Polarization

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Lecture 20

Conditions for Coherent Superposition

- Recall the superposition of two light waves

$$E_{01}\cos(k_1x_1 - \omega_1 t + \phi_1) + E_{02}\cos(k_2x_2 - \omega_2 t + \phi_2).$$

  - The phase difference between the two waves is

$$\delta = (k_2x_2 - k_1x_1) - (\omega_2 - \omega_1)t + (\phi_2 - \phi_1).$$

- To observe interference of the two waves,
  - two beams must have (nearly) the same frequency ω,
  - interfering waves have comparable amplitude, and
  - the phase difference between sources must remains constant.
- In this lecture we consider the direction of $\vec{E}$.

Outline

- Polarization and Its Mathematical Description
- Monochromatic Light and Natural Light
- Polarizing Sheets
- Polarization by Reflection

Polarization

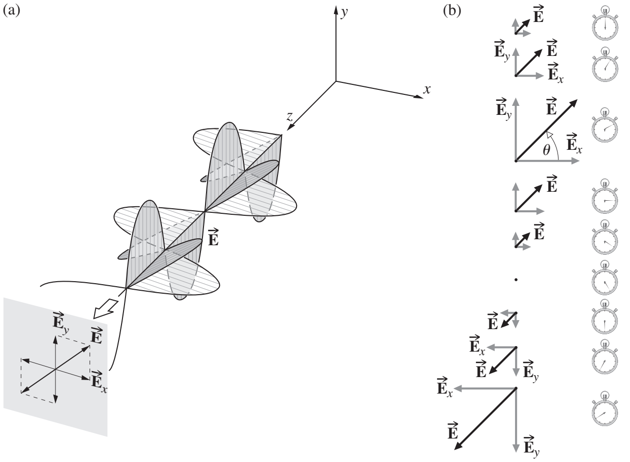
- Light is a transverse electromagnetic wave. Thus far we have considered only light for which the orientation of the electric field is constant, although its magnitude and sign vary in time.
- In general, we can consider two such harmonic lightwaves of the same frequency, moving through the same region of space, in the same direction $\hat{z}$.

$$\vec{E}_x(z,t) = \hat{i} E_{0x} \cos(kz - \omega t)$$

$$\vec{E}_y(z,t) = \hat{j} E_{0y} \cos(kz - \omega t + \epsilon)$$

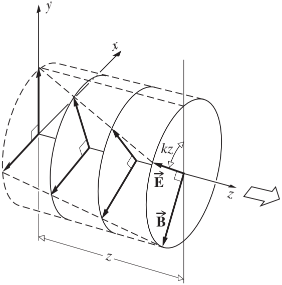
- **Linear polarization:** If ϵ is zero or an integral multiple of $\pm 2\pi$, the resultant wave is

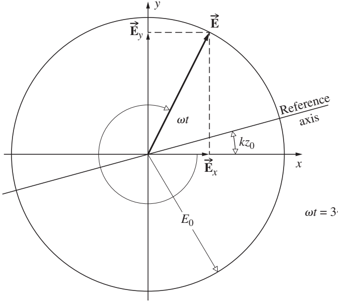
$$\vec{E} = \vec{E}_x + \vec{E}_y = (\hat{\imath} E_{0x} + \hat{\jmath} E_{0y}) \cos(kz - \omega t).$$

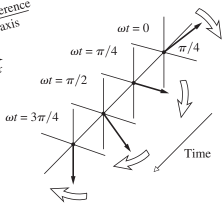


- **Circular polarization:** When both constituent waves have equal amplitudes ($E_{0x} = E_{0y} = E_0$) and their relative phase difference $\epsilon = -\pi/2 + 2m\pi$, where m is an integer, the resultant wave is

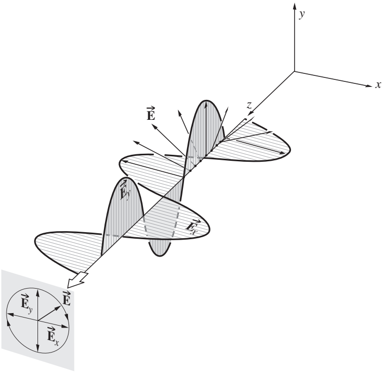
$$\begin{aligned}\vec{E} &= \hat{i}E_0\cos(kz - \omega t) + \hat{j}E_0\cos(kz - \omega t - \pi/2) \\ &= E_0[\hat{i}\cos(kz - \omega t) + \hat{j}\sin(kz - \omega t)].\end{aligned}$$







- The resultant $\vec{E}$ at a fixed z is rotating clockwise at an angular frequency of ω, as seen by an observer toward whom the light is moving. This is referred to as **right-circular** light (from the right-hand rule based on the helix structure of $\vec{E}$ at a fixed time and the direction of propagation).



