Midterm Vocabularly List

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1. Vocabularly from Chapters 1-3

Dot Product in $\mathbb{R}^2$ (1.4 pg. 4)

- The $u \cdot v$ is obtained by multiplying corresponding components and adding the resulting products. The vectors u and v are said to be *orthogonal* (or *perpendicular*) if their dot product is zero –that is, if $u \cdot v = 0$.
- The *dot product* or *inner product* of u and v is defined by:

$$u \cdot v = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

Norm (Length) of a Vector (1.4 pg. 5)

- The *norm* and *length* of a vector u in $\mathbb{R}^n$, denoted by $\|u\|$, is defined to be the nonnegative square root of $u \cdot u$.
- If $u = (a_1, \ldots, a_n)$, then

$$\|u\| = \sqrt{u \cdot u} = \sqrt{a_1^2 + \cdots + a_n^2}$$

- $\|u\|$ is the square root of the sum of squares of each of its components of u. Thus, $\|u\| \geq 0$, and $\|u\| = 0$ if and only if $u = 0$.
- A vector u is called a unit vector if $\|u\| = 1$ or, equivalently, if $u \cdot u = 1$. For any nonzero vector v in $\mathbb{R}^n$, the vector

$$\hat{v} = \frac{v}{||v||}$$

is the unique unit vector in the same direction as v. The process of finding $\hat{v}$ from v is called *normalizing* v.

Distance and Angle Between Vectors (1.4 pg. 6)

- The *distance* between vectors $u = (a_1, \ldots. a_n)$ and $v = (b_1, \ldots, b_n)$ in $\mathbb{R}^n$ is defined by:

$$d(u,v) = ||u - v|| = \sqrt{(a_1 - b_1)^2 + \cdots + (a_n - b_n)^2}$$

- The angle θ between nonzero vectors u, v in $\mathbb{R}^n$ is defined by

$$cos\theta = \frac{u \cdot v}{||u||||v||}$$

Complex Numbers (1.7 pg. 11)

- Formally, a complex number is an ordered pair (a,b) of real numbers.
- The complex number (0,1) is denoted by i. Important Property: $i^2 = -1$
- A complex number can be written $z = (a, b) = a + bi$

Complex Conjugation (1.7 pg. 12)

- Consider the complex number $z = a + bi$. The *conjugate* of z is denoted and defined by: $\overline{z} = \overline{a + bi} = a - bi$.

Absolute Value of a Complex Number (1.7 pg. 12)

- The *absolute value* of z, denoted by $|z|$, is defined to be the nonnegative square root of $z\overline{z}$. Namely, $|z| = \sqrt{z\overline{z}} = \sqrt{a^2 + b^2}$

Dot Product in C^n (1.8 pg. 13)

- The dot or inner product of u and v is denoted by

$$u \cdot v = u_1\overline{v_1} + \cdots + u_n\overline{v_n}$$

Matrix Multiplication (2.5 pg. 31)

$$\begin{aligned} c_{ij} &= a_{i1}b_{1j} + \cdots + a_{ip}b_{pj} \\ &= \sum_{k=1}^{p} a_{ik}b_{kj} \end{aligned}$$

Polynomials in Matrices ()

Invertible Matrices (2.9 pg. 34)

- A square matrix A is said to be *invertible* or *nonsingular* if there exists a matrix B such that:

  $$AB = BA = I$$

  where I is the identity matrix. Such a matrix B is unique. That is, if $AB_1 = B_1A = I$ and $AB_2 = B_2A = I$, then

  $$B_1 = B_1I = B_1(AB_2) = (B_1A)B_2 = IB_2 = B_2$$

- We call such a matrix B the inverse of A and dneote it by A^-1. Observe that the above relatin is symmetric; that is, if B is the inverse A, then A is the inverse of B.

