MEK3230 – Fluidmekanikk Høst 2017

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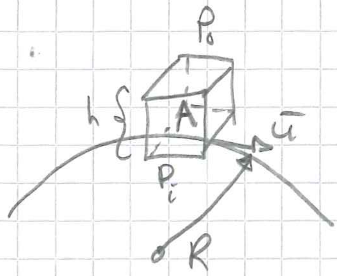
1.1 Vannstråle

1. Dette er feil fordi det er Coanda effekten som virker her. Her får vi strømning på tvers og da er det feil å bruke Bernulli.

Matematisk: (strømlinjene holdes langs en krummede overflate (penn)



trykk endring : $P_0 = P_i + dP$

trykk kraften : $f_p = m\bar{a} = \cancel{\rho} Ah \frac{\partial P}{\partial h} \cdot \frac{1}{\cancel{\rho}}$

$m = \rho Ah$, $\bar{a} = \frac{\partial P}{\partial h}\frac{1}{\rho}$ $\quad = Ah\frac{\partial P}{\partial h}$

Sentripitalkraften : $F_c = m\frac{u^2}{R} = \rho Ah \frac{u^2}{R}$

Ingen bevegelse langs strømlinjene får vi med

$F_c = F_p \longrightarrow \cancel{Ah}\frac{\partial P}{\partial h} = \rho \cancel{Ah}\frac{u^2}{R} \longrightarrow \frac{\partial P}{\partial h} = \rho u^2 \frac{1}{R}$

Ser at den er avhengig av krumning $\frac{1}{R}$, dvs at det er bevegelse på tvers av strømlinjene.

2. Eksperiment som motbeviser dette er?
hvis du blåser på et ark med hår-føner vertikalt som vil du se at det ikke er noen bevegelse i arket, men med en liten bøy i arket vil det bli bevegelse.

1.2 Atmosfærestrømning

$$\rho \frac{D\bar{u}}{Dt} + 2\rho \bar{\Omega} \times \bar{u} + \rho \bar{\Omega} \times \bar{\Omega} \times \bar{r} = -\nabla p + \mu \nabla^2 \bar{u}$$

$$[\bar{\Omega}] = \frac{1}{s}, \quad [\rho] = \frac{kg}{m^3}, \quad [\mu] = Pa \cdot s = \frac{kg}{ms}$$

Parametere / variabler		Dimensjon
1.	$\rho = \frac{kg}{m^3}$	$= ML^{-3}$
2.	$\frac{du}{dt} = \frac{m}{s^2}$	$= LT^{-2}$
3.	$\Omega = \frac{1}{s}$	$= T^{-1}$
4.	$u = \frac{m}{s}$	$= LT^{-1}$
5.	$\nabla p = \frac{kg}{ms^2}$	$= ML^{-1}T^{-2}$
6.	$\mu = \frac{kg}{ms}$	$= ML^{-1}T^{-1}$
7.	$\bar{r} = m$	$= L$

Dimensjon $= 3$
Variable $= 7$
π-grupper $= 7 - 3 = 4$

$$\Pi_1 = \frac{du}{dt} \rho^\alpha \Omega^\beta \mu^\gamma = [LT^{-2}][ML^{-3}]^\alpha [T^{-1}]^\beta [ML^{-1}T^{-1}]^\gamma = M^0L^0T^0$$

$M: \alpha + \gamma = 0 \rightarrow \gamma = -\alpha \rightarrow \underline{\gamma = -\frac{1}{2}}$

$L: 1 - 3\alpha - \gamma = 0 \rightarrow 1 - 3\alpha + \alpha = 0 \rightarrow \underline{\alpha = \frac{1}{2}}$

$T: -2 - \beta - \gamma = 0 \rightarrow -2 - \gamma = \beta \rightarrow \underline{\beta = -\frac{3}{2}}$

$$\underline{\Pi_1} = \frac{du}{dt} \rho^{\frac{1}{2}} \Omega^{-\frac{3}{2}} \mu^{-\frac{1}{2}} = \underline{\frac{\frac{du}{dt}\sqrt{\rho}}{\sqrt{\Omega^3}\sqrt{\mu}}}$$

$$\Pi_2 = u \rho^\alpha \Omega^\beta \mu^\gamma = [LT^{-1}][ML^{-3}]^\alpha [T^{-1}]^\beta [ML^{-1}T^{-1}]^\gamma = M^0L^0T^0$$

