MEK3570 Oblig 2

Soran Hussein Mohmmed / soranhm

Oktober 2017

Kapitel 4.2, Oppgave 2

$$\frac{D}{Dt}\int_{\Omega}\Phi dv = \int_{\Omega}(\dot{\Phi} + \Phi div\vec{v})dv \tag{1}$$

$$\frac{D}{Dt}\int_{\Omega}\Phi dv = \int_{\Omega}\Big(\frac{\partial\Phi}{\partial t} + grad\Phi\cdot\vec{v} + \Phi div\vec{v}\Big) = \int_{\Omega}\Big(div(\Phi\vec{v}) + \frac{\partial\Phi}{\partial t}\Big) \tag{2}$$

$$\frac{D}{Dt}\int_{\Omega}\Phi dv = \int_{\partial\Omega}\Phi\vec{v}\cdot\vec{n}ds + \int_{\Omega}\frac{\partial\Phi}{\partial t}dv \tag{3}$$

$$\frac{D}{Dt}\int_{\Omega}\rho(\vec{x},t)\Psi(\vec{x},t)dv = \int_{\Omega}\rho(\vec{x},t)\dot{\Psi}(\vec{x},t)dv \tag{4}$$

$$div(\vec{u}\otimes\vec{v}) = (grad\vec{u})\vec{v} + \vec{u}div\vec{v} \tag{5}$$

$$\int_{s}\Phi\mathbf{A}\vec{n}ds = \int_{v}div(\Phi\mathbf{A})dv \tag{6}$$

$$(\vec{u}\otimes\vec{v})\vec{w} = \vec{u}(\vec{v}\cdot\vec{w}) = (\vec{v}\cdot\vec{w})\vec{u} \tag{7}$$

Har et romlig vektor $\vec{u} = \vec{u}(\vec{x},t)$, skal bevise:

$$\begin{aligned}
\frac{D}{Dt}\int_{\Omega}\vec{(}u)dv &\overset{1}{=} \int_{\Omega}(\dot{\vec{u}} + \vec{u}div\vec{v})dv \overset{utskriving}{=} \int_{\Omega}\Big(\frac{\partial\vec{u}}{\partial t} + (grad\vec{u})\cdot\vec{v} + \vec{u}div\vec{v}\Big)dv \\
&\overset{5}{=} \int_{\Omega}\Big(\frac{\partial\vec{u}}{\partial t} + div(\vec{u}\otimes\vec{v})\Big)dv = \int_{\Omega}div(\vec{u}\otimes\vec{v})dv + \int_{\Omega}\frac{\partial\vec{u}}{\partial t}dv \\
&\overset{6}{=} \int_{\Omega}\frac{\partial\vec{u}}{\partial t}dv + \int_{\partial\Omega}(\vec{u}\otimes\vec{v})\vec{n}ds \overset{7}{=} \underline{\underline{\int_{\partial\Omega}\vec{u}(\vec{v}\cdot\vec{n})ds + \int_{\Omega}\frac{\partial\vec{u}}{\partial t}dv}}
\end{aligned}$$

for å vise (4.34) bruker jeg (4) og bytter ut $dm = \rho dv$ og regner ut $\frac{D\vec{u}}{Dt}$

$$\frac{D}{Dt}\int_{\Omega}\rho\vec{u}dv = \int_{\Omega}\rho\dot{\vec{u}}dv$$

Kapitel 4.3, Oppgave 1

Anta at $\vec{g}$ som masse senter med vilkårlige område Ω, kroppen β og massen m er definer som:

$$\vec{g}(t) = \frac{1}{m(\Omega)} \int_{\Omega} \rho(\vec{x}, t)\vec{x}dv \tag{8}$$

masse bevaring: $m(\Omega_0) = m(\Omega)$ kan blir presentert som: $\frac{1}{m(\Omega_0)} \int_{\Omega} \rho(\vec{x}, t)\vec{\chi}(\vec{X}, t)dV$

