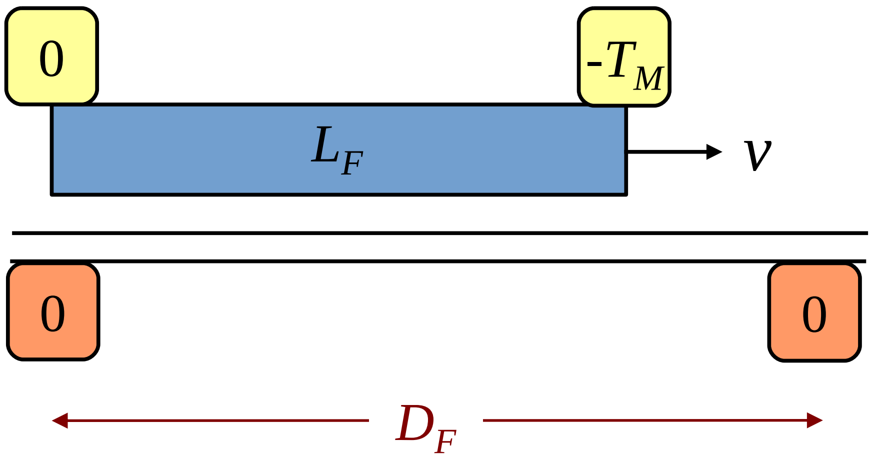
General Physics I

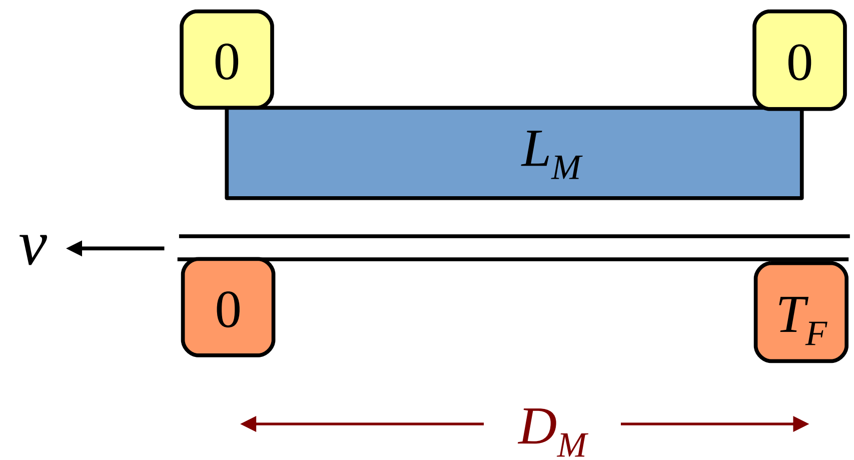
Lecture 19: Moving Clocks and Moving Sticks

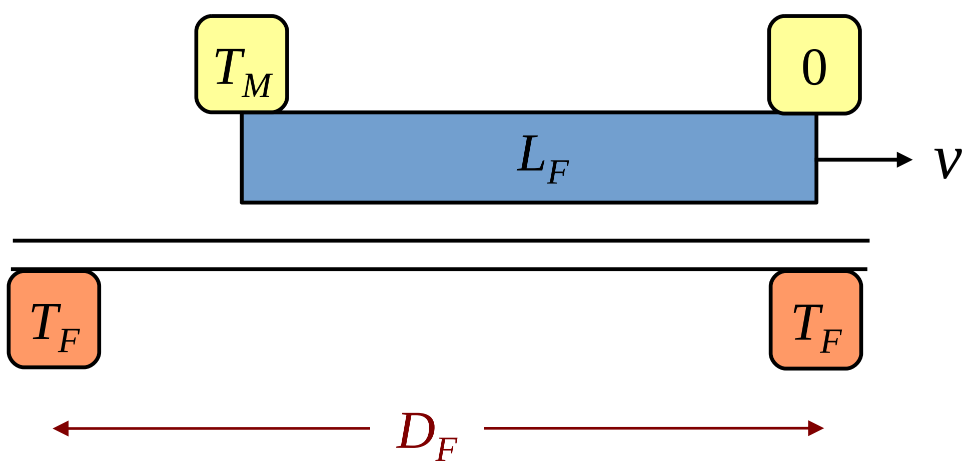
The Tale of Two Frames

Track Frame (Frank's)



Train Frame (Mary's)





$$T_F = D_F v / c^2$$

By the principle of relativity,

$$T_M = L_M v / c^2$$

Rule for Synchronized Clocks

- **If two clocks are synchronized and separated by a distance *D* in their proper frame (in which they do not move), then in a frame in which the clocks move along the line joining them with speed *v*, the reading of the clock in front is behind the reading of the clock in the rear by Dv/c^2, i.e., rear clock ahead.**

While the readings were made in the frame that the two clocks are not synchronized, the time difference is still defined in the frame that they are synchronized.

Mr Tompkins in Wonderland

By George Gamow

True or false?

The streets grew shorter, the windows of the shops began to look like narrow slits, and the policeman on the corner became the thinnest man he had ever seen…

