General Physics I

Lecture 22: More Twists on Space and Time

Outline

- The equivalence principle
- The consequences of general relativity
  - Altering of time scale
  - Bending of light
  - Gravitational wave
- Application: Global navigation satellite systems

The General Theory of Relativity

- **Mass has two seemingly different properties: a gravitational attraction for other masses and an inertial property that resists acceleration.**

Gravitational property	$F_g = m_g g$
Inertial property	$\Sigma F = m_i a$

It appears that gravitational mass and inertial mass may indeed be exactly proportional. (The value for the gravitational constant *G* was chosen to make the magnitudes of the two mass numerically equal.) But why?

Einstein’s Breakthrough

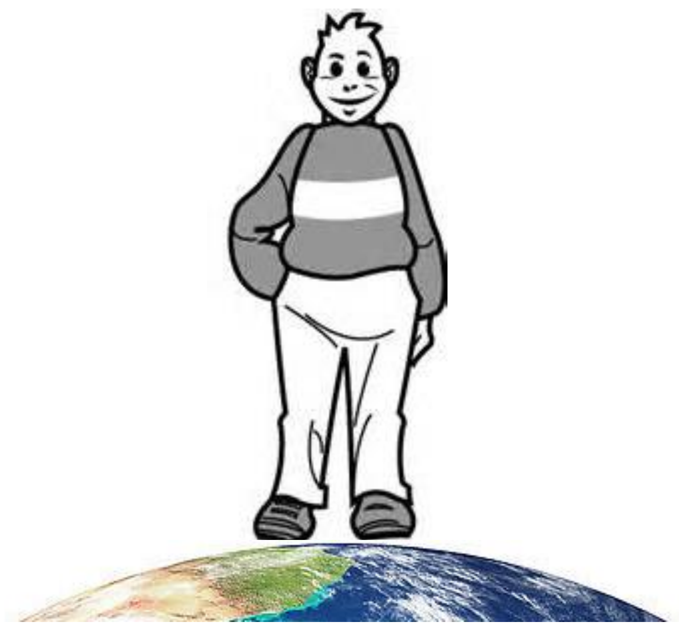
- “The breakthrough came suddenly one day. I was sitting in my patent office in Bern. Suddenly a thought struck me: If a man falls freely, he would not feel his weight. I was taken aback. This simple thought experiment made a deep impression on me. This led me to the theory of gravity. I continued my thought: A falling man is accelerated. Then what he feels and judges is happening in the accelerated frame of reference. I decided to extend the theory of relativity to the reference frame with acceleration. I felt that in doing so I could solve the problem of gravity at the same time. A falling man does not feel his weight because in his reference frame there is a new gravitational field which cancels the gravitational field due to the Earth. In the accelerated frame of reference, we need a new gravitational field.”

Physics Today 35, 8, 45 (1982)

The Equivalence Principle

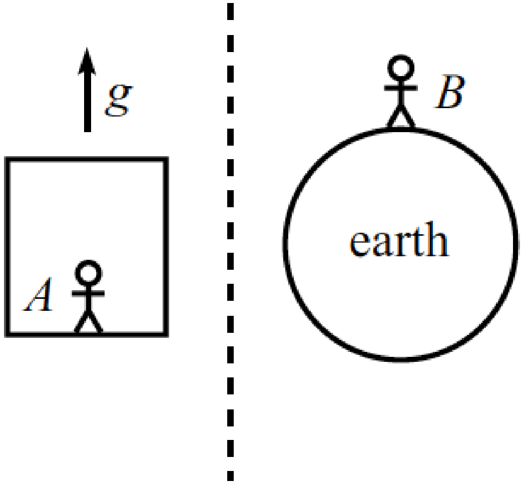
- Einstein proposed that no experiment, mechanical or otherwise, could distinguish between the two cases.

Gravity = Acceleration



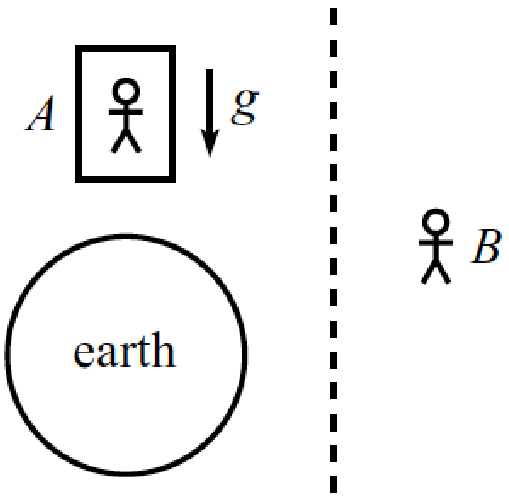
The Postulates

- The two postulates of Einstein's general theory of relativity are
  - **(The principle of relativity)** All the laws of nature have the same form for observers in any frame of reference, whether accelerated or not.
  - **(The principle of equivalence)**
    In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects.



Gravitational Mass = Inertial Mass

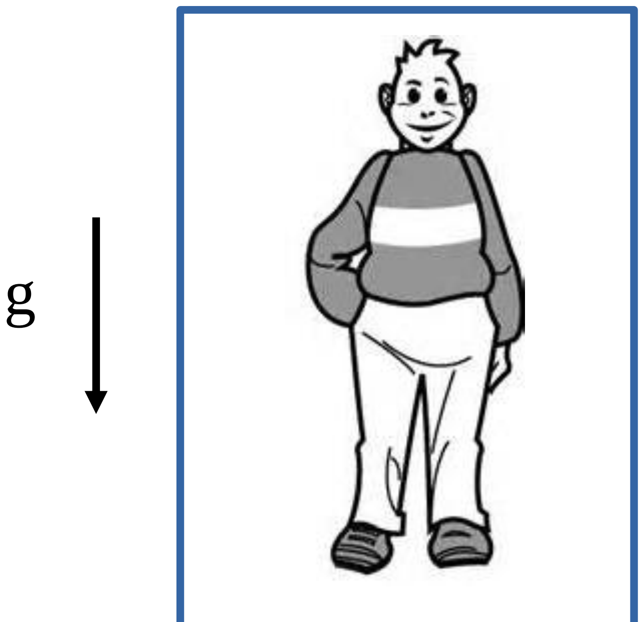
- **Two different masses that start at rest near B will stay right where they are as they float freely in space.**
- **Two different masses that start at rest near A will stay next to each other if and only if their accelerations are equal.**



$$a = (m_g/m_i)g$$

Free Fall

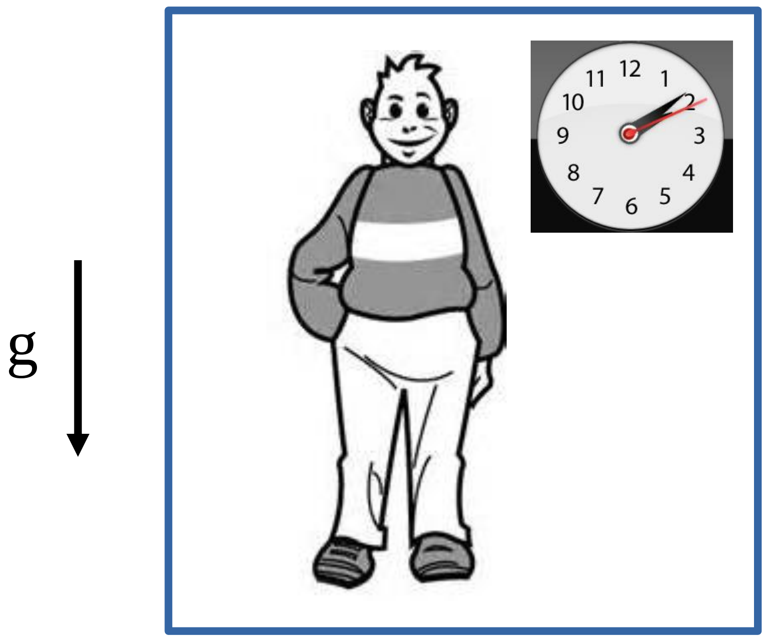
- **In a free-falling elevator, gravity is cancelled by the inertial force.**



