Lab Nr. 6, Numerical Calculus

Lagrange Interpolation I

Fundamental Polynomials; Barycentric Formula

1. Implement Lagrange interpolation, using the classical form (with Lagrange basis polynomials).

2. Implement Lagrange interpolation, using the barycentric formula.

Applications

1. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \frac{x+1}{3x^2+2x+1}.$$

a) Find the Lagrange polynomial $L_9 f$ (in classical form) that interpolates f at 10 equally spaced nodes in $[-2, 4]$;

b) Plot the nodes, f and $L_9 f$, on the same set of axes. What can be noticed?

c) Plot the errors $|f - L_9 f|$ and compute the maximum error on $[-2, 4]$.

d) Approximate $f(1/2)$ by $(L_9 f)(1/2)$. What is the error of this approximation?

2. According to the International Data Base of the U.S. Census Bureau, the following data represents the population of the world between 1980 and 2020:

Year	1980	1990	2000	2010	2020
Pop. (mln.people)	4451	5287	6090	6970	7821

Use the barycentric formula to interpolate these data and approximate the world population in 2005 and 2015. Knowing the actual world population was 6474 million in 2005 and 7405 million people in 2015, how good are these approximations (compute the relative errors)?

3. Use Lagrange interpolation (in either form) to approximate $\sqrt{118}$ with 3 correct decimals.

Optional

4. Consider the function $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = |x| + \frac{1}{2}x - x^2.$$

a) Plot on the same set of axes the function f and its Lagrange interpolant at $m = 10$ equidistant nodes in $[-1, 1]$. Then at $m = 20$ nodes. What do you notice?

b) Repeat point **a)** for $m = 20$ and $m = 100$ Chebyshev nodes of the first kind.

c) Repeat point **b)** for $m = 20$ and $m = 100$ Chebyshev nodes of the second kind.