Coulomb's Law and the Electric Field

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Lecture 1

Outline

- Electric Charge
- Coulomb's Law
- The Electric Field
    - Electric Field Lines
    - Electric Dipole
- Motion of Charged Particles in an Electric Field

Insulators and Conductors

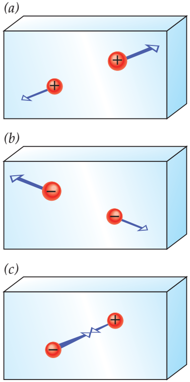
- **Insulators** are materials through which charge cannot move freely. Examples: rubber, plastic, glass, and chemically pure water.
- **Conductors** are materials through which charge can move rather freely. Examples: metals (Cu, Al, . . .), the human body, and tap water.
- **Semiconductors** are materials that are intermediate between conductors and insulators. Examples: Si & GaAs.
- **Superconductors** are materials that are perfect conductors, allowing charge to move without any hindrance.

Why Conductors Conduct?

- Conductors (and insulators) are made up of atoms.
- An electrically neutral atom contains equal numbers of electrons and protons.
- When atoms of a conductor like copper come together to form the solid, some of their electrons become free to wander about within the solid, leaving behind positively charged atoms (positive ions).
- We call the mobile electrons **conduction electrons**.
- We will need quantum mechanics for a complete understanding.

Properties of Electric Charge

- There are two types of electric charge, named by Benjamin Franklin as *positive charge* and *negative charge*.
- When the net charge of an object is zero, the object is said to be (electrically) *neutral*.
- Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other.



Quantized and Conserved Electric Charge

- Particles are the substance and charge happens to be one of their properties, just as mass is. But unlike mass, charge is *quantized*, namely, charge can have only discrete values rather than any value.
- Experiment shows that any positive or negative charge can be written as $q = ne$, where n is an integer. The elementary charge is $e = 1.602 \times 10^{-19}$ C.
- Electron/proton has a charge of magnitude e.
- Charge can be transferred from one body to another, but cannot be created or annihilated *individually*. This is known as the *conservation of charge*.
- **Charge is Lorentz INVARIANT.**

Milliken's Measurement of the Electron Charge

- J. J. Thompson discovered electron in 1897 and measured the charge-to-mass ratio of electron.
- R. A. Milliken, with his graduate student H. Fletcher, measured the charge of an electron ($e = 1.592 \times 10^{-19}$ C, see Appendix 1A) in the "oil drop experiment" (Nobel Prize in Physics, 1923).

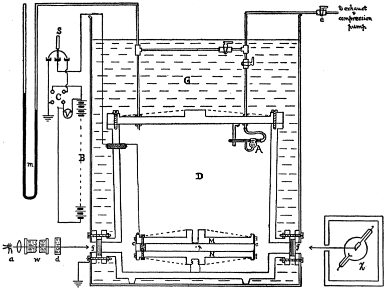


Figure 1: Apparatus shown in R. A. Millikan, Phys. Rev. 32, 349 (1911).

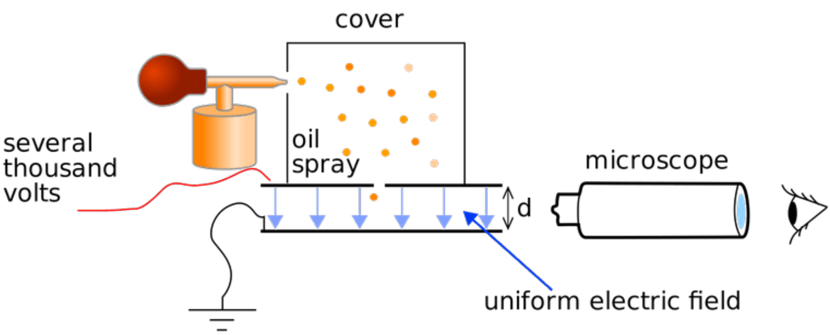


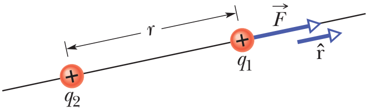
Figure 2: Oil droplets with different charge q are subject to gravitational force, buoyancy force, drag force, and electric force (in the region of the uniform electric field).

Coulomb's Law (1785)

- If two charged particles are brought near each other, they each exert an (attractive or repulsive) electrostatic force on the other.

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$

  where q_1 and q_2 are the charges of the two particles. The separation between the particles is r, and $\hat{r}$ is a unit vector along the axis through the two particles.



