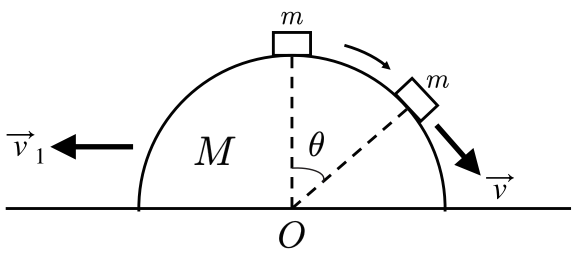
(a) Because the hemisphere can move freely on the frictionless table, it moves to the left when the block slides down. At the moment of detachment, suppose the velocity of the block relative to the hemisphere is $\overrightarrow{v}$, which is in the tangential direction, and the velocity of the hemisphere relative to the table is $\overrightarrow{v}_1$.



We first choose the table as the inertial frame of reference. On the one hand, we consider the block and the hemisphere as a system. The net external force in the horizontal direction is zero for this system, so the system's horizontal momentum is conserved:

$$m(v\cos\theta - v_1) - Mv_1 = 0$$

On the other hand, we consider the block, the hemisphere, and the Earth as a system. There is no non-conservative force doing work in this system, so the system's mechanical energy is conserved:

$$mgR = mgR\cos\theta + \frac{1}{2}m(v\cos\theta - v_1)^2 + \frac{1}{2}m(v\sin\theta)^2 + \frac{1}{2}Mv_1^2$$

The force between the block and the hemisphere is zero when they detach each other, leading to zero acceleration of the hemisphere. Therefore, at the moment of detachment, the hemisphere is also an inertial frame of reference. We switch to this reference frame, and have

$$mg\cos\theta = m\frac{v^2}{R}$$

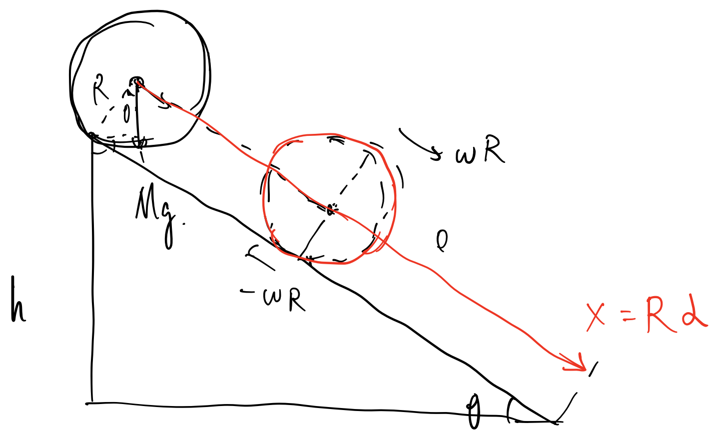
at the moment of detachment.

From the above three equations, we can get

$$\frac{m}{m+M}\cos^3\theta - 3\cos\theta + 2 = 0$$

(b) When $m \ll M$, we have $\cos\theta = 2/3$, thus $\theta = \arccos(2/3) \approx 48.2°$.

When $m \gg M$, we have $\cos^3\theta - 3\cos\theta + 2 = (\cos\theta - 1)^2(\cos\theta + 2) = 0$, thus $\cos\theta = 1$, $\theta = 0°$.



(1) 首先作受力分析

$$\begin{cases} Mg\sin\theta - f = M\ddot{x} \\ Mg\cos\theta - N = 0 \end{cases}$$

并且有 $Rf = I\ddot{\alpha}$，又有 $\underline{\ddot{x} = R\ddot{\alpha}}$

代入有 $\begin{cases} Mg\sin\theta - f = MR\ddot{\alpha} \\ Mg\sin\theta - \frac{I\ddot{\alpha}}{R} = M\ddot{\alpha}R \end{cases} \Rightarrow \ddot{\alpha}\left(RM + \frac{I}{R}\right) = Mg\sin\theta$，可以解出

