Algorithm 1: Binomial Link

```
Input: node: BinNode, child: BinNode
   Output: The root of the binomial heap after linking
1 Function BinomialLink(node, child):
      {f if} node is NULL {f then}
       return child;
3
      if child is NULL then
4
       return node;
5
      {f if}\ node's degree < child's degree\ {f then}
6
          return BinomialLink(child, node);
7
      child's parent \leftarrow node;
8
      child's \ sibling \leftarrow node's \ child;
9
      node's \ child \leftarrow child;
10
      node's degree++;
11
      return node;
12
```

Algorithm 2: Binomial Merge

```
Input: node1: BinNode, node2: BinNode
   Output: The merged binomial heap
 1 Function BinomialMerge(node1, node2):
       // First step, merge them in increasing order;
 2
       BinNode\ pointer1 \leftarrow node1,\ pointer2 \leftarrow node2;
 3
       BinNode begin \leftarrow NULL;
 4
       BinNode node \leftarrow &begin;
 5
       while pointer1 and pointer2 do
 6
           if pointer1's degree \leq pointer2's degree then
 7
               *node \leftarrow pointer1;
 8
               pointer1 \leftarrow pointer1's sibling;
 9
           else
10
               *node \leftarrow pointer2;
11
              pointer2 \leftarrow pointer2's sibling;
12
           node \leftarrow \&((*node) \rightarrow sibling)
13
       if pointer1 == NULL then
14
           *node \leftarrow pointer2;
15
16
       else
           *node \leftarrow pointer1;
17
       if !begin then
18
          return begin;
19
       // Second step, combine subtrees with same degrees;
20
       BinNode prev \leftarrow NULL;
21
       BinNode current \leftarrow begin;
22
       BinNode next \leftarrow current's sibling;
23
       while next do
24
           if current's degree \neq next's degree or (next's sibling and next's
25
            sibling's degree == current's degree) then
               prev \leftarrow current;
26
               current \leftarrow next;
27
           else
28
               if current's degree == next's degree then
29
                   if current's \ value \leq next's \ value then
30
                       current's sibling \leftarrow next's sibling;
31
                       BinNode temp \leftarrow BinomialLink(current, next);
32
                   else
33
                       if prev == NULL then
34
                        begin \leftarrow next;
35
                       else
36
                        | prev's sibling \leftarrow next;
37
                       BinomialLink(next, current);
38
                       current \leftarrow next;
39
           next \leftarrow current's sibling;
40
       return begin;
41
```

```
Algorithm 3: BinGetMin2
      Input: node: BinNode
      Output: An array containing the minimum node and its previous node
   1 Function BinGetMin2(node):
          \mathbf{if} \ \mathit{node} == \mathit{NULL} \ \mathbf{then}
   2
           return NULL;
    3
          BinNode^* min2 \leftarrow (BinNode^*)malloc(2 * sizeof(BinNode));
   4
          if min2 == NULL then
   5
           // Handle memory allocation failure return NULL;
    6
          \min 2[0] \leftarrow \text{NULL};
   7
          \min 2[1] \leftarrow \text{node};
          BinNode prev \leftarrow NULL;
   9
          BinNode current \leftarrow node;
  10
          while current do
  11
              if current \rightarrow value \leq min2/1 \rightarrow value then
  12
                   // Update the min node and its previous node min2[0] \leftarrow
  13
                    prev;
                   min2/1 \leftarrow current;
   14
               prev \leftarrow current;
  15
               \mathit{current} \leftarrow \mathit{current} {\rightarrow} \mathit{sibling}
  16
    17
              return min2;
18
```

