General Physics I

Lecture 16: Fluid Mechanics

Motivations

- Newton's laws for fluid statics?
  - Force → pressure
  - Mass → density
- How to treat the flow of fluid?
  - Difficulties:
    - Continuous medium
    - Change of the shape (still incompressible!)
  - Frame of reference

Density

Density (scalar): mass of a small element of material Δm divided by its volume ΔV

- **For infinitesimally small ΔV:**

$$\rho = \Delta m / \Delta V$$

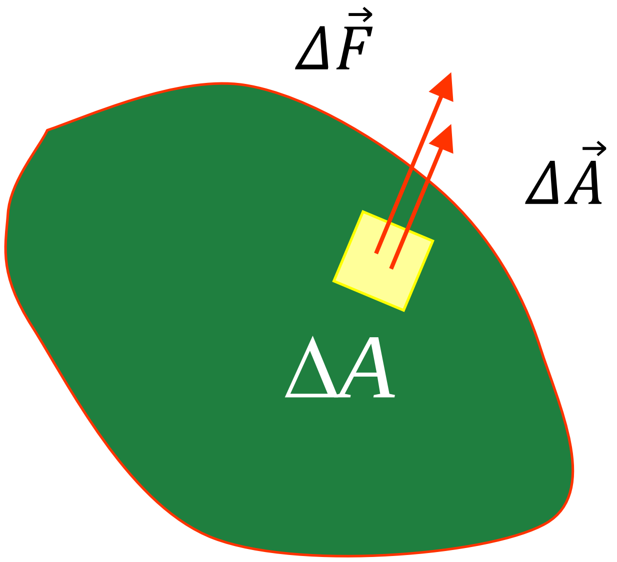
- **For homogeneous material:**

$$\rho = m / V$$

Pressure

Under static condition, force acts normal or perpendicular to the surface – vector.

Pressure: the magnitude of the normal force per unit surface area – scalar.



$$\Delta\vec{F} = p\Delta\vec{A}$$

$$p = \frac{\Delta F}{\Delta A}$$

Common Units

$$1\ pascal = 1\ N/m^2 \quad [\mathrm{SI}]$$

$$1\ atmosphere = 1.01325 \times 10^5\ Pa$$

$$1\ bar = 10^5\ Pa \approx 1\ atm$$

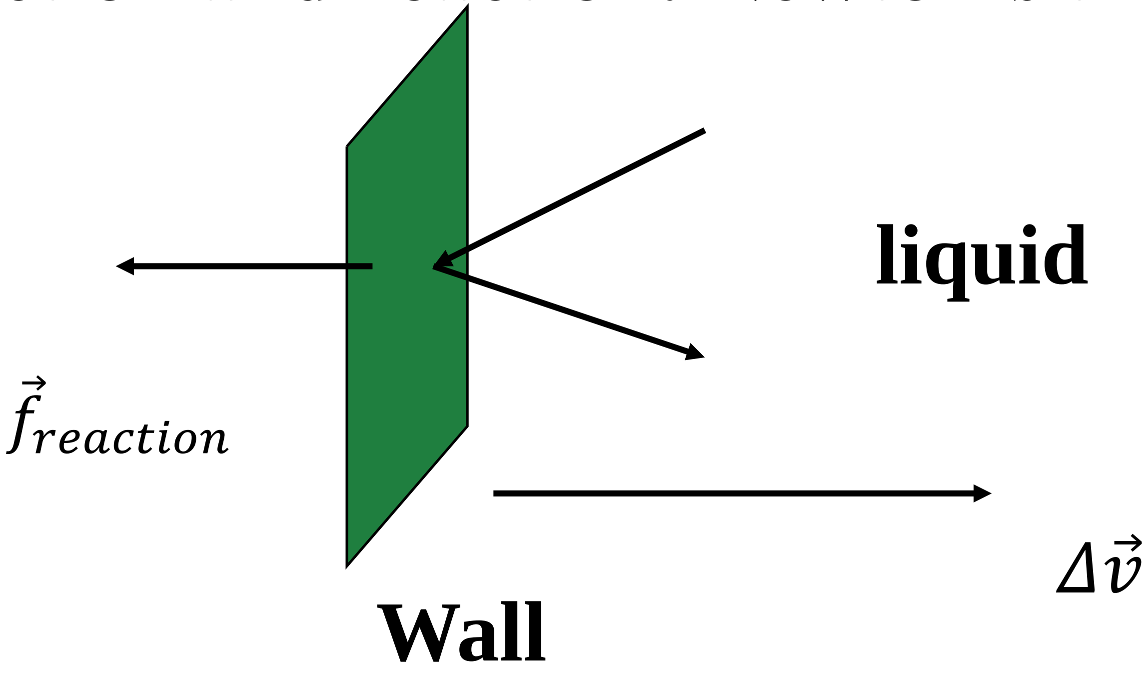
$$1\ atm = 760\ mm\ of\ Hg = 760\ torr$$

$$1\ atm \approx 14.7\ lb/in.^2$$

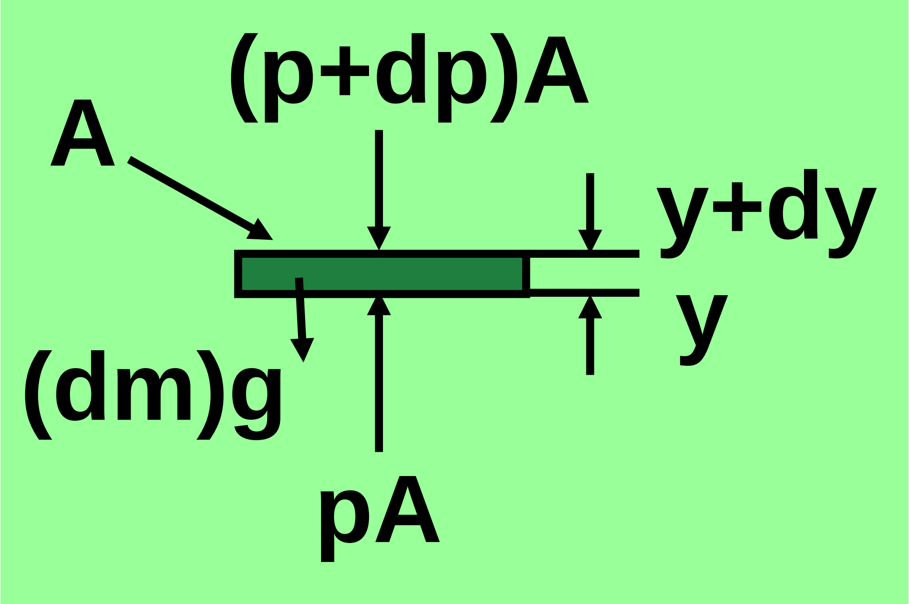
Origin of Pressure

Microscopic origin: collisions of molecules of the fluid with the surface

- **Action and reaction: Newton's third law**



Pressure in a Fluid at Rest



$$\frac{dp}{dy} = -\rho g$$

$$(dm)g = (\rho A dy)g = \rho g A dy$$

$$pA - \rho g A dy - (p + dp)A = 0$$

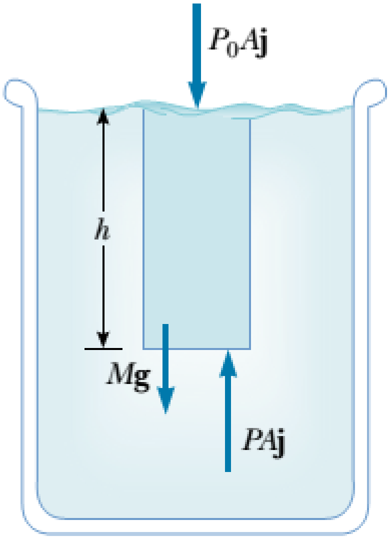
Pressure in a Fluid at Rest

Assumption: ρ, g independent of y

$$\frac{dp}{dy} = -\rho g \quad \Rightarrow \quad \int_{p_0}^{p} dp' = -\rho g \int_{y_0}^{y} dy'$$

