

Lab Nr. 9, Numerical Calculus

Cubic Spline Interpolation; Least Squares Approximation

Matlab functions

- *spline* and *pchip*
- *polyfit* and *polyval*

Applications

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \frac{x+1}{3x^2+2x+1}.$$

Plot on the same set of axes the nodes, f and the Lagrange, Hermite (with double nodes) and deBoor spline interpolants at 7 equally spaced nodes in the interval $[-2, 4]$.

2. Consider the function $f(x) = x \sin(\pi x)$ and the nodes $\{-1, -1/2, 0, 1/2, 1, 3/2\}$.
- a) Obtain the deBoor, complete and piecewise Hermite cubic splines of these data;
 - b) Plot the nodes, f and the three spline functions, on the same set of axes, for $x \in [-1, 3/2]$.

3. The following data is given:

x	0.5	1.5	2	3	3.5	4.5	5	6	7	8
f	5	5.8	5.8	6.8	6.9	7.6	7.8	8.2	9.2	9.9

- a) Draw the scatterplot and find the least squares polynomial approximation that best fits the data;
 - b) Compute the error of this approximation;
 - c) Use this approximation to estimate the value at $x = 4$;
 - d) Plot on the same graph the data and the approximation polynomial.
4. The following table contains values of the pressure P of water vapors (measured in bars) as a function of temperature T (measured in $^{\circ}\text{C}$):

T	0	10	20	30	40	60	80	100
P	0.0061	0.0123	0.0234	0.0424	0.0738	0.1992	0.4736	1.0133

- a) Find the quadratic and cubic least squares approximation polynomials of these data. Which approximation is better?
- b) Approximate the pressure corresponding to a temperature of $T = 45^{\circ}$;
- c) Plot on the same set of axes the data and the two approximating polynomials.

Optional

5. Implement the computation of the *natural* cubic spline approximation. Then plot it together with the complete spline for the function and nodes in Problem 2.