Equivalent Systems of Linear Equations (3.3 pg. 60)

- Two systems of linear equations have the same solutoins if and only if each equation in each system is alinear combination of the equations in the other system.
- Two systems of linear equations are said to be equivalent if they have the same solutions.

Augmented Matrix of a Linear System (3.2 pg. 59)

- Is the matrix of coefficients plus the last column which is made up of constants, sometimes written $M = [A, B]$.

Coefficient Matrix of a Linear System (3.3 pg. 59)

- Is the matrix of coefficients which is associated with a general system of m equations and n unknowns.

Matrix Equation of a system of linear Equations (3.2 pg. 58)

- Is a list of linear equations with the same unknowns – that is, a system of m linear equations and n unknowns.
- The coefficients, constants, and equations can be represented in a matrix as the two above definitions state(augmented or coefficient matrices).

Elementary Row Operations(ERO) (3.3 pg. 61)

- The following operations on a system of linear equations:
    1. Interchange two of the equations.
    2. Replace an equation by a nonzero multiple of itself.
    3. Replace an equation by the sum of a multiple of another equation and itself.

Elementary Matrices (3.12 pg. 84)

- Let e denote an elementary row operation and let $e(A)$ denote the results of applying the operation e to a matrix A. Now let E be the matrix obtained by applying e to the identity matrix I; that is,

$$E = e(I)$$

- Then E is called the *elementary matrix* corresponding to the elementary row operation e. Note that E is always a square matrix

Row Equivalence (3.7 pg. 72)

- A matrix A is said to be *row equivalent* to a matrix B, written A B
- If B can be obtained from A by a sequence of elementary row operations. In the case that B is also an echelon matrix, B is called an *echelon form* of A.

Echelon Form (3.7 pg. 70)

- A matrix A is called an *echelon matrix*, or is said to be in *echelon form*,if the following two conditions hold (where a leading nonzero element of a row A is the first nonzero element in the row):
  1. All zero rows, if any, are at the bottom of the matrix
  2. Each leading nonzero entry in a row is to the right of the leading nonzero entry in the preceding row.

Row Canonical Form (3.7 pg. 71)

- A matrix A is said to be in *row canonical form* if it is an echelon matrix – that is, if it satisfies the above properties (1) and (2), and if it satisfies the following additional two properties
- The major difference between an echelon matrix and a matrix in row canonical form is that in an echelon matrix there must be zeros below the pivots, but in a matrix in row canonical form, each pivot must also equal 1 and there must also be zeros above the pivots.

- The zero matrix of any size and the identity matrix I of any size are important special examples of matrices in row canonical form.

Free Variables and Pivot Variables (3.5 pg. 65)

- The leading unknowns in a system are called *pivot* variables, and the other unknowns are called *free variables*.

Gaussian Elimination (3.8 pg. 73)

- Algorithms to convert matrices to echelon form and row canonical form by using elementary row operations, are simply restatements of the Gaussian elimination as applied to matrices rather than linear equations.

Homogeneous Systems of Linear Equations associated to a Matrix (3.11 pg. 81)

- A system of linear equations is said to be *homogeneous* if all the constant terms are zero. Thus, a homogeneous system has the form $AX = 0$
- Such a system has the zero vector as a solution. We are usually interested in whether or not the system has a nonzero solution.
- Here r denotes the number of equations in echelon form and n denotes the number of unknowns. Thus, the echelon system has $n - r$ free variables.
  1. $r = n$. The system has only the zero solution.
  2. $r < n$. The system has a nonzero solution.

2. Vocabularly from Chapter 4

Vector Space (4.2 pg. 112)

- Let V be a nonempty set with two operations:
  1. Vector Addition: This assigns to any $u, v \in V$ a sum $u + v$ in V.
  2. Scalar Multiplication: This assigns to any $u \in V, k \in K$ a product $ku \in V$.

- There are additional axioms that need to hold located on pg. 113 of the textbook. They can be summarized by saying V is a *commutative group* under addition. Also, *subtraction* in V is defined by $u - v = u + (-v)$, where $-v$ is a unique negative of v.

