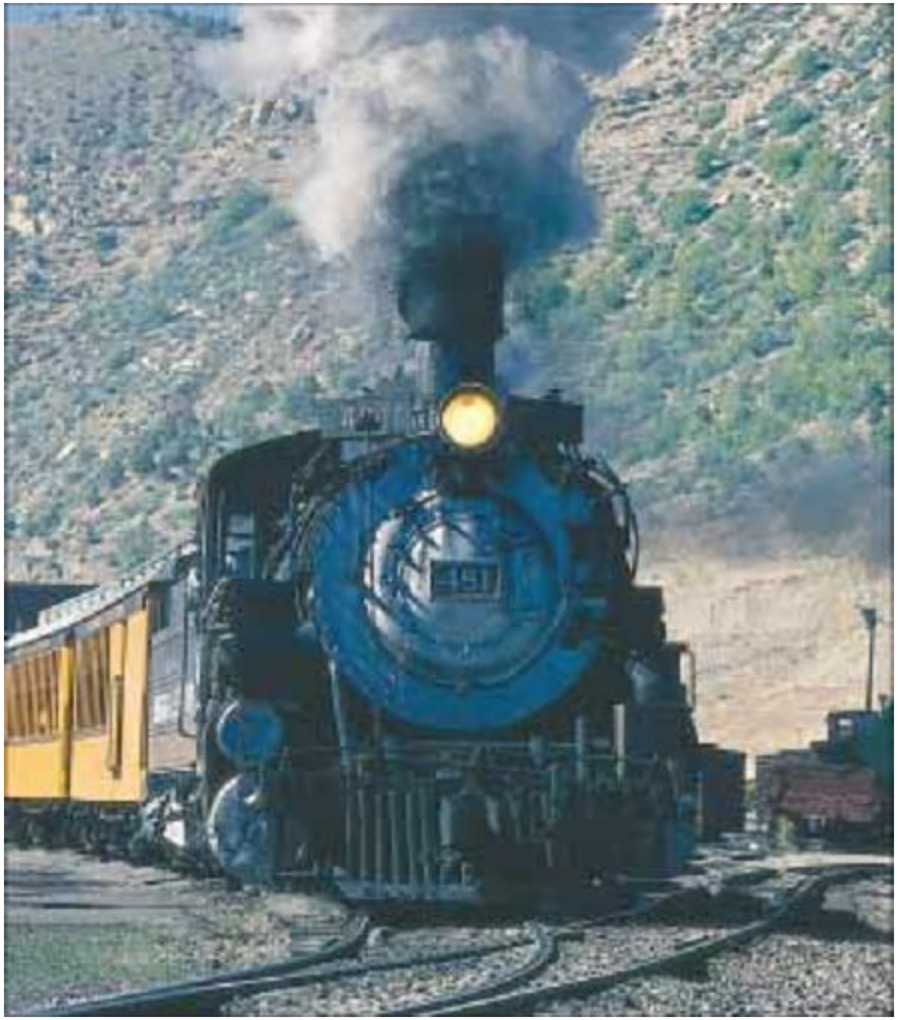
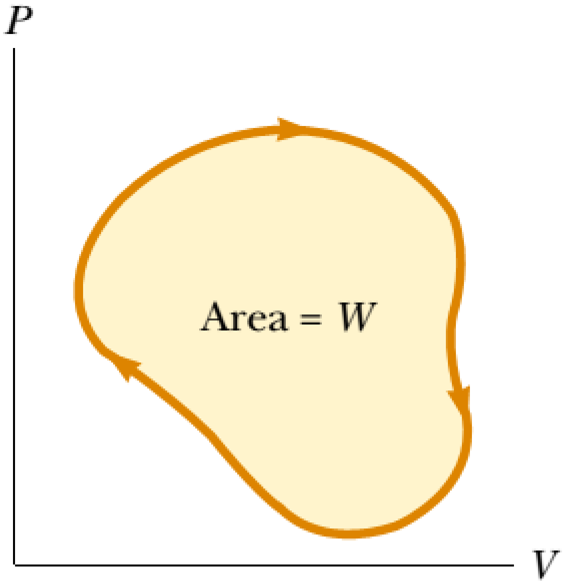
General Physics I