Outline

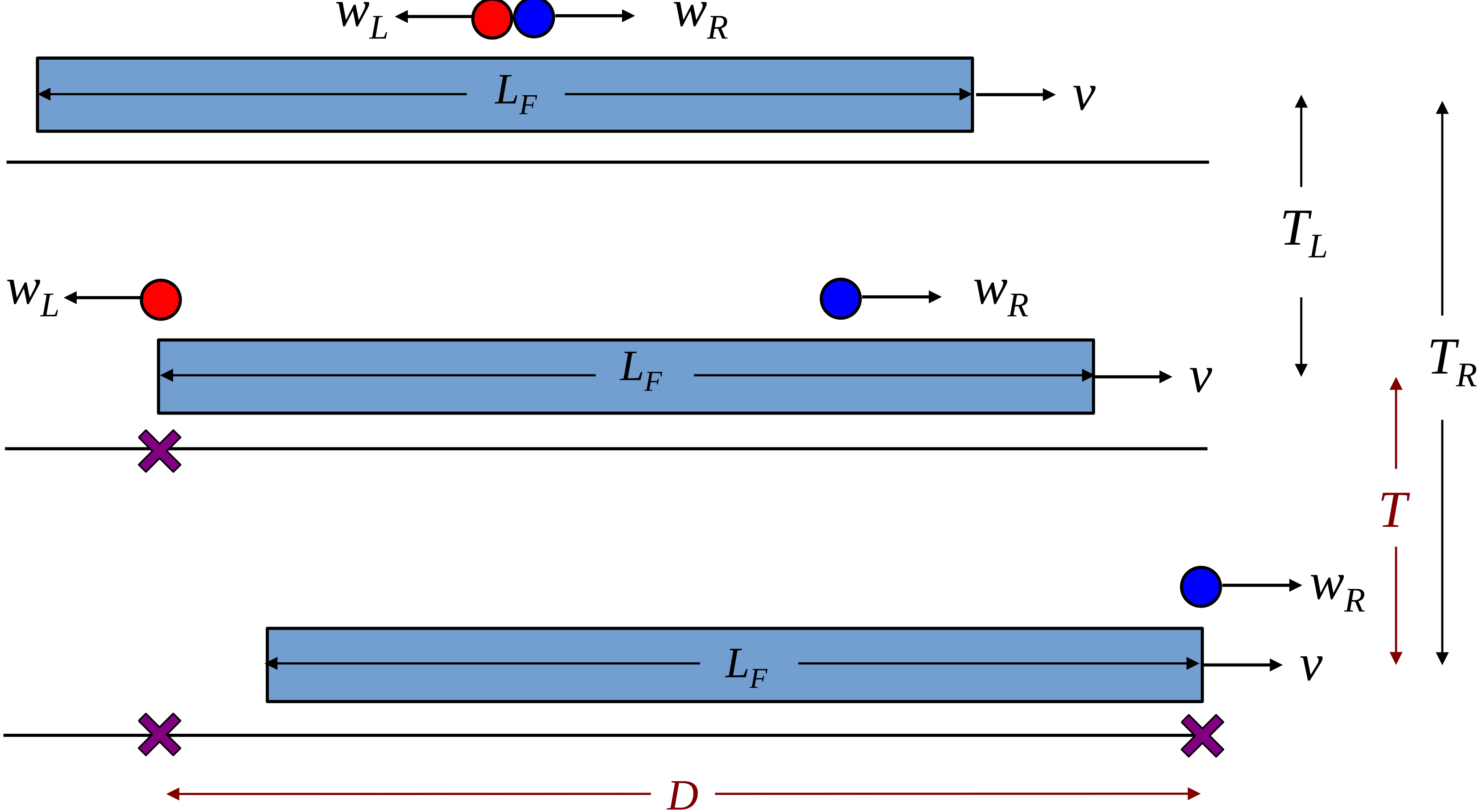
- Synchronizing without light
- Moving clocks run slowly and moving sticks shrink
- Experimental verification of the theory of relativity

Surely You're Cheating, Professor!

- Shouldn't you object, "You are cheating by using the highly peculiar light to synchronize the clock"?
- Am I?
- Are the rule for simultaneous events and the one for synchronized clocks just illusions that should disappear once we use, say, sound to synchronize the clocks?
- Well, let's see. Now we blow a whistle in the middle to synchronized the clocks.

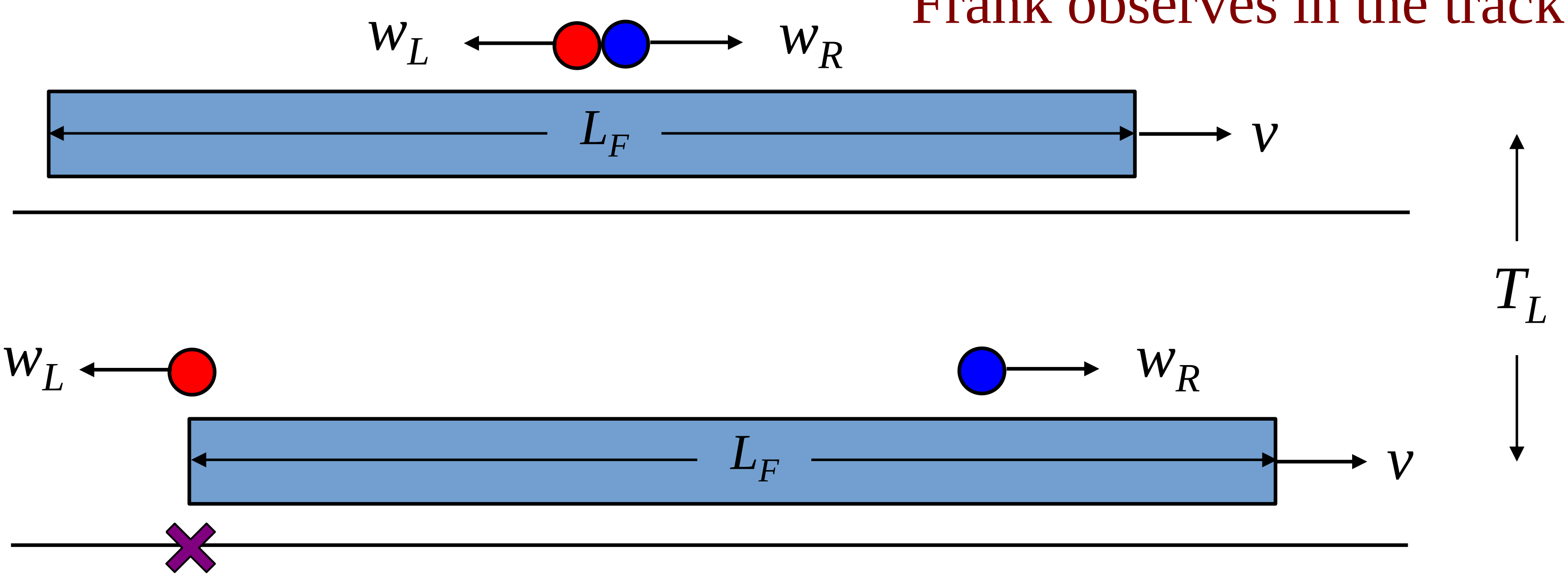
Synchronize with Other Signals

Frank observes in the track frame.



Synchronize with Other Signals

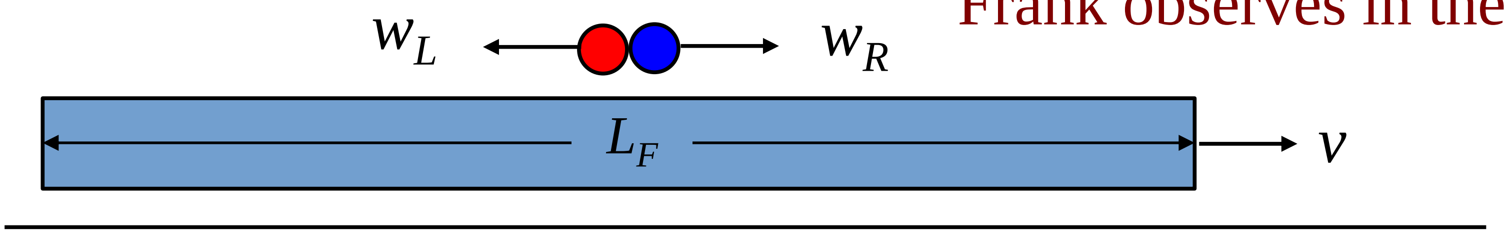
Frank observes in the track frame.