- The $\vec{E}$-vector makes one complete rotation as the wave advances through one wavelength.

- When their relative phase difference $\epsilon = \pi/2 + 2m\pi$, where m is an integer, the resultant wave is

$$\vec{E} = \hat{i}E_0 \cos(kz - \omega t) + \hat{j}E_0 \cos(kz - \omega t + \pi/2)$$
$$= E_0[\hat{i}\cos(kz - \omega t) - \hat{j}\sin(kz - \omega t)].$$

- The amplitude is unaffected, but $\vec{E}$ at a fixed z now rotates counterclockwise, and the wave is **left-circularly polarized**.
- A linearly polarized wave can be synthesized from two oppositely polarized circular waves of equal amplitude.

A Math Description of Polarization

- R. Clark Jones invented a representation of polarized light in 1941.
- The technique he evolved has the advantages of being applicable to coherent beams and at the same time being extremely concise. It uses the electric vector itself.
- Written in column and complex form, this Jones vector is

$$\vec{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} = \begin{bmatrix} E_{0x} e^{i(kz-\omega t+\phi_x)} \\ E_{0y} e^{i(kz-\omega t+\phi_y)} \end{bmatrix}.$$

- In many applications it is not necessary to know the exact amplitudes and phases. We can rewrite the Jones vector by

$$\tilde{E} = \begin{bmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{bmatrix}.$$

- Horizontal and vertical linearly polarized are thus given by

$$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

- Linearly polarized at $+45^\circ$ from the x-axis (diagonal) and at -45° from the x-axis (anti-diagonal) are given by

$$|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$|A\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

- Note that we are only interested in polarization, so the vectors are normalized, or in one unit length (the factor of $1/\sqrt{2}$ is introduced for this purpose).

- Right-circular light is given by

$$|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i\,|V\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}.$$

That is,

$$\vec{E} = \hat{\imath}E_0\cos(kz-\omega t) + \hat{\jmath}E_0\cos(kz-\omega t-\pi/2).$$

- Left-circular light is given by

$$|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i\,|V\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}.$$

- Note that in the lecture on interference, we discussed the addition in a two-dimensional real space $\mathbb{R}^2$.
- The space is also equivalent to a one-dimensional complex space $\mathbb{C}^1$, or a two-component real vector space.

- Now, with polarization, we have generalized the one-dimensional complex space $\mathbb{C}^1$ to a two-dimensional complex space $\mathbb{C}^2 = \mathbb{C}^1 \otimes \mathbb{C}^1$, or a two-component complex vector space (in Jones' vector representation). The additional $\mathbb{C}^1$ space is spanned by the two orthogonal, linearly polarized states $|H\rangle$ and $|V\rangle$.

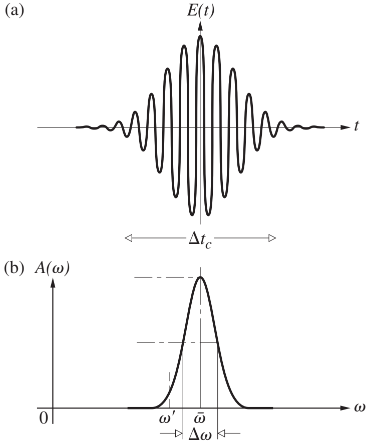
- Two vectors $\vec{A}$ and $\vec{B}$ are said to be orthogonal when $\vec{A} \cdot \vec{B} = 0$; similarly, two complex vectors $\vec{A}$ and $\vec{B}$ are said to be orthogonal when $\langle A|B\rangle \equiv \vec{A}^{*} \cdot \vec{B} = 0$.
- Any polarization state will have a corresponding orthogonal state. Notice that

$$\langle H|V\rangle = \langle D|A\rangle = \langle L|R\rangle = 0.$$

- As we have seen, any polarization state can be described by a linear combination of the vectors in either one of the orthogonal sets. These same ideas are of considerable importance in quantum mechanics, where one deals with orthonormal wave functions.