- The *electrostatic constant* k has a value 8.99×10^9 N·m^2/C^2.
- For simplicity (later in Maxwell's equations) and historical reasons, we often write

$$k = \frac{1}{4\pi\epsilon_0},$$

where $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N·m^2 is known as the *permittivity constant*.

Action at a Distance

- An electrostatic force acts on particle 1 due to the presence of particle 2. Since the particles do not touch, how can particle 2 push on particle 1? How can there be such an *action at a distance*?
- Explanation: Particle 2 sets up an **electric field** at all points in the surrounding space, even if the space is a vacuum. If we place particle 1 at any point in that space, particle 1 knows of the presence of particle 2 because it is affected by the electric field particle 2 has already set up at that point. Thus, particle 2 pushes on particle 1 by means of the electric field it has set up.

What Is a Field?

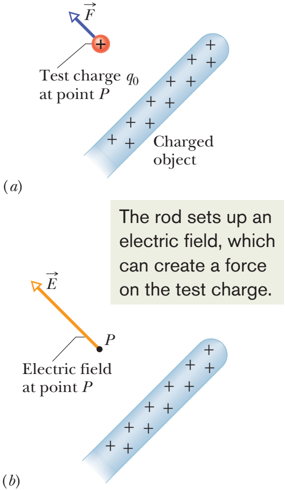
- **Field** is a map, or a function, that represents the distribution of a quantity.
- Scalar fields
  - Temperature field $T(x, y, z)$ – e.g., in heat conduction
  - Pressure field $P(x, y, z)$ – e.g., in Pascal's principle
- Vector fields
  - Velocity field $\vec{v}(x, y, z)$ – e.g., in fluid dynamics
  - Force field $\vec{F}(x, y, z)$
- There can also be *time-varying* fields, e.g., $T(x, y, z, t)$.

The Electric Field

- For a certain collection of charges that exert a force $\vec{F}$ on a positive *test charge* q_0 at a particular point, we define the **electric field** as

$$\vec{E} = \vec{F}/q_0.$$

- We always represent an electric field with an arrow with its tail anchored on the point where the measurement is made.



The Electric Field Due to a Point Charge

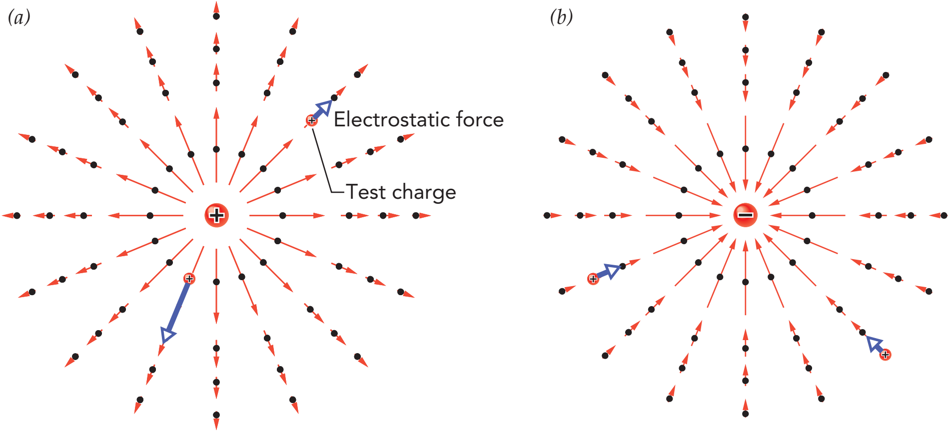
- From Coulomb's law, the force on the test charge due to a charged particle (often called a point charge) with charge q at distance r is

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\hat{r}.$$

- We can now write the electric field set up by the particle (at the location of the test charge) as

$$\vec{E} = \vec{F}/q_0 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}.$$

- The direction of $\vec{E}$ is directly away from the particle if q is positive and directly toward it if q is negative.



Electric field of a positive charge

Electric field of a negative charge

Electric Field Lines

- Michael Faraday introduced the idea of electric fields and envisioned **electric field lines** in the space around any given charged particle or object.

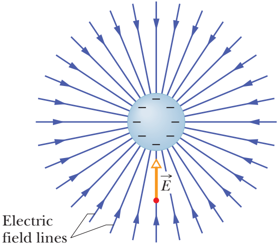
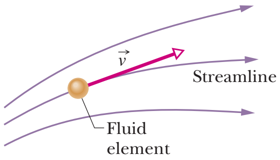
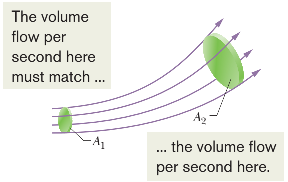


Figure 3: Electric field lines near a negatively charged sphere.