$M: \alpha + \gamma = 0 \rightarrow \alpha = -\gamma \rightarrow \underline{\alpha = \frac{1}{2}}$

$L: 1 - 3\alpha - \gamma = 0 \rightarrow 1 + 3\gamma - \gamma = 0 \rightarrow \underline{\gamma = -\frac{1}{2}}$

$T: -1 - \beta - \gamma = 0 \rightarrow -1 + \frac{1}{2} = \beta \rightarrow \underline{\beta = -\frac{1}{2}}$

$$\underline{\Pi_2} = u \rho^{\frac{1}{2}} \Omega^{-\frac{1}{2}} \mu^{-\frac{1}{2}} = \underline{\frac{u\sqrt{\rho}}{\sqrt{\Omega}\sqrt{\mu}}}$$

$$\Pi_3 = \Delta P\, \rho^{\alpha} \Omega^{\beta} \mu^{\gamma} = [ML^{-1}T^{-2}][ML^{-3}]^{\alpha}[T^{-1}]^{\beta}[ML^{-1}T^{-1}]^{\gamma} = M^0L^0T^0$$

$M: 1 + \alpha + \gamma = 0 \rightarrow \gamma = -\alpha - 1 \rightarrow \underline{\gamma = -1}$

$L: -1 - 3\alpha - \gamma = 0 \rightarrow -1 - 3\alpha + \alpha + 1 = 0 \rightarrow \underline{\alpha = 0}$

$T: -2 - \beta - \gamma = 0 \rightarrow -2 - \beta + 1 = 0 \rightarrow \underline{\beta = -1}$

$$\underline{\Pi_3} = \Delta P\, \rho^{0} \Omega^{-1} \mu^{-1} = \underline{\frac{\Delta P}{\Omega \mu}}$$

$$\Pi_4 = r\, \rho^{\alpha} \Omega^{\beta} \mu^{\gamma} = [L][ML^{-3}]^{\alpha}[T^{-1}]^{\beta}[ML^{-1}T^{-1}]^{\gamma}$$
$$= M^0L^0T^0$$

$M: \alpha + \gamma = 0 \rightarrow \alpha = -\gamma \rightarrow \underline{\alpha = \frac{1}{2}}$

$L: 1 - 3\alpha - \gamma = 0 \rightarrow 1 - 3\gamma - \gamma = 0 \rightarrow \underline{\gamma = -\frac{1}{2}}$

$T: -\beta - \gamma = 0 \rightarrow \beta = -\gamma \rightarrow \underline{\beta = \frac{1}{2}}$

$$\underline{\Pi_4} = r\, \rho^{\frac{1}{2}} \Omega^{\frac{1}{2}} \mu^{-\frac{1}{2}} = \underline{\frac{r\sqrt{\rho}\sqrt{\Omega}}{\sqrt{\mu}}}$$

1.3 Aquaporin kanaler

Kanalen har radius R, lengde L og væske fluidet har en viskositet μ som presser gjennom av et (osmotisk) trykk p i cellen:

Volumrate: $\frac{\partial V}{\partial t} = \left[\frac{m^3}{s}\right]$ $(V = R^2 L)$

Variabler		Dimensjoner
1.	$R = m$	$= L$
2.	$L = m$	$= L$
3.	$\mu = \frac{kg}{ms}$	$= ML^{-1}T^{-1}$
4.	$p = \frac{kg}{s^2}$	$= MT^{-2}$
5.	$\frac{\partial V}{\partial t} = \frac{m^3}{s}$	$= L^3 T^{-1}$
6.	$v = \frac{m}{s}$	$= LT^{-1}$

Variabler = 6

Dimensjoner = 3

Π-grupper: 5 − 3 = 2

trenger bare å finne Π_1

$$\Pi_1 = \frac{\partial V}{\partial t} \rho^\alpha \mu^\beta R^\gamma = [L^3 T^{-1}][MT^{-2}]^\alpha [ML^{-1}T^{-1}]^\beta [L]^\gamma$$
$$= M^0 L^0 T^0$$

$M: \quad \alpha + \beta = 0 \rightarrow \alpha = -\beta \rightarrow \underline{\alpha = -1}$

$L: \quad 3 - \beta + \gamma = 0 \rightarrow 3 - 1 + \gamma = 0 \rightarrow \underline{\gamma = -2}$

