GEL-8000 Pre-doctoral exam, Yannick Hold-Geoffroy
Dates: January 15–29, 2016

This exam has 9 questions on 7 pages (including this page). Make sure you have all the pages! You have until the end of the day on Jan. 15th, 2016, to ask any clarifications about the questions. Make sure you send your answers to `jflalonde@gel.ulaval.ca` before the end of the day on Jan. 29th, 2016.

Good luck!

1 Questions by Denis Laurendeau

Since the Ph.D. project deals with light sources, lighting and surface reconstruction based on various types of light sources, the following questions explore basic radiometric concepts.

1. Strip source illuminating a point on a surface.
   (a) Let us assume a strip light source of uniform luminance L, width W and length H in the yz plane as shown in fig. 1. What is the illuminance E_{strip} at point P lying on a plane parallel to the yz plane and located opposite the middle of the strip and at a distance r?

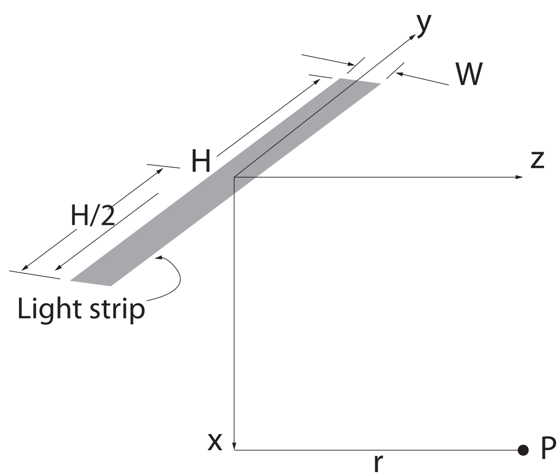

Figure 1: Strip light source of uniform luminance.

   (b) What becomes the expression of E_{strip} found in (a) when the length H of the light strip is very large with respect to the distance r between the center point of the strip and P?

   (c) Let us assume that the light strip in fig. 1 is replaced by a light tube (i.e. a cylinder of radius $\ll H$). What is the expression of the illuminance E_{tube} at P as a function of E_{strip}? What is the expression of this illuminance E_{tube} when r is equal to 0?

2. Rectangular plane light source.

   (a) Let us assume the geometry of fig. 2-(a) showing a light plane of luminance L, width w and height h. A point P located on a plane parallel to the xy plane and at distance q from the corner of the plane at $(0, 0)$ is illuminated by the plane. What is the expression of the illuminance at P?

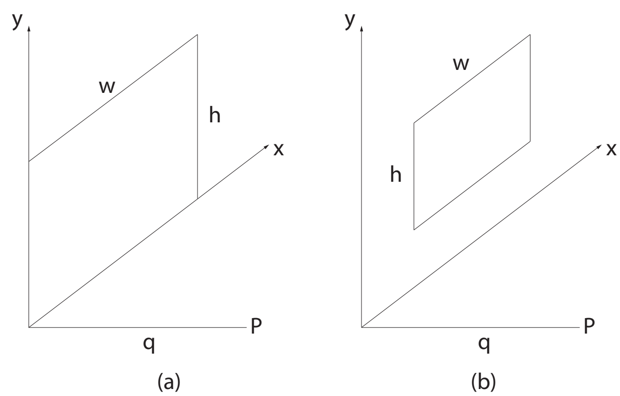

Figure 2: Strip light source of uniform luminance.

   (b) What is the illuminance at P if the light plane is the one shown in fig. 2-(b)?

2 Questions by Philippe Giguère

Impact of estimated light source error vs. estimated normal error (bias).

3. For this problem, we will look at the case where we are performing photometric stereo when the position of the light sources are known. Suppose that we have 3 point light sources of equal intensity located on an equilateral triangle, and shining on a surface of lambertian reflectance, as illustrated in fig. 3. The angle formed between any light source, the center of the surface, and the center of the triangle is r. Let us assume that there is an error of ε_r on the estimation of these angles, and that the error is the same for all 3 sources. In other words, the sources remain on an equilateral triangle, but the triangle estimated to be shrunk or enlarged. How does this error ε_r translate onto an error in the estimation of the surface normal? Draw the error on the estimation of the surface normal as a function of ε_r for a few different ground truth surface normal orientations. You can ignore the effect of albedo.

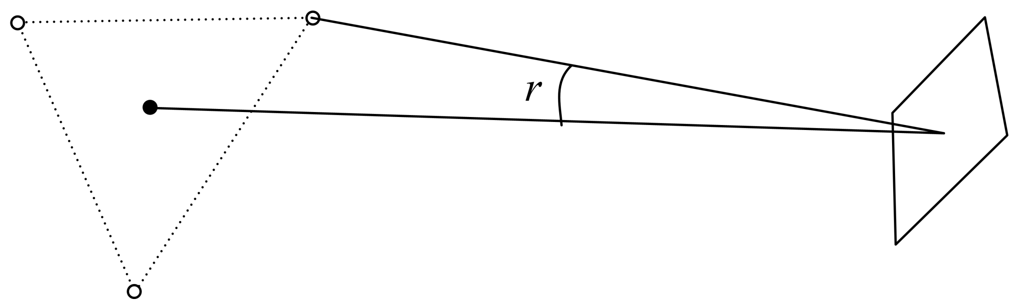

Figure 3: Geometry of the problem. The three light sources on the left are arranged on an equilateral triangle.

Sensitivity study on non-coplanarity vs. pixel noise.