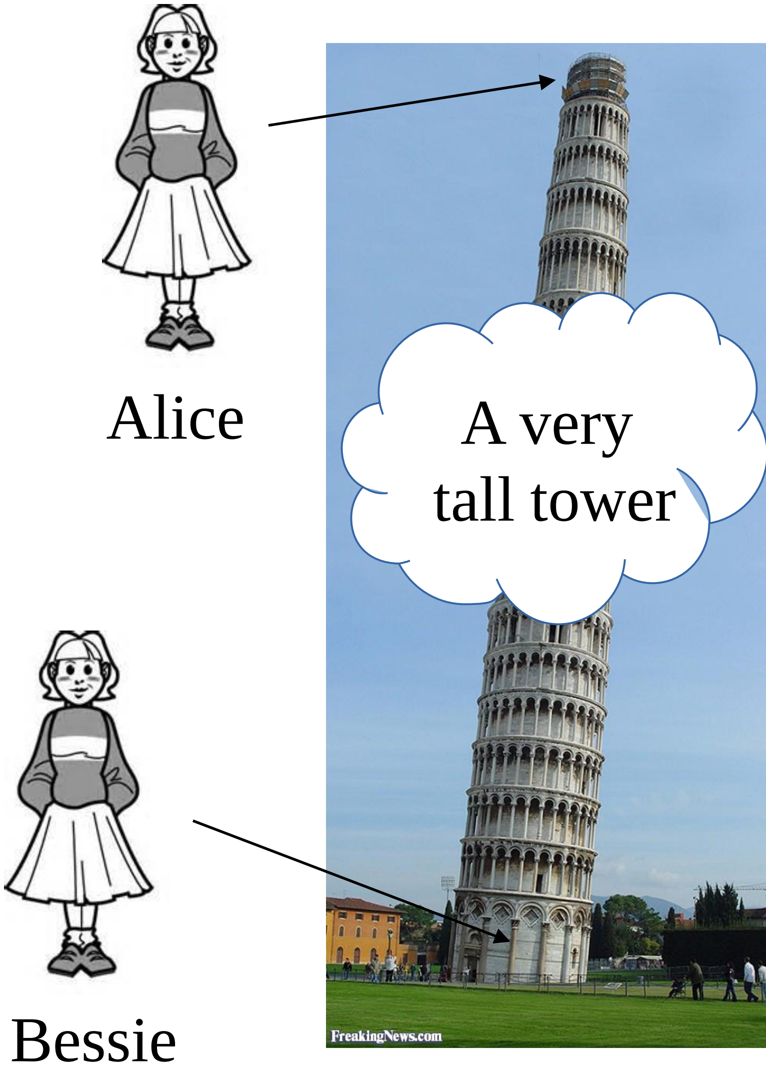
An inertial frame of reference.

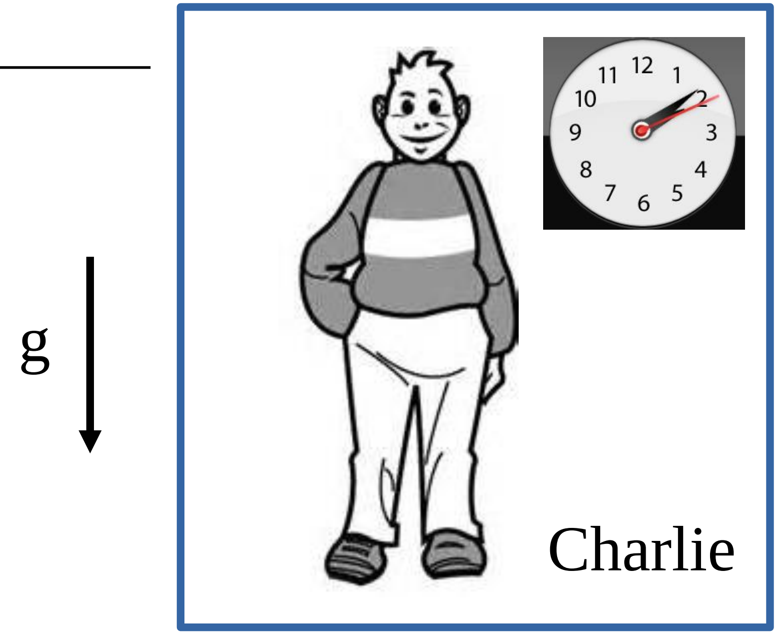
Clock in a Falling Elevator

- In a free-falling elevator, the special theory of relativity is valid. Time elapses with constant intervals.



