浙江大學
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Computer Logic Design Fundamentals

Chapter 1 – Digital Systems and Information

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2023年9月

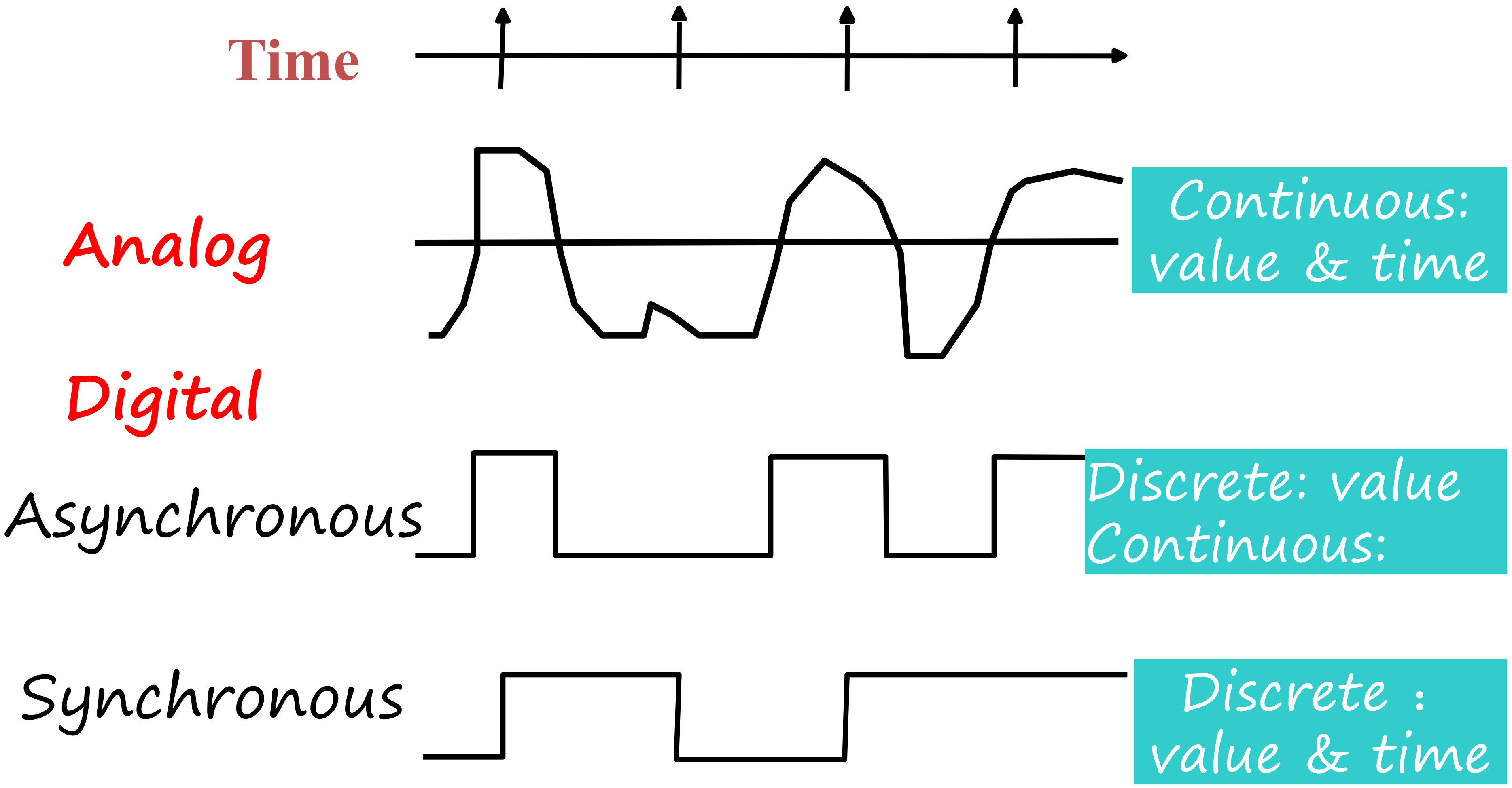
Overview

- **Signal**
- **Digital Systems**
- **Digital Computer**
- **Organization Of Computer**
- **Number Systems & Codes**

Signal

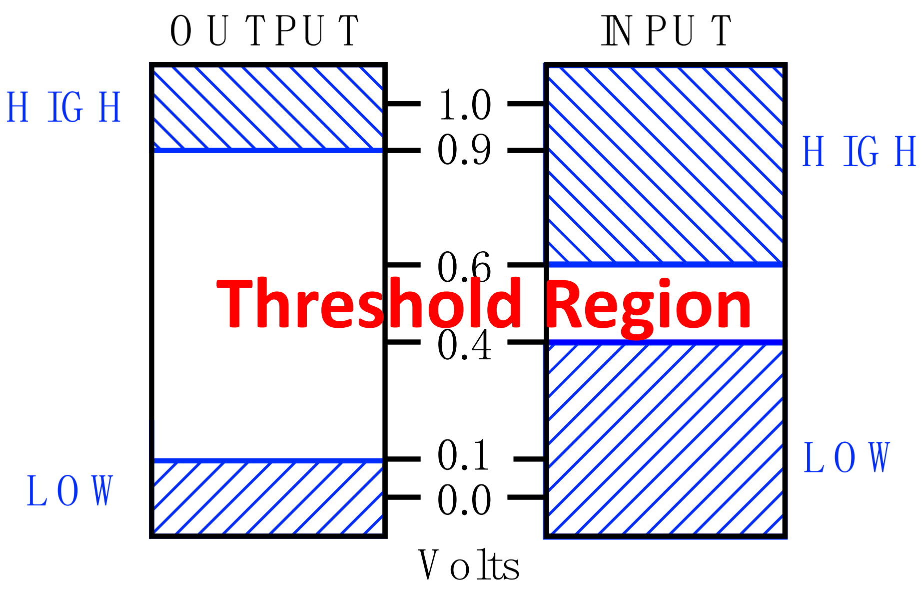
- Information variables
  - represented by physical quantities.
- For digital systems
  - the variables take on discrete values
  - Two level, or binary values are the most prevalent values in digital systems
- Represented abstractly by:
  - digits： 0 / 1
  - symbols：
    - False (F) / True (T)
    - Low (L) / High (H)
    - On / Off
- Binary values are represented by values or ranges of values of physical quantities

