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## SOFTWARE PROJECT

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### overview:

As we know , there are several methods used to solve a circuit such as mesh , cut-set , loop or node analysis , but the most efficient one is a method called the "Modified Nodal analysis " (MNA). using this method , we will be able to extract all the voltages and current flowing through the circuit's elements. by having these amounts , we will be able to compute other things such as different responses and so on .

### Modified Nodal Analysis :

MNA applied to a circuit with only passive elements (resistors) and independent current and voltage sources results in a matrix equation of the form:

$$Ax = z$$

For a circuit with  $n$  nodes and  $m$  independent voltage sources:

- the  $A$  matrix :  $(n + m) \times (n + m)$  in size, and consists only of known quantities. the  $n \times n$  part of the matrix in the upper left: has only passive elements elements connected to ground appear only on the diagonal elements not connected to ground are both on the diagonal and off-diagonal terms. the rest of the  $A$  matrix (not included in the  $n \times n$  upper left part) contains only 1, -1 and 0

- The  $x$  matrix: is an  $(n + m) \times 1$  vector that holds the unknown quantities (node voltages and the currents through the independent voltage sources). the top  $n$  elements are the  $n$  node voltages. the bottom  $m$  elements represent the currents through the  $m$  independent voltage sources in the circuit.

- The  $z$  matrix:

is an  $(n+m) \times 1$  vector that holds only known quantities the top  $n$  elements are either zero or the sum and difference of independent current sources in the circuit. the bottom  $m$  elements represent the  $m$  independent voltage sources in the circuit.

### Generating the MNA matrices

There are three matrices we need to generate, the  $A$  matrix, the  $x$  matrix and the  $z$  matrix. Each of these will be created by combining several individual sub-matrices.

## The $A$ matrix

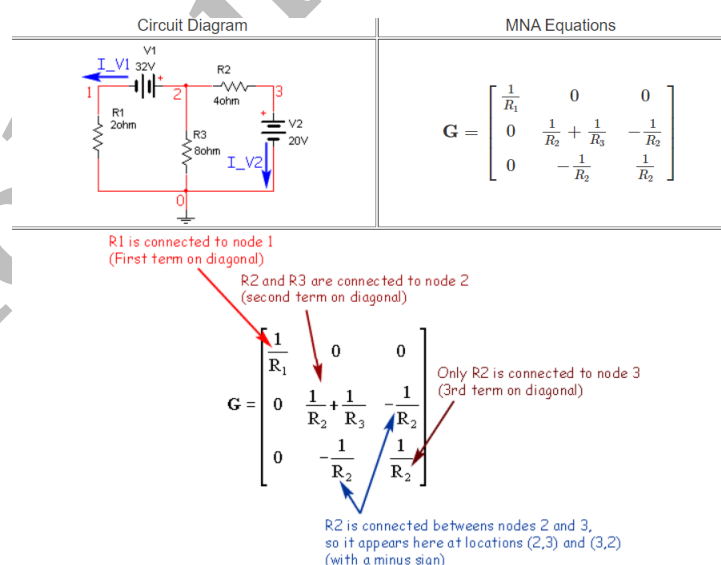
The  $A$  matrix will be developed as the combination of 4 smaller matrices,  $G$ ,  $B$ ,  $C$ , and  $D$ .

The  $A$  matrix is  $(m + n)(m + n)$  ( $n$  is the number of nodes, and  $m$  is the number of independent voltage sources) and:

- the  $G$  matrix is  $nn$  and is determined by the interconnections between the passive circuit elements (resistors)
- the  $B$  matrix is  $nm$  and is determined by the connection of the voltage sources.
- the  $C$  matrix is  $mn$  and is determined by the connection of the voltage sources. ( $B$  and  $C$  are closely related, particularly when only independent sources are considered).
- the  $D$  matrix is  $mm$  and is zero if only independent sources are considered.
- The  $G$  matrix :

