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Study for a lab-made Moiré pattern-based thread counter

Yifan Liu, Jin Wang*, Wenhua Li and Xiaoqing Wen

School of Physics, Nankai University, Tianjin, People's Republic of China

E-mail: wangjin8208@nankai.edu.cn

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Abstract

The 32nd International Young Physicists' Tournament (IYPT), a physics competition for students in schools, settled up a problem for the Moiré thread counter. The problem mentioned to design an overlay to measure the thread count of the fabric. In this paper, we made one kind of Moiré pattern-based thread counter by printing the designed overlay on the plastic paper. As a contrast, we also constructed a self-made CCD camera assisted imaging system and studied the thread count. We used a 1951 USAF resolution test chart and MATLAB software for imaging analysis. Ten kinds of simple cloth were prepared, and their thread count measured. The results and the accuracy of the two methods were discussed.

Keywords: Moiré pattern, thread counter, imaging

(Some figures may appear in colour only in the online journal)

1. Introduction

Moiré fringe is an optical phenomenon first discovered in China and then commercially introduced in France by an English manufacturer in 1754 [1]. From a technical point of view, Moiré fringes are the visual result of interference between two lines or two objects at a constant angle and frequency. When the human eye cannot distinguish between these two lines or two objects, only the interfering patterns can be seen. The pattern in this optical phenomenon is the Moiré pattern.

The Moiré pattern has been widely applied in many different fields [2, 3]. Its magnification feature was used in the microscopy area, which includes scanning tunneling microscopy [4], the transmission electron microscope [5] and super-resolution microscopy [6] etc. It can also be used to study the microscopic strain in materials [7] and for encryption because of the difficulty in cracking its patterns [8, 9].

*Author to whom any correspondence should be addressed.

Figure 1. One kind of magnifier for thread counting.

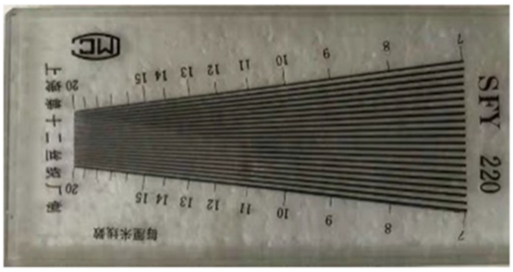

Figure 2. Thread counter based on the Moiré pattern in industry.

In industry, a variety of small magnifiers, often called thread counters are used to determine the thread count for cloth. A thread counter linen tester magnifier (Donegan V388-1/2) is shown in figure 1, which can magnify the cloth by counting the number of threads with the help of a needle along a scale with even distribution. Also, there are varied counters based on the Moiré pattern (figure 2), with uneven distribution of its scale [10].

The 32nd International Young Physicists' Tournament (IYPT) set up a problem for the Moiré thread counter [11]. The problem is 'When a pattern of closely spaced non-intersecting lines (with transparent gaps in between) is overlaid on a piece of woven fabric, characteristic Moiré fringes may be observed. Design an overlay that allows you to measure the thread count of the fabric. Determine the accuracy for simple fabrics (e.g. linen) and investigate if the method is reliable for more complex fabrics (e.g. denim or Oxford cloth).'

In this paper, we used various inverse proportional curves to form a Moiré pattern and then printed the designed pattern on plastic paper to make a thread counter. As a contrast, we also constructed a self-made CCD camera assisted imaging system and studied the thread count. We used a 1951 USAF resolution test chart and Matlab software for imaging analysis. Ten kinds of cloth samples were prepared and their thread counts were measured.

2. Theory

Moiré patterns are large-scale interference patterns that can be produced when an opaque ruled pattern with transparent gaps is overlaid on another similar pattern. For the appearance of the Moiré interference pattern, the overlapped two patterns must not be completely identical, but rather e.g. displaced, rotated or have a slightly different pitch. This kind of pattern can be understood by comparing the 'beat' effect of the grating distribution functions [12]. From the distribution of intersection, we can determine the Moire pattern functions.

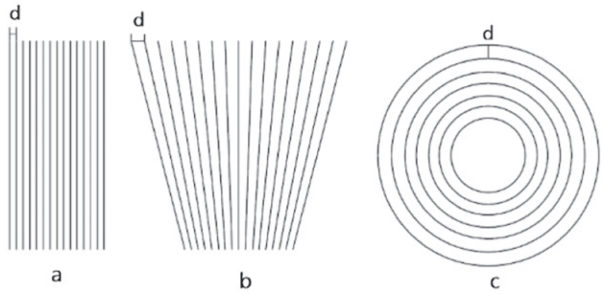

Figure 3. Three kinds of gratings. (a) Horizontal Moiré pattern, (b) vertical Moiré pattern, (c) circular Moiré Pattern.