$$p - p_0 = -\rho g (y - y_0)$$

$$p = p_0 - \rho g (y - y_0) = p_0 - \rho g h$$

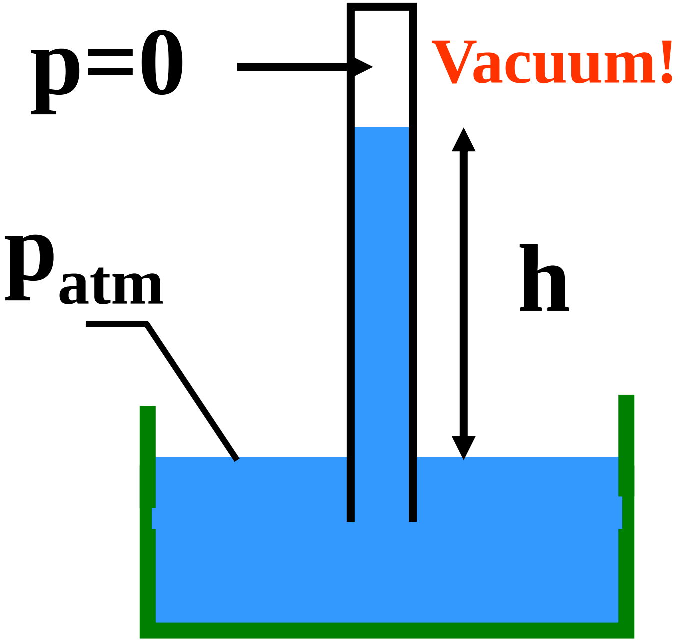


Mercury Barometer

How to measure atmospheric pressure?

E. Torricelli (1608-47)

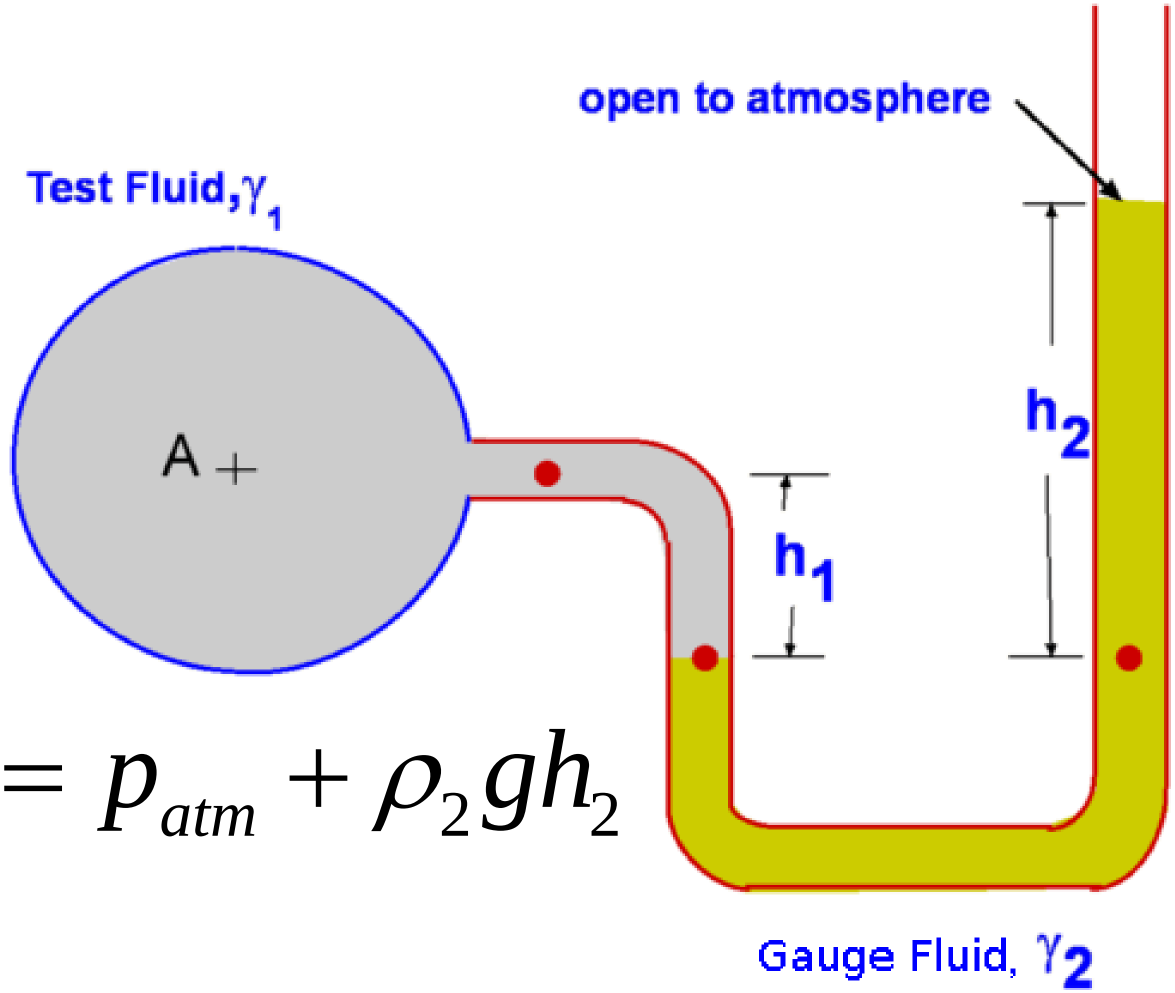
$$P_{atm} = \rho g h \;\Rightarrow\; h = \frac{P_{atm}}{\rho g}$$



For mercury, h = 760 mm.

How high will water rise?

U-Tube Manometer



$$p_A + \rho_1 g h_1 = p_{atm} + \rho_2 g h_2$$

Snorkeling

Can I use a longer snorkel to look closer at the underwater life?

Atmospheric Pressure

Gases are compressible. Thus, ρ varies!

$$p_0 = p_{\text{sea level}} = 1.013 \times 10^5\, Pa$$

$$pV = nRT \quad \Rightarrow \quad \frac{p}{p_0} = \frac{\rho}{\rho_0}, \quad \text{for constant } T$$

$$\frac{dp}{dy} = -\rho g = -\rho_0 g \frac{p}{p_0}$$

Atmospheric Pressure

$$\frac{dp}{dy} = -\rho_0 g \frac{p}{p_0} \Rightarrow \int_{p_0}^{p} \frac{dp'}{p'} = -\frac{\rho_0 g}{p_0} \int_{y_0}^{y} dy'$$

$$\ln p - \ln p_0 = -\frac{\rho_0 g}{p_0}(y - y_0)$$

$$p = p_0 e^{-(\rho_0 g / p_0)h} \xrightarrow{h \to 0} p = p_0 - \rho_0 g h$$

A.P. – an Improved Version

$$pV = nRT$$

$$\Rightarrow \frac{p}{p_0} = \frac{\rho}{\rho_0}\frac{T}{T_0}$$

$$T = T_0 - \gamma(y - y_0)$$

$$\frac{dp}{dy} = -\rho g = -\rho_0 g \frac{p}{p_0}\frac{T_0}{T_0 - \gamma(y - y_0)}$$

