General Physics I

Lecture 13: Wave Motion

What Is a Wave?

- **A bit of gossip starting in Washington reaches New York [by word of mouth] very quickly, even though not a single individual who takes part in spreading it travels between these two cities. There are two quite different motions involved, that of the rumor, Washington to New York, and that of the persons who spread the rumor. The wind, passing over a field of grain, sets up a wave which spreads out across the whole field. Here again we must distinguish between the motion of the wave and the motion of the separate plants, which undergo only small oscillations... The particles constituting the medium perform only small vibrations, but the whole motion is that of a progressive wave. The essentially new thing here is that for the first time we consider the motion of something which is not matter, but energy propagated through matter.**

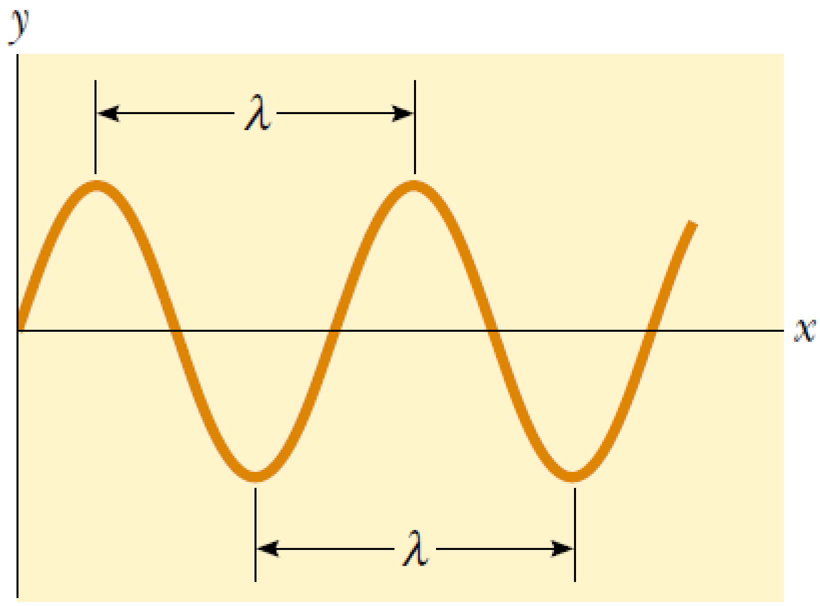
  - **A. Einstein and L. Infeld in *The Evolution of Physics***

Outline

- Basic variables of wave motion
- Longitudinal wave versus transverse wave
- Traveling waves
- Superposition and interference
- Reflection and transmission
- The linear wave equation

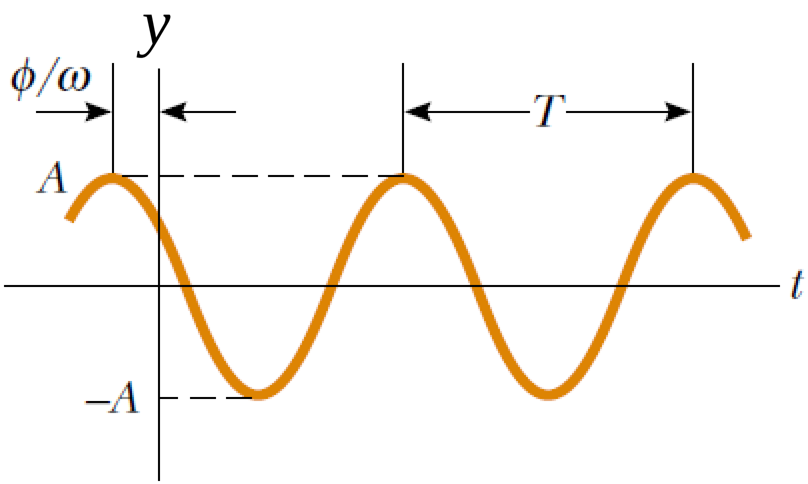
Basic Variables of Wave Motion

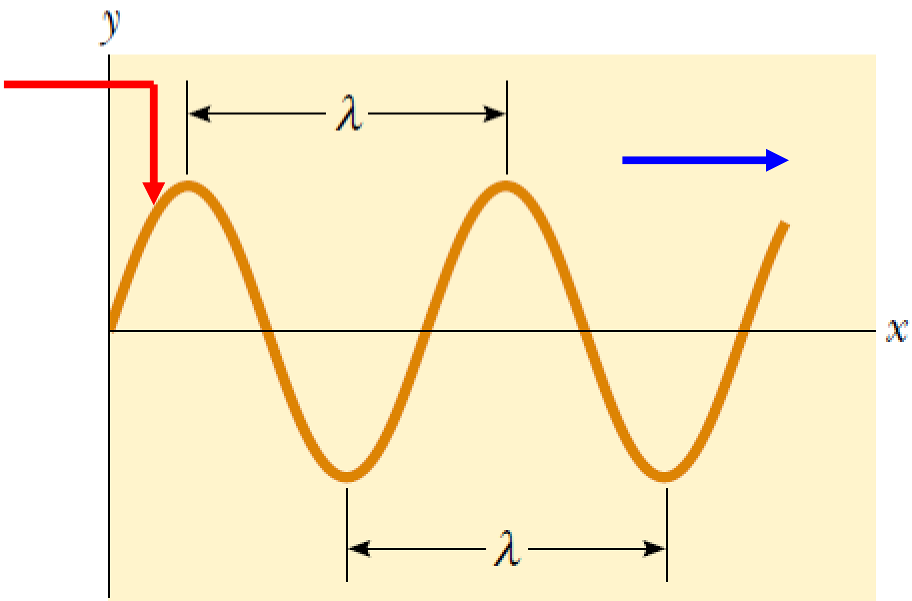
- **Wavelength** λ
- **Period** *T*
- **Frequency** *f*
- **Amplitude** *A*



The wavelength is the minimum distance between any two identical points (such as the crests) on adjacent waves.