- As you can imagine, the idea of electric field lines resembles that of streamlines in an ideal fluid. The velocity vector of the element is tangent to the streamline at every point.



- Fluid must come from sources, and vanish at drains.
- The equation of continuity tells us that the flow speed decreases when we increase the cross-sectional area through which the fluid flows.



Rules for Drawing Field Lines

- Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).
- At any point, the electric field vector *must be tangent* to the electric field line through that point and in the same direction.
- In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field there, with greater density for greater magnitude.

More Examples

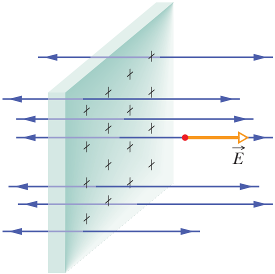


Figure 4: Field lines near a very large, nonconducting sheet. Symmetry dictates that the field lines are perpendicular to the sheet.

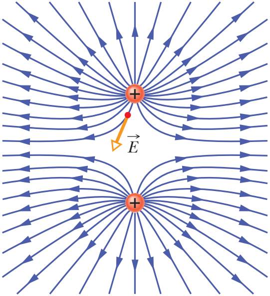
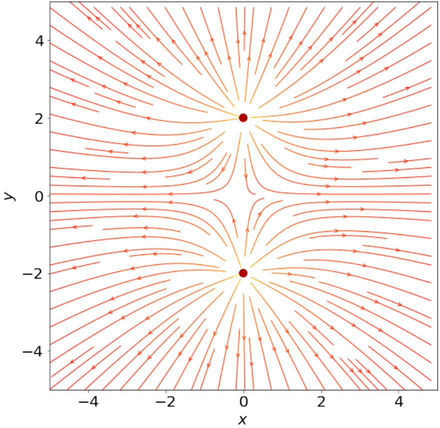


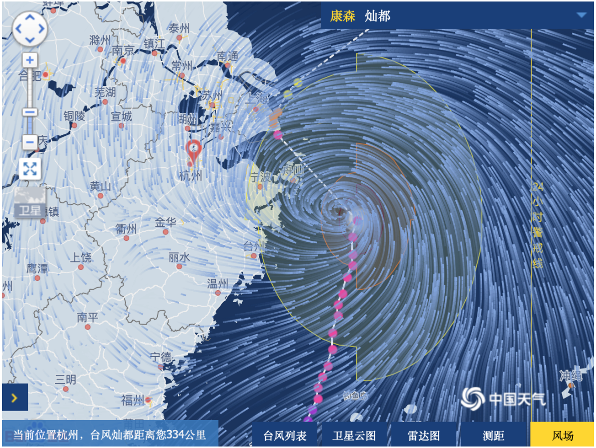
Figure 5: Field lines for two particles with equal positive charge.

Computer Visualization



- In modern times, we probably rely on a computer to visualize the field lines.
- The figure is generated by the streamplot method using Python matplotlib.
- Discuss the difference from the conventional plot (e.g., on the previous page).

Typhoon Wind Field (2021.9.13)



Real-time link

The Electric Field Due to a Dipole

- An **electric dipole** is a pair of two particles that have the same charge magnitude q but opposite signs. The particles are separated by distance d and lie along the dipole axis, an axis of rotation symmetry.
- The product $q\vec{d}$ is known as the *electric dipole moment* $\vec{p}$ of the dipole.

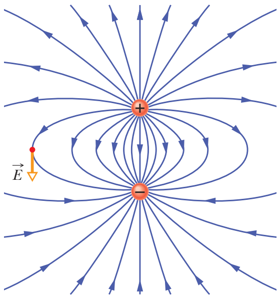


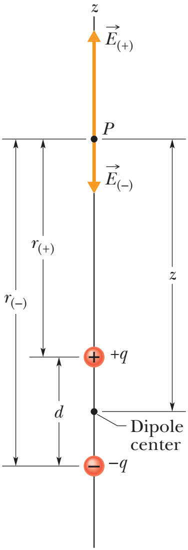
Figure 6: Electric field lines around an electric dipole.

