# 浙江大学 20 20 - 20 21 学年 春夏 学期

## 《 普通物理学 I (H) 》课程期中考试试卷

课程号： R61R0060 ，开课学院： 物理学系

考试试卷：A 卷、B 卷（请在选定项上打√）√

考试形式：闭、开卷（请在选定项上打√），允许带 计算器和字典 入场 √

考试日期： 2021 年 04 月 28 日，考试时间： 110 分钟

**诚信考试，沉着应考，杜绝违纪。**

**考生姓名：______ 学号：______ 所属院系：______**

| 题序 | 一 | 二 | 三 | 四 | 总 分 |
|---|---|---|---|---|---|
| 得分 | | | | | |
| 评卷人 | | | | | |

Instructions:

1. There are 4 problems and all of them are comprehensive questions.
2. Please include the necessary intermediate results, for which you can get partial credits. If you guess the final results, please state clearly and write down why you guess so; otherwise, you may not get any credit.

## Problem 1: Motion of planet with *Discrete Time Interval* method (25 points)

We consider the motion of a planet orbiting around the sun. We suppose that the sun is infinitely massive, so that we can neglect its motion. The gravity force $\vec{F}$ on the planet due to the sun is shown in Fig. 1. With the initial conditions given below, we choose for the planet its plane of motion to be the *x*-*y* plane.

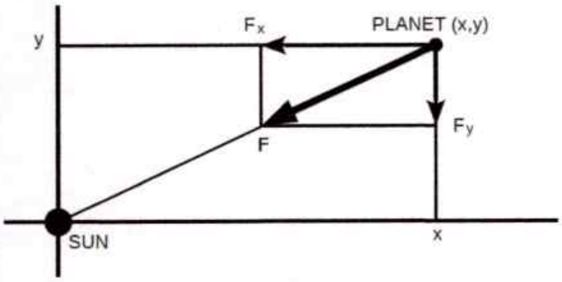



Figure 1

(a) Using Newton's second law and Newton's law of gravitation, write down the equation of motion for the planet in the *x*- and *y*-component forms.

In the following, we set $GM = 1$, where $G$ is the gravitational constant, $M$ is the mass of the sun.

We will now determine its orbit with a *discrete time interval* method. It consists of the following steps:

$$\vec{r}(t+\varepsilon) = \vec{r}(t) + \varepsilon\vec{v}(t+\varepsilon/2),$$
$$\vec{v}(t+\varepsilon/2) = \vec{v}(t-\varepsilon/2) + \varepsilon\vec{a}(t),$$

where $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are the position, velocity and acceleration vectors describing the motion of the planet at time $t$, and where $\varepsilon$ is a constant time interval that we choose to be $\varepsilon = 0.100$ s.

An additional equation for setting up the initial velocity is needed: $\vec{v}(\varepsilon/2) = \vec{v}(0) + (\varepsilon/2)\vec{a}(0)$.

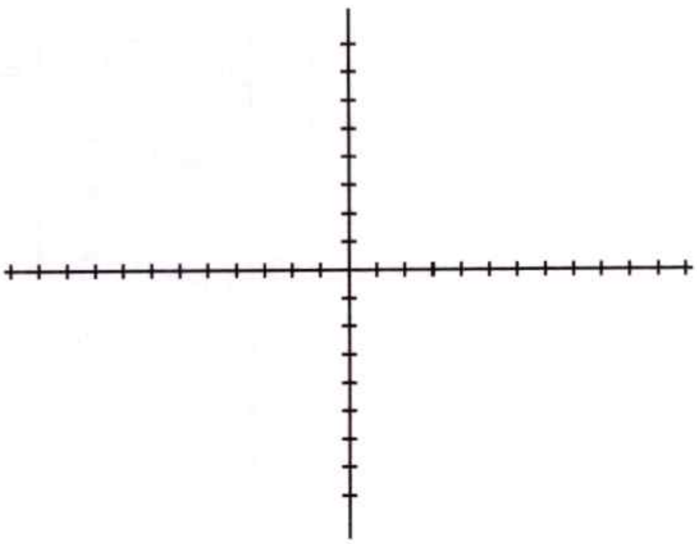
The initial values are given as follows:

$$x(0) = 0.500, y(0) = 0.000;\ \ v_x(0) = 0.000, v_y(0) = +1.630.$$

Table 1 (page 4) tabulates the positions at discrete time step, as well as a complete data of the corresponding velocity and acceleration of the planet.

(b) Following the *discrete time interval* method, fill in the missing data points in shaded boxes (12 boxes). In other words, find the position $\vec{r}(t)$ and all necessary information needed for its computation up to $t = 0.200$ s using the time interval $\varepsilon = 0.100$ s. Write down clearly the steps on how you arrive at your answers.

(c) Using the information available in Table 1, make a plot of the orbit of the planet in the plotting field below.



(d) Determine the period of the orbit. Determine the length of the semimajor axis (one half of the longest axis).

(e) Kepler's third law states that $T^2 = 4\pi R^3/GM$, where $T$ and $R$ are the period and the radius of the planet's orbit, respectively. Demonstrate that your answer in (d) agrees with Kepler's law.

(f) Calculate the total mechanical energy of the planet at three instances of time at your choice. Furthermore, determine the corresponding angular momentum. Provide a physical understanding of your answers.