Time Sequence Signal

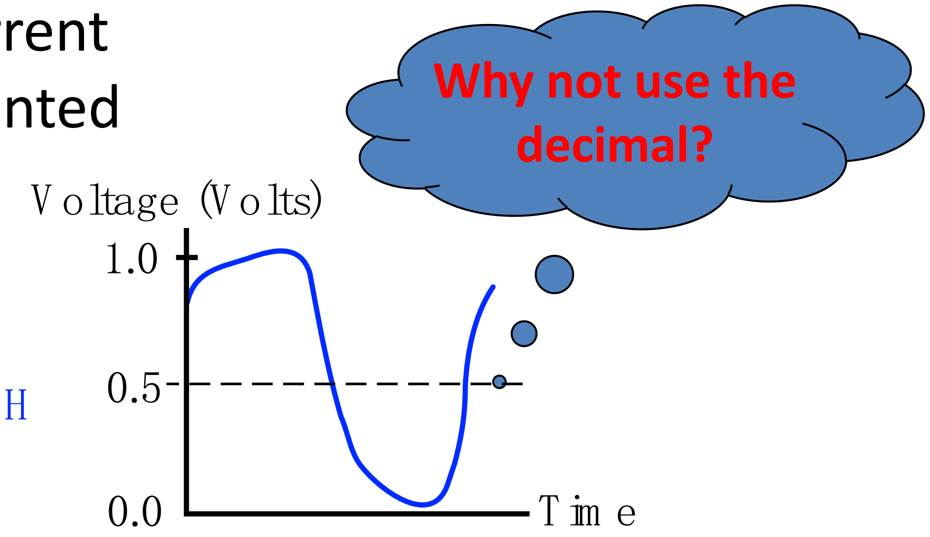


Physical Quantity Example: Voltage

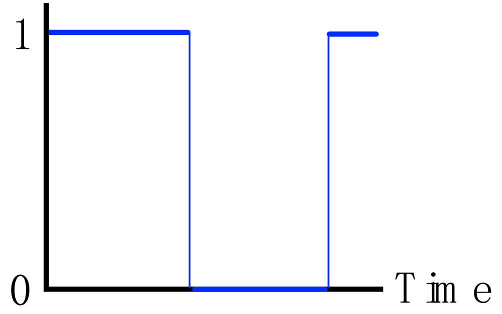
The most commonly used two-valued information is an electrical signal - voltage or current typically two discrete values represented by the voltage range of values



(a) Example voltage ranges



(b) Time-dependent Voltage



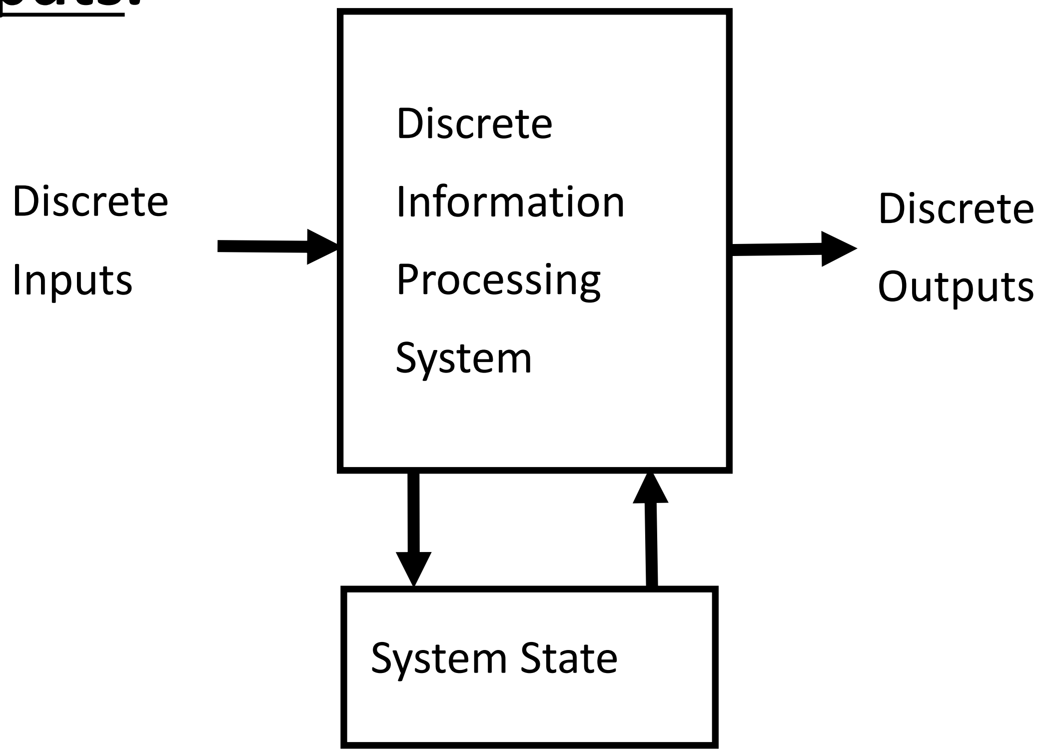
(c) Binary model of time-dependent voltage