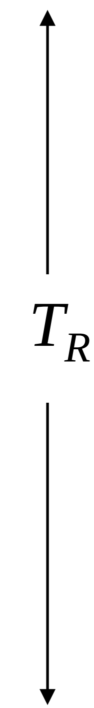


$$w_L T_L = \frac{1}{2} L_F - v T_L \quad \Rightarrow \quad T_L = \frac{L_F/2}{w_L + v}$$

Synchronize with Other Signals

Frank observes in the track frame.

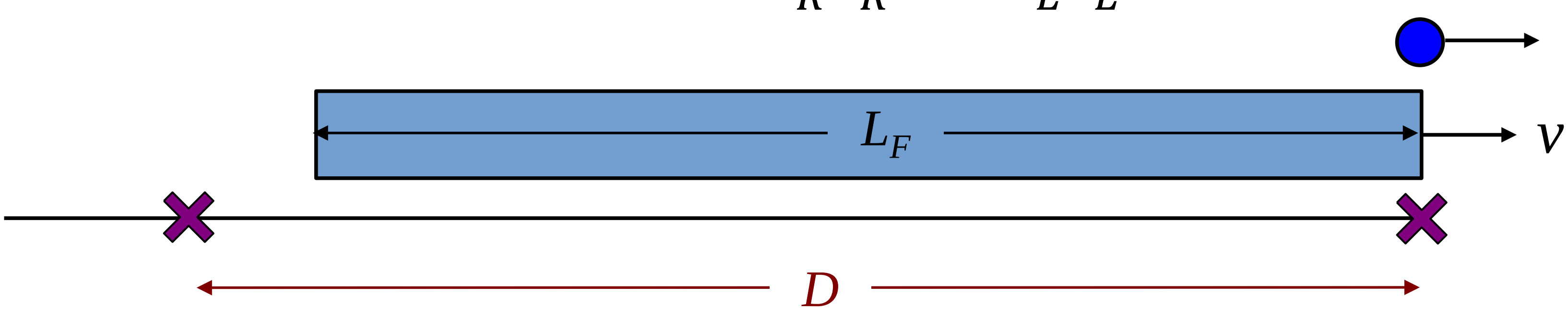




$$w_R T_R = \frac{1}{2} L_F + v T_R \quad \Rightarrow \quad T_R = \frac{L_F/2}{w_R - v}$$

Together with the equation at the previous page

$$D = w_R T_R + w_L T_L$$



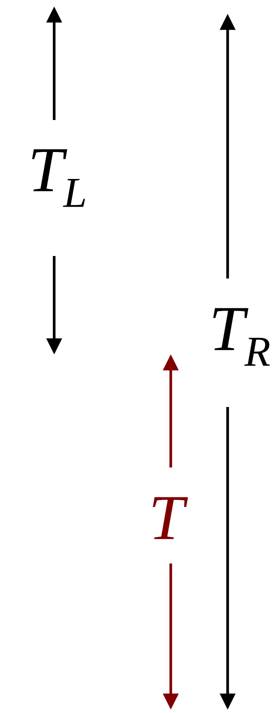
Synchronize with Other Signals

$$T_L = \frac{L_F/2}{w_L + v} \qquad T_R = \frac{L_F/2}{w_R - v} \qquad D = w_R T_R + w_L T_L$$

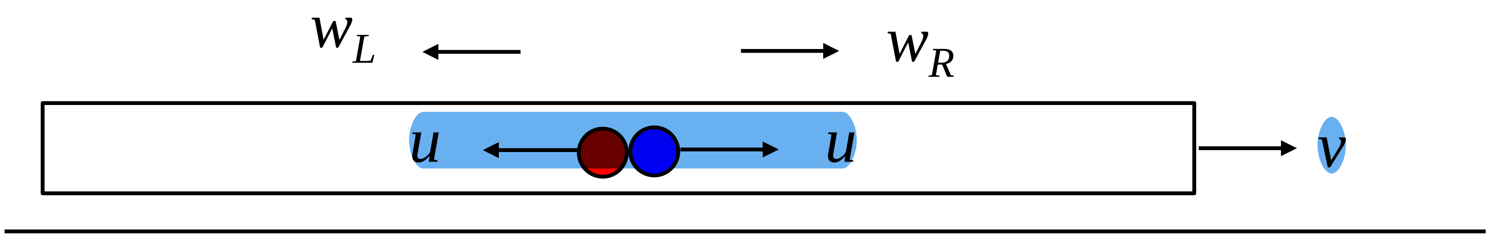
$$\Rightarrow \quad T = \frac{1}{2} L_F \left(\frac{1}{w_R - v} - \frac{1}{w_L + v} \right)$$

$$D = \frac{1}{2} L_F \left(\frac{w_R}{w_R - v} + \frac{w_L}{w_L + v} \right)$$

$$\Rightarrow \quad \frac{T}{D} = \frac{2v - (w_R - w_L)}{2 w_R w_L + v(w_R - w_L)}$$



Synchronize with Other Signals



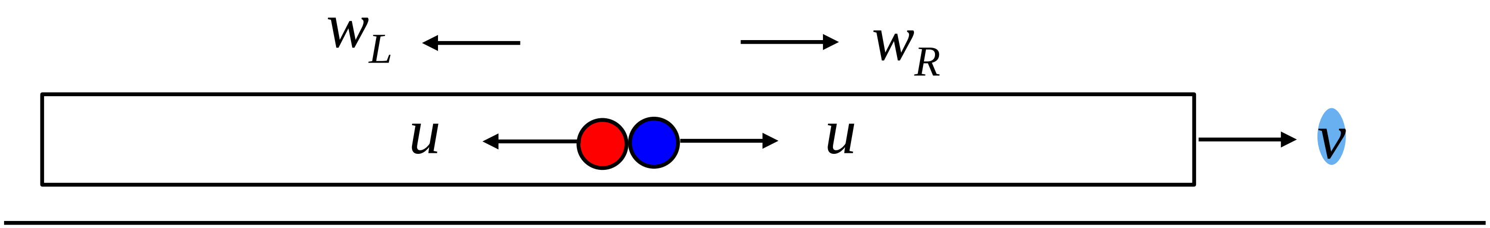
In the classical mechanics,

$$w_R = u + v \qquad\qquad w_L = u - v$$

$$\frac{T}{D} = \frac{2v - (w_R - w_L)}{2w_R w_L + v(w_R - w_L)} = 0$$

This is the nonrelativistic rule: two events that are simultaneous in the train frame are also simultaneous in the track frame.

