Maths 223 Assignment 2

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1 Matrix Products

$$ABC = \begin{pmatrix} -3+6i & -6+7i & -3+6i \\ -1+i & -2+i & -1+i \\ -1 & -1 & -1 \end{pmatrix} \tag{1}$$

$$BAC = \begin{pmatrix} -4+3i & -4+2i & 2i \\ 3i & -1+3i & 1+i \\ -4 & -3-i & -1 \end{pmatrix} \tag{2}$$

$$CAB = \begin{pmatrix} -1+6i & 1+4i \\ -7-i & -5+i \end{pmatrix} \tag{3}$$

2 Lambda Functional Applications

2.1 Applied to Matrix

$$\begin{pmatrix} 6 & 5 \\ 0 & 6 \end{pmatrix} \tag{4}$$

2.2 With Identity Matrix

$$\begin{aligned} f(I\lambda) &= I^2\lambda^2 + I(3\lambda) + 2I &&= 0 && (5)\\ &= (\lambda^2 + 3\lambda + 2)I &&= 0 && (6)\\ &= (\lambda+2)(\lambda+1)I &&= 0 && (7)\\ \lambda &= \{-1, -2\} &&&& (8) \end{aligned}$$

3 Trace

$$A = \begin{pmatrix} 3 & 0 & i & 1 \\ 6 & \pi & 3 & 2 \\ 5 & 2 & -5 & 0 \\ 7 & 6 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} Tr(A) &= 3 + \pi - 5 + 1 && (9)\\ &= \pi - 1 && (10) \end{aligned}$$

4 Representations of Complex Numbers

Let $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Let $g(a+bi) = aI + bJ$

Prove

$g(z_1 z_2) = g(z_1)g(z_2)$

$g(z_1 + z_2) = g(z_1) + g(z_2)$.

$$\begin{aligned} g(a+bi) &= aI + bJ && (11)\\ &= a * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b * \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} && (12)\\ &= \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix} && (13)\\ &= \begin{pmatrix} a & b \\ -b & a \end{pmatrix} && (14) \end{aligned}$$

4.1 For $g(z_1 z_2) = g(z_1)g(z_2)$

$$g(z_1 z_2) = g((a+bi)*(c+di)) \tag{15}$$
$$= g(ac + bidi + adi + cbi) \tag{16}$$
$$= g((ac-bd) + i(bc+ad)) \tag{17}$$
$$= \begin{pmatrix} (ac-bd) & (bc+ad) \\ -(bc+ad) & (ac-bd) \end{pmatrix} \tag{18}$$

$$g(z_1)g(z_2) = g(a+bi)g(c+di) \tag{19}$$
$$= \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} c & d \\ -d & c \end{pmatrix} \tag{20}$$
$$= \begin{pmatrix} (ac-bd) & (bc+ad) \\ -(bc+ad) & (ac-bd) \end{pmatrix} = g(z_1 z_2) \tag{21}$$

4.2 For $g(z_1 + z_2) = g(z_1) + g(z_2)$

$$g(z_1 + z_2) = g(a + c + i(b+d)) \tag{22}$$
$$= \begin{pmatrix} (a+c) & (b+d) \\ -(b+d) & (a+c) \end{pmatrix} \tag{23}$$

$$g(z_1) + g(z_2) = g(a+bi) + g(c+di) \tag{24}$$
$$= \begin{pmatrix} (a+c) & (b+d) \\ -(b+d) & (a+c) \end{pmatrix} = g(z_1 + z_2) \tag{25}$$

5 System of Equations

Let $L_1 = 2x_1 + 4x_2 = 6$.
Let $L_2 = x_1 - x_2 = 0$.

5.1 Find solutions to $\{L_1, L_2\}$

$$\left(\begin{array}{cc|c} 2 & 4 & 6 \\ 1 & -1 & 0 \end{array}\right) \tag{26}$$
$$x_1 = 1, x_2 = 1 \tag{27}$$

5.2 Write out $3L_1 - L_2$ explicitly

Let $L_3 = 3L_1 - L_2$

$$L_3 = \{3(2x_1 + 4x_2) - (x_1 - x_2) = 18 - 0\} \tag{28}$$
$$= \{5x_1 + 13x_2 = 18\} \tag{29}$$

5.3 Prove Equivalent Solutions

In this specific case (general case seems to be the next question):

$$L_3 = \{5x_1 + 13x_2 = 18\} \tag{30}$$
$$\text{has solution } \; x_1 = \frac{18 - 13x_2}{5} \tag{31}$$
$$\{L_1, L_2\}\text{has solution } \{x_1, x_2\} = \{1, 1\} \tag{32}$$
$$solution(\{L_1, L_2\}) \in solution(L_3) \tag{33}$$