Figure 4. (a) Superposing two gratings with constant spacing. (b) Superposing a grating with constant spacing and one with linear changing spacing.

As shown in figure 3, common optical gratings include parallel linear gratings, radial gratings, concentric gratings, etc, which are all made up of a sequence of slits. The appearance of the formed pattern is relevant to the distance between the slits and the angle formed by them.

By overlaying two parallel linear gratings, we can get simple Moiré patterns in the following forms:

(a) Horizontal Moiré pattern (with fixed spacing, and a non-zero angle).
(b) Vertical Moiré pattern (with changing spacings, and a zeroth angle).
(c) Oblique Moiré pattern (with changing spacings, and a non-zero angle).

4. Other Moiré patterns.

In figure 4, we simulated two kinds of Moiré pattern. Figure 4(a) shows the pattern with changing spacings and a zero angle. Figure 4(b) shows the pattern with changing spacings and a non-zero angle.

In order to discuss the magnification feature of the Moiré pattern, the detail of the pattern is shown in figure 5. In a triangle with three sides of a, $b$and$c$, by law of cosines we have:

$$a^2 = b^2 + c^2 - 2bc\ \cos\ \theta, \tag{1}$$

where θ is the angle between $b$and$c$. By the geometric relationship, we have:

$$d = c\ \sin\ \theta \tag{2}$$

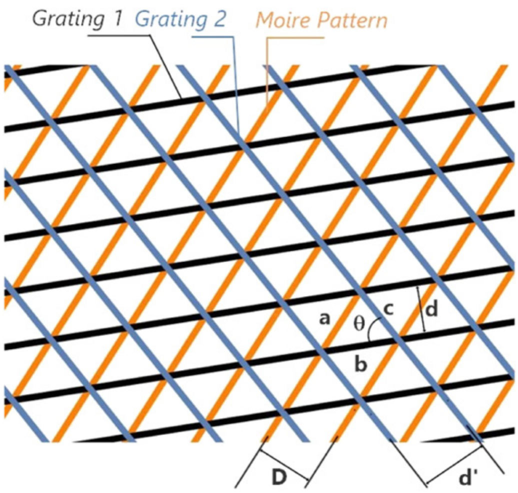

Figure 5. The detail of the Moiré pattern. (Black: grating 1, blue: grating 2, orange: Moiré pattern).

$$d' = b \ \sin \ \theta \tag{3}$$

$$\frac{bc}{2} \ \sin \ \theta = S\triangle = aD/2. \tag{4}$$

Then we can get the Moire stripe spacing D, which is:

$$D = dd'/\sqrt{d^2 + d'^2 - 2dd' \ \cos \ \theta}. \tag{5}$$

Setting $\theta = \pi/2$, we have:

$$D = \frac{dd'}{|d - d'|}. \tag{6}$$

With a changing spacing d, we can obtain a changing D. When d equals to d', D is singular. Therefore, by treating the weaves as parallel lines, we can use a designed grating to 'magnify' the small spacing and obtain the thread counts from the formed pattern.

Here we mainly discuss two kinds of gratings, a discrete linear grating and a hyperbolic grating. First, for a discrete linear grating shown in figure 4(b), the mathematical form of it is:

$$y = \alpha kx \quad k = 1, 2, \ldots, k_{\max}, \tag{7}$$

where α is a coefficient which indicates the slope of the first line.

The parallel simulated cloth pattern is designed as:

$$y = jd \quad j = 1, 2, \ldots \tag{8}$$

Setting $k = j \pm \Delta$, gives the pattern function:

$$y = \frac{\Delta}{|1/\alpha x - 1/d|} \quad \Delta = 1, 2, \ldots \tag{9}$$

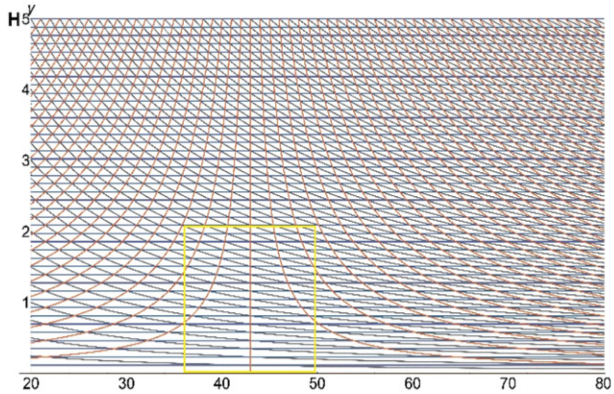

Figure 6. Graphical representation of the fringes formed by a curved-line overlay. Black: counter fringes, blue: simulated cloth pattern, orange: Moiré pattern's centre.

