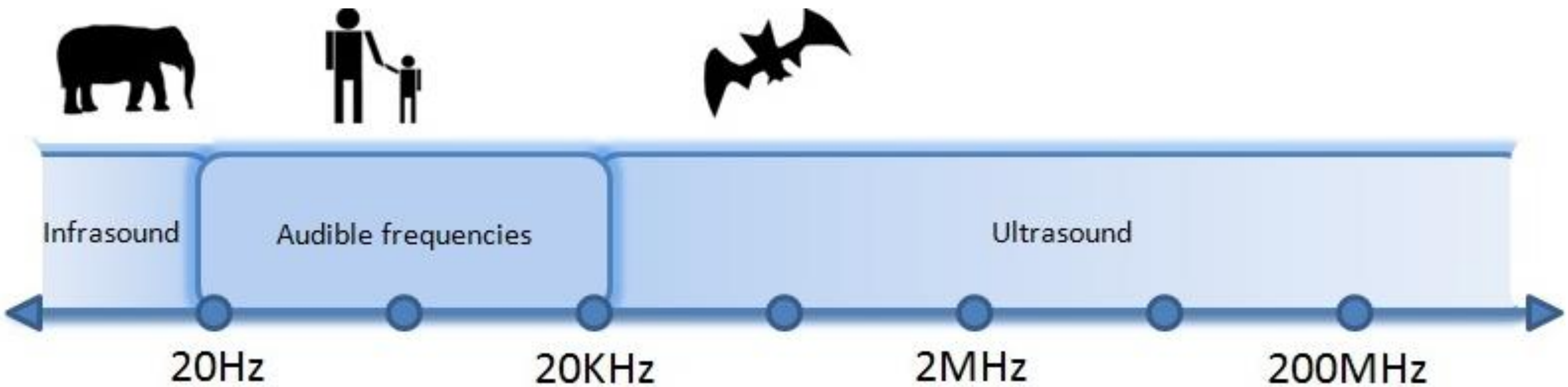
General Physics I

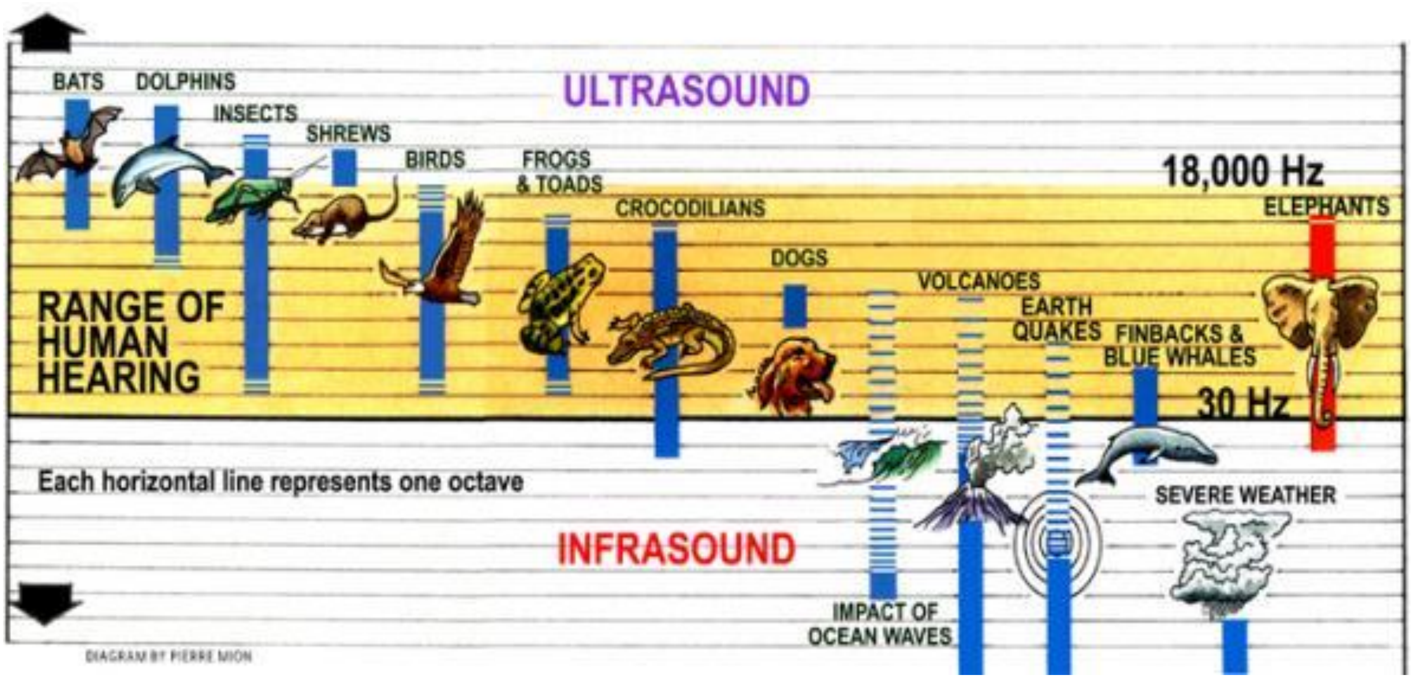
Lecture 15: Sound Waves

Outline

- Sound frequency and sound level
- Speed of sound waves
- The physics of piano
- The Doppler effect
- Shock waves

Categories of Sound Waves





Sound Intensity

- **We define the intensity I of a wave, or the power per unit area, to be the rate at which the energy being transported by the wave flows through a unit area A perpendicular to the direction of travel of the wave.**

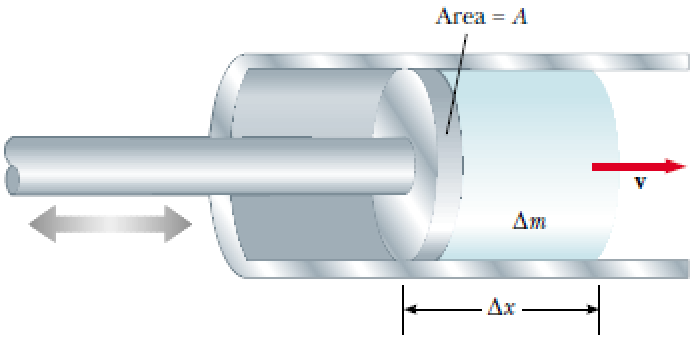
$$I = \frac{\mathcal{P}}{A} = \frac{1}{2}\rho v(\omega s_{\max})^2$$

density

speed of sound

frequency

amplitude



Definition of Sound Level

- **Because the range of sound intensities is so wide, it is convenient to use a logarithmic scale, where the sound level is defined by the equation**

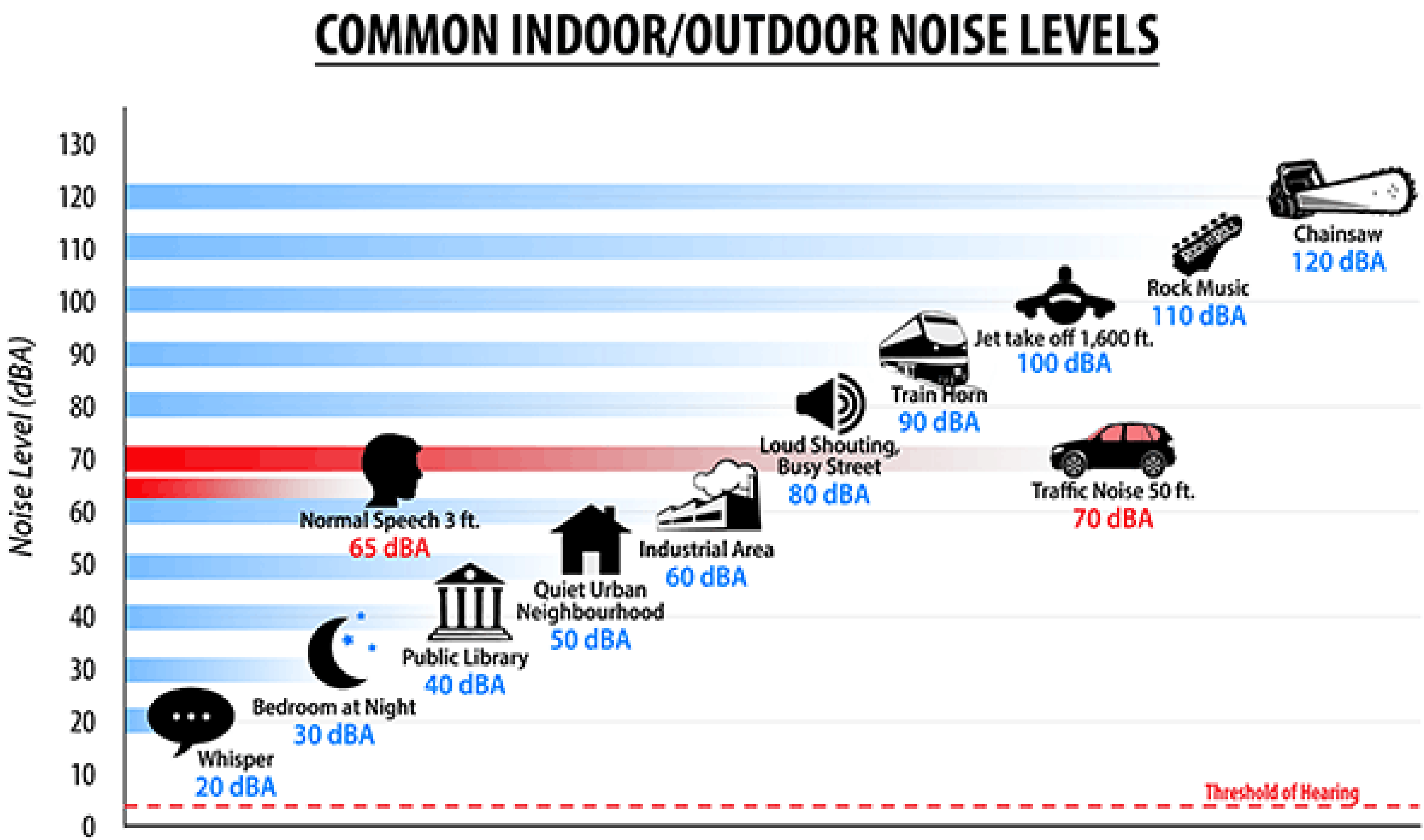
$$\boxed{\beta = 10 \log\left(\frac{I}{I_0}\right)} \quad \text{in } \textbf{decibels} \text{ (dB)}$$

$$I_0 = 1.00 \times 10^{-12}\ \mathrm{W/m^2} \quad \text{(threshold of hearing)}$$

