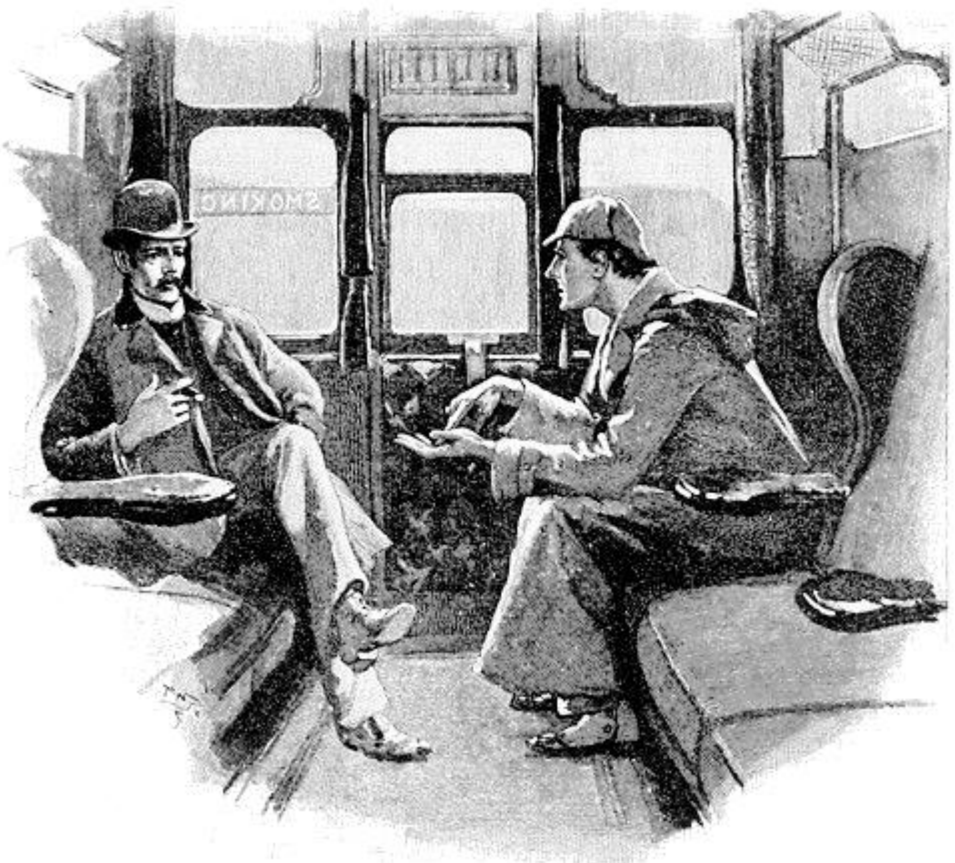
General Physics I

Lecture 18: Combining Velocities and Synchronizing Clocks

The Train of Reasoning



You have been in Afghanistan, I perceive.

– Sherlock Holmes

The train of reasoning ran: "Here is a gentleman of a medical type, but with the air of a military man. Clearly an army doctor then. He has just come from the tropics, for his face is dark, and that is not the natural tint of his skin, for his wrists are fair. He has undergone hardships and sickness, as his haggard face says clearly. His left arm has been injured. He holds it in a stiff and unnatural manner. Where in the tropics could an English army doctor have seen much hardship and got his arm wounded? Clearly in Afghanistan."

Outline

- Finding suspect
- Combining velocities
- Events and simultaneous events
- Synchronizing clocks

With Respect to What?

- **The answer seems to be "with respect to any inertial frame you like."**
- **But how can this even be possible? It is highly counterintuitive. No, it seems impossible.**

-Frank notes in each second the light moves 299,792 km to his left, and Mary is moving 208 km (we assume this for simple illustration) to his right. Light is moving closer to Mary at 300,000 km/s, isn't it? But Mary should measure the light coming at her with a speed 299,792 km. Who is wrong?

It came to me that time was suspect! *–Albert Einstein*

Implicit Assumption

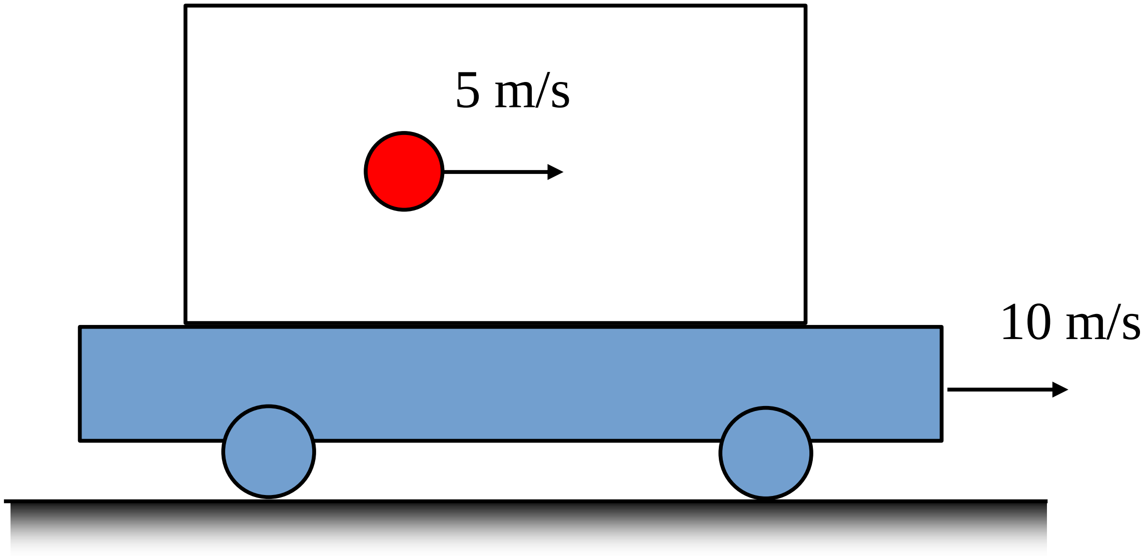
- **Besides using the principle of relativity, we repeatedly made implicit use of the nonrelativistic velocity addition law**

$$v_{XZ} = v_{XY} + v_{YZ}$$

- v_{XZ} **is the velocity of X with respect to Z.**
- **Aha! Here comes the "with respect to".**
- **How many of you find the addition law not so obvious? Why?**

Innocent or Guilty?

