Lab Nr. 5, Numerical Calculus

Linear Systems II

Iterative Methods: Jacobi, Gauss-Seidel, SOR;

Conditioning of Linear Systems

Implement the Jacobi and Gauss-Seidel iterative methods, in the form

$$[x, nit] = Name(A, b, x0, err, maxnit),$$

where
A is the system matrix,
b is the right-hand side of the system,
$x0$ is the initial solution,
err is the desired error,
$maxnit$ is the maximum number of iterations allowed (in case the method does not converge),
x is the iterative solution of the system,
nit is the number of iterations.

Applications

1. Use both iterative methods to approximate the solution of the linear system (Problem 1, Lab Nr. 4)

$$\begin{cases} 5x_1 - x_2 = 4 \\ -x_{j-1} + 5x_j - x_{j+1} = 3, \quad j = \overline{2, n-1}, \\ - x_{n-1} + 5x_n = 4 \end{cases}$$

for $n = 1000$. Compare the number of iterations needed for an error of 10^{-5}.

2. (Wilson's Example) Consider the system $Ax = b$, where

$$A = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}, \quad b = \begin{bmatrix} 32 \\ 23 \\ 33 \\ 31 \end{bmatrix}.$$

a) Solve the system.

b) Perturb b to $\tilde{b} = \begin{bmatrix} 32.1 \\ 22.9 \\ 33.1 \\ 30.9 \end{bmatrix}$. Solve the system $Ax = \tilde{b}$. What are the input and output relative errors?

c) The same question for the perturbed system matrix $\tilde{A} = \begin{bmatrix} 10 & 7 & 8.1 & 7.2 \\ 7.8 & 5.04 & 6 & 5 \\ 8 & 5.98 & 9.89 & 9 \\ 6.99 & 4.99 & 9 & 9.98 \end{bmatrix}$.

d) Explain the phenomenon.

Optional

3. Implement the successive overrelaxation (SOR) method, in the form

$$[x, nit] = Name(A, b, \omega, x0, err, maxnit),$$

where ω is the optimal relaxation parameter. Then use it to approximate the solution of the system (Problem 3, Lab Nr. 4)

$$\begin{bmatrix} 5 & -1 & 0 & -1 & \dots & & \dots & 0 \\ -1 & 5 & -1 & 0 & -1 & \dots & \dots & \vdots \\ 0 & -1 & 5 & -1 & \ddots & \ddots & \vdots & \vdots \\ -1 & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 & 5 & -1 & 0 & -1 \\ 0 & \dots & \ddots & 0 & -1 & 5 & -1 & 0 \\ 0 & \dots & & -1 & 0 & -1 & 5 & -1 \\ 0 & \dots & & & -1 & 0 & -1 & 5 \end{bmatrix} x = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ \vdots \\ 1 \\ 2 \\ 2 \\ 3 \end{bmatrix},$$

for $n = 10$ and $n = 1000$. Compare the number of iterations needed in all three iterative methods, for a precision of 10^{-5}.