General Physics II

Solution #6

2021/12/08

P6-1. A thin flake of mica ($n = 1.58$) is used to cover one slit of a double-slit interference arrangement. The central point on the viewing screen is now occupied by what had been the seventh bright side fringe ($m = 7$). If $\lambda = 550$ nm, what is the thickness of the mica?

Solution: Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness x is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where λ_m is the wavelength in the mica and n is the index of refraction of the mica.

The relationship $\lambda_m = \lambda/n$ is used to substitute for λ_m. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

$$x = \frac{7\lambda}{n-1} = \frac{7 \times (550 \times 10^{-9})}{1.58 - 1} = 6.64 \times 10^{-6}\ \mathrm{m}.$$

P6-2. Add the quantities $y_1 = 10 \sin \omega t$, $y_2 = 15 \sin(\omega t + 30^\circ)$, and $y_3 = 5.0 \sin(\omega t - 45^\circ)$ using the phasor method.

Solution: In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ and add them as vectors:

$$y_h = 10\cos 0^\circ + 15\cos 30^\circ + 5.0\cos(-45^\circ) = 26.5$$

$$y_v = 10\sin 0^\circ + 15\sin 30^\circ + 5.0\sin(-45^\circ) = 4.0$$

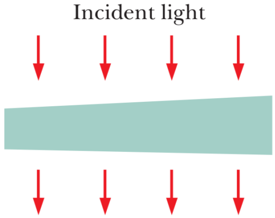
so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 26.8 \approx 27$$

$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 8.5^\circ.$$

Thus, $y = y_1 + y_2 + y_3 = y_R \sin(\omega t + \beta) = 27\sin(\omega t + 8.5^\circ)$.

P6-3. A broad beam of light of wavelength 630 nm is incident at 90° on a thin, wedge-shaped film with index of refraction 1.50. Transmission gives 10 bright and 9 dark fringes along the film's length. What is the left-to-right change in film thickness?



Solution: Assume the wedge-shaped film is in air, so the wave reflected from one surface undergoes a phase change of π rad while the wave reflected from the other surface does not. At a place where the film thickness is L, the condition for fully constructive interference is $2nL = (n + \frac{1}{2})\lambda$ where n is the index of refraction of the film, λ is the wavelength in vacuum, and m is an integer. The ends of the film are bright. Suppose the end where the film is narrow has thickness L_1 and the bright fringe there corresponds to $m = m_1$. Suppose the end where the film is thick has thickness L_2 and the bright fringe there corresponds to $m = m_2$.

Since there are ten bright fringes, $m_2 = m_1 + 9$. Subtract $2nL_1 = (m_1 + \frac{1}{2})\lambda$ from $2nL_2 = (m_1 + 9 + \frac{1}{2})\lambda$ to obtain $2n\Delta L = 9\lambda$, where $\Delta L = L_2 - L_1$ is the change in the film thickness over its length. Thus,

$$\Delta L = \frac{9\lambda}{2n} = \frac{9 \times (630 \times 10^{-9})}{2 \times 1.50} = 1.89 \times 10^{-6}\ \mathrm{m}.$$

P6-4. (a) Show that the values of α at which intensity maxima for single-slit diffraction occur can be found exactly by differentiating $I(\theta) = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$ with respect to α and equating the result to zero, obtaining the condition $\tan\alpha = \alpha$. To find values of α satisfying this relation, plot the curve $y = \tan\alpha$ and the straight line $y = \alpha$ and then find their intersections, or use a calculator to find an appropriate value of α by trial and error. Next, from $\alpha = (m + \frac{1}{2})\pi$, determine the values of m associated with the maxima in the single-slit pattern. (These m values are *not* integers because secondary maxima do not lie exactly halfway between minima.) What are the (b) smallest α and (c) associated m, the (d) second smallest α and (e) associated m, and the (f) third smallest α and (g) associated m?

Solution:

(a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2 \alpha}{\alpha^2}.$$

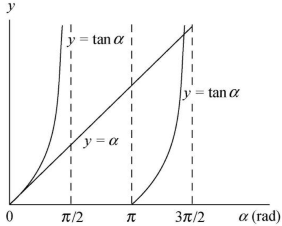
To locate the extrema, we set the derivative of I with respect to α equal to zero and solve for α. The derivative is

$$\frac{dI}{d\alpha} = 2I_m \frac{\sin \alpha}{\alpha^3}(\alpha \cos \alpha - \sin \alpha).$$

The derivative vanishes if $\alpha = m\pi$, where m is a nonzero integer. These are the intensity minima: $I = 0$ for $\alpha = m\pi$. The derivative also vanishes for $\alpha \cos \alpha - \sin \alpha = 0$. This condition can be written $\tan \alpha = \alpha$. These implicitly locate the maxima.