a)

deriverer $\vec{g}$ mhp tid, og finner $\dot{\vec{g}}$ og $\ddot{\vec{g}}$:

$$\frac{D\vec{g}(t)}{Dt} = \frac{1}{m(\Omega)} \frac{D}{Dt} \int_{\Omega} \rho(\vec{x}, t)\vec{x}dv \overset{dm=\rho dv}{=} \frac{1}{m(\Omega)} \int_{\Omega} \dot{\vec{x}}dm = \frac{1}{m(\Omega)} \int_{\Omega} \frac{\partial \vec{x}}{\partial t} + (grad\vec{x})\vec{v}dm$$

$$\dot{\vec{g}}(t) \overset{grad\vec{x}=0}{=} \frac{1}{m(\Omega)} \int_{\Omega} \rho(\vec{x}, t)\frac{\partial \vec{x}}{\partial t}dv = \frac{1}{m(\Omega)} \int_{\Omega} \rho(\vec{x}, t)\vec{v}(\vec{x}, t)dv$$

Gjør det samma igjen for $\ddot{\vec{g}}$:

$$\ddot{\vec{g}}(t) = \frac{1}{m(\Omega)} \int_{\Omega} \rho(\vec{x}, t)\dot{\vec{v}}(\vec{x}, t)dv$$

b)

$$\vec{L}(t) = \int_{\Omega} \rho\vec{v}(\vec{x}, t)dv \tag{9}$$

$$\dot{\vec{L}}(t) = \int_{\Omega} \rho\dot{\vec{v}}dv = \vec{F}(t) \tag{10}$$

$$\begin{aligned}
\vec{L}(t) =& m(\Omega)\dot{\vec{g}} \quad \overset{a)}{=} \cancel{m(\Omega)} * \frac{1}{\cancel{m(\Omega)}} \int_{\Omega} \rho\vec{v}(\vec{x}, t)dv = \int_{\Omega} \rho\vec{v}(\vec{x}, t)dv \\
\dot{\vec{L}}(t) =& m(\Omega)\ddot{g}(t) \quad \overset{a)\&10}{=} \cancel{m(\Omega)} * \frac{1}{\cancel{m(\Omega)}} \int_{\Omega} \rho\dot{\vec{v}}(\vec{x}, t)dv = \int_{\Omega} \rho\dot{\vec{v}}(\vec{x}, t)dv \\
=& \vec{F}(t)
\end{aligned}$$

Kapitel 6.3, Oppgave 1

$$\sigma_a = -p + 2\Big(\lambda_a^2 \frac{\partial \Psi}{\partial I_1} - \frac{1}{\lambda_a^2}\frac{\partial \Psi}{\partial I_2}\Big) \tag{11}$$

For å vise a) og b) bruker jeg (11), først finner jeg p ved hjelp av en av σ som er lik 0, og bruker dette til å finne de andre σ'ene.

a)

For oppgitt at : $\lambda_1 = \lambda, \lambda_2 = \lambda_3 = \lambda^{-\frac{1}{2}}, \sigma_1 = \sigma, \sigma_2 = \sigma_3 = 0$

$$\sigma_3 = -p + 2\Big(\lambda_3^2 \frac{\partial \Psi}{\partial I_1} - \frac{1}{\lambda_3^2}\frac{\partial \Psi}{\partial I_2}\Big) = 0 \to p = 2\Big(\frac{1}{\lambda}\frac{\partial \Psi}{\partial I_1} - \lambda \frac{\partial \Psi}{\partial I_2}\Big)$$
$$\sigma_1 = -p + \Big(\lambda_1^2 \frac{\partial \Psi}{\partial I_1} - \frac{1}{\lambda_1^2}\frac{\partial \Psi}{\partial I_2}\Big) = -2\Big(\frac{1}{\lambda}\frac{\partial \Psi}{\partial I_1} - \lambda \frac{\partial \Psi}{\partial I_2}\Big) + \Big(\lambda^2 \frac{\partial \Psi}{\partial I_1} - \frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_2}\Big)$$
$$\overset{mellomregning}{=} 2\Big(\lambda^2 - \frac{1}{\lambda}\Big)\Big(\frac{\partial \Psi}{\partial I_1} + \frac{1}{\lambda}\frac{\partial \Psi}{\partial I_2}\Big)$$