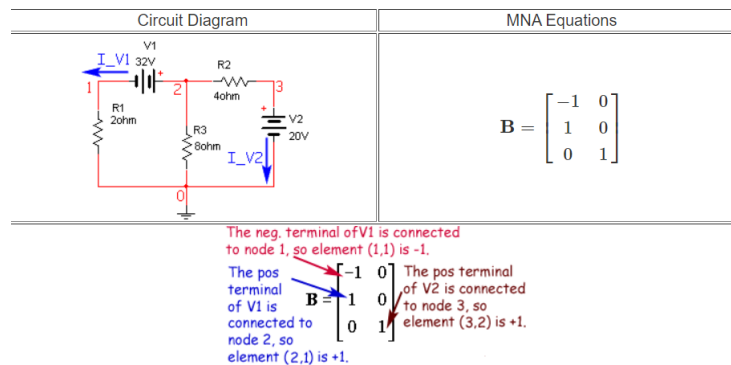
The  $G$  matrix is an  $nn$  matrix formed in two steps:

Each element in the diagonal matrix is equal to the sum of the conductance (one over the resistance) of each element connected to the corresponding node. So the first diagonal element is the sum of conductances connected to node 1, the second diagonal element is the sum of conductances connected to node 2, and so on. The off diagonal elements are the negative conductance of the element connected to the pair of corresponding node. Therefore a resistor between nodes 1 and 2 goes into the  $G$  matrix at location (1,2) and locations (2,1).



### •• The $B$ matrix :

The  $B$  matrix is an  $n \times m$  matrix with only 0, 1 and -1 elements. Each location in the matrix corresponds to a particular voltage source (first dimension) or a node (second dimension). If the positive terminal of the  $i$ th voltage source is connected to node  $k$ , then the element  $(i,k)$  in the  $B$  matrix is a 1. If the negative terminal of the  $i$ th voltage source is connected to node  $k$ , then the element  $(i,k)$  in the  $B$  matrix is a -1. Otherwise, elements of the  $B$  matrix are zero.



### •• The $C$ matrix :

The  $C$  matrix is an  $m \times n$  matrix with only 0, 1 and -1 elements. Each location in the matrix corresponds to a particular node (first dimension) or voltage source (second dimension). If the positive terminal of the  $i$ th voltage source is connected to node  $k$ , then the element  $(k, i)$  in the  $C$  matrix is a 1. If the negative terminal of the  $i$ th voltage source is connected to node  $k$ , then the element  $(k,i)$  in the  $C$  matrix is a -1. Otherwise, elements of the  $C$  matrix are zero for example , in the circuits shown in top , the matrix will be :

$$C = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### •• The $D$ matrix :

The  $D$  matrix is an  $m \times m$  matrix that is composed entirely of zeros.

## The $x$ matrix:

The  $x$  matrix holds our unknown quantities and will be developed as the combination of 2 smaller matrices  $v$  and  $j$ . It is considerably easier to define than the  $A$  matrix.

The  $x$  matrix is  $(m + n) \times 1$  ( $n$  is the number of nodes, and  $m$  is the number of independent voltage sources) and:

the  $v$  matrix is  $n \times 1$  and hold the unknown voltages the  $j$  matrix is  $m \times 1$  and holds the unknown currents through the voltage sources.

### •• the $v$ matrix:

The  $v$  matrix is an  $n \times 1$  matrix formed of the node voltages. Each element in  $v$  corresponds to the voltage at the equivalent node in the circuit (there is no entry for ground – node 0).