Binary Values: Other Physical Quantities

- What are other physical quantities represent 0 and 1?
  - CPU — Voltage
  - Disk — Magnetic Field Direction
  - CD — Surface Pits/Light
  - Dynamic RAM — Electrical Charge

Digital System

- **Takes a set of discrete information inputs and discrete internal information (system state) and generates a set of discrete information outputs.**



Types of Digital Systems

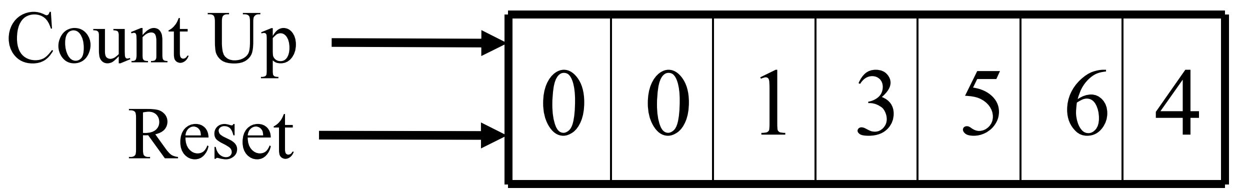
- **Combinational Logic System**
  - **No state present**
  - **Output function：** f_{Output} **= Function(Input)**

    General multivariate, multi-output function
- **Sequential System**
  - **State present**
  - **State updated at discrete times**
    - **Synchronous Sequential System**
  - **State updated at any time**
    - **Asynchronous Sequential System**
  - **State Function：** f_{State} **= Function (State, Input)**
  - **Output Function：** f_{Output} **= Function (State)**

    Or f_{Output} **=Function (State, Input)**

Digital System Example

A Digital Counter (e. g., odometer):

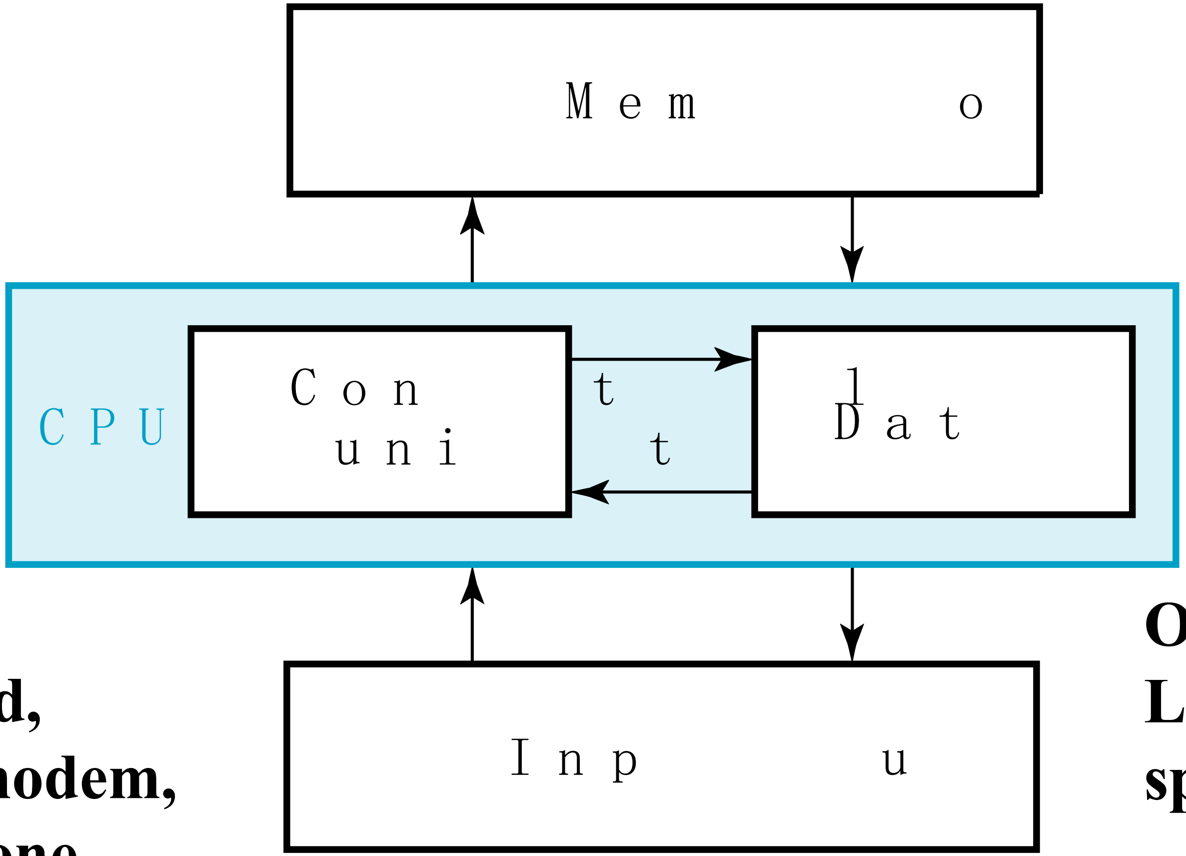


Inputs: Count Up, Reset
Outputs: Visual Display
State: "Value" of stored digits

Synchronous or Asynchronous?