- **A train moves to the right at 10 m/s in the track frame. In the train frame a ball moves to the right at 5 m/s. Hence, the ball is moving 15 m/s to the right in the track frame. Any doubt?**



Now the Cumbersome Version

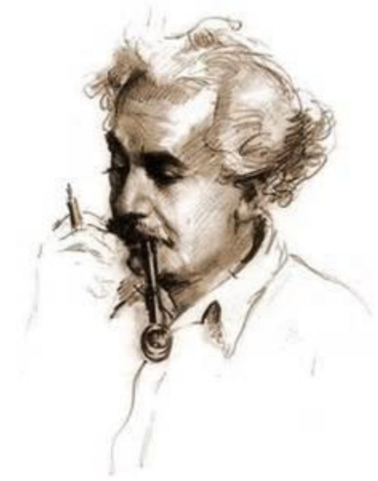
- If the ball moves to the right in the train frame at 5 m/s, then in one second according to the train time its gets 5 meters further down the train.
- If the train moves to the right at 10 m/s in the track frame, then in one second according to the track time it gets 10 meters further right along the track.
- So in one second the ball gets 15 meters further right along the track – the 5 it gains on the train and the additional 10 the train gains on the track.
- But what does in one second mean? It makes no sense unless the track time and the train time are the same.

From Newton to Einstein

Absolute, true, and mathematical time, of itself and from its own nature, flows equably without relation to anything external.

– *Isaac Newton*

It came to me that time was suspect!
– *Albert Einstein*



Combing Velocities

- The nonrelativistic velocity addition law $w = u + v$
- should be replaced, when the velocities is comparable with the speed of light, by the relativistic velocity addition law

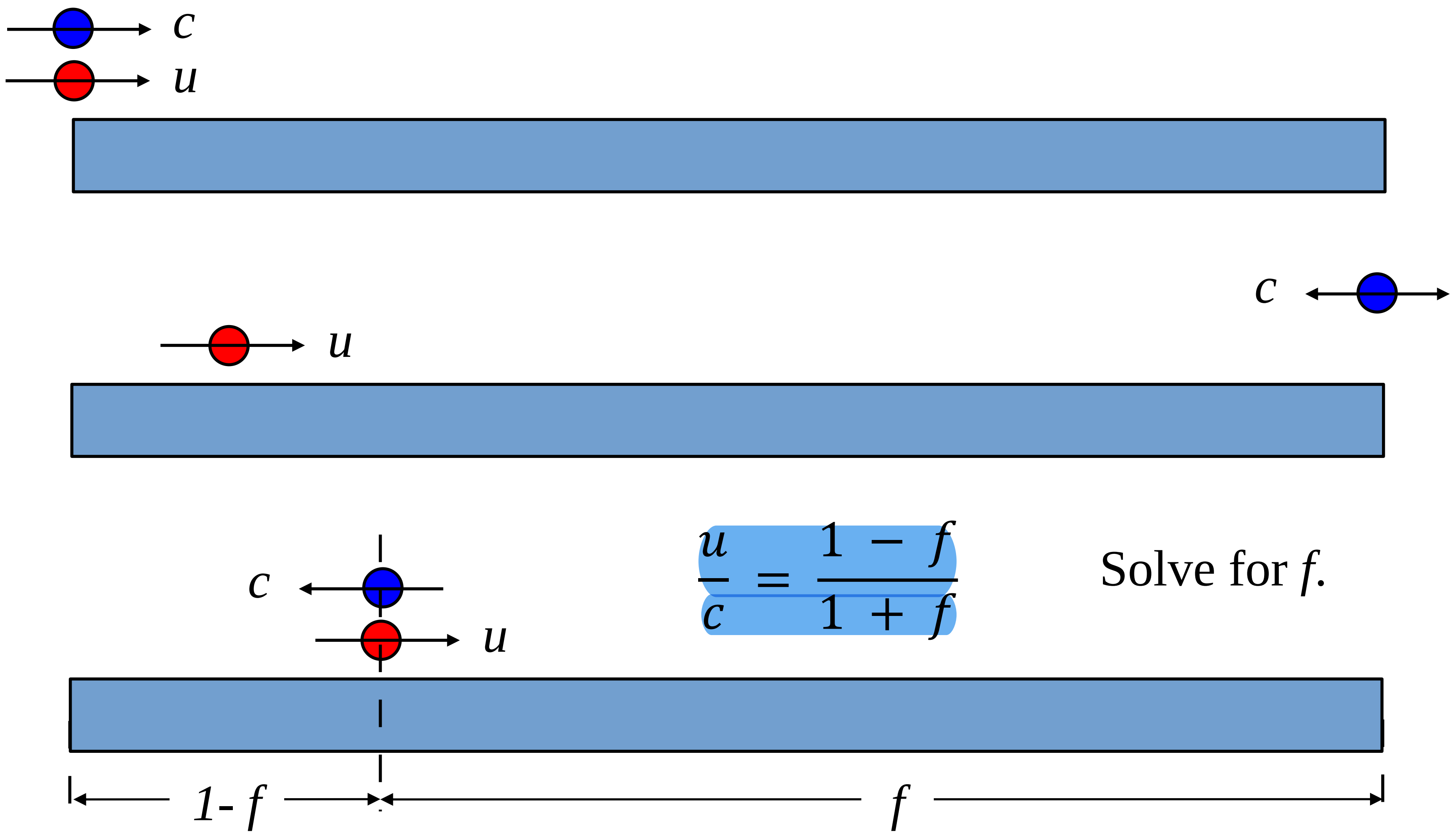
$$w = \frac{u + v}{1 + \left(\frac{u}{c}\right)\left(\frac{v}{c}\right)}$$

- In fact, the frame independence of the speed of light c and the nonrelativistic velocity addition law turn out to be special cases of a very general rule for combining velocities whether or not the speeds involved are small compared to the speed of light. We now derive on the general ground.