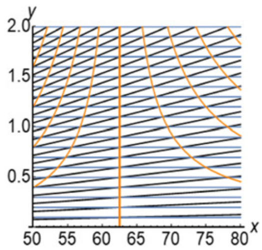

Figure 7. Magnified square in figure 6. Black: counter fringes, blue: simulated cloth pattern, orange: Moiré pattern's centre.

Then the vertical line, where αx equals to d, indicates the position where the space between adjacent cloth lines equals to d.

Though this kind of pattern has already been used in industry for line counting, it is obvious that the Moiré pattern is not symmetric, and the space between adjacent Moiré lines do not meet. This will make it unfavourable for the readout of inverse proportional scale.

The second type of grating was formed by a series of inverse proportional curves, as shown in figure 6. We use horizontal lines (black) to represent the weft, and inverse proportional curves (blue) to represent the overlay pattern. The intersecting lines (orange) indicate expecting Moiré pattern. The yellow square in figure 6 is magnified and shown in figure 7.

We assume that the height of the overlay as H. Then the mathematical form of the counter fringes is:

$$y = kH/x \quad k = 1, 2, \ldots \tag{10}$$

The parallel simulated cloth pattern is designed as:

$$y = jd \quad j = 1, 2, \ldots \tag{11}$$

Setting $k = j \pm \Delta$, gives the pattern function (intersection):

$$y = \frac{\Delta}{|x/H - 1/d|}. \tag{12}$$

Then the vertical line indicates the area that the space between the curves equals to d (or $kd, d/k$ $k = 2, 3, \ldots$). By the scale on the top of the counter, we can read out the thread count easily. Whereas the first one, which forms an inverse proportional scale, is not easy for indication or the analysis of the accuracy, and will not be used in our work.

3. Materials and methods

3.1. Materials

We prepared ten kinds of clothes varying in thread counts from a local market, all of them are simple fabrics weaved by intertwining the warp and the wefts most simply. All the sample are uniform in weave density. The materials of the thread are a binding of hards (about 55%) and cotton (about 45%).

3.2. Methods by Moiré thread counter

The counter we used here is made by printing the fringes onto an apparent negative. The fringe is generated by following Mathematica commands

```
x1=50; x2=150; H=5; f[x_,k_]:=H*k/x; f[j_]:=j*H/80;
overlay=Plot[Table[f[x,k],{k,1,150}],{x,x1,x2},PlotRange-
>{0,H},PlotStyle->Thickness[0.001], ImageSize-
>{Automatic, 1000},AspectRatio->1/3]
```

The method has been discussed in detail by Gary Reich [13], here we changed some parameters. In order to get a proper size, the final counter image was adjusted to fit the actual size. The height we used here is 5 cm, so the calculated density range is between 10 lines/cm and 30 lines/cm.

Measurement are taken by the following procedures. First, the cloth sample was spread on a flat surface, but not stretched or pulled to get a relatively even threads distribution. A good light source was shining on the sample. As shown in figure 8, we superpose the overlay onto the cloth sample. Then when the pattern appears, we adjust the position of the counter to find the place for the clearest vertical line and read out the number correspond-ing to it. In figure 8, the x-coordinate of the black line indicate the thread number. Here, we need to mark the measured area on the cloth, which will be used exactly for the CCD method.

3.3. Method by CCD imaging

Figure 9 shows the experimental setup of a self-made CCD imaging system. A microscope with a CCD camera (A) and an annular illumination light source (B) was fixed on a three-direction translation stage (C). Sample (D) is fixed on a sample holder (E). The signal is processed by a computer (F). By adjusting the three-direction translation stage and the height of the sample holder, the centre of the illuminated area is coincident with the marked area tested by the thread counter. The distance between the CCD and the cloth is fixed at about 25.0 cm. We can adjust the external cylinder wheel of the microscope to get a clear image.