Temperature decreases 6^{o}C for each 1,000 meters of elevation

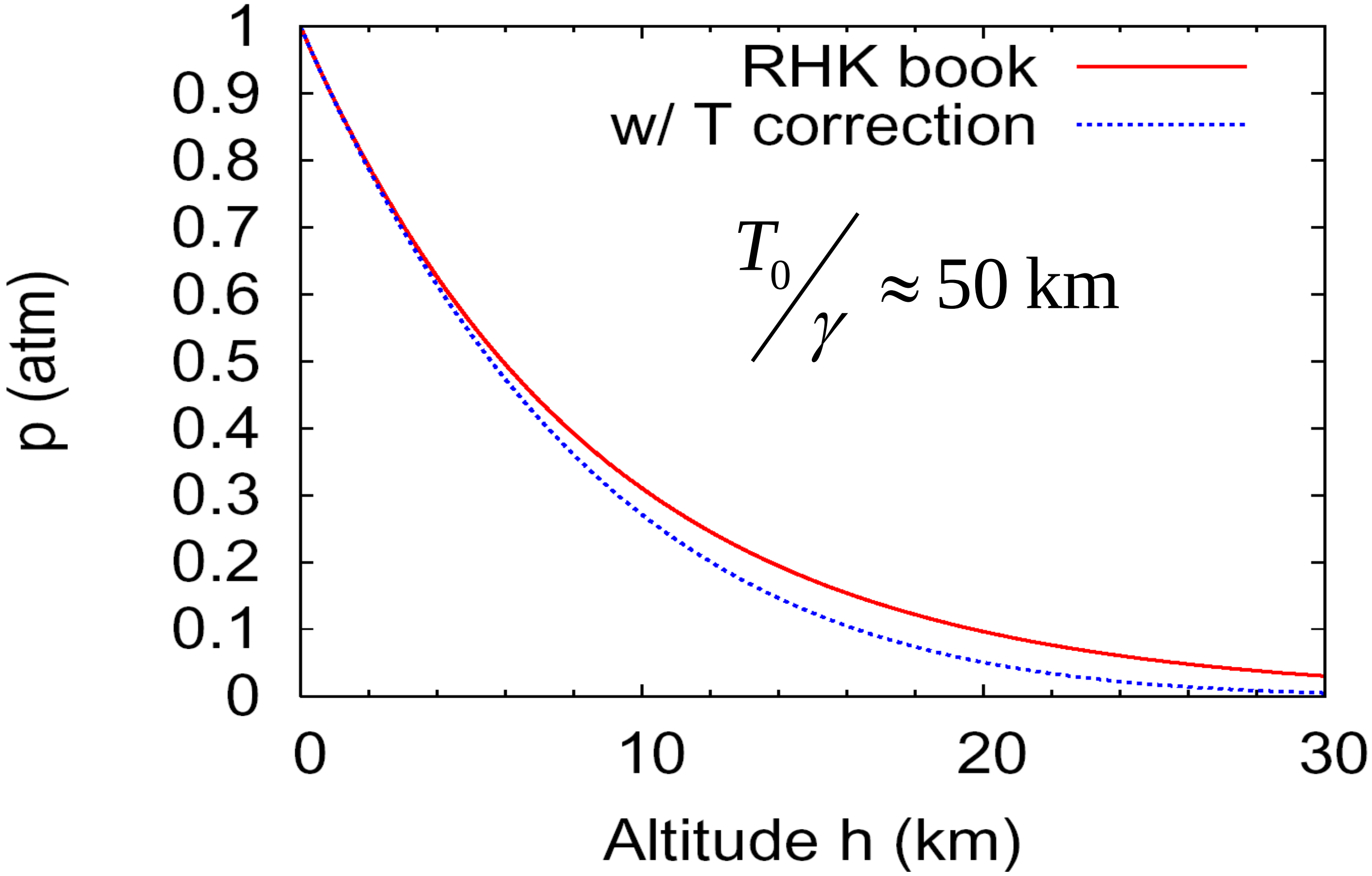
A.P. – an Improved Version

$$\frac{dp}{p} = -\frac{\rho_0 g}{p_0} \frac{T_0 dy}{T_0 - \gamma(y - y_0)}$$

$$\Rightarrow \quad \int_{p_0}^{p} \frac{dp'}{p'} = \frac{\rho_0 g}{p_0} \frac{T_0}{\gamma} \int_{y_0}^{y} \frac{-\gamma dy'}{T_0 - \gamma(y' - y_0)}$$

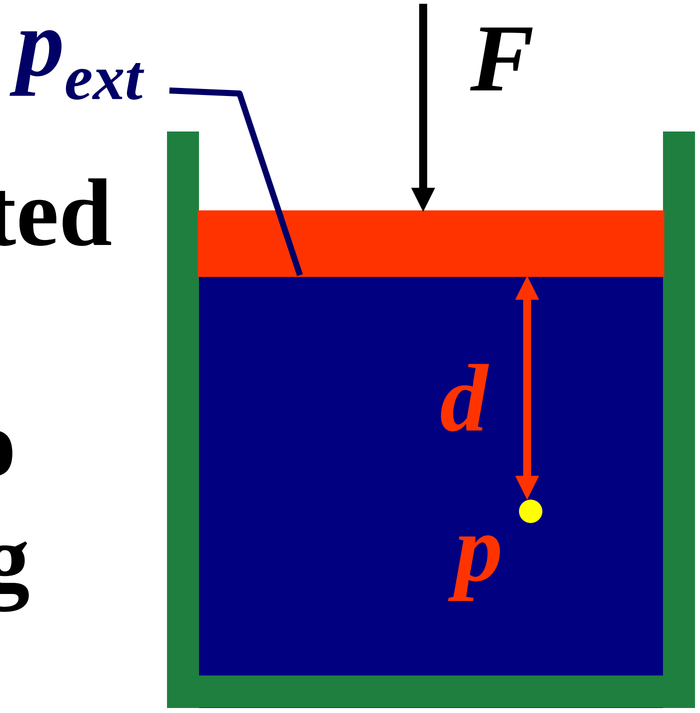
$$p = p_0 \left\{ \left[1 - \gamma h \big/ T_0 \right]^{-T_0/\gamma} \right\}^{-\frac{\rho_0 g}{p_0}} \xrightarrow{\gamma \to 0} p_0 e^{-\frac{\rho_0 g}{p_0} h}$$