Subspace (4.5 pg. 117)

- Let V be a vector space over field K and let W be a subset of V. Then W is a *subspace* of V if W is itself a vector space over K with respect to the operations of vector addition and scalar multiplication on V.
- The way you would show that any set W is a vector space is to show that W satisfies the eight axioms of a vector space. However, if W is a subset of a vector space V, then some of the axioms automatically hold in W, because they already hold in V.
- To identify a vector space suppose W is a subset of a vector space V. Then W is a subspace of V if the following two conditions hold:
  1. The zero vector 0 belongs to W
  2. For ever $u, v \in W, k \in K$: (i) The sume $u + v \in W$ (ii) The multiple $ku \in W$.

Linear Combination (4.4 pg. 115)

- A vector v in V is a linear combination of vectors $u_1, u_2, \ldots, u_m$ in V if there exists scalars $a_1, a_2, \ldots, a_m$ in K such that

$$v = a_1 u_1 + a_2 u_2 + \cdots + a_m u_m$$

- Alternatively, v is a linear combination of $u_1, u_2, \ldots, u_m$ if there is a solution to the vector equation

$$v = x_1 u_1 + x_2 u_2 + \cdots + x_m u_m$$

  where $x_1, x_2, \ldots, x_m$ are unknown scalars.
- A system can have a unique solution, many solutions, or no solution. No solution means that the system cannot be written as a linear combination.

Spanning Set (4.4 pg. 116)

- Let V be a vector space over K. Vectors $u_1, u_2, \ldots, u_m$ in V are said to span V or to form a spanning set of V if every v in V is a linear combination of the vectors $u_1, u_2, \ldots, u_m$ - that is, if there exists scalars $a_1, a_2, \ldots, a_m$ in K such that

$$v = a_1 u_1 + a_2 u_2 + \cdots + a_m u_m$$

Span of a Set of Vectors ()

Row Space of a Matrix ()

Column Space of a Matrix ()

- Otherwise, we say that the vectors are *linearly independent*.

Linear Independence (4.7 pg. 121

- Consider the vector equation

$$x_1 v_1 + \cdots + x_m v_m = 0$$

- where the x's are unknown scalars in K. This equation always has the *zero solution* $x_1 = 0, \cdots, x_m = 0$. Suppose this is the only solution; that is, suppose we can show:

$$x_1 v_1 + \cdots + x_m v_m = 0$$

  implies

$$x_1 = 0, \cdots, x_m = 0$$

- Then the vectors $v_1, \cdots, v_m$ are *linearly independent*

Basis (4.8 pg. 124)

- Definition 1: A set $S = \{u_1, \cdots, u_n\}$ of vectors is a *basis* of V if it has the following two properties: (1) S is linearly independent (2) S spans V.
- Definition 2: A set $S = \{u_1, \cdots, u_n\}$ of vectors is a *basis* of V if every $v \in V$ can be written uniquely as a linear combination of the basis vectors.

Dimension ()

Rank of Matrix (4.9 pg. 126)

- The *rank* of a matrix A, written rank(A), is equal to the maximum number of linearly independent rows of A or, equivalently, the dimension of the row space of A.
- The rank can be equal to the number of pivots in echelon form.
- The rank can correspond to the nonzero rows in echelon form, which form a basis for the row space of the original vector.

Sum of two Subspaces (4.10 pg. 129)

- Suppose U and W are subspaces of V. Then one can shows that $U+W$ is a subspace of V. Recall that $U \cap W$ is also a subspace of V.
- Suppose U and W are finite-dimensional subspaces of a vector space V. Then $U+W$ has finite dimension and

$$dim(U+W) = dimU + dimW - dim(U \cap W)$$

Direct Sum (4.10 pg. 129)

- The vector space V is said to be the *direct sum* of its subspaces U and W, denoted by

$$V = U \oplus W$$

- if every $vinV$ can be written in one and only one way as $v = u + w$ where $u \in U$ and $w \in W$.