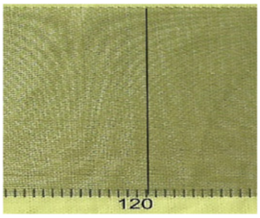

Figure 8. Image of the designed thread counter.

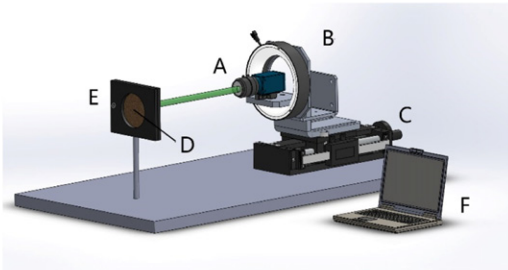

Figure 9. Experimental setup consisting of the following elements: A: CCD, B: annular illumination light, C: sample support, D: the cloth sample, E: shifting bench, F: PC.

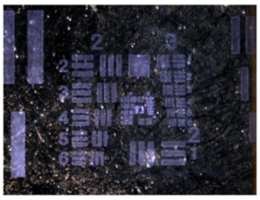

Figure 10. Image of USAF resolution chart for calibration.

We first use a 1951 USAF resolution test chart [14] as a sample for calibration. The chart consists of ‘groups’ of six ‘elements’ each. The group numbers are located at the top of the group and the element numbers are located at the sides of the groups. Each element consists of three horizontal and three vertical bars. The image of the enlarged centre of the USAF resolution chart we used is shown in figure 10. We can find the detail of the line width of by the group number and element number. By comparing the captured image and the standard of the resolution chart, we obtain the resolution of the CCD system, which is about 1.69 um for one pixel.

Then we replace the resolution chart with the cloth sample and take images. The marked area former measured by the Moiré thread counter was located along the centre of the microscope to minimize the error caused by the nonuniform thread count of the cloth.

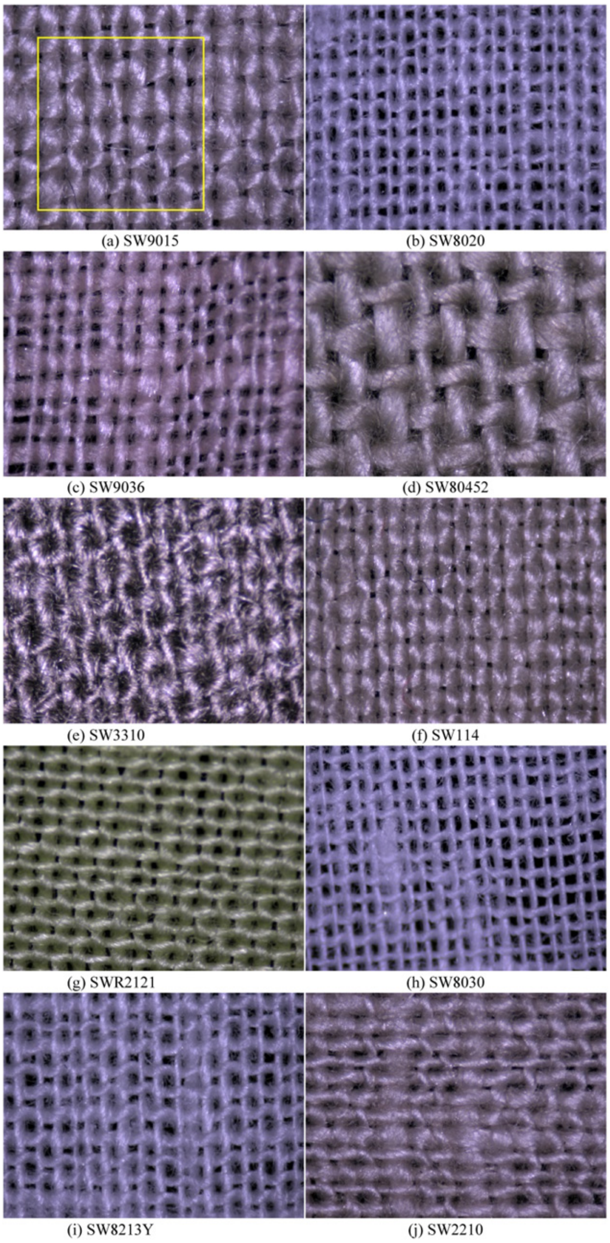

Figure 11. Imaging Results. (a) SW9015 (b) SW8020 (c) SW9036 (d) SW80452 (e) SW3310 (f) SW114 (g) SWR2121 (h) SW8030 (i) SW8213Y (j) SW2210.