Lecture 26: Engines and The 2nd Law of Thermodynamics

Steam-Driven Locomotive

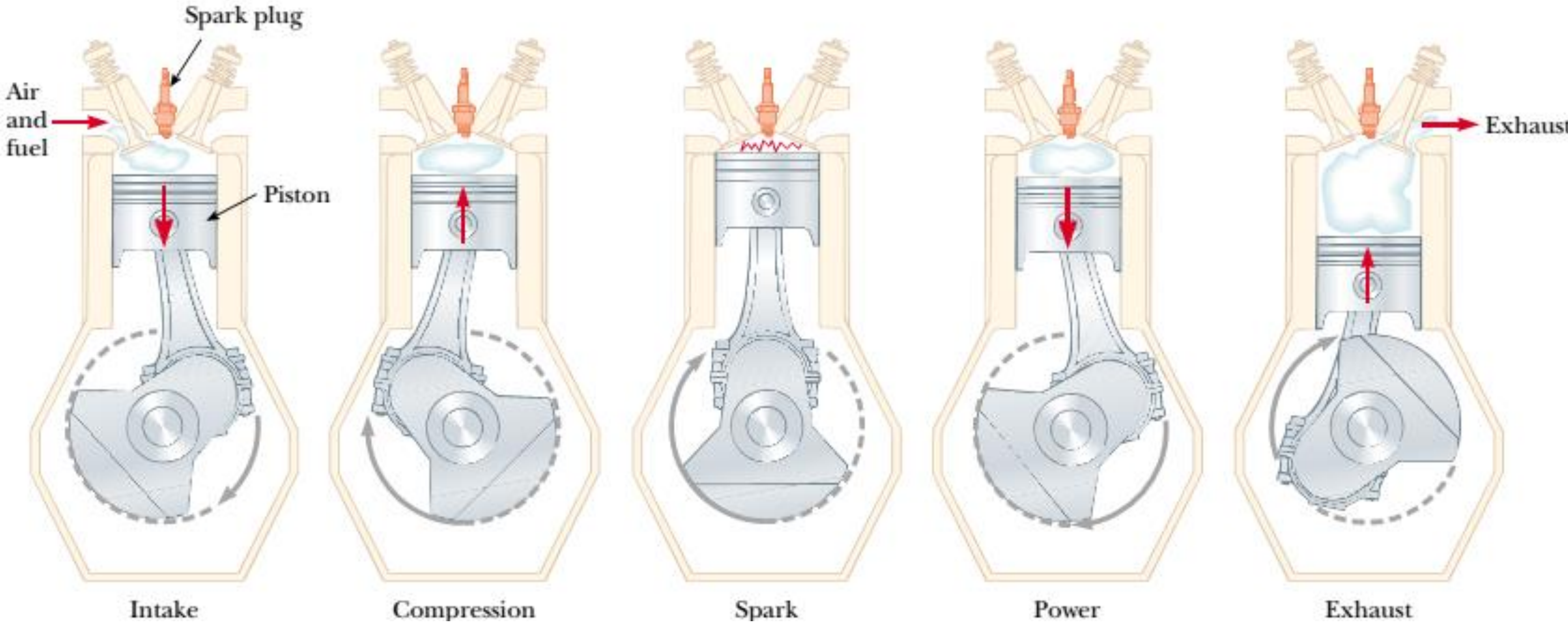




$$Q_{in} - Q_{out} = W$$

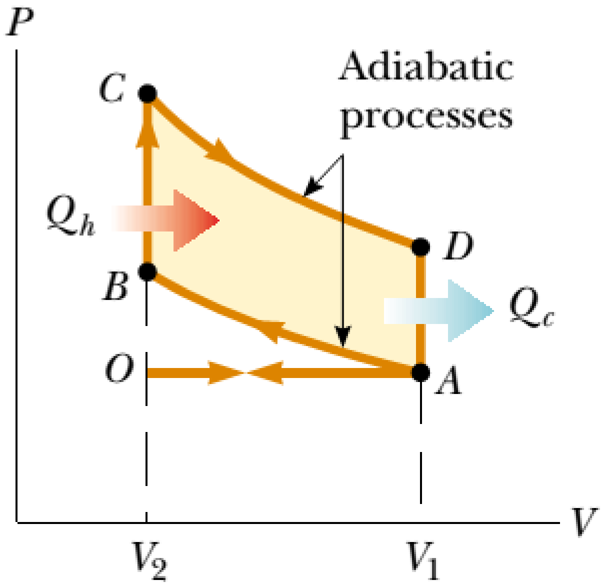
$$e = \frac{\text{efficiency}}{\text{cost}} = \frac{W}{Q_{\text{in}}}$$

Gasoline Engine



For a given cycle, the piston moves up and down twice. This represents a four-stroke cycle consisting of two upstrokes and two downstrokes.

Efficiency of the Otto Cycle



$$W = Q_h - Q_c$$

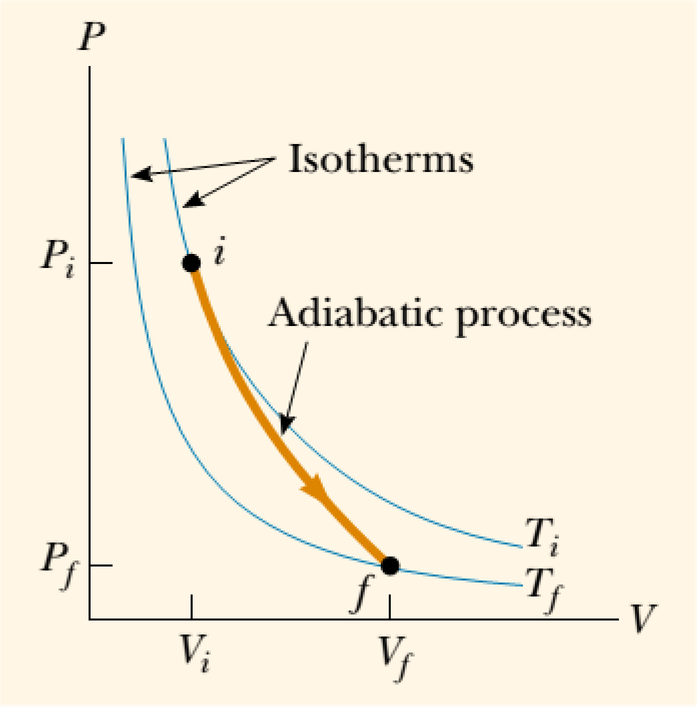
$$Q_h = nC_V(T_C - T_B)$$

$$Q_c = nC_V(T_D - T_A)$$

$$e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_D - T_A}{T_C - T_B}$$

Why not using isothermal compression and expansion?

Adiabatic Expansion



adiabatic $đQ = 0$

$$C_V \mathrm{dT} = \mathrm{dU} = -\mathrm{PdV}$$

$$\xrightarrow{P=\frac{\mathrm{Nk}_B \mathrm{T}}{V}} \frac{C_V \mathrm{dT}}{T} = -\frac{\mathrm{Nk}_B \mathrm{dV}}{V}$$

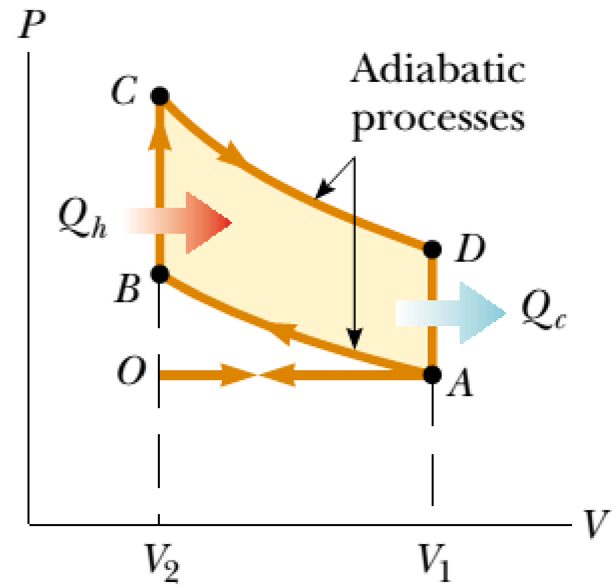
$$\mathrm{TV}^{\gamma-1} = \mathrm{const}$$

or $\mathrm{PV}^{\gamma} = \mathrm{const}$

$$\gamma = C_P / C_V = 1 + \mathrm{Nk}_B / C_V$$

Efficiency of the Otto Cycle

$$e = 1 - \frac{T_D - T_A}{T_C - T_B}$$



$$TV^{\gamma-1} = \text{constant}$$

$$A \longrightarrow B: \qquad T_A V_1{}^{\gamma-1} = T_B V_2{}^{\gamma-1}$$

$$C \longrightarrow D: \qquad T_D V_1{}^{\gamma-1} = T_C V_2{}^{\gamma-1}$$

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_A}{T_B} = \frac{T_D}{T_C}$$

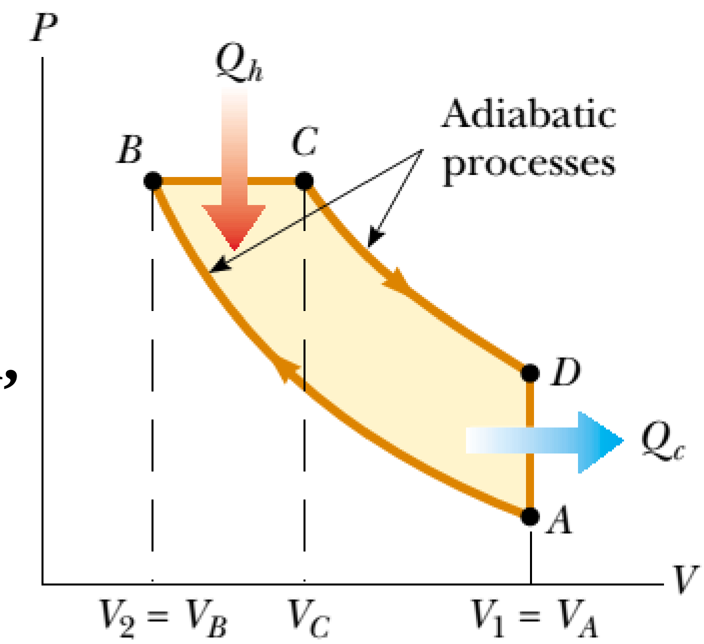
$$e = 1 - \frac{T_A}{T_B} = 1 - \frac{T_D}{T_C}$$

$$e = 1 - \frac{1}{(V_1 / V_2)^{\gamma-1}}$$

Diesel Engine

In a diesel engine, only air (and no fuel) is present in the cylinder at the beginning of the compression. In the idealized diesel cycle:

- **(A-B) air adiabatic compression**
- **(B-C) fuel injected at B, fuel-air mixture constant-pressure expansion to V_C**
- **(C-D) high T causing combustion, adiabatic expansion to V_D**
- **(D-A) exhaust valve opened, constant-volume output of energy occurs**



VW EA288 4-Cylinder Diesel Engine

2.0 Liter, 112 kW (or 150 horsepower) @ 3500 rpm

Simplified 2.0L Diesel Engine

$$\frac{T_A}{T_B} = \left(\frac{V_B}{V_A}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1}$$

$$\frac{T_C}{T_B} = \frac{V_C}{V_B} = r_c$$

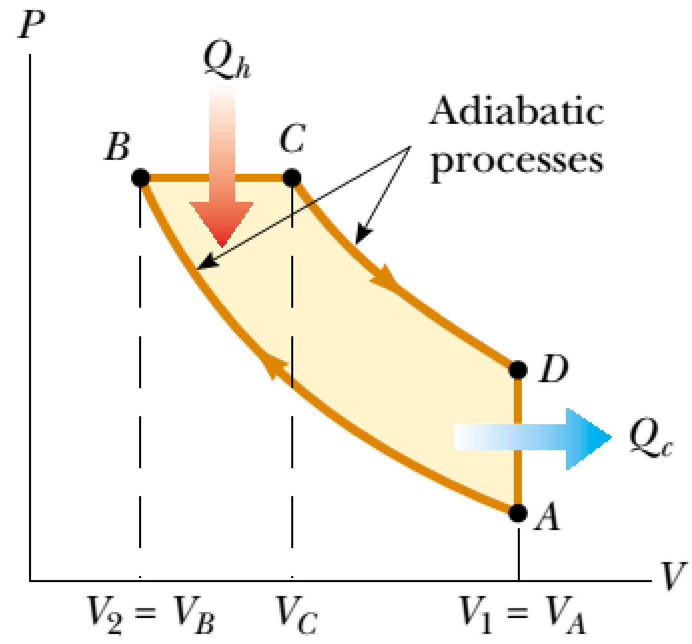
$$r = \frac{V_A}{V_B} \approx 20$$

$$r_c = \frac{V_C}{V_B} \approx 2$$

$$\frac{T_D}{T_C} = \left(\frac{V_C}{V_D}\right)^{\gamma-1} = \left(\frac{r_c V_B}{V_A}\right)^{\gamma-1} = \left(\frac{r_c}{r}\right)^{\gamma-1}$$

$$T_{A,B,C,D} \approx 300, 1000, 2000, 800\ K$$

Typical ambient T



Simplified 2.0L Diesel Engine

$$n = \frac{2\ \text{L}}{24\ \text{L/mol}}$$

$$W_{\text{cycle}} = n\left[C_p(T_C - T_B) - C_V(T_D - T_A)\right]$$

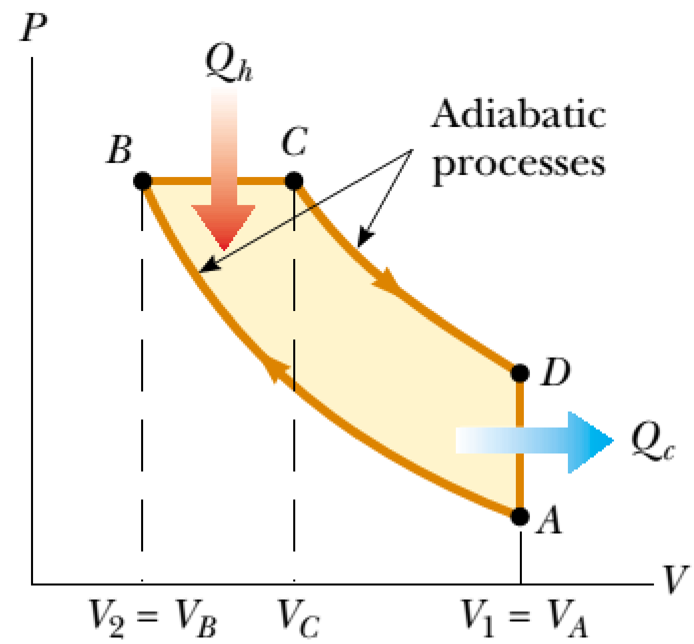
$$C_P = C_V + R \qquad C_V = 5R/2$$

$$W_{\text{cycle}} = 1.6\ \text{kJ}$$

$$P = \frac{3500/60\ \text{(rev/s)}}{2\ \text{(rev/cycle)}} \times 1.6\ \text{kJ} = 46\ \text{kW}$$

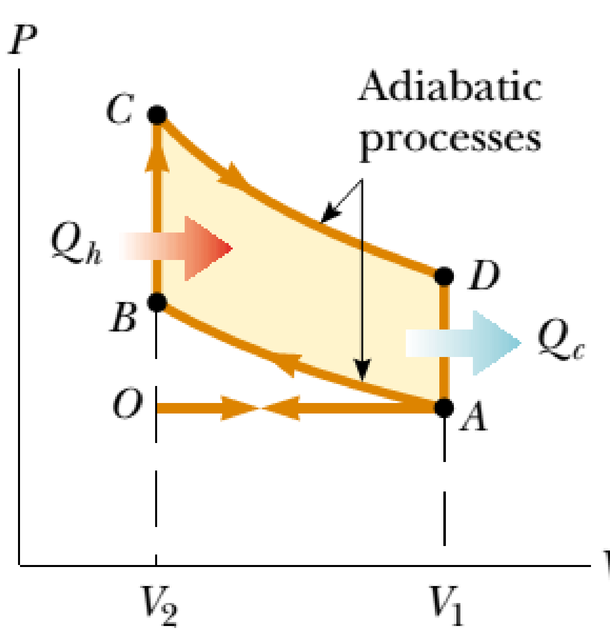
$$Q_{\text{in}} = \text{n}C_p(T_C - T_B)$$

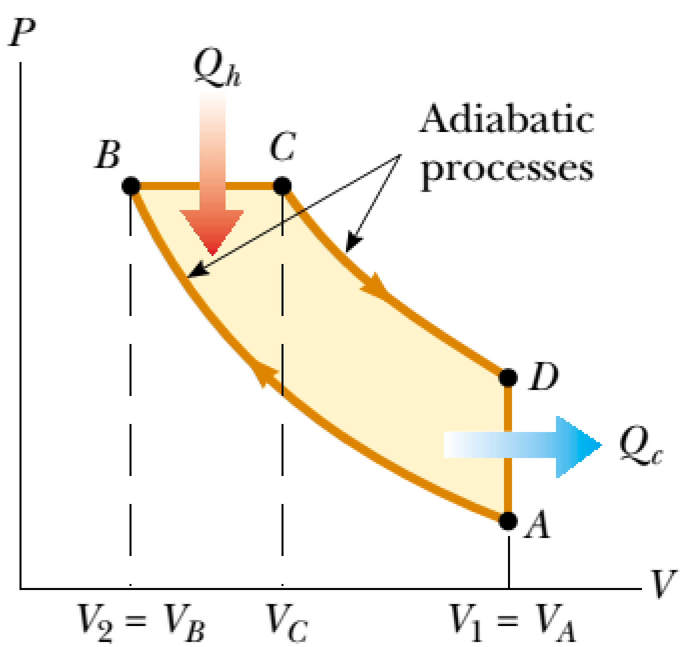
$$Q_{\text{out}} = \text{n}C_V(T_D - T_A)$$