$$\ddot{\alpha} = \frac{RMg\sin\theta}{MR^2+I} \qquad \begin{cases} \alpha = \frac{1}{2}\frac{RMg\sin\theta}{MR^2+I}t^2 \\ x = \frac{1}{2}\frac{R^2Mg\sin\theta}{MR^2+I}t^2 \end{cases} \Rightarrow \frac{h}{\sin\theta} = \frac{1}{2}\frac{R^2Mg\sin\theta}{MR^2+I}t^2$$

$$t^2 = \frac{2h(MR^2+I)}{(\sin^2\theta MgR^2)} \qquad I \propto MR^2$$

(2) 由(1)可以看出，$I\uparrow$ $t\uparrow$. 如果 $I \propto kMR^2$, MR 给定, $k\uparrow$

$I \propto k+1$ 也增大。即转动惯量增大，

转动需要的时间变长。

(3) $\mu N = f \Rightarrow \mu \cdot Mg\cos\theta = \dfrac{I(\cancel{R}Mg\sin\theta)}{\cancel{R}(MR^2+I)}$

$$\mu = \left(\frac{I}{MR^2+I}\right)\tan\theta.$$

(1)

$$\begin{cases} m_1\ddot{x}_1 = k(x_2-x_1) \\ m_2\ddot{x}_2 = -k(x_2-x_1)+k(x_3-x_2) \\ m_1\ddot{x}_3 = -k(x_3-x_2) \end{cases}$$ +5.

447

假设 $x_i = A_i e^{i(\omega t+\varphi)}$

$$\begin{cases} -m_1\omega^2 A_1 = k(A_2-A_1) \\ -m_2\omega^2 A_2 = -k(A_2-A_1)+k(A_3-A_2) \\ -m_1\omega^2 A_3 = -k(A_3-A_2) \end{cases}$$ +3

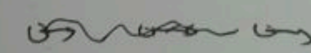
$$\begin{pmatrix} \frac{k}{m_1}-\omega^2 & -\frac{k}{m_1} & 0 \\ -\frac{k}{m_2} & \frac{2k}{m_2}-\omega^2 & \frac{-k}{m_2} \\ 0 & -\frac{k}{m_1} & \frac{k}{m_1}-\omega^2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = 0$$

$$\Rightarrow \left| \quad \right| = 0 \Rightarrow \omega^2\left[\omega^2-\frac{k}{m_1}-\frac{2k}{m_2}\right]\left[\omega^2-\frac{k}{m_1}\right]=0$$ +3

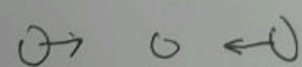
17

$$\Rightarrow \omega_1 = 0, \quad \omega_2 = \sqrt{\frac{k}{m_1}}, \quad \omega_3 = \sqrt{\frac{k}{m_1}+\frac{2k}{m_2}}$$

(1) $\omega_1 = 0 \quad \therefore A_1 = A_2 = A_3$ +2 反向



(2) $\omega_2 = \sqrt{\frac{k}{m_1}} \quad A_2 = 0, \quad A_1 + A_3 = 0$ +2



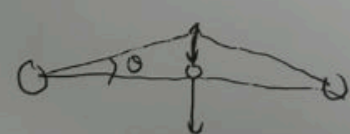
(3) $\omega_3 = \sqrt{\frac{k}{m_1}+\frac{2k}{m_2}} \quad A_1 = A_3 = -\frac{m_2}{2m_1}A_2$ +2

17+

[2] $F(x) = -2k(\sqrt{a_0^2+y^2}-a_0)\sin\theta$

$\sin\theta = \frac{y}{\sqrt{a_0^2+y^2}}$ +4

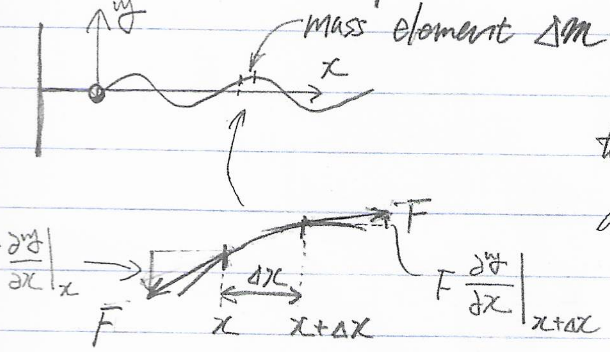
8



$$\therefore F(y) = -2k(\sqrt{a_0^2+y^2}-a_0)\frac{y}{\sqrt{a_0^2+y^2}}$$
$$= -2k\left[y - a_0 y\left(1+\frac{y^2}{a_0^2}\right)^{-\frac{1}{2}}\right]$$ +2
$$= -2k\left[y - y\left(1-\frac{y^2}{2a_0^2}\right)\right] = -ky^3/2a_0^2$$
$$\neq -ky$$ +2 (Not)

Galilean transformation of the wave equation. No. 1

An oscillatory wave traveling on a string with line density σ.