$T: \quad -1 - 2\alpha - \beta = 0 \rightarrow -1 + 2\beta - \beta = 0 \rightarrow \underline{\beta = 1}$

$$\underline{\Pi_1} = \frac{\partial V}{\partial t} \rho^{-1} \mu^1 R^{-2} = \underline{\frac{\frac{\partial V}{\partial t}\mu}{\rho R^2}}$$

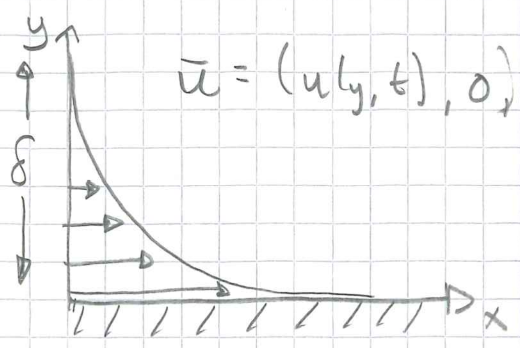
$$\frac{\partial V}{\partial t} = \frac{\rho R^2}{\mu} \Pi_1$$ (kan bruke Π_2 til å finne Π_1)

1.4 Hastighetsfelt

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}, \quad \omega_z = (\nabla \times \bar{u}) \cdot \bar{e}_z$$

$$\Gamma = \int_\Omega \omega \, dx = \int_C \bar{u} \cdot d\bar{c}$$

$\bar{u} = (u(y,t), 0, 0)$



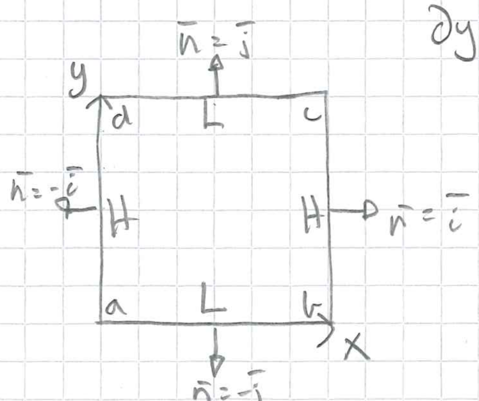
i) $\frac{\partial u}{\partial t} = \frac{\partial u(y,t)}{\partial t} > 0$

ii) $u\frac{\partial u}{\partial x} = u(y,t)\frac{\partial u(y,t)}{\partial x} = 0$

iii) $v\frac{\partial u}{\partial y} = 0 \cdot \frac{\partial u(y,t)}{\partial y} = 0$

iv) $$\omega_z = (\nabla \times \bar{u}) \cdot \bar{e}_z = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \cdot e_z$$

$$= -\frac{\partial u}{\partial y} e_z \cdot e_z = -\frac{\partial u}{\partial y} < 0$$



$\bar{u} = (u, 0, 0)$

- ingen strømning i y
- konstant strøm i x

$v = 0, \quad \frac{\partial}{\partial x} = 0$

$d\bar{c} = \bar{t}\,\bar{n}\,dr, \quad d\bar{x} = dx, dy, dz$

$$\underline{\Gamma} = \int_\Omega \omega \, dx = \int_C \bar{u} \cdot d\bar{c} = \int_0^L u(y,t)\,\bar{i} \cdot \bar{i} \, dx$$

$$= [Ux]_0^L = \underline{UL} \quad \text{(de andre blir 0)}$$

1.5 Impulslignngen

Impulslignngen: $\int_\tau \frac{\partial}{\partial t}(\rho \bar{v})\, d\tau + \int_\sigma \rho \bar{v}\bar{v}\cdot d\bar{\sigma} = \underline{P} + \underline{F}$

τ: er et geometrisk avgrenset volum

σ: er begrensnings flaten til det geometriske avgrenset volumet.

$\rho \bar{v}$: bevegelses mengden

$\frac{\partial}{\partial t}(\rho \bar{v})$: endring av bevegelses mengden med hensyn på tid

$\int_\tau \frac{\partial}{\partial t}(\rho \bar{v})\, d\tau$: —"— innenfor et fast geometrisk avgrenset volum

$\rho \bar{v}\bar{v}$: netto strøm av bevegelses mengden

$\int_\sigma \rho \bar{v}\bar{v}\cdot d\bar{\sigma}$: Strømmen av bevegelsesmengden gjennom kontrollflaten

$\left.\begin{array}{l}\underline{P}: \text{flate kraften} \\ \underline{F}: \text{volumkraften}\end{array}\right\}$ $\underline{P} + \underline{F}$ = Flate- og volum kraften som virker på fluidet innenfor volumet.

