Zhejiang University

Department of Physics

General Physics (H) Problem Set #14

1. In order to take a nice warm bath, you mix 50 liters of hot water at 55°C with 25 liters of cold water at 10°C. How much new entropy have you created by mixing the water?

2. Calculate the change in entropy for a process in which 2 moles of an ideal gas undergoes a free expansion to three times its initial volume.

3. Experimental measurements of the heat capacity of aluminum at low temperatures (below about 50 K) can be fit to the formula
$$C_V = aT + bT^3,$$
where C_V is the heat capacity of one mole of aluminum, and the constants a and b are approximately $a = 0.00135$ J/K^2 and $b = 2.48 \times 10^{-5}$ J/K^4. From this data, find a formula for the entropy of a mole of aluminum as a function of temperature (assuming $S = 0$ at 0 K). Evaluate your formula at $T = 1$ K and at $T = 10$ K.

4. Derive the efficiency of the Otto cycle
$$e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1},$$
where V_1/V_2 is the compression ratio and γ is the adiabatic exponent.

5. A bit of computer memory is some physical object that can be in two different states, often interpreted as 0 and 1. A byte is eight bits, a kilobyte is $1024 = 2^{10}$ bytes, a megabyte is 1024 kilobytes, and a gigabyte is 1024 megabytes. (i) Suppose that your computer erases or overwrites one gigabyte of memory, keeping no record of the information that was stored. Explain why this process must create a certain amount of entropy, and calculate how much. (ii) If the entropy is dumped into an environment at room temperature, how much heat must come along with it? Is this amount of heat significant?

6.

Entropy and the Carnot cycle

Consider the Carnot cycle operating with a hot and cold heat baths whose temperatures are T_h and T_c ($T_h > T_c$), respectively. Working substance means the gas in the engine, and we consider a gas in general for the working substance.

(a) Suppose the amount of heat exchange during the isothermal process with the hot (cold) heat bath is Q_h (Q_c), and determine the entropy change ΔS_h and ΔS_c of the working substance in the respective process. Then, specify ΔS_h and ΔS_c are positive or negative. Here, take the positive sign of Q_h and Q_c for the heat input from the heat bath to the working substance.

(b) Figure 1 shows the pressure-volume (PV) diagram of the Carnot cycle. Draw a temperature-entropy (TS) diagram of the corresponding Carnot cycle. Here, take the temperature T for the vertical axis, and the entropy S for the horizontal axis. Specify the direction and what kind of quasi-static process (i.e., isothermal/adiabatic/isobaric/isovolumetric; compression/expansion) it is for each stroke.

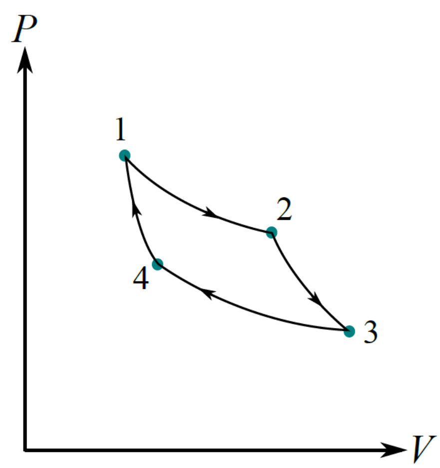

Figure 1: PV diagram of the Carnot cycle.

(c) Using the result of (a), derive the efficiency η of the Carnot cycle in terms of T_h and T_c. Here, the efficiency is defined as $\eta \equiv W/Q_h$, where W is the total work output through the cycle.

(d) Now we use an ideal gas as the working substance and consider a free expansion process. Suppose the initial temperature of the gas is T_i and the volume of the gas increases from V_i to V_f. Determine the heat input Q_{fe} and the entropy change ΔS_{fe} of the working substance through the free expansion process.

(e) By replacing a quasi-static isothermal expansion process in the Carnot cycle by a free expansion process, it seems that it is possible to construct a cycle with a single heat bath. Does this fact violate the second law of thermodynamics? Answer by yes or no, then explain your answer using the case of the Carnot cycle.