Oscillation vs Wave

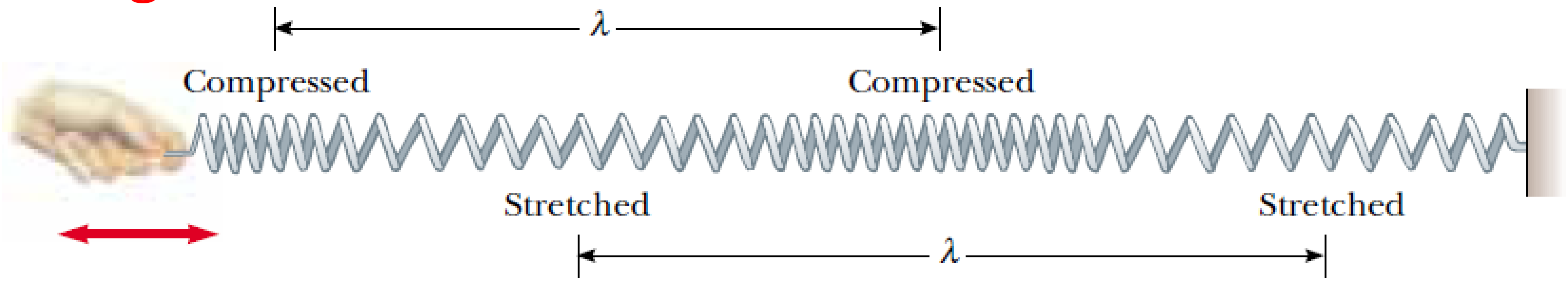




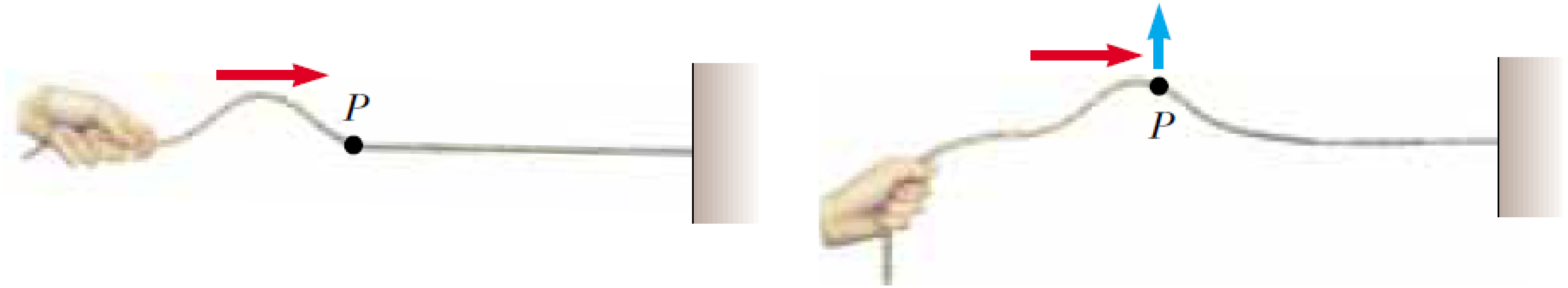
Mechanical waves need
(1) some source of disturbance,
(2) a medium that can be disturbed, and
(3) some physical connection through which adjacent portions of the medium can influence each other.

Longitudinal vs Transverse

- **A wave that causes the particles of the medium to move parallel to the direction of wave motion is called a longitudinal wave.**