6 Equivalent Systems of Equations

Let $\{L_1,\ L_2, \ldots, L_m\}$ be a system of linear equations in the n unknowns $x_1, \ldots x_n$, i.e.
Let $L_i = a_{i1}x_1 + \cdots + a_{in}x_n = b_i$.
Let $\{L'_1, \ldots, L'_k\}$ be a set of linear equations each of which can be expressed as a linear combination of the equations L_i, i.e. there exists scalar $\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{im}$ such that

$$L'_i = \lambda_{i1}L_1 + \lambda_{i2}L_2 + \cdots + \lambda_{im}L_m.$$

Show that a solution of the system $\{L_1, L_2, \ldots, L_m\}$ is also a solution of $\{L'_1, \ldots, L'_k\}$. Prove that if each equation L_i can also be expressed as a linear combination of equations in the set $\{L'_1, \ldots, L'_k\}$, then the systems $\{L_1, L_2, \ldots, L_m\}$ and $\{L'_1, \ldots, L'_k\}$ are equivalent.

And we'll do it through some substitution that hopefully doesn't have any major typos:

$$(a_{i1}x_1 + \cdots + a_{in}x_n)k_1 + \cdots + (a_{i1}x_1 + \cdots + a_{in}x_n)k_n \tag{34}$$

$$= a_{i1}(x_1k_1 + \cdots + a_{in}x_nk_n) + \cdots + a_{in}(x_1k_n + \cdots + x_nk_n) \tag{35}$$

$$= a_{i1}b_1 + \cdots + a_{in}b_i \tag{36}$$

7 Block Matrix Computations

Let $M = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$,

$C = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$, and $D = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$.

$$M = \begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 \end{pmatrix} \tag{37}$$

$$M^2 = \begin{pmatrix} 1 & 6 & 0 & 2 \\ -2 & 3 & -2 & 0 \\ 1 & -1 & 2 & 4 \\ 0 & -3 & 0 & 4 \end{pmatrix} \tag{38}$$

$$N = \left(\begin{array}{c|c} A^2 + BC & AB + BD \\ \hline CA + DC & CB + D^2 \end{array}\right) \tag{39}$$

$$N = \begin{pmatrix} 1 & 6 & 0 & 2 \\ -2 & 3 & -2 & 0 \\ 1 & -1 & 2 & 4 \\ 0 & -3 & 0 & 4 \end{pmatrix} = M^2 \tag{40}$$

8 2.86 from Book

Let $M = diag(A_i)\ andN = diag(B_i)$

Since M and N are both regular matrices, block matrices or no, all the regular properties of matrices apply to them. Each operation is thus trivial to demonstrate assuming I don't have to prove the basic properties of matrices. I'd normally prove each of these properties but it's really late, I have hefty assignments due in both my graduate classes, and I'd kill for some sleep. Sorry for the lapse in thoroughness.

We will show the following by the properties of matrix addition.

$$M + N = diag(A_i) + diag(B_i) \tag{41}$$

$$M + N = diag(A_i + B_i) \tag{42}$$

We will show the following by properties of matrix multiplied by constant.

$$kM = k * diag(A_i) \tag{43}$$

$$kM = diag(kA_i) \tag{44}$$

We will show the following by properties of matrix multiplication.

$$MN = diag(A_i) * diag(B_i) \tag{45}$$

$$MN = diag(A_iB_i) \tag{46}$$

For any polynomial $f(x)$

$$f(M) = f(diag(A_i)) \tag{47}$$

$$f(M) = diag(f(A_i)) \tag{48}$$

9 More Matrix Operations

Let

$$A = \begin{bmatrix} 1 & 3.5 & 1 \\ 3 & 11 & 0 \\ 7 & 5 & -1 \end{bmatrix}$$

Let $b_2 = [0,1,0]^T$.

$$Ab_2 = (3.5, 11, 5)^T \tag{49}$$

Let A be an $n \times n$ matrix, and let b_i denote the $n \times 1$ column vector whose i-th entry is equal to 1 and whose other entries are equal to 0. Prove that Ab_i is equal to the i-th column of A.

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \tag{50}$$

$$\mathbf{b_i} = \{b_1 \dots b_n\}^T \tag{51}$$

$$A\mathbf{b_i} = C_i = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} \tag{52}$$

$$c_k = \sum_{j=1}^{n} b_j * a_{kj} \tag{53}$$

Let I_n denote the $n \times n$ identity matrix. Using the fact that Ab_i is equal to the i-th column of A, prove that $AI_n = A$.

and hopefully everything was done correctly
and hopefully you know who Kronk is
otherwise you should look up Kronk
or Emperor's New Groove
because it's awesome
squirrels