COMPM012 - Coursework 1

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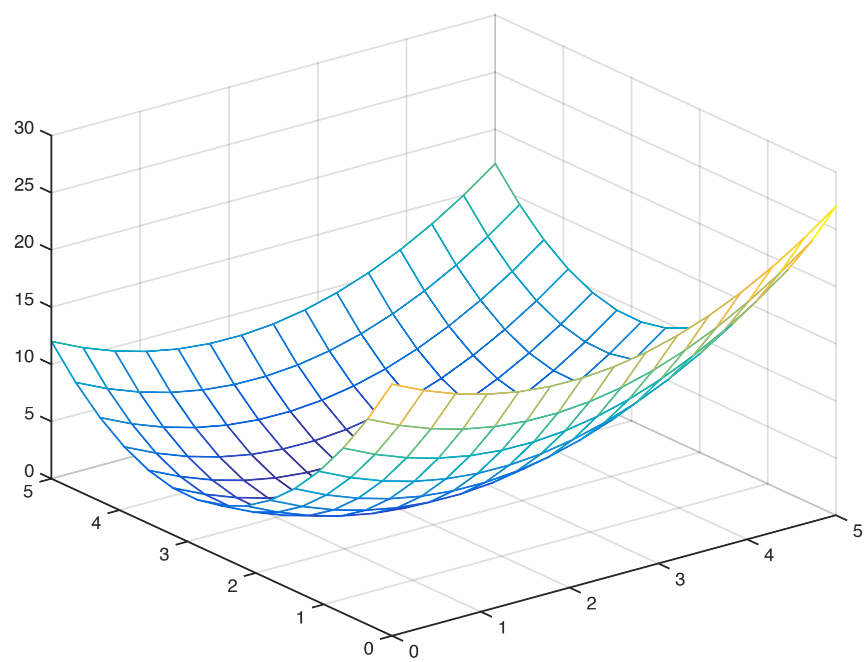
1 Gradient Descent Visualisation

a) To produce the plot the following commands are used:

```
>> [X,Y] = meshgrid(linspace(0,5,15),linspace(0,5,15));
>> mesh(X,Y, fcarg(X,Y));
```

Which gives us the plot



b) i) The modified graddesc function becomes:

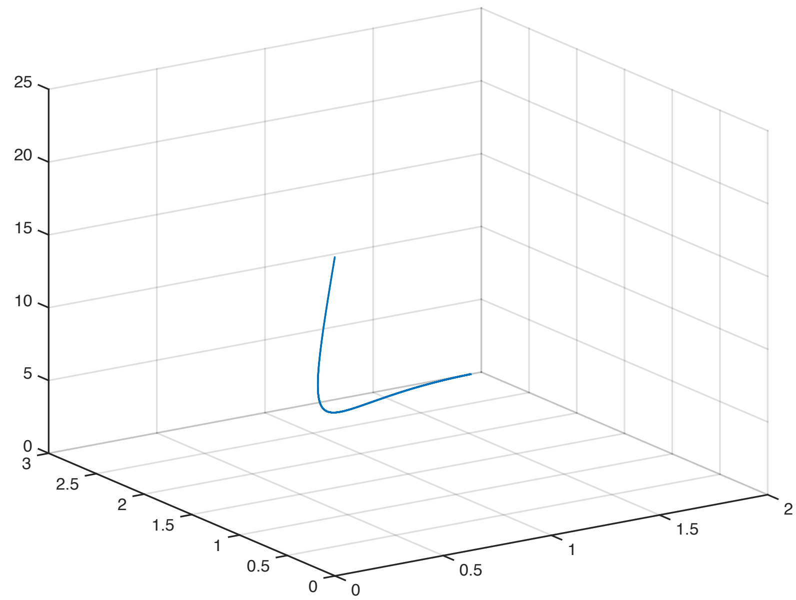
```
function [soln, X, Y, Z] = graddesc(f, g, i,e, t)
    gi = feval(g,i) ;
    X = [];
    Y = [];
    Z = [];
    while(norm(gi)>t)  % crude termination condition
        i = i - e .* feval(g,i) ;
        gi = feval(g,i);
        X = [X i(1)];
        Y = [Y i(2)];
        Z = [Z fc(i)];
    end
    soln = i ;
end
```

ii) We produce the plot with the commands

```
% Gradient descent
[result, X, Y, Z] = graddesc('fc','dfc',[0,0],0.001,0.1);

% Plot descent path
hold on
plot3(X, Y, Z)
grid on
hold off
```