- **Threshold of pain:**

$$10 \log[(1\ \mathrm{W/m^2})/(10^{-12}\ \mathrm{W/m^2})] = 10 \log(10^{12}) = 120\ \mathrm{dB}$$

Sound Levels



Speed of Sound in a Solid

- **If a solid bar is struck at one end with a hammer, a longitudinal pulse propagates down the bar with a speed**

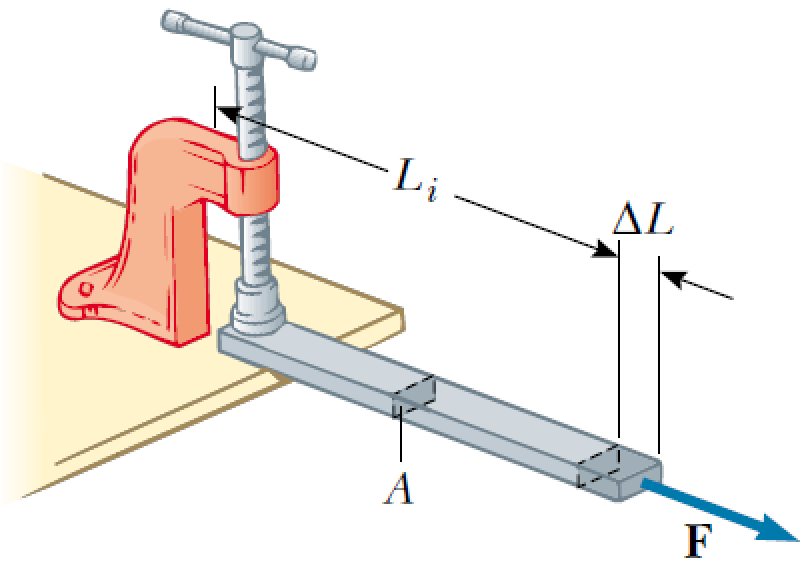
$$v = \sqrt{Y/\rho}$$

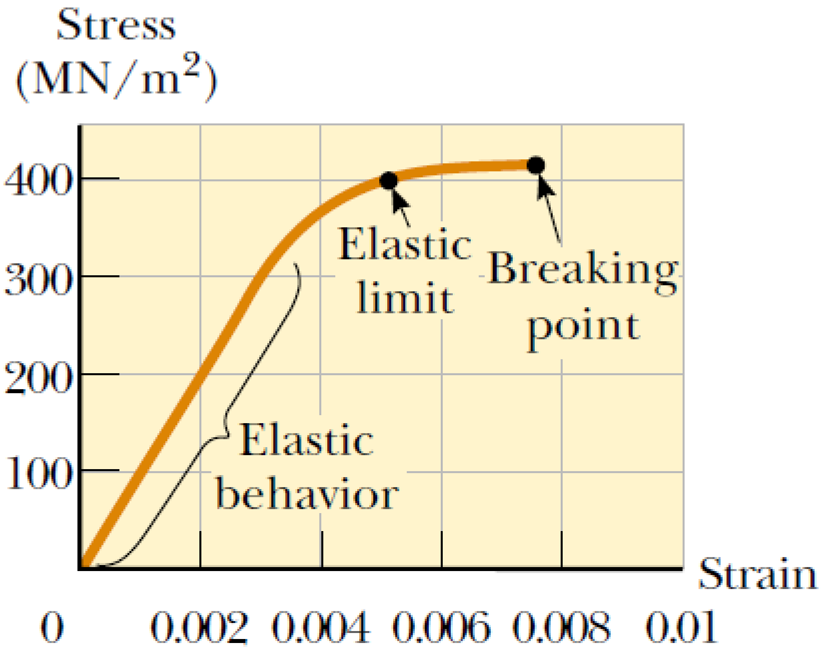
- **where *Y* is the Young's modulus for the material and ρ the density of material.**

Elasticity in Length

- **Young's Modulus:**

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$





The Speed of Sound Wave

- **In the continuous limit (in a solid),**

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} \qquad \textbf{where} \qquad v = a\sqrt{\frac{K}{M}}$$

v

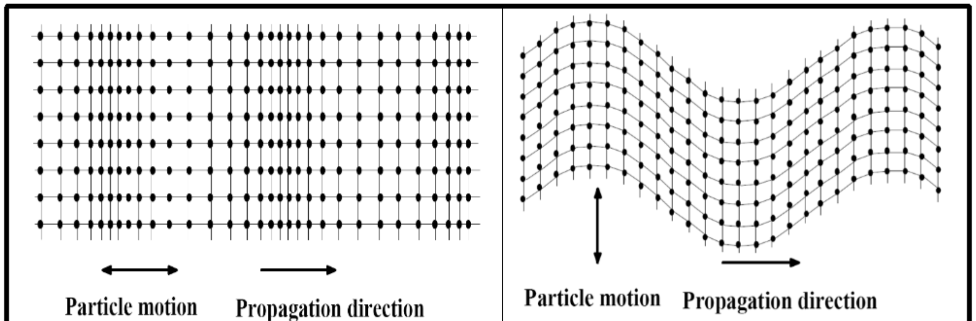


Figure: longitudinal waves- particles move along direction of propagation

Figure: Transverse waves- particles move perpendicular to direction of propagation

Speed of Sound for Several Common Solids

Solid	Structure Type	Nearest Neighbor Distance (A°)	Density ρ (kg/m³)	Elastic bulk modulus Y (10^{10} N/m²)	Calculated Wave Speed (m/s)	Observed speed of sound (m/s)
Sodium	B.C.C	3.71	970	0.52	2320	2250
Copper	F.C.C	2.55	8966	13.4	3880	3830
Aluminum	F.C.C	2.86	2700	7.35	5200	5110
Lead	F.C.C	3.49	11340	4.34	1960	1320
Silicon	Diamond	2.35	2330	10.1	6600	9150
Germanium	Diamond	2.44	5360	7.9	3830	5400
NaCl	Rocksalt	2.82	2170	2.5	3400	4730

Most calculated V_L values are in reasonable agreement with measurements. Sound speeds are of the order of **5000 m/s** in typical metallic, covalent & ionic solids :

Now, ...

- Can you show that

$$v = \sqrt{Y/\rho}$$

(macroscopic)

$$v = a\sqrt{\frac{K}{M}}$$

(microscopic)

- are two equivalent forms of the speed of sound in the solid?

Speed of Sound in a Fluid

- **The speed of all mechanical waves follows an expression of the general form**

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

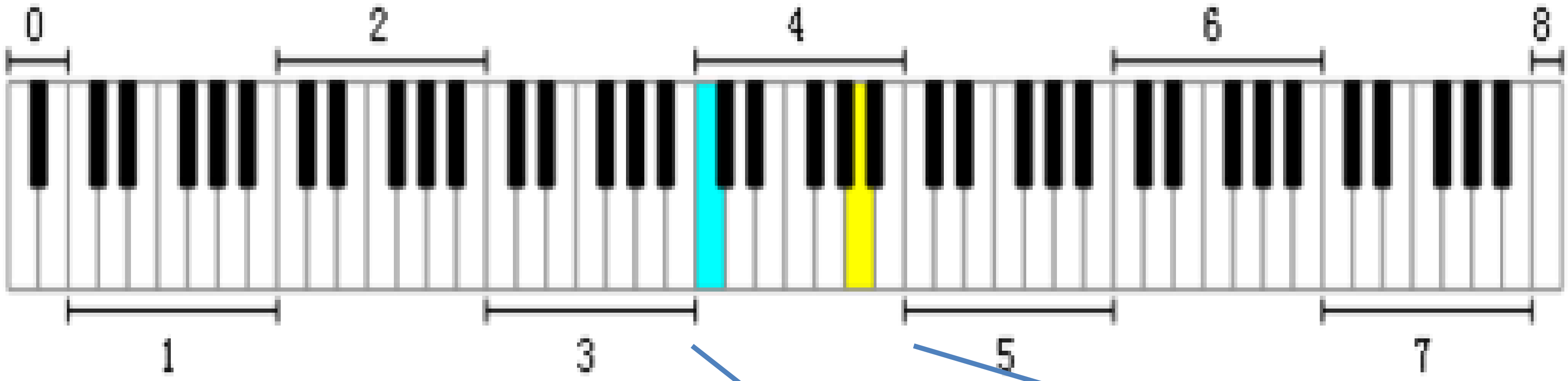
- **We will, hopefully, come back to this issue in the part of thermodynamics for a complete understanding.**

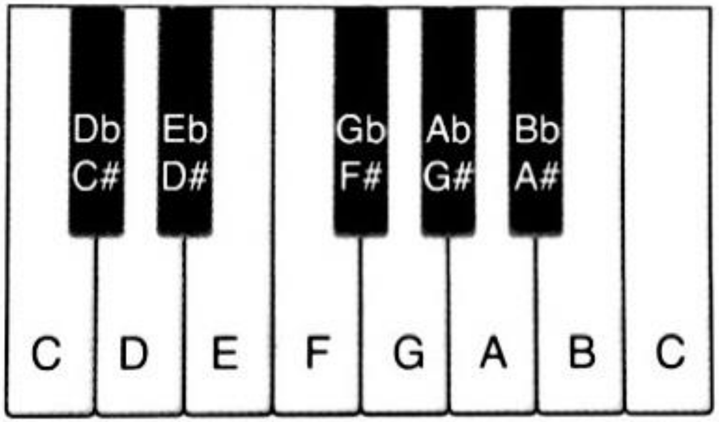
声速 $c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\kappa R T}$