A Tale of Two Races

- Now we derive the velocity addition law by taking advantage of the fact that we do know, at least, the speed of light. We avoid using clocks, rulers, etc.
- We run two races between a ball and a pulse of light (a photon in quantum mechanics), one with the train at rest on the track and another with a moving train.

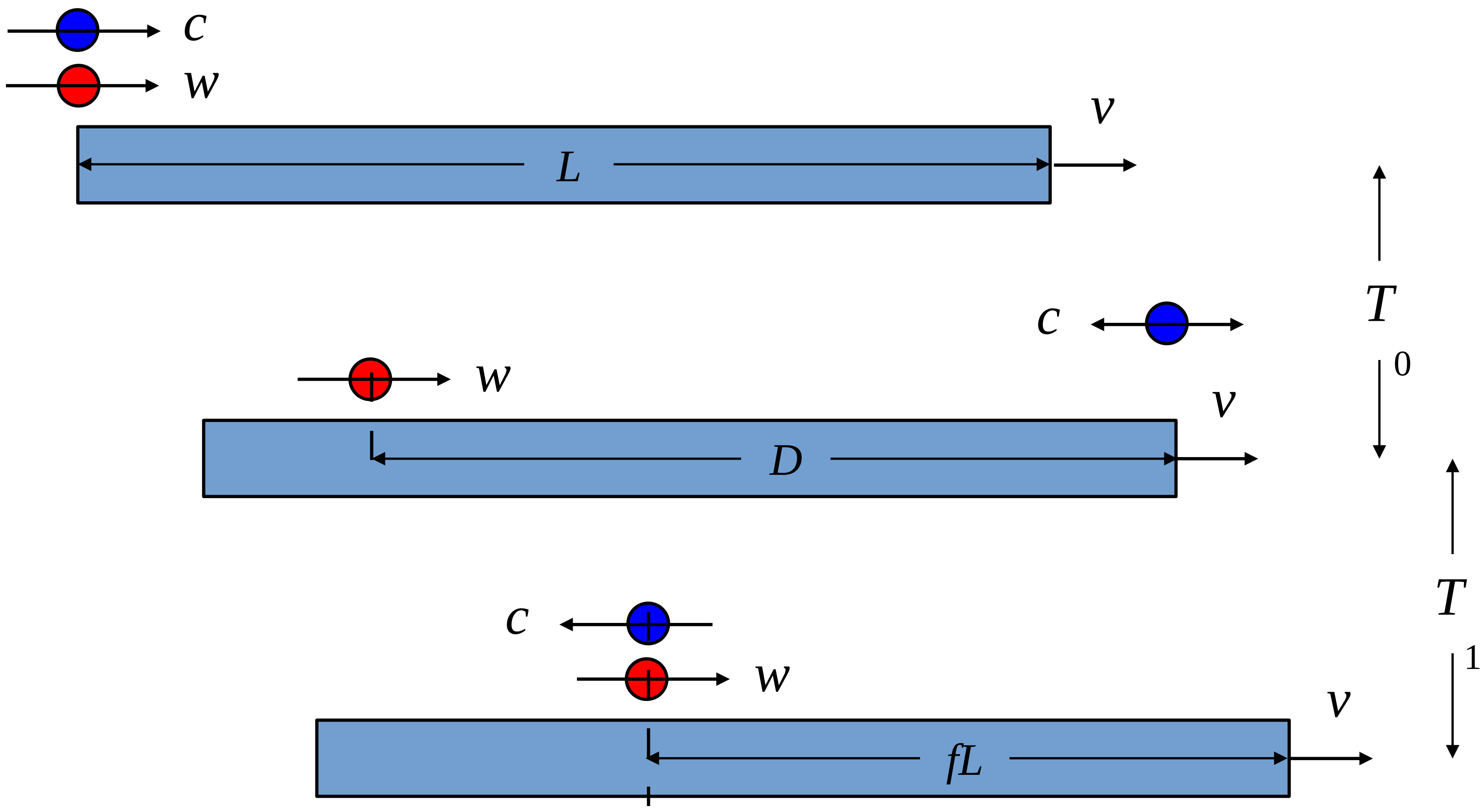
The First Race



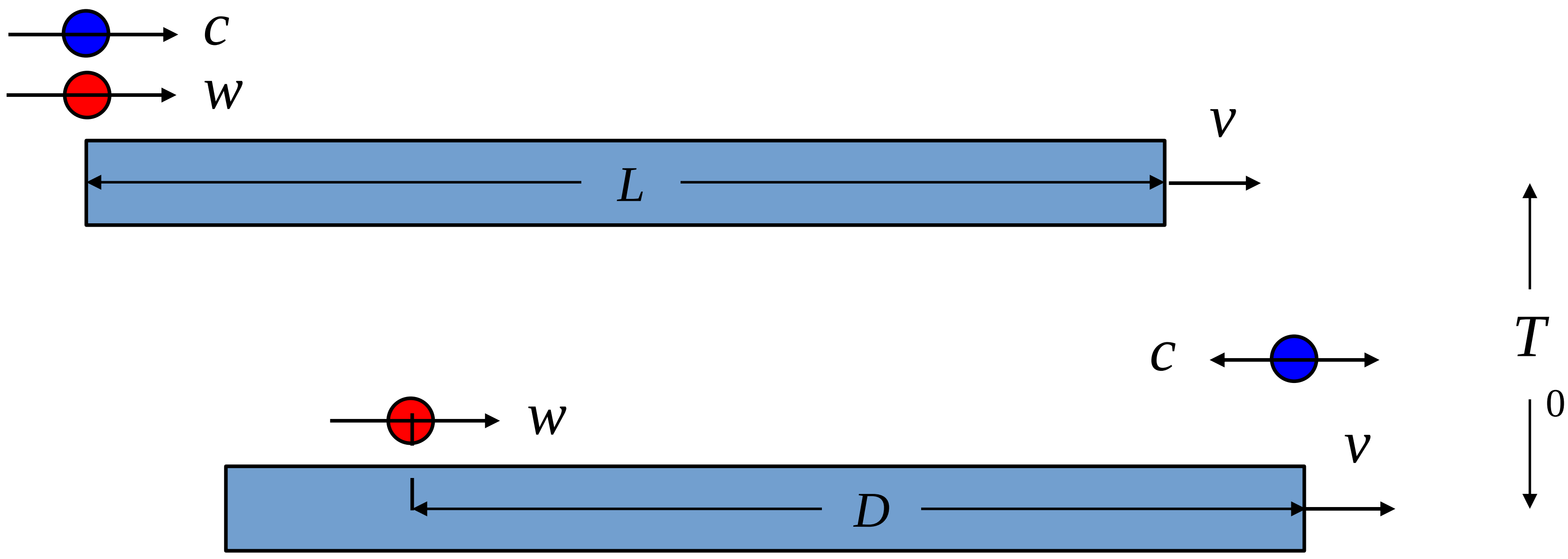
$$\frac{u}{c} = \frac{1 - f}{1 + f}$$

Solve for *f*.

The Second Race



The Second Race



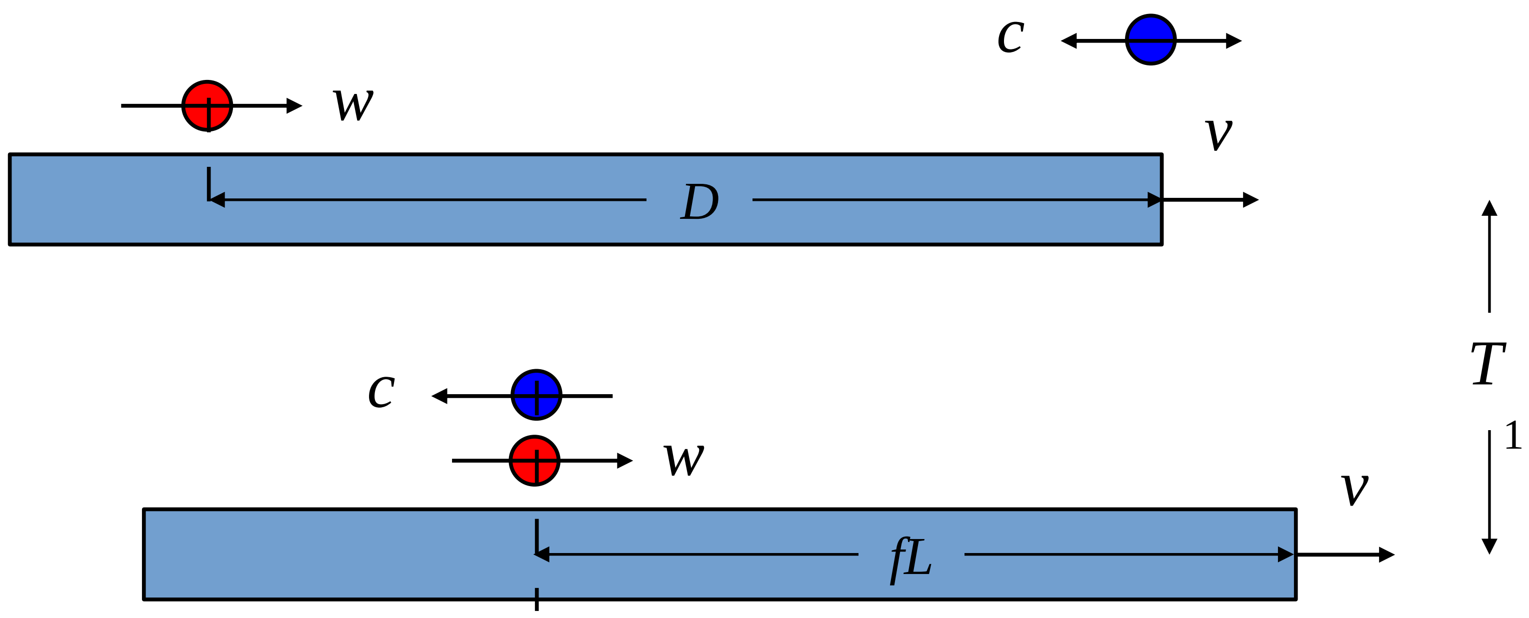
$$D = cT_0 - wT_0$$

$$L = cT_0 - vT_0$$

The Second Race

$$D = cT_1 + wT_1$$

$$fL = cT_1 + vT_1$$



Relativistic Velocity Addition Law

$$D = cT_0 - wT_0 \qquad D = cT_1 + wT_1$$

$$L = cT_0 - vT_0 \qquad fL = cT_1 + vT_1$$

All the times and distances are unknown track-frame values, but since the problematic quantities all drop out in the final result, this should not cause any difficulty.

Now solve for *f* again. Together with the result from the first race, you should conclude

$$\frac{c - w}{c + w} = \left(\frac{c - u}{c + u}\right)\left(\frac{c - v}{c + v}\right)$$

Comments

- **The nonrelativistic velocity addition law is recovered when both *u* and *v* are small compared to the speed of light.**
- **If *u* = *c*, then *w* is required to be *c*, whatever the value of *v* may be. The speed the light remains constant in all inertial frames of reference.**
- **How to combine three (and more) velocities?**
- **How to include negative velocities?**

Additional Comments

- Objects moving at the speed of light behave in some strange (meaning beyond classical mechanics) ways, the behavior of objects moving at speeds comparable to the speed of light can be just as peculiar.
- The peculiarity of motion at the speed of light is just a special case of a more general peculiarity of all motion, which becomes prominent only at extremely high speeds.
- The general peculiarity is an elementary but precise rule.