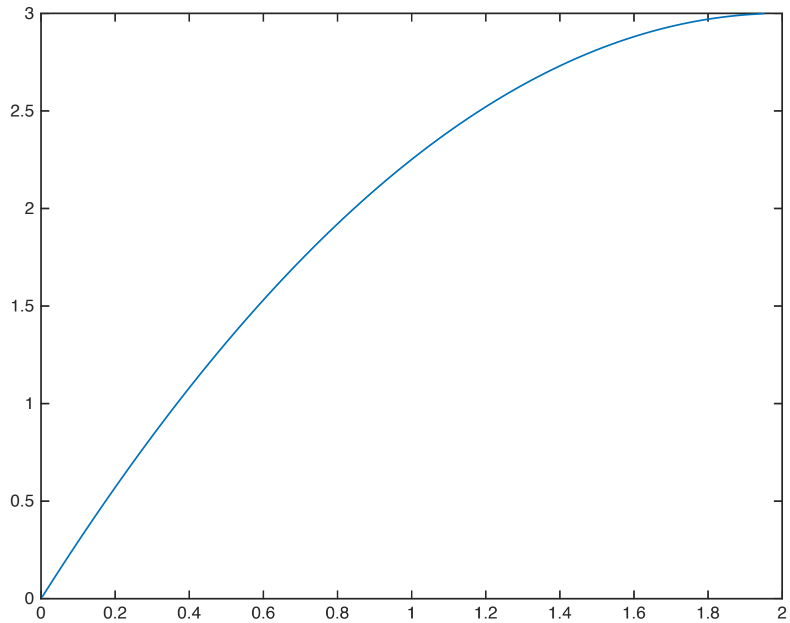
The obtained plot looks like



iii) We can plot the projection to the XY-plane by simply plotting the X, Y values and ignore the Z values. We do this with the following commands

```
% Plot projection to XY plane
plot(X, Y)
```

Which produces the following plot



2 Gradient Descent for Linear Regression

a) My implementation of gradient descent is

```
function [sol, guesses] = mygraddesc(A, b, guess, step, tol)
    mse = @(w) (A*w - b).' * (A*w - b);
    dmsedw = @(w) (-2*A.' * b) + (2*A.' * A * w);
    guesses = [guess; mse(guess)]; % Used for visualisation

    while norm(dmsedw(guess)) > tol
        guess = guess - dmsedw(guess)*step;
        guesses = [guesses [guess; mse(guess)]];
    end

    sol = guess;
end
```

b) We can find the solution to the system using the gradient descent implementation by putting the coefficients for x_1 and x_2 in a matrix A and the right-hand-sides of the equations in a vector b. Passing A and

b as arguments to the gradient descent function along with an initial guess, step size, and tolerance gives us the solution to the system. In Matlab we do:

```
A = [1  -1;  1  1;  1  2]
b = [1;  1;  3];
guess = [0;  0];

mygraddesc(A,  b,  guess,  0.01,  0.0001)
```

Which rerturns

```
ans =
      1.2857
      0.5714
```

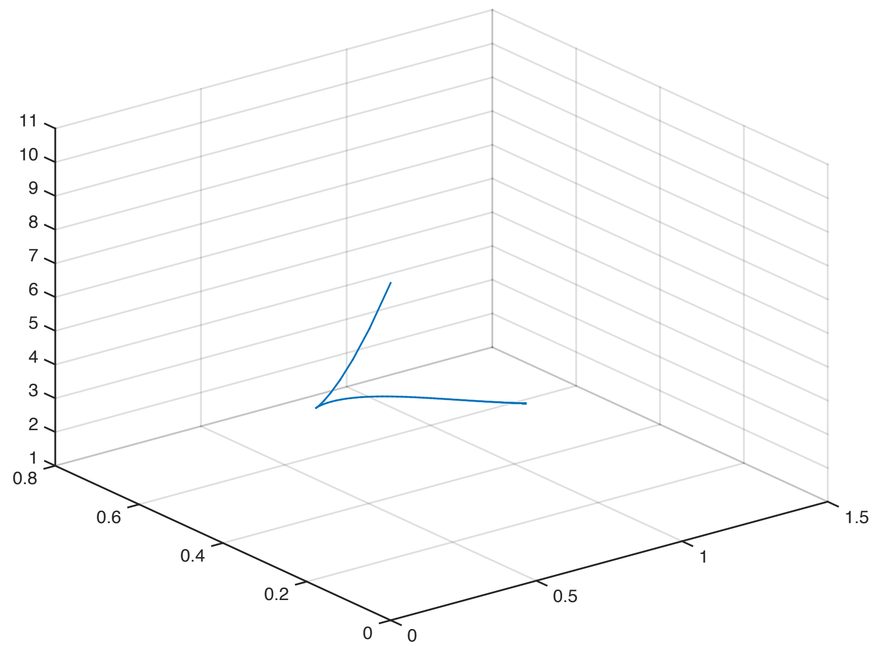
So $x_1 = 1.2857$ and $x_2 = 0.5714$ is the least squares solution to the system.

c) My gradient descent implementation lets us retrieve the guesses for each iteration of the descent. We plot this as follows

```
[~,  guesses] = mygraddesc(A,  b,  guess,  0.01,  0.001);

plot3(guesses(1,:),  guesses(2,:),  guesses(3,:));
hold on
grid on
hold off
```