Synchronize with Other Signals



With the relativistic velocity addition law,

$$w_R = \frac{u + v}{1 + \left(\frac{u}{c}\right)\left(\frac{v}{c}\right)} \qquad w_L = \frac{u - v}{1 - \left(\frac{u}{c}\right)\left(\frac{v}{c}\right)}$$

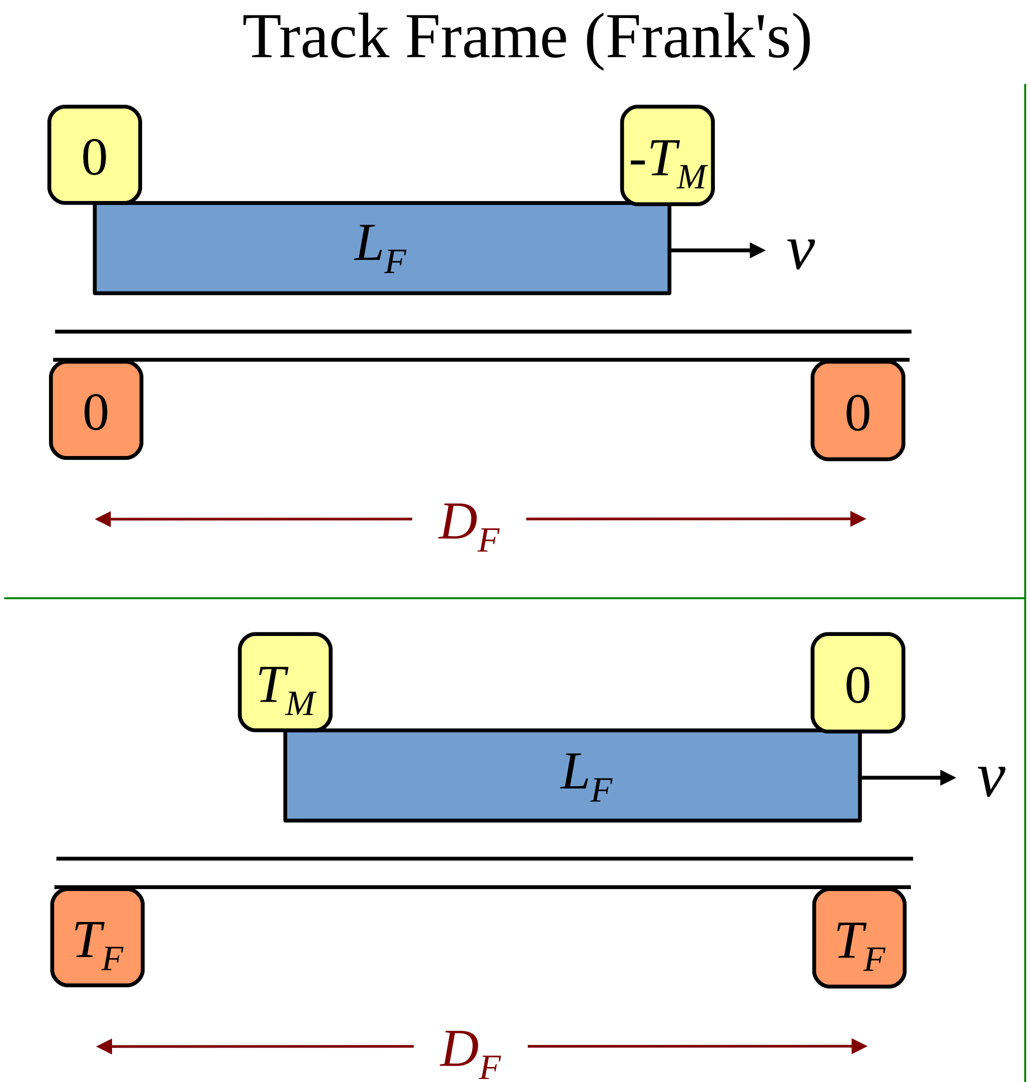
$$\frac{T}{D} = \frac{2v - (w_R - w_L)}{2w_R w_L + v(w_R - w_L)} = \frac{v}{c^2}$$

We recover the result obtained with light, $D = D_F$ and $T = T_F$.

Compare Time & Length

- The principle of relativity guarantees that **the rule for synchronized clocks must be valid in any inertial frame of reference.**
- So, we can compare clocks synchronized in Mary's frame and clocks synchronized in Frank's frame. We shall deduce that moving clocks must run slowly.
- Likewise, moving trains (or tracks) must shrink along the direction of their motion.

The Slowing-Down Factor

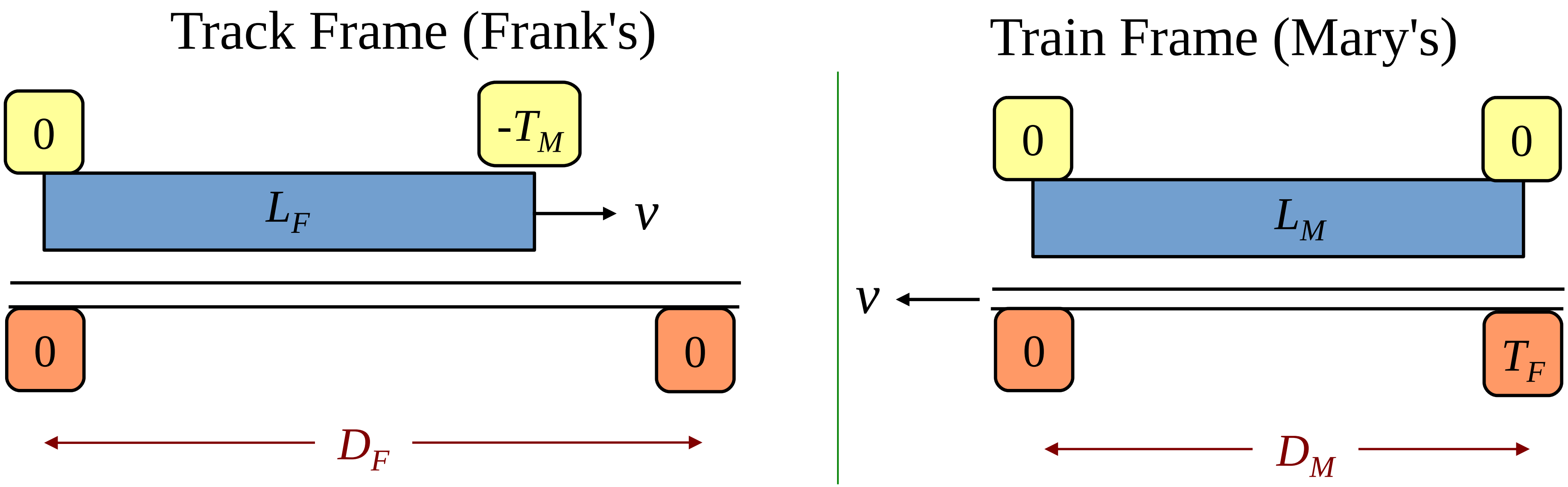


Between the two pictures, both Frank's (track) clocks advance by a time T_F and both Mary's (train) clocks advance by a time T_M. Therefore, the slowing-down factor is

$$\frac{T_M}{T_F} = \frac{L_M}{D_F} = \frac{D_M}{D_F} = s$$

Note that the train-frame picture is a single-moment snapshot (as is evident from the same reading of the train clocks), hence $L_M = D_M$.

