0] + dtV + \frac{1}{2}dt^2 f(t[0]) \\
u[n+1] =& dt^2 f(t[n]) + 2u[n] - u[n-1] - dt^2 w^2 u[n]
\end{aligned}
$$

g)

Adding a test function of the quadratic solution to the file. Choosing the variables and test the numerical with the exact solution shows that there is no error in the quadratic solution as long u use tol ¿ 1E-16. It gives a error equal to: 8.881784197001252e-16.

```
1x-193-157-183-247:Oblig1 soranhussein$ nosetests vib_undamped_verify_mms.py
.
----------------------------------------------------------------------
Ran 1 test in 0.028s

OK
```

```
1x-193-157-183-247:Oblig1 soranhussein$ nosetests vib_undamped_verify_mms.py
F
======================================================================
FAIL: Verify solver with quadratic, transform from symbol to
----------------------------------------------------------------------
Traceback (most recent call last):
  File "/Library/Python/2.7/site-packages/nose/case.py", line 197, in runTest
    self.test(*self.arg)
  File "/Users/soranhussein/Dropbox/01_master/IN5270/Oblig1/vib_undamped_verify_mms.py", line
92, in test_quadratic_solution
    assert success, msg
AssertionError: ('Error in quadratic solution:', 8.8817841970012523e-16)

----------------------------------------------------------------------
Ran 1 test in 0.021s

FAILED (failures=1)
```

Figure 2: Run of the test function