Maths 223 Assignment 1

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February 16, 2014

1 Prove the Triangle Inequality

$\|u+v\| \leq \|u\| + \|v\|$

Proof 1 of Triangle Inequality.

$$\begin{aligned}
\|u+v\|^2 &= (u+v)^2 && (1)\\
&= u^2 + 2uv + v^2 && (2)\\
&\leq \|u\|^2 + 2\|u\| \cdot \|v\| + \|v\|^2 \quad \text{Cauchy-Schwarz} && (3)\\
&\leq (\|u\| + \|v\|)^2 && (4)
\end{aligned}$$

□

2 Operations in 3 Dimensional Space

For all $u = (2, -1, 1)$, $v = (1, 3, 3)$, $w = (1, 0, -1)$ find the below[1].

2.1. $\langle \|u\|, \|v\|, \|w\| \rangle$

$$\begin{aligned}
\langle \|u\|, \|v\|, \|w\| \rangle &= \langle \sqrt{4+1+1}, \sqrt{1+9+9}, \sqrt{1+1} \rangle && (5)\\
&= \langle \sqrt{6}, \sqrt{19}, \sqrt{2} \rangle && (6)
\end{aligned}$$

2.2. $\langle u \cdot v, v \cdot w, w \cdot u, u \cdot w \rangle = \langle 2, -2, 1, 1 \rangle$.

2.3. $\cos(\theta)$, where $\theta = \measuredangle uw$

$$\begin{aligned}
\cos(\theta) &= \frac{u \cdot w}{\|u\|\|w\|} && (7)\\
&= \frac{1}{2\sqrt{3}} && (8)
\end{aligned}$$

2.4. $u + 2v - 3w = (2, -1, 1) + (2, 6, 6) + (-3, 0, 3) = (1, 5, 10)$

2.5. $d(v, w) = \|v - w\| = \|(0, 3, 4)\| = \sqrt{9+16} = 5$

2.6. $\text{proj}(u, w) = \frac{u \cdot w}{\|w\|} = \frac{1}{\sqrt{2}}$.

[1] My answers are provided as tuples for compactness.

3 Expression Simplification

3.1. $(3-2i)(4+5i) = 12 - 8i - 10i^2 + 15i = 10 + 7i + 12 = 22 + 7i$

3.2. $(1+3i)^2 = 1 + 6i + 9i^2 = -8 + 6i$

3.3. $(3-4i)^{-1} = \frac{3+4i}{(3-4i)(3+4i)} = \frac{3+4i}{25}$

3.4. $\frac{2+3i}{3-5i} = \frac{(2+3i)(3+5i)}{(3-5i)(3+5i)} = \frac{-9+19i}{34}$

3.5. $(1+i)^3 = -2 + 2i$

4 *More* Expression Simplification

4.1. $1/(3i) = 3i^{-1} * \frac{-i}{-i} = -\frac{i}{3}$

4.2. $\frac{1+2i}{1-2i} = \frac{1+2i}{1-2i} * \frac{1+2i}{1+2i} = \frac{-3+4i}{5}$

4.3. $\langle i^5, i^{50}, i^{2014}\rangle = \langle i, -1, -1\rangle$

4.4. $(1/(i-2))^2 = \frac{1}{i-2} * \frac{i+2}{i+2} = \frac{3+4i}{25}$

5 Operations with Complex Numbers

For all $z = 2-3i$ and $w = 1+2i$ find the below.

5.1. $z + w = 2 - 3i + 1 + 2i = 3 - i$

5.2. $zw = (2-3i)(1+2i) = 8 + i$

5.3. $z/w = \frac{2-3i}{1+2i} = \frac{-4-7i}{5}$

5.4. $z^2 + 2w = (2-3i)^2 + 2(1+2i) = -3 - 8i$

5.5. $\langle \overline{z}, \overline{w}\rangle = \langle 2+3i, 1-2i\rangle$

5.6. $\langle |z|, |w|\rangle = \langle \sqrt{13}, \sqrt{5}\rangle$

6 Correctness Proofs

Show that for any complex number z:

6.1. $\text{Re}(z) = \frac{1}{2}(z + \overline{z})$

Proof.

$$z = a + bi \tag{9}$$
$$Re(z) = a \tag{10}$$
$$\frac{1}{2}(z + \overline{z}) = (a + bi + a - bi)/2 \tag{11}$$
$$= a = Re(z) \tag{12}$$

□

6.2. $\text{Im}(z) = \frac{1}{2i}(z - \overline{z})$

Proof.

$$z = a + bi \tag{13}$$
$$Im(z) = b \tag{14}$$
$$\frac{z - \overline{z}}{2i} = \frac{a + bi - a + bi}{2i} \tag{15}$$
$$= b = Im(z) \tag{16}$$

□

6.3. $zw = 0$ implies that either $z = 0$ or $w = 0$.

Proof. [2]

$$z = a + bi \tag{17}$$
$$w = c + di \tag{18}$$
$$zw = (a+bi)(c+di) = 0 \tag{19}$$
$$= ac + adi + cbi - bd \tag{20}$$
$$= (ac - bd) + i(ad + cb) = 0 \tag{21}$$
$$ac = bd,\ ad = -cb \tag{22}$$
$$\langle a, b\rangle = \langle 0, 0\rangle \vee \langle c, d\rangle = \langle 0, 0\rangle \tag{23}$$

□

7 Operations in Complex 3 Dimensional Space

Let $u = (1+3i, 2-3i)$ and $v = (3-i, 2i)$. Find:

7.1. $u + v$

$$u + v = (1+3i, 2-3i) + (3-i, 2i) \tag{24}$$
$$= (4+2i, 2-i) \tag{25}$$

7.2. $(2+i)u$

$$(2+i)u = (2+i)(1+3i, 2-3i) \tag{26}$$
$$= (-1+7i, 7-4i) \tag{27}$$

7.3. $(1-i)u + (-1-3i)v$

$$(1-i)u = (4+2i, -1-5i) \tag{28}$$
$$(-1-3i)v = (-6-8i, 6-2i) \tag{29}$$
$$(1-i)u + (-1-3i)v = (-2-6i, 5-7i) \tag{30}$$
$$= (-2-6i, 5-7i) \tag{31}$$

[2] The logic that ac = bd *and* ad = -cb can be further followed to prove the statement following it. This would be done through substitution of a = -cb/d, but this is trivial.

7.4. $u \cdot v$

$$u \cdot v = (1+3i)(3-i) + (2-3i)(2i) \tag{32}$$
$$= 12 + 12i \tag{33}$$

7.5. $\|u\|$ and $\|v\|$

$$\|u\| = \sqrt{\sqrt{1+9}^2 + \sqrt{4+9}^2} \tag{34}$$
$$= \sqrt{23} \tag{35}$$
$$\|v\| = \sqrt{\sqrt{9+1}^2 + \sqrt{4}^2} \tag{36}$$
$$= \sqrt{14} \tag{37}$$

Mission Accomplished

AND HOPEFULLY EVERYTHING WAS DONE CORRECTLY