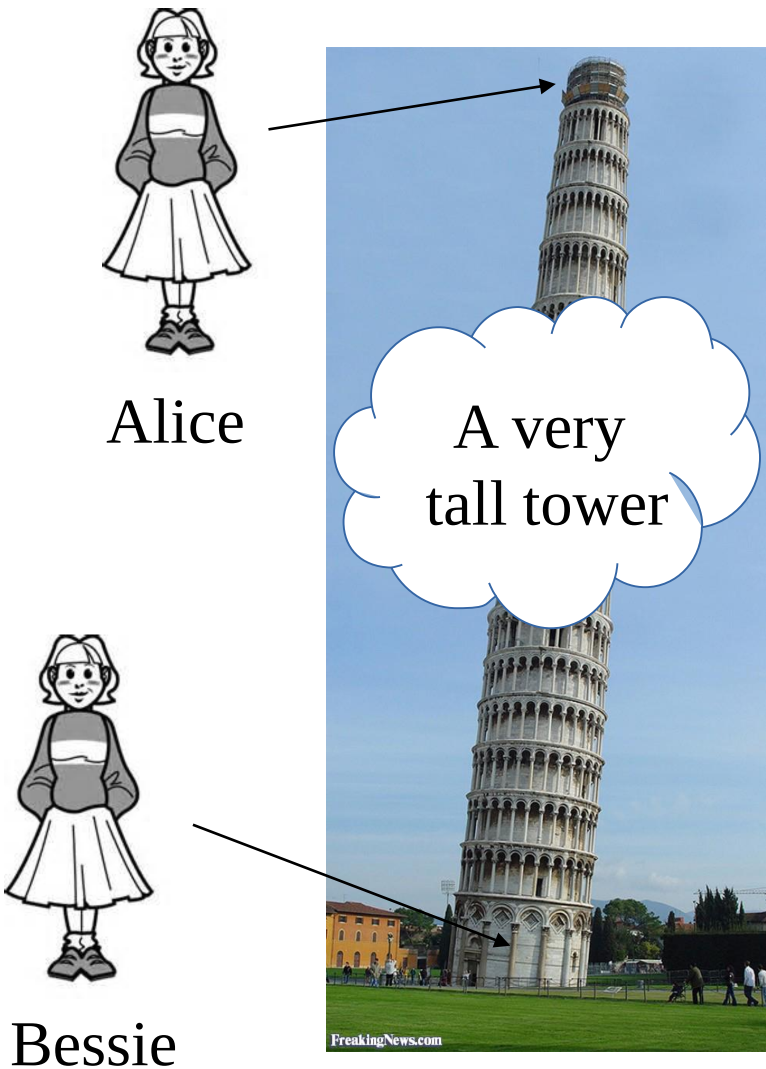
Synchronize with Alice



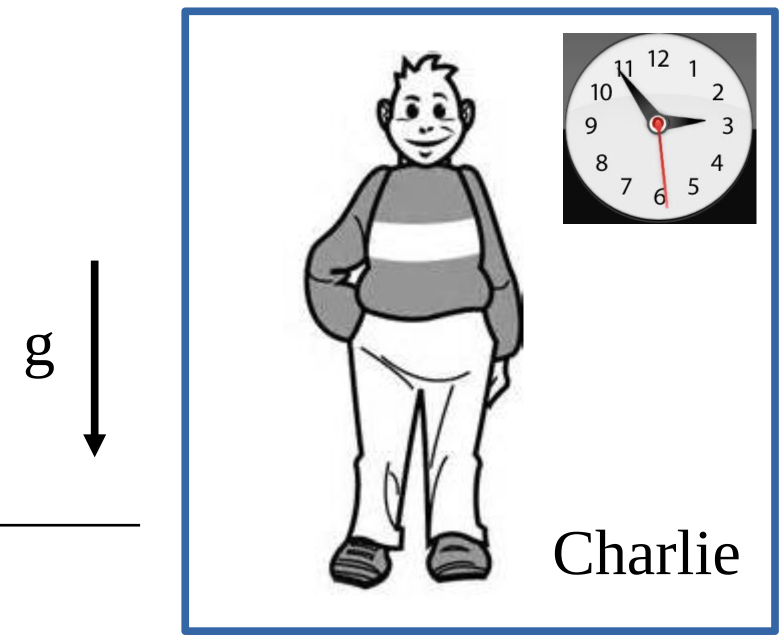


Charlie starts at the same height with Alice. Their clocks are adjusted to tick at the same speed.

Compare with Bessie



When Charlie meets Bessie, she is moving up relatively with a finite speed. So Bessie's clock is slower than Charlie's.



Clocks Upstairs and Downstairs

- By comparison, Alice's clock is faster than Bessie's clock.
- When Charlie is at the same height as Bessie

$$\frac{1}{2}mv^2 = \Delta U = mgh$$

- Bessie's clock is slower by a factor

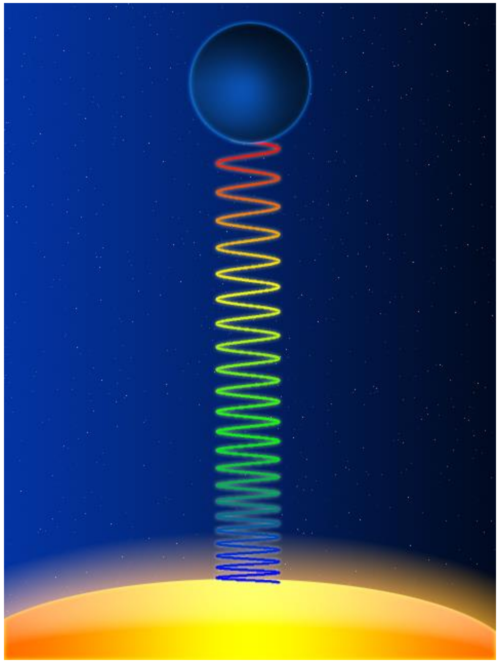
$$s = \sqrt{1 - v^2/c^2} \approx 1 - \frac{\Delta U}{mc^2}$$

If you live in a second-story apartment, and the thought of the universe stealing a split second of your life keeps you awake at night, science offers a solution. Just run in circles to keep your watch in synch with clocks on ground floor.

Clock Runs Slow in a G-Field

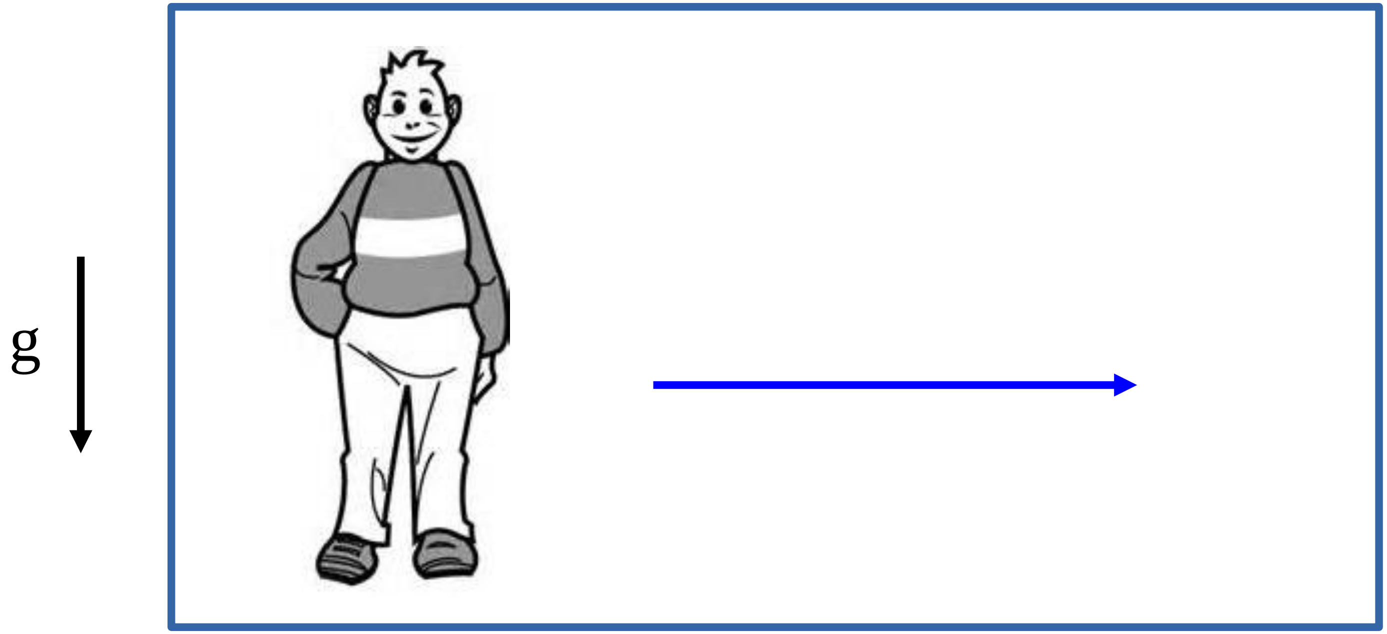
- The interesting effect predicted by the general theory is, therefore, that time scales are altered by gravity. A clock in the presence of gravity runs more slowly than one located where gravity is negligible.

