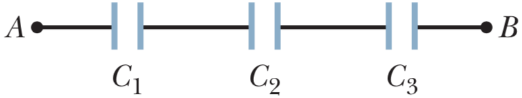
General Physics II

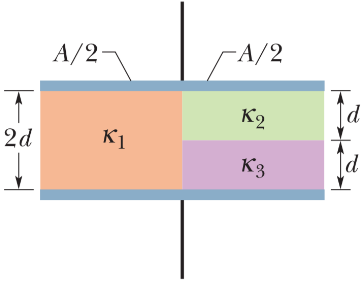
Homework #2

Due 2021/10/16

P2-1. In a three-capacitor, $C_1 = 10.0\ \mu\mathrm{F}$, $C_2 = 20.0\ \mu\mathrm{F}$, and $C_3 = 25.0\ \mu\mathrm{F}$. If no capacitor can withstand a potential difference of more than 100 V without failure, what are (a) the magnitude of the maximum potential difference that can exist between points A and B and (b) the maximum energy that can be stored in the three-capacitor arrangement?



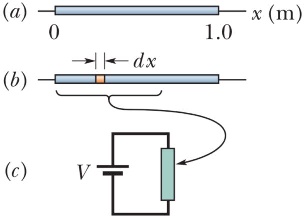
P2-2. A parallel-plate capacitor of plate area $A = 10.5\ \mathrm{cm}^2$ and plate separation $2d = 7.12$ mm. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 21.0$; the top of the



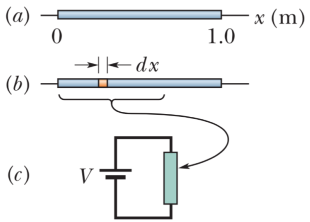
right half is filled with material of dielectric constant $\kappa_2 = 42.0$; the bottom of the right half is filled with material of dielectric constant $\kappa_3 = 58.0$. What is the capacitance?

P2-3. The rain-soaked shoes of a person may explode if ground current from nearby lightning vaporizes the water. The sudden conversion of water to water vapor causes a dramatic expansion that can rip apart shoes. Water has density 1000 kg/m^3 and requires 2256 kJ/kg to be vaporized. If horizontal current lasts 2.00 ms and encounters water with resistivity 150 $\Omega\cdot$m, length 12.0 cm, and vertical cross-sectional area 15×10^{-5} m^2, what average current is required to vaporize the water?

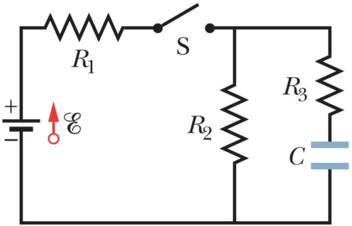
P2-4. There is a rod of resistive material (Figure a). The resistance per unit length of the rod increases in the positive direction of the x axis. At any position x along the rod, the resistance dR of a narrow (differential) section of width dx is given by $dR = 5.00x\ dx$, where dR is in ohms and x is in meters. Figure b shows such a narrow section.



You are to slice off a length of the rod between $x = 0$ and some position $x = L$ and then connect that length to a battery with potential difference $V = 5.0$ V (Figure c). You want the current in the length to transfer energy to thermal energy at the rate of 200 W. At what position $x = L$ should you cut the rod?

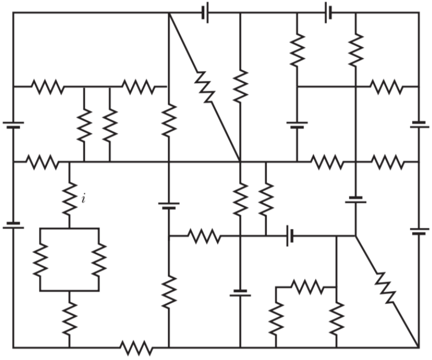


P2-5. In a circuit, $\mathcal{E} = 1.2$ kV, $C = 6.5\ \mu$F, $R_1 = R_2 = R_3 = 0.73$ MΩ. With C completely uncharged, switch S is suddenly closed (at $t = 0$).



At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At $t = \infty$ (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$? (i) Sketch V_2 versus t between these two extreme times.

P2-6. What are the (a) size and (b) direction (up or down) of current i, where all resistances are 4.0 Ω and all batteries are ideal and have an emf of 10 V? (*Hint: Find a special loop such that you can answer by mental calculation only.*)



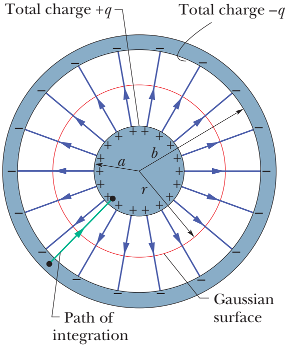
P2-7. A metal sphere of radius 15 cm has a net charge of 3.0×10^{-8} C.

(a) What is the electric field at the sphere's surface?
(b) If $V = 0$ at infinity, what is the electric potential at the sphere's surface?
(c) At what distance from the sphere's surface has the electric potential decreased by 500 V?

P2-8. Consider two concentric spherical shells, of radii a and b. Show that the capacitance of the shells is

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}.$$

What is the capacitance to a single isolated spherical conductor of radius R, then?



P2-9. Show that the curl of a central force $\vec{F}(\vec{r}) = f(r)\hat{r}$ is zero, i.e.,

$$\nabla \times \vec{F}(\vec{r}) = 0.$$

Hence, central forces are conservative.

P2-10. Consider a two-dimensional electric field

$$\vec{E}(x, y) = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}.$$

(a) Calculate the curl of the field $\nabla \times \vec{E}$.

(b) Show that the circulation of the field

$$\Gamma = \oint_C \vec{E} \cdot d\vec{s} = 2\pi$$

around a unit circle centered at origin.

Therefore, a vanishing curl does not implies, in general, that the force is conservative. They are equivalent only when the space is simply connected.