A Misconception

- **Prior to Einstein in 1905 everyone implicitly believed that there is an absolute meaning to the simultaneity of two events that happen in different places, independent of the frame of reference in which the events are described.**
- **How do we know that they happen at the same time?**

Event

- Here we define an event to be something that happens at a definite place at a definite time. It is the space-time generalization of the purely spatial geometric notion of point.
- An event is an idealization.
  - Instantaneous: zero extension in time
  - Pointlike: zero extension in space
- In reality, its temporal and spatial extensions in a given frame of reference are small compared with other quantities of interest.

Making Marks on the Tracks

- **To be concrete, suppose one event consists of quickly making a tiny mark on the tracks.**
- **This way, we can discuss whether two different events, happening in different places simultaneous in the train frame are also simultaneous in the track frame.**
- **Alternatively, one can also consider simultaneous events in the track frame, ask whether they are simultaneously in the train frame.**

Simultaneity in the Train Frame

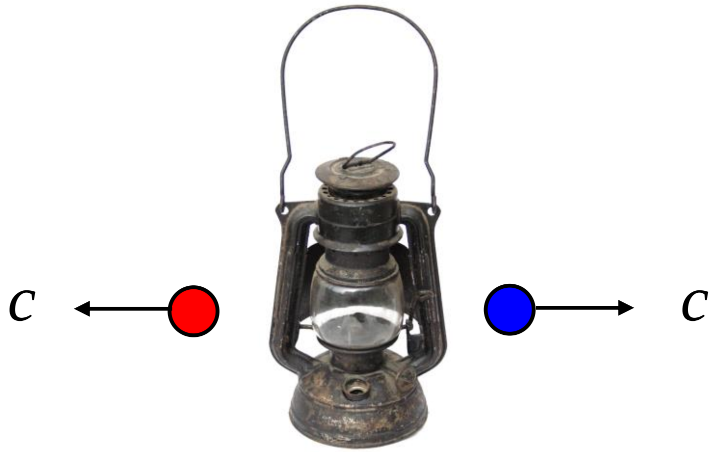
- How does Mary, who uses the train frame, make sure that making a tiny mark on the tracks from the front of the train and doing the same from the rear of the train happen at the same time?
- Make sure both ends of the train have a clock and the two clocks are synchronized. But how do you know they are properly synchronized?

Solution for Mary

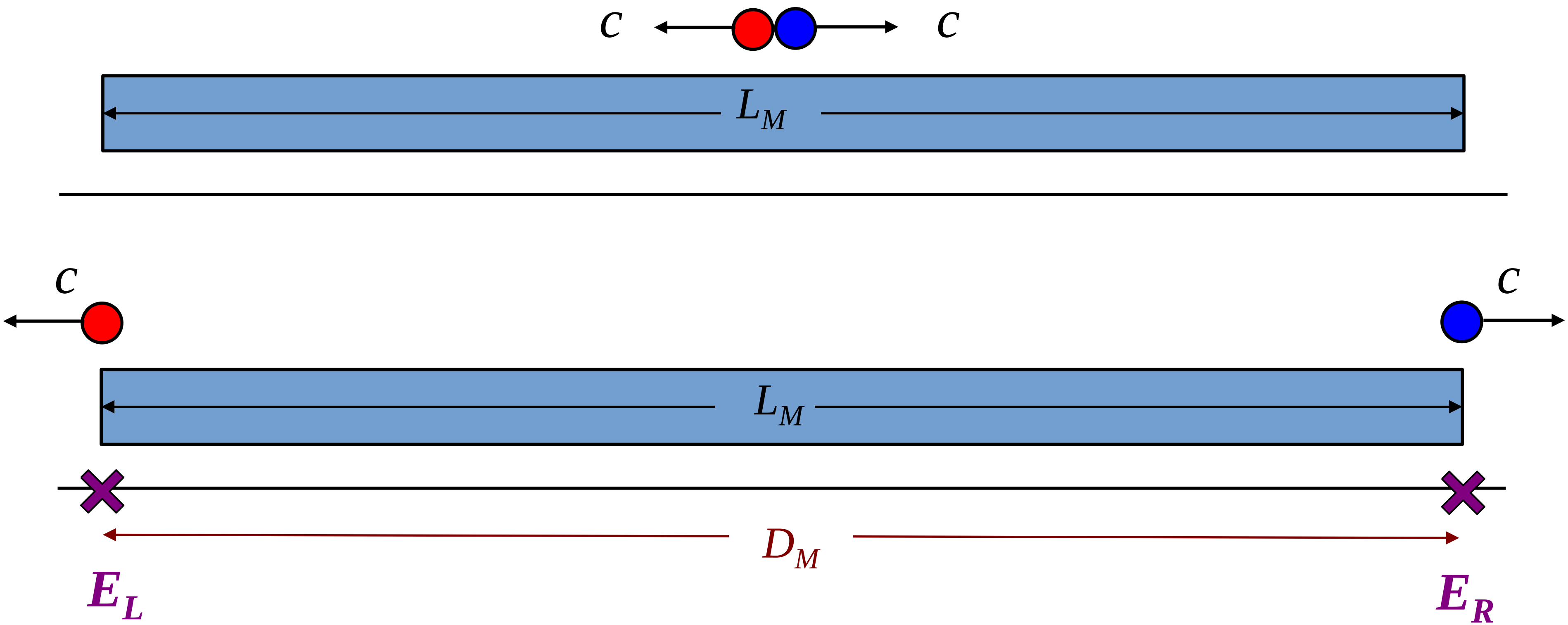
- **Mary brings two clocks together, synchronize them, and then have them carried to the two ends of the train.**
- **But how does she know how fast the two clocks run as they get carried away?**
- **Well, start two clocks at the exact middle. Carry them to the ends in exactly the same way except for their opposite directions. (The same peculiarity will happen to both clocks if there is one.)**

Using Light to Synchronize

- We can replace the clock moving with the use of light signals for the synchronization. After all, a light pulse travels at the speed of light regardless of the direction it is moving in and the frame of reference in which the speed is measured.