Physics of the Piano

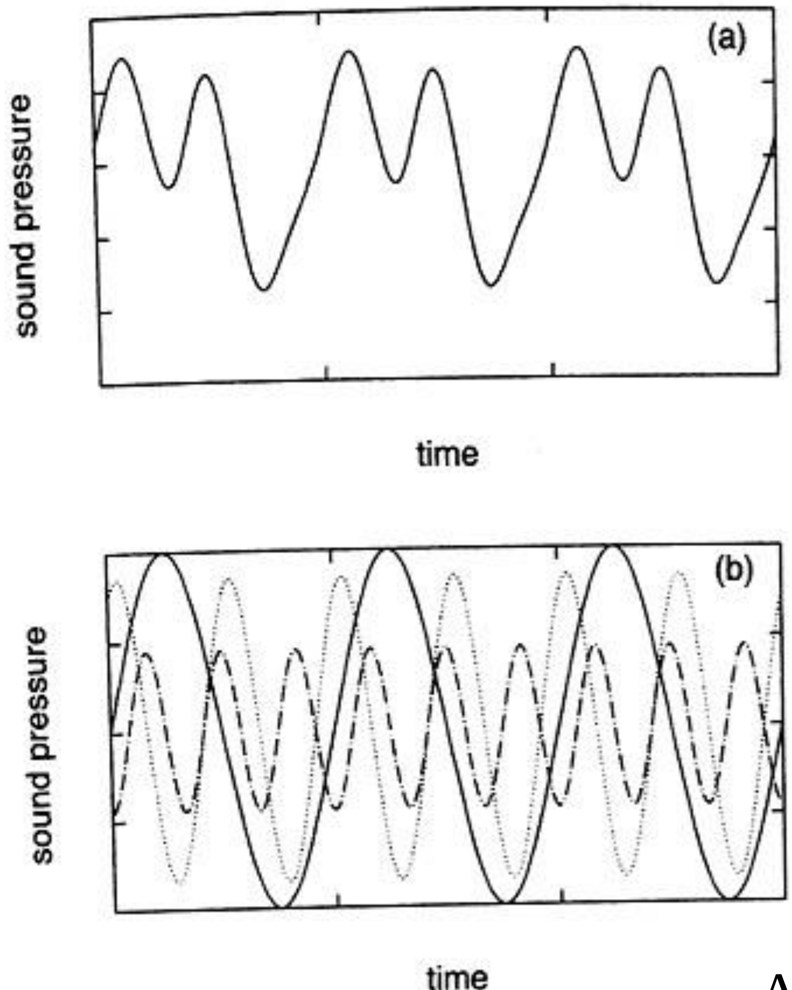
Beauty is all about the frequency of the sound.

Octave (8 Notes)

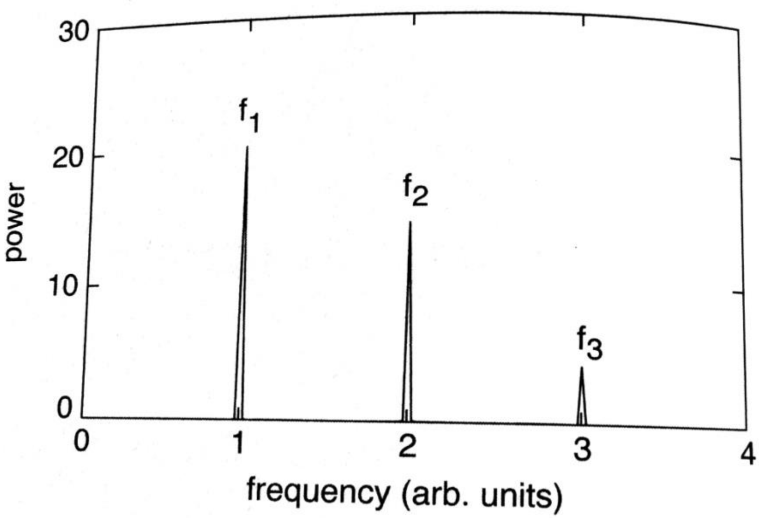




Tone and Pitch

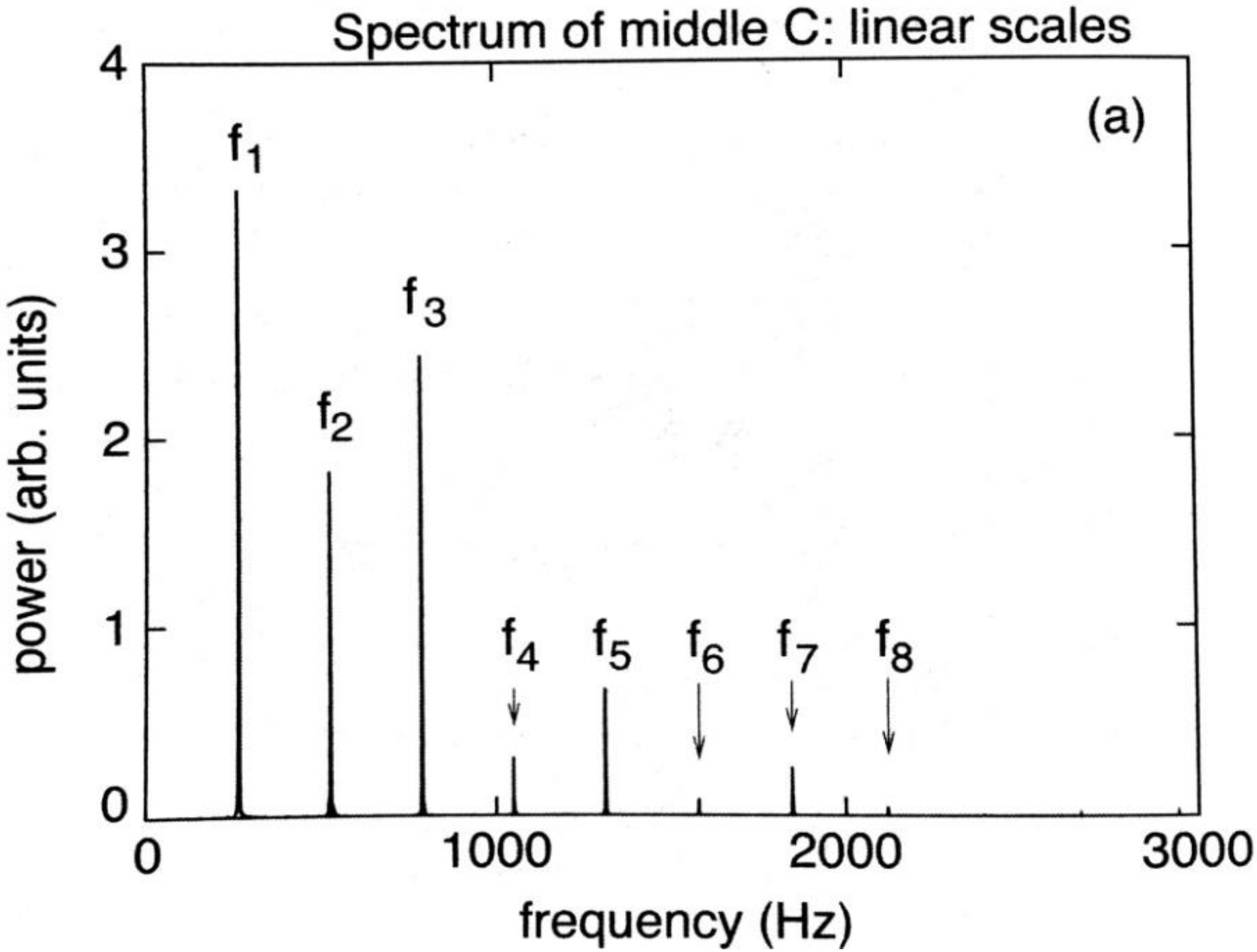


A musical tone is a steady periodic sound. A musical tone is characterized by its duration, pitch, intensity (or loudness), and timbre (or quality).

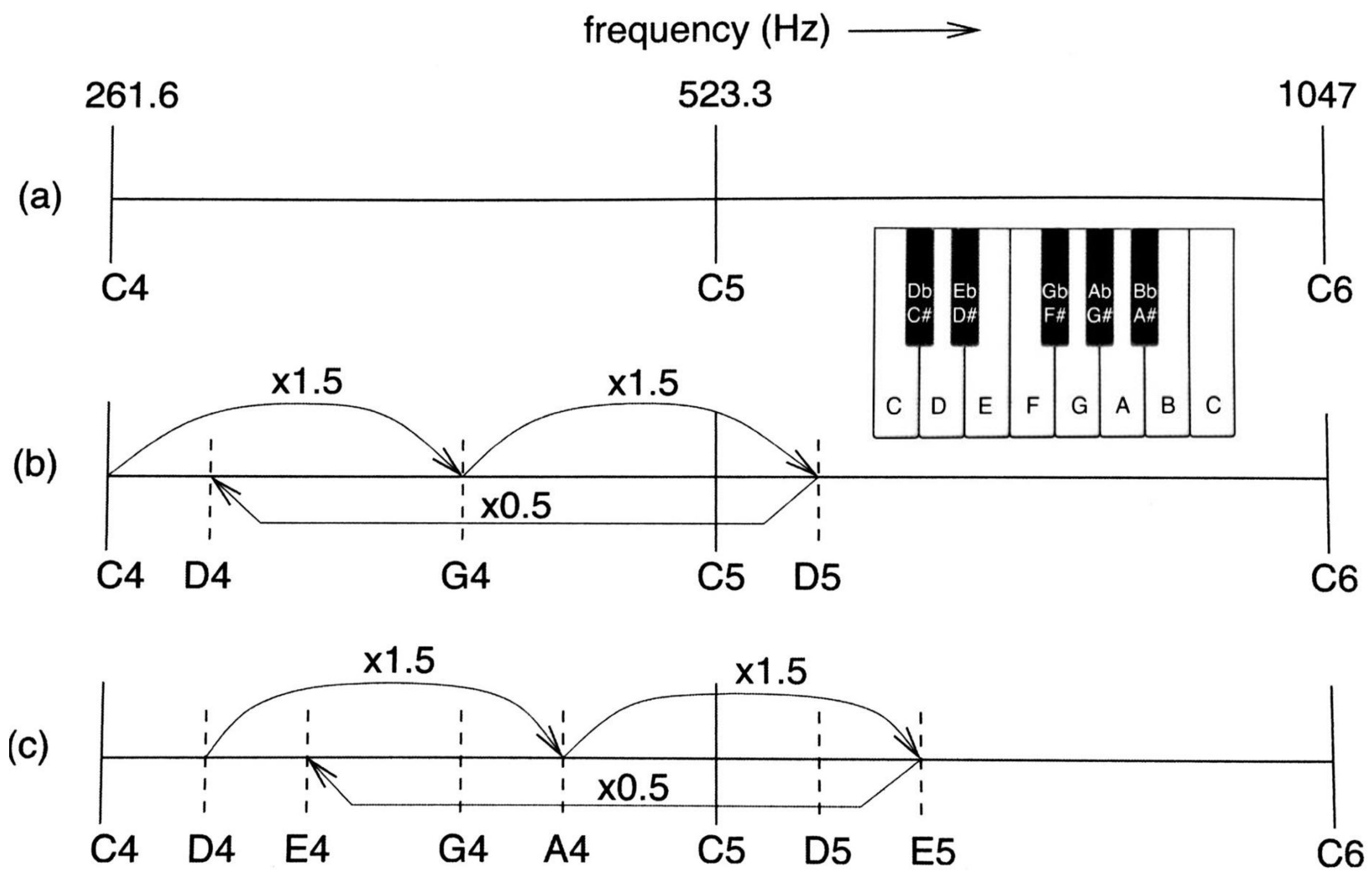


A pitch is a particular frequency of sound.