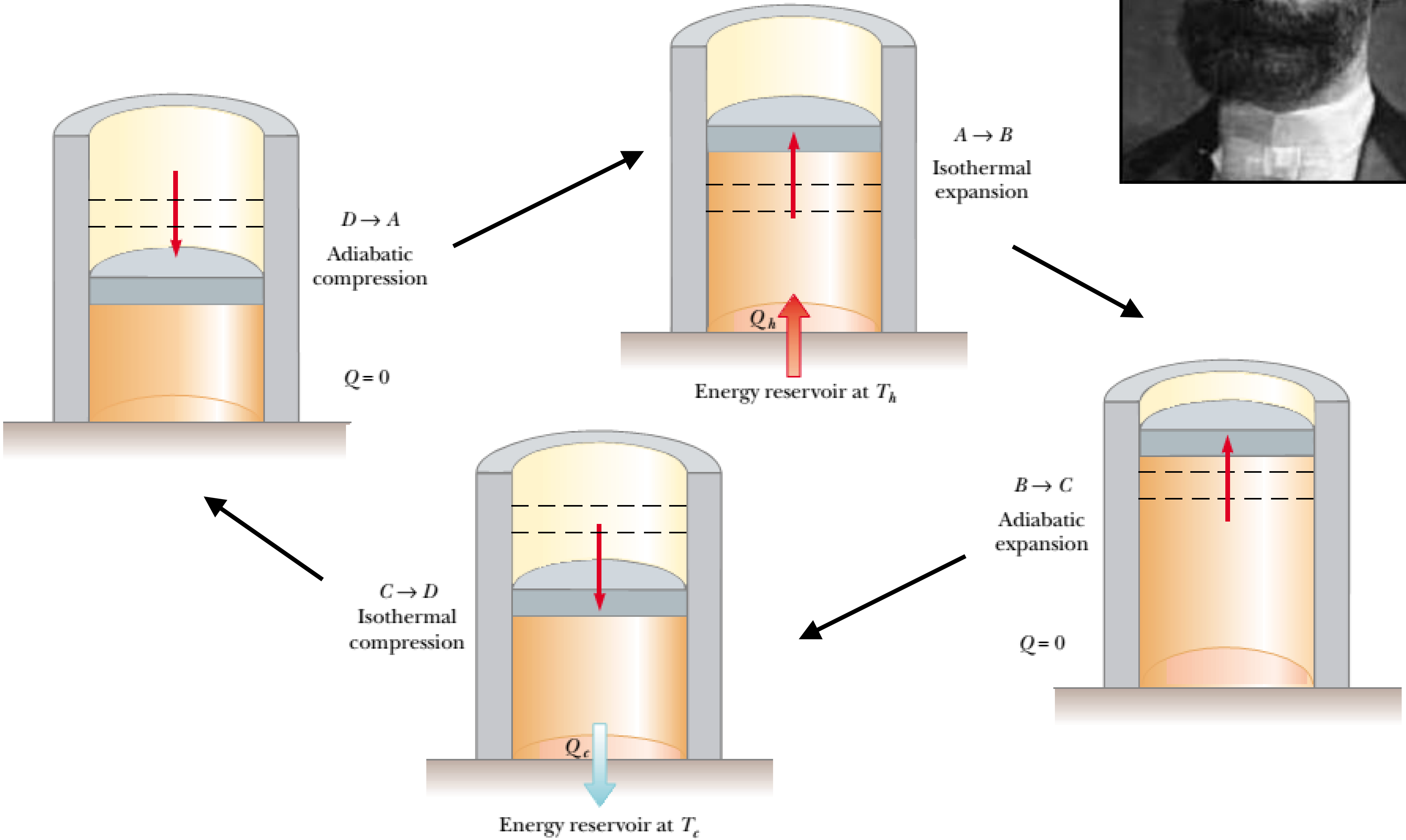
Heat Engine in General

Work is done on the gas during compression, but significantly more work is done on the piston by the mixture as the products of combustion expand in the cylinder.