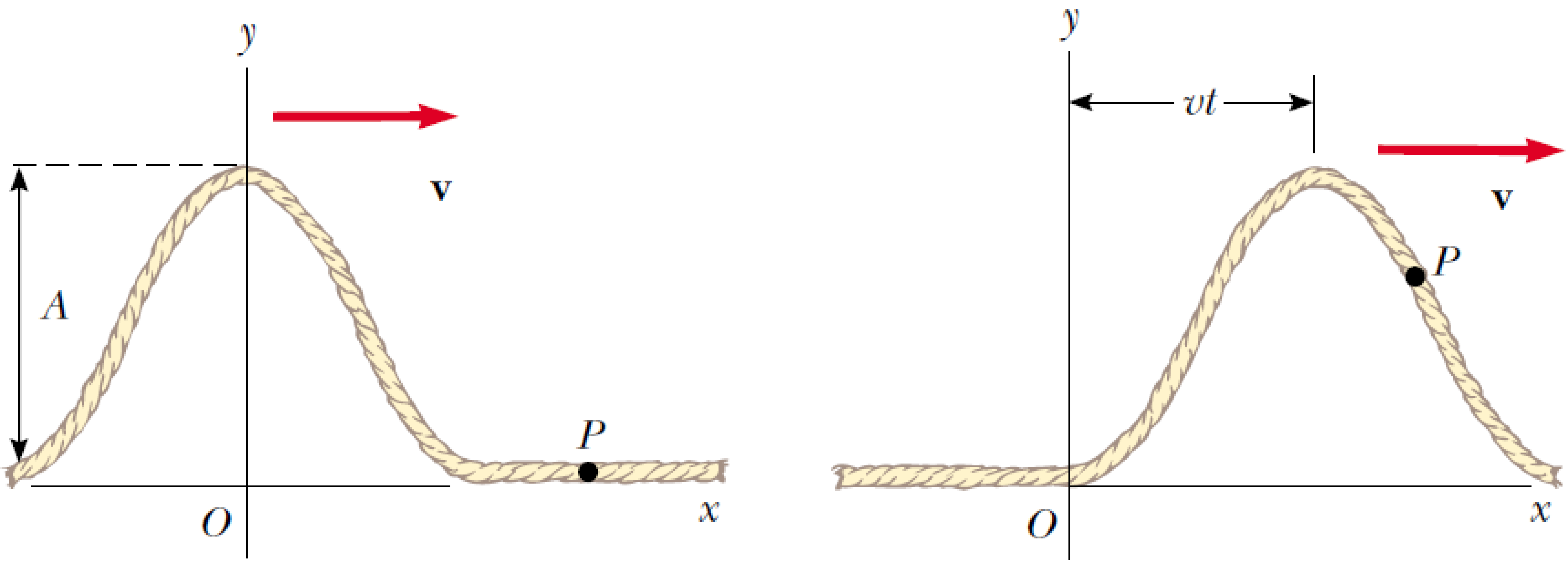
- **A wave that causes the particles of the disturbed medium to move perpendicular to the wave motion is called a transverse wave.**



1D Traveling Waves

- The displacement y, sometimes called the **wave function**, depends on both x and t. For this reason, it is often written y(x, t), which is read "y as a function of x and t."