(b) The values of α that satisfy $\tan\alpha = \alpha$. can be found by trial and error on a pocket calculator or computer. Each of them is slightly less than one of the values $(m + \frac{1}{2})\pi$ rad, so we start with these values. They can also be found graphically. As in

the diagram that follows, we plot $y = \tan\alpha$ and $y = \alpha$ on the same graph. The intersections of the line with the $\tan\alpha$ curves are the solutions. The smallest α is $\alpha = 0$.



(c) We write $\alpha = (n + \frac{1}{2})\pi$ for the maxima. For the central maximum, $\alpha = 0$ and $m = -1/2 = -0.500$.

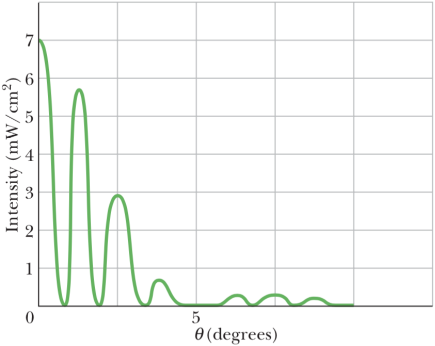
(d) The next one can be found to be $\alpha = 4.493$ rad.

(e) For $\alpha = 4.4934$, $m = 0.930$.

(f) The next one can be found to be $\alpha = 7.725$ rad.

(g) For $\alpha = 7.7252$, $m = 1.96$.

P6-5. Light of wavelength 440 nm passes through a double slit, yielding a diffraction pattern whose graph of intensity I versus angular position θ is shown in figure. Calculate (a) the slit width and (b) the slit separation. (c) Verify the displayed intensities of the $m = 1$ and $m = 2$ interference fringes.



Solution:

(a) The first minimum of the diffraction pattern is at 5.00°, so

$$a = \frac{\lambda}{\sin\theta} = \frac{0.440}{\sin 5.00^\circ} = 5.05\ \mu\mathrm{m}.$$

(b) Since the fourth bright fringe is missing,
$d = 4a = 4 \cdot 5.05 = 20.2\ \mu\mathrm{m}.$
(c) For the $m = 1$ bright fringe,

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi \cdot 5.05 \cdot \sin 1.25^\circ}{0.440} = 0.787\ \mathrm{rad}.$$

Consequently, the intensity of the $m = 1$ fringe is

$$I = I_m \left(\frac{sin\alpha}{\alpha} \right)^2 = 7.0 \cdot \left(\frac{\sin 0.787}{0.787} \right)^2 = 5.7 \ \mathrm{mW/cm^2},$$

which agrees with figure. Similarly for m = 2, the intensity is $I = 2.9 \ \mathrm{mW/cm^2}$, also in agreement with figure.

P6-6. Derive this expression for the intensity pattern for a three-slit "grating":

$$I = \frac{1}{9} I_m (1 + 4\cos\phi + 4\cos^2\phi),$$

where $\phi = (2\pi d \sin\theta)/\lambda$ and $a \ll \lambda$.

Solution: Since the slit width is much less than the wavelength of the light, the central peak of the single-slit diffraction pattern is spread across the screen and the diffraction envelope can be ignored. Consider three waves, one from each slit. Since the slits are evenly spaced, the phase difference for waves from the first and second slits is the same as the phase difference for waves from the second and third slits. The electric fields of the waves at the screen can be written as

$$E_1 = E_0 \sin(\omega t)$$

$$E_2 = E_0 \sin(\omega t + \phi)$$

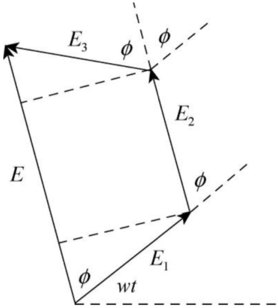
$$E_3 = E_0 \sin(\omega t + 2\phi)$$

where $\phi = (2\pi d/\lambda)\sin\theta$. Here d is the separation of adjacent slits and λ is the wavelength.

The phasor diagram is shown on the right. It yields

$$E = E_0(1 + 2\cos\phi)$$

for the amplitude of the resultant wave. Since the intensity of a wave is proportional to the square of the electric field, we may write



$I = AE_0^2(1 + 2\cos\phi)^2$, where A is a constant of proportionality. If I_m is the intensity at the center of the pattern, for which $\phi = 0$, then $I_m = 9AE_0^2$. We take A to be $I_m/9E_0^2$ and obtain

$$I = \frac{I_m}{9}(1 + 2\cos\phi)^2 = \frac{I_m}{9}(1 + 4\cos\phi + 4\cos^2\phi).$$