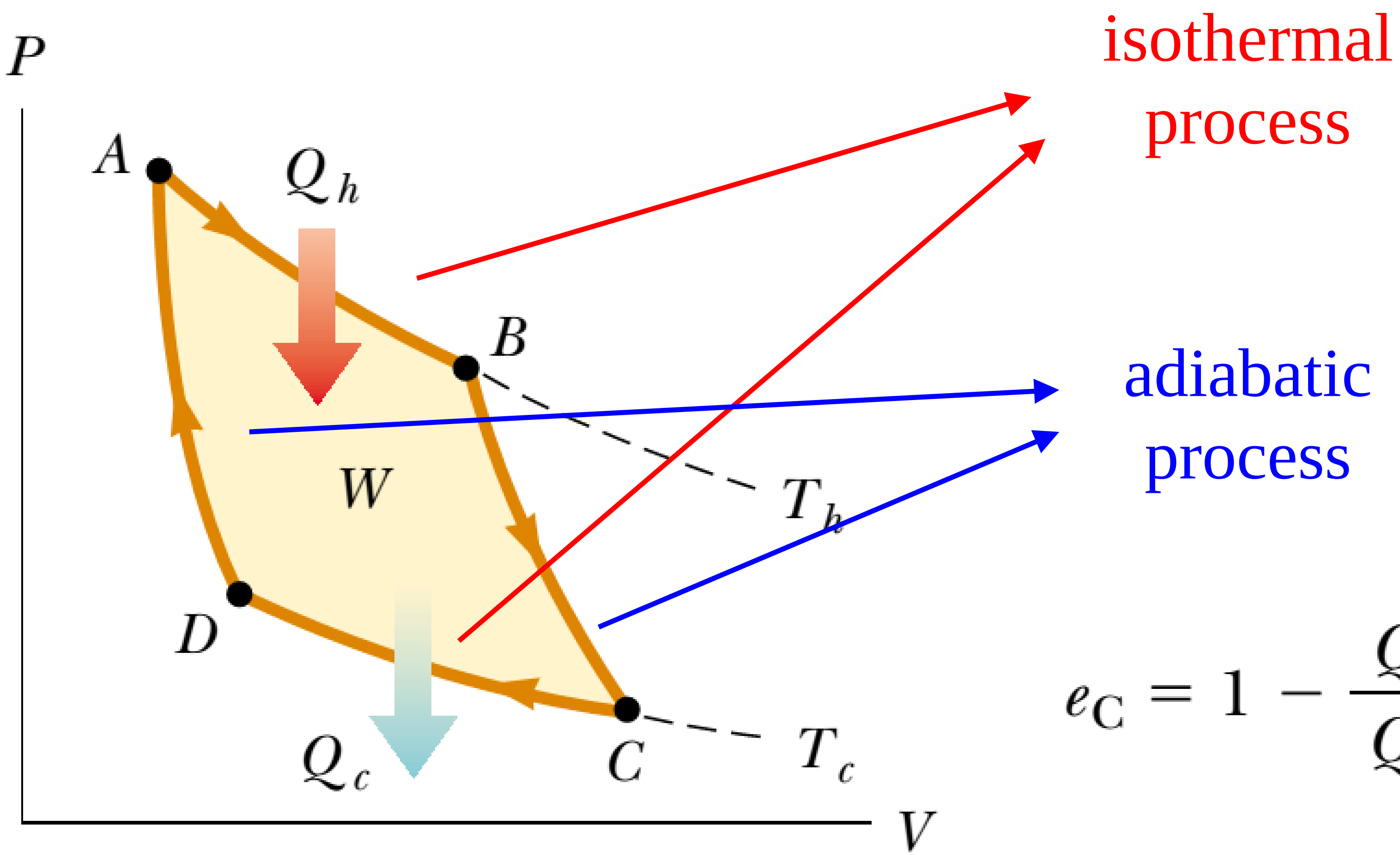




Carnot's Engine



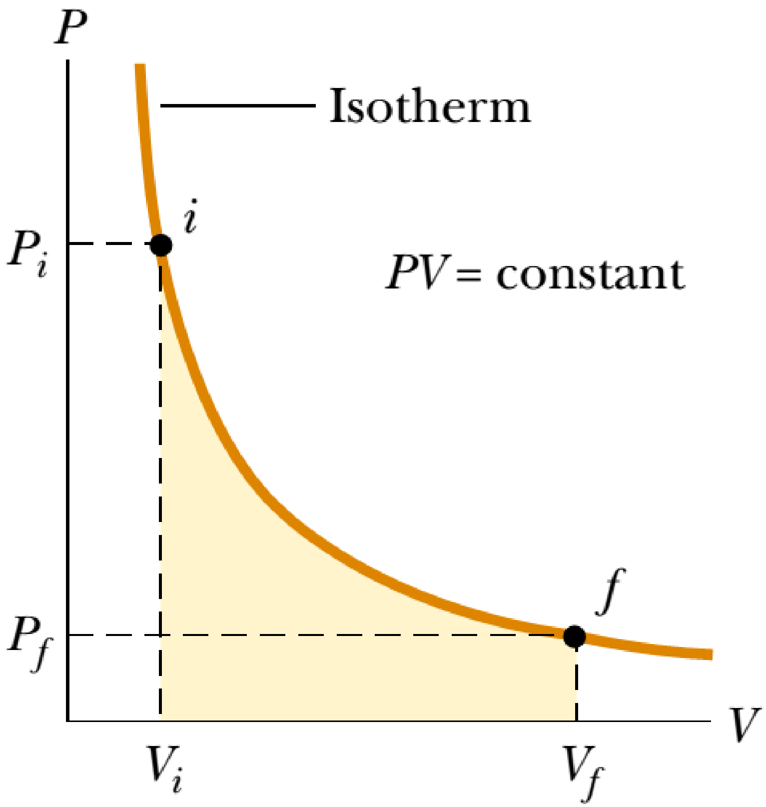
Efficiency of a Carnot Engine



$$e_C = 1 - \frac{Q_c}{Q_h}$$

Reminder: Isothermal Expansion

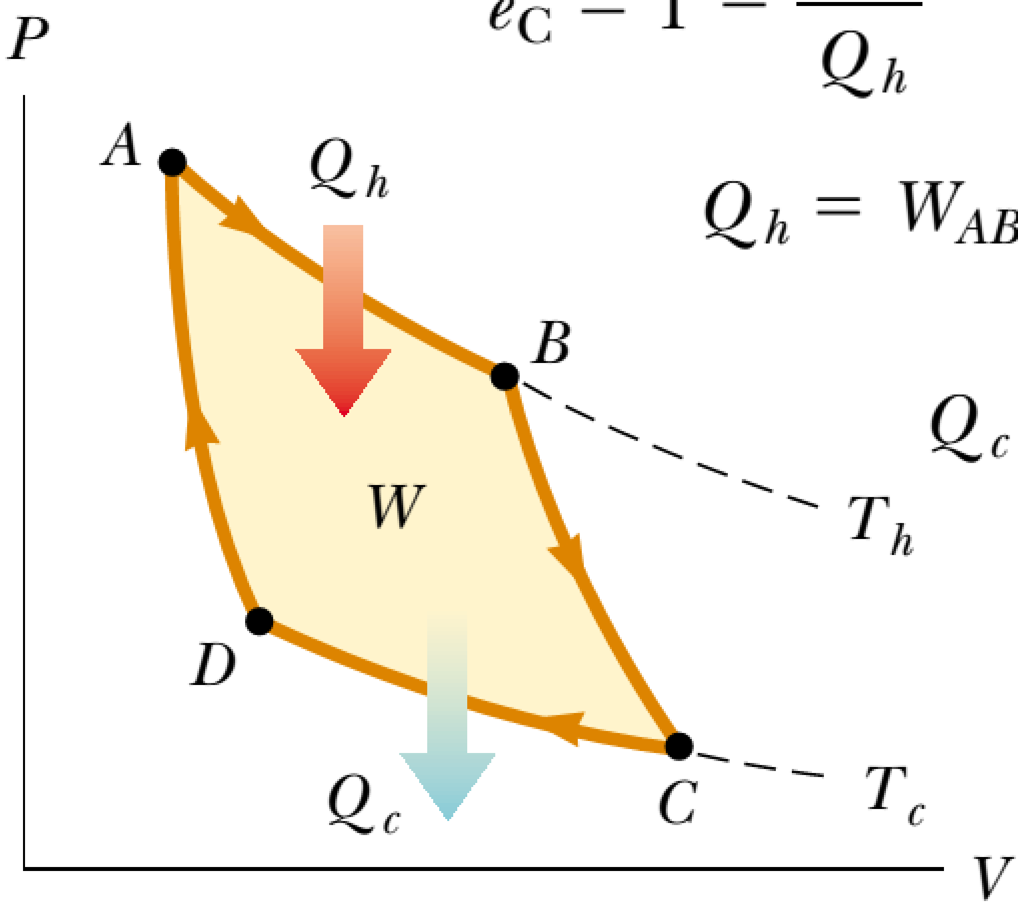
$$W = \int \mathrm{PdV} = \int \frac{\mathrm{N}k_B T}{V}\mathrm{dV} = \mathrm{N}k_B T \ln\frac{V_f}{V_i}$$