- **[Gravitational Redshift]** Consequently, the frequencies of radiation emitted by atoms in the presence of a strong gravitational field are red-shifted to lower frequencies when compared with the same emissions in the presence of a weak field.



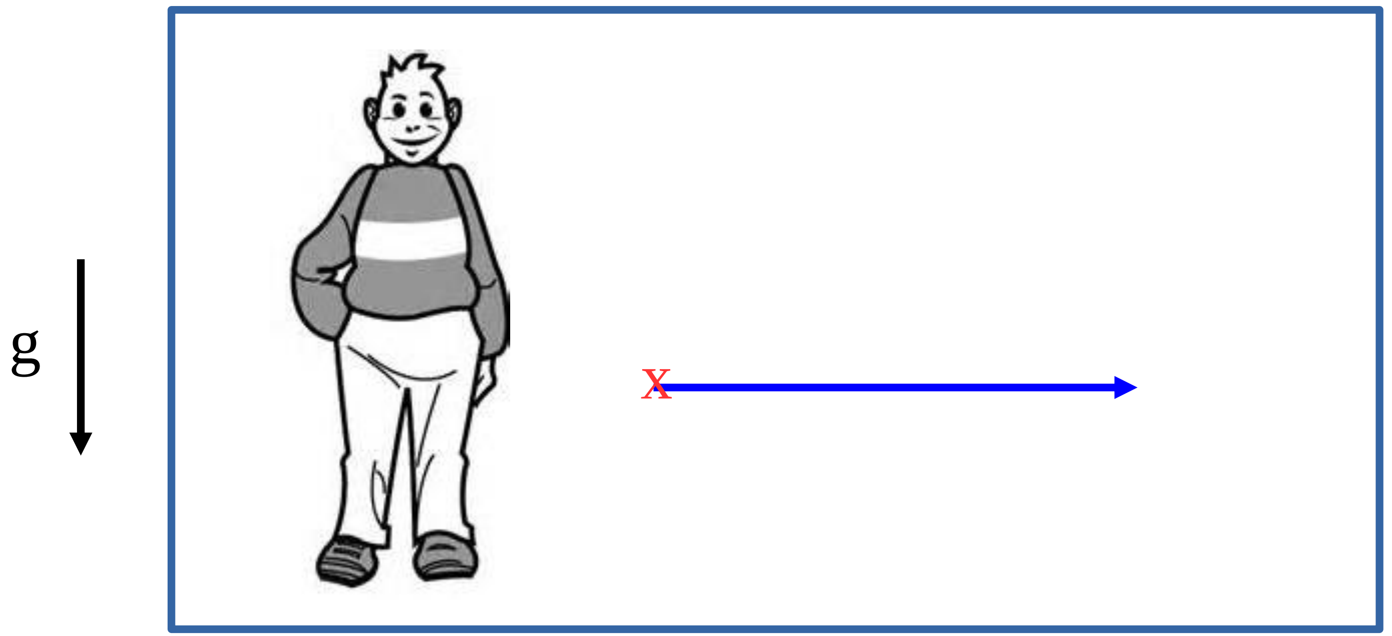
Light in a Falling Elevator

- In a free-falling elevator, according to the special theory of relativity, light propagates along a straight line in the speed of light.



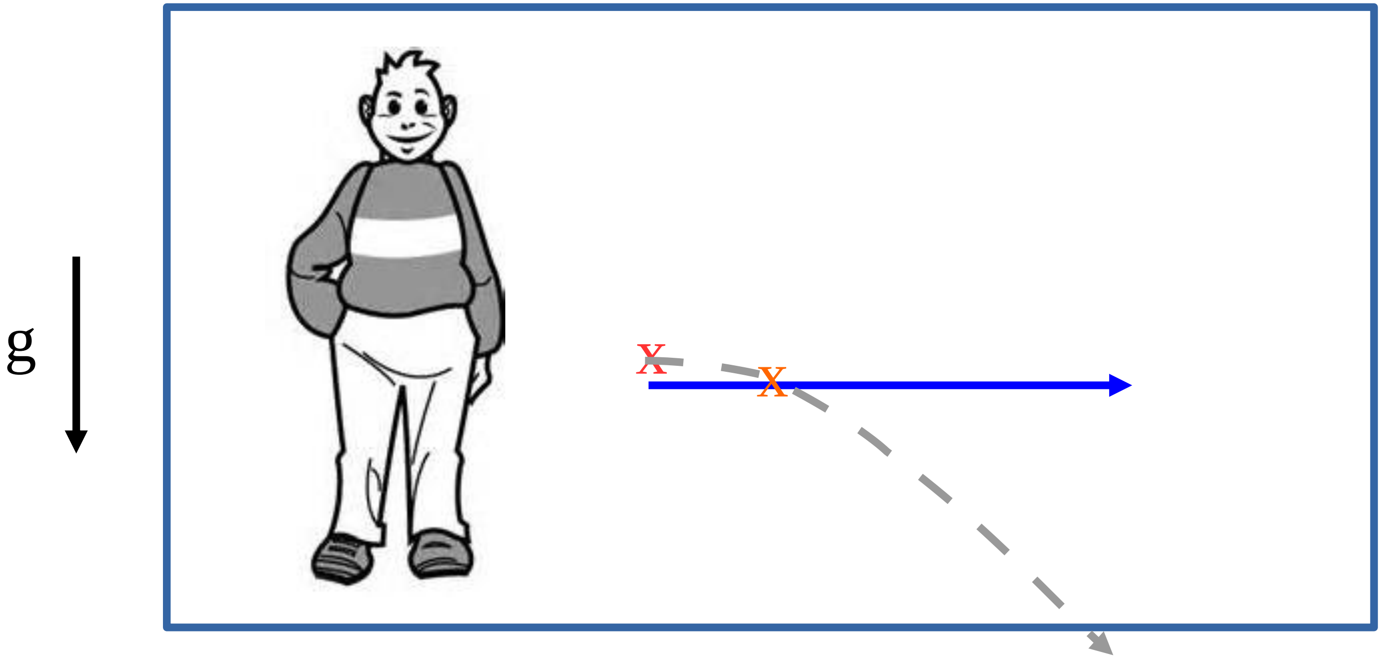
Light in a Falling Elevator

- To a outside observer (t = 0)