Digital Computer Example

A common system used on the discrete elements in the information processing



Inputs: Keyboard, mouse, modem, microphone

Outputs: CRT, LCD, modem, speakers

Synchronous or Asynchronous?

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Digital Computer

1. Features: commonality, flexibility, versatility

- A common System for processing the discrete elements of the information

2. Information representation within Computer

- used the binary numerical system: 0 and 1
- A binary signal is represents one bit
- Multi-digit bit used to represent data & Instructions can be executed in the computer
- Analog done automatically converted into a digital-value used on analog-to-digital conversion apparatus

Organization Of Computer

1. Memory

- **Can be stored Program and Data from input & output, and intermediate results**
  - Main Memory
  - The external memory (as part of the peripheral)
  - Cache

2. Datapath(BUS)

- **The channel between the processor, memory, and input / output device (connection)**
  - Processor bus （within CPU）
  - I/O BUS: Different data transfer rates of the two buses, different bus data communication through the completion of the bus interface hardware

Organization Of Computer-2

3. Control unit

- **Monitoring the exchange of information between the different parts**

4. CPU(Central processor Unit)

- **Composed by the data path and control unit. The modern processor comprises 4 functional modules：CPU，FPU，MMU & Internal cache**
  - FPU(Floating-point unit): specific to the implementation of floating-point operations 。
  - MMU (Memory Management Unit): see the CPU storage device the size of multi-size larger than the actual physical RAM.

5. Input / output device （I/O）

- **Device for information processing systems interact with each other**

And Beyond – Embedded Systems

- **Specific computer systems**
  - Computers as integral parts of other products
- **Examples of embedded computers**
  - Microcomputers
  - Microcontrollers
  - Digital signal processors

Examples of Embedded

- **Examples of Embedded Systems Applications**
  - Cell phones
  - Automobiles
  - Video games
  - Copiers
  - Dishwashers
  - Flat Panel TVs
  - Global Positioning Systems

NUMBER SYSTEMS and Code

The rule of the number system that constraints on the number Value. The most commonly encountered daily in the decimal counting system, in Digital system widely used in the computer binary, octal and hexadecimal.

1、 radix and cardinality

Cardinality—Represents the number of digital collection (basic symbols) within Counting system

radix—Size of the collection(base)

e.g.: assume radix: R

R basic symbols， 0， 1， 2……,R

Every R Carry in 1

2、Weight

Bit weights： Determine the **digit position**(Weight value of Digit at some position)

Each Weight is a **Power** of R corresponding to the digit’s position

E.g.： 2*3*56 in Decimal， 3’ **Weight** is 10^2

for 8421 Coder first bit **Weight** is 8

3、 Representation Method for R-ary

（N Bit digits from left to right, Size: m+n）

represented by a string of digits $0 \le A_i \le R$

$$(N)_R = (A_{n-1} A_{n-2} A_{n-3} \ldots A_1 A_0 \bullet A_{-1} A_{-2} A \ldots A_{-m+1} A_m)_R$$

MSB LSB

represents the power series

$$(N)_R = \left(\sum_{i=-m}^{n-1} A_i R^i \right)_R$$

Instance represents a decimal number

radix： R=10

basic symbols： 0,1,2,3…9

Weight： $W_i = 10^i$

Representation：

$$(N)_{10} = \left(\sum_{i=-m}^{n-1} A_i 10^i\right)_{10} =$$

$$A_{n-1} \bullet 10^{n-1} + A_{n-2} \bullet 10^{n-2} + \ldots + A_1 \bullet 10^1 + A_0 \bullet 10^0 + A_{-1} \bullet 10^{-1} + \ldots + A_{-m} \bullet 10^{-m}$$

e.g. $(123.45)_{10} = 1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0 + 4 \cdot 10^{-1} + 5 \cdot 10^{-2}$

Instance represents a binary number

1、 Representation

radix： R=2

basic symbols： 0，1

Weight： Wi＝2^i

Representation：

$$(N)_2 = \left(\sum_{i=-m}^{n-1} A_i 2^i\right)_2 =$$

$$A_{n-1} \bullet 2^{n-1} + A_{n-2} \bullet 2^{n-2} + \ldots + A_1 \bullet 2^1 + A_0 \bullet 2^0 + A_{-1} \bullet 2^{-1} + \ldots\ldots + A_{-m} \bullet 2^{-m}$$

e.g.： $(1011.101)_2 = 1\cdot2^3+0\cdot2^2+1\cdot2^1+1\cdot2^0+1\cdot2^{-1}+0\cdot2^{-2}+1\cdot2^{-3}$

2. Rules of operation

ADD: $0+0=0$ $\quad 0+1=1$

$1+0=1$ $\quad 1+1=10$

MUL: $0\times 0=0$ $\quad 0\times 1=0$

$1\times 0=0$ $\quad 1\times 1=1$

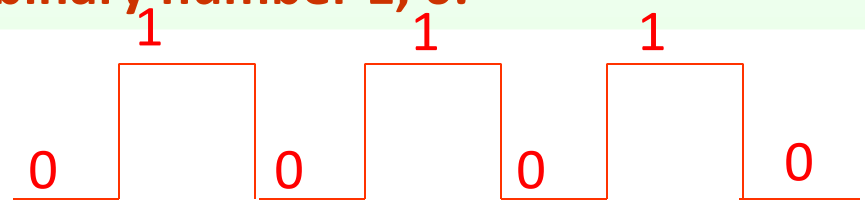
3. Physical representation：

Convenience： with transistors or magnetic

Trouble： written 、 memory –

Octal or hexadecimal abbreviations

The high / low voltage can be used to represent the binary number 1, 0.