σ_2 er samma som σ_3 dermed får vi at $\sigma_2 = \sigma_3$ der begge er lik null. også har vi fått oppgitt at $\sigma = \sigma_1$ og dette er det jeg har fått over.
$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \lambda^2 + 2\lambda^{-1}$
$I_2 = \lambda_1^2\lambda_2^2 + \lambda_1^2\lambda_3^2 + \lambda_2^2\lambda_3^2 = 2\lambda + \lambda^{-2}$
Gjør det samma for andre del med:
$\lambda_1 = \lambda_2 = \lambda, \lambda_3 = \lambda^{-2}$, $\sigma_1 = \sigma_2 = \sigma, \sigma_3 = 0$

$$\sigma_3 = -p + 2\Big(\frac{1}{\lambda^4}\frac{\partial \Psi}{\partial I_1} - \lambda^4 \frac{\partial \Psi}{\partial I_2}\Big) = 0 \to p = 2\Big(\frac{1}{\lambda^4}\frac{\partial \Psi}{\partial I_1} - \lambda^4 \frac{\partial \Psi}{\partial I_2}\Big)$$
$$\sigma_1 = -2\Big(\frac{1}{\lambda^4}\frac{\partial \Psi}{\partial I_1} - \lambda^4 \frac{\partial \Psi}{\partial I_2}\Big) + 2\Big(\lambda^2 \frac{\partial \Psi}{\partial I_1} - \frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_2}\Big)$$
$$\sigma_2 = -2\Big(\frac{1}{\lambda^4}\frac{\partial \Psi}{\partial I_1} - \lambda^4 \frac{\partial \Psi}{\partial I_2}\Big) + 2\Big(\lambda^2 \frac{\partial \Psi}{\partial I_1} - \frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_2}\Big)$$
$$\sigma = \sigma_1 = \sigma_2 = 2\Big(\lambda^2 - \frac{1}{\lambda^4}\Big)\Big(\frac{\partial \psi}{\partial I_1} + \lambda^2 \frac{\partial \psi}{\partial I_2}\Big)$$

Finner invariandsene:
$I_1 = 2\lambda^2 + \lambda^{-4}$
$I_1 = 2\lambda^{-2} + \lambda^4$

b)

$\lambda_1 = \lambda, \lambda_2 = 1, \lambda_3 = \lambda^{-1}$ siden vi har en ren skjærdeformasjon er $\sigma_3 = 0$.

$$\sigma_3 = -p + 2\Big(\frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_1} - \lambda^2 \frac{\partial \Psi}{\partial I_2}\Big) = 0 \to p = 2\Big(\frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_1} - \lambda^2 \frac{\partial \Psi}{\partial I_2}\Big)$$
$$\sigma_1 = -2\Big(\frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_1} - \lambda^2 \frac{\partial \Psi}{\partial I_2}\Big) + 2\Big(\lambda^2 \frac{\partial \Psi}{\partial I_1} - \frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_2}\Big)$$
$$\overset{mellomregning}{=} 2\Big(\lambda^2 - \frac{1}{\lambda^2}\Big)\Big(\frac{\partial \Psi}{\partial I_1} + \frac{\partial \Psi}{\partial I_2}\Big)$$
$$\sigma_2 = -2\Big(\frac{1}{\lambda^2}\frac{\partial \Psi}{\partial I_1} - \lambda^2 \frac{\partial \Psi}{\partial I_2}\Big) + 2\Big(\frac{\partial \Psi}{\partial I_1} - \frac{\partial \Psi}{\partial I_2}\Big)$$
$$\overset{mellomregning}{=} 2\Big(1 - \frac{1}{\lambda^2}\Big)\Big(\frac{\partial \Psi}{\partial I_1} + \lambda^2 \frac{\partial \Psi}{\partial I_2}\Big)$$