Temperature Corrections



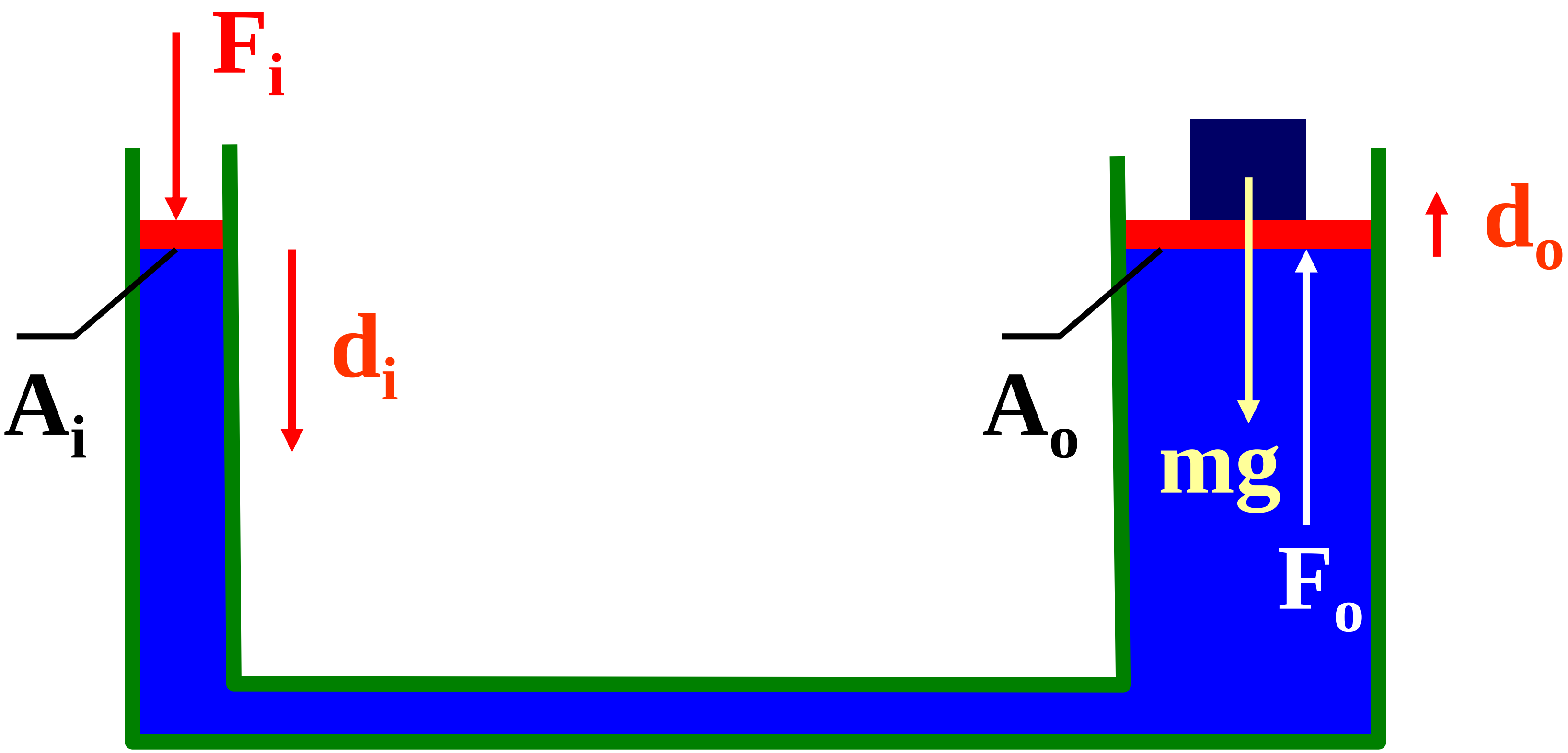
Pascal's Principle

Pressure applied to an enclosed fluid is transmitted undimished to every portion of the fluid and to the walls of the containing vessel.



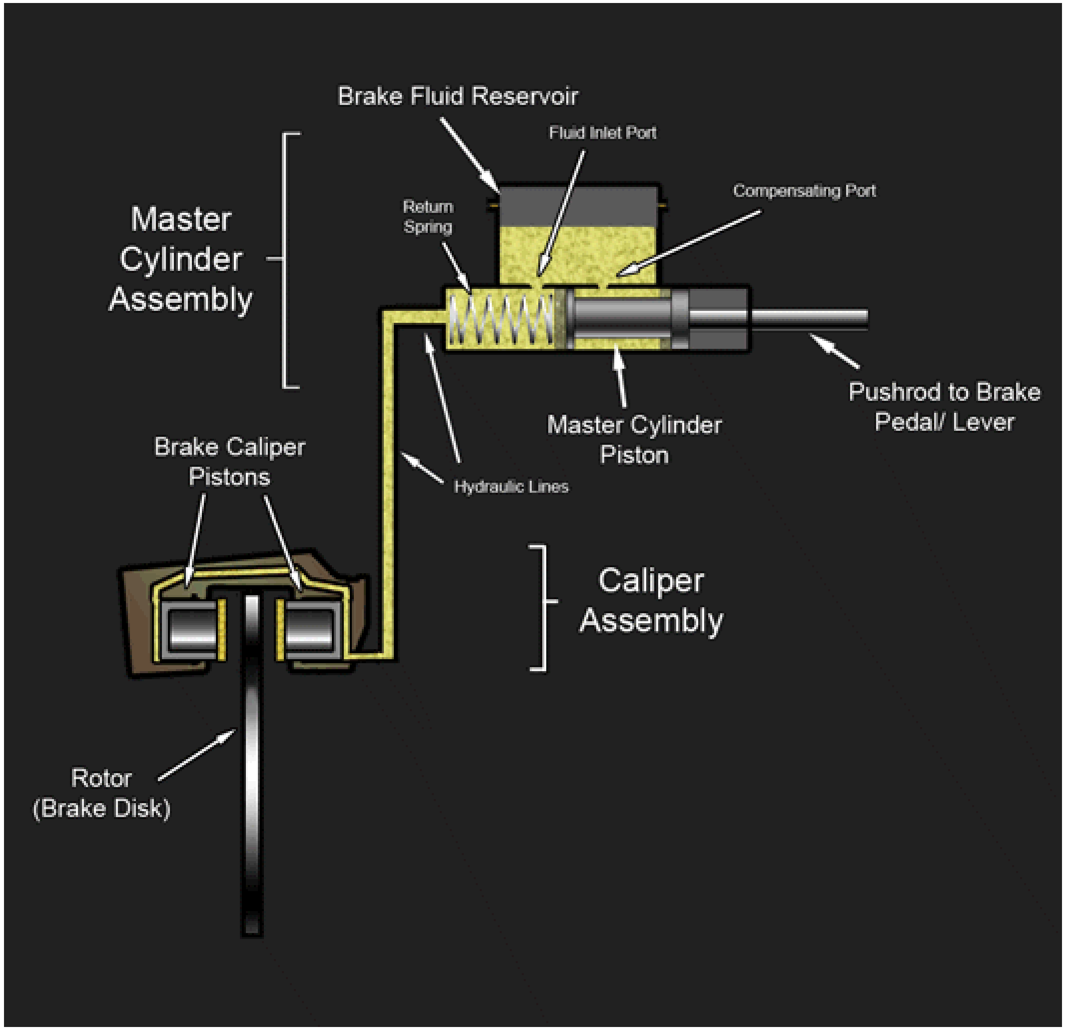
$$p = p_{ext} + \rho g d \Rightarrow \Delta p = \Delta p_{ext}$$

Hydraulic Lever



$$F_i / A_i = F_o / A_o, \quad A_i d_i = A_o d_o$$

Hydraulic Brake



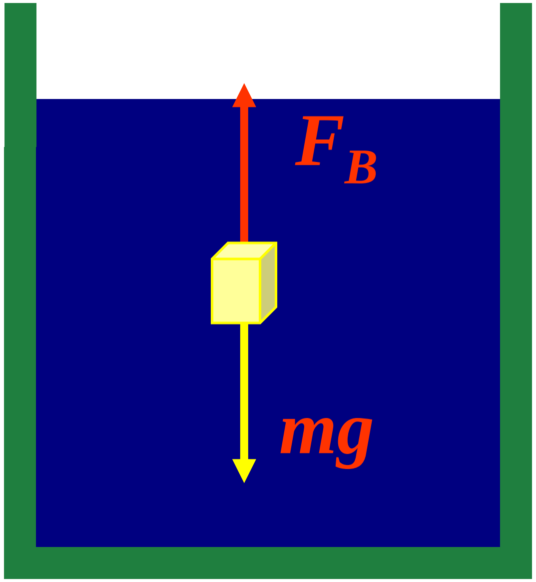
Fred Duesenberg originated hydraulic brakes on his 1914 racing cars and Duesenberg was the first automotive marque to use the technology on a passenger car in 1921. In 1918 Malcolm Lougheed (who later changed the spelling of his name to Lockheed) developed a hydraulic brake system.

Archimedes' Principle

A body wholly or partially immersed in a fluid is buoyed up by a force equal in magnitude to the weight of the fluid displaced by the body.

$$F_B = \rho_{fluid} V g = m_{fluid} g$$

$$mg = \rho_{object} V g$$



Two Comments

Buoyant force ⟺ volume/density

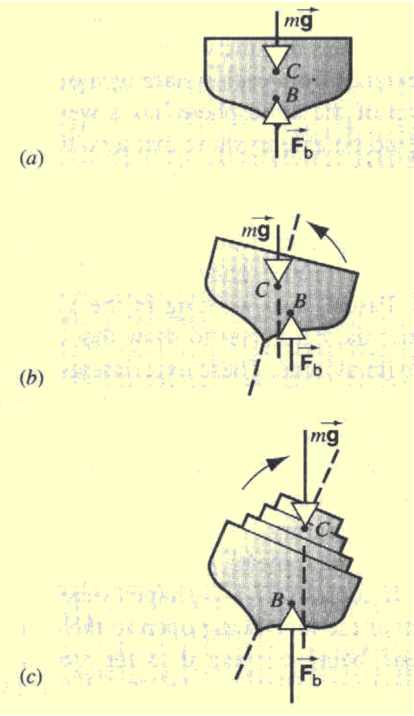
$$F_B = \rho_{fluid} V g = m_{fluid}\, g$$