$$\Delta U = 0$$

$$Q = W = \mathrm{N}k_B T \ln\frac{V_f}{V_i}$$

Efficiency of a Carnot Engine



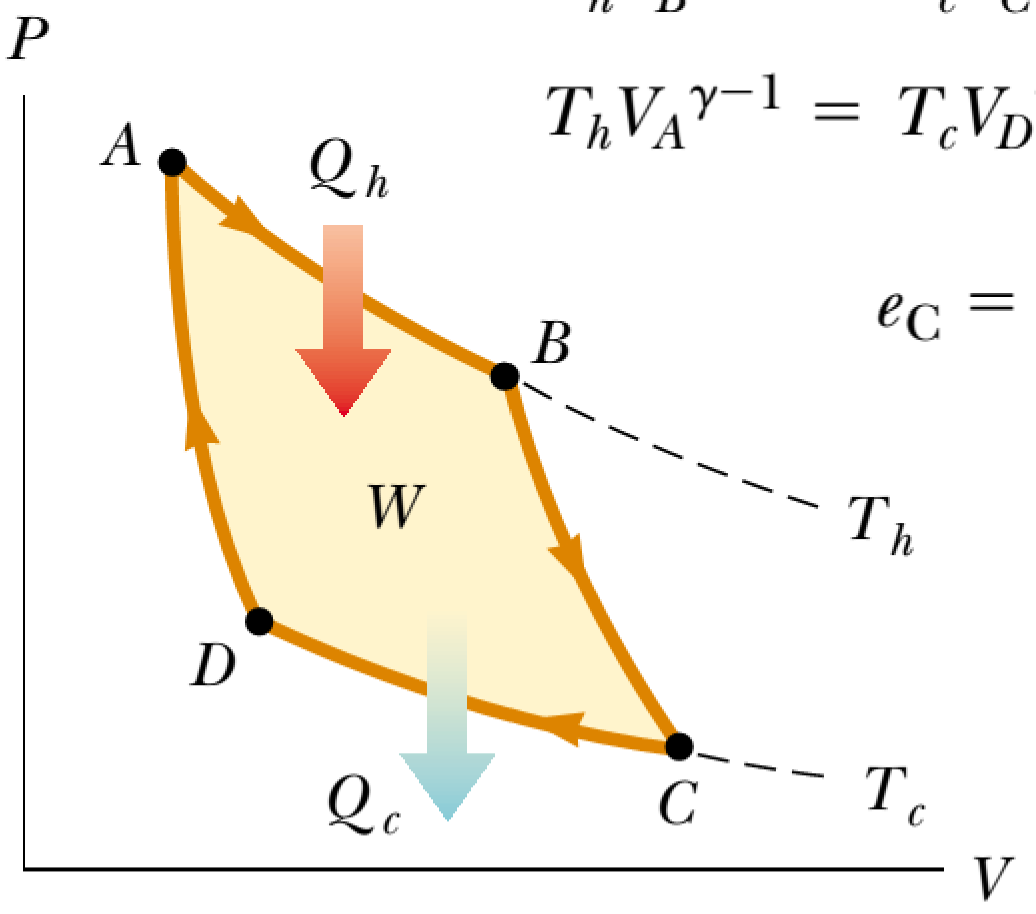
$$e_C = 1 - \frac{Q_c}{Q_h}$$

$$Q_h = W_{AB} = nRT_h \ln \frac{V_B}{V_A}$$

$$Q_c = |W_{CD}| = nRT_c \ln \frac{V_C}{V_D}$$

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h} \frac{\ln(V_C / V_D)}{\ln(V_B / V_A)}$$

Efficiency of a Carnot Engine



$$T_h V_B^{\gamma-1} = T_c V_C^{\gamma-1}$$

$$T_h V_A^{\gamma-1} = T_c V_D^{\gamma-1}$$

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

$$e_C = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$$

All Carnot engines operating between the same two temperatures have the same efficiency.

Carnot Cycle

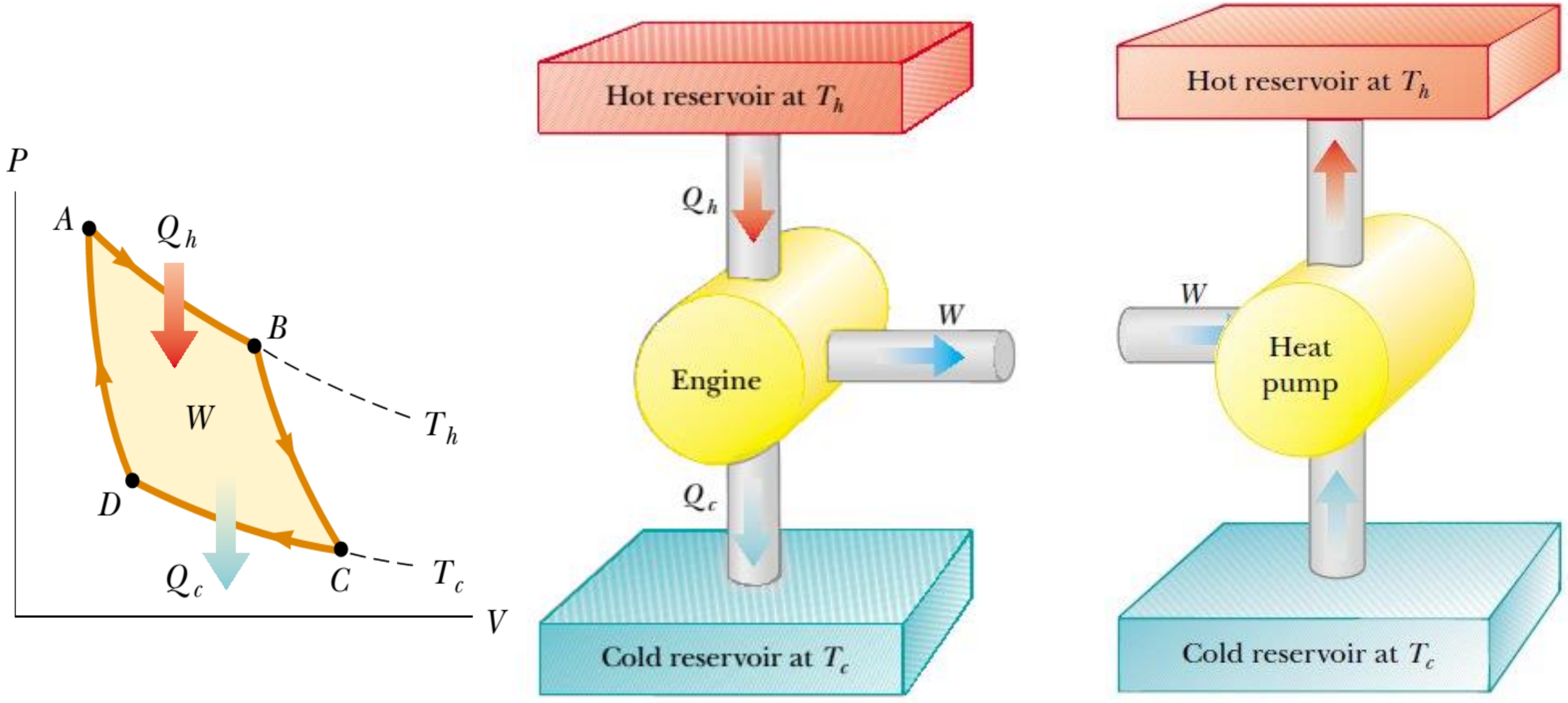
The efficiency of the Carnot cycle only depends on the temperatures T_c & T_h.

The efficiency approaches 1 when $T_c/T_h \rightarrow 0$, or $T_c \rightarrow 0$.