The Shrinking Factor

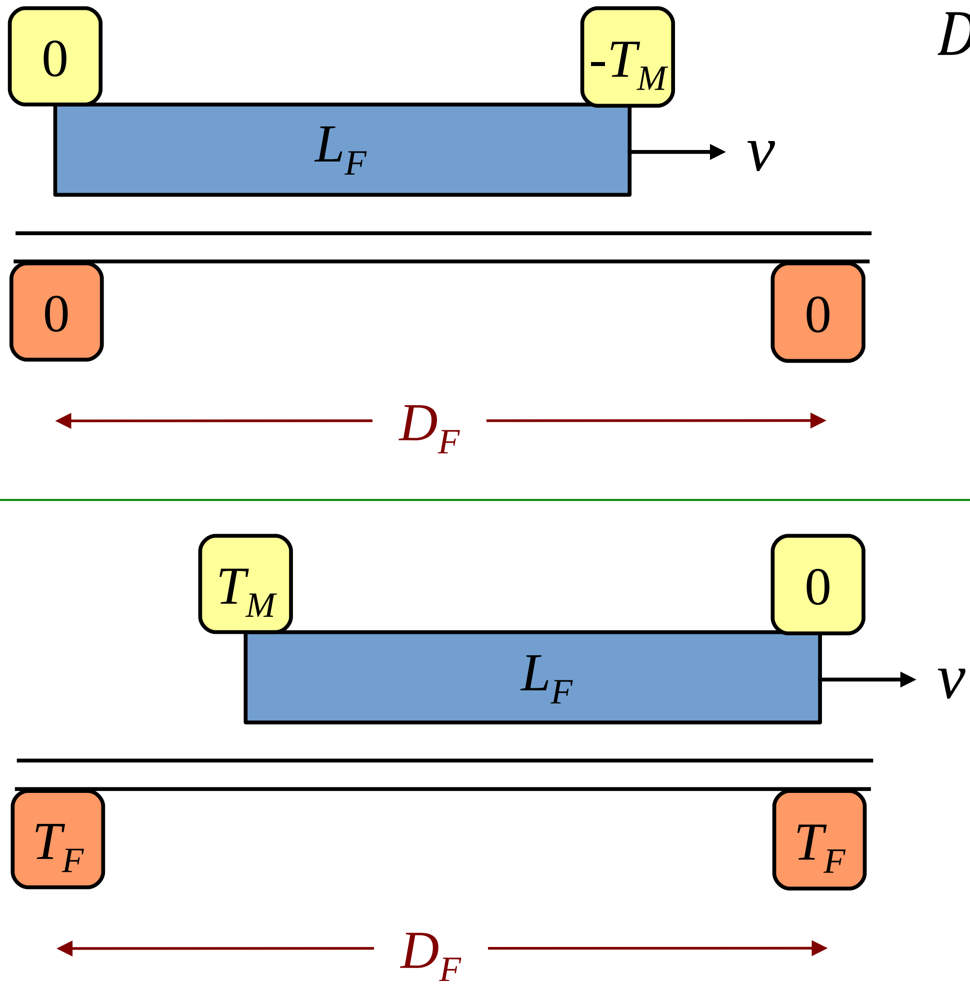


- The shrinking factor for a moving object

$$\frac{L_F}{L_M} = \frac{D_M}{D_F} \equiv s$$

Solving the s-Factor

Track Frame (Frank's)



$$D_F = L_F + vT_F = L_F + \frac{v^2 D_F}{c^2}$$

$$\Rightarrow \quad D_F = \frac{L_F}{1 - v^2/c^2}$$

- From the previous page

$$L_F = sL_M = sD_M = s^2 D_F$$

$$\Rightarrow \quad s = \sqrt{1 - v^2/c^2}$$

Note that s < 1 justifies the names of the slowing-down and shrinking factor.

Nomenclature

- **Time dilation** refers to the slowing down of the moving clock, although time itself (or whatever that might be) is not really expanding – it is the relation between the two sets of clocks.
- The shrinking of moving objects along the direction of their motion is called the **Lorentz-Fitzgerald contraction** (in the context of moving relatively to the ether).
- In the literature $\gamma = 1/s$ is quite often used.

New Puzzle #1

- **How can Mary conclude that Frank's clocks are running slowly, while Frank concludes that it is Mary's clocks that are running slowly?**

Note that in the theory of relativity consistency, rather than the everyday experience, is more important. Do you see any inconsistency?