Harmonic Spectrum

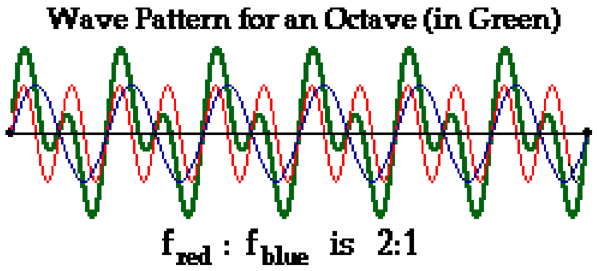


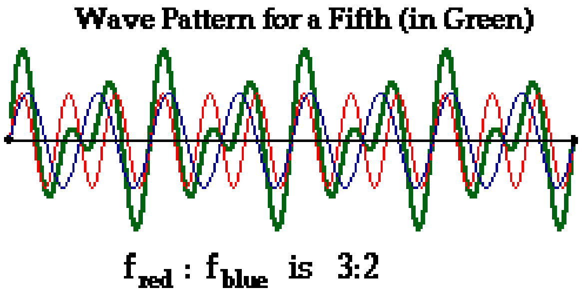
Musical Scale (Pythagoras)

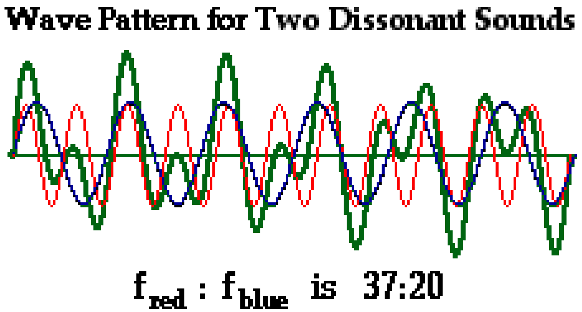


Musical Scale

- A4, fundamental frequency of 440 Hz
- $2^{1/12}$ = 1.05946… (equal temperament)
- Perfect 5th, $2^{7/12}$ = 1.4983…≈ 1.5



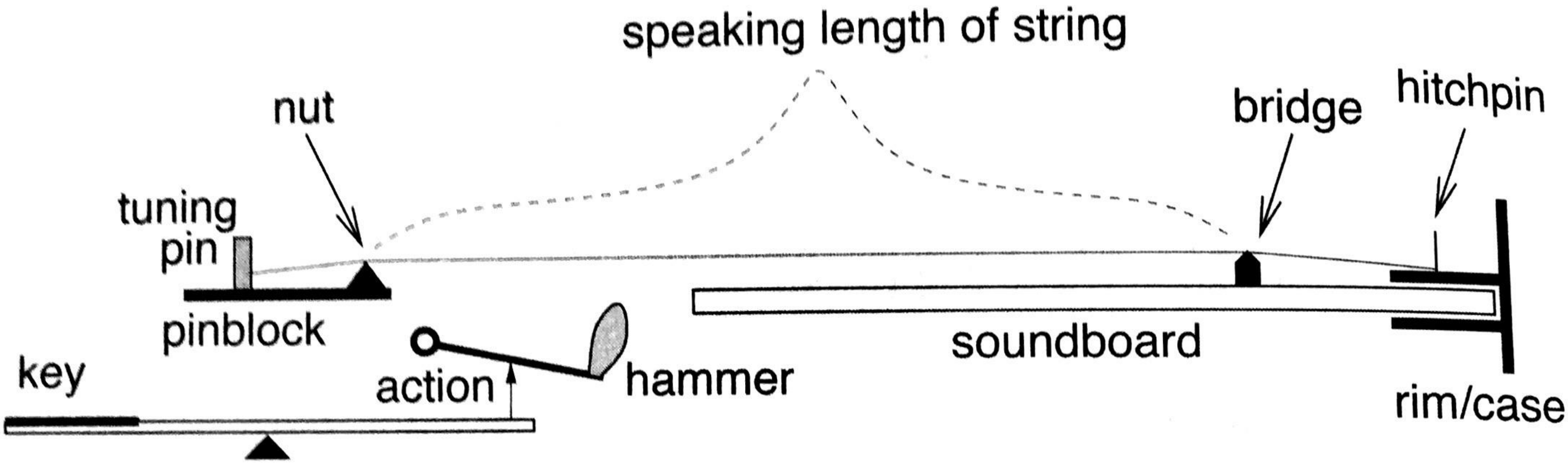




What's Inside?

Physics in piano when making music.

Vibrating Strings



$$f = v/\lambda$$

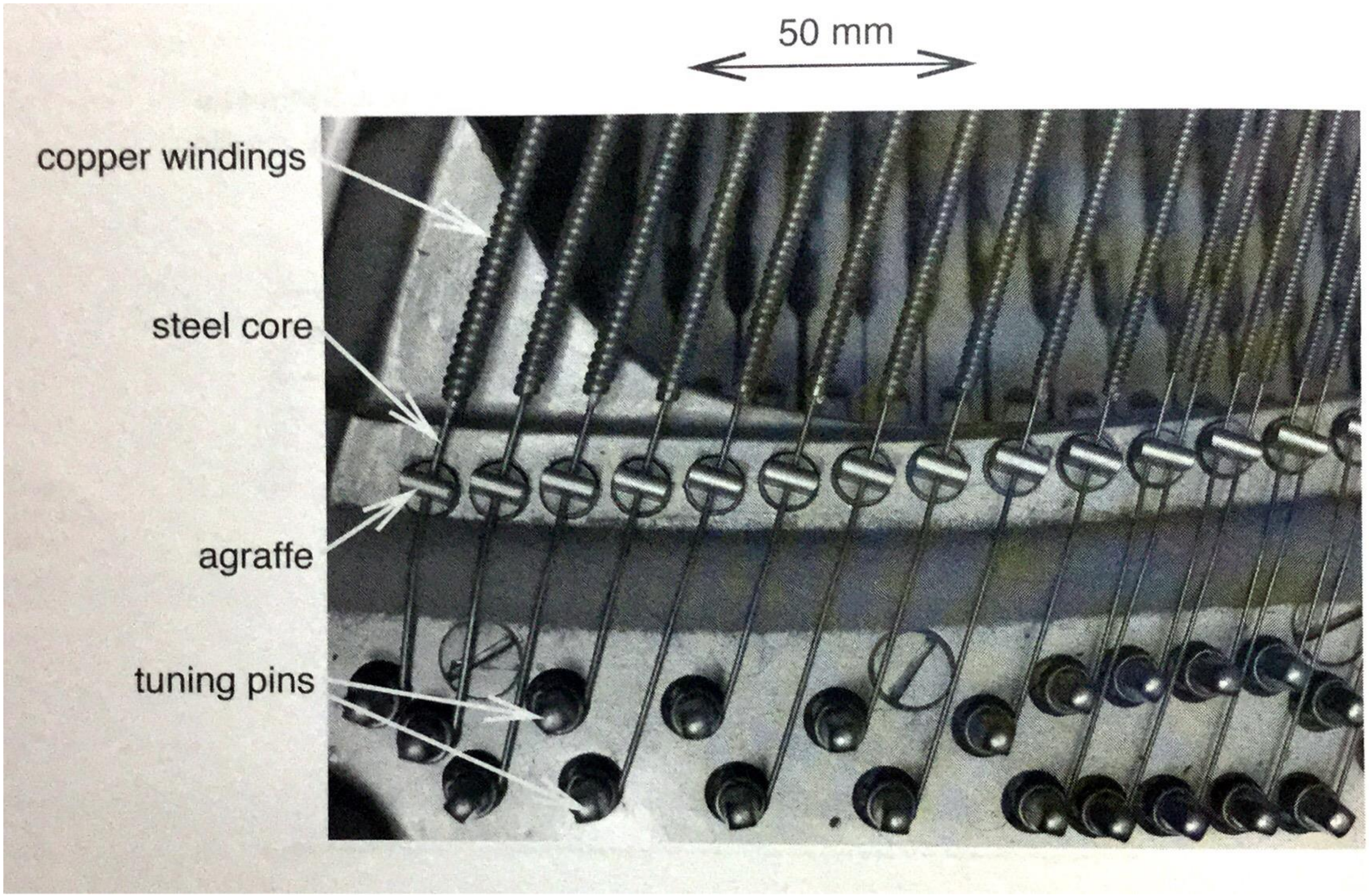
$$\lambda = L/n$$

$$f \propto v/L$$

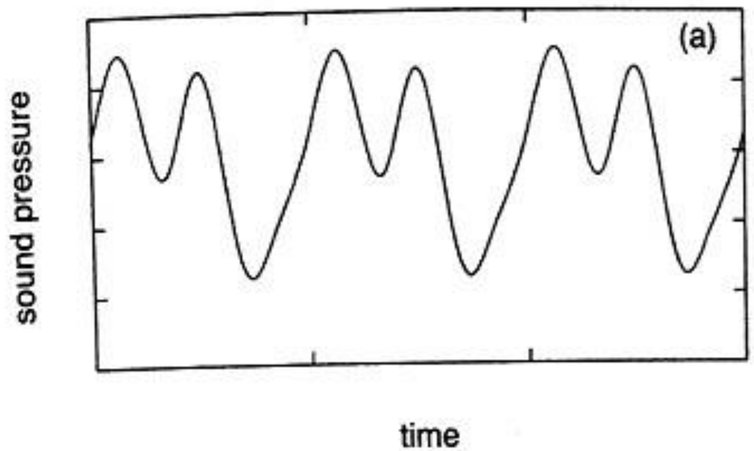
Can you determine frequency by dimension analysis?