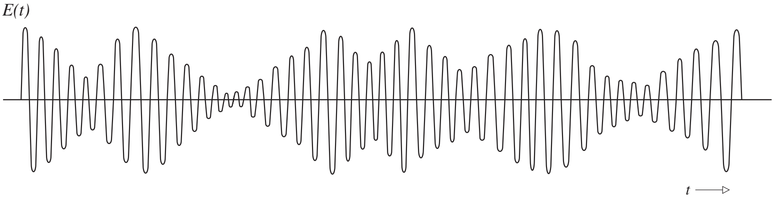
Light Trains and Monochromatic Light

- In reality, a non-laser source emits, to the best, quasimonochromatic light trains, whose frequency can be represented by a bell-shaped Gaussian function.
- That is, the irradiance (hence its square root, the amplitude) versus frequency is found to be Gaussian with a width $\Delta\omega = 2\pi\Delta\nu$.



- Quasimonochromatic light resembles a series of randomly phased finite wavetrains. Such a disturbance is nearly sinusoidal, although the frequency does vary slowly about some mean value. Moreover, the amplitude fluctuates as well, but this too is a comparatively slow variation.
- The average constituent wavetrain exists roughly for the **coherence time**

$$\Delta t_c = 1/\Delta\nu.$$



- An idealized **monochromatic plane wave** must be depicted as an infinite wavetrain. If this disturbance is resolved into two orthogonal components perpendicular to the direction of propagation, they, in turn, must have the same frequency, be infinite in extent, and therefore be mutually coherent (i.e., $\epsilon =$ constant).

$$\vec{E}_x(z,t) = \hat{i}E_{0x}\cos(kz - \omega t)$$

$$\vec{E}_y(z,t) = \hat{j}E_{0y}\cos(kz - \omega t + \epsilon)$$

- *A perfectly monochromatic plane wave is always polarized.*

- The most spectacular of all present-day sources is the laser. Under optimum conditions, with temperature variations and vibrations meticulously suppressed, a laser was actually operated at quite close to its theoretical limit of frequency constancy.
- For example, a short-term frequency stability of about 8 parts per 10^{14} was attained with a He–Ne continuous gas laser at $\lambda_0 = 1153$ nm [Jaseja et al., Phys. Rev. Lett. 10, 165 (1963)]. That corresponds to a remarkably narrow bandwidth of about $\Delta\nu = 20$ Hz.

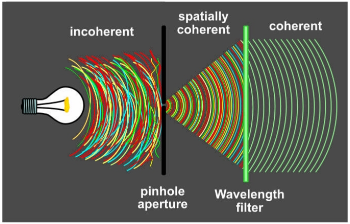
Natural Light

- **Natural light** is composed of a rapidly varying succession ($\sim 10^{-8}$ s) of the different polarization states. It is also known as **unpolarized** or **randomly polarized** light.
- We can mathematically represent natural light in terms of two arbitrary, incoherent, orthogonal, linearly polarized waves of equal amplitude (i.e., waves for which the relative phase difference varies rapidly and randomly).

- We mentioned coherence or incoherence several times. What exactly is coherence?

- Coherence is a measure of the correlation between the phases measured at different (temporal and spatial) points on a wave.
- **Temporal coherence** is a measure of the correlation of light wave's phase at different points along the direction of propagation – it tells us how monochromatic a source is. (Think about the description of quasimonochromatic light.)
- **Spatial coherence** is a measure of the correlation of light wave's phase at different points transverse to the direction of propagation – it tells us how uniform the phase of the wavefront is. (Think about Young's interference experiment.)

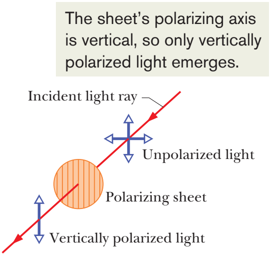
- The figure illustrates how to prepare a monochromatic wave that is both *temporally and spatially coherent* from incoherent natural light.



- In reality, light is generally neither completely polarized nor completely unpolarized. More often, the electric-field vector varies in a way that is neither totally regular nor totally irregular, and such an optical disturbance is **partially polarized**. One useful way of describing this behavior is to envision it as the result of the superposition of specific amounts of natural and polarized light.

Polarizing Sheets

- Unpolarized visible light can be transformed into polarized light by sending it through a polarizing sheet, or a Polaroid sheet, which was invented in 1932 by, then, an undergraduate student Edwin Land.

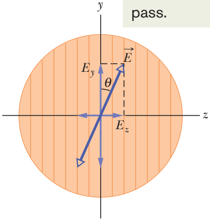


- A polarizing sheet consists of certain long molecules embedded in plastic. When light is then sent through the sheet, *the electric field component parallel to the polarizing direction is passed (transmitted); the component perpendicular to it is absorbed*.