For example if a circuit has three nodes, the  $v$  matrix is:

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

●● The  $J$  matrix:

The  $j$  matrix is an  $m1$  matrix, with one entry for the current through each voltage source. So if there are two voltage sources  $V1$  and  $V2$ , the  $j$  matrix will be:

$$j = \begin{bmatrix} i_{V1} \\ i_{V2} \end{bmatrix}$$

Demonstration for the  $X$  matrix:

Circuit Diagram	MNA Equations
	$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \mathbf{j} = \begin{bmatrix} i_{V1} \\ i_{V2} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \mathbf{v} \\ \mathbf{j} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_{V1} \\ i_{V2} \end{bmatrix}$

## The $Z$ matrix :

The  $z$  matrix holds our independent voltage and current sources and will be developed as the combination of 2 smaller matrices  $i$  and  $e$ . It is quite easy to formulate.

$$z = \begin{bmatrix} i \\ e \end{bmatrix}$$

The  $z$  matrix is  $(m + n)1$  ( $n$  is the number of nodes, and  $m$  is the number of independent voltage sources) and:

the  $i$  matrix is  $n1$  and contains the sum of the currents through the passive elements into the corresponding node (either zero, or the sum of independent current sources).

the  $e$  matrix is  $m1$  and holds the values of the independent voltage sources. ●● The  $i$  matrix :

The  $i$  matrix is an  $n1$  matrix with each element of the matrix corresponding to a particular node.

The value of each element of  $i$  is determined by the sum of current sources into the corresponding node. If there are no current sources connected to the node, the value is zero. ●● The  $e$  matrix :

The  $e$  matrix is an  $m1$  matrix with each element of the matrix equal in value to the corresponding independent voltage source.

Demonstrations for the  $z$  matrix:

Circuit Diagram	MNA Equations
	$\mathbf{i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{e} = \begin{bmatrix} V1 \\ V2 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V1 \\ V2 \end{bmatrix}$

To conclude , we can solve the  $Ax = z$  equation using steps we have top ,and for our simulation we used the same method to compute voltage and current and use them to plot the bode diagram .

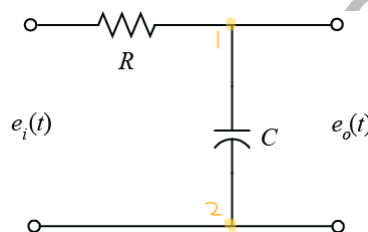
Circuit Diagram	MNA Equations
	$\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & 1 & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_{V1} \\ i_{V2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V1 \\ V2 \end{bmatrix}$

## Matlab Code :

To analyse a circuit with the simulation , we need 3 parameter for each element :

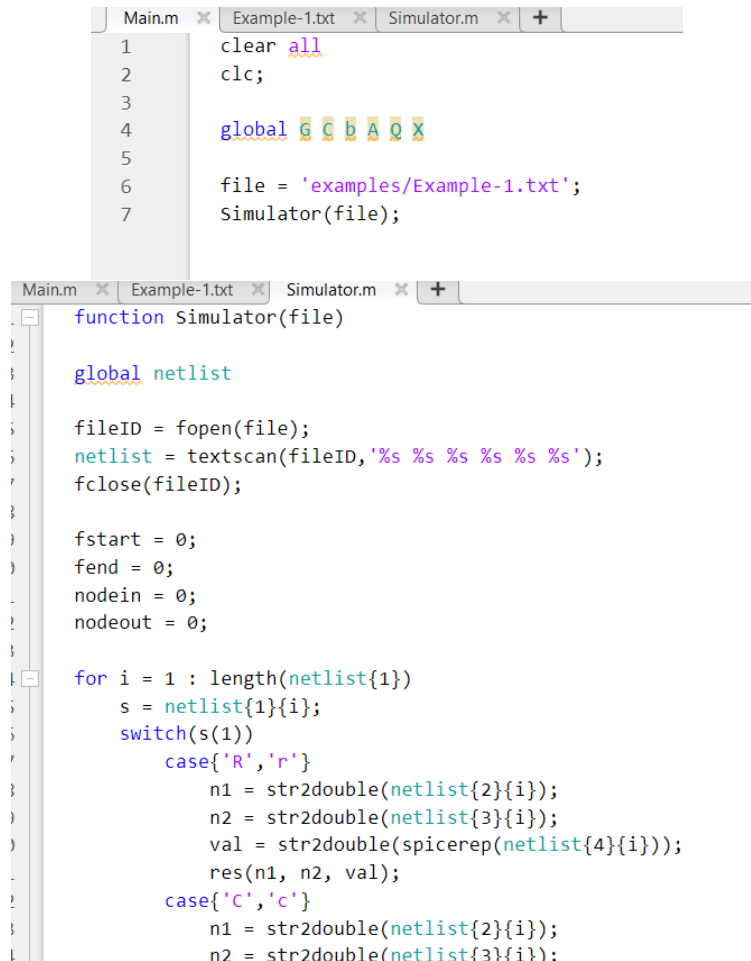
- 1- positive node
- 2- negative node
- 3 amount