4. May use Boolean algebra

Common values

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Instance represents a octal number

radix： R=8

basic symbols： 0、1、3…、7

Weight: $W_i = 8^i$

Representation：

$$(N)_8 = \left(\sum_{i=-m}^{n-1} A_i 8^i\right)_8 =$$

$$A_{n-1} \bullet 8^{n-1} + A_{n-2} \bullet 8^{n-2} + \ldots + A_1 \bullet 8^1 + A_0 \bullet 8^0 + A_{-1} \bullet 8^{-1} + \ldots\ldots + A_{-m} \bullet 8^{-m}$$

e.g.: $(567.125)_8$

Instance represents a Hexadecimal number

radix： R=16

basic symbols： 0、1、3…、9, A、B …、F

Weight： $W_i = 16^i$

Representation：

$$(N)_{16} = \left(\sum_{i=-m}^{n-1} A_i 16^i\right)_{16} =$$

$$A_{n-1}\bullet 16^{n-1} + A_{n-2}\bullet 16^{n-2} + \ldots + A_1\bullet 16^1 + A_0\bullet 16^0 + A_{-1}\bullet 16^{-1} + \ldots + A_{-m}\bullet 16^{-m}$$

e.g.: $(5AF.9B)_{16}$

If remember then benefit

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

1. Adds and subtracts

example 1： $1100+10001=11101$

$$\begin{array}{r} 01100 \\ +\ \underline{10001} \\ 11101 \end{array}$$

example 2： $10011-11110=-01011$

$$\begin{array}{r} 11110 \\ -\ \underline{10011} \\ -\ 01011 \end{array}$$

（ Set minus sign for difference if Small minus big number） example 3：

$$(59F)_{16}+(E46)_{16}=(13E5)_{16}$$

$$\begin{array}{r} 59F \\ +\quad E46 \\ \hline 13E5 \end{array}$$

2. Multiplication

example 1：

$$(762)_{8}\times(45)_{8}=(43772)_{8}$$

$$\begin{array}{r} 762 \\ \times\quad 45 \\ \hline 4672 \\ 3710 \\ \hline 43772 \end{array}$$

3. BCD adder

example :

$$448+489=(0100\ 0100\ 1000)_{BCD}+(0100\ 1000\ 1001)_{BCD}$$

$$\begin{array}{rr} & 0100\ 0100\ 1000 \\ + & 0100\ 1000\ 1001 \\ \hline & 1000\ 1101\ 0001 \\ + & 0110\ 0110 \\ \hline & 1001\ 0011\ 0111 \end{array}$$

illustrate：When each the sum is greater than 9 or Carry, need to adjustments with using **plus 6**

Converting from 2、8 、16

1. The right expansion method

2. Fractional shift method

3. Digital replacement method

Converting between binary and octal

原理：由于 $2^3 = 8$　　故三位二进制能表示为一位八进制数

方法：以小数点为中心——整数右对齐

——小数左对齐

例：　$(67.731)_8 = (110\ 111\ .111\ 011\ 001)_2$

$(312.64)_8 = (011\ 001\ 010\ .\ 110\ 1)_2$

$(11\ 111\ 101\ .\ 010\ 011\ 11)_2 = (375.236)_8$

$(10\ 110.11)_2 = (26.6)_8$

Converting between binary and Hexadecimal

原理：由于 $2^4 = 16$ 故四位二进制能表示为一位十六进制数

方法：以小数点为中心——整数右对齐

——小数左对齐

例： $(3AB4.1)_{16} =（0111\ 1010\ 1011\ 100\ .0001)_2$

$(21A.5)_{16}=(0010\ 0001\ 1010\ .\ 0101)_2$

$(1001101.01101)_2= (\ 0100\ \ 1101.\ 0110\ 1000\ )_2=(4D.68)_{16}$

$(111\ 1101\ .\ 0100\ 1111)_2=(7D.4F)_{16}$

$(110\ 0101.101)_2=(65.A)_{16}$

Converting between binary and decimal

Converting Binary to Decimal

原理：权展开表达式

方法：权相加——权展开十进制相加

例：$(110\ 0101.101)_2 = 1*2^6+1*2^5+0*2^4+0*2^3+$

$+1*2^2+0*2^1+1*2^0+1*2^{-1}+0*2^{-2}+1*2^{-3}$

$=(813.625)_{10}$

Converting Decimal to Binary

原理：整数——权展开式除2，余数构成最低位

小数——权展开式乘2，整数构成最高位

方法：整数——除2取余

小数——乘2取整

例：$(725.678) = (10\ 1101\ 0101.1010\ 1101\ 1001)_2$

$= (2D5.AD9)_{16}$

1. The integer portion: divided by 2, to take the remainder

e.g.: **Converting** $(725)_{10}$ **to Binary**

$$(725)_{10} = (k_{n-1}\ k_{n-2} \cdots k_1\ k_o)_2$$

$$= k_{n-1}\times 2^{n-1} + k_{n-2}\times 2^{n-2} + \cdots k_1\times 2^1 + k_o\times 2^0$$

$$= 2(k_{n-1}\times 2^{n-2} + k_{n-2}\times 2^{n-3} + \cdots k_1) + k_o$$

1

$$(362+\tfrac{1}{2})_{10} = k_{n-1}\times 2^{n-2} + k_{n-2}\times 2^{n-3} + \cdots k_1 + \frac{k_o}{2}$$