Synchronize with Light

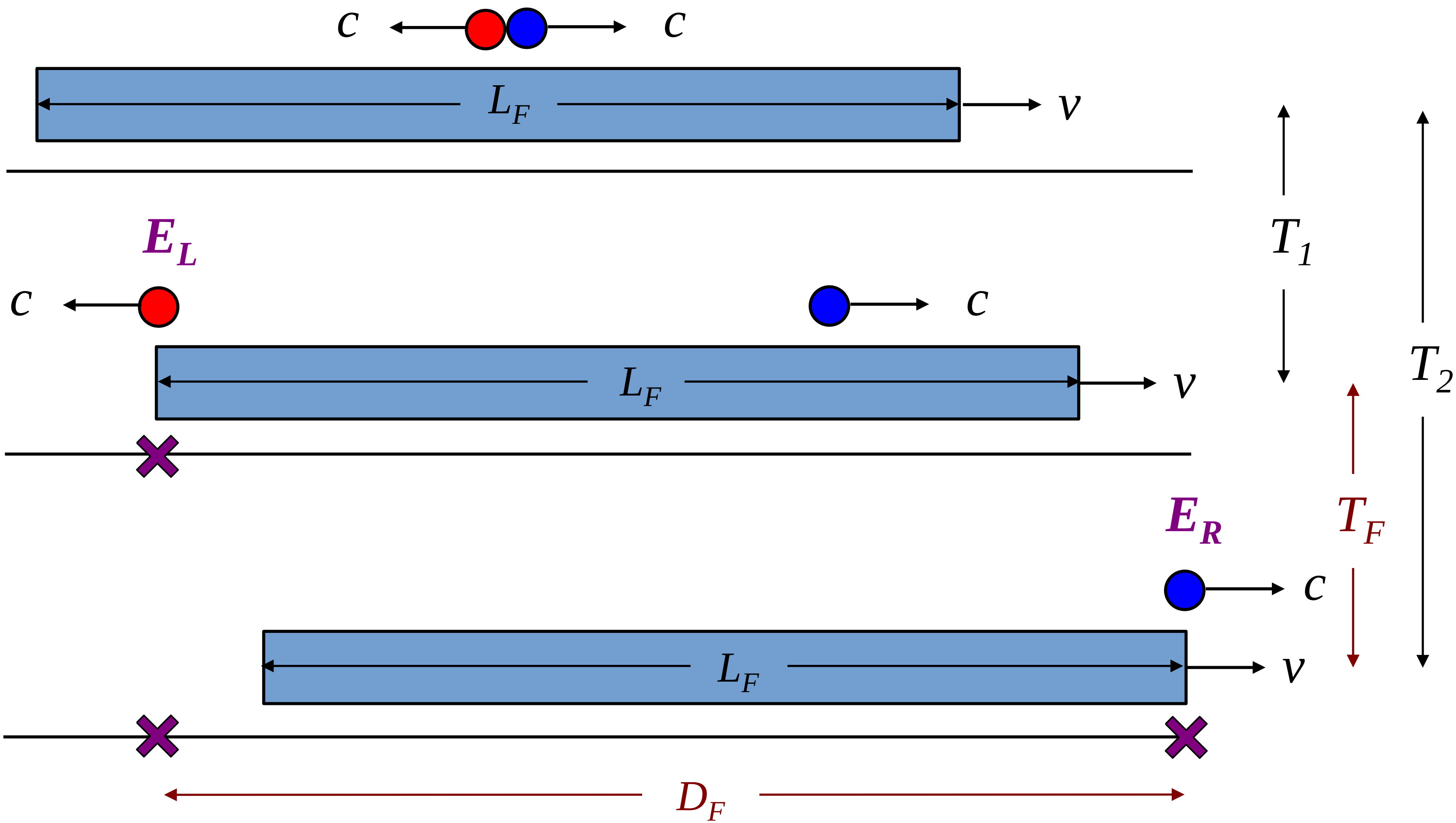


Simultaneous events E_L and E_R in the train frame. Mary also concludes that the distance between the two marks on the tracks is $D_M = L_M$.

