Maths 223 Assignment 3

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Note before you read this!!! I'm a computer science major... and I use Lisp a lot, which (surprisingly) helps a lot with Linear Algebra since the concept of folding and reducing data becomes very natural. A lot of the proofs here can be approached in a Lispic manner to make proofs very simple, but I've defined some custom "functions" in order to do this! They should be really straightforward, but I'm sorry if it's a little odd- still learning how to do this in an idiomatic maths fashion.

1 Prove Not 0 Matrix

Let A be an invertible matrix.
Let v be a non-zero vector.
Prove that Av is not equal to zero.

Proof by contradiction.

$$A = M_{invertible} \tag{1}$$
$$Av = 0 \tag{2}$$
$$v = [v_1, v_2, \dots, v_n] \tag{3}$$
$$BA = I \tag{4}$$
$$BAv = Iv \tag{5}$$
$$B(Av) = v \tag{6}$$
$$B(0) = 0 \tag{7}$$

2 Prove Invertibility of Product of Invertible Matrices

Prove that a product of invertible matrices is invertible.

Proof by demonstrating through result of multiplication of inverted matrices.

$$A = Set(M_{invertible}) \tag{8}$$
$$B = Set(M_{invertible}) \tag{9}$$
$$(A_n^{-1} B_n^{-1})(B_n A_n) \tag{10}$$
$$(A_n^{1})(I)(A_n) \tag{11}$$
$$I_{invertible} \tag{12}$$

3 Prove 0 Conditions of Matrix? No clue what to call this section

Let A be a matrix.
Let B be the row canonical form of A.
Show that for any given vector v, $Av = 0$ if and only if $Bv = 0$.

Welcome to proofs hell. I'm sorry to do this. Not sure how else to do it!!

$$\forall reduce(\lambda) \implies fold(product(\lambda)) \tag{13}$$
$$\forall comb(\lambda) \implies \text{linear\_combinations}(\lambda) \tag{14}$$
$$Av = 0 = (reduce(ElementaryMatrix))v \tag{15}$$
$$B = list(comb(A)) \implies v = comb(comb(B)) \tag{16}$$
$$B = list(comb(A)) \iff A = list(comb(B)) \tag{17}$$
$$\forall Av = 0, \;\; Bv = 0 \tag{18}$$
$$\forall Bv = 0, \;\; Av = 0 \tag{19}$$

In words, the rows of A is a linear combination of the rows of B and the rows of B are thus a linear combination of the rows of A. $Av = 0$ means that v is also some linear combination of the rows of A. Since v is thus by extension some combination of the rows of B, similarly $Bv = 0$ means that v is some linear combination of the rows of B and by extension a linear combination of the rows of A. Since these are all linear combinations of each other given both situations, $Av = 0 \iff Bv = 0$.

4 Prove Invertibility of Row Canonical Matrix

Prove that if A is not invertible, then the row canonical form of A is not the identity matrix.

$$A = Matrix_{invertible} \tag{20}$$
$$B = list(comb(A)) \implies \exists B^{-1} * A = I \tag{21}$$
$$A \neq M_{invertible} \implies \neg\exists list(comb(A)) = I \tag{22}$$

In words, if A is an invertible matrix and B is some list of linear combinations of A. Since B must be an invertible list, there is some B where the inverse of B (which is just the list of linear combinations folded in reverse) * A is the identity matrix, so A must be invertible. If A is then *not* invertible, it cannot reduce to the identity matrix.

5 Pivots in a square matrix in row canonical form

Explain why a square matrix in row canonical form that is not the identity matrix must have a column without a pivot.

By definition of a pivot, each pivot in a row reduced matrix must have a leading 1. A square matrix that is not the identity matrix must have at least one pair of linearly dependent rows. One of a pair of linearly dependent rows always reduces to a zero row through row reduction. The produced zero row will not have a pivot.

6 More on pivots

$$v = \{-1, 3, -1, 0\} \tag{23}$$
$$Mv = 0 \tag{24}$$

I have no idea how this works and don't have the time to figure it out :-(, but damn that's cool.

7 Combining Above Work

Sadly, this relies on the above problem, which I haven't finished.

8 Matrix Multiplication to Zero

The heavenly practical or computational problem appears. We will apply the same damned cool trick as used in problem 6.

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$
$$v = \{1, 1, -1, 0\} \tag{26}$$
$$Av = 0 \tag{27}$$

9 Identity Matrix Pivot Points

This was explained in problem 5. It can be succinctly described as a result of linear independence.

An identity matrix is a combination of n linear independent rows. These n linear independent rows must thus have n entries. Given n rows of n entries that are all linearly independent, every row (and thus column) will be a pivot point.

10 Finding the Inverse Through Row Reduction

Sweet! This is how I've always found the inverse matrix, tends to be faster. For the sake of page space and typing, I'll only type beginning and ending steps. It should be evident the middle was solved correctly.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 0 & 1 & 0 \\ 1 & 2 & 2 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \tag{28}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \tag{29}$$

$$\vdots \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1/2 & -1/2 & -1/2 & 1 \\ 0 & 0 & 1 & -1 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1/2 & 1/2 & -1/2 & 0 \end{array}\right] \tag{30}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1/2 & -1/2 & -1/2 & 1 \\ 0 & 0 & 1 & 0 & 1/2 & 3/2 & 1/2 & -1 \\ 0 & 0 & 0 & 1 & -1/2 & -1/2 & 1/2 & 0 \end{array}\right] \tag{31}$$

11 Problem 3.36 in the Book

The answer for 3.36 appears to be in the book. Even easier to find online!

I suppose we're to provide an answer all the same, and it's just a two liner so not so bad.

A and B must both be n x n matrices as they must be multiplied to be an identity matrix both ways.

$$B = BI = B(AB) = (BA)B \tag{32}$$

$$(I - BA)B = 0 \implies I = BA \tag{33}$$

And, as always, hopefully everything was done correctly. Didn't finish as much of this assignment as one would hope. Though given the last assignment perhaps I should be hoping things were done in sufficient detail instead.