Both Are Correct

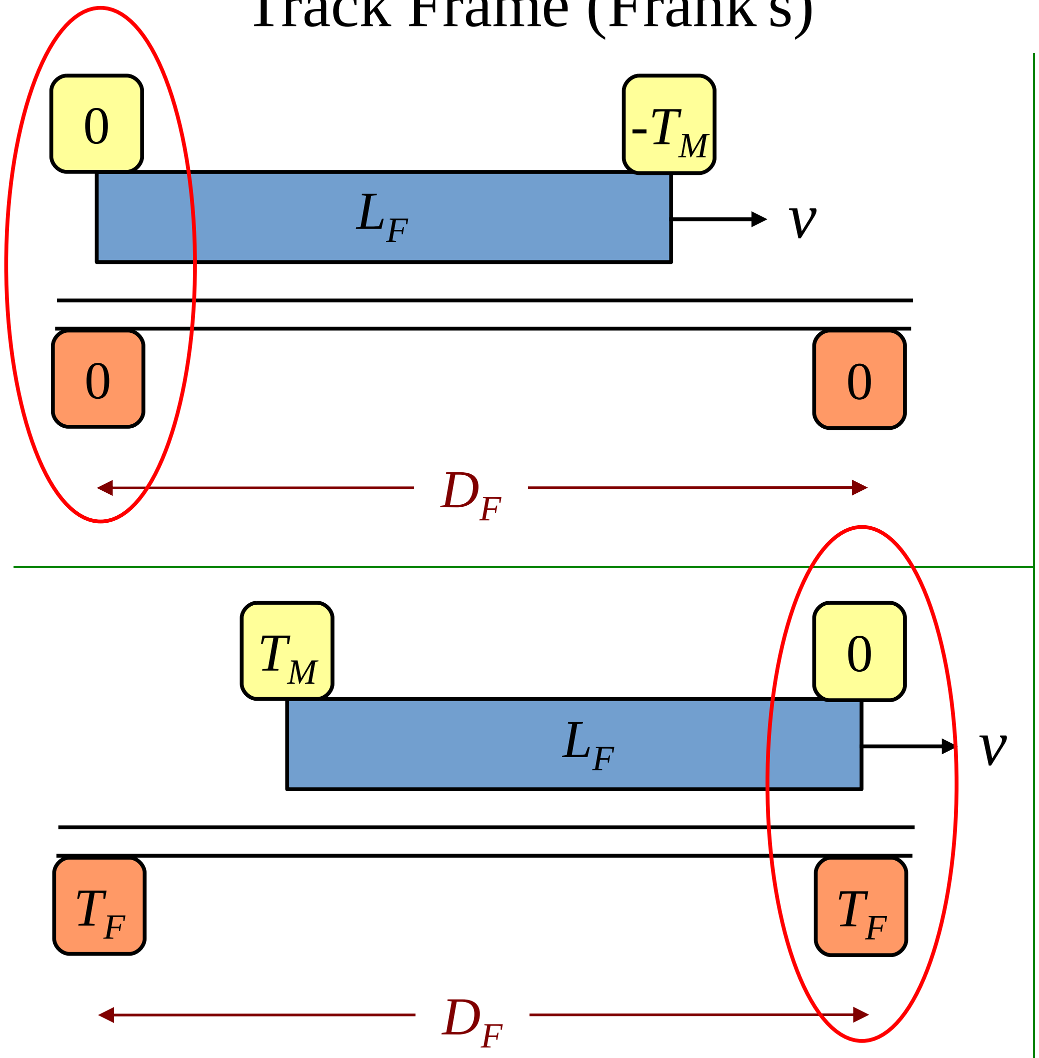
- To determine the rate of a moving clock, it is necessary to compare two of the readings of the moving clock with the readings of two stationary clocks that are next to it when it shows those readings. This requires one to use two synchronized clocks in two difference places.
- To determine the length of a moving stick, one must determine where its two ends are at the same time. This requires the judgment about whether spatially separated events at the two ends of the stick are simultaneous.
- Both Mary and Frank can point out that the other fails to synchronize her/his clocks properly. So both are correct.

Fundamental Disagreement

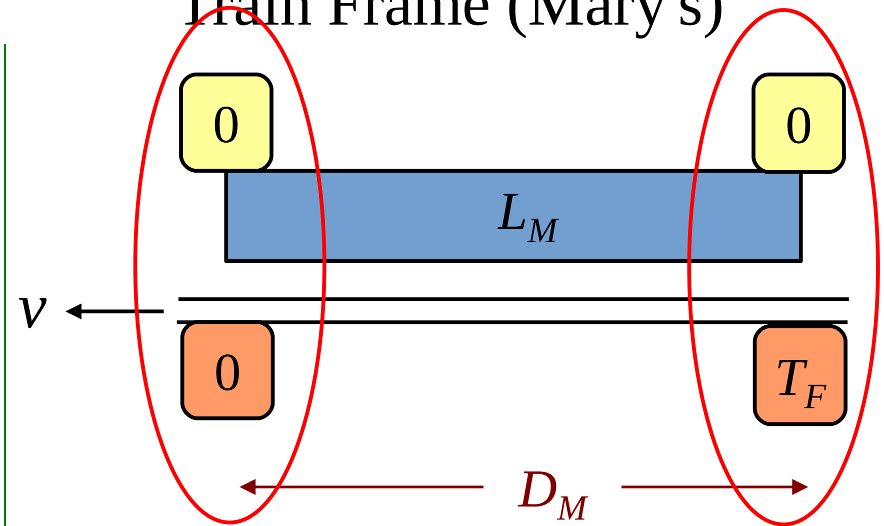
- The fundamental disagreement is on whether two events in different places happen at the same time or, equivalently, on whether two clocks in different places are synchronized.
- Time dilation and length contraction are merely conventions about how we describe the behavior of clocks and measuring sticks.

What Mary and Frank Agree

Track Frame (Frank's)



Train Frame (Mary's)



$$T_F = D_F\, v/c^2$$

$$T_M = L_M\, v/c^2$$

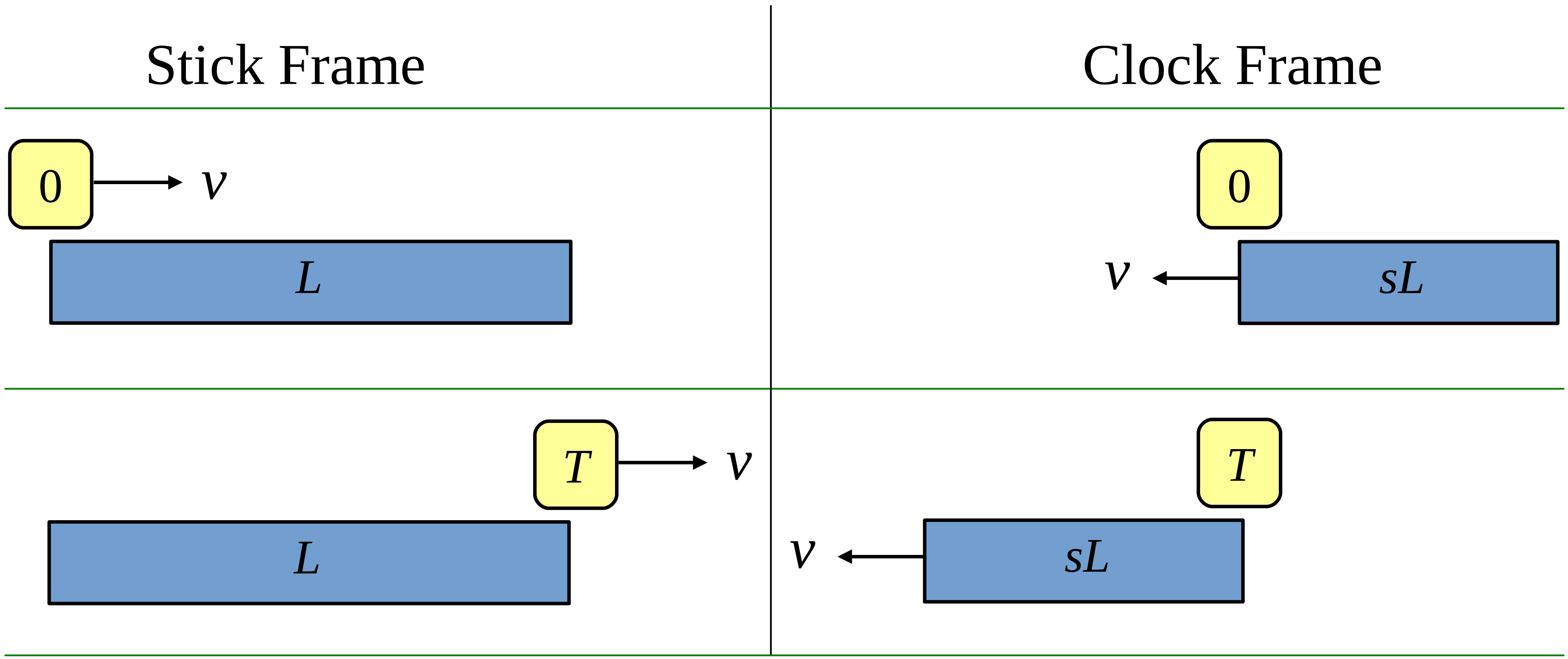
New Puzzle #2

- **How comes the slowing-down factor *s* for moving clocks is the same as the shrinking factor *s* for moving sticks?**

$$s = \sqrt{1 - v^2/c^2}$$

Again, do you see any inconsistency?