Algorithm 4: BinDeleteMin

```
Input: node: BinNode
   Output: The root node after deleting the minimum node
 1 Function BinDeleteMin(node):
       if node == NULL then
 \mathbf{2}
           return NULL;
 3
       // Get the min node and its previous node. BinNode* min2 \leftarrow
 4
         BinGetMin2(node);
       BinNode prev \leftarrow \min 2[0];
 5
       BinNode min \leftarrow min2[1];
 6
        // Remove the min node from the list. if prev == NULL then
 7
 8
           node \leftarrow min \rightarrow sibling
 9
           prev-\dot{\epsilon}sibling \leftarrow \min \rightarrow sibling
10
        // Reverse the child list of min node. BinNode childlist ← NULL;
11
       BinNode child \leftarrow min\rightarrow child while child do
12
            BinNode temp \leftarrow child\rightarrow sibling if !childlist then
13
                // If childlist is empty, create a new list. childlist \leftarrow child;
14
                childlist \rightarrow sibling \leftarrow NULL;
15
                childlist \rightarrow parent \leftarrow NULL;
16
            else
17
                // If childlist is not empty, insert the new node to the head
18
                 of the list. childlist \rightarrow parent \leftarrow NULL;
                child \rightarrow sibling \leftarrow childlist;
19
                childlist \leftarrow child;
20
           child \leftarrow temp;
21
       free(min 2);
22
       return BinomialMerge(node, childlist);
\mathbf{23}
```

Algorithm 5: BinDeleteMin Input: node: BinNode

```
Input: node: BinNode
   Output: The root node after deleting the minimum node
 1 Function BinDeleteMin(node):
       if node == NULL then
 \mathbf{2}
           return NULL;
 3
       // Get the min node and its previous node. BinNode* min2 \leftarrow
 4
         BinGetMin2(node);
       BinNode prev \leftarrow \min 2[0];
 5
       BinNode min \leftarrow min2[1];
 6
        // Remove the min node from the list. if prev == NULL then
 7
 8
           node \leftarrow min \rightarrow sibling
 9
           \text{prev} \rightarrow sibling \leftarrow \min \rightarrow sibling
10
        // Reverse the child list of min node. BinNode childlist ← NULL;
11
       BinNode child \leftarrow min\rightarrow child while child do
12
            BinNode temp \leftarrow child\rightarrow sibling if !childlist then
13
                // If childlist is empty, create a new list. childlist \leftarrow child;
14
                childlist \rightarrow sibling \leftarrow NULL;
15
                childlist \rightarrow parent \leftarrow NULL;
16
            else
17
                // If childlist is not empty, insert the new node to the head
18
                 of the list. childlist \rightarrow parent \leftarrow NULL;
                child \rightarrow sibling \leftarrow childlist;
19
                childlist \leftarrow child;
20
           child \leftarrow temp;
21
       free(min 2);
22
       return BinomialMerge(node, childlist);
\mathbf{23}
```

Algorithm 6: BinDecrease

```
Input: binnode: BinNode, value: int, NodeArray: BinNode
 1 Function BinDecrease(binnode, value, NodeArray):
         binnode\rightarrow value \leftarrow value;
 3
         BinNode parent, child;
         parent \leftarrow binnode\rightarrow parent child\leftarrow binnode;
 4
         while parent \neq NULL and parent \rightarrow value > child \rightarrow value do
 5
               // Exchange the position between child and parent.
 6
               int temp_value, temp_vertex;
 7
               BinNode\ temp \leftarrow NodeArray[child \rightarrow vertex]
 8
               NodeArray[child \rightarrow vertex] \leftarrow NodeArray[parent \rightarrow vertex]
 9
               NodeArray[parent \rightarrow vertex] \leftarrow temp;
10
               temp\_value \leftarrow \mathit{child} \rightarrow \mathit{value}
11
               temp\_vertex \leftarrow child \rightarrow vertex
12
               \mathit{child} \!\!\to \mathit{value} \!\!\leftarrow \mathit{parent} \!\!\to \mathit{value}
13
              \mathit{child} \!\!\to \mathit{vertex} \!\!\leftarrow \mathit{parent} \!\!\to \mathit{vertex}
14
               parent \rightarrow value \leftarrow temp\_value;
15
               parent \rightarrow vertex \leftarrow temp\_vertex;
16
               child \leftarrow parent;
17
              parent \leftarrow parent \rightarrow parent
18
```

Algorithm 7: IsBinEmpty

```
Input: node: BinNode
```

Output: 1 if the binomial heap is empty, 0 otherwise

- 1 Function IsBinEmpty(node):
- **return** node == NULL;