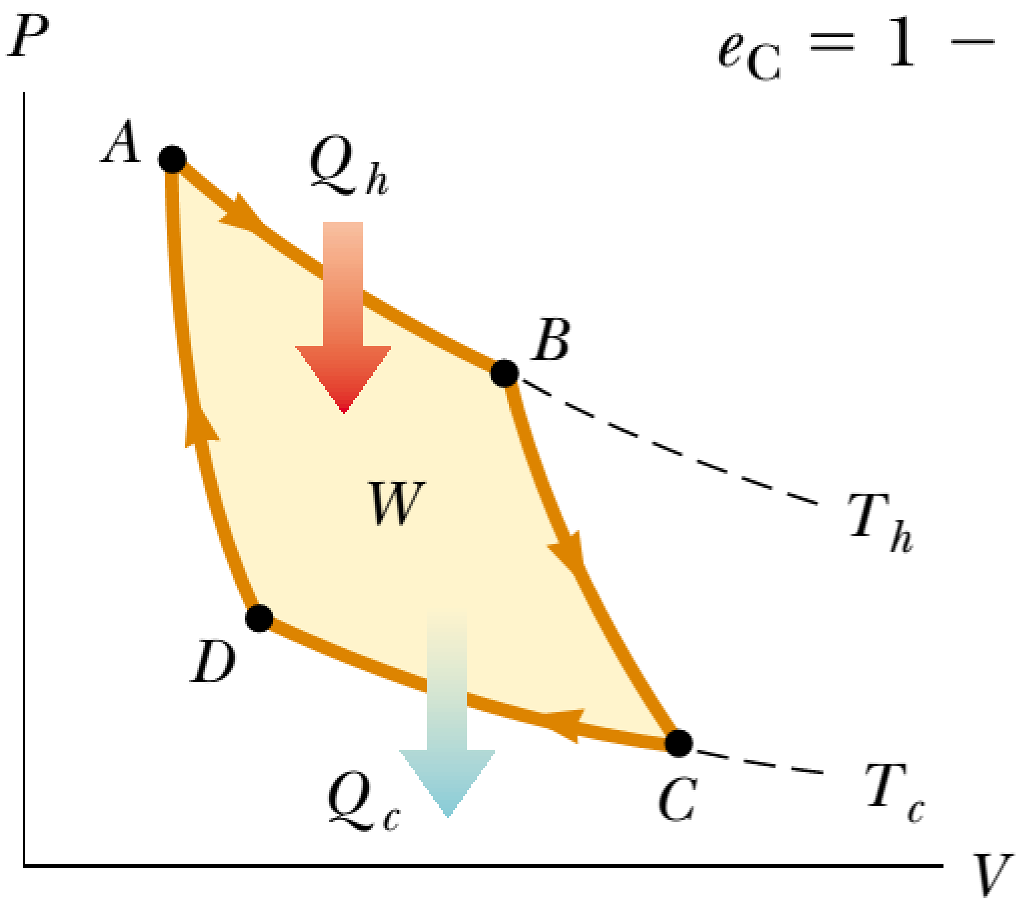
***T* is the absolute energy scale, in units of K.**

Carnot cycle is a reversible cycle – quasistatic with no dissipation.

Reverse Carnot Cycle



Carnot's Theorem



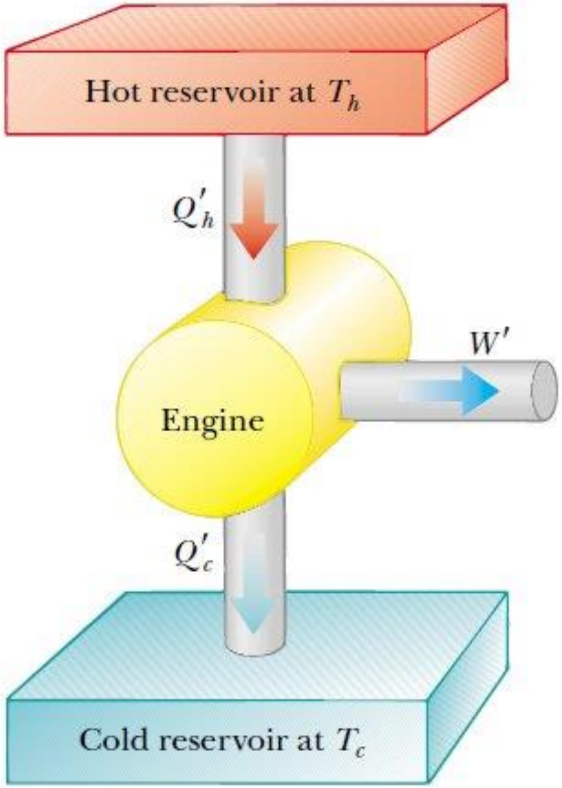
$$e_C = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$$

No real heat engine operating between two energy reservoirs can be more efficient than my engine operating between the same two reservoirs.

-- Sadi Carnot

What if not?

If we suppose $\varepsilon' = \frac{W'}{Q'_h} > \varepsilon_{\text{carnot}} = \frac{W}{Q_h}$



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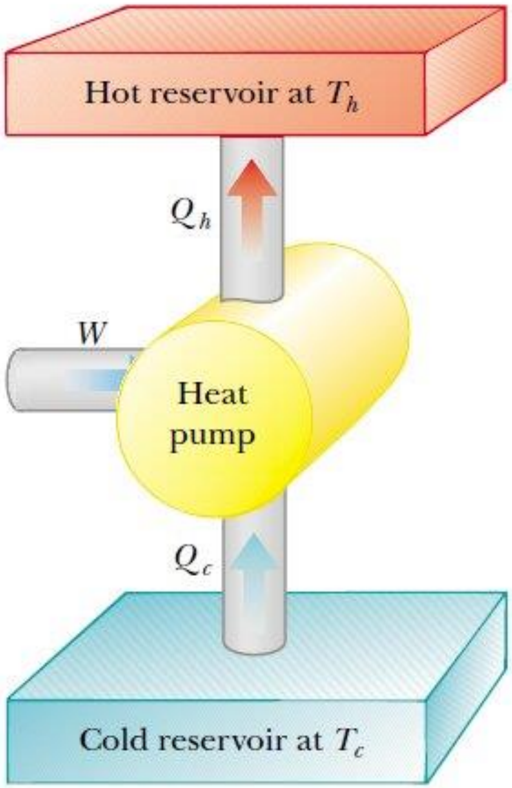
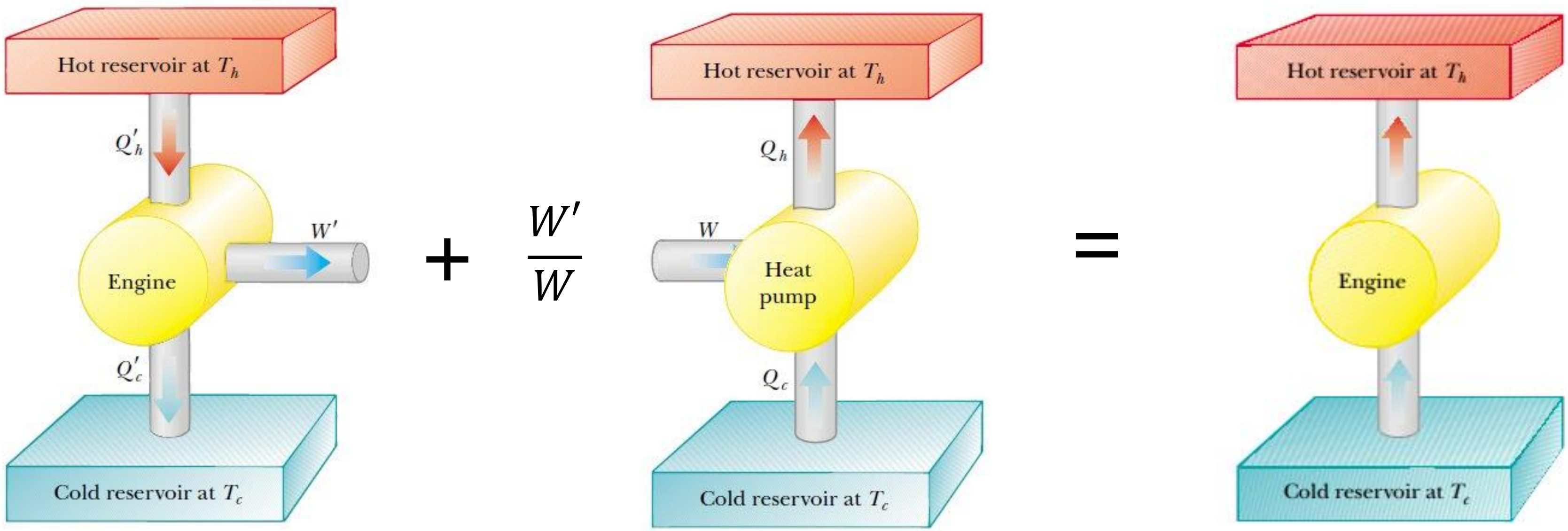