Both Factors Are the Same



$$T = s'(L/v)$$

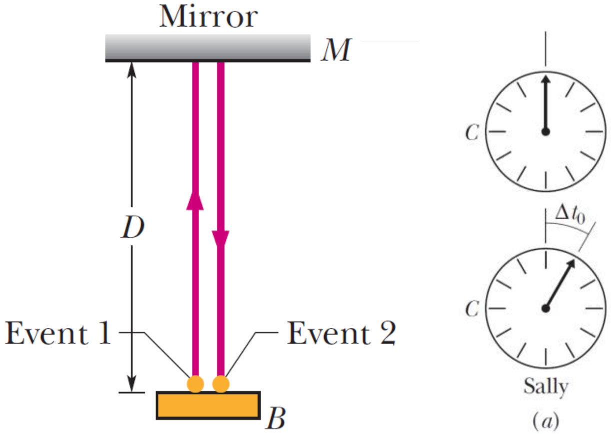
time dilation

$$T = (sL)/v$$

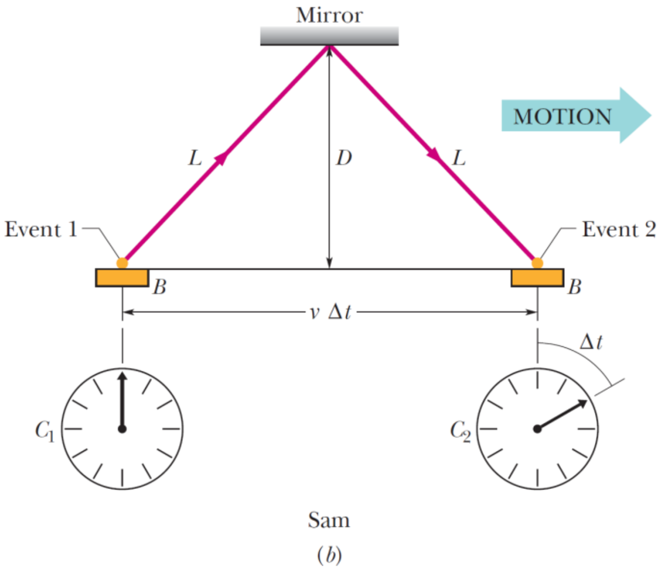
length contraction

Time T on the clock must be agreed since it is about things at a single place and time.

Time Dilation



$$\Delta t_0 = \frac{2D}{c} \quad \text{(Sally)}.$$

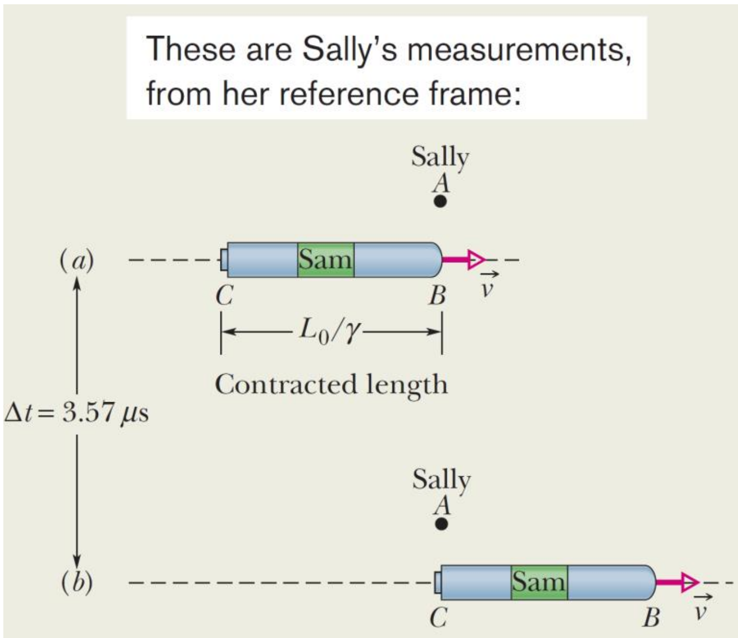


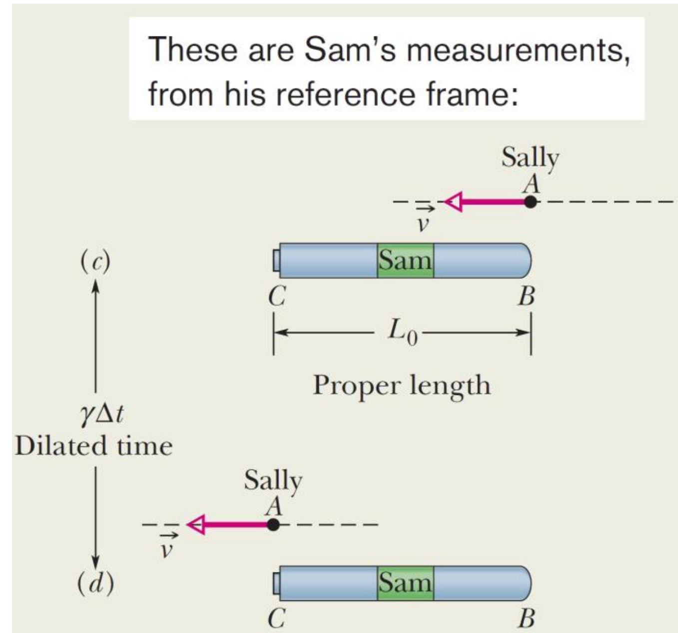
$$\Delta t = \frac{2L}{c} \quad \text{(Sam)},$$

$$L = \sqrt{(\tfrac{1}{2}v\ \Delta t)^2 + D^2}.$$

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - (v/c)^2}}.}$$

Length Contraction





$$L = v\,\Delta t_0 \qquad \text{(Sally)}$$

$$L_0 = v\,\Delta t \qquad \text{(Sam).}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}.$$