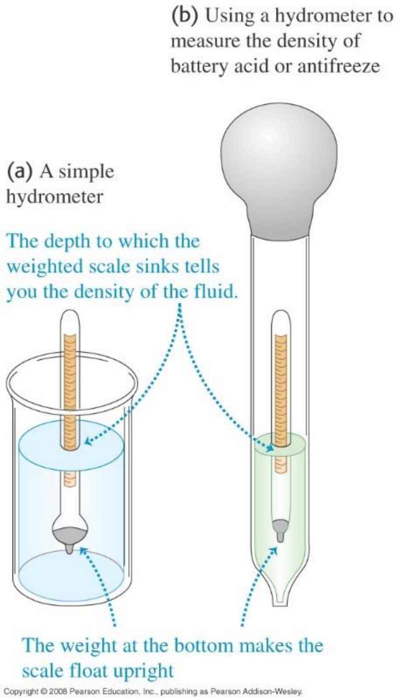
$$mg = \rho_{object} V g$$

Center of buoyancy (not fixed)

Center of gravity

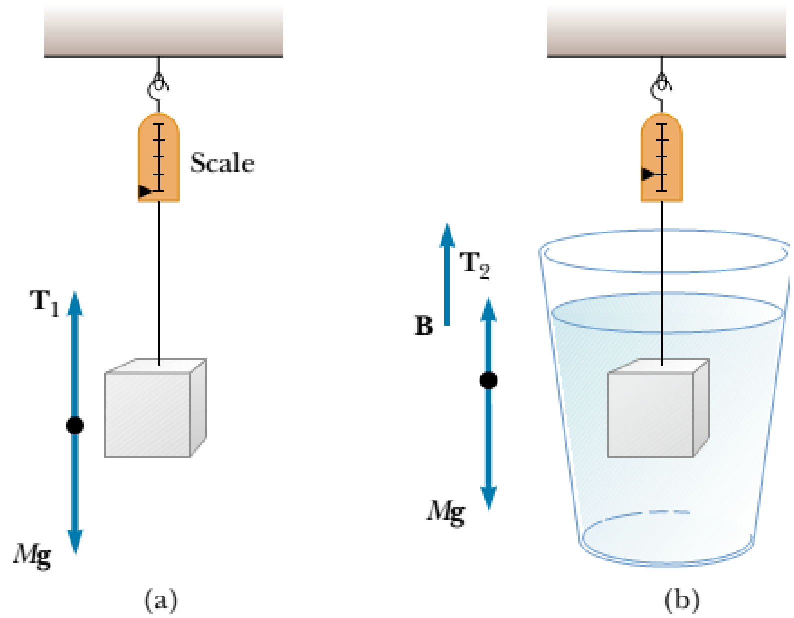
Video—MIT center of gravity or buoyancy!





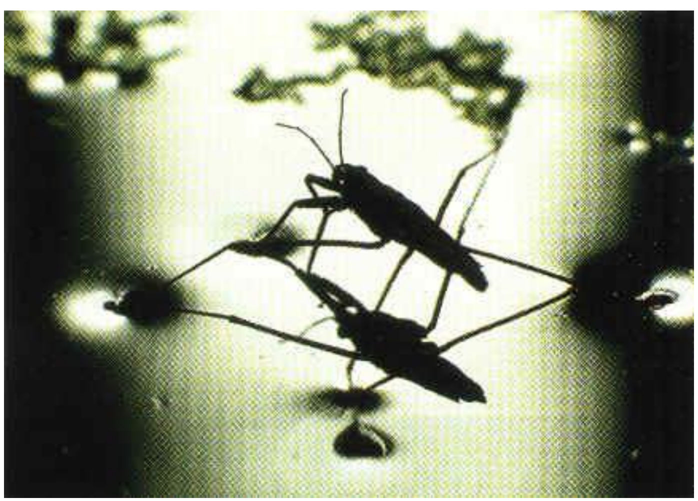
Question: Galileo's Scale

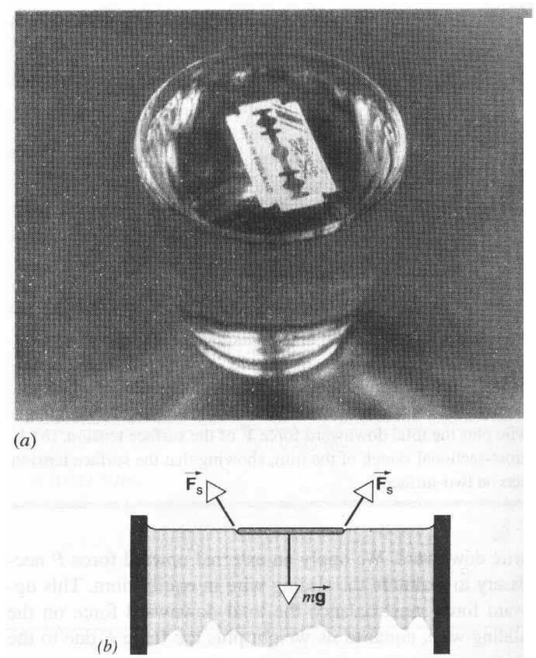
Can you invent a scale that one can use to read the percentage of silver in the gold crown without explicit calculations?