4. You have deployed an immobile probe on planet YHG666, and you want to get the 3D normal of a slanted monolith at the surface of this planet. This planet revolves around a binary star system. One of the stars has similar characteristics as the sun. The other star is a small black hole that does not emit radiation in the visible spectrum, but sends copious amounts of gamma radiation that generates random gaussian noise σ_{CCD} on the CCD.

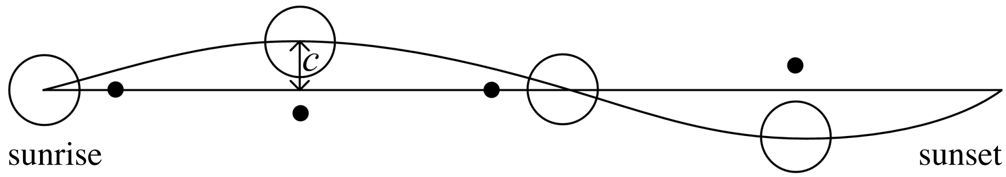

Figure 4: Deviation from a normal planar trajectory for the visible sun for planet YHG666 (viewing direction is parallel to the plane). The small black dot is the invisible black hole, and the larger disk is the visible sun. The binary system is illustrated for different moments of the day.

Because of this binary star configuration, the visible sun will have a non-coplanar trajectory in the sky during a day. We will assume that the non-coplanar component is simply a sine of amplitude c, where c is an angle in the sky, with a period such that a complete oscillation is visible during daytime. This trajectory is sketched in fig. 4.

Using a sensitivity analysis (and/or empirical experimentation), comment on the distribution of the errors on the estimation of the normal for a flat surface of the monolith, a function of this angular deviation c and image pixel noise σ_{CCD}. Use a sensible orientation for this flat surface and the trajectory of the sun in the sky (and justify briefly why they are a good choice for this question.)

3 Questions by Paulo Gotardo

5. Answer the following questions related to linear algebra.
   (a) Let $\mathbf{A} \in \mathbb{R}^{m \times n}$, with $m \gg n$. Show how the eigenvalues and (unit) eigenvectors of $\mathbf{A}\mathbf{A}^T$ can be obtained from the eigen decomposition of the smaller matrix $\mathbf{A}^T\mathbf{A}$.
   (b) Describe the soft-thresholding algorithm for solving an over-determined system of linear equations with the L_1-norm loss function,
   $$\mathbf{x}^* = \arg\min_{\mathbf{x}} \sum_{i=1}^{N} \left| b_i - \mathbf{a}_i^T \mathbf{x} \right|, \quad \mathbf{x} \in \mathbb{R}^m, \tag{1}$$
   for given (constant) scalars b_i and vectors $\mathbf{a}_i \in \mathbb{R}^m$ ($i = 1, \ldots, N$). When is this method more appropriate than least squares (L_2-norm loss function)? When is least squares preferable (i.e., optimal)?
   (c) Propose and discuss two different optimization strategies (with their advantages and disadvantages) for the following problem:
   $$\mathbf{x}^* = \arg\min_{\mathbf{x}} \sum_{i=1}^{N} \left(b_i - \sum_{j=1}^{L} \max(0, \boldsymbol{\omega}_j^T \mathbf{x}) \right)^2, \quad \mathbf{x} \in \mathbb{R}^m, \tag{2}$$
   for given scalars b_i ($i = 1, \ldots, N$) and vectors $\boldsymbol{\omega}_j \in \mathbb{R}^m$ ($j = 1, \ldots, L$). *Hint: revisit soft-thresholding.*

6. What are the differences between temporally-multiplexed and spectrally-multiplexed photometric stereo? How does temporally-multiplexed outdoor photometric stereo benefit from the color spectrum of natural outdoor illumination?

7. Describe an algorithm for computing an HDR image that operates linearly on the input data. Assume the N input images are perfectly aligned and taken at different, known exposure values e_i ($i = 1, \ldots, N$). In addition, assume the pixels in the i-th input image are corrupted by additive, i.i.d. Gaussian noise $\mathcal{N}(0, \sigma_i^2)$, and are linearly related to the scene radiance. Ignore any over-(under-)saturation effects. Derive a model for the noise distribution of the resulting HDR image. How can this model be used to compute better solutions in outdoor photometric stereo?

4 Questions by Jean-François Lalonde

As mentioned in the proposal document, modeling outdoor illumination will be important when tackling the uncalibrated outdoor photometric stereo case. These questions explore various ways of modeling lighting.

8. A common way of modeling lighting is to use spherical harmonics (SH).
   (a) Given a known lighting environment map $\mathbf{L}$ (e.g. in latitude-longitude representation), explain how SH coefficients s can be estimated from $\mathbf{L}$.
   (b) A common problem is that the signal reconstructed from SH coefficients s can be negative, which is physically inconsistent when the signal represents lighting. Explain how SH coefficients s_{nonneg} can be estimated from $\mathbf{L}$, under the constraint that the reconstructed lighting be positive.
   (c) As their name indicates, SH model full *spherical* signals. However, outdoor illumination is emitted from the sky *hemisphere*. Describe Hemispherical Harmonics (HH), and how they differ from SH.
   (d) In general, the convolution $f * g$ of two spherical signals f and g cannot be represented on the sphere, but rather in the rotation group SO(3). However, if f or g have a specific property, then the result of $f * g$ *can* in fact be represented on the sphere. What is this property? How can the result of the convolution be efficiently computed using SH? Give an example of where this can be used in a practical application. *Hint: think about the rendering equation.*

9. Explain why spherical harmonics may not be well-suited for modeling high-frequency signals such as the sun. Describe an alternative representation that could better represent sunlight.