Which produces the plot



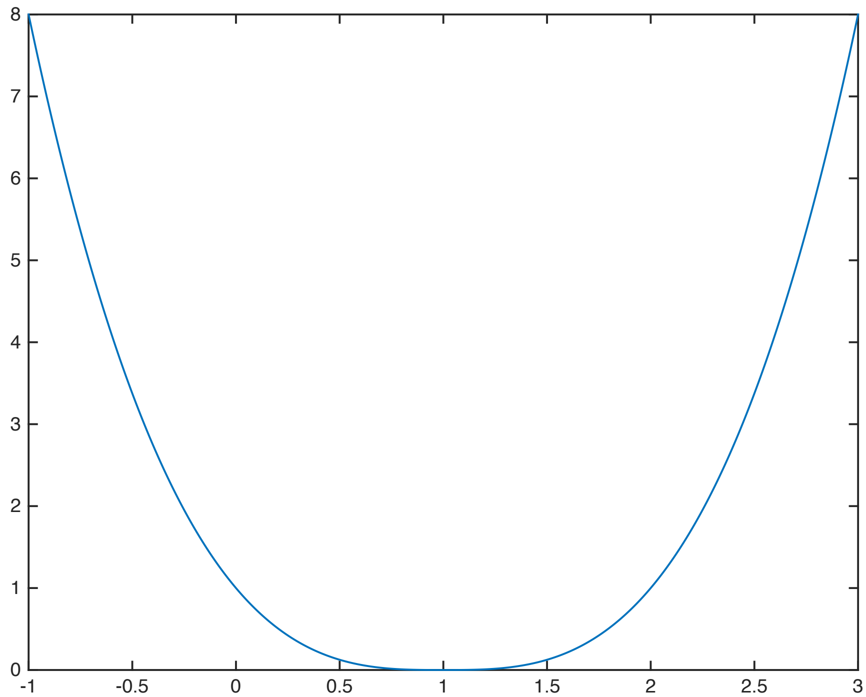
3 Convergence of Gradient Descent in a Single Variable

a) We can plot the function to get an intuition.

```
domain = [-1:0.01:3];
range = arrayfun(@(x) abs(x-1)^3, domain);

plot(domain, range);
```

and we get



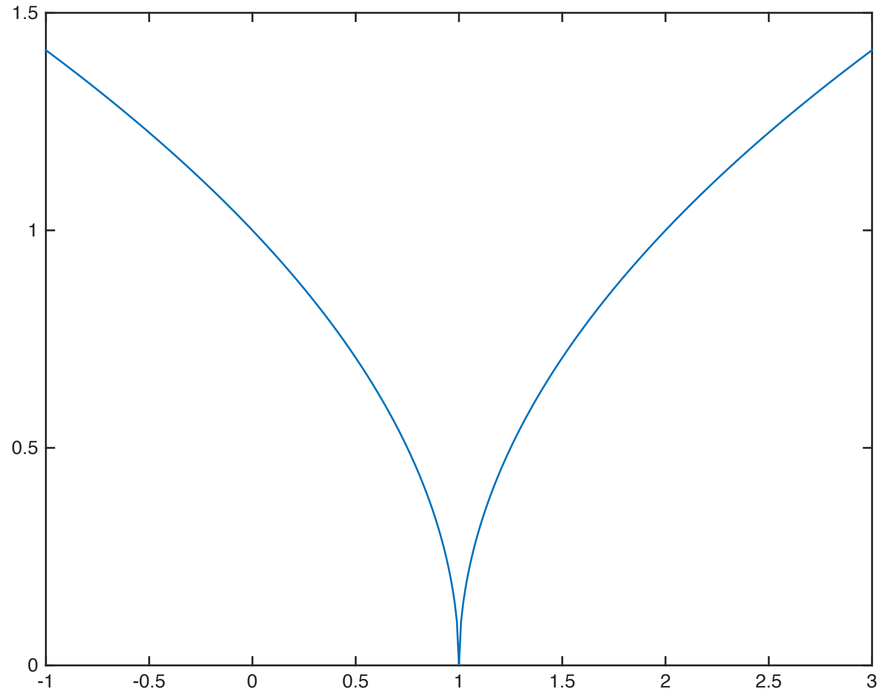
The function is clearly convex and gradient descent will converge for any x_0 and a small step size, say $\lambda = 0.01$.

b) Again, we plot the function

```
domain = [-1:0.01:3];
range = arrayfun(@(x) sqrt(abs(x-1)), domain);

plot(domain, range);
```

And obtain



This function is clearly not convex and gradient descent will not converge for any x_0 and λ unless $x_0 = 1$.

c) $x^4 + 5x^2$ is a convex function with one minimum. We can show this by observing that x^4 is convex and x^2 is convex and $x^4 + 5x^2$ is therefore a convex combination of two convex functions. Therefore gradient descent will converge.