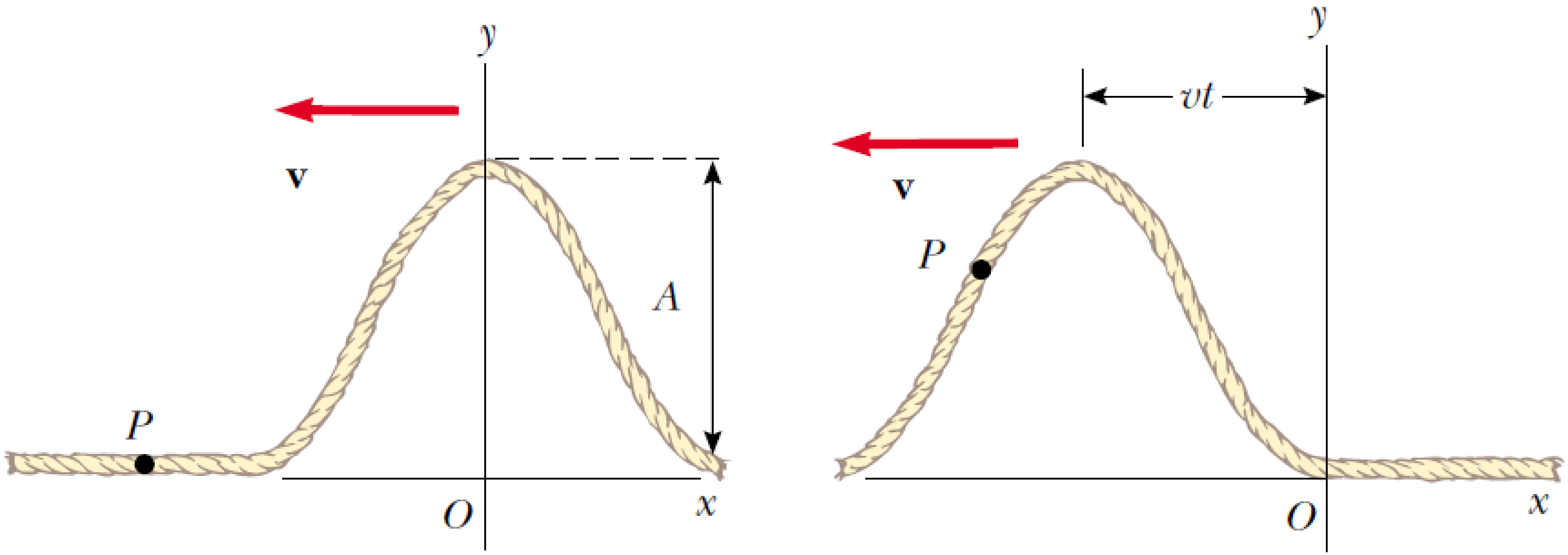
$$y = f(x - vt)$$ right-moving



The Wave Speed

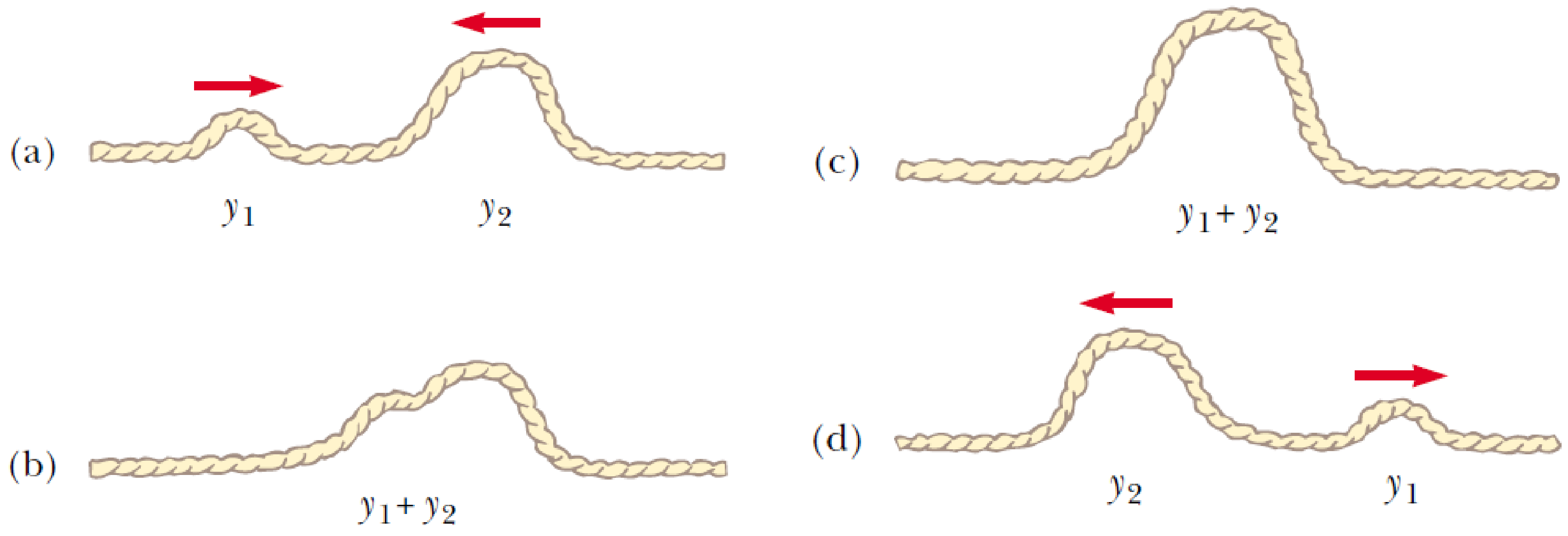
- The **wave speed** *v* is how fast the shape of the pulse travels, not how fast an underlying particle (of the medium) is moving.

$$y = f(x + vt)$$ **left-moving**



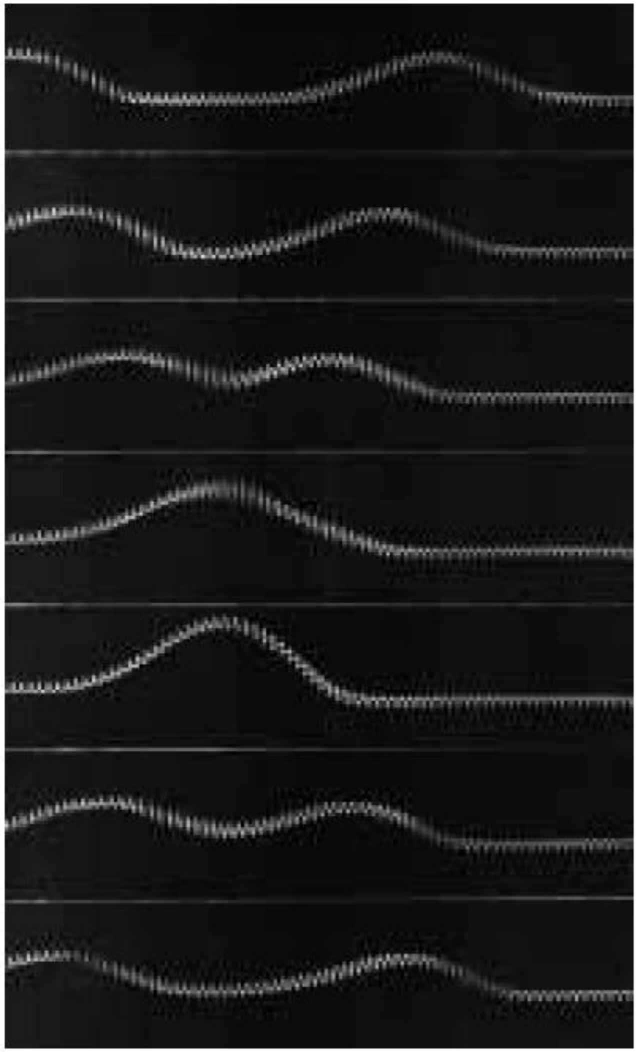
Superposition Principle

- If two or more traveling waves are moving through a medium, the resultant wave function at any point is the **algebraic sum of the wave functions** of the individual waves.



