General Physics II

Homework #7

2021/12/15

P7-1. At a beach the light is generally partially polarized due to reflections off sand and water. At a particular beach on a particular day near sundown, the horizontal component of the electric field vector is 2.3 times the vertical component. A standing sunbather puts on polarizing sunglasses; the glasses eliminate the horizontal field component. (a) What fraction of the light intensity received before the glasses were put on now reaches the sunbather's eyes? (b) The sunbather, still wearing the glasses, lies on his side. What fraction of the light intensity received before the glasses were put on now reaches his eyes?

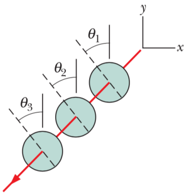
Solution: (a) The fraction of light that is transmitted by the glasses is

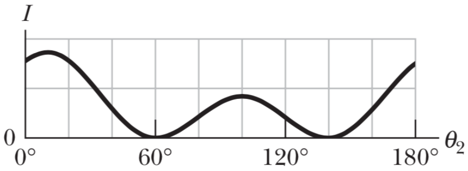
$$\frac{I_f}{I_0} = \frac{E_f^2}{E_0^2} = \frac{E_v^2}{E_v^2 + E_h^2} = \frac{E_v^2}{E_v^2 + (2.3E_v)^2} = 0.16.$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0} = \frac{E_h^2}{E_v^2 + E_h^2} = \frac{(2.3E_v)^2}{E_v^2 + (2.3E_v)^2} = 0.84.$$

P7-2. In the left figure, unpolarized light is sent into a system of three polarizing sheets. The angles θ_1, θ_2, and θ_3 of the polarizing directions are measured counterclockwise from the positive direction of the y axis (they are not drawn to scale). Angles θ_1 and θ_3 are fixed, but angle θ_2 can be varied. The right figure gives the intensity of the light emerging from sheet 3 as a function of θ_2. (The scale of the intensity axis is not indicated.) What percentage of the light's initial intensity is transmitted by the three-sheet system when $\theta_2 = 90°$?





Solution: We note the points at which the curve is zero ($\theta_2 = 60^\circ$ and 140°) in the right figure. We infer that sheet 2 is perpendicular to one of the other sheets at $\theta_2 = 60^\circ$, and that it is perpendicular to the *other* of the other sheets when $\theta_2 = 140^\circ$. Without loss of generality, we choose $\theta_1 = 150^\circ$, $\theta_3 = 50^\circ$. Now, when $\theta_2 = 90^\circ$, it will be $|\Delta\theta| = 60^\circ$ relative to sheet 1 and $|\Delta\theta'| = 40^\circ$ relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 7.3\%.$$

P7-3. A beam of partially polarized light can be considered to be a mixture of polarized and unpolarized light. Suppose we send such a beam through a polarizing filter and then rotate the filter through 360° while keeping it perpendicular to the beam. If the transmitted intensity varies by a factor of 5.0 during the rotation, what fraction of the intensity of the original beam is associated with the beam's polarized light?

Solution: Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is fI_0. After transmission, this portion contributes $fI_0 \cos^2 \theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1 - f)I_0$ and after transmission, this portion contributes $(1 - f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = fI_0 \cos^2 \theta + \frac{1}{2}(1 - f)I_0.$$

As the filter is rotated, $\cos^2\theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\min} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\max} = fI_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0.$$

The ratio of $I_{\max}$ to $I_{\min}$ is

$$\frac{I_{\max}}{I_{\min}} = \frac{1+f}{1-f}.$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.

P7-4. A special kind of lightbulb emits monochromatic light of wavelength 630 nm. Electrical energy is supplied to it at the rate of 60 W, and the bulb is 93% efficient at converting that energy to light energy. How many photons are emitted by the bulb during its lifetime of 730 h?

Solution: The total energy emitted by the bulb is $E = 0.93Pt$, where $P = 60$ W and

$$t = 730 \text{ h} = 2.628 \times 10^6 \text{ s}.$$

The energy of each photon emitted is $E_{\text{ph}} = hc/\lambda$. Therefore, the number of photons emitted is

$$\begin{aligned} N &= \frac{E}{E_{\text{ph}}} = \frac{0.93Pt}{hc/\lambda} \\ &= \frac{0.93 \cdot 60 \cdot 2.628 \times 10^6}{6.63 \times 10^{-34} \cdot 2.998 \times 10^8 / (630 \times 10^{-9})} \\ &= 4.7 \times 10^{26}. \end{aligned}$$

P7-5. In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300 nm and a stopping potential of 0.820 V for a wavelength of 400 nm. From these data find (a) a value for the Planck constant, (b) the work function Φ for sodium, and (c) the cutoff wavelength λ_0 for sodium.