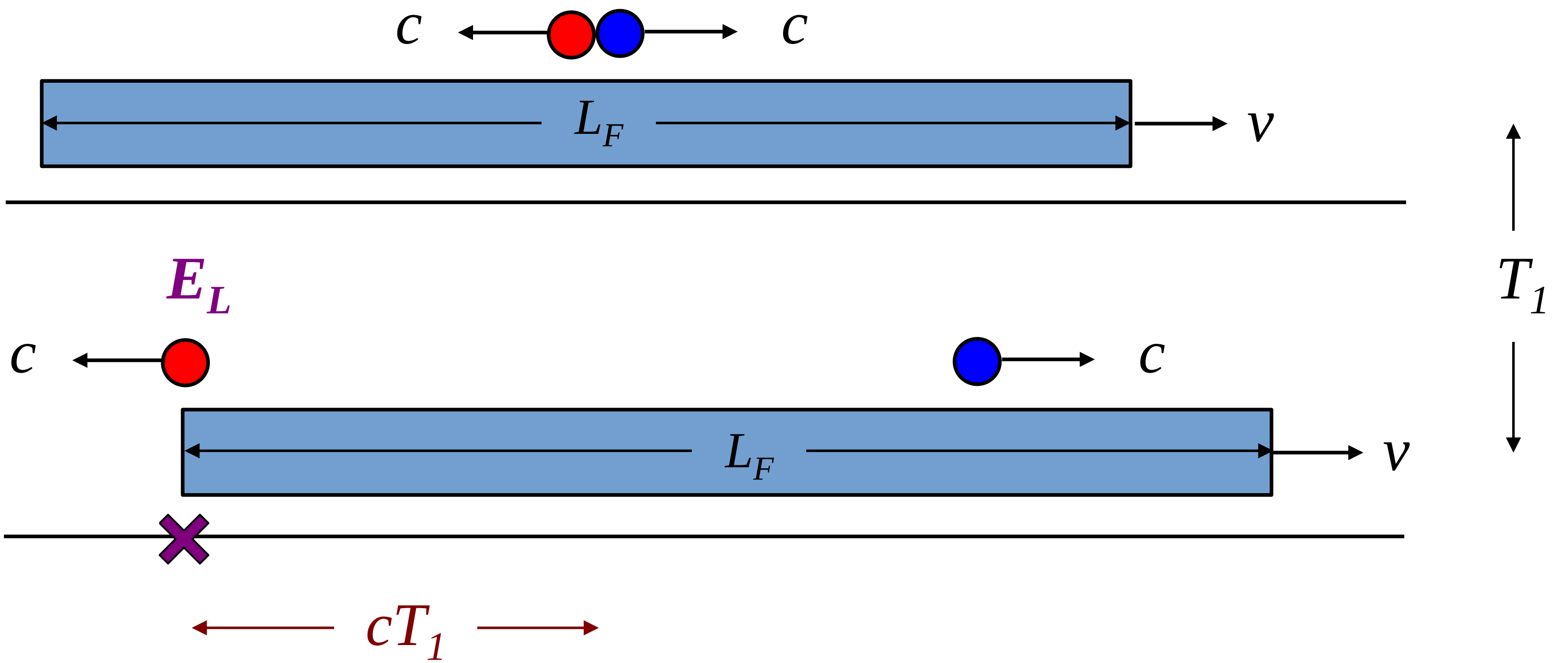
Observation from Frank

- **Would Frank, using the track frame, agree that the two clocks are properly synchronized?**
- **Frank would agree that the lamp is at the center of the train and the speed of light in either direction is *c*.**
- **But Frank must conclude then the light reaches the rear of the train before it reaches the front, as the train is moving forward. Hence the mark in the rear is made before the mark in the front.**