Light in a Falling Elevator

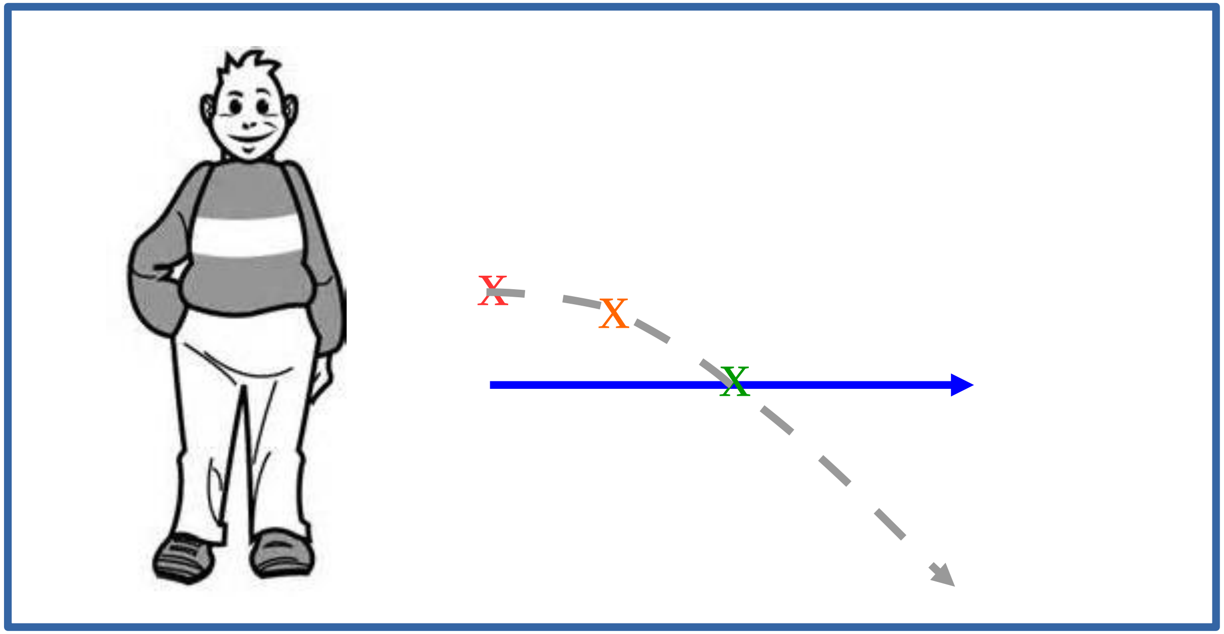
- To a outside observer (t = 1)



Light in a Falling Elevator

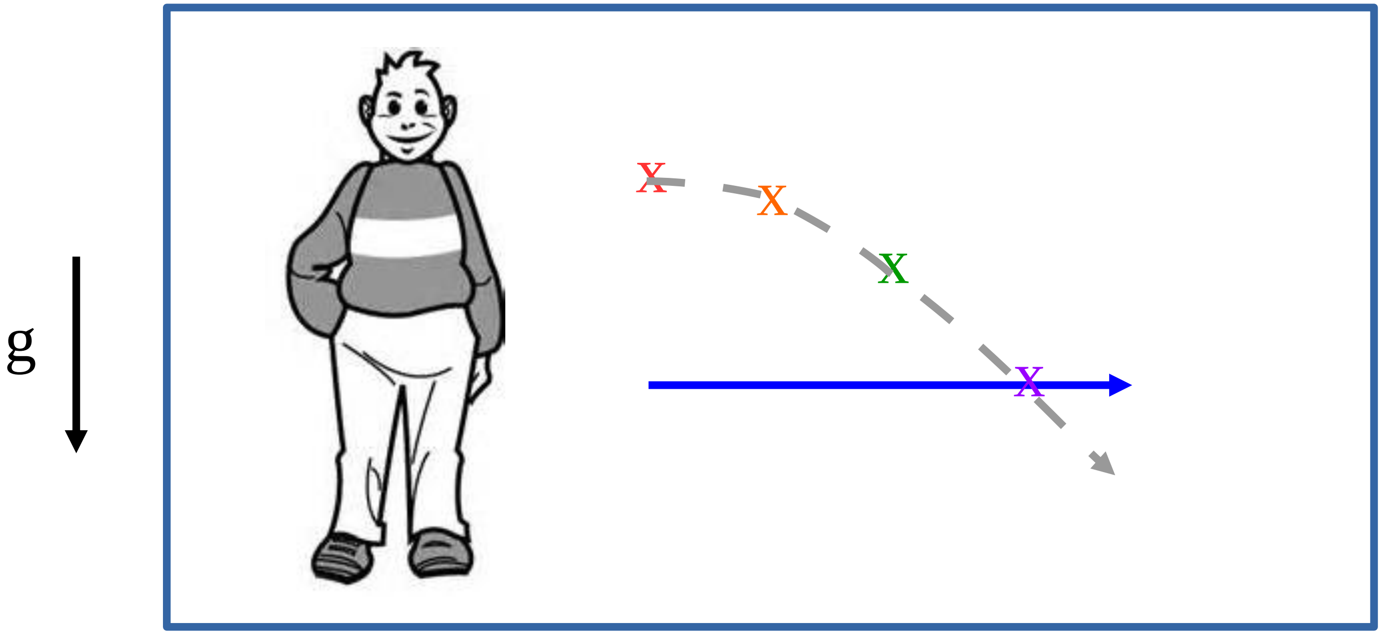
- To a outside observer (t = 2)

g



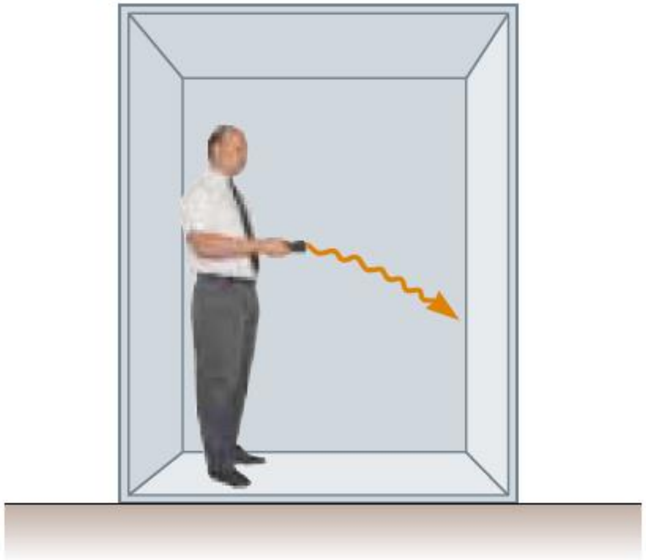
Light in a Falling Elevator

- To a outside observer (t = 3)
  - A beam of light is bent downward by the gravitational field.

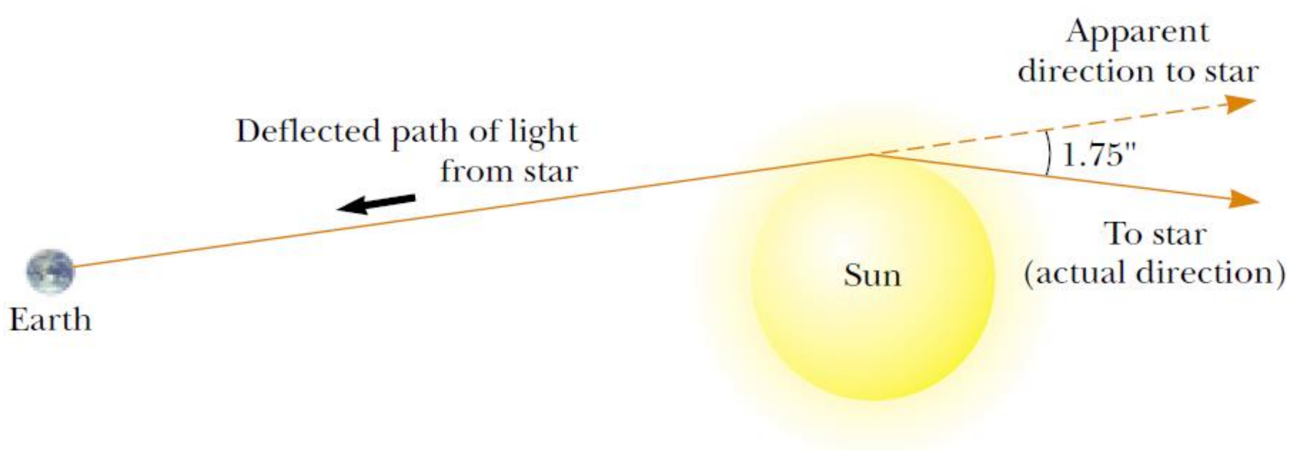


An Alternative View

- Suppose that a light pulse is sent horizontally across the elevator, while the elevator accelerates upward. This causes the light to arrive at a location lower on the wall than the spot it would have hit if the elevator were not accelerating. Thus, in the frame of the elevator, the trajectory of the light pulse bends downward as the elevator accelerates upward to meet it.
- Because the accelerating elevator cannot be distinguished from a nonaccelerating one located in a gravitational field, Einstein proposed that a beam of light should also be bent downward by a gravitational field.