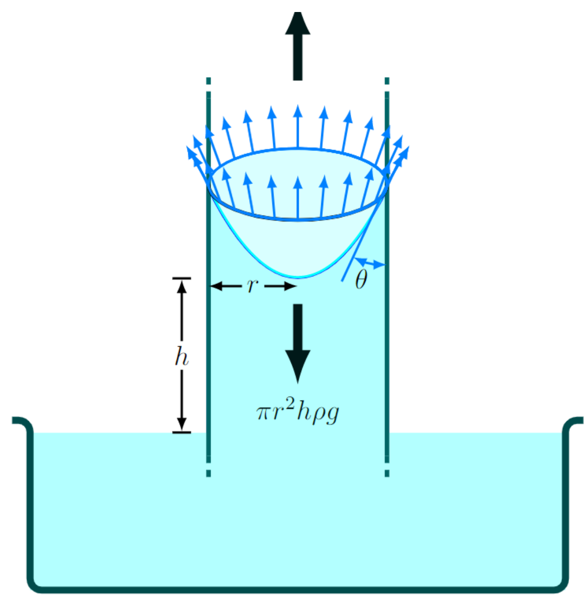
Surface Tension*

$$F = \gamma l \qquad \gamma = \frac{F}{l}$$





$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

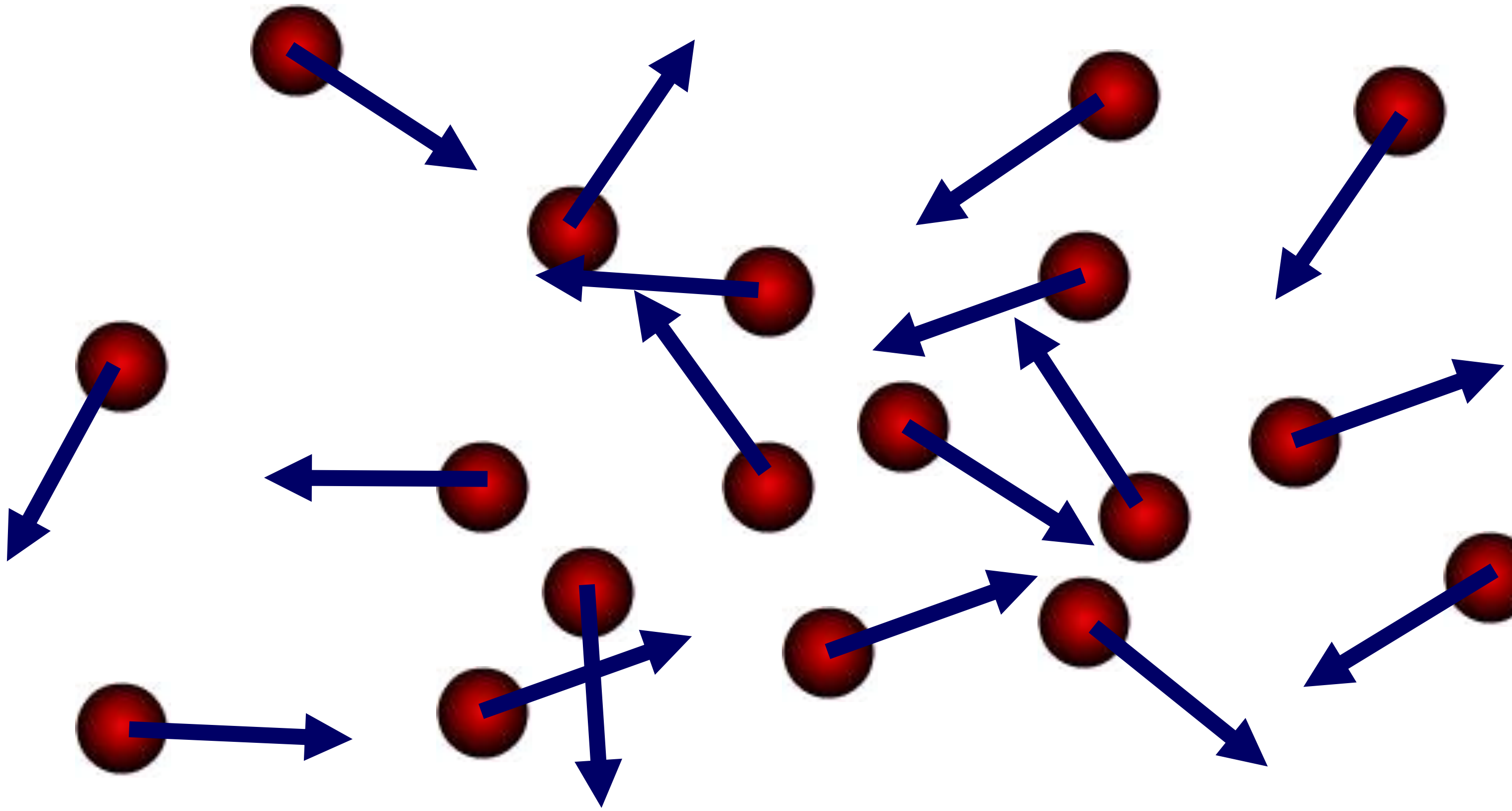


Fluid Dynamics

- **Aerodynamics (gases in motion)**
- **Hydrodynamics (liquids in motion)**
  - **Blaise Pascal**
  - **Daniel Bernoulli, *Hydrodynamica* (1738)**
  - **Leonhard Euler**
  - **Lagrange, d’Alembert, Laplace, von Helmholtz**
- **Airplane, petroleum, weather, traffic**

The Microscopic Approach

N particles $r_i(t)$, $v_i(t)$; interaction $V(r_i\text{-}r_j)$



The Macroscopic Approach

A fluid is regarded as a continuous medium.

Any small volume element in the fluid is always supposed to be so large that it still contains a very large number of molecules.

When we speak of the displacement of fluid at a point, we meant not the displacement of an individual molecule, but that of a volume element containing many molecules, though still regarded as a point.

Euler's Solution

For fluid at a point at a time: **Field**

$$\rho(x, y, z, t), \qquad \vec{v}(x, y, z, t)$$

State of the fluid: described by parameters p**,** T**.**

However, laws of mechanics applied to particles, not to points in space.

Ideal Fluids

Steady: velocity, density and pressure not change in time; no turbulence

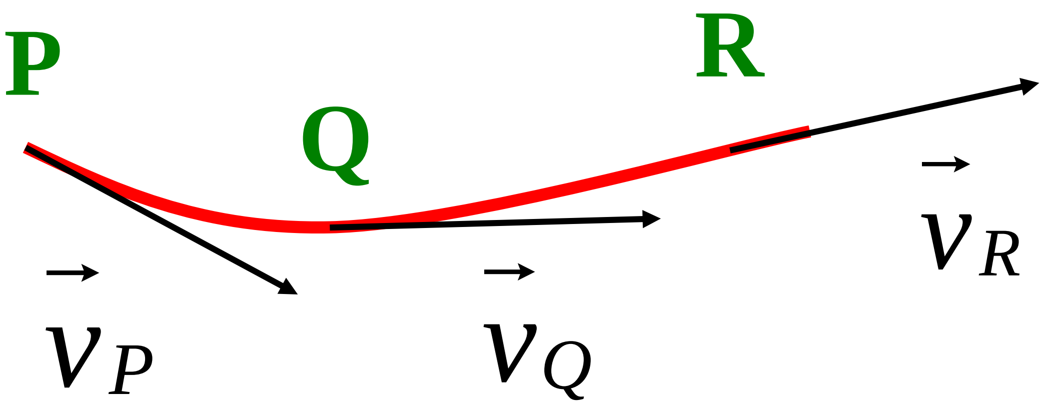
Incompressible: constant density

Nonviscous: no internal friction between adjacent layers

Irrotational: no particle rotation about the center of mass

Streamlines

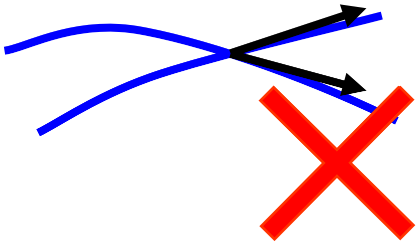
Paths of particles



P → Q → R

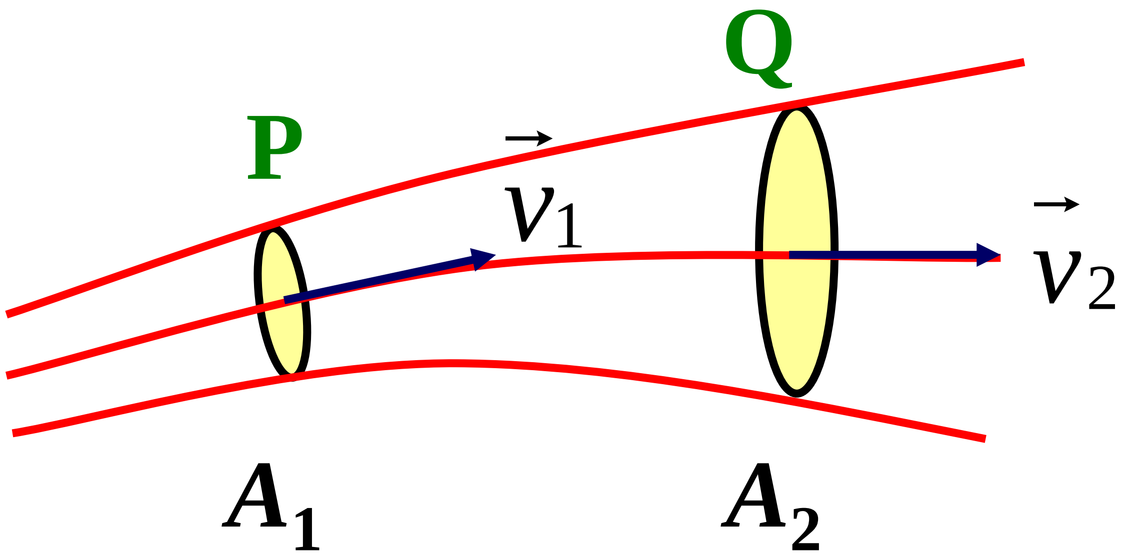
***v* tangent to the streamline**

No crossing of streamlines



Mass Flux

Tube of flow: bundle of streamlines



$$\delta m_1 = \rho_1 A_1 v_1 \delta t \;\Rightarrow\; \text{mass flux } \frac{\delta m_1}{\delta t_1} = \rho_1 A_1 v_1$$