Linear Waves

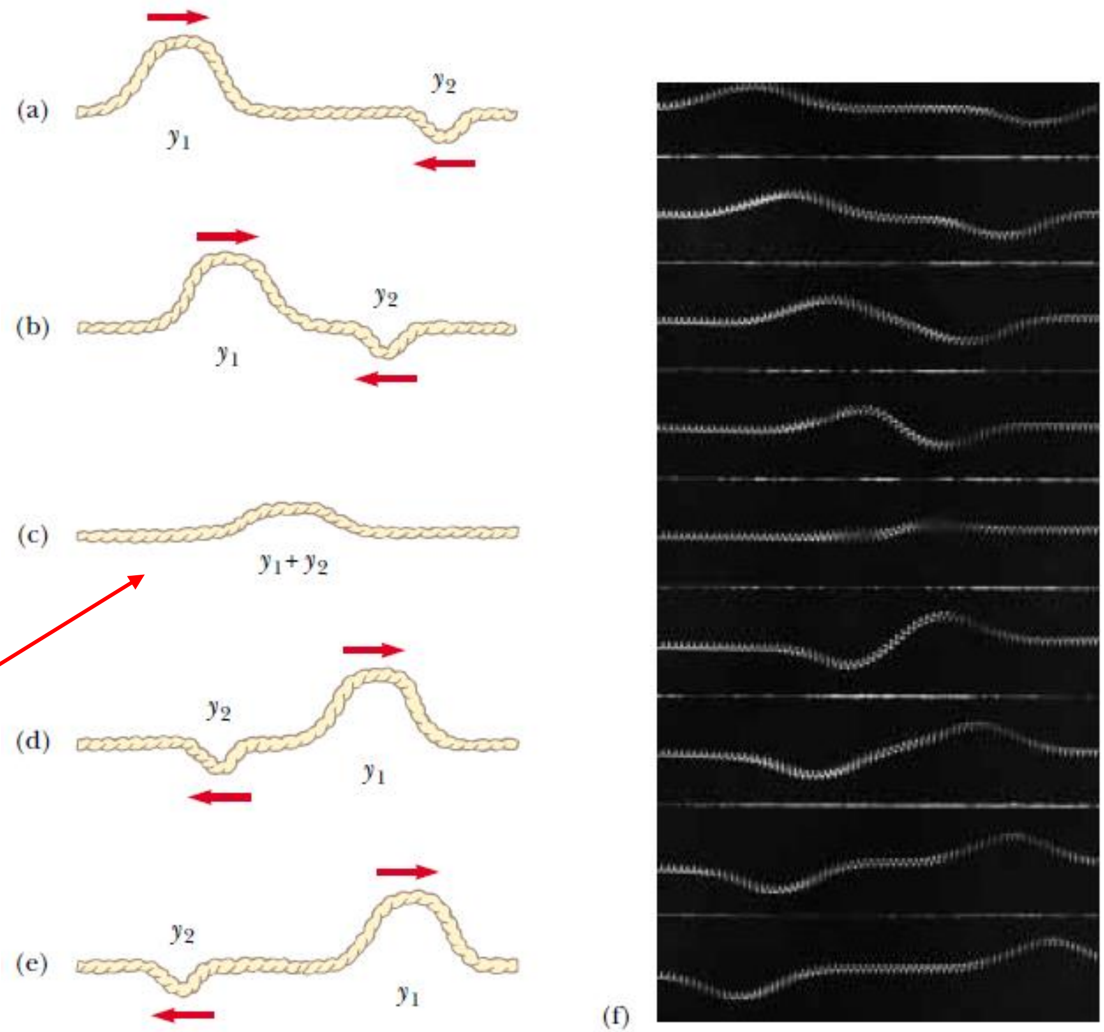
- Waves that obey the superposition principle are called linear waves and are generally characterized by small amplitudes. Waves that violate the superposition principle are called nonlinear waves and are often characterized by large amplitudes.
- We deal only with linear waves. That is, two traveling waves can pass through each other without being destroyed or even altered.