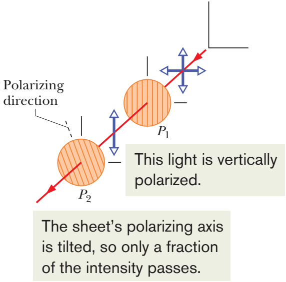
- Electric field oscillations of unpolarized light can resolve into two components with equal intensity. Therefore, the intensity I of the polarized light emerging from a polarizing sheet is then half the intensity I_0 of the original light, $I = I_0/2$.
- For polarized light, only the component ($E_y = E\cos\theta$) parallel to the polarizing direction of the sheet can be transmitted. Therefore, the intensity of the emerging wave is

$$I = I_0 \cos^2\theta.$$

The sheet's polarizing axis is vertical, so only vertical components of the electric fields pass.



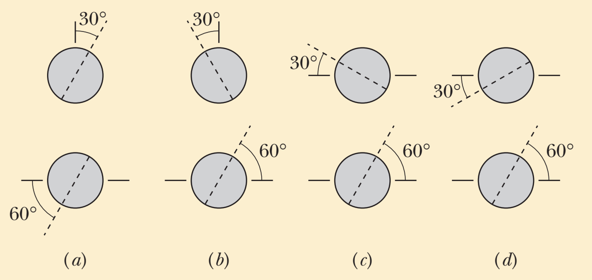
- Initially unpolarized light is sent through two polarizing sheets P_1 (polarizer) and P_2 (analyzer). In general, some of the light transmitted by P_1 will be transmitted by P_2.



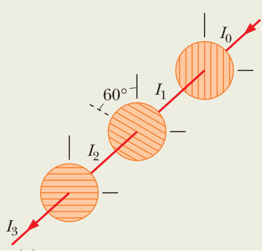
- If their polarizing directions are parallel, all the light passed by the first sheet is passed by the second sheet.
- If those directions are perpendicular (the sheets are said to be crossed), no light is passed by the second sheet.

Quiz 23-1: Polarizing Sheets

1. The figure below shows four pairs of polarizing sheets. Each pair is mounted in the path of initially unpolarized light. The polarizing direction of each sheet (indicated by the dashed line) is referenced to either a horizontal x axis or a vertical y axis. Rank the pairs according to the fraction of the initial intensity that they pass, greatest first.



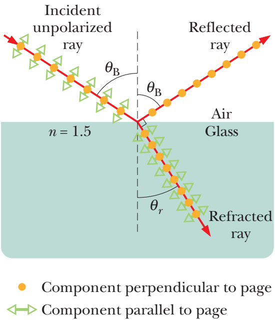
2. The following figure shows a system of three polarizing sheets in the path of initially unpolarized light. The polarizing direction of the first sheet is parallel to the y axis, that of the second sheet is at an angle of 60° counter-clockwise from the y axis, and that of the third sheet is parallel to the x axis. What fraction of the initial intensity of the light emerges from the three-sheet system?



Polarization by Reflection

- One of the most common sources of polarized light is the ubiquitous process of reflection from dielectric media.
- Consider a ray of unpolarized light incident on a glass surface. The field $\vec{E}$ of the incident light can be decomposed into two components of equal magnitude, one perpendicular and another parallel to the plane of incidence.
- In general, the reflected light is *partially polarized*.

- When the light is incident at a particular incident angle, called the **Brewster angle** θ_B, the reflected light is fully polarized.
- One finds experimentally that at the incident angle θ_B the reflected and refracted rays are perpendicular to each other:



$$\theta_B + \theta_r = 90^\circ.$$

- For example, when the incident beam is in air ($n_i = 1$) and the transmitting medium is glass ($n_r = 1.5$), the Brewster angle is about 56°.

- According to Snell's law

$$n_i \sin\theta_B = n_r \sin\theta_r,$$

we obtain

$$n_i \sin\theta_B = n_r \sin\theta_r = n_r \sin(90^\circ - \theta_B) = n_r \cos\theta_B,$$

or

$$\theta_B = \tan^{-1}\frac{n_r}{n_i}.$$

- If the incident and reflected rays travel in air, we can approximate n_i as unity, so

$$n_r = \tan\theta_B.$$

Summary

- Understand polarization as the superposition of coherent waves.
  - Distinguish linearly polarized and circularly polarized light.
  - Familiar yourself with Jones' vector representation.
- Distinguish unpolarized (or randomly polarized), partially polarized, and polarized light.
  - How to polarize natural light with polarizing sheets or reflection?
  - What is the relative intensity of light transmitted by a polarizing sheet?
  - What is the Brewster angle?

Reading

Halliday, Resnick & Krane:

- Chapter 44: Polarization