Coordinates (4.11 pg. 130)

- Let V be an n-dimensional vector space over K with basis $S = \{u_1, \cdots, u_n\}$. Then any vector $v \in V$ can be expressed uniquely as a linear combination of the basis vectors ini S, say

$$v = a_1 u_1 + \cdots + a_n u_n$$

- Then n scalars $a_1, \cdots, a_n$ are called the *coordinates* of v relative to the basis S, and they form a vector $[a_1, \cdots, a_n]$ in K^n called the *coordinate vector* of v relative to S.
- We denote this vector by $[v]_S$, or simple $[v]$, when S is understood. Thus,

$$[v]_S = [a_1, \cdots, a_n]$$

Isomorphism of vector spaces (4.11 pg. 130)

3. Vocabularly from Chapters 5 and 6

Function/Mapping (5.2 pg. 164)

- Let A and B be arbitrary sets. Suppose to each element in $a \in A$ there is assigned a unique element of B; called the *image* of a.
- The collection of f of such assignments is called a *mapping* from A to B, and is denoted by

$$f : A \to B$$

- The set A is called the *domain* of the mapping, and B is called the *target set*.
- One may also view a mapping $f : A \to B$ as a computer that, for each input value $a \in A$, produces a unique output $f(a) \in B$.

Range/Image (5.2 pg. 164)

- Is a subset of the *target set* that the function actually maps to.

- Sometimes the "barred" arrow $\mapsto$ is used to denote the range of an arbitrary element $x \in A$ under a mapping $f : A \to B$ by writing $x \mapsto f(x)$.

One-to-One Mapping (5.2 pg. 166)

- A mapping $f : A \to B$ is said to be *one-to-one* (or 1-1 or injective) if different elements of A have distinct images; that is,

$$\text{If } f(a) = f(a'), \text{ then } a = a'$$

Onto Mapping (5.2 pg. 166)

- A mapping $f : A \to B$ is said to be *onto* (or f maps A onto B or *surjective*) if ever $b \in B$ is the range of at least one $a \in A$.

Linear Transformation (5.3 pg. 167)

- Let V and U be vector spaces over the same field K. A mapping $F : V \to U$ is called a *linear mapping* or *linear transformation* if it satisfies the following two conditions:
  1. For any vectors $v, w \in V, F(v + w) = F(v) + F(w)$.
  2. For any scalar k and vector $v \in V, F(kv) = kF(v)$.
- Basically, $F : V \to U$ is linear if it "preserves" the two basic operations of a vector space, that of vector addition and that of scalar multiplication.

Kernal (5.4 pg. 169)

- Let $F : V \to U$ be a linear transformation/mapping. The *kernel* of F, written Ker F, is the set of elements in V that map into the zero vector 0 in U; that is,

$$\text{Ker } F = \{v \in V : F(v) = 0\}$$

Image (5.4 pg. 169)

- The *image* of F, written Im F, is the set of image points in U; that is,

$$\text{Im } F = \{u \in U : thereexistsv \in V forwhichF(v) = u\}$$

Rank (5.4 pg. 171)

- Let $F : V \to U$ be a linear mapping. The *rank* of F is defined to be the dimension of its image.

$$\text{rank(F)} = \text{dim(Im F)}$$

Nullity (5.4 pg. 171)

- The *nullity* of F is defined to be the dimension of its kernel

$$\text{nullity(F)} = \text{dim(Ker F)}$$

Linear Transformation associated to a Matrix (Matrix Mapping) ()

Singular Linear Transformation (5.5 pg. 172)

- Let $F : V \to U$ be a linear mapping. Recall that $F(0) = 0$. F is said to be *singular* if the image of some nonzero vector v is 0 – that is, if there exists $v \neq 0$ such that $F(v) = 0$.

Matrix Representation of Linear Transformation ()