2.1 Potensialstrømning (Matlab lagt ved)

$$\omega(z) = \left(\frac{m - i\Gamma}{2\pi}\right) \ln z \;, \quad z = x + iy = re^{i\theta}$$

m og Γ er reelle konstanter

1. $$w(re^{i\theta}) = \left(\frac{m - i\Gamma}{2\pi}\right) \ln(re^{i\theta})$$

$$= \left(\frac{m - i\Gamma}{2\pi}\right)(\ln r + i\theta) = \frac{m}{2\pi}\ln r + i\frac{m\theta}{2\pi}$$

$$-\frac{i\Gamma}{2\pi}\ln r + \frac{\Gamma\theta}{2\pi}$$

(Setter $\psi = \psi_0$ og finner $\theta =$ for så plotte dette)

$$= \left(\frac{m}{2\pi}\ln r + \frac{\Gamma\theta}{2\pi}\right) + i\left(\frac{m\theta - \Gamma\ln r}{2\pi}\right)$$

$$\Phi = \frac{m\ln r + \Gamma\theta}{2\pi} \;, \quad \psi = \left(\frac{m\theta - \Gamma\ln r}{2\pi}\right)$$

2. $$\underline{u_\theta} = \frac{1}{r}\frac{d\Phi}{\partial\theta} = \frac{1}{r}\frac{d}{\partial\theta}\left(\frac{m\ln r + \Gamma\theta}{2\pi}\right) = \frac{1}{r}\left(\frac{\Gamma}{2\pi}\right) = \underline{\frac{\Gamma}{2\pi r}}$$

$$\underline{u_r} = \frac{\partial\psi}{\partial r} = \frac{\partial}{\partial r}\left(\frac{m\theta - \Gamma\ln r}{2\pi}\right) = \underline{\frac{m}{2\pi r}}$$

$$\underline{w'(z)} = \left(\frac{m - i\Gamma}{2\pi}\right)\frac{1}{z} = \left(\frac{m - i\Gamma}{2\pi}\right)\frac{1}{re^{i\theta}} = \underline{\left(\frac{m - i\Gamma}{2\pi}\right)\frac{1}{r}e^{-i\theta}}$$

$$= \left(\frac{m}{2\pi r} - i\frac{\Gamma}{2\pi r}\right)e^{-i\theta} = (u_\theta - iu_r)e^{-i\theta}$$

3. $$u_r = \frac{dr}{dt} \rightarrow \frac{m}{2\pi r} = \frac{dr}{dt} \rightarrow \frac{m}{2\pi} = r\frac{dr}{dt}$$

$$\frac{r^2}{2} = \frac{m}{2\pi}t + C_1 \rightarrow r(t) = \sqrt{\frac{mt}{\pi} + C_1}$$

Randbetingelse $r(0) = r_0$;

$$r(0) = \sqrt{C_1} = r_0 \rightarrow C_1 = r_0^2$$

$$\underline{r(t) = \sqrt{\frac{mt}{\pi} + r_0^2}}$$

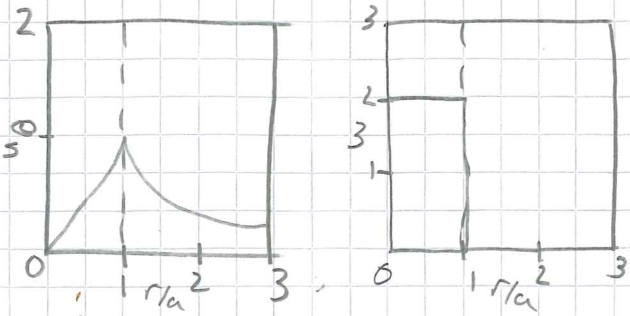
$$u_\theta = r\frac{d\theta}{dt} \rightarrow \frac{d\theta}{dt} = \frac{\Gamma}{2\pi r^2} = \frac{\Gamma}{2\pi}\left(\frac{\pi}{mt} + \frac{1}{r_0^2}\right)$$

$$\theta(t) = \frac{\Gamma}{2\pi}\left(\frac{\pi}{m}\ln t + \frac{t}{r_0^2}\right) + C_2 \;, \quad t = 0 \rightarrow \theta = \theta_0$$

$$\theta(0) = C_2 = \theta_0 \rightarrow \underline{\theta(t) = \frac{\Gamma}{2\pi}\left(\frac{\pi}{m}\ln t + \frac{t}{r_0^2}\right) + \theta_0}$$