Interference

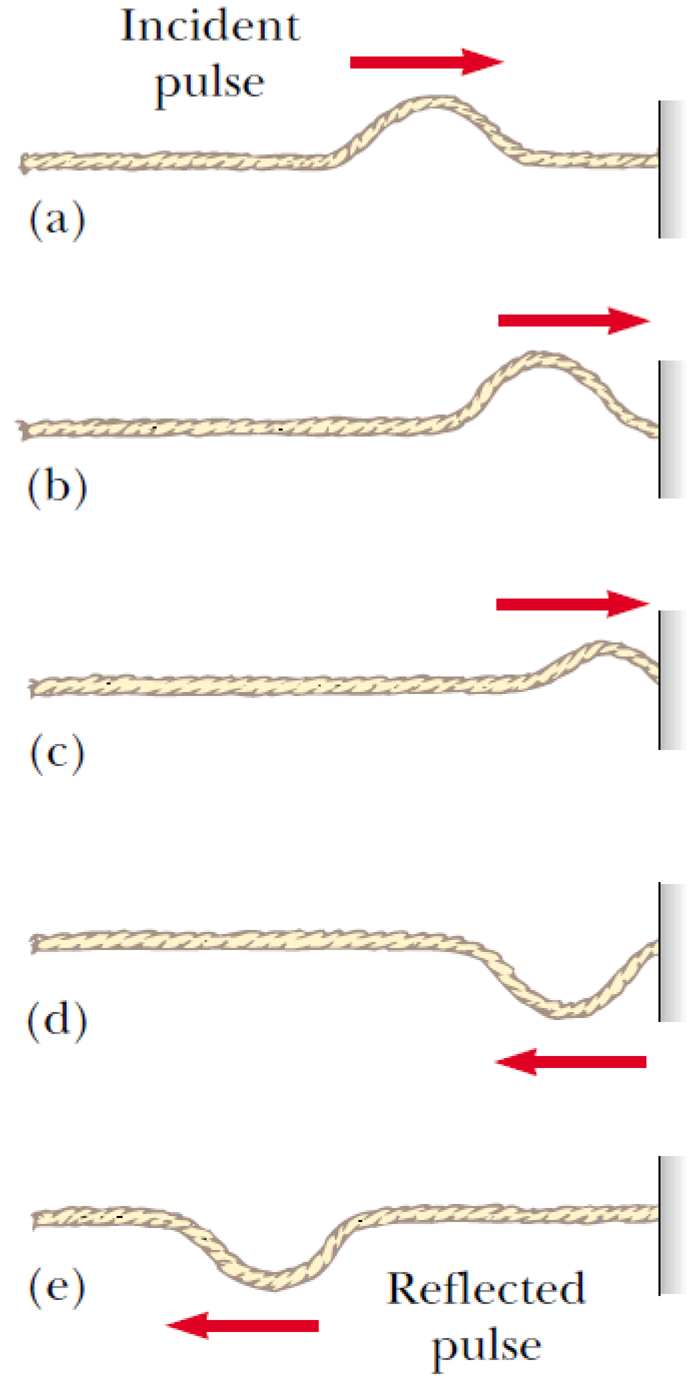
- **The combination of separate waves in the same region of space to produce a resultant wave is called interference.**

- **Constructive interference (the previous example)**
- **Destructive interference**



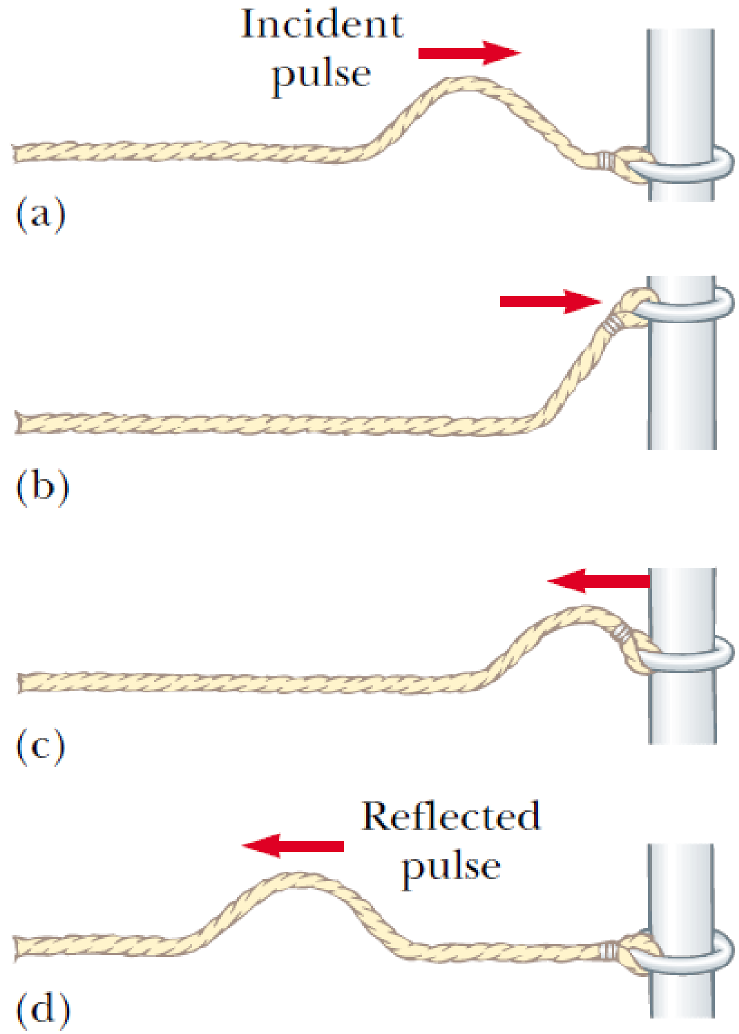
Reflection

- When the pulse reaches the support, a severe change in the medium occurs—the string ends.
- The wave undergoes reflection—that is, the pulse moves back along the string in the opposite direction.
- When the pulse reaches the fixed end of the string, the string produces an upward force on the support. By Newton's third law, the support must exert an equal and opposite (downward) reaction force on the string. This downward force causes the pulse to invert upon reflection.