Solution:

(a) For the first and second case (labeled 1 and 2) we have

$$eV_{01} = hc/\lambda_1 - \Phi, \qquad eV_{02} = hc/\lambda_2 - \Phi,$$

from which h and Φ can be determined. Thus,

$$\begin{aligned} h &= \frac{e(V_1 - V_2)}{c(\lambda_1^{-1} - \lambda_2^{-1})} \\ &= \frac{1.85\text{eV} - 0.820\text{eV}}{(3.00 \times 10^{17}\text{nm/s})[(300\text{nm})^{-1} - (400\text{nm})^{-1}]} \\ &= 4.12 \times 10^{-15}\ \text{eV} \cdot \text{s}. \end{aligned}$$

(b) The work function is

$$\Phi = \frac{3(V_2\lambda_2 - V_1\lambda_1)}{\lambda_1 - \lambda_2} = \frac{0.820 \cdot 400 - 1.85 \cdot 300}{300 - 400}$$
$$= 2.27 \text{ eV}.$$

(c) Let $\Phi = hc/\lambda_{\max}$ to obtain

$$\lambda_{\max} = \frac{hc}{\Phi} = \frac{1240}{2.27} = 545 \text{ nm}.$$

P7-6. Show that when a photon of energy E is scattered from a free electron at rest, the maximum kinetic energy of the recoiling electron is given by

$$K_{\max} = \frac{E^2}{E + mc^2/2}.$$

Solution: Referring to Sample Problem — "Compton scattering of light by electrons," we see that the fractional change in photon energy is

$$\frac{E - E_n}{E} = \frac{\Delta\lambda}{\lambda + \Delta\lambda} = \frac{(h/mc)(1 - \cos\phi)}{(hc/E) + (h/mc)(1 - \cos\phi)}.$$

Energy conservation demands that $E \breve{} E' = K$, the kinetic energy of the electron. In the maximal case, $\phi = 180^\circ$, and we find

$$\begin{aligned}\frac{K}{E} &= \frac{(h/mc)(1 - \cos 180^\circ)}{(hc/E) + (h/mc)(1 - \cos 180^\circ)} \\ &= \frac{2h/mc}{(hc/E) + (2h/mc)}.\end{aligned}$$

Multiplying both sides by E and simplifying the fraction on the right-hand side leads to

$$K = E\frac{2/mc}{c/E + 2/mc} = \frac{E^2}{mc^2/2 + E}.$$

P7-7. The highest achievable resolving power of a microscope is limited only by the wavelength used; that is, the smallest item that can be distinguished has dimensions about equal to the wavelength. Suppose one wishes to "see" inside an atom. Assuming the atom to have a diameter of 100 pm, this means that one must be able to resolve a width of, say, 10 pm. (a) If an electron microscope is used, what minimum electron energy is required? (b) If a light microscope is used, what minimum photon energy is required? (c) Which microscope seems more practical? Why?

Solution: (a) Setting $\lambda = h/p = h/\sqrt{E/c^2 - m_e^2 c^2}$, we solve for $K = E\breve{\ } m_e c^2$:

$$
\begin{aligned}
K &= \sqrt{\left(\frac{hc}{\lambda}\right)^2 + m_e^2 c^4} - m_e c^2 \\
&= \sqrt{\left(\frac{1240 \mathrm{eV} \cdot \mathrm{nm}}{10 \times 10^{-3} \mathrm{nm}}\right)^2 + (0.511 \mathrm{MeV})^2} - 0.511 \mathrm{MeV} \\
&= 15 \ \mathrm{keV}.
\end{aligned}
$$

(b) Using the value $hc = 1240 \ \mathrm{eV} \cdot \mathrm{nm}$,

$$
E = \frac{hc}{\lambda} = \frac{1240 \mathrm{eV} \cdot \mathrm{nm}}{10 \times 10^{-3} \mathrm{nm}} = 1.2 \times 10^5 \mathrm{eV} = 120 \ \mathrm{keV}.
$$

(c) The electron microscope is more suitable, as the required energy of the electrons is much less than that of the photons.