- We restrict our interest to the electric field $\vec{E}$ at an arbitrary point P along the dipole axis, at distance z from the dipole's center,

$$E = \frac{q}{4\pi\epsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\epsilon_0(z + \frac{1}{2}d)^2}.$$

- For $z \gg d$, we obtain (see Appendix 1B for hint)

$$E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}.$$



- The electric field of a dipole decreases more rapidly with distance than does the electric field of a single charge. It turns out that E for a dipole varies as $1/r^3$ for all distant points, regardless of whether they lie on the dipole axis.
- The physical reason for this rapid decrease in electric field for a dipole is that from distant points a dipole looks like two particles that almost — but not quite — coincide. Thus, because they have charges of equal magnitude but opposite signs, their electric fields at distant points cancel each other at the leading order $(1/r^2)$.

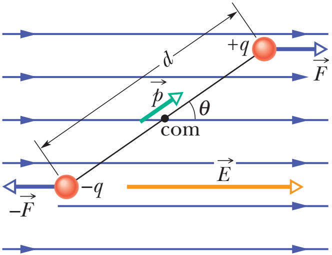
A Point Charge in an Electric Field

- An electrostatic force $\vec{F} = q\vec{E}$ acts on the particle, in which q is the charge of the particle (including its sign) and $\vec{E}$ is the **external field**, i.e., the electric field that other charges have produced at the location of the particle.
- The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

A Dipole in a Uniform Electric Field

- In a uniform electric field, the net force on the dipole from the field is zero and the center of mass of the dipole does not move.
- However, the forces on the charged ends do produce a net torque

$$\tau = -F\frac{d}{2}\sin\theta - F\frac{d}{2}\sin\theta$$
$$= -Fd\sin\theta.$$

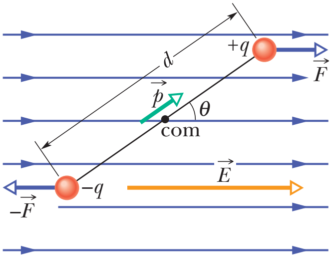


- In terms of the magnitudes of the electric field E and the dipole moment $p = qd$, we find

$$\tau = -Fd\sin\theta = -pEsin\theta$$

- In vector form (see Appendix 1C for a reminder),

$$\vec{\tau} = \vec{p} \times \vec{E}.$$



- The torque tends to rotate $\vec{p}$ into the direction of field $\vec{E}$, thereby reducing θ.

Quiz 1-1

Summary

- Concepts: electric field, electric field lines, electric dipole
- Key formulas:

$$\vec{E} = \vec{F}/q_0 \qquad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$E_z = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3} \qquad \vec{p} = q\vec{d}$$

$$\vec{F} = q\vec{E} \qquad \vec{\tau} = \vec{p} \times \vec{E}$$

- What are the similarities and differences between Coulomb's law and Newton's law of universal gravitation?

Reading

Halliday, Resnick & Krane:

- Chapter 25. Electric Charge and Coulomb's Law
- Chapter 26. The Electric Field

Appendix 1A: Feynman on Cargo Cult Science

“We have learned a lot from experience about how to handle some of the ways we fool ourselves. One example: Millikan measured the charge on an electron by an experiment with falling oil drops and got an answer which we now know not to be quite right. It’s a little bit off, because he had the incorrect value for the viscosity of air. It’s interesting to look at the history of measurements of the charge of the electron, after Millikan. If you plot them as a function of time, you find that one is a little bigger than Millikan’s, and the next one’s a little bit bigger than that, and the next one’s a little bit bigger than that, until finally they settle down to a number which is higher.

“Why didn’t they discover that the new number was higher right away? It’s a thing that scientists are ashamed of—this history—because it’s apparent that people did things like this: When they got a number that was too high above Millikan’s, they thought something must be wrong—and they would look for and find a reason why something might be wrong. When they got a number closer to Millikan’s value they didn’t look so hard. And so they eliminated the numbers that were too far off, and did other things like that. We’ve learned those tricks nowadays, and now we don’t have that kind of a disease.”

The takeaway message in Feynman's commencement address at Caltech is: Don't aim for fast results with superficial learning.

"But this long history of learning how to not fool ourselves—of having utter scientific integrity—is, I'm sorry to say, something that we haven't specifically included in any particular course that I know of. We just hope you've caught on by osmosis.