Free Boundary Condition

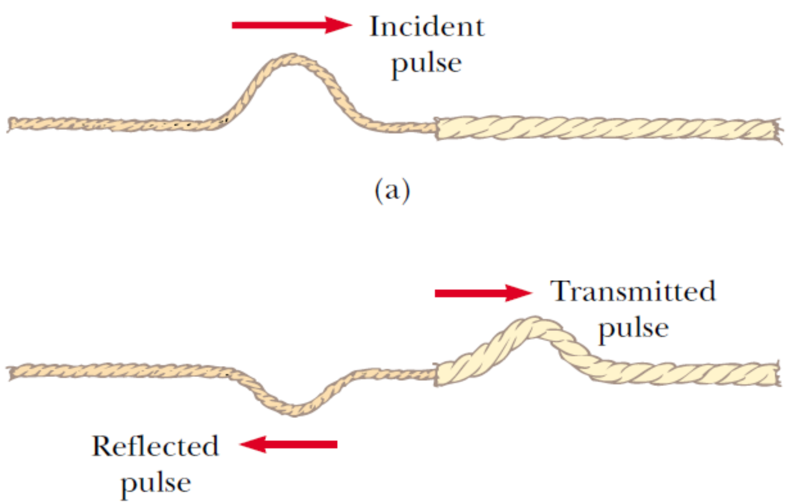
- The tension at the free end is maintained because the string is tied to a ring of negligible mass that is free to slide vertically on a smooth post. Again, the pulse is reflected, but this time it is not inverted. When it reaches the post, the pulse exerts a force on the free end of the string, causing the ring to accelerate upward. The ring overshoots the height of the incoming pulse, and then the downward component of the tension force pulls the ring back down.



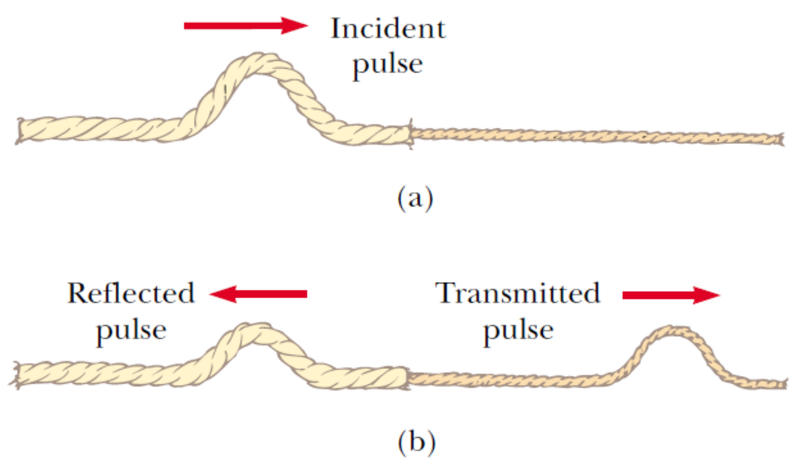
Transmission

- We may have a situation in which the boundary is intermediate between these two extremes. In this case, part of the incident pulse is reflected and part undergoes **transmission**—that is, some of the pulse passes through the boundary.

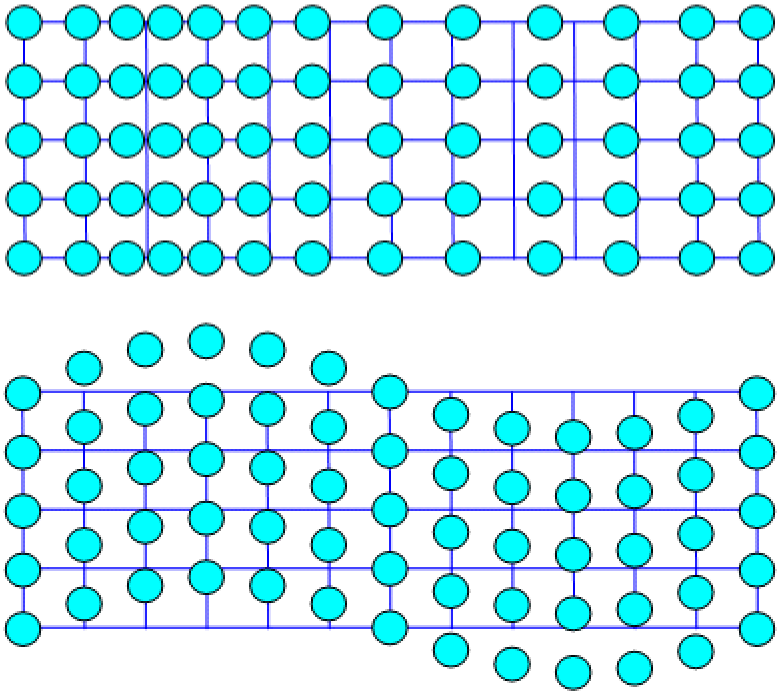
more like “fixed B.C.”



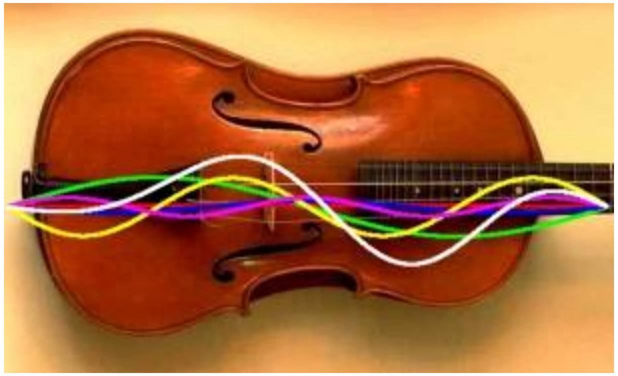
more like “free B.C.”