Conservation of Mass

IF: no sources and no sinks/drains

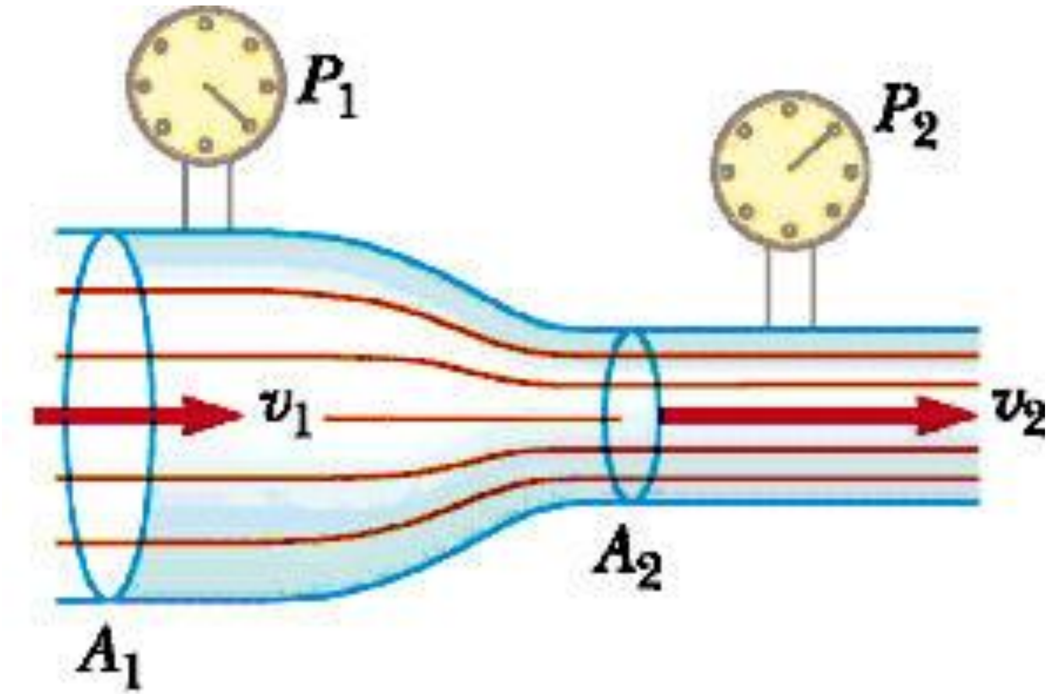
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \text{constant}$$

$$A_1 v_1 = A_2 v_2 = \text{constant, for incompressible fluid}$$

- **Narrower tube == larger speed, fast**
- **Wider tube == smaller speed, slow**

Example of equation of continuity.
Also conservation of charge in E&M

What Accelerates the Fluid?

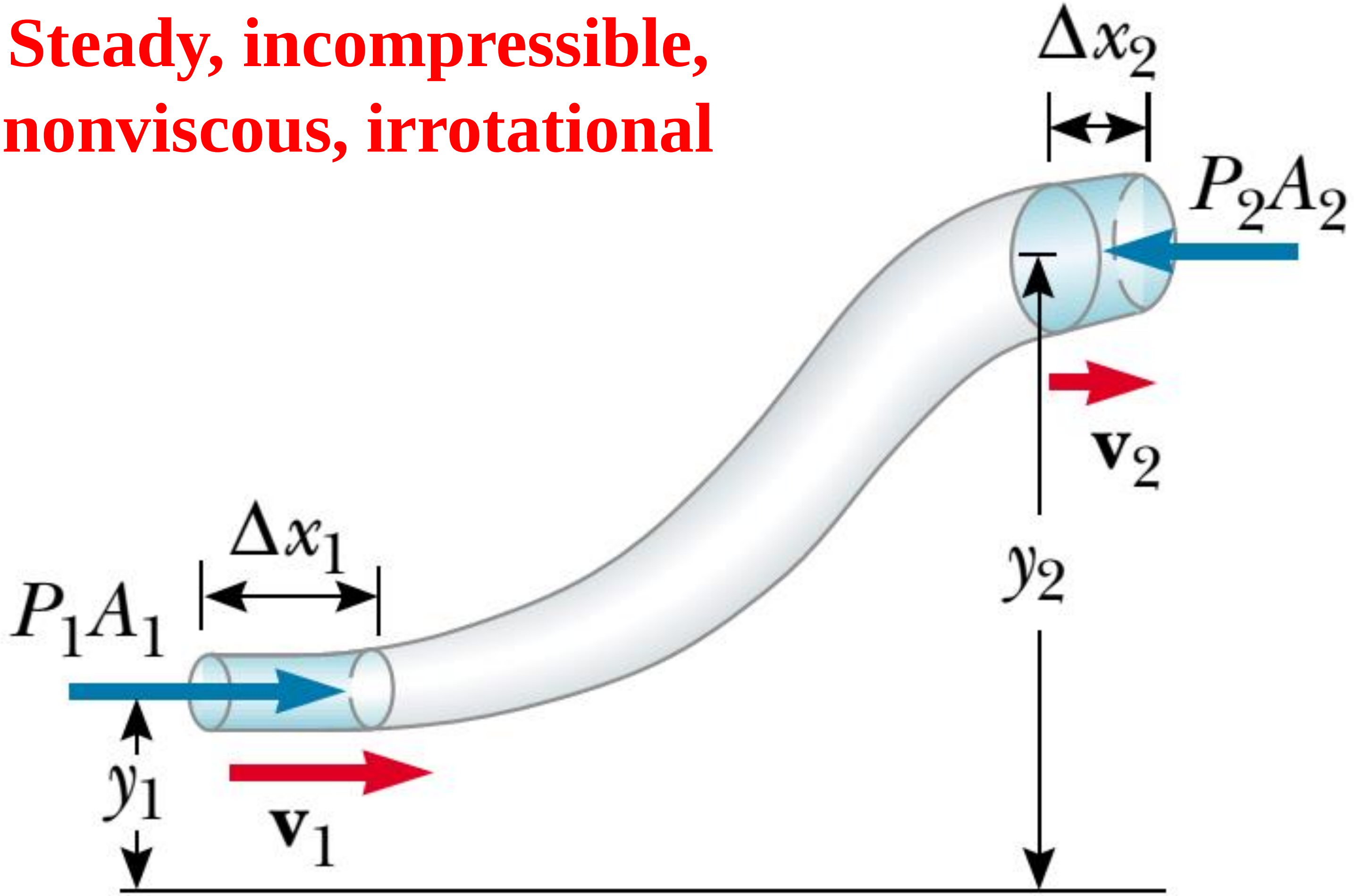


Acceleration due to pressure difference.

Bernoulli's Principle = Work-Energy Theorem

Work-Energy Theorem

Steady, incompressible, nonviscous, irrotational



Bernoulli's Equation

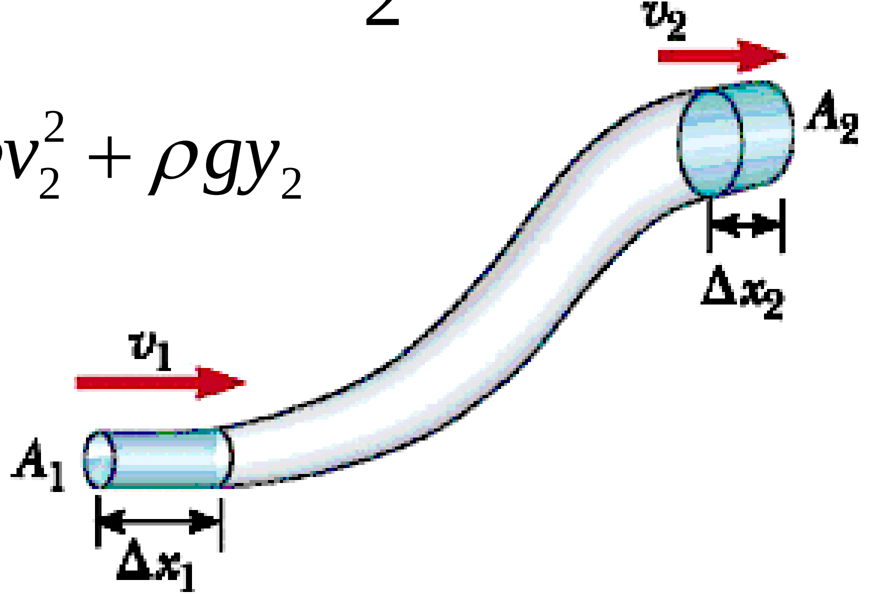
kinetic E, potential E, external work

$$\delta m = \rho A_1 \delta x_1 = \rho A_2 \delta x_2$$

$$p_1 A_1 \delta x_1 - p_2 A_2 \delta x_2 = \frac{1}{2}\delta m v_2^2 + \delta m g y_2 - \frac{1}{2}\delta m v_1^2 - \delta m g y_1$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$



