Maths 223
Lecture Notes Winter 2014

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Contents

First Lecture

1. Cauchy-Schwarz Inequality

1.1 Theorem

$$\forall u, v \in \mathbb{R}^n |u \cdot v| \leq \|u\| \cdot \|v\| \tag{1}$$

1.2 Proof

Let $tu + v$ be a family of vectors where t is in $\mathbb{R}$.

$$\begin{aligned}
\forall w \in \mathbb{R}^n 0 \leq w \cdot w && (2)\\
0 \leq (tu + v) \cdot (tu + v) && (3)\\
0 \leq tu \cdot v + 2u \cdot v + v \cdot tu + v \cdot v && (4)\\
\forall a = u \cdot u, b = 2u \cdot v : 0 \leq t^2 && (5)
\end{aligned}$$

//TODO CJB REREAD THE FIRST PAGE (having trouble in low light)

2. Triangle Inequality

2.1 Definition

$$\|u + v\| \leq \|u\| + \|v\| \tag{6}$$

2.2 Proof

We show that $\|u + v\|^2 \leq (\|u\| + \|v\|)^2$.

$$\begin{aligned}
\|u + v\|^2 &= (u + v) \cdot (u + v) = u \cdot u + 2u \cdot v + v\dot{v} && && (7)\\
&= \|u\|^2 + 2u \cdot v + \|v\|^2 && && (8)\\
&\leq \|u\|^2 + 2\|u \cdot v\| + \|v\|^2 && && (9)\\
&\leq \|u\|^2 + 2\|u|\ \|v\| + \|v\|^2 && \text{by Cauchy} && (10)\\
&\leq (\|u\| + \|v\|)^2 && && (11)
\end{aligned}$$