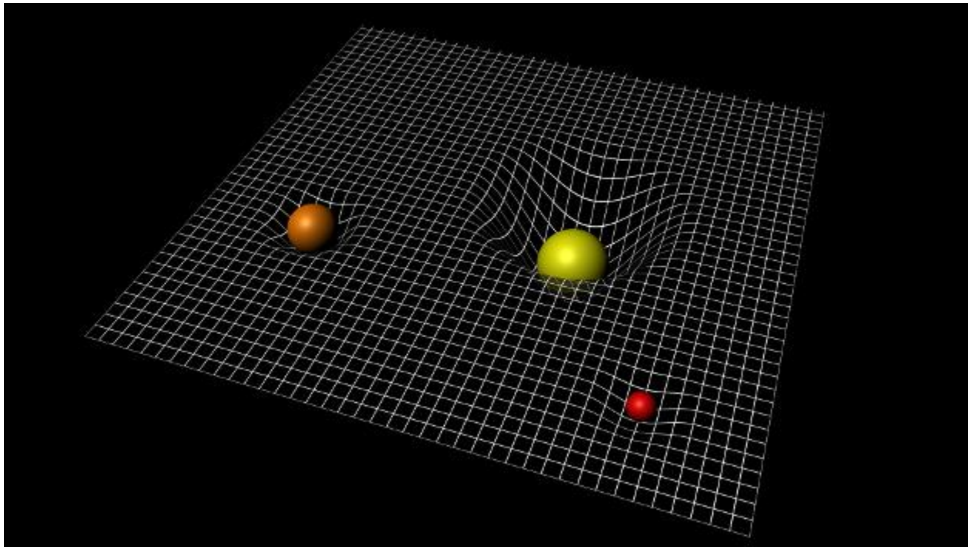
Experimental Verification



In 1979, John Wheeler summarized Einstein’s general theory of relativity in a single sentence: “Space tells matter how to move and matter tells space how to curve.”

(Update 12 February 2016: high-frequency gravitational waves (i.e. ripples in the spacetime continuum), emitted by a pair of merging black holes, were directly detected for the first time with the Advanced Laser Interferometer Gravitational-Wave Observatory.)

Spacetime Curvature

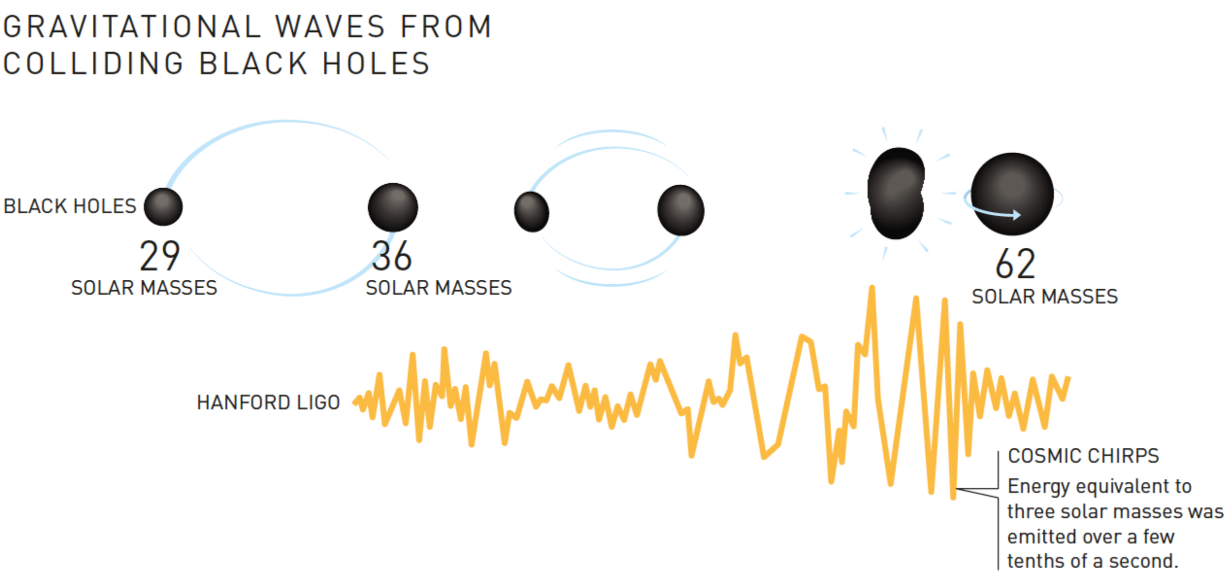


In general relativity, spacetime is not 'flat' but is curved by the presence of massive bodies. The distortion caused by each sphere is proportional to its mass.

The curvature of spacetime influences the motion of massive bodies within it; in turn, as massive bodies move in spacetime, the curvature changes and the geometry of spacetime is in constant evolution. Gravity then provides a description of the dynamic interaction between matter and spacetime.

Gravitational Waves

- Einstein (1916): Gravitational waves are ripples in the metric of spacetime that propagate at the speed of light.
- On February 11, 2016, the Advanced LIGO team announced that they had directly detected gravitational waves from a pair of black holes merging.