med $I_1 = I_2 = \lambda^2 + \lambda^-2 + 1$

Kapitel 6.4, Oppgave 4

Bruker (6.104) for å vise (6.118). Det å komme fra (6.104) fra $\sigma_{iso} = \mathcal{P} : \overline{\sigma}$ til (6.104) er ukjent.

$$\begin{aligned}
\overline{\sigma} =& 2 * J^{-1} \frac{\partial \Psi_{iso}(\overline{b})}{\partial \overline{b}} \overline{b} = 2 * J^{-1} \Big(\frac{\partial \Psi}{\partial I_1} \frac{\partial I_1}{\partial \overline{b}} + \frac{\partial \Psi}{\partial I_2} \frac{\partial I_2}{\partial \overline{b}} \Big) & (12) \\
=& 2 * J^{-1} \Big(\frac{\partial \Psi}{\partial I_1} \underline{I} + \frac{\partial \Psi}{\partial I_2} (\overline{I_1} \underline{I} - \overline{b}) \Big) \overline{b} & (13) \\
=& 2 * J^{-1} \Big(\frac{\partial \Psi}{\partial I_1} \underline{I} + \frac{\partial \Psi}{\partial I_2} \overline{I_1} \underline{I} - \frac{\partial \Psi}{\partial I_2} \overline{b} \Big) \overline{b} & (14) \\
\overline{\sigma} =& 2 * J^{-1} \Big(\overline{\gamma_1} - \overline{\gamma_2} \overline{b} \Big) \overline{b} = \underline{\underline{2 * J^{-1} \Big(\overline{\gamma_1} \overline{b} - \overline{\gamma_2} \overline{b}^2 \Big)}} & (15)
\end{aligned}$$

bruker det jeg får oppgitt i oppg3 (6.117) og siden $\overline{C}$ og $\overline{b}$ har samme egenverdier så har vi:

$$\frac{\partial \overline{I_1}}{\partial \overline{b}} = \frac{\partial \overline{I_1}}{\partial \overline{C}} = \underline{I}, \quad \frac{\partial \overline{I_2}}{\partial \overline{b}} = \frac{\partial \overline{I_2}}{\partial \overline{C}} = \overline{I_1} \underline{I} - \overline{C} = \overline{I_1} \underline{I} - \overline{b}$$

Kapitel 6.5, Oppgave 2

Sfærisk ballong med inkompressible hyperelastisk material ($\lambda_1 \lambda_2 \lambda_3 = 1$).

$$\Psi = \frac{\mu}{\alpha} (\lambda_1^\alpha + \lambda_2^\alpha + \lambda_3^\alpha - 3), \alpha = 2$$

a)

Ved likevekt har vi at $r^2 \pi p_i = 2 r \pi h \sigma \to p_i = 2 \frac{h}{r} \sigma$. Har $\sigma_1 = \sigma_2 = \sigma$ mens $\sigma_3 = 0$.

$$\sigma_3 = -p + \sum_{p=1}^{N} \mu_p \lambda_3^{\alpha p} = 0 \to p = \sum_{p=1}^{N} \mu_p \lambda_3^{\alpha p}$$

har at N = 1. ved vloum bevaring $4 \pi r^2 h = 4 \pi R^2 H$, ut fra dette kan vi finne $\frac{h}{r}$ og $\lambda_3 = \frac{h}{H}$. $4 \pi r^2 h = 4 \pi R^2 H \to \frac{h}{H} = \frac{r^2}{R^2} = \frac{1}{\lambda^2}$, $\frac{h}{r} = \frac{R^2 H}{r^3} = \frac{R^3}{r^3} \frac{H}{R} = \frac{1}{\lambda^3} \frac{H}{R}$, da har jeg også $\lambda_3 = \frac{1}{\lambda^2} \to \lambda_1 = \lambda_2 = \lambda$