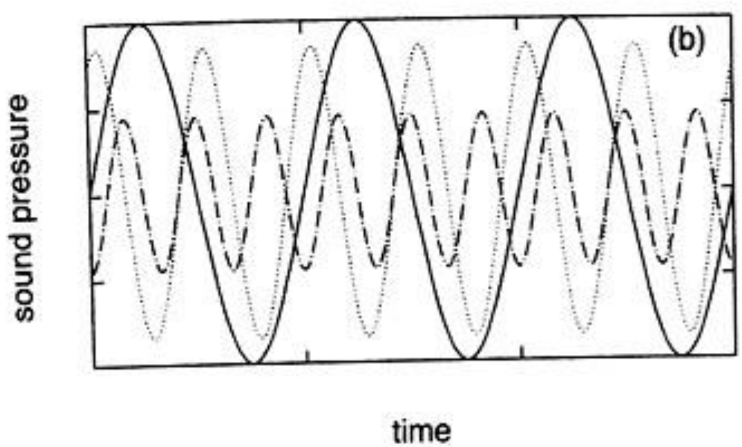
$$f \, \mu \, f(T, r, L)$$

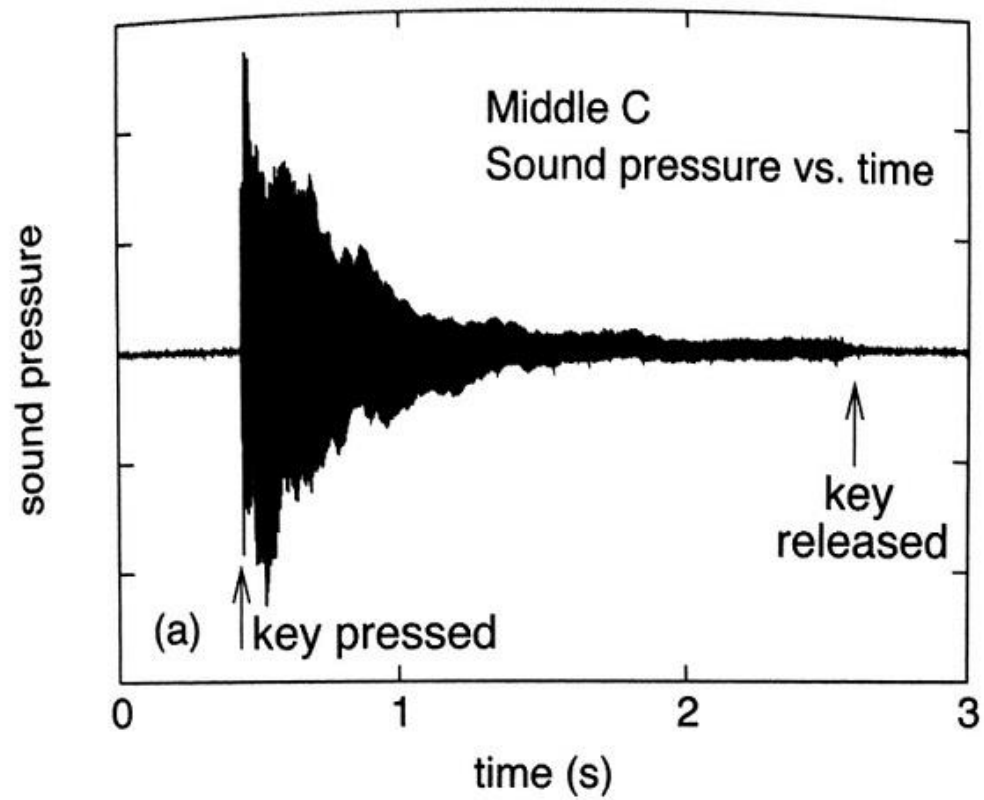
Vibrating Strings



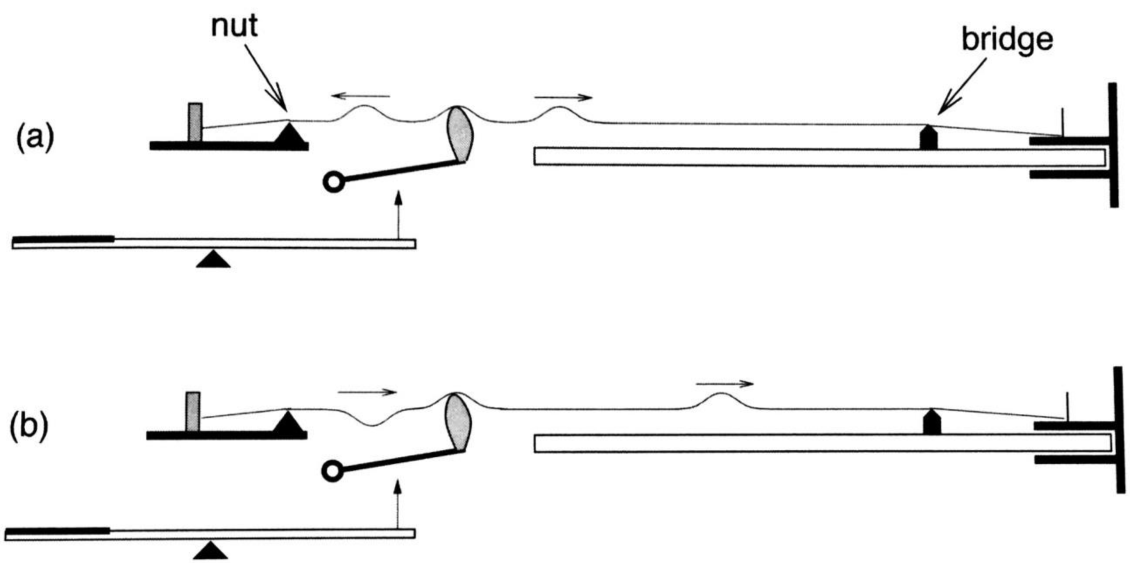
Hammers that Hit the Strings

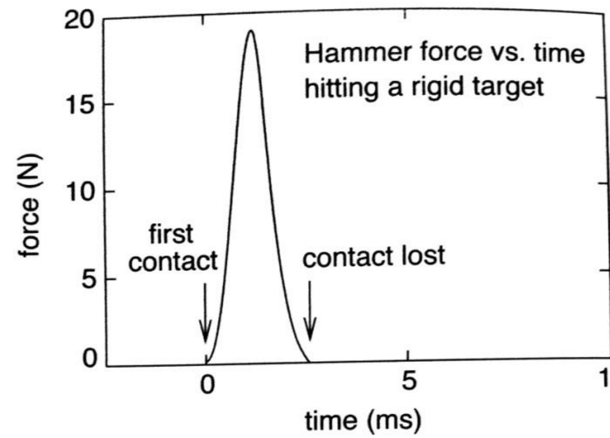


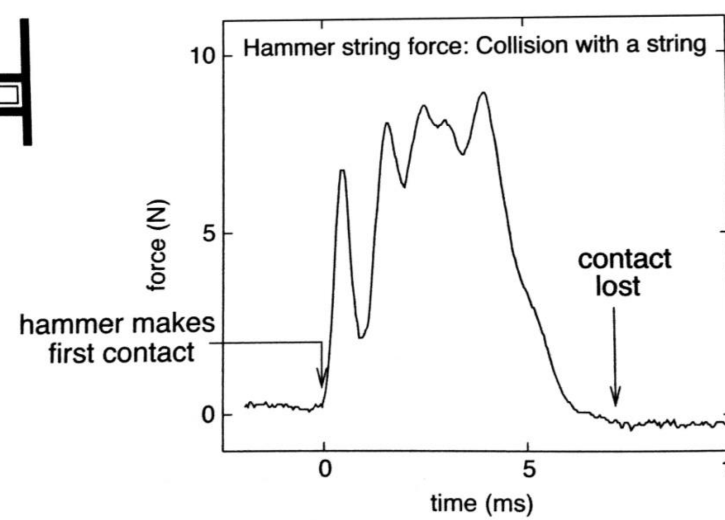




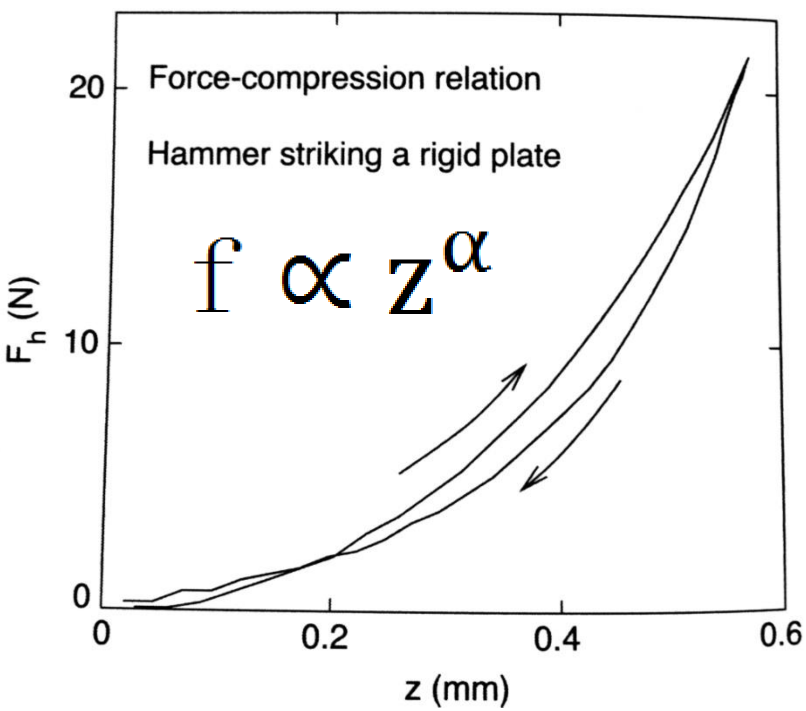
Hammers that Hit the Strings

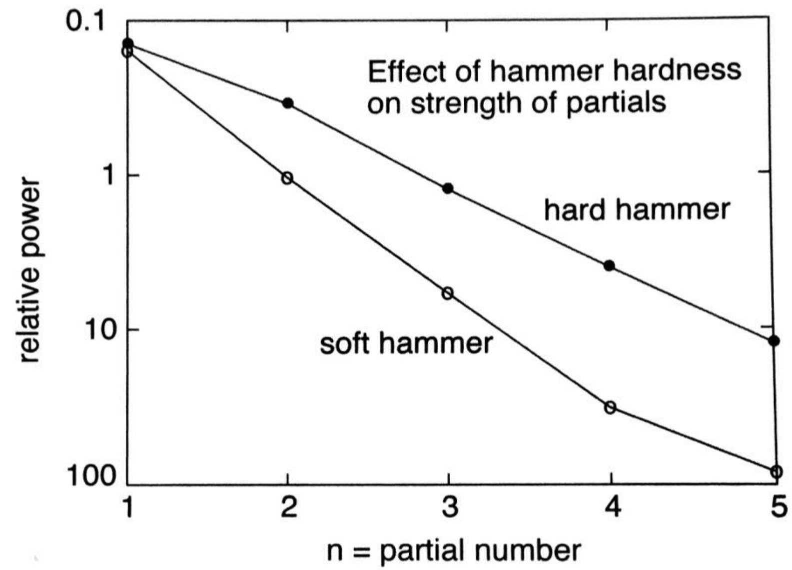