In the Track Frame

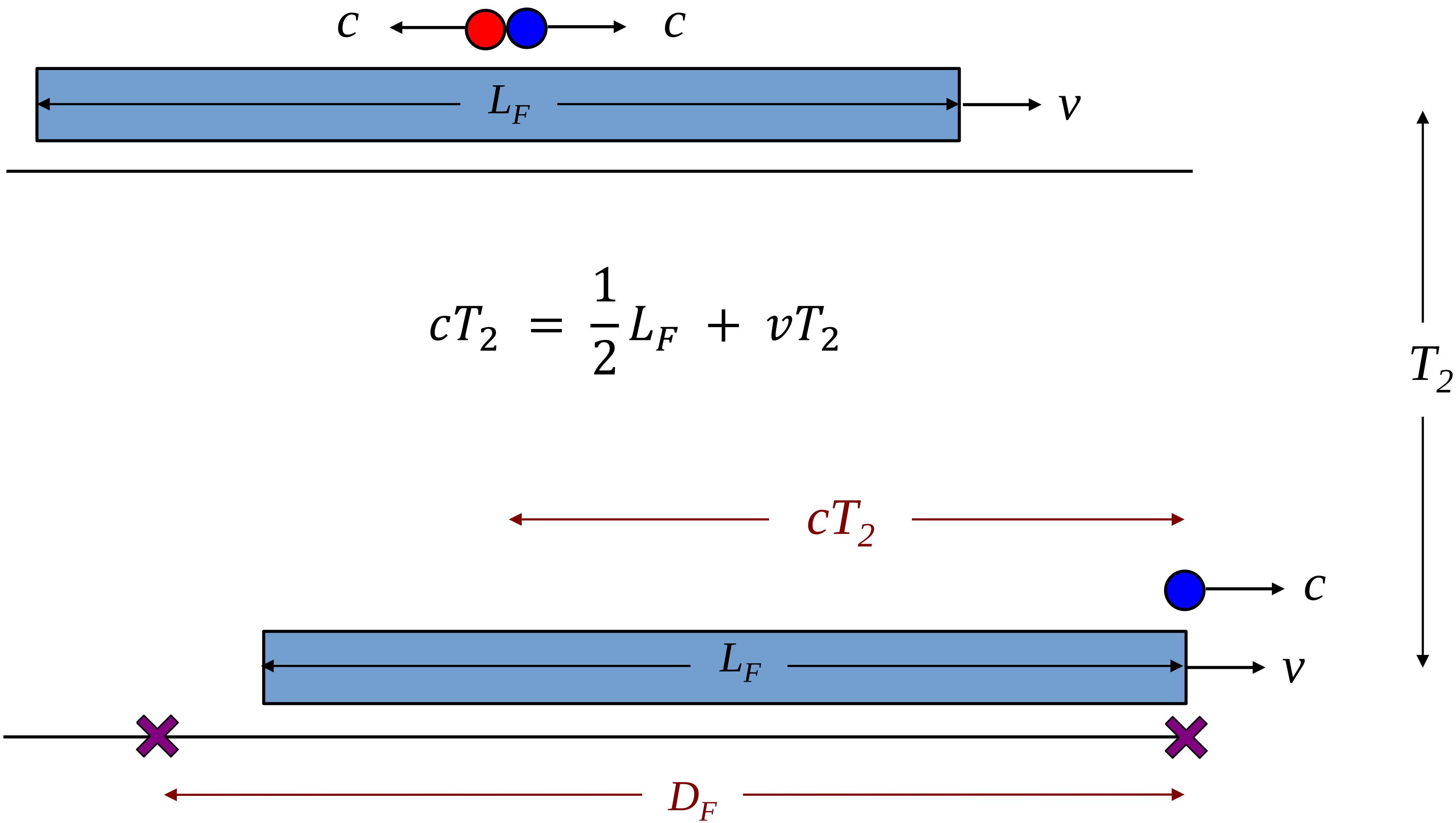


In the Track Frame



$$cT_1 = \frac{1}{2}L_F - vT_1$$

In the Track Frame



$$cT_2 = \frac{1}{2} L_F + vT_2$$

In the Track Frame

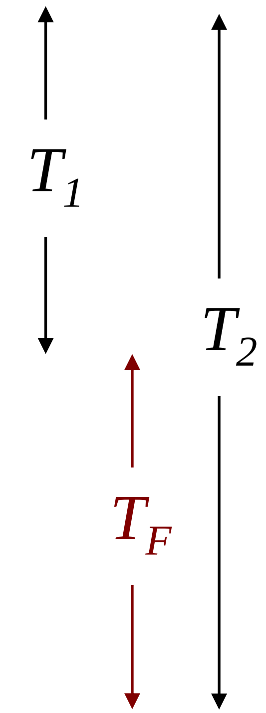
$$cT_1 = \frac{1}{2}L_F - vT_1$$

$$cT_2 = \frac{1}{2}L_F + vT_2$$

$$cT_F = c(T_2 - T_1) = v(T_2 + T_1)$$

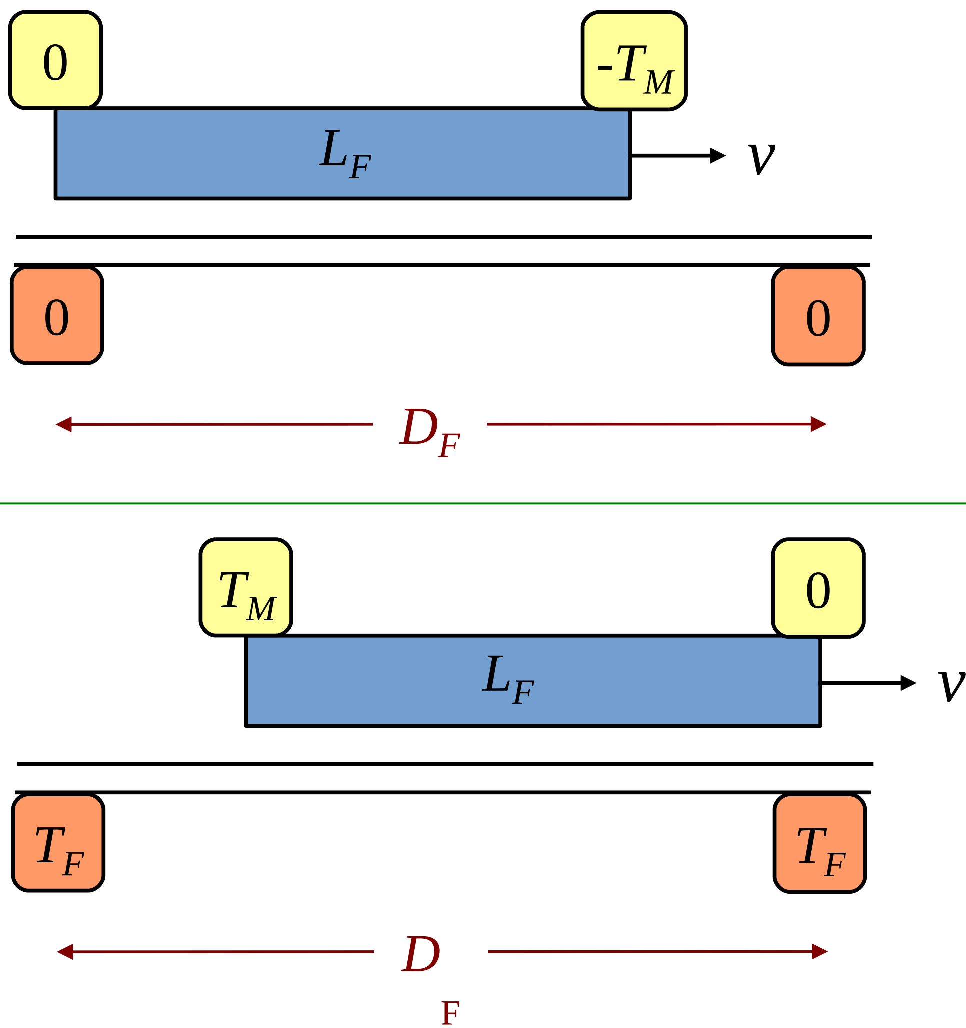
$$D_F = c(T_2 + T_1)$$

$$T_F = D_F\, v/c^2$$

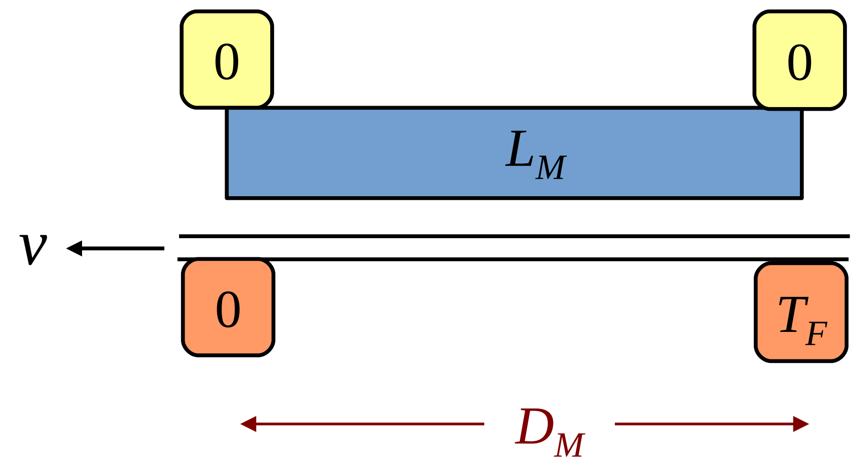


The Tale of Two Frames

Track Frame (Frank's)



Train Frame (Mary's)



$$T_F = D_F\, v/c^2$$

Mary and Frank Found

- **About simultaneous events: If events E_L and E_R are simultaneous in one frame of reference (M), then in a second frame (F) that moves with speed v in the direction pointing from E_R to E_L, the event E_L happens at a time Dv/c^2, in the second frame (F), earlier than the event E_R, where D is the distance between the two events also in the second frame (F).**

How big an effect is this in everyday life?