A Wave of Atoms



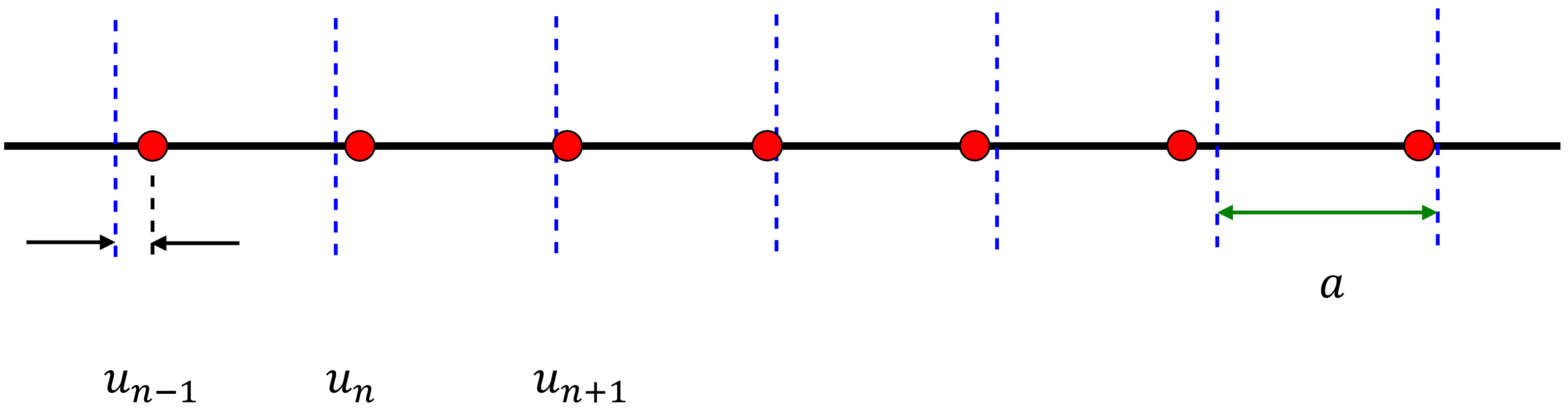
Collective motion of the atoms oscillating around their equilibrium positions are wave-like excitations, also known as phonons.



Wave is different from oscillation in the sense that the particles oscillate as a function of both time and position.

Model of Monoatomic Crystal

- **One-dimensional example:**



Equilibrium positions: $X_n = na$

Deviations from the equilibrium: $u_n = x_n - X_n$

$$u_{n+1} - u_n = [x_{n+1} - (n+1)a] - [x_n - na] = x_{n+1} - x_n - a$$

Interatomic Potential

- **We consider nearest-neighbor interactions only.**

$$\phi(x_{n+1} - x_n) = \phi_0 + \frac{1}{2}K(x_{n+1} - x_n - a)^2 + \cdots$$

$$= \phi_0 + \frac{1}{2}K(u_{n+1} - u_n)^2 + \cdots$$

First derivative vanishes at the equilibrium!

In the harmonic approximation,

$$U^{harm} = \frac{1}{2}K\sum_n (u_{n+1} - u_n)^2$$

Equations of Motion

$$M\ddot{u}_n = -\frac{dU^{harm}}{du_n} = K[u_{n+1} - u_n] - K[u_n - u_{n-1}]$$

$$= (Ka)\left.\frac{\Delta u}{\Delta x}\right|_{X_n+\frac{a}{2}} - (Ka)\left.\frac{\Delta u}{\Delta x}\right|_{X_n-\frac{a}{2}}$$

Note $\left.\frac{\Delta u}{\Delta x}\right|_{X_n+\frac{a}{2}} = \frac{u_{n+1} - u_n}{a} \xrightarrow{\Delta x \to 0} \left.\frac{\partial u}{\partial x}\right|_{X_n+\frac{a}{2}}$

Equations of Motion

$$M\ddot{u}_n = -\frac{dU^{harm}}{du_n} = K[u_{n+1} - u_n] - K[u_n - u_{n-1}]$$

$$= (Ka)\frac{\Delta u}{\Delta x}\bigg|_{X_n+\frac{a}{2}} - (Ka)\frac{\Delta u}{\Delta x}\bigg|_{X_n-\frac{a}{2}}$$

For wavelength much greater than *a*,

$$M\left(\frac{\partial^2 u}{\partial t^2}\right)_{X_n} = (Ka)\left[\left(\frac{\partial u}{\partial x}\right)_{X_n+\frac{a}{2}} - \left(\frac{\partial u}{\partial x}\right)_{X_n-\frac{a}{2}}\right]$$

$$= (Ka^2)\left(\frac{\partial^2 u}{\partial x^2}\right)_{X_n}$$

The Linear Wave Equation

The linear wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} \qquad \textbf{where} \qquad v = a\sqrt{\frac{K}{M}}$$

The linear wave equation applies in general to various types of waves. For waves on strings, *u* represents the vertical displacement of the string. For sound waves, *u* corresponds to displacement of air molecules from equilibrium or variations in either the pressure or the density of the gas through which the sound waves are propagating. In the case of electromagnetic waves, *u* corresponds to electric or magnetic field components.