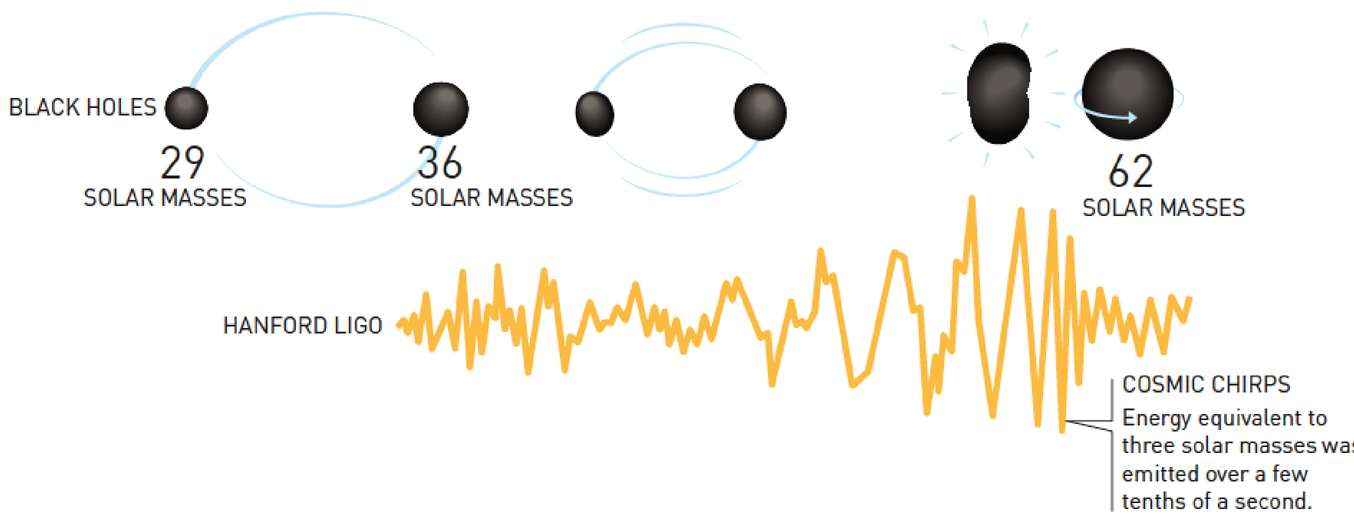


Nobel Prize in Physics 2017

"for decisive contributions to the LIGO detector and the observation of gravitational waves".

Spacetime Vibrates

…. With every cycle, they swept spacetime into a spiral, a spacetime disturbance which propagated further and further out into space in the form of gravitational waves. The waves carried away energy, causing the black holes to move closer to one another. The closer their spiraling motion brought them, the faster the black holes rotated and the more energy was sent out in an accelerating dance that continued for many millions of years.



The Fateful Moment

It was completely dark. But not completely still. Tremors from two black holes colliding shook all of spacetime. Like ripples from a pebble thrown into water, gravitational waves from the impact spread through the cosmos. It took time for them to reach us. Despite moving at the speed of light, the fastest possible, it took more than a thousand million years for these waves to arrive here on Earth. On 14 September 2015, at 11.51 CET, a gentle wobble in the light pattern at America's twin LIGO laboratories revealed the drama that unfolded long ago and far away, 1.3 billion lightyears from Earth.

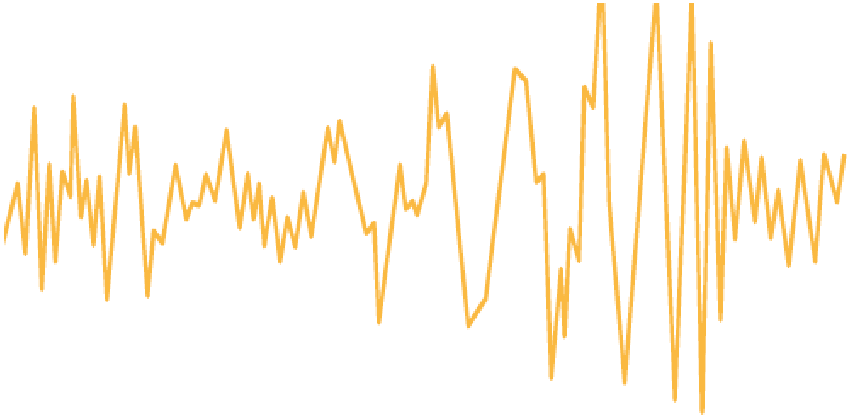


Figure 1. The first gravitational wave ever detected.

Satellite Navigation

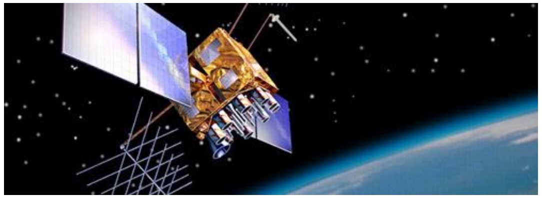
- **Motivation and the Beginning**

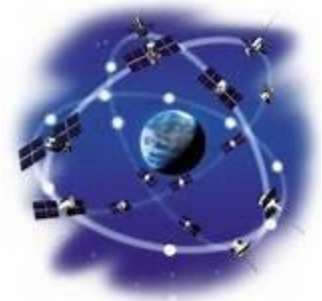
- **The USSR launched the first satellite Sputnik in 1957.**
- **Two American physicists, William Guier and George Weiffenbach, at Johns Hopkins's Applied Physics Laboratory (APL), decided to monitor Sputnik's radio transmissions. Within hours they realized that, because of the Doppler effect, they could pinpoint where the satellite was along its orbit.**
- **The next spring, Frank McClure, the deputy director of the APL, asked Guier and Weiffenbach to investigate the problem— pinpointing the user's location given that of the satellite.**

GPS: Some Milestones

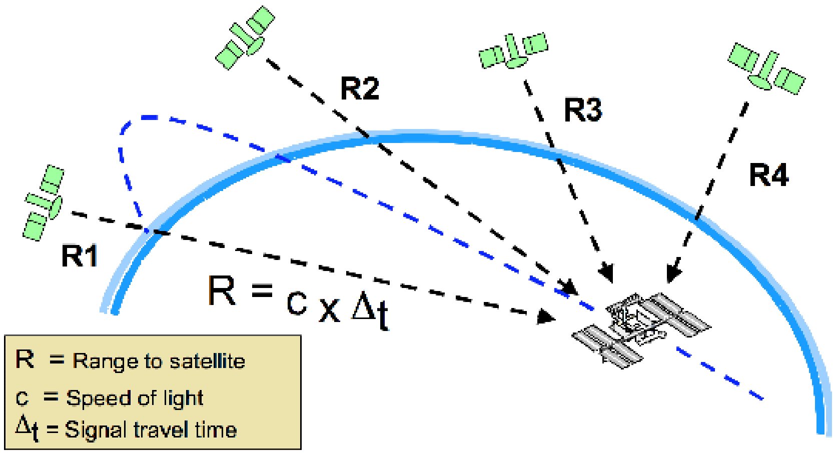
US Global Positioning System: http://www.gps.gov/systems/gps/

- **The Gulf War from 1990 to 1991 was the first conflict in which the military widely used GPS.**
- **In 1991, a project to create a miniature GPS receiver successfully ended, replacing the previous 50 pound military receivers with a 2.75 pound handheld receiver.**
- **November 2004, Qualcomm announced successful tests of assisted GPS for mobile phones.**