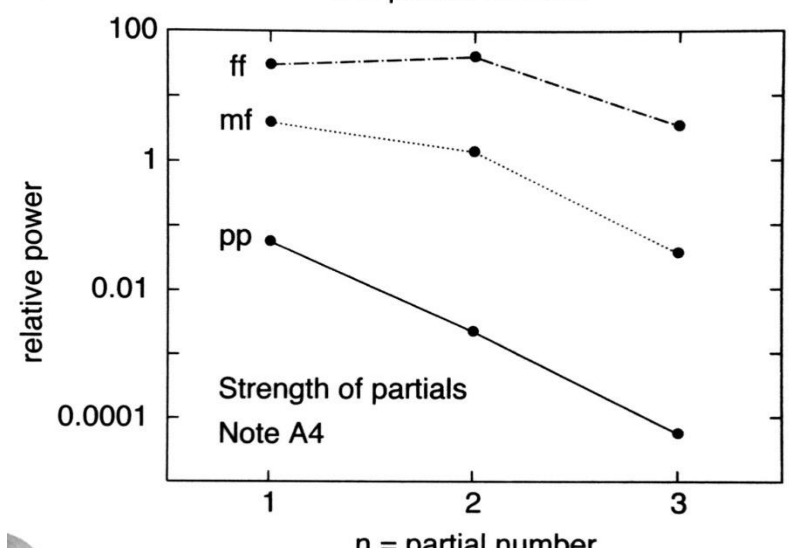




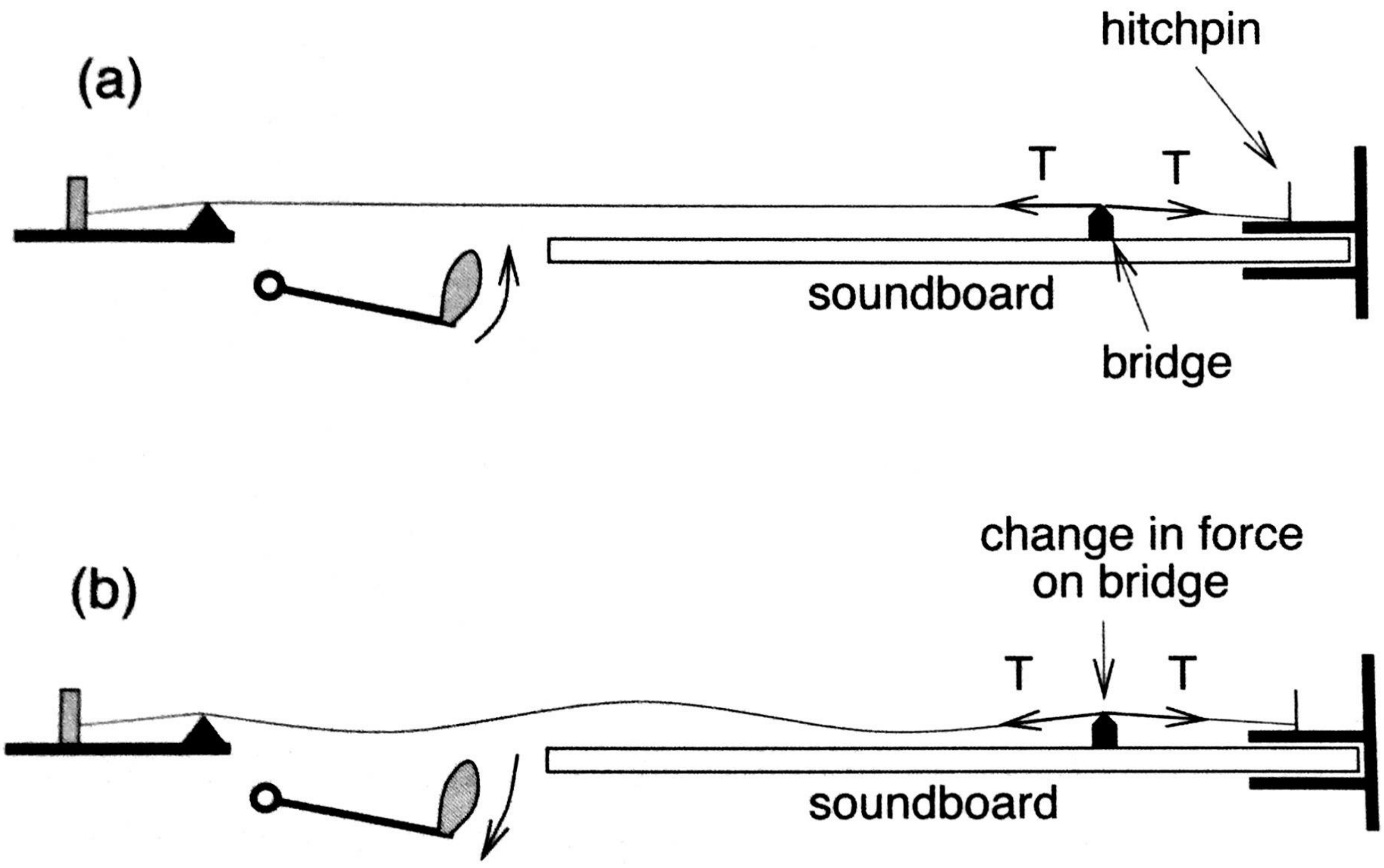
Hammers That Hit the Strings







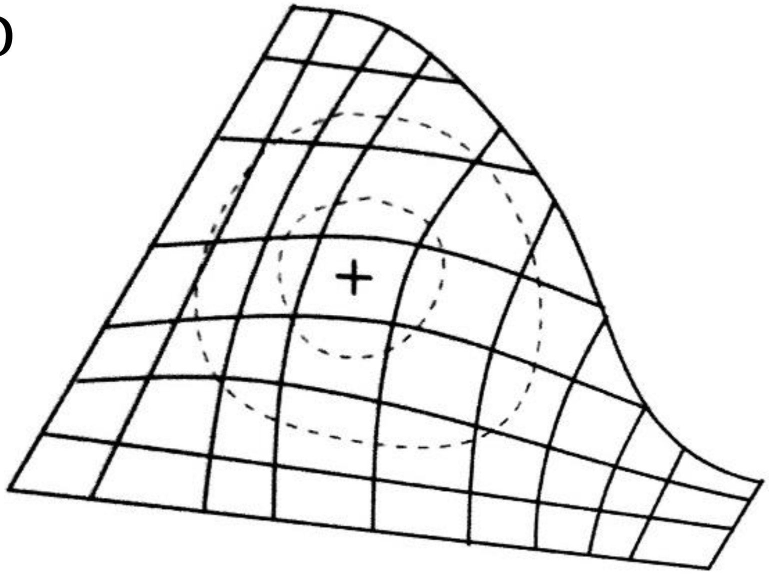
Soundboard Producing Sound

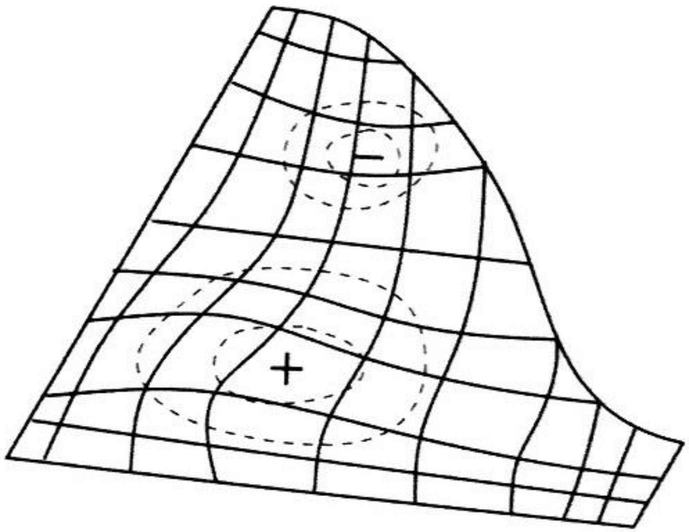


Forced oscillations

Soundboard Producing Sound

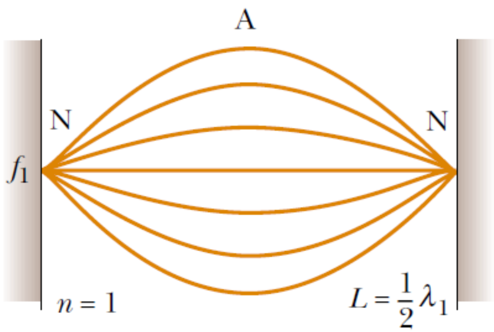
2D



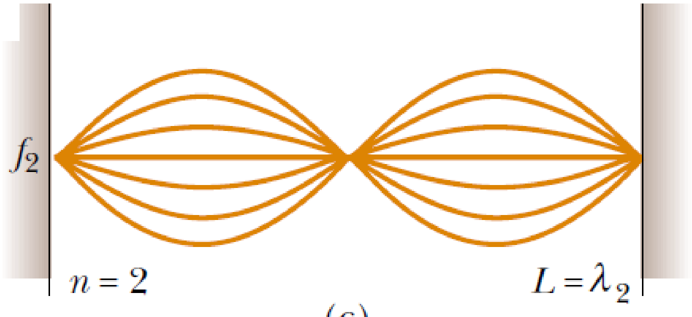


(b)