at the end we need to identify the frequency domain and first and last node. for example a proper input for the circuit below will be :



```
R, 1, 2, 10k
C, 2, 0, 100u
Vin, 1, 0, 1
.0.01, 100, 1, 2
```

as we run the program , the main file will be guided to a class named "simulation" , where the file will be read . then the different parts and elements will be separated and each each data will be stored in a cell.



```

Main.m
1 clear all
2 clc;
3
4 global G C b A Q X
5
6 file = 'examples/Example-1.txt';
7 Simulator(file);

Simulator.m
function Simulator(file)

global netlist

fileID = fopen(file);
netlist = textscan(fileID, '%s %s %s %s %s %s');
fclose(fileID);

fstart = 0;
fend = 0;
nodein = 0;
nodeout = 0;

for i = 1 : length(netlist{1})
    s = netlist{1}{i};
    switch(s(1))
        case{'R', 'r'}
            n1 = str2double(netlist{2}{i});
            n2 = str2double(netlist{3}{i});
            val = str2double(spicerep(netlist{4}{i}));
            res(n1, n2, val);
        case{'C', 'c'}
            n1 = str2double(netlist{2}{i});
            n2 = str2double(netlist{3}{i});
    end
end

```

after separating the amounts , its time to generate the matrixes mentioned before.

\*Note: the global functions are the matrixes we need .

```

function ind(n1, n2, val)

global G C b

d = size(G, 1);
xr = d + 1;
b(xr, 1) = 0;

G(xr, xr) = 0;
C(xr, xr) = 0;

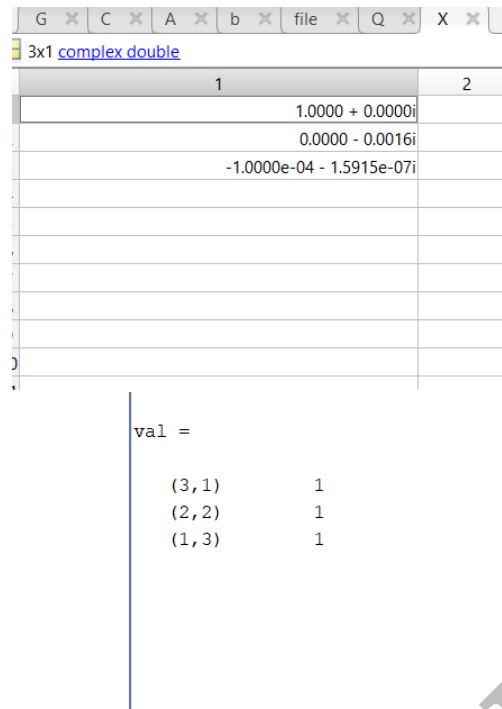
if (n1 ~= 0)
    G(n1, xr) = 1;
    G(xr, n1) = 1;
end

if (n2 ~= 0)
    G(n2, xr) = -1;
    G(xr, n2) = -1;
end

C(xr, xr) = -val;

```

after the program is done computing , the outputs of the given circuits would be :



the next part has asked to change the frequency domain , as said before , we can change the input frequency domain . we will see that the values remain the same but the program is not able to handle the bode diagram .