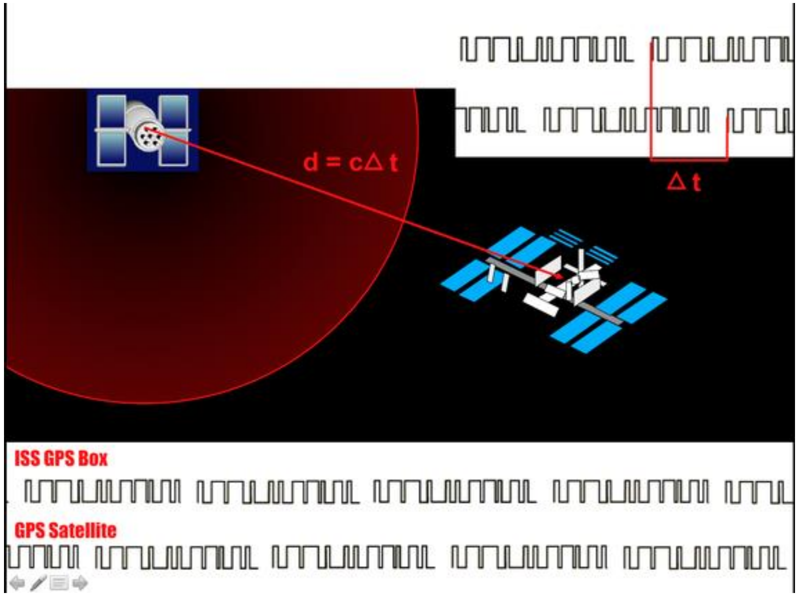


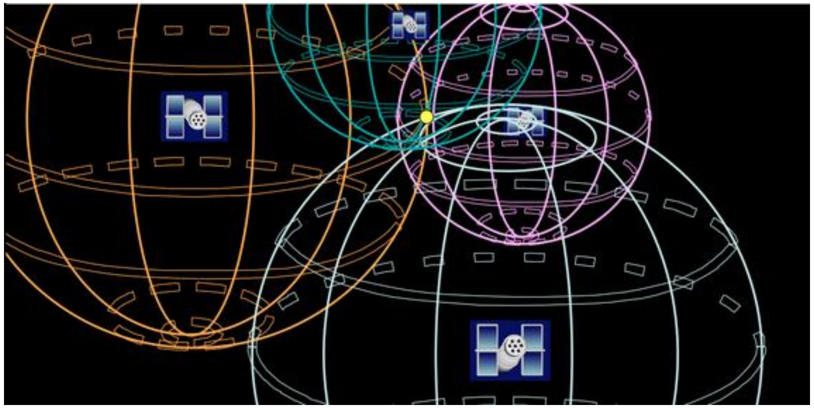


How to Locate a Position?



$$(x_{iss}-x_1)^2+(y_{iss}-y_1)^2+(z_{iss}-z_1)^2=(r_1-\varepsilon_{clock})^2$$
$$(x_{iss}-x_2)^2+(y_{iss}-y_2)^2+(z_{iss}-z_2)^2=(r_2-\varepsilon_{clock})^2$$
$$(x_{iss}-x_3)^2+(y_{iss}-y_3)^2+(z_{iss}-z_3)^2=(r_3-\varepsilon_{clock})^2$$
$$(x_{iss}-x_4)^2+(y_{iss}-y_4)^2+(z_{iss}-z_4)^2=(r_4-\varepsilon_{clock})^2$$





http://www.beidou.gov.cn/

- **According to the China National Space Administration, the development of the system would be carried out in three steps:**
  - **2000–2003: experimental BeiDou navigation system consisting of 3 satellites**
  - **By 2012: regional BeiDou navigation system covering China and neighboring regions**
  - **By 2020: global BeiDou navigation system**

On March 30, 2015, the first new-generation BeiDou Navigation satellite (and the 17th overall) was successfully set to orbit by a Long March 3C rocket.

http://www.beidou.gov.cn/

Why do we need the corrections from the theory of relativity in GPS navigation?

www.beidou.gov.cn/2012/01/05/20120105b32bf90e15e94e26a119fd799788104e.html

狭义相对论认为高速移动物体的时间流逝得比静止的要慢。每个GPS卫星时速为1.4万千米，根据狭义相对论，它的星载原子钟每天要比地球上的钟慢7微秒。另一方面，广义相对论认为引力对时间施加的影响更大，GPS卫星位于距离地面大约2万千米的太空中，由于GPS卫星的原子钟比在地球表面的原子钟重力位高，星载时钟每天要快45微秒。两者综合的结果是，星载时钟每天大约比地面钟快38微秒。一般说来，GPS接受器准确度在30米之内就意味着它已经利用了相对论效应。

Corrections for GPS

- **The GPS space segment consists of a constellation of satellites transmitting radio signals to users. The United States is maintaining the availability of at least 24 operational GPS satellites, 95% of the time.**
- **A GPS satellite orbits the earth with a period of 12 hours.**
- **According to Lecture 7, the orbit radius (you should work it out!)**

$$r = \left(\frac{gR_e^2T^2}{4\pi^2}\right)^{1/3} = 2.66\times10^7\ m$$

$$v = \omega r = \left(\frac{2\pi g R_e^2}{T}\right)^{1/3} = 3.87\times10^3\ m/s$$

每个GPS卫星时速为1.4万千米

The Special Relativity Effect

- **According to time dilation (e.g., from Lecture 19, Slide *Ex: Time Dilation*)**

$$\frac{\Delta\tau_s}{\Delta t} = \sqrt{1 - v^2/c^2} \approx 1 - \frac{v^2}{2c^2}$$

- **After one day, the time difference is**

$$\Delta\tau_s \approx -\frac{v^2}{2c^2}\Delta t = -7.18 \times 10^{-6}\ s$$

根据狭义相对论，它的星载原子钟每天要比地球上的钟慢7微秒。

The General Relativity Effect

- **Due to the difference in gravity**

$$\frac{\Delta\tau_g}{\Delta t} = 1 + \frac{\Delta U}{mc^2} = 1 + \frac{gR_e^2}{c^2}\left(\frac{1}{R_e} - \frac{1}{r}\right)$$

- **After one day, the time difference is**

$$\Delta\tau_g \approx \frac{gR_e^2}{c^2}\left(\frac{1}{R_e} - \frac{1}{r}\right)\Delta t = 4.55 \times 10^{-5}\ s$$

由于GPS卫星的原子钟比在地球表面的原子钟重力位高，星载时钟每天要快45微秒。

Importance of the Corrections

- **The total effect is that the satellite clock is faster than a clock on the earth's surface**

$$\Delta\tau = \Delta\tau_g + \Delta\tau_s = 3.83 \times 10^{-5}\, s$$

- **The error in position due to the effect of relativity is**

$$\Delta L = c\Delta\tau = 1.15 \times 10^{4}\, m = 11.5\, km$$

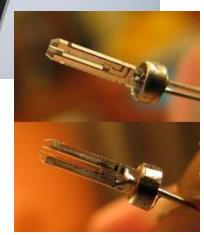
两者综合的结果是，星载时钟每天大约比地面钟快38微秒。一般说来，GPS接受器准确度在30米之内就意味着它已经利用了相对论效应。(?)

Clocks with improving accuracy

Pendulum clock

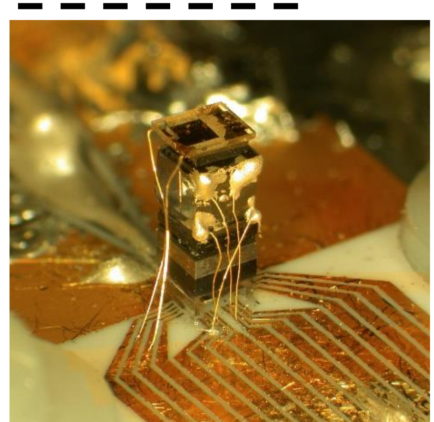
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Quartz clock:
<u>piezoelectric material</u>



$$f = \frac{1.875104^2}{2\pi}\frac{a}{l^2}\sqrt{\frac{E}{12\rho}},$$

Chip-scale
atomic clocks



Atomic clock

Louis Essen (R) & Jack Parry (L) the world's first caesium-133 atomic clock.