$$\frac{L}{L_0} = \frac{v\,\Delta t_0}{v\,\Delta t} = \frac{1}{\gamma},$$

$$L = \frac{L_0}{\gamma},$$

Take-Home Message

- **Rule for synchronized clocks**

- *If two clocks are stationary, synchronized, and separated by a distance D in one frame of reference, then in a second frame, in which they are moving with speed v along the line joining them, the clock in the front lags behind the clock in the rear by*

$$T = Dv/c^2$$

- **Rule for shrinking of moving sticks or slowing down of moving clocks**

- *The shrinking (or slowing-down) factor s associated with a speed v is given by*

$$s = \sqrt{1 - v^2/c^2}$$

Muon Decay

- **Cosmic rays interact with particles in the earth's outer atmosphere and create particles called muons. Muons are unstable with half-life 1.52 μs in their own rest frame, i.e.**

$$N = N_0 2^{-t/t_{1/2}}$$

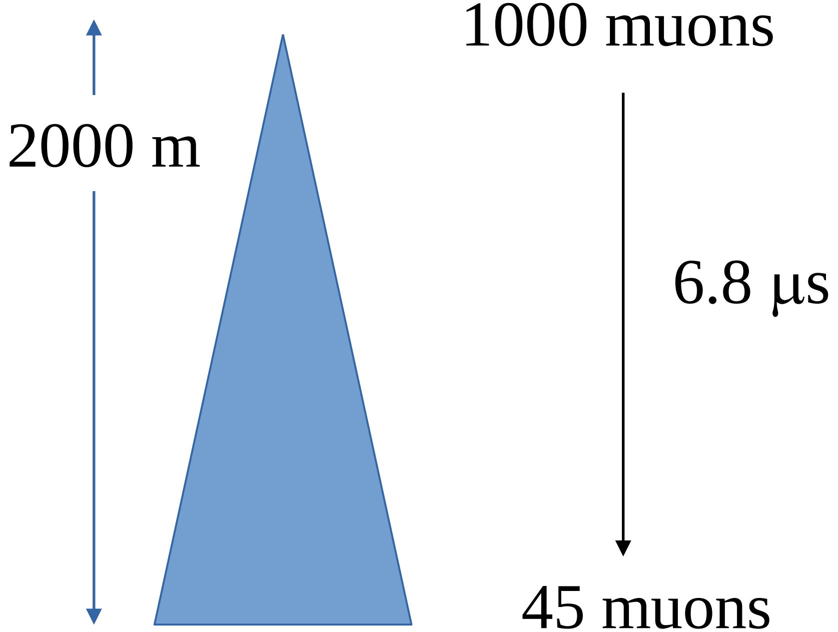
- **A detector on top of a 2000-m mountain counts 1000 muons traveling at *v* = 0.98*c* over a given period of time.**

- **Now at the sea level, how many muons do you expect to detect at *v* = 0.98*c* over the same period of time?**

See B. Rossi and D. B. Hall, Phys. Rev. 59, 223 (1941).

Nonrelativistic View

- **Classically, muons travel 2000 m in 6.8 μs. Expect only 45 muons to survive.**



$$\frac{2000m}{0.98 \cdot 3 \times 10^8\, m/s} = 6.8\, \mu s$$

$$1000 \times 2^{-6.8/1.52} = 45$$

Experimental measurement indicates 542 muons survive, a factor of 12 more. Why?

See B. Rossi and D. B. Hall, Phys. Rev. 59, 223 (1941).

Relativistic, Earth Frame

- **At the high speed *v* = 0.98*c* of the muons relative to the experimenters fixed on the earth, we conclude that they observe the muons' clock to be running slower by a slowing-down factor *s* = 0.2.**
- **In the earth frame, the halflife of a muon is not 1.52 μs, but 7.6 μs.**

$$1000 \times 2^{-6.8/7.6} = 538$$

- **The radioactive decay law predicts that 538 muons survive. Hence the measurement (542 muons) verified the special theory of relativity within experimental uncertainties.**

Relativistic, Muon Frame

- **In the muons' rest frame, the mountain is moving at the high speed v = 0.98c relative to the muons. We conclude that muons observe the mountain’s height shrinks by a shrinking factor 0.2. So the height of the mountain is not 2000 m, but 400 m.**
- **The flight time over 400 m is 1.36 μs. Again, we obtain**

$$1000 \times 2^{-1.36/1.52} = 538$$

Comparison

	Relativistic, Muon Frame	Relativistic, Earth Frame	Nonrelativistic
Distance	400 m	2000 m	2000 m
Travel time	1.36 μs	6.8 μs	6.8 μs
Half-life	1.52 μs	7.6 μs	1.52 μs
Halflives	0.895	0.895	4.47
Surviving	538	538	45

The relativistic approach yields agreement with experiment and is greatly different from the non-relativistic result. Note that the muon and ground frames do not agree on the distance and time, but they agree on the final result. One observer sees time dilation, the other sees length contraction, but neither sees both.

Atomic Clock Time Measurement

- **Cesium-133 atom has a well-defined transition at a frequency of 9,192,631,770 Hz and can be used as an accurate measurement of a time period.**

In October 1971, Joseph C. Hafele and Richard E. Keating took four cesium-beam atomic clocks aboard commercial airliners. They flew twice around the world, first eastward, then westward, and compared the clocks against others that remained at the US Naval Observatory.

J.C. Hafele and R. E. Keating, Science 177, 166 (1972)

Atomic Clock Time Measurement

- General relativity predicts an increase in gravitational potential due to altitude further speeds the clocks up. That is, clocks at higher altitude tick faster than clocks on Earth's surface.

	nanoseconds gained			
	predicted			measured
	gravitational (general relativity)	kinematic (special relativity)	total	
eastward	144±14	−184 ± 18	−40 ± 23	−59 ± 10
westward	179±18	96±10	275±21	273±7

J.C. Hafele and R. E. Keating, Science 177, 166 (1972)

An Educated Protocol

- **Mary prepares two identical stationary clocks in the same place, say at the rear of the train.**
- **She synchronizes them to 0 by direct comparison in the same place.**
- **She sets one of them into uniform motion with speed u toward the front of the train.**
- **She stops the motion of the clock at the front of the train, at which moment the clock reads t.**
- **Mary resets the clock to** $t/\sqrt{1-u^2/c^2}$

Questions

- **Are the two clocks synchronized in Mary's frame?**
- **Assume, again, Mary and the train is moving with respect to Frank on the ground with a speed *v*. Does Frank agree that Mary's clocks are synchronized? If not, how much does the two clocks differ in Frank's frame?**
- **Show explicitly, through the slowing-down factors s_u, s_v, s_w, that the result can be calculated in Frank's frame. Here, *w* refers to the combined velocity of *u* and *v*.**