2.2 Rankne-virvel - en forenklet tornado

I kjernen av tornadoet har virvling, r=0.
Utover vil vinkelhastigheten minke desto lengre vi forflytter oss



$u_\theta = \Gamma r$ når $r < a$

$u_\theta = \frac{\Gamma a^2}{r}$ når $r > a$

$u_r = u_z = 0$

inkompressibel friksjonsfri strømning

1. Bruker formelen som er oppgitt i heftet, og leddene jeg står igjen med er:

r-komp: $-\rho \frac{u_\theta^2}{r} = -\frac{\partial P}{\partial r}$

θ-komp: $0 = -\frac{1}{r}\frac{\partial P}{\partial \theta} \rightarrow P = 0$ (konstant)

z-komp: $0 = -\frac{\partial P}{\partial z} - \rho g$

Dette gir:

$P(z) = -\rho g z + f(r)$

$r < a$: $P(r) = \int \rho \frac{\Gamma^2 r^{\cancel{2}}}{\cancel{r}} dr = \frac{\rho \Gamma^2}{2} r^2 + f(z)$

$r > a$: $P(r) = \int \rho \frac{\left(\frac{\Gamma a^2}{r}\right)^2}{r} dr = -\frac{\rho \Gamma^2 a^4}{2}\frac{1}{r^2} + f(z)$

for $r < a$: $P_1(r, z) = \frac{\rho \Gamma^2}{2} r^2 - \rho g z + C_1$

for $r > a$: $P_2(r, z) = -\frac{\rho \Gamma^2 a^4}{2}\frac{1}{r^2} - \rho g z + C_2$

2 Det jeg skal finne her er trykk differansen altså $P_2 - P_1$

$r = 0$: $P_1(0, z) = -\rho g z + C_1$

$r = \infty$: $P_2(0, z) = -\rho g z + C_2$

finner P_2 og P_1 i $r = a$ (da er $P_2 = P_1$)

$$\frac{\rho \Gamma a^2}{2} - \cancel{\rho g z} + \cancel{C_1}_{\to 0} = -\frac{\rho \Gamma^2 a^2}{2} - \cancel{\rho g z} + C_2$$

$$C_2 = \frac{\rho \Gamma^2 a^2}{2} + \rho \frac{\Gamma^2 a^2}{2} = \rho \Gamma a^2$$

Dermed har vi trykk differansen: (i $r=0$ og $r \to \infty$)

$$P_2 - P_1 = -\cancel{\rho g z} + \cancel{\rho g z} + C_2 = \rho \Gamma^2 a^2$$

$$\underline{\underline{P_2 - P_1 = \rho \Gamma^2 a^2}}$$

$$\left(\begin{array}{ll} P_1 = \frac{\rho \Gamma^2}{2} r^2 - \rho g z & r < a \\ P_2 = -\frac{\rho \Gamma^2 a^4}{2} \frac{1}{r^2} - \rho g z + \rho \Gamma^2 a^2 & r > a \end{array} \right)$$

3. I midten $r = a$ er trykket ute lik trykket inne.

$r < a$: $P_0 = \frac{\rho \Gamma^2}{2} r^2 - \rho g z \to z = \frac{\Gamma^2}{2g} r^2 - \frac{P_0}{\rho g}$

$r > a$: $P_0 = -\frac{\rho \Gamma^2 a^4}{2} \frac{1}{r^2} - \rho g z + \rho \Gamma^2 a^2$

$$z = \frac{\Gamma^2 a^2}{g} - \frac{\Gamma^2 a^4}{2g} \frac{1}{r^2} - \frac{P_0}{\rho g}$$