## Problem 2: Rotation of a rigid body (25 points)

Consider a rod with mass $m$ and length $L$ standing straight on the friction-less ground. When we release the rod, it will fall from the unstable equilibrium position:

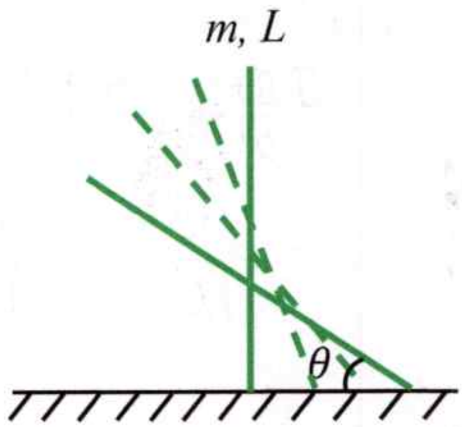



Figure 2

(a) Calculate the angular velocity of the rod, when it has an angle of $\theta$ with respect to the ground as illustrated in Figure 2.

(b) What is the final angular velocity $\omega_1$ of the rod before it hits the ground?

(c) If the same rod is leaning to a frictionless wall with an initial angle of $\alpha$ to the frictionless ground (see Figure 3), what is the final angular velocity $\omega_2$ of the rod before it hits the ground?

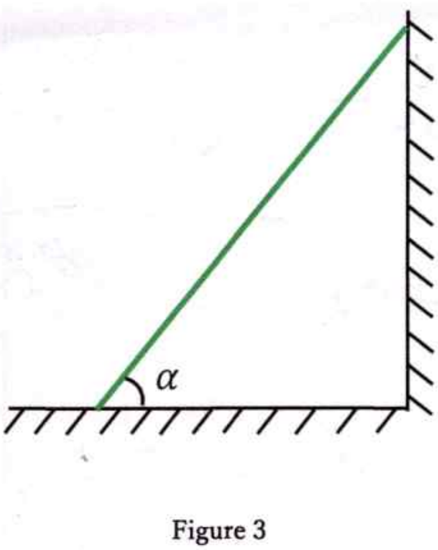



Figure 3

## Problem 3: Waves (20 points)

Two plane waves propagate in a homogeneous elastic medium, one along the $x$ axis and the other along the $y$ axis: $\xi_1 = A\cos(\omega t - kx)$ and $\xi_2 = A\cos(\omega t - ky)$. Suppose the waves are transverse with the same oscillation direction.

(a) Determine the location of the nodes in the $x$-$y$ plane.

(b) Determine the location of the antinodes in the $x$-$y$ plane.

(c) Provide a sketch of the wave motion pattern.

(d) Do you expect a different wave motion pattern in the case of longitudinal waves? (In order to justify your answer, describe the difference between transversal and longitudinal waves and link it to the expected wave motion pattern.)

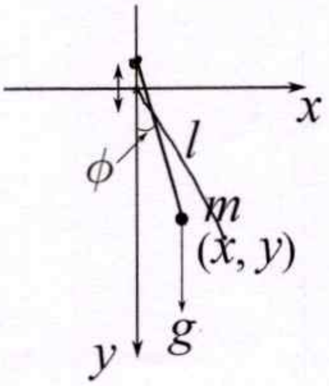
**Trigonometric relations:**

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$
$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

## Problem 4: Kapitza pendulum (30 points)

Consider a rigid pendulum whose pivot is movable in the vertical direction: A point mass $m$ is attached to one end of a massless rigid bar with length $l$, and the other end of the bar is movable in $y$ direction (see the figure). Suppose we oscillate the position $y_0$ of the pivot at the angular frequency $\Omega$ and the amplitude $A$ as

$$y_0(t) = A \sin \Omega t.$$




Here, we consider the case such that the oscillation frequency $\Omega$ is much larger than the natural frequency $\omega_0 = \sqrt{g/l}$ of the pendulum, and the amplitude $A$ is much smaller than the length of the bar $l$ while their product $\Omega l$ is finite.

In this problem, the angle measured from the vertical downward line is denoted by $\phi$ and the gravitational acceleration is $g$. The angle $\phi$ is not necessarily small.

(a) Derive an equation of motion of $\phi$. (Hint: Focus on the component in the tangential direction of the arc [the length of the arc $s$ is given by $s = l\,\phi$].)

(b) Split $\phi(t)$ into a slowly oscillating part $\phi_s(t)$ and a fast oscillating part $\phi_f(t)$ as

$$\phi(t) = \phi_s(t) + \phi_f(t).$$

Assuming that $\phi_s(t)$ is almost constant in time and $|\phi_f(t)| \ll 1$, determine $\phi_f(t)$ up to the leading order based on the equation of motion derived in (a).

(Hint: Keep in mind that $|\phi_f(t)| \ll 1$, $\Omega$ is very large while $\Omega l$ is finite.)

(c) Derive an equation of motion of the slowly changing variable $\phi_s(t)$ by averaging over the rapid oscillation at frequency $\Omega$. This derivation can be done in the following way: Substitute $\phi_f(t)$ obtained in (b) into the equation of motion of $\phi(t)$ obtained in (a). Then, take a time average over the period of the fast oscillation at frequency $\Omega$.

[Hint: The time average of a function $f(t)$ over the period $T$ is given by $T^{-1}\int_0^T f(t)\,dt$.]

(d) From the equation of motion of $\phi_s(t)$ derived in (c), determine the effective potential $V_{eff}(\phi_s)$ defined by

$$ml\ddot{\phi}_s = -\frac{dV_{eff}}{d(l\phi_s)}.$$

(e) Derive the condition in order that the inverted case ($\phi_s = \pi$) becomes a stable equilibrium.