Generalization of the 1D cases



(b)



(c)

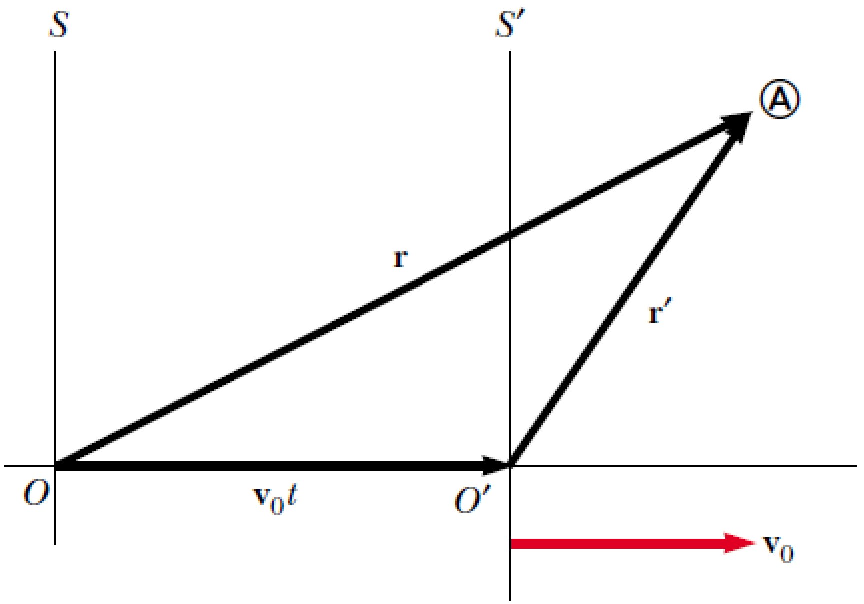
Galilean Transformation

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t$$

$$\frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{v}_0$$

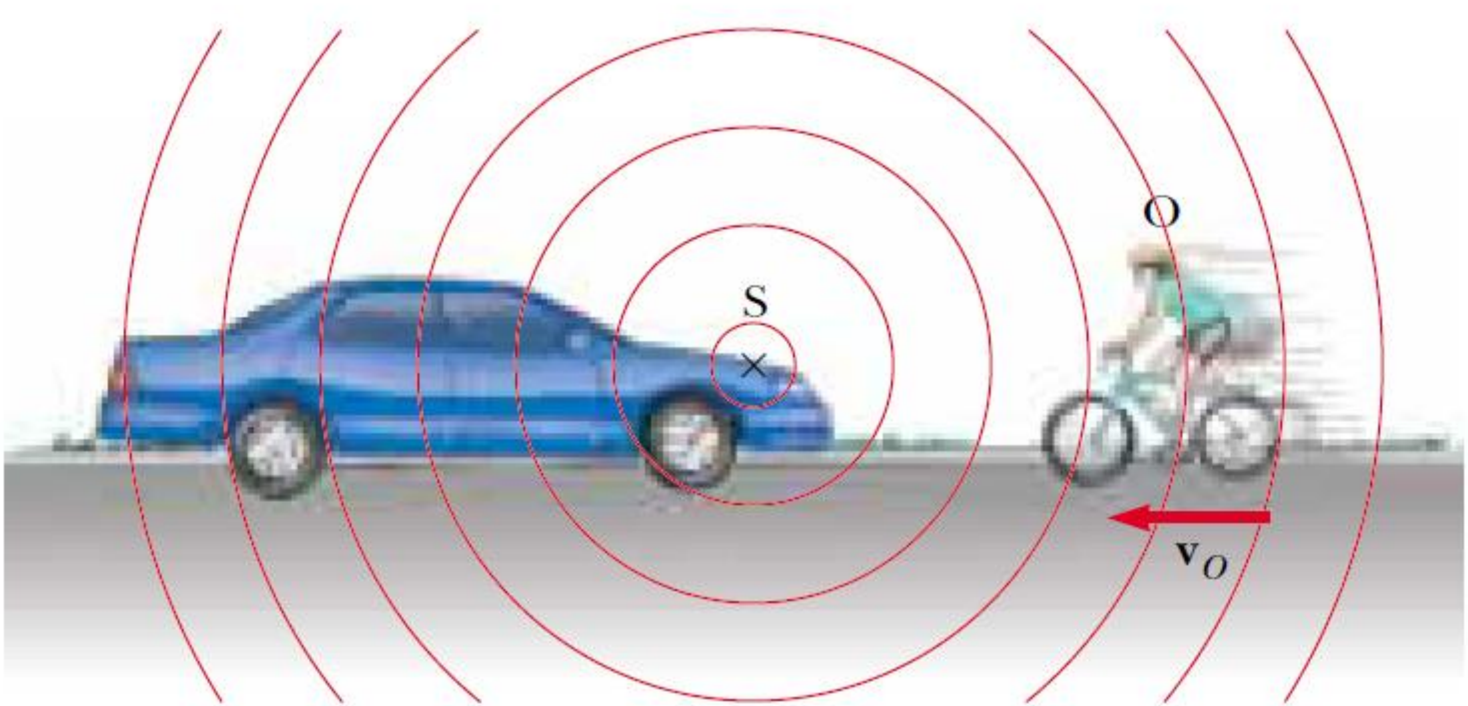
$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0$$

$$\frac{d\mathbf{v}'}{dt} = \frac{d\mathbf{v}}{dt} - \cancel{\frac{d\mathbf{v}_0}{dt}} \Rightarrow \mathbf{a}' = \mathbf{a}$$



The two inertial observers agree on measurements of acceleration.

Moving Observer



We take the frequency of the source to be *f*, the wavelength to be λ, and the speed of sound to be *v*.

Analyze the Moving Observer

- **The speed of the waves relative to the observer is**

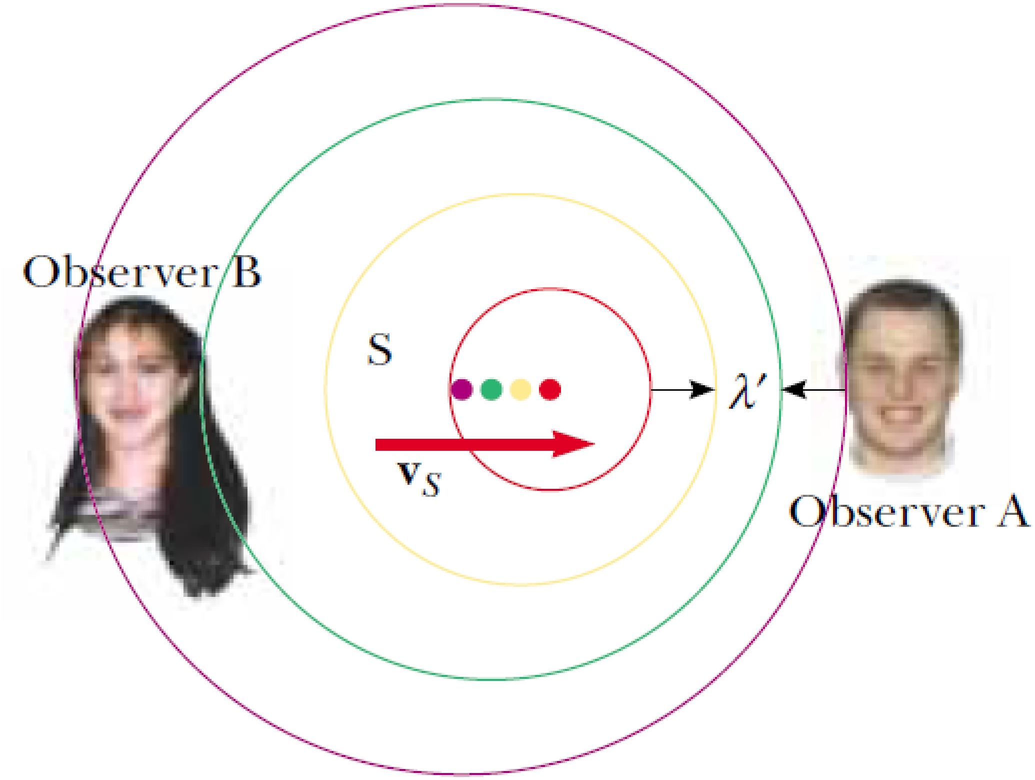
$$v' = v + v_O$$

- **The wavelength λ is unchanged.**

$$f' = \frac{v'}{\lambda} = \frac{v + v_O}{\lambda} = \left(1 + \frac{v_O}{v}\right) f$$

Positive v_o for observer moving toward source, and
negative v_o for observer moving away from source.

Moving Source



During each vibration, which lasts for a time T (the period), the source moves a distance

$$v_S T = v_S / f$$

Analyze Moving Source

- For observer A, the wavelength is shortened to

$$\lambda' = \lambda - \Delta\lambda = \lambda - \frac{v_S}{f}$$

- The frequency heard by observer A is

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - \frac{v_S}{f}} = \left(\frac{1}{1 - \frac{v_S}{v}}\right) f$$

- For observer B, simply use a negative v_S.