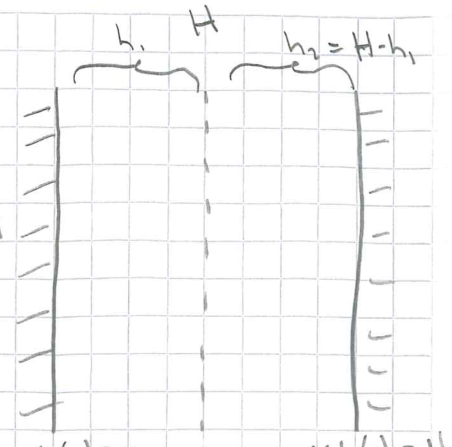
$r = a$: $P_0 = \rho \Gamma^2 a^2 - \rho g z$

$$z = \frac{\Gamma^2 a^2}{g} - \frac{P_0}{\rho g}$$

(Matlab på slutten)

2.3 Viskøs strømning

- Inkompressible viskøse fluid
- Stasjonær
- Ingen trykkgradient



$$w_1(h_1) = w_2(h_2)$$

$$\underbrace{-gk}\quad \frac{\partial w_1}{\partial x} = -\frac{\partial w_2}{\partial x}$$

1. har $\bar{u} = (u, 0, w)$
Ut fra de 3 betingelsene over gir Navier Stoke:

$$\rho\Big(\underbrace{\frac{\partial u}{\partial t}}_{\to 0} + \underbrace{\bar{u}\cdot\nabla\bar{u}}_{\to 0}\Big) = -\underbrace{\nabla p}_{\to 0} + \mu\nabla^2\bar{u} + \bar{g}$$

$$\bar{u}\cdot\nabla\bar{u} = u\frac{\partial\bar{u}}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial\bar{u}}{\partial z} = 0$$

$$0 = \frac{\mu}{\rho}\frac{\partial^2 w}{\partial x^2} - g \quad (\text{Ref. feil står med igjen})$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\rho g}{\mu} \Rightarrow \frac{\partial w_1}{\partial x} = \frac{\rho_1 g}{\mu}x + C_1$$

total krafter:

$$\frac{\partial w_2}{\partial x} = \frac{\rho_2 g}{\mu}x + C_3$$

$$\underline{\tau_1} = \mu\frac{\partial w_1}{\partial x} = \mu\frac{\rho_1 g}{\mu}x + C_1\mu = \underline{\rho_1 g x + C_1\mu}$$

$$\underline{\tau_2} = -\mu\frac{\partial w_2}{\partial x} = -\mu\frac{\rho_2 g}{\mu}x - C_3\mu = -\underline{\rho_2 g x - C_3\mu}$$

2. $\frac{\partial w_1}{\partial x} = \frac{\rho_1 g}{\mu} x + C_1 \rightarrow \underline{w_1(x) = \frac{\rho_1 g}{2\mu} x^2 + x C_1 + C_2}$

$\frac{\partial w_2}{\partial x} = \frac{\rho_2 g}{\mu} x + C_3 \rightarrow \underline{w_2(x) = \frac{\rho_2 g}{2\mu} x^2 + x C_3 + C_4}$

Randbetingelse: $w_1(0) = 0$, $w_2(H) = 0$, $w_1(h_1) = w_2(h_1)$

$\frac{\partial w_1}{\partial x} = -\frac{\partial w_2}{\partial x}$

$w_1(0) = \frac{\rho_1 g}{2\mu} \cdot 0^2 + C_1 \cdot 0 + C_2 = 0 \rightarrow \underline{C_2 = 0}$

$w_2(H) = \frac{\rho_2 g}{2\mu} H^2 + H C_3 + C_4 = 0$

$\underline{C_4 = H\left(-\frac{\rho_2 g}{2\mu} H - C_3\right)}$

$\left.\frac{\partial w_1}{\partial x}\right|_{h_1} = -\left.\frac{\partial w_2}{\partial x}\right|_{h_1} \rightarrow \frac{\rho_1 g}{\mu} h_1 + C_1 = -\frac{\rho_2 g}{\mu} h_1 - C_3$

$\underline{C_3} = -\frac{\rho_2 g}{\mu} h_1 - \frac{\rho_1 g}{\mu} h_1 - C_1 = \underline{-\frac{g}{\mu} h_1 (\rho_2 + \rho_1) - C_1}$

$\underline{w_2(x)} = \frac{\rho_2 g}{2\mu}(x^2 + H^2) + \frac{g}{\mu} h_1 (\rho_2 - \rho_1)(H - x) + C_1 (H - x)$

$w_1(h_1) = w_2(h_2)$ (bruker $h_2 = H - h_1$)