To plot the bode diagram , all we need is to use the formula and write a function as below :

```
function ret = resultfunc(f, nodein, nodeout)

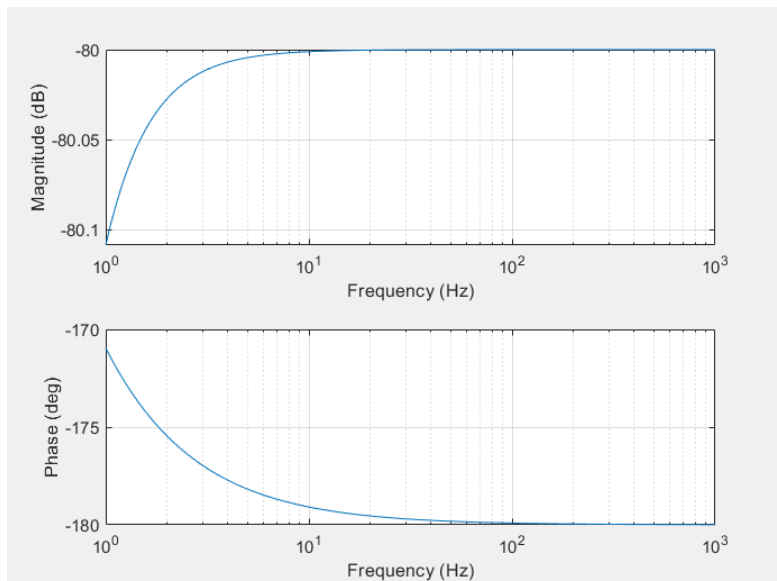
global G C b A Q X

for i = 1 : length(f)
    w = 2*pi*f(i, 1);
    s = (1j)*w;
    A = (G + s.*C);

    thresh = [0.1, 0.001];
    [L,U,P,Q] = lu(sparse(A), thresh);
    z = L\(P*b);
    y = U\z;
    X = Q*y;

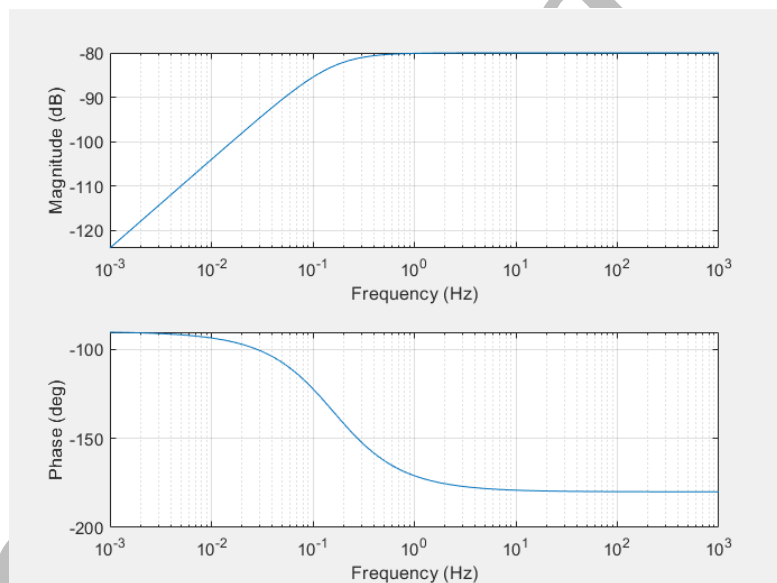
    ret(1, i) = 20*log10( abs(X(nodeout,1)/X(nodein,1)) );
    ret(2, i) = rad2deg( angle(X(nodeout,1)/X(nodein,1)) );
end
end
```

the bode diagram of the circuit given ,with a frequency domain of 1-100 will be:



as we make the lower limit of the domain closer to 0 , the line becomes more leaner :

(0.001 -1000):



(0.00001 - 1000)