0

$$(181+\frac{0}{2})_{10} = k_{n-1}\times 2^{n-3} + k_{n-2}\times 2^{n-4} + \cdots k_2 + \frac{k_1}{2}$$

$$\vdots$$

$(725)_{10}$ = **(10 1101 0101)**$_2$

- ***short division:*** **The integer portion:**

divided by 2, to take the remainder

$$(725)_{10} = (\mathbf{10\ 1101\ 0101})_2$$

```
2|725
 2|362............1
 2|181............0
   2|90...........1
   2|45...........0
   2|22...........1
   2|11...........0
     2|5..........1
     2|2..........1
     2|1..........0
     2|0..........1
```

2. Fractional part: multiplied by 2，

to take the integral numb

1

e.g.： Converting $(0.678)_{10}$ **to Binary**

1

$$(0.678)_{10} = \frac{k_{-1}}{2} + \frac{1}{2}(k_{-2} \times 2^{-1} + \cdots + k_{-m} \times 2^{-m+1})$$

0

$$(1+0.356)_{10} = k_{-1} + (k_{-2} \times 2^{-1} + \cdots + k_{-m} \times 2^{-m+1})$$

$$(0+0.712)_{10} = k_{-2} + (k_{-3} \times 2^{-1} + \cdots + k_{-m} \times 2^{-m+2})$$

$$(1+0.424)_{10} = a_{-3} + (k_{-4} \times 2^{-1} + \cdots + k_{-m} \times 2^{-m+3})$$

$$\vdots$$

$(0.678)_{10} = (0.\mathbf{1010\ 1101\ 1001})_2$

Note: **can not be accurately converted**

Fractional part: multiplied by 2，

to take the integral number

$2 \times 0.678 \ldots\ldots\ldots\ldots = 1.356$

$2 \times 0.356 \ldots\ldots\ldots\ldots = 0.712$

$2 \times 0.712 \ldots\ldots\ldots\ldots = 1.424$

$2 \times 0.424 \ldots\ldots\ldots\ldots = 0.848$

$2 \times 0.848 \ldots\ldots\ldots\ldots = 1.696$

$2 \times 0.696 \ldots\ldots\ldots\ldots = 1.392$

$2 \times 0.392 \ldots\ldots\ldots\ldots = 0.784$

$2 \times 0.784 \ldots\ldots\ldots\ldots = 1.568$

$2 \times 0.568 \ldots\ldots\ldots\ldots = 1.136$

$2 \times 0.136 \ldots\ldots\ldots\ldots = 0.272$

$2 \times 0.272 \ldots\ldots\ldots\ldots = 0.544$

$2 \times 0.544 \ldots\ldots\ldots\ldots = 1.088$

3.3位二进制数为1位十进制数

$$(0.678)_{10} = (0.1010\ 1101\ 1001)_2$$

Binary Numbers and Binary Coding

- **Flexibility of representation**
  - **Within constraints below, can assign any binary combination (called a code word) to any data as long as data is uniquely encoded.**
- **Information Types**
  - **Numeric**
    - **Must represent range of data needed**
    - **Very desirable to represent data such that simple, straightforward computation for common arithmetic operations permitted**
    - **Tight relation to binary numbers**
  - **Non-numeric**
    - **Greater flexibility since arithmetic operations not applied.**
    - **Not tied to binary numbers**

Non-numeric Binary Codes

- Given *n* binary digits (called bits), a binary code is a mapping from a set of represented elements to a subset of the 2^n binary numbers.
- Example: A binary code for the seven colors of the rainbow
- Code 100 is not used

Color	Binary Number
Red	000
Orange	001
Yellow	010
Green	011
Blue	101
Indigo	110
Violet	111

Number of Bits Required

- **Given M elements to be represented by a binary code, the minimum number of bits, *n*, needed, satisfies the following relationships:**

  $$2^n > M >= 2^{(n-1)}$$

  $n = \lceil \log_2 M \rceil$ where $\lceil x \rceil$, called the *ceiling function*, is the integer greater than or equal to x.

- **Example: How many bits are required to represent decimal digits with a binary code?**

Number of Elements Represented

- **Given n digits in radix r, there are r^n distinct elements that can be represented.**
- **But, you can represent m elements, m < r^n**
- **Examples:**
  - **You can represent 4 elements in radix r = 2 with n = 2 digits: (00, 01, 10, 11).**
  - **You can represent 4 elements in radix r = 2 with n = 4 digits: (0001, 0010, 0100, 1000).**
  - **This second code is called a "<u>one hot</u>" code.**

DECIMAL CODES - Binary Codes for Decimal Digits

There are over 8,000 ways that you can chose 10 elements from the 16 binary numbers of 4 bits. A few are useful:

Decimal	8,4,2,1	Excess3	8,4, -2, -1	Gray
0	0000	0011	0000	0000
1	0001	0100	0111	0100
2	0010	0101	0110	0101
3	0011	0110	0101	0111
4	0100	0111	0100	0110
5	0101	1000	1011	0010
6	0110	1001	1010	0011
7	0111	1010	1001	0001
8	1000	1011	1000	1001
9	1001	1100	1111	1000

Binary Coded Decimal (BCD)

- **The BCD code is the 8,4,2,1 code.**
- **8, 4, 2, and 1 are weights**
- **BCD is a *weighted* code**
- **This code is the simplest, most intuitive binary code for decimal digits and uses the same powers of 2 as a binary number, but only encodes the first ten values from 0 to 9.**
- **Example: 1001 (9) = 1000 (8) + 0001 (1)**
- **How many "invalid" code words are there?**
- **What are the "invalid" code words?**

Excess 3 Code and 8, 4, –2, –1 Code

Decimal	Excess 3	8, 4, –2, –1
0	0011	0000
1	0100	0111
2	0101	0110
3	0110	0101
4	0111	0100
5	1000	1011
6	1001	1010
7	1010	1001
8	1011	1000
9	1100	1111

What interesting property is common to these two codes?