$$\begin{aligned}
\sigma_1 =& -\mu \lambda_3^\alpha + \mu \lambda_1^\alpha = \mu(\lambda^\alpha - \lambda^{-2\alpha}) \\
\sigma_1 = \sigma_2 = \sigma =& \mu(\lambda^\alpha - \lambda^{-2\alpha}) \\
p_i =& 2\mu \frac{H}{R} \lambda^- 3 \Big(\lambda^\alpha - \lambda^{-2\alpha} \Big) = \underline{\underline{2\mu \frac{H}{R} \Big(\lambda^{\alpha-3} - \lambda^{-2\alpha-3} \Big)}}
\end{aligned}$$

b)

Fro å finne hvor den har maks og min så derivere jeg og setter p_i lik null, for så finne hva λ er lik, og setter inn dette inn i dobbel derivert der jeg og ser hvor jeg får maks og min. Maks vil være min og min vil være maks i dobblederiverte.

$$\begin{aligned}
p_i'(\lambda) =& 2\mu\frac{H}{R}\Big((\alpha-3)\lambda^{\alpha-4} - (-2\alpha-3)\lambda^{-2\alpha-4}\Big) \\
=& 2\mu\frac{H}{R}\Big((\alpha-3)\lambda^{\alpha-4} + (2\alpha+3)\lambda^{-2\alpha-4}\Big) \\
p_i'(\lambda) =& 2\mu\frac{H}{R}\Big((\alpha-3)\lambda^{\alpha-4} + (2\alpha+3)\lambda^{-2\alpha-4}\Big) = 0 \\
\lambda =& \Big(-\frac{2\alpha+3}{\alpha-3}\Big)^{\frac{1}{3\alpha}} \\
p_i''(\lambda) =& 2\mu\frac{H}{R}\Big((\alpha^2-7\alpha+12)\lambda^{\alpha-5} - (4\alpha^2+14\alpha+12)\lambda^{-2\alpha-5}\Big)
\end{aligned}$$

plotter dette og ser at jeg får maks i $0 < \alpha < 3$ og min i $-\frac{3}{2} < \alpha < 0$:

```
a = linspace(-2,3,100);
my = 1; H = 1; R = 1;
k = 2*my*H/R;

g = (-(2*a+3)./(a-3)).^(1./(3*a));

p1 = (a.^2 - a*7 + 12).*g.^(a-5);

p2 = (4*a.^2 + a*14 + 12).*g.^(-a*2-5);
p = k*(p1 - p2)
plot(a,p)
```

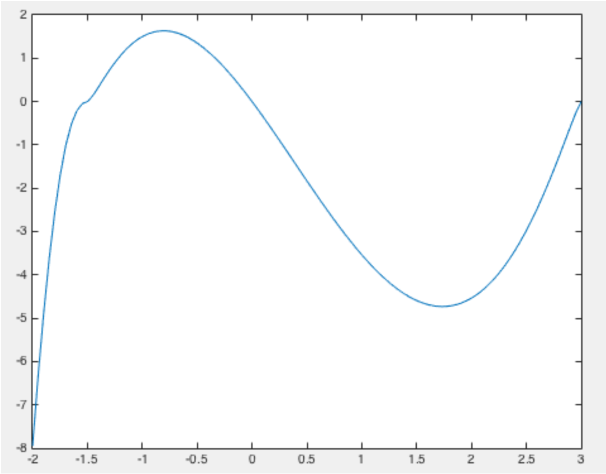

Figure 1: Plottet, ser at fra og med -3/2 går mot $-\infty$ og etter og med 3 går mot ∞