"The first principle is that you must not fool yourself—and you are the easiest person to fool. So you have to be very careful about that. After you've not fooled yourself, it's easy not to fool other scientists. You just have to be honest in a conventional way after that."

http://calteches.library.caltech.edu/51/2/CargoCult.htm

The story of Cargo Cult: "... In the South Seas there is a Cargo Cult of people. During the war they saw airplanes land with lots of good materials, and they want the same thing to happen now. So they've arranged to make things like runways, to put fires along the sides of the runways, to make a wooden hut for a man to sit in, with two wooden pieces on his head like headphones and bars of bamboo sticking out like antennas—he's the controller—and they wait for the airplanes to land. They're doing everything right. The form is perfect. It looks exactly the way it looked before. But it doesn't work. No airplanes land. So I call these things Cargo Cult Science, because they follow all the apparent precepts and forms of scientific investigation, but they're missing something essential, because the planes don't land."

Appendix 1B: Polynomial Expansion in Physics

- In physics, we are often interested in the properties $f(x)$ of systems with a parameter x, which is small. In such cases, we expand the properties in terms of polynomials of x in the following fashion (Maclaurin series)

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots .$$

  The expansion is a special example of the Taylor series.
- In most cases, the expansion can be terminated at either the linear term (x) or the quadratic term (x^2).

- Some useful expansions are

$$1/(1+x) = 1 - x + x^2 + \cdots$$

$$\sqrt{1+x} = 1 + x/2 + \cdots$$

$$(1+x)^n = 1 + nx + \cdots$$

$$e^x = 1 + x + x^2/2 + \cdots$$

$$\sin x = x - x^3/6 + \cdots$$

$$\cos x = 1 - x^2/2 + \cdots$$

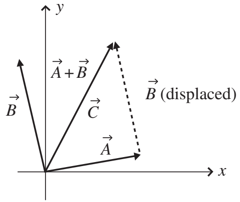
$$\ln(1+x) = x - x^2/2 + \cdots$$

- We often encounter the case when a parameter, say z, is large. Then we can expand with $1/z$. For example, when $z \gg a$,

$$
\begin{aligned}
\frac{1}{(z+a)^2} &= \frac{1}{z^2(1+a/z)^2} \\
&= \frac{1}{z^2}\frac{1}{(1+a/z)^2} \\
&= \frac{1}{z^2}\left[1 - 2(a/z) + \cdots\right] \\
&= \frac{1}{z^2} - \frac{2a}{z^3} + \cdots
\end{aligned}
$$

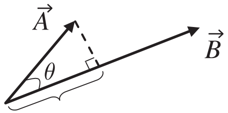
Appendix 1C: Vector Algebra

- **Vectors** have direction as well as magnitude. The magnitude of a vector $\vec{A}$ is written as $|\vec{A}|$ or A.
- Addition
  - *Commutative*: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
  - *Associative*: $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$
  - Subtraction: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



- Multiplication by a scalar a
  - *Distributive*: $a(\vec{A} + \vec{B}) = a\vec{A} + a\vec{B}$

- **Dot product** of two vectors:



$$\vec{A} \cdot \vec{B} \equiv AB\cos\theta.$$

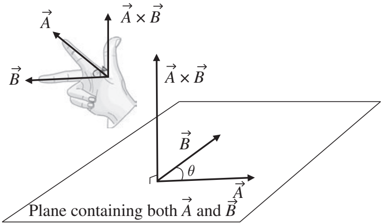
- Geometrically, $\vec{A} \cdot \vec{B}$ is the product of B times the projection of $\vec{A}$ along $\vec{B}$ (or the product of A times the projection of $\vec{B}$ along $\vec{A}$).
  - In particular, for any vector $\vec{A}$, $\vec{A} \cdot \vec{A} = A^2$.
  - If $\vec{A}$ and $\vec{B}$ are perpendicular, then $\vec{A} \cdot \vec{B} = 0$.

- *Commutative*: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- *Distributive*: $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

- **Cross product** of two vectors:

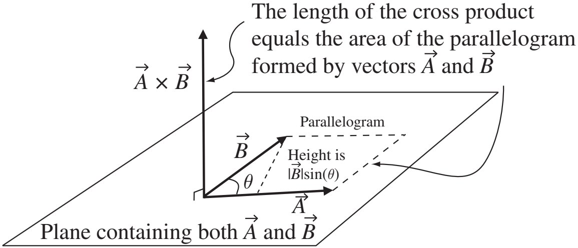
$$\vec{A} \times \vec{B} \equiv AB \sin\theta \hat{n},$$

where $\hat{n}$ is a unit vector whose direction is determined by the *right-hand rule*.



- *Anti-commutative*: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

- Geometrically, $|\vec{A} \times \vec{B}|$ is the area of the parallelogram generated by $\vec{A}$ and $\vec{B}$.
  - In particular, $\vec{A} \times \vec{A} = 0$.



- *Distributive*: $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$