Doppler Effect

- **Finally, if both source and observer are in motion, we find the following general relationship for the observed frequency:**

$$f' = \frac{v + v_O}{v - v_S} f$$

The word toward is associated with an increase in observed frequency. The words away from are associated with a decrease in observed frequency.

Doppler Effect

Example: During a train passed a station, an observer on the station hear the frequency of the siren of the train varied from 1200Hz to 1000Hz. Find the speed of the train.(the speed of sound in air is 330 m/s)

Solution:

$$f_1' = f\frac{v}{v - v_s} \qquad f_2' = f\frac{v}{v + v_s}$$

$$v_s = \frac{f_1' - f_2'}{f_1' + f_2'} v = \frac{1200 - 1000}{1200 + 1000} \cdot 330 = 30\text{m/s}$$

Video—MIT Doppler effect

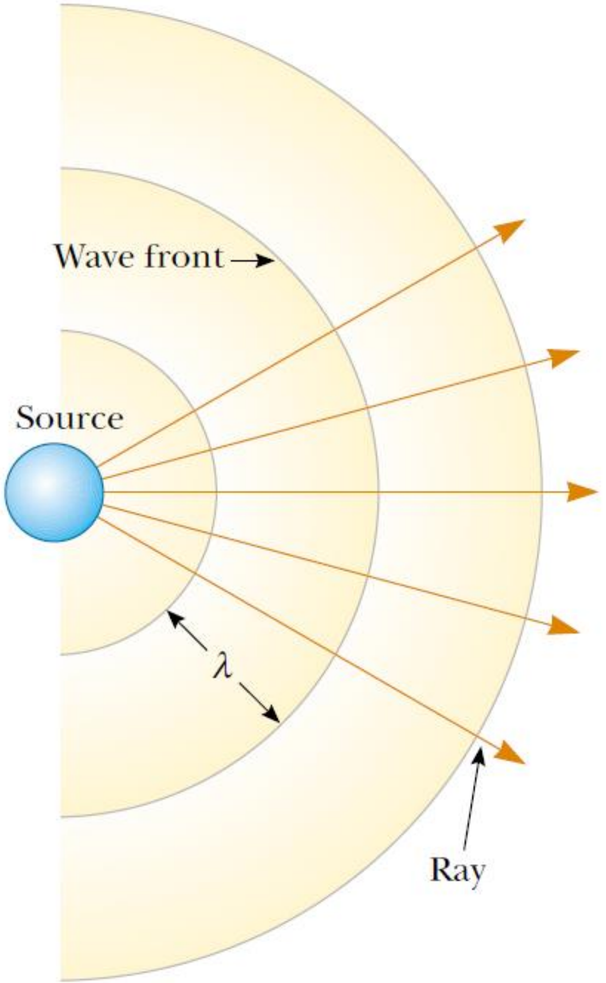
Spherical Waves

- The wave intensity at a distance *r* from the source is

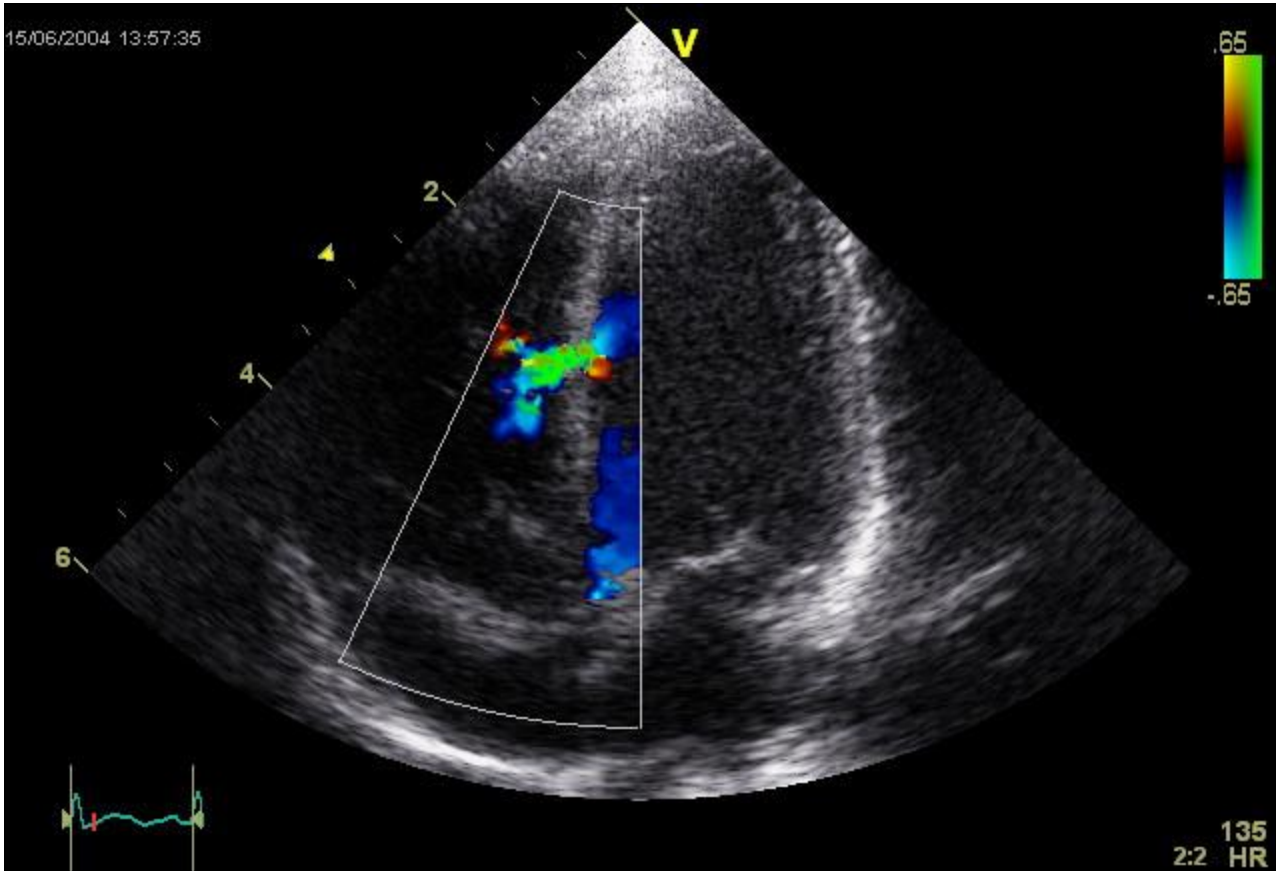
$$I = \frac{\mathcal{P}_{\text{av}}}{A} = \frac{\mathcal{P}_{\text{av}}}{4\pi r^2}$$

- The intensity is proportional to the square of the amplitude. Hence,

$$\psi(r,\ t) = \frac{s_0}{r}\sin(kr - \ \omega t)$$



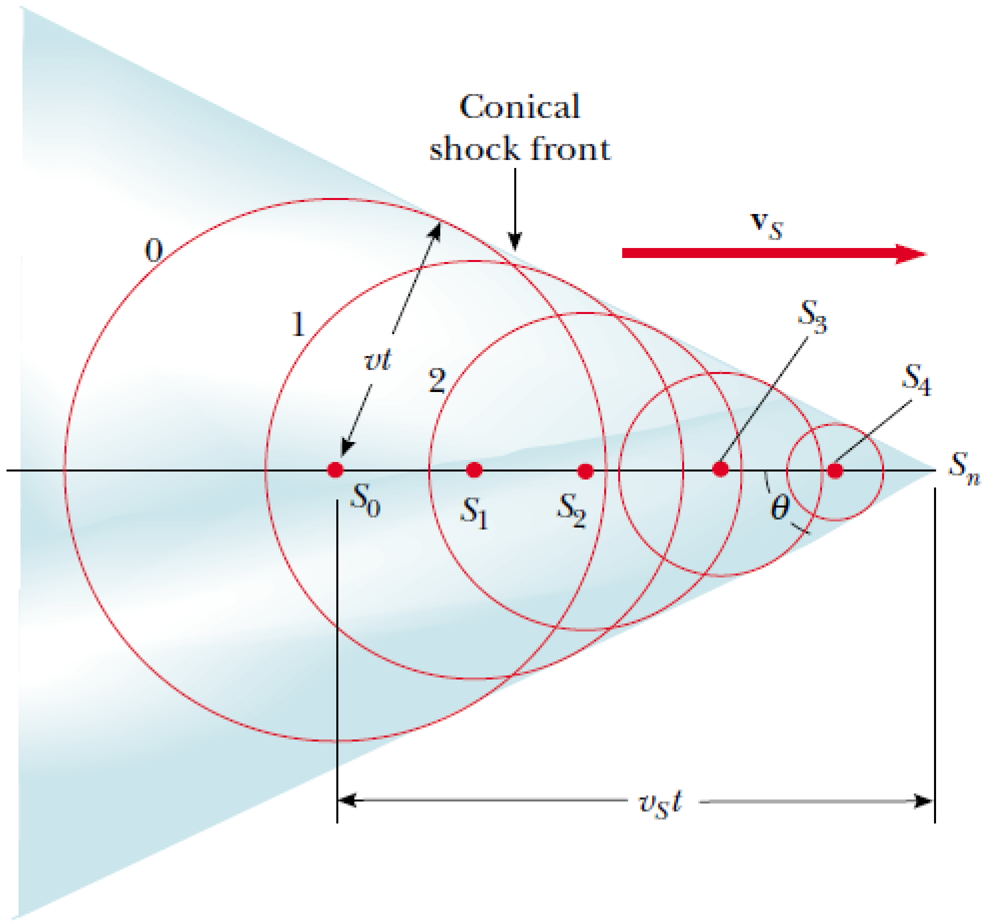
Echocardiogram (ECG)



https://baike.baidu.com/item/%E8%B6%85%E5%A3%B0%E5%BF%83%E5%8A%A8%E5%9B%BE

http://en.volupedia.org/wiki/Doppler_echocardiography

Shock Waves $(v_S > v)$



Mach number: v_S / v

$$\sin\theta = \frac{vt}{v_S t} = \frac{v}{v_S}$$

Doppler effect