Warning: Conversion or Coding?

- **Do NOT mix up conversion of a decimal number to a binary number with coding a decimal number with a BINARY CODE.**
- **$13_{10} = 1101_2$ (This is conversion)**
- **13 $\Leftrightarrow$ 0001|0011 (This is coding)**

BCD Arithmetic

Given a BCD code, we use binary arithmetic to add the digits:

8	1000	Eight
+5	+0101	Plus 5
13	1101	is 13 (> 9)

Note that the result is MORE THAN 9, so must be represented by two digits!

To correct the digit, subtract 10 by adding 6 modulo 16.

8	1000	Eight
+5	+0101	Plus 5
13	1101	is 13 (> 9)
	+0110	so add 6
carry = 1	0011	leaving 3 + cy
	0001 | 0011	Final answer (two digits)

If the digit sum is > 9, add one to the next significant digit

BCD Addition Example

- **Add 2905_{BCD} to 1897_{BCD} showing carries and digit corrections.**

$$
\begin{array}{rcccc}
 & & & & 0 \\
 & 0001 & 1000 & 1001 & 0111 \\
+ & \underline{0010} & \underline{1001} & \underline{0000} & \underline{0101} \\
 & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_
\end{array}
$$

ALPHANUMERIC CODES - ASCII Character Codes

- **American Standard Code for Information Interchange (Refer to Table 1 -4 in the text)**
- **This code is a popular code used to represent information sent as character-based data. It uses 7-bits to represent:**
  - **94 Graphic printing characters.**
  - **34 Non-printing characters**
- **Some non-printing characters are used for text format (e.g. BS = Backspace, CR = carriage return)**
- **Other non-printing characters are used for record marking and flow control (e.g. STX and ETX start and end text areas).**

ASCII Properties

ASCII h

Digits 0 ... 39_{16}

Upper c

Lower c

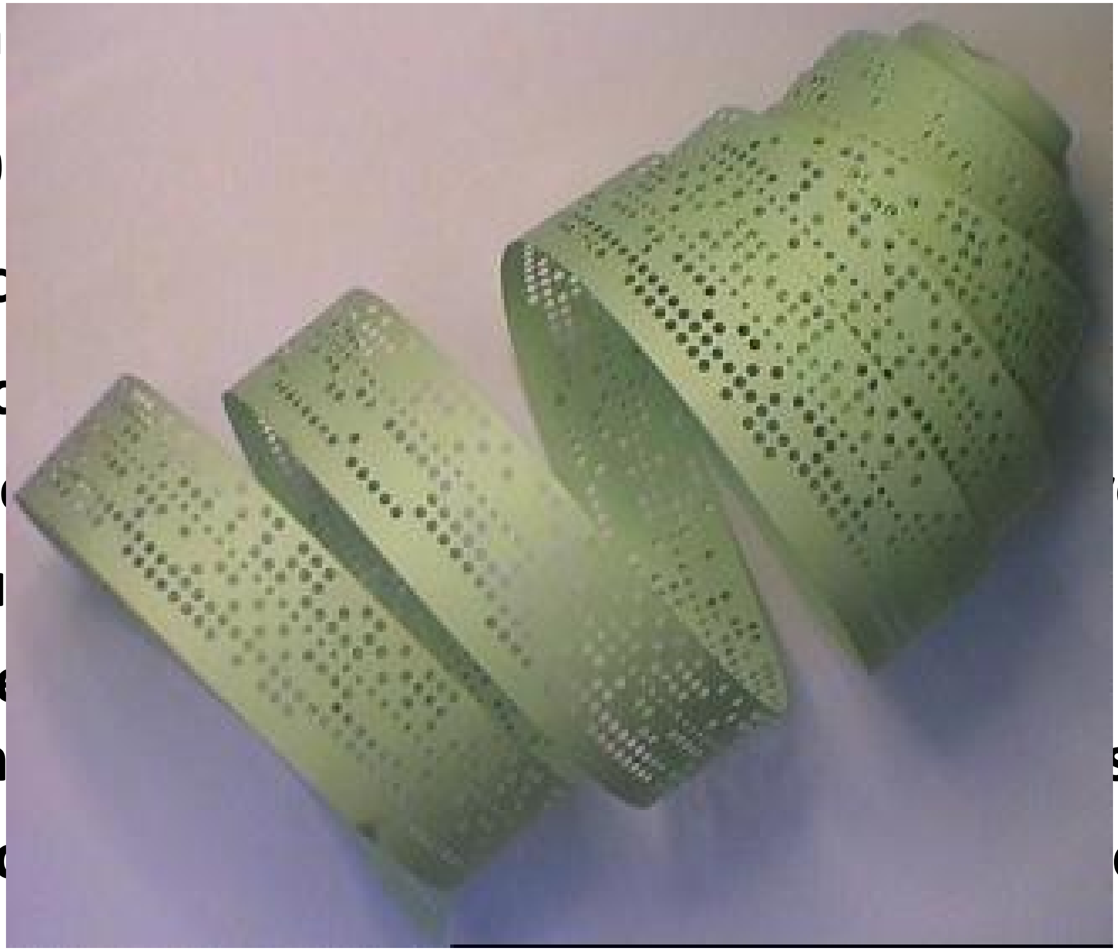
- Low ... ersa)

  occu

  Delete ... when

  pun ... ssages.

  Punc ... e!