Assume that the magnitude F of the tension is const. throughout the string, and the amplitude of the oscillation is small.

(a) For mass element Δm at x,

Newton's eq. of y component reads.

$$\Delta m \frac{\partial^2 y}{\partial t^2} = F \left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - F \left.\frac{\partial y}{\partial x}\right|_{x} = F\left(\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_{x} \right).$$

Since $\Delta m \simeq \sigma \Delta x$.

$$\frac{\partial^2 y}{\partial t^2} \simeq \frac{F}{\sigma} \frac{1}{\Delta x}\left(\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_{x} \right)$$

$$\simeq \frac{F}{\sigma}\frac{\partial^2 y}{\partial x^2}$$

$$\therefore \frac{\partial^2 y}{\partial t^2} - \frac{F}{\sigma}\frac{\partial^2 y}{\partial x^2} = 0 \quad \text{——— ①}$$

(b) For a traveling wave $y = y(x-ct)$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left(-c\frac{dy(s)}{ds}\right) = c^2 \frac{d^2 y}{ds^2} \qquad \text{with } s \equiv x - ct$$

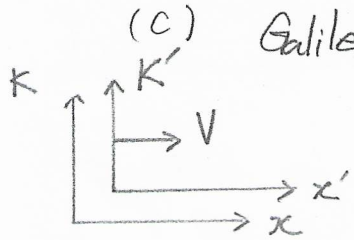
$$\frac{\partial^2 y}{\partial x^2} = \frac{d^2 y(s)}{ds^2}$$

Substituting them into Eq. ①, we get.

$$\left(c^2 - \frac{F}{\sigma}\right)\frac{d^2 y}{ds^2} = 0 \quad \text{for } \forall s \Rightarrow c^2 - \frac{F}{\sigma} = 0$$

$$\therefore c = \sqrt{\frac{F}{\sigma}} \quad /\!/ \quad \text{——— ②}$$

(c) Galilean transf.



$$\begin{cases} x' = x - Vt \\ t' = t. \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial x}\frac{\partial}{\partial t'} = \frac{\partial}{\partial x'} \quad //$$

$$\frac{\partial}{\partial t} = \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} = -V\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'} \quad //$$

(d) $$\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x'^2}$$

$$\frac{\partial^2}{\partial t^2} = \left(-V\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}\right)^2 = V^2\frac{\partial^2}{\partial x'^2} - 2V\frac{\partial^2}{\partial x'\partial t'} + \frac{\partial^2}{\partial t'^2}$$

$\therefore$ Wave eq. ① $\dfrac{\partial^2 y}{\partial x^2} - \dfrac{1}{c^2}\dfrac{\partial^2 y}{\partial t^2} = 0$ becomes

$$0 = \frac{\partial^2 y(x',t')}{\partial x'^2} - \frac{1}{c^2}\left[V^2\frac{\partial^2}{\partial x'^2} - 2V\frac{\partial^2}{\partial x'\partial t'} + \frac{\partial^2}{\partial t'^2}\right] y(x',t')$$

$$= \frac{\partial^2 y'}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 y'}{\partial t'^2} + \frac{2V}{c^2}\frac{\partial^2 y'}{\partial x'\partial t'} - \frac{V^2}{c^2}\frac{\partial^2 y'}{\partial x'^2} \quad //$$

$$\neq \frac{\partial^2 y'}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 y'}{\partial t'^2}$$

$\therefore$ wave eq. is not Galilean invariant.

(e) Wave speed is equal to c in the rest frame of the medium. //

The propagation speed c is specific to the rest frame of the medium.