Comments

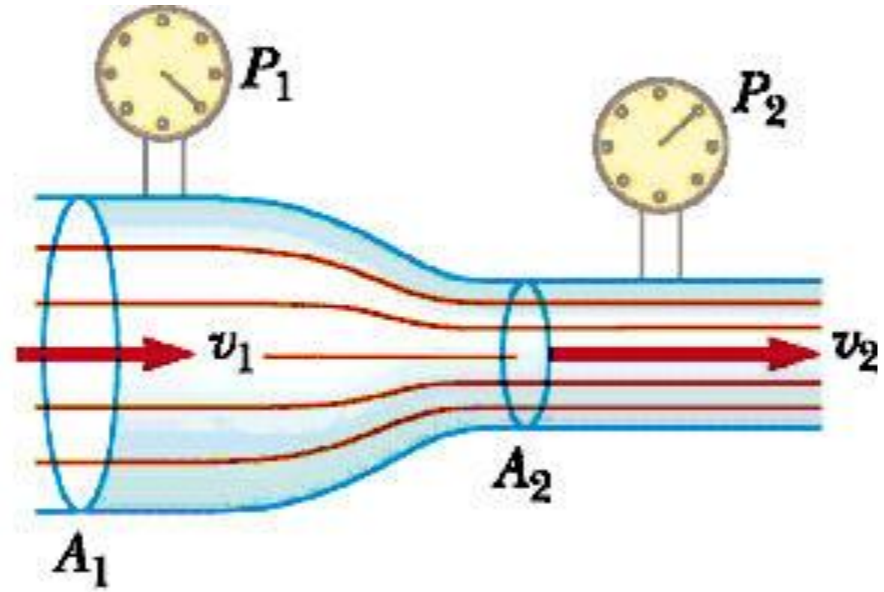
Restrictions of the Bernoulli's equation:

- **Points 1 and 2 lie on the same streamline.**
- **The fluid has constant density.**
- **The flow is steady.**
- **There is no friction.**

When streamlines are parallel the pressure is constant across them (if we ignore gravity and assume velocity is constant over the cross-section).

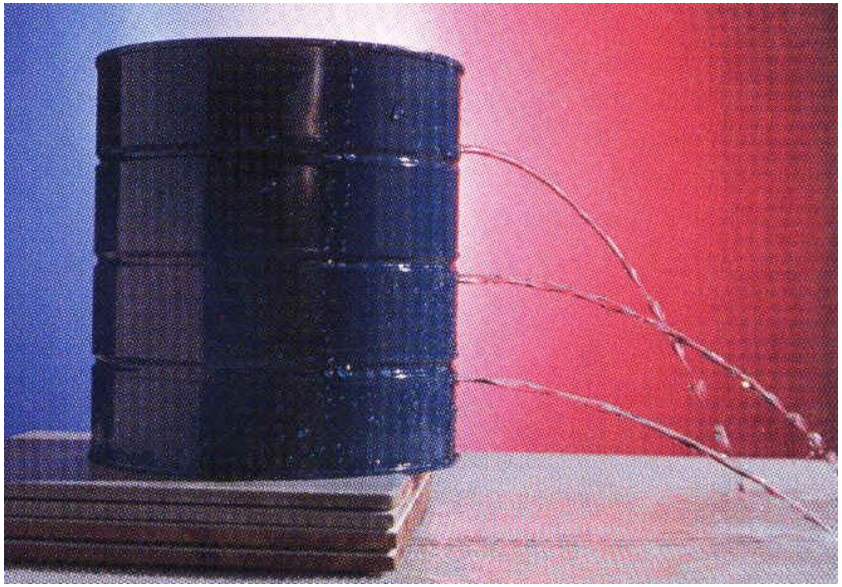
The Venturi Meter

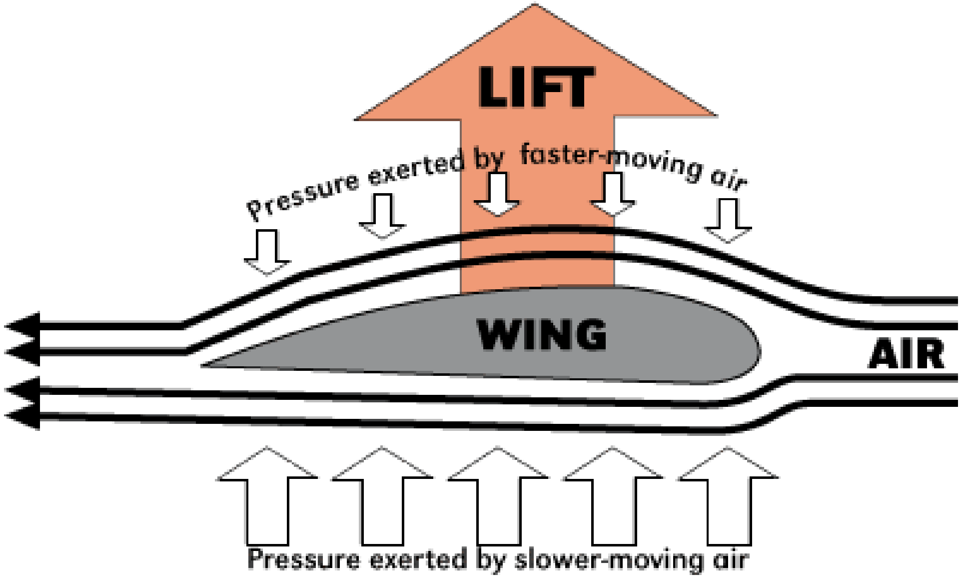
Speed changes as diameter changes. Can be used to measure the speed of the fluid flow.



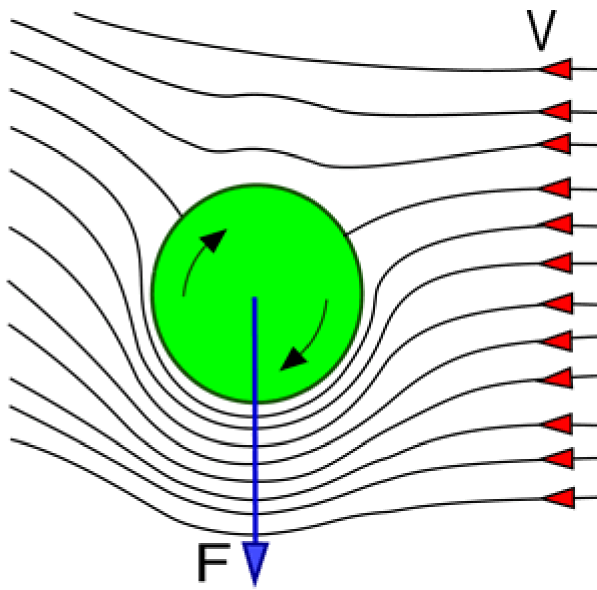
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2, \qquad v_1 A_1 = v_2 A_2$$

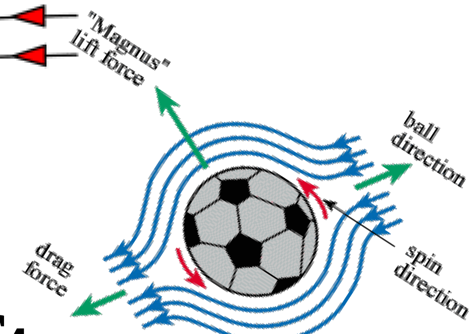
Bernoulli's Equation





Bend it like Beckham*





Dynamic lift

- http://www.tudou.com/programs/view/qLaZ-A0Pk_g/

Banana Free Kick*

Distance 25 m

Initial v = 25 m/s

Flight time 1s

Spin at 10 rev/s

Lift force ~ 4 N

Ball mass ~ 400 g

$a = 10\ \mathrm{m/s^2}$

A swing of 5 m!