Parity Bit Error-Detection Codes

- <u>Redundancy</u> (e.g. extra information), in the form of extra bits, can be incorporated into binary code words to detect and correct errors.
- A simple form of redundancy is <u>parity</u>, an extra bit appended onto the code word to make the number of 1's odd or even. Parity can detect all single-bit errors and some multiple-bit errors.
- A code word has <u>even parity</u> if the number of 1's in the code word is even.
- A code word has <u>odd parity</u> if the number of 1's in the code word is odd.

4-Bit Parity Code Example

- **Fill in the even and odd parity bits:**

Even Parity Message	- Parity	Odd Parity Message	- Parity
000	-	000	-
001	-	001	-
010	-	010	-
011	-	011	-
100	-	100	-
101	-	101	-
110	-	110	-
111	-	111	-

- **The codeword "1111" has <u>even parity</u> and the codeword "1110" has <u>odd parity</u>. Both can be used to represent 3-bit data.**

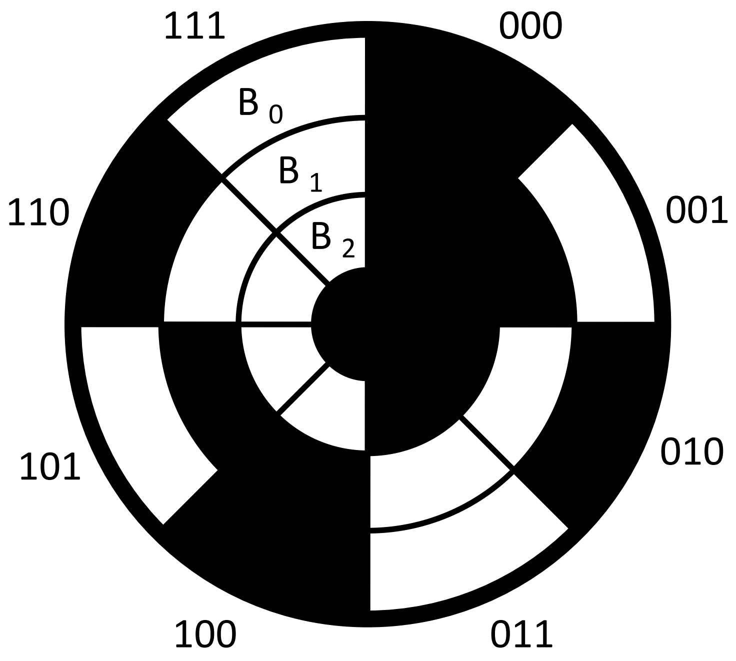
GRAY CODE – Decimal

Decimal	8,4,2,1	Gray
0	0000	0000
1	0001	0100
2	0010	0101
3	0011	0111
4	0100	0110
5	0101	0010
6	0110	0011
7	0111	0001
8	1000	1001
9	1001	1000

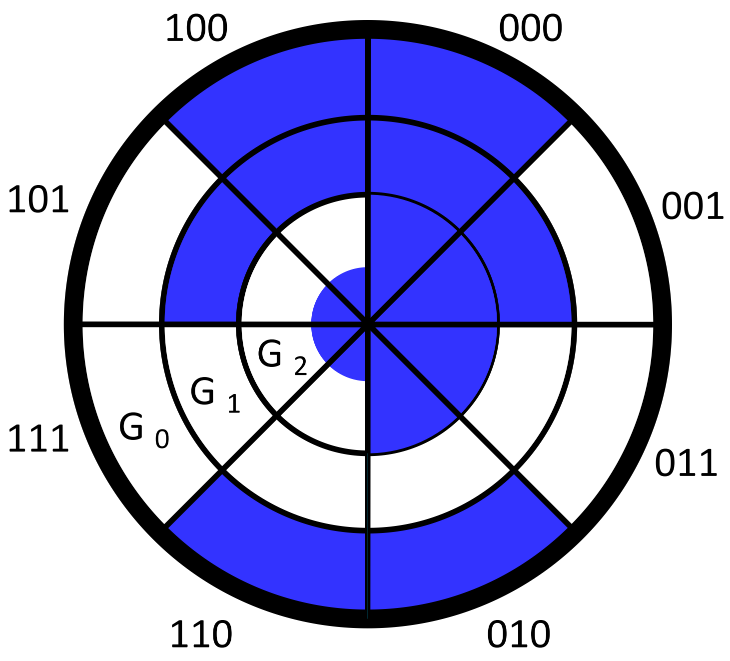
- **What special property does the Gray code have in relation to adjacent decimal digits?**

Optical Shaft Encoder

- **Does this special Gray code property have any value?**
- **An Example: Optical Shaft Encoder**



(a) Binary Code for Positions 0 through 7



(b) Gray Code for Positions 0 through 7

Shaft Encoder (Continued)

- **How does the shaft encoder work?**

- **For the binary code, what codes may be produced if the shaft position lies between codes for 3 and 4 (011 and 100)?**

- **Is this a problem?**

Shaft Encoder (Continued)

- **For the Gray code, what codes may be produced if the shaft position lies between codes for 3 and 4 (010 and 110)?**

- **Is this a problem?**

- **Does the Gray code function correctly for these borderline shaft positions for all cases encountered in octal counting?**

UNICODE

- **UNICODE extends ASCII to 65,536 universal characters codes**
  - **For encoding characters in world languages**
  - **Available in many modern applications**
  - **2 byte (16-bit) code words**
  - **See Reading Supplement – Unicode on the Companion Website http://www.prenhall.com/mano**

Assignment

See Assignment Section of Website

Too simple

Thank You !