$\frac{\rho_1 g}{2\mu} h_1^2 + h_1 C_1 = \frac{\rho_2 g}{2\mu}(h_1^2 + H^2) + \frac{g}{\mu} h_1 \overbrace{(H - h_1)}^{h_2} + C_1 \overbrace{(H - h_1)}^{h_2}$

$$\underline{C_1 = \frac{\frac{g h_1^2}{2\mu}(\rho_2 - \rho_1) + \frac{g h_1 h_2}{\mu}(\rho_1 - \rho_2) - \frac{\rho_2 g}{2\mu} H^2}{(h_1 - h_2)}}$$

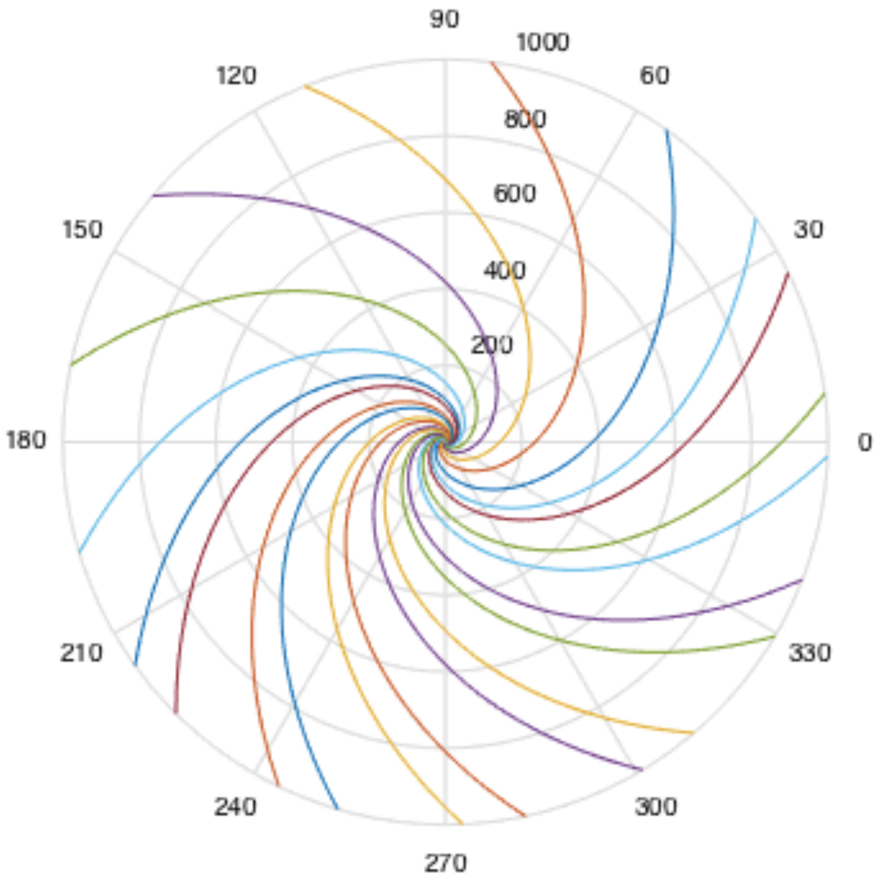
$\underline{w_1(x)} = \frac{\rho_1 g}{2\mu} x^2 + x C_1$

Kunne skrevet ut og forkortet litt.

```
% OPPGAVE 2.1 - 1)

n = 20;
r = linspace(1,1000);
m = 1; gamma = 1;
w0 = linspace(0.5,2,n);

for i = 1:n
    kons = (2*pi*w0(i))/m;
    polar(kons + (gamma/m)*log(r),r);
    hold on
end
```



```
% OPPGAVE 2.2 - 3)

g = 9.81; gamma = 1; n = 100;
rho = 1; p0 = 1; a = 1;
r = linspace(0,2,n);
z = zeros(1,n);

for i = 1:n
    if r(i) > a
        z(i) = (gamma^2 * a^2)/g - (gamma^2 * a^4)/(2*g*r(i)^2) - p0/(rho*g);
    elseif r == a
        z(i) = - p0/(g*rho) + (gamma^2*a^2)/g
    else
        z(i) = (gamma^2 * r(i)^2)/(2*g) - p0/(rho*g);
    end
end

plot(r,z)
```