4. Results and discussions

4.1. Results

The CCD images of the ten cloth samples are shown in figure 11.

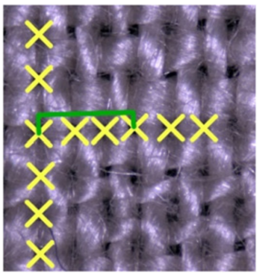

Figure 12. Magnified square in figure 11(a).

Table 1. Weft Thread counts (lines/inch).

Label	SW9015	SW8020	SW80452	SW9036	SW3310
Index	**a**	**b**	**d**	**c**	**e**
CCD	38.9	58.4	27.9	51.1	38.2
Counter	38.9	58.5	28.4	49.9	38.4
Relative error	0.1%	0.2%	1.7%	2.3%	0.4%
Label	SW114	SWR2121	SW8030	SW8213Y	SW2210
Index	**f**	**g**	**h**	**i**	**j**
CCD	46.3	63.9	55.1	65.3	62.6
Counter	46.7	61.5	55.5	65.0	61.2
Relative error	0.9%	3.8%	0.8%	0.4%	2.2%

Take figure 11(a) for example, the magnified area is shown in figure 12. First, we marked the centre of holes of the cloth, then we mark some adjacent holes and use MATLAB software to get the coordinates of the centre of the holes. As shown in figure 12, we can derive the average distance by the successive difference method from multiple points in a line. The thread count of the sample is related to the distance between the adjacent holes. Using 1.68 um/pixel for iteration, we can calculate the thread count by the CCD method. The weft and warp thread counts measured by the Moiré thread counter and CCD imaging method are listed in tables 1 and 2 respectively.

4.2. Discussions

Transforming equation (12) gives that:

$$x = \frac{H}{d} \pm \frac{\Delta H}{y} \quad \Delta = 0, 1, 2, \ldots \tag{13}$$

For chosen y, the x distance between adjacent curves is:

$$\Delta x = H/y. \tag{14}$$

This is the distance between two adjacent Moire patterns.

When $y = H$, $\Delta x = 1$. This is the distance between two adjacent Moire patterns at the top of the counter, where our graduation locates. Since the line we read locates between the tick marks,

Table 2. Warp Thread counts (lines/inch).

Label	SW9015	SW8020	SW9036	SW80452	SW3310
Index	**a**	**b**	**c**	**d**	**e**
CCD	53.5	70.8	60.0	33.6	53.0
Counter	50.0	70.9	59.7	32.8	52.6
Relative error	6.4%	0.0%	0.5%	2.4%	0.9%
Label	SW114	SWR2121	SW8030	SW8213Y	SW2210
Index	**f**	**G**	**h**	**i**	**j**
CCD	68.5	57.6	73.7	51.1	42.1
Counter	67.3	56.6	72.9	50.3	42.7
Relative error	1.8%	1.7%	1.1%	1.5%	1.4%

We get the resolution of the counter. The resolution equals to 1/H. And the uncertainty of the counter can be defined as a ruler, which is:

$$u_b = \Delta/\sqrt{3}, \tag{15}$$

where Δ is the resolution of the equipments. u_b is the class b uncertainty of the measurement. In our experiment, the theoretical class b uncertainty is $u_b = 1/\sqrt{3}$ lines/inch.

Form tables 1 and 2, we can see that, for some clothes like (a), (g), (j), there are relatively larger errors than for others, which may originate from the nonuniform thread count in different parts. The relative errors range from 0.1% to 6.4%. However, in figures 11(b), (c) and (e), the thread counts are much more uniform). Compared to the CCD method, the counter method take the advantage of easiness without losing the accuracy of the readouts. The relative errors range from 0.0% to 2.3%.

5. Conclusion

We provide an easy way for a lab-made Moiré pattern-based thread counter. A self-made CCD imaging system was also used for the thread count. We measured cloth samples by these two methods and studied the accuracy. We found that this Moiré pattern-based thread counter is easy to use. With the help of the CCD imaging system, we can verify the accuracy of it, which presents its great practicability in industrial, scientific and teaching usage.

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ORCID iDs

Yifan Liu https://orcid.org/0000-0003-4374-4037
Jin Wang https://orcid.org/0000-0002-6447-5262

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