= ?

Match Work First

If we suppose $\varepsilon' = \dfrac{W'}{Q'_h} > \varepsilon_{\text{carnot}} = \dfrac{W}{Q_h}$



$$Q'_h - \frac{W'}{W} Q_h = Q'_h \left(1 - \frac{W'}{Q'_h} \frac{Q_h}{W} \right) = Q'_h (1 - \varepsilon' / \varepsilon_{\text{carnot}}) < 0$$

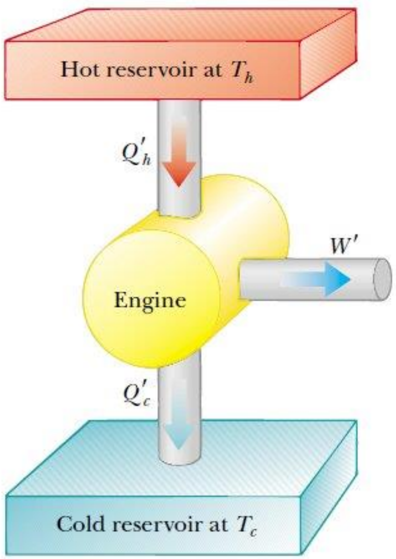
Clausius Statement

It is impossible to construct a cyclical machine whose sole effect is the continuous transfer of energy from one object to another object at a higher temperature without the input of energy by work.

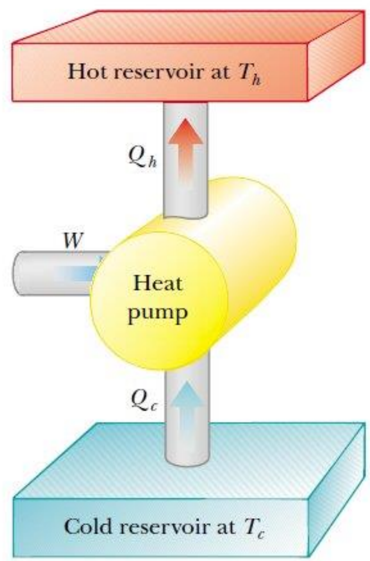
No perfect refrigerator!

Match Heat at the Cold End

If we suppose $$\varepsilon' = \frac{W'}{Q'_h} > \varepsilon_{\text{carnot}} = \frac{W}{Q_h}$$



$$+ \ \frac{Q'_c}{Q_c}$$



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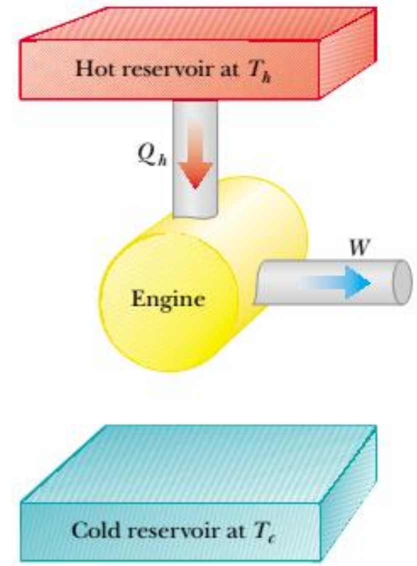


$$Q'_h - \frac{Q_c'}{Q_c} Q_h = Q'_h \left(1 - \frac{Q_c'}{Q'_h}\frac{Q_h}{Q_c}\right) = Q'_h \left(1 - \frac{1-\varepsilon'}{1-\varepsilon_{\text{carnot}}}\right) > 0$$

Kelvin Statement

It is impossible to construct a heat engine that, operating in a cycle, produces no effect other than the absorption of energy from a reservoir and the performance of an equal amount of work.

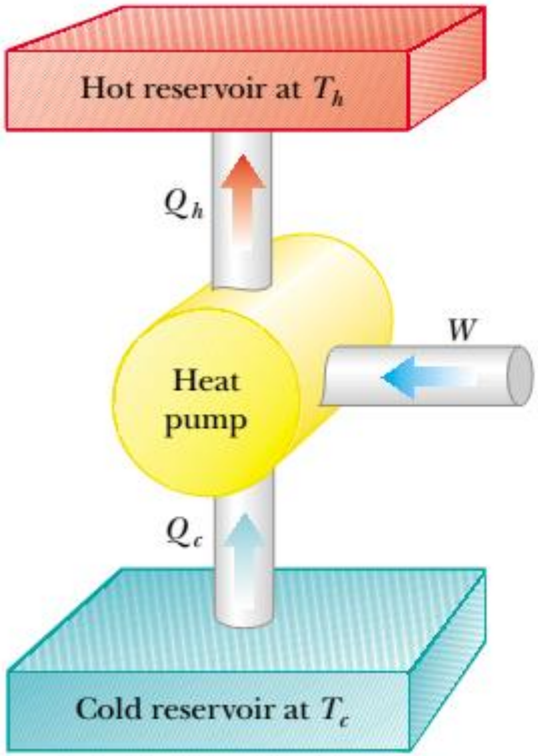
No perfect heat engine!

Laws in Plain Language

The 1st and 2nd laws of thermodynamics can be summarized as follows:

- **The first law specifies that we cannot get more energy out of a cyclic process by work than the amount of energy we put in.**
- **The second law states that we cannot break even because we must put more energy in, at the higher temperature, than the net amount of energy we get out by work.**

Heat Pumps & Refrigerators



Carnot cycle in reverse:

$$\text{COP}_\text{C}(\text{heating mode}) = \frac{Q_h}{W}$$

$$= \frac{Q_h}{Q_h - Q_c} = \frac{1}{1 - \frac{Q_c}{Q_h}} = \frac{1}{1 - \frac{T_c}{T_h}} = \frac{T_h}{T_h - T_c}$$

$$\text{COP}_\text{C}\ (\text{cooling mode}) = \frac{T_c}{T_h - T_c}$$

$$\text{COP (cooling mode)} = \frac{Q_c}{W}$$

$$\text{COP (heating mode)} \equiv \frac{\text{Energy transferred at high temperature}}{\text{Work done by pump}} = \frac{Q_h}{W}$$

Heat Engine & Refrigerator